

THE PLOT THICKENS

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1. CLOSED UNION

Prove that the class of regular languages is closed under the union (\cup) operation. (Hint: Use proof by construction.)

2. IF A MAN ATTEMPTS TO SOLVE A PROBLEM USING REGULAR EXPRESSIONS, THEY NOW HAVE TWO PROBLEMS

Provide the regular expression that corresponds to each of the following language descriptions. Assume that Σ is $\{0, 1\}$:

- (1) $\{w|w \text{ has exactly a single } 1\}$
 $0^* \cdot \{1\} \cdot 0^*$
- (2) $\{w|w \text{ contains the string } 001 \text{ as a substring}\}$
 $\Sigma^* \cdot \{001\} \cdot \Sigma^*$
- (3) $\{w|w \text{ is a string of even length}\}$
 $(\{0, 1\} \cdot 0, 1)^*$
- (4) $\{w|w \text{ starts and ends with the same symbol}\}$

3. REGEX TO NFA

Convert the regular expression $(a \cup b)$ to a nondeterministic finite state automaton. Note that you do not have to draw the NFA if you do not wish to do so. You may provide the mathematical specification $M = (Q, \Sigma, \delta, q_0, F)$. For δ you may provide a table of the transitions.

4. PUMP IT UP

Using the pumping lemma prove that $F = \{ww|w \in \{0, 1\}^*\}$ is nonregular.