8.6

Volumes by Disc & Washer Methods Homework

Name Key

Date ______________ Period ________

Problems 1 – 12, Find the volume of the region bounded by the given functions and revolved about the given axis.

1. $y = x^2 - 4x$; $y = 0$ about the x-axis

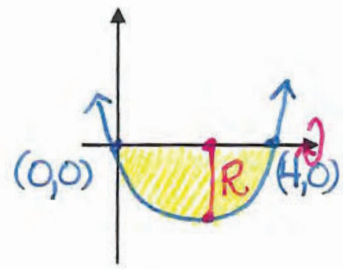
$$V = \pi \int_0^4 (x^2 - 4x)^2 \, dx$$

$$V = \pi \int_0^4 (x^4 - 8x^3 + 16x^2) \, dx$$

$$V = \pi \left[\frac{x^5}{5} - 2x^4 + \frac{16}{3}x^3 \right]_0^4$$

$$V = \pi \left[\frac{1024}{5} - 512 + \frac{1024}{3} \right]$$

$$\underline{V = \frac{512\pi}{15}}$$



$$\underline{\underline{V = \frac{512\pi}{15}}}$$

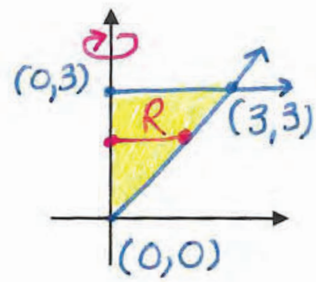
2. $y = x$; $y = 3$; $x = 0$ about the y-axis

$$V = \pi \int_0^3 y^2 \, dy$$

$$V = \pi \left. \frac{1}{3}y^3 \right|_0^3$$

$$V = \frac{\pi}{3}\left[3^3 - 3^0 \right]$$

$$\underline{\underline{V = 9\pi}}$$



$$\underline{\underline{V = 9\pi}}$$

3. $y = \sqrt{9 - x^2}$; $y = 0$; $x = 0$ about the x-axis

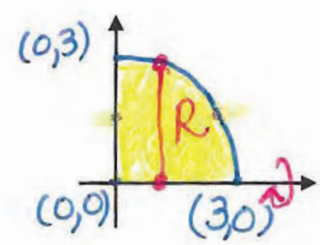
$$V = \pi \int_0^3 \left(\sqrt{9 - x^2}\right)^2 dx$$

$$V = \pi \int_0^3 (9 - x^2) \, dx$$

$$V = \pi \left[9x - \frac{1}{3}x^3 \right]_0^3$$

$$V = \pi [27 - 9]$$

$$\underline{\underline{V = 18\pi}}$$



$$\underline{\underline{V = 18\pi}}$$

4. $y = x^3$; $x = -2$; $y = 0$ about the x-axis

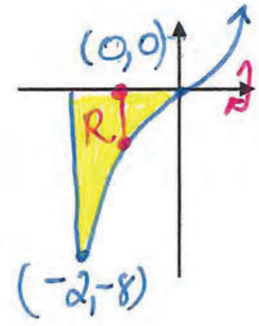
$$V = \pi \int_{-2}^{0} (x^3)^2 \, dx$$

$$V = \pi \int_{-2}^{0} x^6 \, dx$$

$$V = \pi \left[\frac{1}{7} x^7 \right]_{-2}^{0}$$

$$V = \frac{\pi}{7} [0 + 128]$$

$$V = \frac{128\pi}{7}$$



$$V = \frac{128\pi}{7}$$

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5. $y = \sqrt{x}$, $y = 3$ about the y-axis

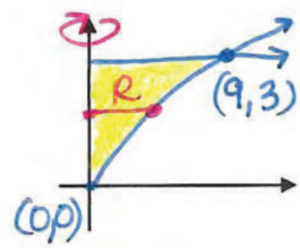
$$V = \pi \int_{0}^{3} (y^2)^2 \, dy$$

$$V = \pi \int_{0}^{3} y^4 \, dy$$

$$V = \pi \left[\frac{1}{5} y^5 \right]_{0}^{3}$$

$$V = \frac{\pi}{5} (243 - 0)$$

$$V = \frac{243\pi}{5}$$



$y = \sqrt{x}$
$x = y^2$

$$V = \frac{243\pi}{5}$$

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6. $y^2 = x$; $2y = x$ about the y-axis

$$V = \pi \int_{0}^{2} \left[(2y)^2 - (y^2)^2 \right] dy$$

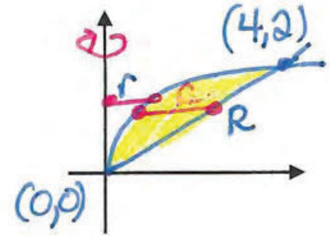
$$V = \pi \int_{0}^{2} (4y^2 - y^4) \, dy$$

