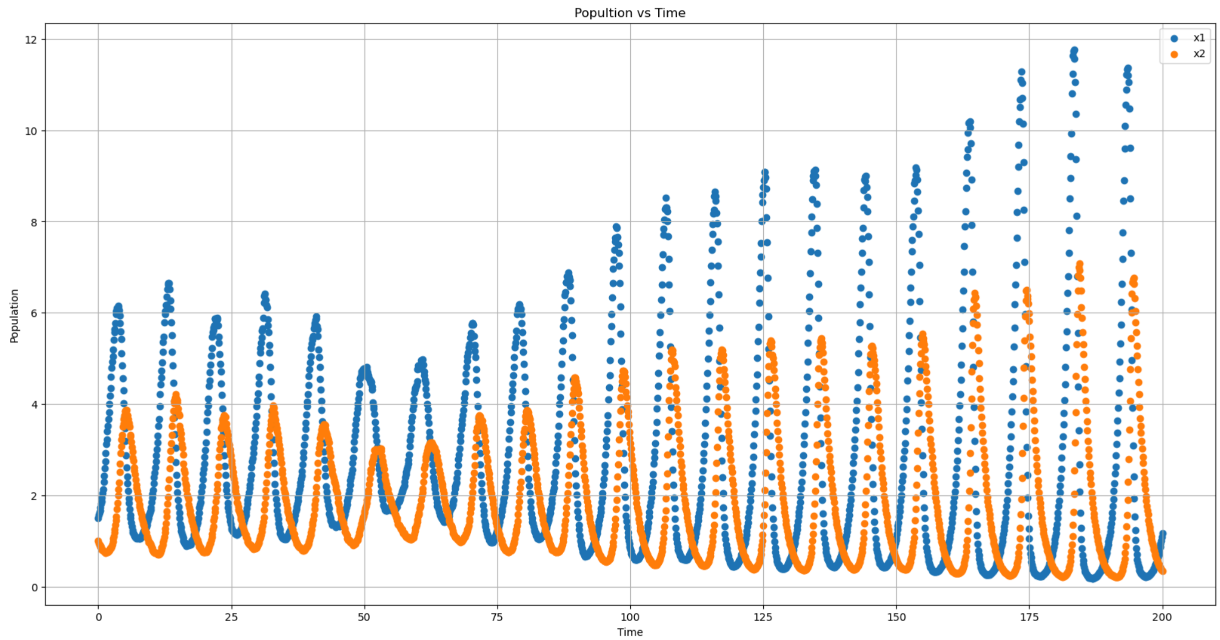
ME228 - Hackathon 22b2429

Part a.

In this question, we want to obtain the governing ordinary differential equation for the evolution of population of two creatures.
Let their populations be x1 and x2 , then what we desire to find is the following:
dx1/dt = f(x1, x2, x1^2, x2^2, $x1 * x2$) and
dx2/dt = g(x1, x2, x1^2, x2^2, $x1 * x2$)
What we are given is the values of x1 and x2, ie the respective population of leoplanet and paldore at time intervals of 0.1.
The scatter plots of their respective population are generated using the below block of code:

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import statsmodels.api as sm

pop = pd.read_csv("29.csv")
time=[]
for x in range(len(pop['x1'].tolist())):
    time.append(x/10)

plt.figure(figsize=(20, 10))
plt.scatter(time, pop['x1'], label='x1')
plt.scatter(time, pop['x2'], label='x2')
plt.title("Popultion vs Time")
plt.xlabel('Time')
plt.ylabel('Population')
plt.legend()
plt.grid(True)
plt.show()
```



Part b :

Now, We will be using Multiple Linear Regression to find the governing ODE in both the cases.
So basically, our Hypothesis Set will be as follows :
dx1/dt = $a * x1 + b * x2 + c * x1^2 + d * x2^2 + e * x1 * x2 + f$ where (a,b,c,d,e,f) belong to Real No
and
dx2/dt = $u * x1 + v * x2 + w * x1^2 + x * x2^2 + y * x1 * x2 + z$, where (u,v,w,x,y,z) belong to Real No
So, basically what we will do intially is add new columns to our dataframe including those of the
derivatives and the non linear terms.
Then We will split the data into train and test data sets (thrice using forward chaining as explained in the next blck in detail).
After this, in each of the train data, We will train my MLR model, once for x1_derivative and secondly for x2_derivative and carry out Feature Redn using p values.(p value elimination explained in next blck in detail)
Now , We will asscess the performance of my model on the testing data by checking the Mse and also the curves of predicted vs actual values.
We will then obtain the governing ODEs for both creatures using the co-efficients given by our model.

Part c. Pseudo Code:

1.Import csv file
2.plot the data to check for any noise(minimal in this check by manual inspection)

3.now add new columns to this datset which contains the derivatives of x1 and x2 wrt time using central difference method

And also add the non linear terms upto the second degree.

4.Now split the data into train and test sets thrice using forward chaining.

Forward chaining while splitting basically makes sure of the fact That more accurately models the situation you'll

see at prediction time, where you'll model on past data and predict on forward-looking data. It also will give you

a sense of the dependence of your modeling on data size.

Hence , These are 3 splits :

First 40 perecnt train ,40-60 percent test

Then First 60 percent train ,60-80 percent test

Then First 80 percent train ,80-100 percent test

5.Now in each of these splits, use the train data to model the ODE Using MLR

applied twice, once to fit dx1/dt and then to fit dx2/dt.

6.Now look at the p values of your co-efficients as given by the model, and eliminate those features one by one whose

p values is more than 0.05 as they have almost no relationship with the output label.

7.Now looking at the R_2 , gauge the performance of your model in sample.

8.Using this model get the prediced label on the test data

and gauge your model performance out of sample by lokking at the curves of y_predicted vs y_actual

wrt each of the features and aso observe the MSE values.

9.Now repeat this thrice for each of the splits.

The code which starts from here basically does the work starting from Step 3 to step 9 as mentioned above.

In [ ]:

```
t = 0.1

#  TODO - Calculate derivatives for x1 and x2 (excluding first and last rows)
lisx1=pop['x1'].tolist()
lisx2=pop['x2'].tolist()
lisdx1=[0 for x in lisx1]
lisdx2=[0 for x in lisx2]
for i in range(1,len(lisx1)-1):
    lisdx1[i]=(lisx1[i+1]-lisx1[i-1])/0.2
    lisdx2[i]=(lisx2[i+1]-lisx2[i-1])/0.2

pop['dx1']=lisdx1  # Time derivative of x1
pop['dx2']=lisdx2  # TIme derivative of x2
pop['x1_squared'] = pop['x1'] ** 2
pop['x2_squared'] = pop['x2'] ** 2
pop['x1_times_x2'] = pop['x1'] * pop['x2']

pop.to_csv("29.csv", index=False)
```

```
print(pop.head())
```

```
         x1        x2  x1_squared  x2_squared  x1_times_x2       dx1       dx2
0  1.505066  1.001880    2.265225    1.003764     1.507897  0.000000  0.000000
1  1.569910  0.948006    2.464616    0.898716     1.488284  0.524076 -0.373664
2  1.609882  0.927147    2.591719    0.859602     1.492598  0.522625 -0.201146
3  1.674435  0.907777    2.803732    0.824060     1.520014  0.605508 -0.244769
4  1.730983  0.878194    2.996303    0.771224     1.520139  0.677403 -0.190437
```

The following two blocks of code are for the Fisrt Split as illustrated in the pseudo code abive. Written soln of part d is at the end, once all the splits are completed.

Part d and e.

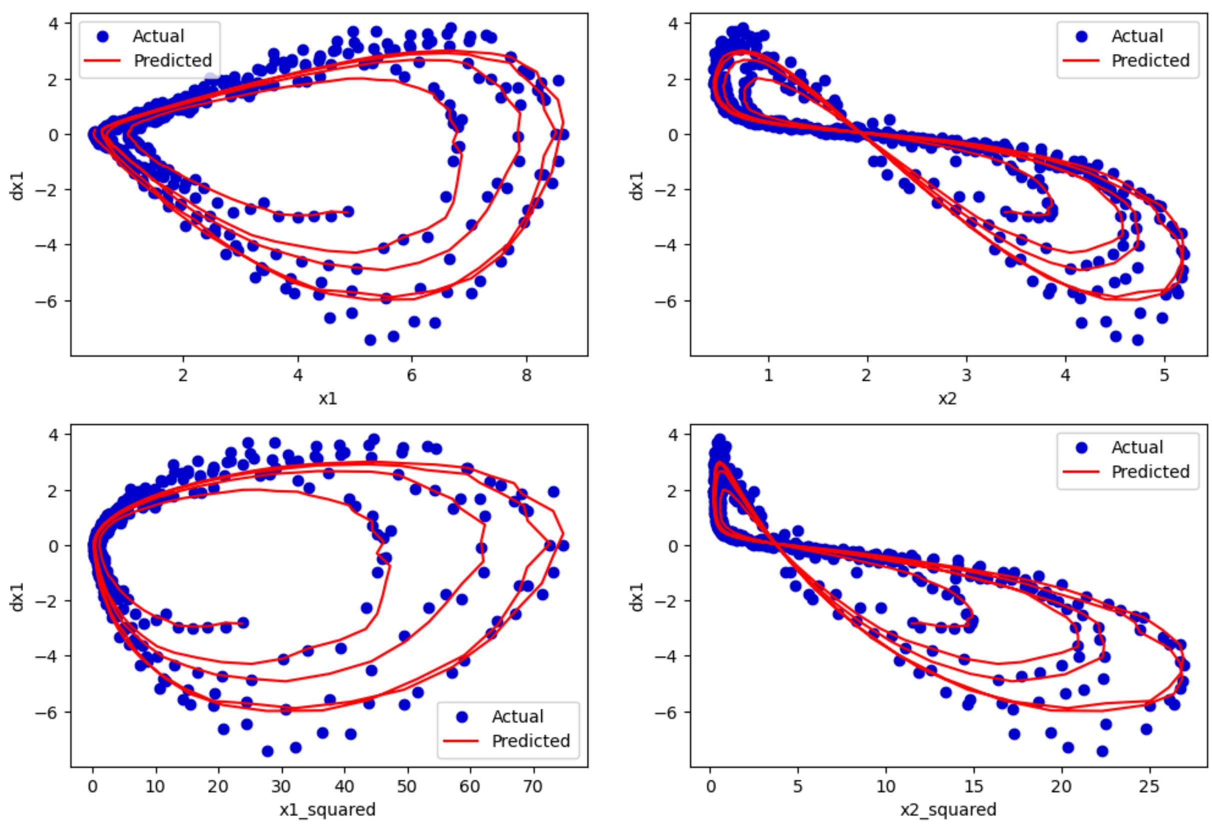
Split 1 : $dx1/dt$ (Leopanet) :

In [ ]:

```
total_rows = len(pop)
train_end_index = int(total_rows * 0.4)
validation_end_index = int(total_rows * 0.6)

train_data = pop[:train_end_index]
validation_data = pop[train_end_index:validation_end_index]

# TODO START

# Define features (X) and target variable (y)
X_train = train_data[['x1', 'x2', 'x1_squared', 'x2_squared', 'x1_times_x2']]
y_train = train_data['dx1']

# Add a constant to the features (for intercept term in MLR)
X_train = sm.add_constant(X_train)

# Apply Multiple Linear Regression
model = sm.OLS(y_train, X_train)
results = model.fit()

# Get coefficients, p-values, and R-squared
coefficients = results.params
p_values = results.pvalues
R_squared = results.rsquared

print("Coefficients:")
print(coefficients)
print("\nP-values:")
print(round(p_values,4))
print("\nR-squared:")
print(R_squared)

# Predicted values using coefficients
X_validation = validation_data[['x1', 'x2', 'x1_squared', 'x2_squared', 'x1_times_x
X_validation = sm.add_constant(X_validation)
y_pred = results.predict(X_validation)
```

```
fig, axes = plt.subplots(2, 2, figsize=(12, 8))
# Plot for x1 vs dx1 (actual vs predicted)
axes[0, 0].scatter(validation_data['x1'], validation_data['dx1'],color='mediumblue'
axes[0, 0].plot(validation_data['x1'], y_pred,color='red',label='Predicted')
axes[0, 0].set_xlabel('x1')
axes[0, 0].set_ylabel('dx1')
axes[0, 0].legend()

# Plot for x2 vs dx1 (actual vs predicted)
axes[0, 1].scatter(validation_data['x2'], validation_data['dx1'],color='mediumblue'
axes[0, 1].plot(validation_data['x2'], y_pred,color='red', label='Predicted')
axes[0, 1].set_xlabel('x2')
axes[0, 1].set_ylabel('dx1')
axes[0, 1].legend()

