KE2A Peace. 8th June 2025.

Calculate the eigen values of the following matrix :

$$\begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix}$$

$$Av = \lambda v$$
$$Av - \lambda v = 0$$
$$v(A - \lambda) = 0$$

$$\det(A - \lambda I) = 0$$

$$\det A - \lambda I = \begin{pmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{pmatrix}$$

Let the first row be: U_{11} U_{12} U_{13} U_{14}

then: $\det(A - \lambda I) = (4-\lambda)U_{11} - 8 \cdot U_{12} + (-1)U_{13} - (-2)U_{14}$

$$U_{11} = \det \begin{pmatrix} -9-\lambda & -2 & -4 \\ 10 & +5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{pmatrix}$$

$$= (-9-\lambda)\begin{pmatrix} 5-\lambda & -10 \\ -14 & -13-\lambda \end{pmatrix} - (-2)\begin{pmatrix} 10 & -10 \\ -13 & -13-\lambda \end{pmatrix} + (-4)\begin{pmatrix} 10 & 5-\lambda \\ -13 & -14 \end{pmatrix}$$

$$= (-9-\lambda)\left[(5-\lambda)(-13-\lambda) - (140)\right] - (-2)\left[(-130-10\lambda) - (130)\right] + (-4)\left((-140) - (-65+13\lambda)\right)$$

$$= (-9-\lambda)\left[-65 - 5\lambda + 13\lambda + \lambda^2 - 140\right] - (-2)(-130 - 10\lambda - 130) + (-4)\left[(-140 + 65 - 13\lambda)\right]$$

$$= (-9-\lambda)(\lambda^2 + 8\lambda - 205) - (-2)(-10\lambda - 260) + (-4)(-13\lambda - 75)$$

$$= -9\lambda^2 - 72\lambda + 1845 - \lambda^3 - 8\lambda^2 + 205\lambda - 20\lambda + 520 + 52\lambda + 300$$

$$= -\lambda^3 - 17\lambda^2 + 165\lambda + 1625.$$

$$U_{12} = \begin{pmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{pmatrix}$$

$$= -2 \begin{pmatrix} 5-\lambda & -10 \\ -14 & -13-\lambda \end{pmatrix} - (-2) \begin{pmatrix} 0 & -10 \\ -1 & -13-\lambda \end{pmatrix} + (-4) \begin{pmatrix} 0 & 5-\lambda \\ -1 & -14 \end{pmatrix}$$

$$= (-2)[(5-\lambda)(-13-\lambda) - (140)] - (-2)(-10) + (-4)(-(-5+\lambda))$$

$$= (-2)[-65 - 5\lambda + 13\lambda + \lambda^2 - 140) - (20) + (-4)(5-\lambda)$$

$$= (-2)(\lambda^2 + 8\lambda + 205) - 20 - 20 + 4\lambda$$

$$= -2\lambda^2 - 16\lambda - 410 - 20 - 20 + 4\lambda$$

$$= 370 - 12\lambda - 2\lambda^2$$

$$U_{13} = \begin{pmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{pmatrix}$$

$$= (-2) \begin{pmatrix} 10 & -10 \\ -13 & -13-\lambda \end{pmatrix} - (-9-\lambda) \begin{pmatrix} 0 & -10 \\ -1 & -13-\lambda \end{pmatrix} + (-4) \begin{pmatrix} 0 & 10 \\ -1 & -13 \end{pmatrix}$$

$$= (-2)(-130 - 10\lambda - 130) - (-9-\lambda)(-10) + (-4)(+10)$$

$$= -2(-260 - 10\lambda) - (90 + 10\lambda) + (-40)$$

$$= +520 + 20\lambda - 90 - 10\lambda - 40$$

$$= 390 + 10\lambda$$

$$U_{14} = \begin{pmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{pmatrix}$$

$$= (-2) \begin{pmatrix} 10 & 5-\lambda \\ -13 & -14 \end{pmatrix} - (-9-\lambda) \begin{pmatrix} 0 & 5-\lambda \\ -1 & -14 \end{pmatrix} + (-2) \begin{pmatrix} 0 & 10 \\ -1 & -13 \end{pmatrix}$$

