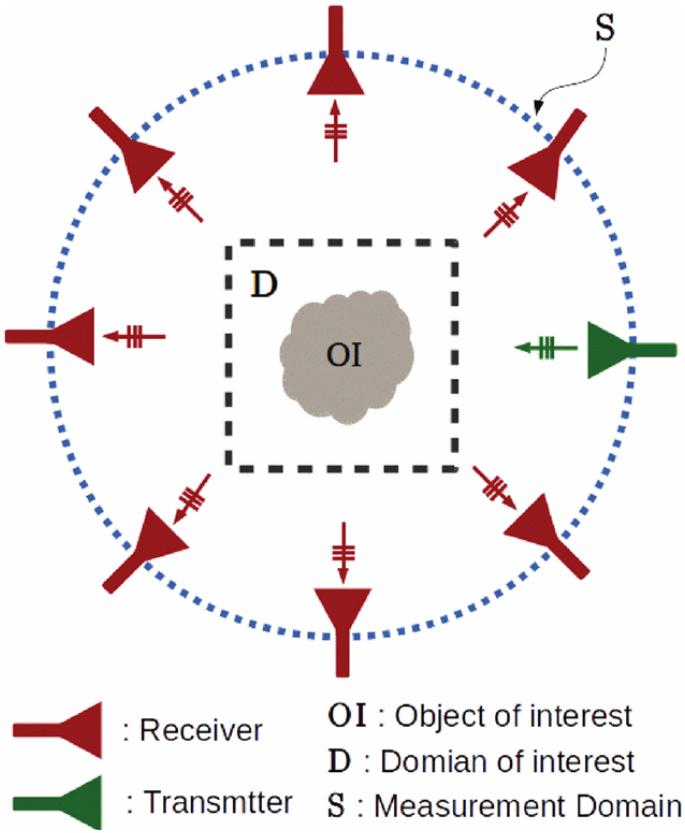
Embedding Deep Learning in Inverse Scattering Solutions

Yash Sanghvi[1], Yaswanth Kalepu[1], and Uday Khankhoje[1]

[1]Department of Electrical Engineering
Indian Institute of Technology Madras

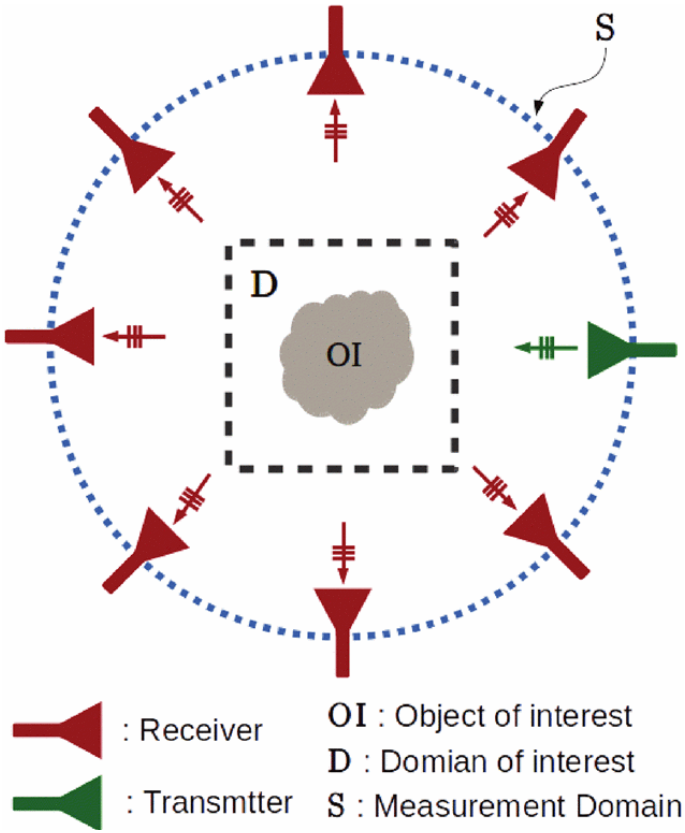
Asia Pacific Radio Science Conference (AP-RASC), March 2019