# Plot for x1_squared vs dx1 (actual vs predicted)
axes[1, 0].scatter(validation_data['x1_squared'], validation_data['dx1'],color='med
axes[1, 0].plot(validation_data['x1_squared'], y_pred,color='red', label='Predicted
axes[1, 0].set_xlabel('x1_squared')
axes[1, 0].set_ylabel('dx1')
axes[1, 0].legend()

# Plot for x2_squared vs dx1 (actual vs predicted)
axes[1, 1].scatter(validation_data['x2_squared'], validation_data['dx1'],color='med
axes[1, 1].plot(validation_data['x2_squared'], y_pred,color='red', label='Predicted
axes[1, 1].set_xlabel('x2_squared')
axes[1, 1].set_ylabel('dx1')
axes[1, 1].legend()

# Calculate Mean Squared Error in test data
mse = np.mean((y_pred - validation_data['dx1']) ** 2)
print("\nMean Squared Error:", mse)


# TODO END
```

```
Coefficients:
const           0.112602
x1              0.688620
x2              0.049579
x1_squared      0.002106
x2_squared     -0.040505
x1_times_x2    -0.366603
dtype: float64

P-values:
const           0.3891
x1              0.0000
x2              0.5602
x1_squared      0.7496
x2_squared      0.0266
x1_times_x2     0.0000
dtype: float64

R-squared:
0.9445176639068287

Mean Squared Error: 0.12969082824471637
```



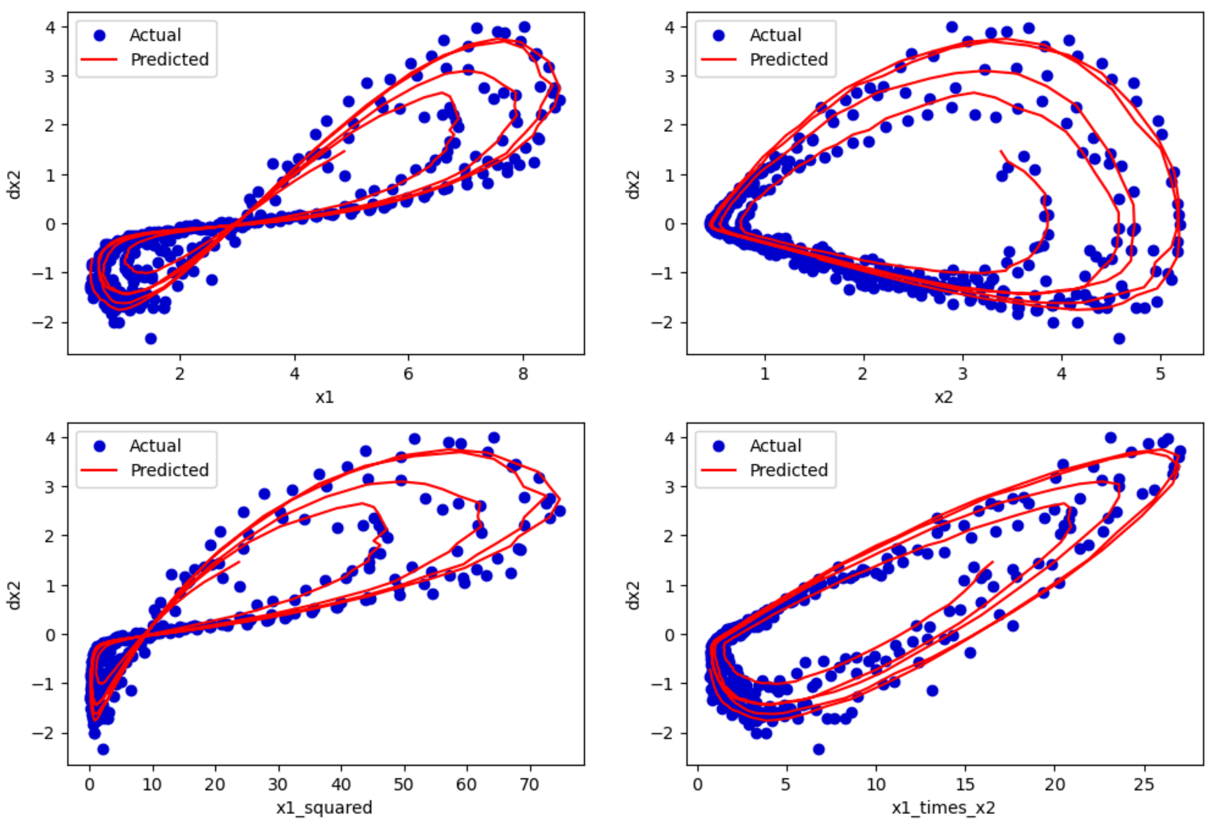
Split 1 : $dx2/dt$ (Paldore) :

```
total_rows = len(pop)
train_end_index = int(total_rows * 0.4)
validation_end_index = int(total_rows * 0.6)

train_data = pop[:train_end_index]
```

```
validation_data = pop[train_end_index:validation_end_index]

# Define features (X) and target variable (y)
X_train = train_data[['x1', 'x2', 'x1_squared', 'x2_squared', 'x1_times_x2']]
y_train = train_data['dx2']

# Add a constant to the features (for intercept term in MLR)
X_train = sm.add_constant(X_train)

# Apply Multiple Linear Regression
model = sm.OLS(y_train, X_train)
results = model.fit()

# Get coefficients, p-values, and R-squared
coefficients = results.params
p_values = results.pvalues
R_squared = results.rsquared

print("Coefficients:")
print(coefficients)
print("\nP-values:")
print(round(p_values,4))
print("\nR-squared:")
print(R_squared)

# Predicted values using coefficients
X_validation = validation_data[['x1', 'x2', 'x1_squared', 'x2_squared', 'x1_times_x
X_validation = sm.add_constant(X_validation)
y_pred = results.predict(X_validation)

fig, axes = plt.subplots(2, 2, figsize=(12, 8))
# Plot for x1 vs dx2 (actual vs predicted)
axes[0, 0].scatter(validation_data['x1'], validation_data['dx2'],color='mediumblue'
axes[0, 0].plot(validation_data['x1'], y_pred,color='red', label='Predicted')
axes[0, 0].set_xlabel('x1')
axes[0, 0].set_ylabel('dx2')
axes[0, 0].legend()

# Plot for x2 vs dx2 (actual vs predicted)
axes[0, 1].scatter(validation_data['x2'], validation_data['dx2'],color='mediumblue'
axes[0, 1].plot(validation_data['x2'], y_pred,color='red', label='Predicted')
axes[0, 1].set_xlabel('x2')
axes[0, 1].set_ylabel('dx2')
axes[0, 1].legend()