$$= (-2)(-140 - (-5+\lambda)) - (-9-\lambda)(-(-5+\lambda)) + (-2)(10)$$

$$= (-2)(-140 + 5 - \lambda) - (-9-\lambda)(5-\lambda) + (-20)$$

$$= 280 - 10 + 2\lambda - (-45 + 9\lambda - 5\lambda + \lambda^2) - 20$$

$$= 280 - 10 + 2\lambda + 45 - 9\lambda + 5\lambda - \lambda^2 - 20$$

$\cancel{U_{14}} \neq \cancel{X}$

$$U_{14} = \begin{pmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{pmatrix}$$

$$= (-2)\begin{pmatrix} 10 & 5-\lambda \\ -13 & -14 \end{pmatrix} - (-9-\lambda)\begin{pmatrix} 0 & 5-\lambda \\ -1 & -14 \end{pmatrix} + (-2)\begin{pmatrix} 0 & 10 \\ -1 & -13 \end{pmatrix}$$

$$= (-2)\left(-140 - (-13(5-\lambda))\right) - (-9-\lambda)(5-\lambda) + (-2)(10)$$

$$= (-2)\left(-140 - (-65+13\lambda)\right) - \cancel{(-9-\lambda)}(-45+9\lambda-5\lambda+\lambda^2) - 20$$

$$= (-2)(-140+65-13\lambda) + 45 - 9\lambda + 5\lambda - \lambda^2 - 20$$

$$= 150 + 26\lambda - \cancel{+}45 + 4\lambda + \lambda^2 - 20$$

$$= \lambda^2 + 30\lambda + 85$$

$$\det(A - \lambda I) = (4-\lambda)U_{11} - 8(U_{12}) + (-1)U_{13} - (-2)U_{14}.$$

$$= (4-\lambda)(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625) - 8(360 - 12\lambda - \cancel{1}2\lambda^2) +$$
$$(-1)(390 + 10\lambda) - (-2)(\lambda^2 + 30\lambda + 85)$$

term 1: $(4-\lambda)(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625)$

$$= \cancel{-4\lambda^3} - \cancel{68\lambda^2} + 660\lambda + 6500 + \lambda^4 + \cancel{17\lambda^3} - \cancel{165\lambda^2} - \cancel{+}1625\lambda$$

$$= \lambda^4 + 13\lambda^3 - 233\lambda^2 - 965\lambda + 6500.$$

term 2: $8(360 - 12\lambda - 2\lambda^2)$

$$= 2880 - 96\lambda - 16\lambda^2$$

term 3: $-1(390 + 10\lambda)$

$$= -390 - 10\lambda$$

term 4: $(-2)(\lambda^2 + 30\lambda + 85)$

$$= +2\lambda^2 + 60\lambda + 170$$

Put the equations together.

$$Det(A-\lambda I) = (6500 - 965\lambda - 233\lambda^2 + 13\lambda^3 + \lambda^4) + (-2960 + 96\lambda + 16\lambda^2 + (-390 - 10\lambda) + (170 + 60\lambda + 2\lambda^2)$$

$$= 6500 - 965\lambda - 233\lambda^2 + 13\lambda^3 + \lambda^4 - 2960 + 96\lambda + 16\lambda^2 - 390 - 10\lambda + 170 + 60\lambda + 2\lambda^2.$$

$$\cancel{= \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3520}$$

$$Det(A-\lambda I) = \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500.$$

Eigen Values.

