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CS275 Probability and Statistics Problem Set 1

1) Assuming all digits different → 6! combinations

$$6! = 720$$

However 5 exists twice so half the combinations will be duplicates since the '5's can be interchanged without changing the number

Distinct numbers using 1234 55 → $\frac{720}{2} = 360$

To find sum of all combinations need to consider each digit at each position

If all 6 digits were different each digit would be found at a particular position 120 times because $\frac{720}{6} = 120$

However due to 5 we ignore half the combinations so each digit will only be found 60 times at a particular position

For one position,

$$60(1+2+3+4+5+5) = 60 \times 20 = 1200$$

Considering all positions,

$$\text{Sum} = 1200 \times (10^0 + 10^1 + 10^2 + 10^3 + 10^4 + 10^5)$$
$$= 1200 \times 111111$$

2) Type 1 $\rightarrow$ Only colts $\quad P(T1) = 0.1$

Type 2 $\rightarrow$ Only fillies $\quad P(T2) = 0.1$

Type 3 $\rightarrow$ Either $\quad P(T3) = 0.8$

	C	F
T1	1	0
T2	0	1
T3	0.5	0.5

$$P(\text{at least one colt}) = P(T1CC) + P(T3CC) + P(T3CF) + P(T3FC)$$
$$= (0.1 \times 1 \times 1) + ((0.8 \times 0.5 \times 0.5) \times 3)$$
$$= 0.1 + (3 \times 0.2)$$
$$= 0.1 + 0.6$$
$$= 0.7$$

$$P(\text{two colts}) = P(T1CC) + P(T2CC)$$
$$= (0.1 \times 1 \times 1) + (0.8 \times 0.5 \times 0.5)$$
$$= 0.1 + 0.2$$
$$= 0.3$$

~~P(two colts at least one colt~~

$$P(\text{two colts if at least one colt}) = \frac{0.3}{0.7}$$
$$= \frac{3}{7}$$

3)

	H	D
P	0.95	0.05
N	0.05	0.95

P → Positive   N → Negative
H → Have   D → Don't have

$$P(H|P) = \frac{P(H \cap P)}{P(P)} = \frac{0.001 \times 0.95}{(0.001 \times 0.95) + (0.999 \times 0.05)}$$

$$= \frac{0.00095}{0.0509}$$

$$= \frac{95}{5090}$$

$$= \frac{19}{1018}$$

4) a) For there to be a difference of 4, the first die must have rolled either 0 or 8. However, this is not possible.
So, E_1 and E_2 are dependent since E_1 occurring means the chance of E_2 happening is 0.

b) Yes because either one event occurring does not change the probability of the other

5) $P(51) = 52C51 \times 0.95^{51} \times 0.05 = 52 \times 0.95^{51} \times 0.05$

$P(52) = 0.95^{52}$

$P(\text{seat for all}) = 1 - P(51) - P(52)$
$= 1 - (52 \times 0.95^{51} \times 0.05) - (0.95^{52})$
$= 0.740503$
≈ 0.741 (3sf)