THE PLOT THICKENS

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1. Closed Union

Prove that the class of regular languages is closed under the union ($\cup$) operation. (Hint: Use proof by construction.)

2. ...now has two problems

Provide the regular expression that corresponds to each of the following language descriptions. Assume that Σ is {0, 1}:

(1) $\{w|w$ has exactly a single 1$\}$
$$0^* \cdot \{1\} \cdot 0^*$$
(2) $\{w|w$ contains the string 001 as a substring$\}$
$$\Sigma^* \cdot \{001\} \cdot \Sigma^*$$
(3) $\{w|w$ is a string of even length$\}$
$$(\{0,1\} \cdot \{0,1\})^*$$
(4) $\{w|w$ starts and ends with the same symbol$\}$

3. Regex to NFA

Convert the regular expression $(a \cup b)^*$ to a nondeterministic finite state automaton. Note that you do not have to draw the NFA if you do not wish to do so. You may provide the mathematical specification $M = (Q, \Sigma, \delta, q_0, F)$. For δ you may provide a table of the transitions.

This seems almost deceptively simple, but it would appear that this is a rather simple Regex to model. $M = (Q, \Sigma, \delta, q_0, F)$, where:

$Q = 1$
$\Sigma = a,\ b$
$\delta = \{1\}_a \rightarrow \{1\}, \{1\}_b \rightarrow \{1\}$
$q_0 = 1$
$F = 1.$

4. Pump it up

Using the pumping lemma prove that $F = \{ww|w\ \epsilon\ \{0,1\}\ \}$ is nonregular.

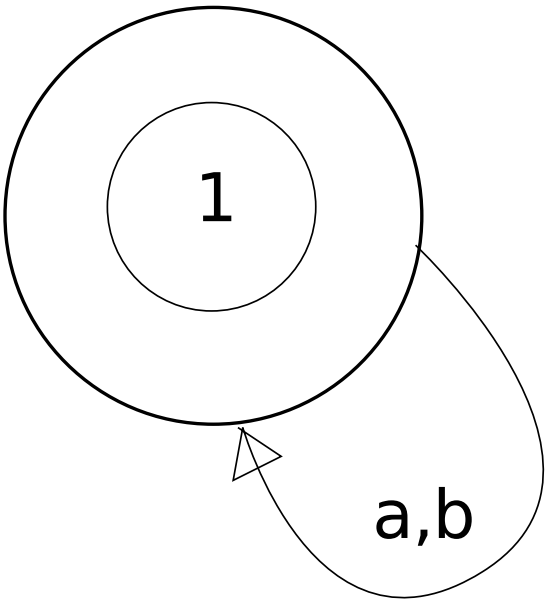

FIGURE 1. T$(a \cup b)^*$