# Plot for x1_squared vs dx2 (actual vs predicted)
axes[1, 0].scatter(validation_data['x1_squared'], validation_data['dx2'],color='med
axes[1, 0].plot(validation_data['x1_squared'], y_pred,color='red', label='Predicted
axes[1, 0].set_xlabel('x1_squared')
axes[1, 0].set_ylabel('dx2')
axes[1, 0].legend()

# Plot for x1_times_x2 vs dx2 (actual vs predicted)
axes[1, 1].scatter(validation_data['x1_times_x2'], validation_data['dx2'],color='me
axes[1, 1].plot(validation_data['x1_times_x2'], y_pred,color='red', label='Predicte
axes[1, 1].set_xlabel('x1_times_x2')
```

```python
axes[1, 1].set_ylabel('dx2')
axes[1, 1].legend()

# Calculate Mean Squared Error in test data
mse = np.mean((y_pred - validation_data['dx2']) ** 2)
print("\nMean Squared Error:", mse)

plt.show()
```

```
Coefficients:
const           0.080224
x1             -0.099348
x2             -0.577766
x1_squared      0.016255
x2_squared     -0.011039
x1_times_x2     0.215595
dtype: float64

P-values:
const           0.2810
x1              0.0003
x2              0.0000
x1_squared      0.0000
x2_squared      0.2878
x1_times_x2     0.0000
dtype: float64

R-squared:
0.9493596496694813

Mean Squared Error: 0.032708459710699946
```

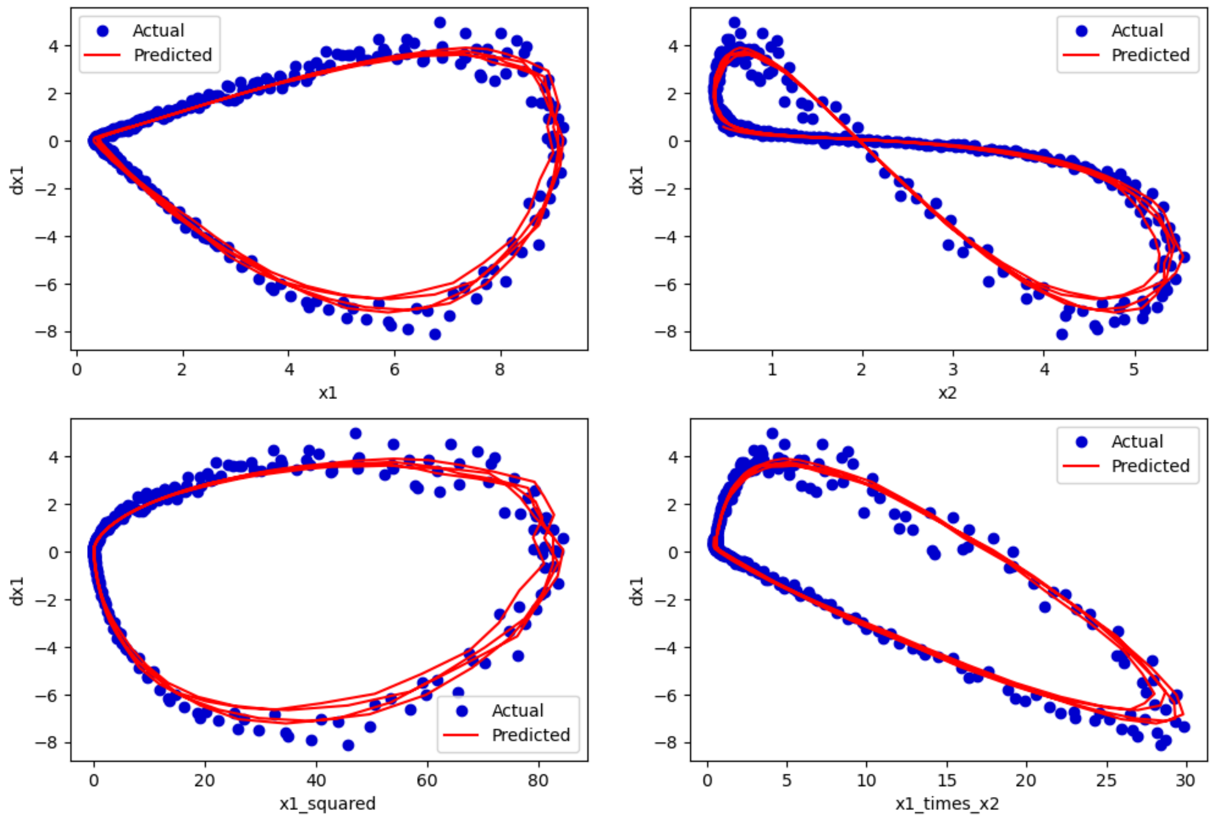


Split 2 : $dx1/dt$ (Leopanet) :

In [ ]:
```
total_rows = len(pop)
train_end_index = int(total_rows * 0.6)
validation_end_index = int(total_rows * 0.8)

train_data = pop[:train_end_index]
validation_data = pop[train_end_index:validation_end_index]

# Define features (X) and target variable (y)
X_train = train_data[['x1', 'x2', 'x1_squared', 'x2_squared', 'x1_times_x2']]
y_train = train_data['dx1']

# Add a constant to the features (for intercept term in MLR)
X_train = sm.add_constant(X_train)

# Apply Multiple Linear Regression
model = sm.OLS(y_train, X_train)
results = model.fit()

# Get coefficients, p-values, and R-squared
coefficients = results.params
p_values = results.pvalues
R_squared = results.rsquared

print("Coefficients:")
print(coefficients)
print("\nP-values:")
print(round(p_values,4))
print("\nR-squared:")
print(R_squared)

# Predicted values using coefficients
X_validation = validation_data[['x1', 'x2', 'x1_squared', 'x2_squared', 'x1_times_x
X_validation = sm.add_constant(X_validation)
y_pred = results.predict(X_validation)

fig, axes = plt.subplots(2, 2, figsize=(12, 8))
# Plot for x1 vs dx1 (actual vs predicted)
axes[0, 0].scatter(validation_data['x1'], validation_data['dx1'], color='mediumblue
axes[0, 0].plot(validation_data['x1'], y_pred,color='red', label='Predicted')
axes[0, 0].set_xlabel('x1')
axes[0, 0].set_ylabel('dx1')
axes[0, 0].legend()

