Control System

EE18BTECH11024
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Question 4

For a unity feedback control system with the forward path transfer function

$$G(s) = \frac{k}{s(s+2)}$$

The peak resonant magnitude M_r of the closed loop frequency is 2. The corresponding value of the gain K is

solution

Given:

For a unity feedback control system $G(s) = \frac{k}{s(s+2)}$ and resonant

1.

peak M_r=2 (2).jpg

we can find its closed loop transfer function as,

$$C(s) = [R(S) - H(S)C(S)]G(S) = R(S)G(S) - H(S)C(S)G(S) \tag{1}$$

$$C(S) + H(S)C(S)G(S) = R(S)G(S) \tag{2}$$

$$C(S)[1 + G(S)H(S)] = R(S)G(S \tag{3}$$

$$\frac{C(s)}{R(s)} = T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{\dfrac{K}{s(s+2)}}{1 + \dfrac{K}{s(s+2)*1}} = \frac{K}{s^2 + 2s + K} \tag{4}$$

standard equation of $T(s) = \dfrac{\omega_n^2}{s^2 + 2S\xi_n^\omega + \omega_n^2}$

formula for resonant peak as $M_r = \dfrac{1}{2\xi(1-(\xi)^2)^{\frac{1}{2}}}$

given resonant peak M_r=2

$$\frac{DCGain}{2\xi(1-(\xi)^2)^{\frac{1}{2}}} = 2$$

here DC gain is 1 (5)

squaring on both sides
$16\xi^2(\xi^2 - 1) = 1$
$putting\ \xi^2 = x$

$$16x^2 - 16x + 1 = 0 \tag{6}$$

$$x = \xi^2 = \frac{2-\sqrt{3}}{4} \tag{7}$$

$$x = \xi^2 = \frac{2+\sqrt{3}}{4} \tag{8}$$

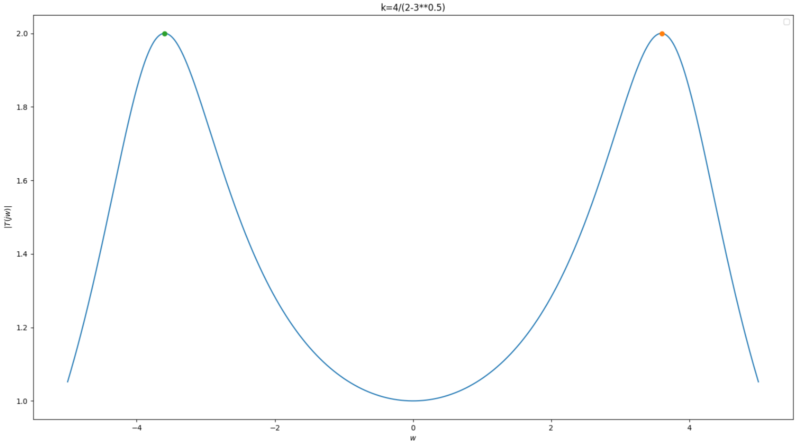
characteristic equation is

$$s^2 + 2s + k$$

comparing it with standard equation we get ${\omega_n}^2 = k$
$2\xi\omega_n$=2

$$\xi = \frac{1}{\omega_n} = \frac{1}{k^{\frac{1}{2}}}$$

$$k = \frac{1}{\xi^2} = \frac{4}{2 - \sqrt{3}} = 14.92$$


k=4/(2-3**0.5)
2.0
1.8
1.6
1.4
1.2
1.0
|T(jw)|
−4
−2
0
2
4
w