$\lambda_1 = -27.125.$

$\lambda_2 = -5.604$

$\lambda_3 = 2.675$

$\lambda_4 = 11.054.$

$$A-\lambda I = \begin{pmatrix} 25.125 & 8 & -1 & -2 \\ -2 & 12.125 & -2 & -4 \\ 0 & 10 & 26.125 & -10 \\ -1 & -13 & -14 & 8.125 \end{pmatrix}$$

<u>Linear Equation by Gaussian Elimination</u>:

$$\begin{pmatrix} 25.125 & 8 & -1 & -2 \\ -2 & 12.125 & -2 & -4 \\ 0 & 10 & 26.125 & -10 \\ -1 & -13 & -14 & 8.125 \end{pmatrix} \times 0.04$$

$$\begin{pmatrix} 1 & 0.318 & -0.04 & -0.08 \\ -2 & 12.125 & -2 & -4 \\ 0 & 10 & 26.125 & -10 \\ -1 & -13 & -14 & 8.125 \end{pmatrix} \begin{array}{l} \times 2 \\ \\ \\ \cancel{\times 1} \end{array}$$

$$\begin{pmatrix} 1 & 0.318 & -0.04 & -0.08 \\ 0 & 12.761 & -2.08 & -4.15 \\ 0 & 10 & 26.125 & -10 \\ -1 & -13 & -14 & 8.125 \end{pmatrix} \begin{array}{l} \cancel{\times 0.078} \\ \times 1 \end{array}$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 \\ 0 & 12.761 & -2.080 & -4.159 \\ 0 & 10 & 26.125 & -10 \\ 0 & -12.682 & -14.040 & 8.045 \end{pmatrix} \begin{array}{l} \times 0.078 \\ \cancel{\times (-10)} \end{array}$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 \\ 0 & 1 & -0.163 & -0.326 \\ 0 & 10 & 26.125 & -10 \\ 0 & -12.682 & -14.040 & 8.045 \end{pmatrix} \times (-10)$$

$$\begin{pmatrix} 1 & 0.318 & -0.04 & -0.08 \\ 0 & 1 & -0.163 & -0.326 \\ 0 & 0 & 27.754 & -6.741 \\ 0 & -12.682 & -14.040 & 8.045 \end{pmatrix} \times (12.682)$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 \\ 0 & 1 & -0.163 & -0.326 \\ 0 & 0 & 27.754 & -6.741 \\ 0 & 0 & -16.106 & 3.912 \end{pmatrix} \times (0.036)$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 \\ 0 & 1 & -0.163 & -0.326 \\ 0 & 0 & 1 & -0.243 \\ 0 & 0 & -16.106 & 3.912 \end{pmatrix} \times (16.106)$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 \\ 0 & 1 & -0.163 & -0.326 \\ 0 & 0 & 1 & -0.243 \\ 0 & 0 & 0 & 0 \end{pmatrix} \times (0.163)$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 \\ 0 & 1 & -0.163 & -0.326 \\ 0 & 0 & 1 & -0.243 \\ 0 & 0 & 0 & 0 \end{pmatrix} \times (0.040)$$

$$\begin{pmatrix} 1 & 0.318 & -0.040 & -0.080 \\ 0 & 1 & 0 & -0.326 \\ 0 & 0 & 1 & -0.243 \\ 0 & 0 & 0 & 0 \end{pmatrix} \times (-0.318)$$

$$\begin{pmatrix} 1 & 0.318 & 0 & -0.089 \\ 0 & 1 & 0 & -0.365 \\ 0 & 0 & 1 & -0.243 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0.027 \\ 0 & 1 & 0 & -0.365 \\ 0 & 0 & 1 & -0.243 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Eigen Vectors. when $\lambda_1 = -21.125$.

$$= \begin{pmatrix} -0.027 \\ 0.365 \\ 0.243 \\ 1 \end{pmatrix}$$

$$\lambda_1 = -21.125$$

$$A - \lambda_1 I = \begin{bmatrix} 4+21.125 & -2 & 0 & -1 \\ 8 & -9+21.125 & 10 & -13 \\ -1 & -2 & 5+21.125 & -14 \\ -2 & -4 & -10 & -13+12.125 \end{bmatrix}$$

$$A - \lambda_1 I = \begin{bmatrix} 25.125 & -2 & 0 & -1 \\ 8 & 12.125 & 10 & -13 \\ -1 & -2 & 26.125 & -14 \\ -2 & -4 & -10 & 8.125 \end{bmatrix}$$

$$(A - \lambda_1 I)\vec{v} = 0$$

$$\vec{v} = [x_1\ x_2\ x_3\ x_4]$$

$$\begin{bmatrix} 25.125 & -2 & 0 & -1 \\ 8 & 12.125 & 10 & -13 \\ -1 & -2 & 26.125 & -14 \\ -2 & -4 & -10 & 8.125 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

