Project Logistic Module

Software Development

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Logistic module

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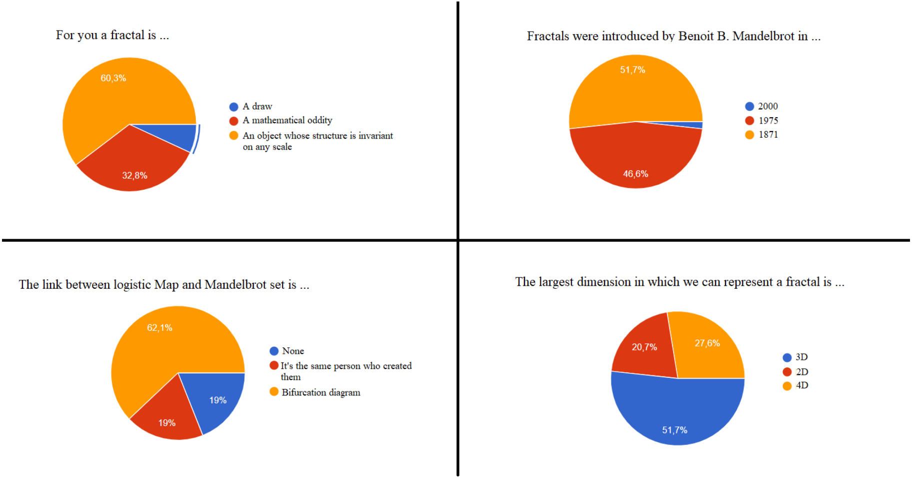
Introduction

Figure: Survey results on 85 individuals



Logistic Map

Definition of Logistic map

Logistic map is defined by the following recurrence :

$$x_{n+1} = rx_n(1 - x_n)$$

where :

- r is the growth ratio and is defined and $r \in [0, 4]$
- $x_0 \in [0, 1]$

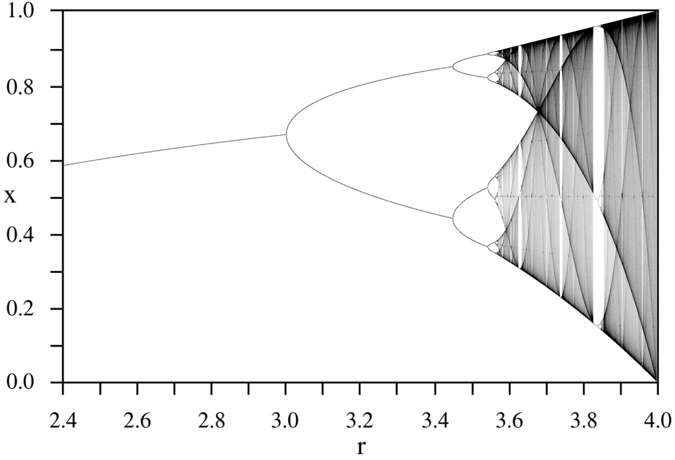
Behaviour of logistic map

The behaviour of this map is controlled by r. Indeed, when :

- $r \in [0, 1]$ the population die ;
- $r \in]1, 3[$ the population approach the value $\frac{r-1}{r}$;
- $r \in [3, 3.56995[$ the population oscillate between two values ;
- $r \geq 3.56995$ the population be chaotic.

Bifurcation Diagram

Figure: Bifurcation Diagram



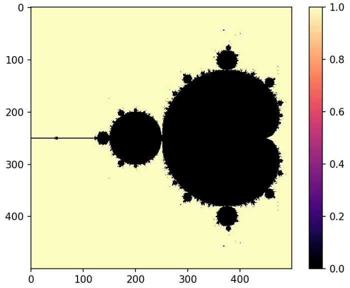
Mandelbrot 2D

The Mandelbrot set is a fractal set defined as the assembly of points c in the complex plan for which the sequence of complex number defined by :

$$\begin{cases} z_0 = 0 \\ z_{n+1} = z_n^2 + c \end{cases}$$

is bounded.