# Plot for x2 vs dx1 (actual vs predicted)
axes[0, 1].scatter(validation_data['x2'], validation_data['dx1'], color='mediumblue
axes[0, 1].plot(validation_data['x2'], y_pred,color='red', label='Predicted')
axes[0, 1].set_xlabel('x2')
axes[0, 1].set_ylabel('dx1')
axes[0, 1].legend()

# Plot for x1_squared vs dx1 (actual vs predicted)
axes[1, 0].scatter(validation_data['x1_squared'], validation_data['dx1'],color='med
```

```
axes[1, 0].plot(validation_data['x1_squared'], y_pred,color='red', label='Predicted
axes[1, 0].set_xlabel('x1_squared')
axes[1, 0].set_ylabel('dx1')
axes[1, 0].legend()

# Plot for x1_times_x2 vs dx1 (actual vs predicted)
axes[1, 1].scatter(validation_data['x1_times_x2'], validation_data['dx1'],color='me
axes[1, 1].plot(validation_data['x1_times_x2'], y_pred,color='red', label='Predicte
axes[1, 1].set_xlabel('x1_times_x2')
axes[1, 1].set_ylabel('dx1')
axes[1, 1].legend()

plt.show()

# Calculate Mean Squared Error in test data
mse = np.mean((y_pred - validation_data['dx1']) ** 2)
print("\nMean Squared Error:", mse)
```

```
Coefficients:
const          -0.090670
x1              0.791410
x2              0.157339
x1_squared      0.000860
x2_squared     -0.046267
x1_times_x2    -0.409795
dtype: float64

P-values:
const           0.1308
x1              0.0000
x2              0.0001
x1_squared      0.7389
x2_squared      0.0000
x1_times_x2     0.0000
dtype: float64

R-squared:
0.9649728638533256
```



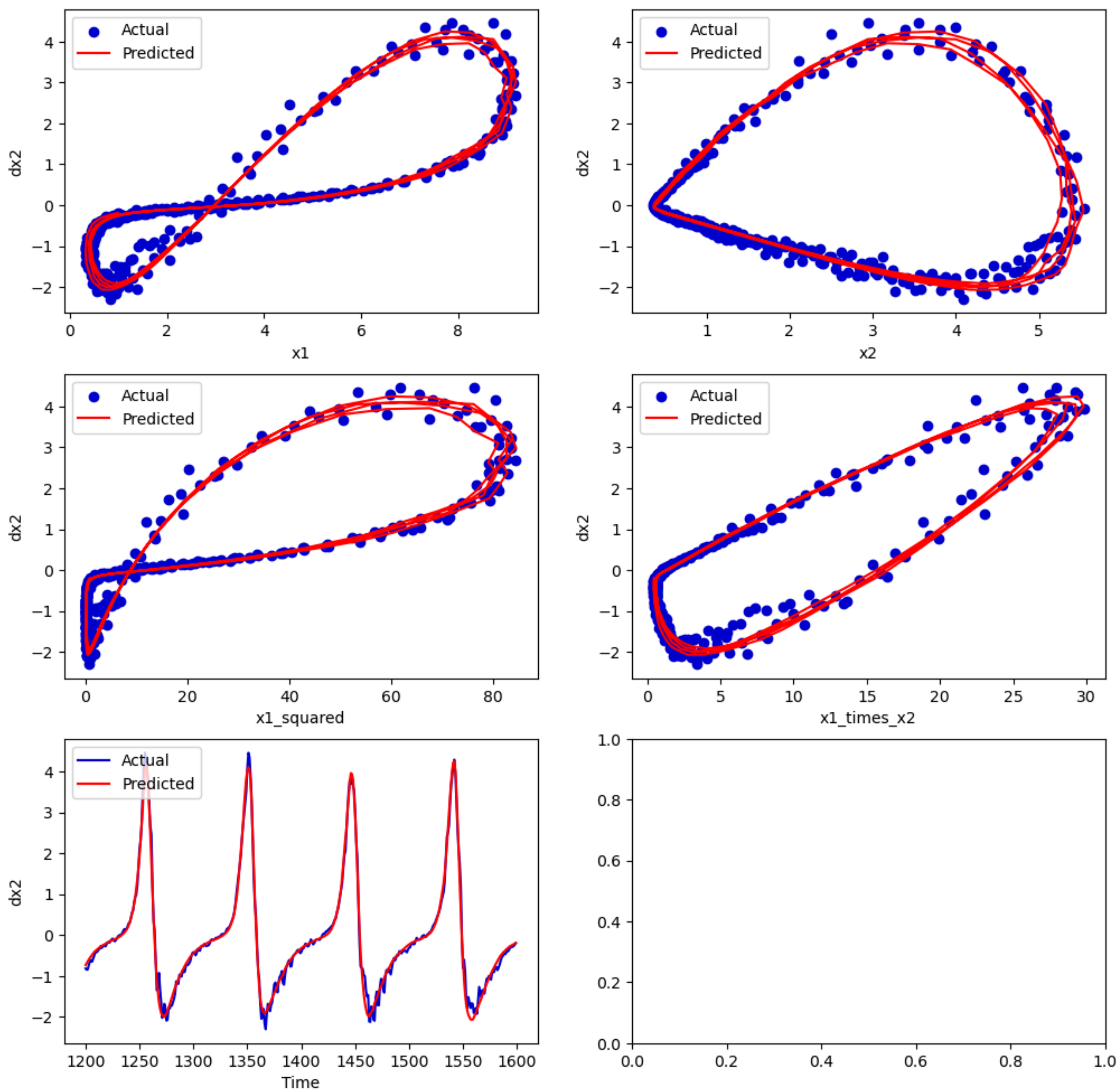

```
Mean Squared Error: 0.105805017181356
```

Split 2 : $dx2/dt$ (Paldore) :

```
In [ ]:
total_rows = len(pop)
train_end_index = int(total_rows * 0.6)
validation_end_index = int(total_rows * 0.8)

train_data = pop[:train_end_index]
validation_data = pop[train_end_index:validation_end_index]

# Define features (X) and target variable (y)
X_train = train_data[['x1', 'x2', 'x1_squared', 'x2_squared', 'x1_times_x2']]
y_train = train_data['dx2']

# Add a constant to the features (for intercept term in MLR)
X_train = sm.add_constant(X_train)

# Apply Multiple Linear Regression
model = sm.OLS(y_train, X_train)
results = model.fit()

# Get coefficients, p-values, and R-squared
coefficients = results.params
p_values = results.pvalues
R_squared = results.rsquared

print("Coefficients:")
print(coefficients)
print("\nP-values:")
```

```
print(round(p_values,4))
print("\nR-squared:")
print(R_squared)