$$V = \pi \left[\frac{4}{3} y^3 - \frac{1}{5} y^5 \right]_{0}^{2}$$

$$V = \pi \left[\frac{32}{3} - \frac{32}{5} \right]$$

$$V = \pi \left[\frac{160 - 96}{15} \right]$$

$$V = \frac{64\pi}{15}$$

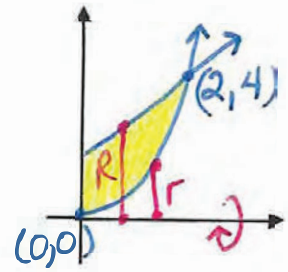


$R(y) = 2y$
$r(y) = y^2$

$$V = \frac{64\pi}{15}$$

7. $y = x^2; y = x + 2; x = 0, y = 0$ about the x-axis

$R(x) = x+2$
$r(x) = x^2$



$$V = \pi \int_0^2 \left[(x+2)^2 - (x^2)^2\right] dx$$

$$V = \pi \int_0^2 (x^2 + 4x + 4 - x^4)\, dx$$

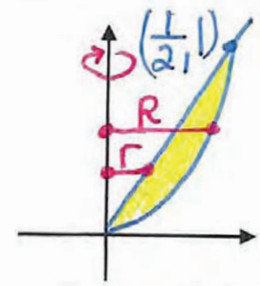
$$V = \pi \left[\tfrac{1}{3}x^3 + 2x^2 + 4x - \tfrac{1}{5}x^5\right]_0^2$$

$$V = \pi \left[\frac{40 + 120 + 120 - 96}{15}\right]$$

$$\underline{\underline{V = \frac{184\pi}{15}}}$$

$x^2 = x+2$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$\{2, -1\}$

8. $y = 2x; y = 4x^2$ about the y-axis



$$V = \pi \int_0^1 \left[\left(\tfrac{1}{2}\sqrt{y}\right)^2 - \left(\tfrac{1}{2}y\right)^2\right] dy$$

$$V = \pi \int_0^1 \left(\tfrac{1}{4}y - \tfrac{1}{4}y^2\right) dy$$

$$V = \frac{\pi}{4}\left[\tfrac{1}{2}y^2 - \tfrac{1}{3}y^3\right]_0^1$$

$$V = \frac{\pi}{4}\left[\tfrac{1}{2} - \tfrac{1}{3}\right]$$

$$\underline{V = \frac{\pi}{24}}$$

$R(y) = \frac{1}{2}\sqrt{y}$
$r(y) = \frac{1}{2}y$

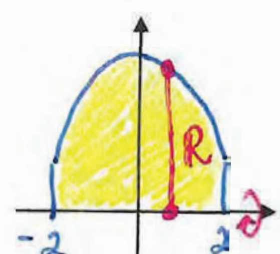
$\underline{V = \frac{\pi}{24}}$

9. $y = 6 - x^2, x = -2, x = 2$, and the x-axis about the x- axis.
Find both exact and approximation.

$$V = \pi \int_{-2}^2 (6 - x^2)^2\, dx$$

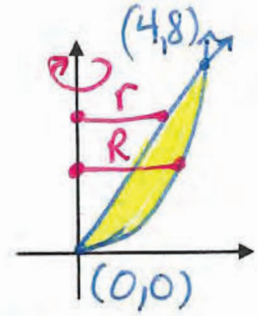
or by symmetry

$$V = 2\pi \int_0^2 (6 - x^2)^2\, dx$$



$$\underline{\underline{V = \frac{464\pi}{5}}} \qquad \underline{V \approx 291.5397}$$

10. $y = 2x$; $y = \frac{1}{8}x^3$, $x = 0$, and $y = 0$ about the y-axis



$$V = \pi \int_0^8 \left[(2\sqrt[3]{y})^2 - \left(\tfrac{1}{2}y\right)^2 \right] dy$$

$$V = \pi \int_0^8 \left(4y^{2/3} - \tfrac{1}{4}y^2\right) dy$$

$$V = \pi \left[\tfrac{12}{5}y^{5/3} - \tfrac{1}{12}y^3 \right]_0^8$$

$$V = \pi \left[\frac{1152 - 640}{15} \right]$$

$$V = \frac{512\pi}{15}$$

$R(y) = 2\sqrt[3]{y}$

$r(y) = \frac{1}{2}y$

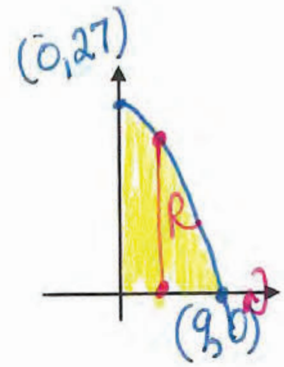
$V = \frac{512\pi}{15}$

11. Let R be the region in the first quadrant bounded by the graph of $y = 27 - x^{3/2}$, the x-axis, and the y-axis. Find an approximation of the volume of the solid generated when R is revolved about the x-axis. Show the integral used to calculate your answer.

$$V = \pi \int_0^9 (27 - x^{3/2})^2 \, dx$$

$$V = \frac{59049\pi}{20}$$

$$V \approx 9275.39523$$



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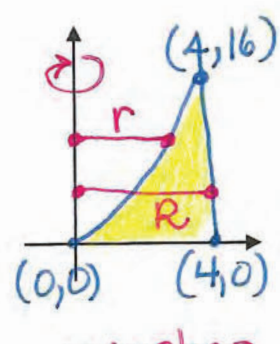
12. Let R be the region enclosed by the graph of $y = x^2$, the line $x = 4$, and the x-axis. Write an integral expression that will approximate the volume of the solid generated when R is revolved about the y-axis. Find an approximation of the volume.

$R(y) = 4 \quad r(y) = \sqrt{y}$

$$V = \pi \int_0^{16} \left(4^2 - \sqrt{y}^{\,2}\right) dy$$

$$V = \pi \int_0^{16} [16 - y] \, dy$$

$$V = 128\pi \approx 402.1238$$



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