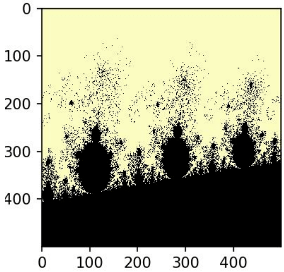
Figure: Mandelbrot set



A fractal set

The previous picture represents the set as we see it most of the time, but changing the arguments in the function can make it a little different. We can already see it, but the set is a fractal: we can see a dozen of mini-Mandelbrot surrounding the big one, but zooming on the set makes it more obvious. Here is a picture obtained by zooming on the upper part of the picture :

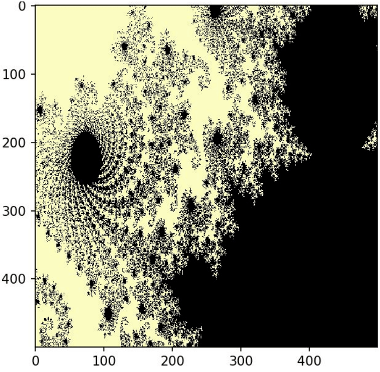
Figure: Mini-Mandelbrots



Visualization of some characteristic plots

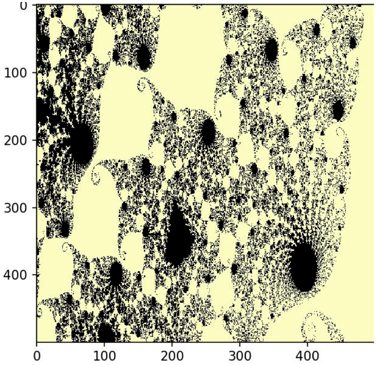
Zooming on the set does not only shows mini-Mandelbrots, it also shows some characteristic patterns as the "Triple spiral valleys" or the "Elephant valleys" :

Figure: Triple spiral valley



Visualization of some characteristic plots

Figure: Elephant valley



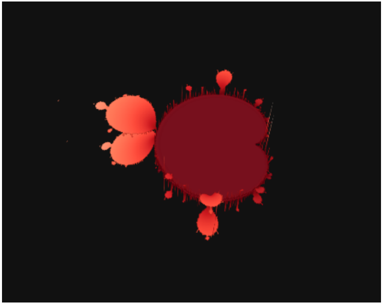
Mandelbrot 3D

As stated before, the Mandelbrot set is defined by this equation :

$$z_{n+1} = z_n^2 + c$$

for each c point in the complex plan.
By iterating this same equation hundreds times and plotting the values it takes on the z-axis, we can visualize the Mandelbrot set in 3D.

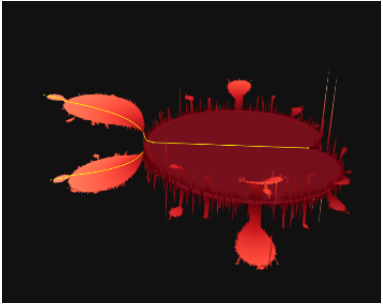
Figure: Mandelbrot set in 3D



Rotation of the 3D Mandelbrot set

As far we know that the Bifurcation diagram is a fractal and the Mandelbrot set also represents a fractal.
What interest us is the connection between these two fractals.
By looking at the 3D Mandelbrot from the side, we can actually see that the Bifurcation diagram is part of the Mandelbrot set.

Figure: Bifurcation diagram in the Mandelbrot set



Conclusion

Through this project we were able to modelize some remarkable mathematical phenomena and have a better undestanding of them.

First, we observed how the final state of a population was controlled by its growth ratio and how chaotic behaviour could appear starting from a simple equation.

Then, we were introduced to the Mandelbrot set.
Seeing how it was defined and looking into some of its many famous characteristic patterns allowed us to better grasp the concept of fractal.

Finally, by looking at the Mandelbrot set in 3D, we could highlight how the fractal was related to the Bifurcation diagram thus showing the link with mathematical results and natural phenomena.