~~Rooo~~ $R_1 = \frac{1}{25.125} \times R_1$

$$R_1 = [1, -0.0796, 0, -0.0398]$$

$$R_2 = R_2 - 8 \times R_1$$

$$R_2 = [0, 12.761, 10, -12.682]$$

$$R_3 = R_3 + R_1$$

$R_3 = [0, -2.0796, 26.125, -14.0398]$

$R_4 = R_4 + 2R_1$

$R_4 = [0, -4.1592, -10, 8.0454]$

$$\text{Updated matrix} = \left[\begin{array}{cccc|c} 1 & -0.0796 & 0 & -0.0398 & 0 \\ 0 & 12.761 & 10 & -12.682 & 0 \\ 0 & -2.0796 & 26.125 & -14.0398 & 0 \\ 0 & -4.1592 & -10 & 8.0454 & 0 \end{array}\right]$$

Normalize R_2

~~$R_2 = R_2 / 12.761 =$~~

$$R_2 = \frac{R_2}{12.761} = [0, 1, 0.7835, 0.9938]$$

$R_3 = R_3 + 2.0796 R_2$

$R_4 = R_4 + 4.1592 R_2$

$R_3 = [0, 0, 27.753, -6.741]$

$R_4 = [0, 0, -16.106, 3.912]$

Normalise R_3

$$R_3 = \frac{R_3}{27.753} = [0, 0, 1 -0.243]$$

$R_4 = R_4 + 16.106 R_3$

$R_4 = [0, 0, 0, 0]$

Back - Substitute

$R_2 = R_2 - 0.7835 R_3$

$R_2 = [0, 1, 0, 0.365]$

$R_1 = R_1 + 0.0398 R_3$

$R_1 = [1, -0.0796, 0, -0.089]$

Then eliminate the y term

$R_1 = R_1 + 0.0796 R_2 -$

$R_1 = [1, 0, 0, 0.027]$

Final Reduced Form

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0.027 & 0 \\ 0 & 1 & 0 & -0.365 & 0 \\ 0 & 0 & 1 & -0.243 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Expressing the solution

$x_1 + 0.027 x_4 = 0$

$x_1 = -0.027 x_4$

$$x_2 - 0.365x_4 = 0$$

$$x_2 = 0.365x_4$$

$$x_3 - 0.234x_4$$

$$x_3 = 0.234x_4$$

let $x_4 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = t \cdot \begin{bmatrix} -0.027 \\ 0.365 \\ 0.243 \\ 1 \end{bmatrix}$$

So when λ_1 is -21.125, the eigenvector is

$$\vec{V} \begin{bmatrix} -0.027 \\ 0.365 \\ 0.243 \\ 1 \end{bmatrix}$$

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Finding the eigen vector for the value

$\lambda = 2.675$

$(A - \lambda I)\vec{V} = 0$

Substitute $\lambda = 2.675$

$$\Rightarrow A - 2.675 I = \begin{bmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & -13 & -14 & -15.675 \end{bmatrix}$$

$$\text{Let } \vec{V} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Setting up the Augmented matrix.

$$\left[\begin{array}{cccc|c} 1.325 & 8 & -1 & -2 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right]$$

performing the Gaussian Elimination.

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$R_1 : 1.325 \Rightarrow \left[\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5094 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.625 & 0 \end{array}\right]$

Eliminating below R_1.