# Predicted values using coefficients
X_validation = validation_data[['x1', 'x2', 'x1_squared', 'x2_squared', 'x1_times_x
X_validation = sm.add_constant(X_validation)
y_pred = results.predict(X_validation)

fig, axes = plt.subplots(3, 2, figsize=(12, 12))
# Plot for x1 vs dx2 (actual vs predicted)
axes[0, 0].scatter(validation_data['x1'], validation_data['dx2'],color='mediumblue'
axes[0, 0].plot(validation_data['x1'], y_pred,color='red', label='Predicted')
axes[0, 0].set_xlabel('x1')
axes[0, 0].set_ylabel('dx2')
axes[0, 0].legend()

# Plot for x2 vs dx2 (actual vs predicted)
axes[0, 1].scatter(validation_data['x2'], validation_data['dx2'],color='mediumblue'
axes[0, 1].plot(validation_data['x2'], y_pred,color='red', label='Predicted')
axes[0, 1].set_xlabel('x2')
axes[0, 1].set_ylabel('dx2')
axes[0, 1].legend()

# Plot for x1_squared vs dx2 (actual vs predicted)
axes[1, 0].scatter(validation_data['x1_squared'], validation_data['dx2'],color='med
axes[1, 0].plot(validation_data['x1_squared'], y_pred,color='red', label='Predicted
axes[1, 0].set_xlabel('x1_squared')
axes[1, 0].set_ylabel('dx2')
axes[1, 0].legend()

# Plot for x1_times_x2 vs dx2 (actual vs predicted)
axes[1, 1].scatter(validation_data['x1_times_x2'], validation_data['dx2'],color='me
axes[1, 1].plot(validation_data['x1_times_x2'], y_pred,color='red', label='Predicte
axes[1, 1].set_xlabel('x1_times_x2')
axes[1, 1].set_ylabel('dx2')
axes[1, 1].legend()

# Plot time evolution of dx2 (actual vs predicted)
axes[2, 0].plot(validation_data.index, validation_data['dx2'],color='mediumblue', l
axes[2, 0].plot(validation_data.index, y_pred,color='red', label='Predicted')
axes[2, 0].set_xlabel('Time')
axes[2, 0].set_ylabel('dx2')
axes[2, 0].legend()

plt.show()

# Calculate Mean Squared Error in test data
mse = np.mean((y_pred - validation_data['dx2']) ** 2)
print("\nMean Squared Error:", mse)
```

```
Coefficients:
const           0.036528
x1             -0.056786
x2             -0.604578
x1_squared      0.007706
x2_squared     -0.010382
x1_times_x2     0.225973
dtype: float64

P-values:
const           0.2706
x1              0.0000
x2              0.0000
x1_squared      0.0000
x2_squared      0.0114
x1_times_x2     0.0000
dtype: float64

R-squared:
0.9684898537038136
```



```
Mean Squared Error: 0.02665175669604008
```

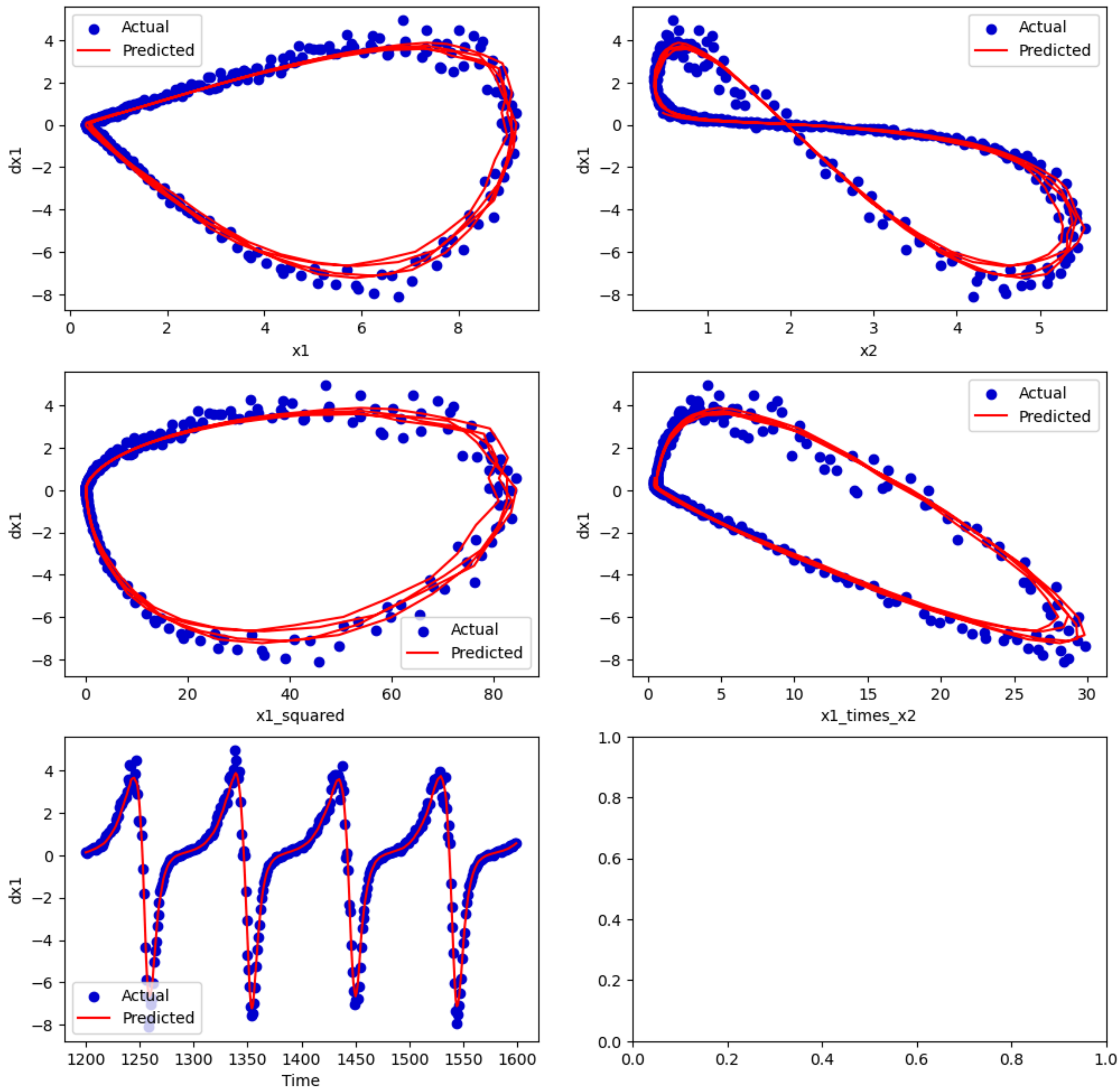
Split 3 : $dx1/dt$ (Leopanet) :

In [ ]:

```
# Define features (X) and target variable (y)
X_train = train_data[['x1', 'x2', 'x1_squared','x2_squared', 'x1_times_x2']]
y_train = train_data['dx1']

# Add a constant to the features (for intercept term in MLR)
X_train = sm.add_constant(X_train)

# Apply Multiple Linear Regression
model = sm.OLS(y_train, X_train)
results = model.fit()

# Get coefficients, p-values, and R-squared
coefficients = results.params
p_values = results.pvalues
R_squared = results.rsquared

print("Coefficients:")
print(coefficients)
print("\nP-values:")
print(round(p_values,4))
print("\nR-squared:")
print(R_squared)

# Predicted values using coefficients
X_validation = validation_data[['x1', 'x2', 'x1_squared', 'x2_squared', 'x1_times_x
X_validation = sm.add_constant(X_validation)
y_pred = results.predict(X_validation)

fig, axes = plt.subplots(3, 2, figsize=(12, 12))
# Plot for x1 vs dx1 (actual vs predicted)
axes[0, 0].scatter(validation_data['x1'], validation_data['dx1'],color='mediumblue'
axes[0, 0].plot(validation_data['x1'], y_pred,color='red', label='Predicted')
axes[0, 0].set_xlabel('x1')
axes[0, 0].set_ylabel('dx1')
axes[0, 0].legend()

# Plot for x2 vs dx1 (actual vs predicted)
axes[0, 1].scatter(validation_data['x2'], validation_data['dx1'],color='mediumblue'
axes[0, 1].plot(validation_data['x2'], y_pred,color='red', label='Predicted')
axes[0, 1].set_xlabel('x2')
axes[0, 1].set_ylabel('dx1')
axes[0, 1].legend()

# Plot for x1_squared vs dx1 (actual vs predicted)
axes[1, 0].scatter(validation_data['x1_squared'], validation_data['dx1'],color='med
axes[1, 0].plot(validation_data['x1_squared'], y_pred,color='red', label='Predicted
axes[1, 0].set_xlabel('x1_squared')
axes[1, 0].set_ylabel('dx1')
axes[1, 0].legend()

# Plot for x1_times_x2 vs dx1 (actual vs predicted)
```

```
axes[1, 1].scatter(validation_data['x1_times_x2'], validation_data['dx1'],color='me
axes[1, 1].plot(validation_data['x1_times_x2'], y_pred,color='red', label='Predicte
axes[1, 1].set_xlabel('x1_times_x2')
axes[1, 1].set_ylabel('dx1')
axes[1, 1].legend()

# Plot time evolution of dx1 (actual vs predicted)
axes[2, 0].scatter(validation_data.index, validation_data['dx1'],color='mediumblue'
axes[2, 0].plot(validation_data.index, y_pred,color='red', label='Predicted')
axes[2, 0].set_xlabel('Time')
axes[2, 0].set_ylabel('dx1')
axes[2, 0].legend()

plt.show()

# Calculate Mean Squared Error in test data
mse = np.mean((y_pred - validation_data['dx1']) ** 2)
print("\nMean Squared Error:", mse)
```

```
Coefficients:
const          -0.090670
x1              0.791410
x2              0.157339
x1_squared      0.000860
x2_squared     -0.046267
x1_times_x2    -0.409795
dtype: float64

P-values:
const           0.1308
x1              0.0000
x2              0.0001
x1_squared      0.7389
x2_squared      0.0000
x1_times_x2     0.0000
dtype: float64

R-squared:
0.9649728638533256
```




```
Mean Squared Error: 0.105805017181356
```

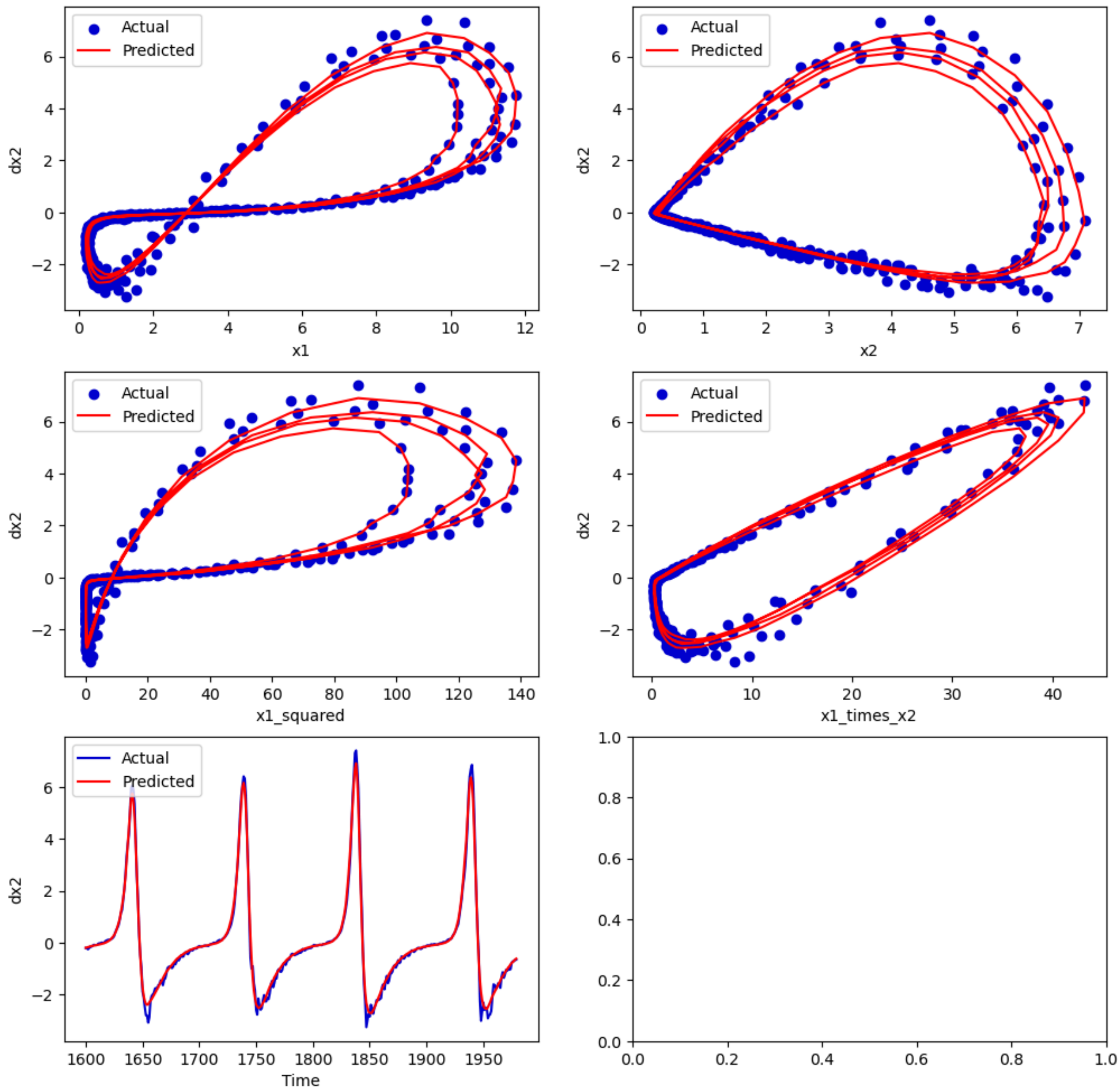
Split 3 : $dx2/dt$ (Paldore) :

In [ ]:
```
total_rows = len(pop)
train_end_index = int(total_rows * 0.8)
validation_end_index = int(total_rows * 0.99)