$R_1 + 2R_1$

$R_4 + R_1$

New matrix.

$$\left[\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5094 & 0 \\ 0 & 0.4004 & -3.5094 & -7.0184 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.9623 & -14.7547 & -17.1844 & 0 \end{array}\right]$$

Divide R_2 by 0.4004:

$R_2 : 0.4004 = \left[\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5094 & 0 \\ 0 & 1 & -8.766 & -17.533 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.9623 & -14.7547 & -17.4444 & 0 \end{array}\right]$

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Eliminating using new R_2

$R_3 - 10R_2$

$R_4 + 6.9623 \cdot R_2$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5044 & 0 \\ 0 & 1 & -8.768 & -17.533 & 0 \\ 0 & 0 & 90.005 & 165.33 & 0 \\ 0 & 0 & 46.257 & 105.3 & 0 \end{array}\right]$$

Substution from the row reduced system

Row 3: $90.005z + 165.33w = 0 \Rightarrow z = \frac{-165.3}{90.005}w$

$\Rightarrow z = -1.837w$

Row 2: $y - 8.768z - 17.533w = 0$

$\Rightarrow y = 8.768z + 17.533w$

substuting $z = 1.837w$

$\Rightarrow y = 8.768(-1.837w) + 17.533w = 16.107w$

R_1: $x + 6.0377y - 0.754z - 1.5044w = 0$

$-6.0377y + 0.7547z + 1.5044w$.

$\Rightarrow x = -6.0377y + 0.7547(-1.837w)$

$\Rightarrow x = -6.0377y(-16.107w) + 0.7547(-1.837w)$

$+ 1.5094w$.

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$x = 97.23w - 1.387w + 1.509w$

$\cancel{x = 9}$ $x = 97.352\,w.$

General Solution:

Let $w = 1$

$$\vec{V} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 97.352 \\ -16.107 \\ -1.837 \\ 1 \end{pmatrix}$$

Normalizing eigen vector $\vec{V_3}$

$$\|\vec{v}\| = \sqrt{(97.352)^2 + (-16.107)^2 + (-1.837)^2 + 1^2}$$

$$\Rightarrow \sqrt{9741.1} = 98.7$$

Normalized eigen vector $\vec{V_3}$

$$\vec{V_3} = \frac{1}{98.7} \begin{pmatrix} 97.352 \\ -16.107 \\ -1.837 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.987 \\ -0.1683 \\ -0.019 \\ 0.010 \end{pmatrix}$$

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Eigen Vector for Value $\lambda = 2.675$

$$\vec{V}_3 \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 97.352 \\ -16.107 \\ -1.837 \\ 1 \end{pmatrix}$$

Normalized Eigen vector is

$$\vec{V}_3^{\,norm} \begin{pmatrix} 0.987 \\ -0.163 \\ -0.019 \\ 0.010 \end{pmatrix}$$

$$\lambda_3 = 2.675 \quad \vec{V}_3 = \begin{pmatrix} 97.325 \\ -16.107 \\ -1.837 \\ 1 \end{pmatrix} \quad V_3^{nom} \begin{pmatrix} 0.987 \\ -0.163 \\ -0.019 \\ 0.010 \end{pmatrix}$$

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Eigenvector Calculation for $\lambda = 4$

$$A = \begin{bmatrix} 4 & 8 & -1 & 2 \\ 2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

Step 1: $(A - \lambda I)$ for $\lambda = 4$

$$A - 4I = \begin{bmatrix} 4-4 & 8 & -1 & 2 \\ -2 & -9-4 & -2 & -4 \\ 0 & 10 & 5-4 & -10 \\ -1 & 13 & -14 & -13-4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 8 & -1 & 2 \\ -2 & -13 & -2 & -4 \\ 0 & 10 & 1 & -10 \\ -1 & -13 & -14 & -17 \end{bmatrix}$$

Step 2: check if $\lambda = 4$ is actually an eigenvalue $\det(A - 4I) = 0$

$(A - 4I)v = 0 \Rightarrow v = 0$

Step 3: Homogeneous system

Swap to get pivot

$$\left[\begin{array}{cccc|c} 0 & 8 & -1 & 2 & 0 \\ -2 & -13 & -2 & -4 & 0 \\ 0 & 10 & 1 & -10 & 0 \\ -1 & -13 & -14 & -17 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} -1 & -13 & -14 & -17 & 0 \\ -2 & -13 & -2 & -4 & 0 \\ 0 & 10 & 1 & -10 & 0 \\ 0 & 8 & -1 & 2 & 0 \end{array}\right]$$

Step 4: Gaussian Elimination

~~$R_1 = R_1/-1$~~

~~$1 \; 13 \; 14 \; 17 \; 0$~~

$\Rightarrow R_2 = R_2 - 2R_1$

$$\left(\begin{array}{cccc|c} -1 & -13 & -14 & -17 & 0 \\ 0 & 13 & 26 & 30 & 0 \\ 0 & 10 & 1 & -10 & 0 \\ 0 & 8 & -1 & 2 & 0 \end{array}\right) \Rightarrow$$

$\Rightarrow$ Divide R_1 by -1

$$\left(\begin{array}{cccc|c} 1 & 13 & 14 & 17 & 0 \\ 0 & 13 & 26 & 30 & 0 \\ 0 & 10 & 1 & -10 & 0 \\ 0 & 8 & -1 & 2 & 0 \end{array}\right)$$

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$\rightarrow R_2 = R_2/13 \qquad \rightarrow R_3 = R_3 - 10R_2$

$$\left(\begin{array}{cccc|c} 1 & 13 & 14 & 17 & 0 \\ 0 & 1 & 2 & 30/13 & 0 \\ 0 & 10 & 1 & -10 & 0 \\ 0 & 8 & -1 & 2 & 0 \end{array}\right)$$

$\rightarrow R_4 = R_4 - 8 * R_2$

$$\left(\begin{array}{cccc|c} 1 & 13 & 14 & 17 & 0 \\ 0 & 1 & 2 & 30/13 & 0 \\ 0 & 0 & -19 & -430/13 & 0 \\ 0 & 0 & -17 & -214/13 & 0 \end{array}\right)$$

$\rightarrow R_3 = R_3/(-19)$

$$\left(\begin{array}{cccc|c} 1 & 13 & 14 & 17 & 0 \\ 0 & 1 & 2 & 30/13 & 0 \\ 0 & 0 & 1 & 430/247 & 0 \\ 0 & 0 & -17 & -214/13 & 0 \end{array}\right)$$

$(3244/247)v_4 = 0$

$v_4 = 0$

$\rightarrow R_4 = R_4 + 17 * R_3$

$$\left(\begin{array}{cccc|c} 1 & 13 & 14 & 17 & 0 \\ 0 & 1 & 2 & 30/13 & 0 \\ 0 & 0 & 1 & 430/247 & 0 \\ 0 & 0 & 0 & 3244/247 & 0 \end{array}\right)$$

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From R_3: $x_3 + (430/247)x_4 = 0 \Rightarrow x_3 = 0$

From R_2: $x_2 + 2x_3 + (30/13)x_4 = 0$

$x_2 = 0$

From R_1: $x_1 + 13x_2 + 14x_3 + 17x_4 = 0$

$x_1 + 13(0) + 14(0) + 17(0)$

$x_1 = 0$

$x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $\lambda = 4$, is not eigenvalue of given matrix

Eigenvalue Importance (Proportional variance)

$\lambda_1 = -21.125$

$\lambda_2 = -5.604$

$\lambda_3 = 2.675$

$\lambda_4 = 11.054$

$$\text{Importance of } \lambda_i = \frac{|\lambda_i|}{\sum_{j=1}^{n} |\lambda_j|}$$

$|\lambda_1| = 21.125$, $|\lambda_2| = 20.643$, $|\lambda_3| = 16.053$, $|\lambda_4| = 9.429$

Total magnitude $= 21.125 + 20.643 + 16.053 + 9.429 = 67.25$

$\% \text{ of } \lambda_1 = \frac{21.125}{67.25} \times 100$

$= 0.3141 \times 100$

$= 31.41\%$

$\% \text{ of } \lambda_2 = \frac{20.643}{67.25} \times 100$

$= 0.3069 \times 100$

$= 30.69\%$

$\% \text{ of } \lambda_3 = \frac{16.053}{67.25} \times 100$

$= 0.2387 \times 100$

$= 23.87$

$\% \text{ of } \lambda_4 = \frac{9.429}{67.25} \times 100$

$= 0.1402$

$= 14.02\%$