train_data = pop[:train_end_index]
validation_data = pop[train_end_index:validation_end_index]

# Define features (X) and target variable (y)
X_train = train_data[['x1', 'x2', 'x1_squared', 'x2_squared', 'x1_times_x2']]
y_train = train_data['dx2']

# Add a constant to the features (for intercept term in MLR)
X_train = sm.add_constant(X_train)

# Apply Multiple Linear Regression
```

```
model = sm.OLS(y_train, X_train)
results = model.fit()

# Get coefficients, p-values, and R-squared
coefficients = results.params
p_values = results.pvalues
R_squared = results.rsquared

print("Coefficients:")
print(coefficients)
print("\nP-values:")
print(round(p_values,4))
print("\nR-squared:")
print(R_squared)

# Predicted values using coefficients
X_validation = validation_data[['x1', 'x2', 'x1_squared', 'x2_squared', 'x1_times_x
X_validation = sm.add_constant(X_validation)
y_pred = results.predict(X_validation)

fig, axes = plt.subplots(3, 2, figsize=(12, 12))
# Plot for x1 vs dx2 (actual vs predicted)
axes[0, 0].scatter(validation_data['x1'], validation_data['dx2'],color='mediumblue'
axes[0, 0].plot(validation_data['x1'], y_pred, color='red',label='Predicted')
axes[0, 0].set_xlabel('x1')
axes[0, 0].set_ylabel('dx2')
axes[0, 0].legend()

# Plot for x2 vs dx2 (actual vs predicted)
axes[0, 1].scatter(validation_data['x2'], validation_data['dx2'],color='mediumblue'
axes[0, 1].plot(validation_data['x2'], y_pred,color='red', label='Predicted')
axes[0, 1].set_xlabel('x2')
axes[0, 1].set_ylabel('dx2')
axes[0, 1].legend()

# Plot for x1_squared vs dx2 (actual vs predicted)
axes[1, 0].scatter(validation_data['x1_squared'], validation_data['dx2'],color='med
axes[1, 0].plot(validation_data['x1_squared'], y_pred, color='red',label='Predicted
axes[1, 0].set_xlabel('x1_squared')
axes[1, 0].set_ylabel('dx2')
axes[1, 0].legend()

# Plot for x1_times_x2 vs dx2 (actual vs predicted)
axes[1, 1].scatter(validation_data['x1_times_x2'], validation_data['dx2'],color='me
axes[1, 1].plot(validation_data['x1_times_x2'], y_pred,color='red', label='Predicte
axes[1, 1].set_xlabel('x1_times_x2')
axes[1, 1].set_ylabel('dx2')
axes[1, 1].legend()

# Plot the time evolution of dx2 (actual vs predicted)
axes[2, 0].plot(validation_data.index, validation_data['dx2'],color='mediumblue', l
axes[2, 0].plot(validation_data.index, y_pred,color='red', label='Predicted')
axes[2, 0].set_xlabel('Time')
axes[2, 0].set_ylabel('dx2')
axes[2, 0].legend()
```

```python
plt.show()

# Calculate Mean Squared Error in test data
mse = np.mean((y_pred - validation_data['dx2']) ** 2)
print("\nMean Squared Error:", mse)
```

```
Coefficients:
const           0.015649
x1             -0.040252
x2             -0.622602
x1_squared      0.006515
x2_squared     -0.002634
x1_times_x2     0.222780
dtype: float64

P-values:
const           0.4659
x1              0.0000
x2              0.0000
x1_squared      0.0000
x2_squared      0.3213
x1_times_x2     0.0000
dtype: float64

R-squared:
0.9770231733430178
```



Mean Squared Error: 0.041477005059446216

Part d: Written Note

The following are my final hypotheses for the rate of change of two creature populations as given by my model:
Considering the splits 2 and 3, we can observe that for x1_derivative, x1_squared, and the constants are not much significant and can be neglected. And for x2_derivative, the constant is not at all significant in any of the three models, but x2_squared is only significant in split-2.

Now we can neglect those terms which have very small coefficients as they won't impact the output much. Hence, refined hypothesis:
For coefficients of x1_derivative:

- const: NEGLECTED
- x1: 0.791410
- x2: 0.157339

- x1_squared: NEGLECTED
- x2_squared: -0.046267
- x1_times_x2: -0.409795

For coefficients of x2_derivative:

- const: NEGLECTED
- x1: -0.040252
- x2: -0.622602
- x1_squared: 0.006515
- x2_squared: NEGLECTED
- x1_times_x2: 0.222780

Part e : (Written Note)

The refined MLR models are really good at explaining the given data, with R-squared values between 0.949 and 0.977, showing they can predict changes in creature populations well. By focusing on important variables and ignoring less important ones (shown by low p-values), these models do a great job predicting how populations change over time.

Thank You !!