Inverse Scattering Problem



Inverse Scattering
Measurement Setup

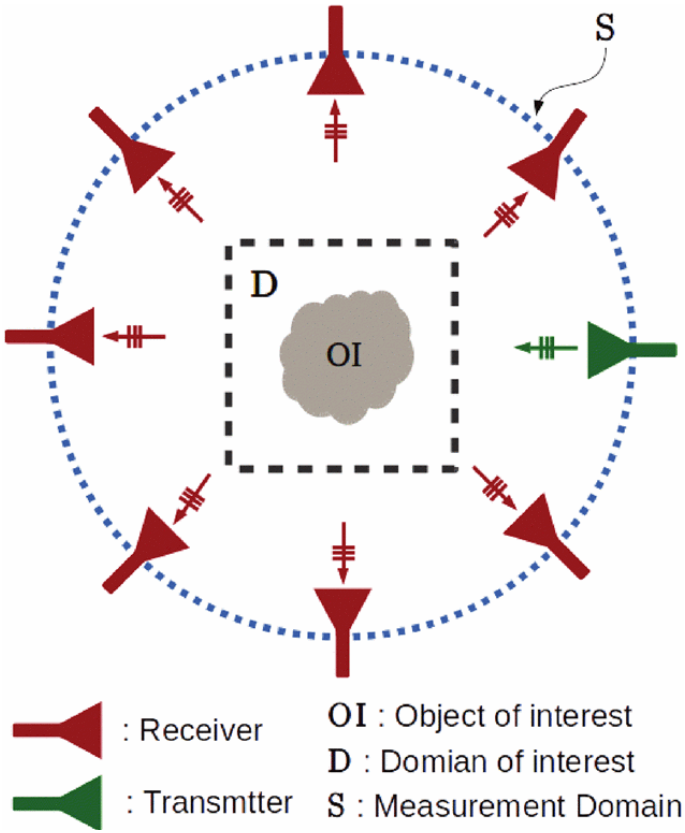
Inverse Scattering Problem



Inverse Scattering
Measurement Setup

$$E_{scat}(r) = \int G(r, r')E_{total}(r')\chi(r')dr'$$

Inverse Scattering Problem

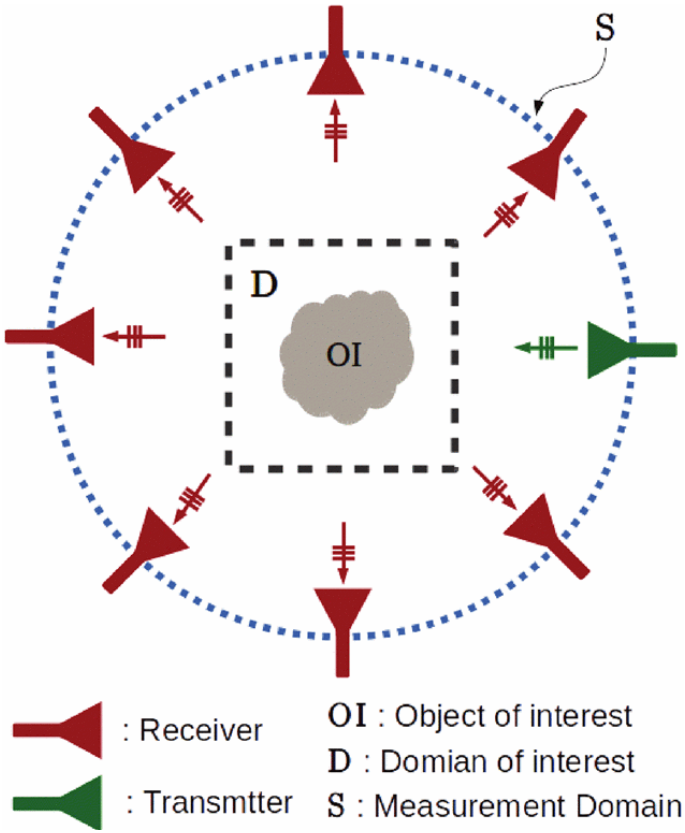


Inverse Scattering Measurement Setup

$$E_{scat}(r) = \int G(r, r')E_{total}(r')\chi(r')dr'$$

Knowns: Scattered Fields (s_v) and
Unknowns: Contrast (x) and
Contrast Source (w_v) where $(w_v)_i = (x_i \cdot d_i)$

Inverse Scattering Problem



Inverse Scattering Measurement Setup

$$E_{scat}(r) = \int G(r, r') E_{total}(r') \chi(r') dr'$$

Knowns: Scattered Fields (s_v) and
Unknowns: Contrast (x) and
Contrast Source (w_v) where $(w_v)_i = (x_i \cdot d_i)$

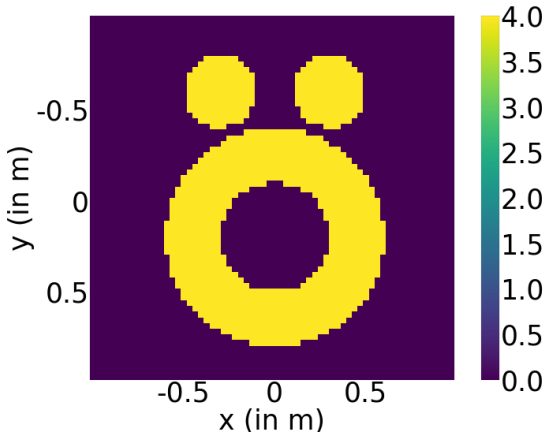
$$s_v = G_S \, w_v \qquad \text{(Data Equation)}$$

$$w_v = x \cdot e_v + x \cdot (G_D w_v) \quad \text{(Current Equation)}$$

Recovering High Contrast Objects

[1] V. Berg, et al. "Contrast source inversion method: State of art." 2001

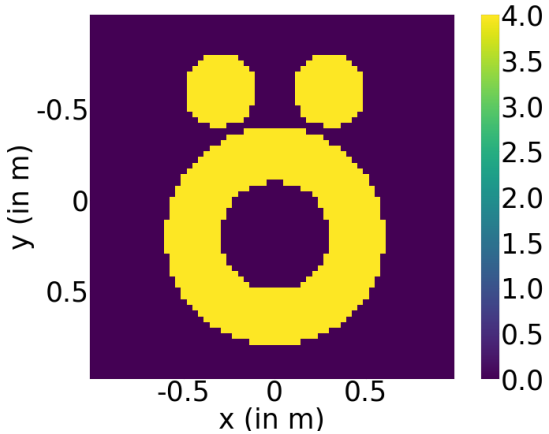
Recovering High Contrast Objects



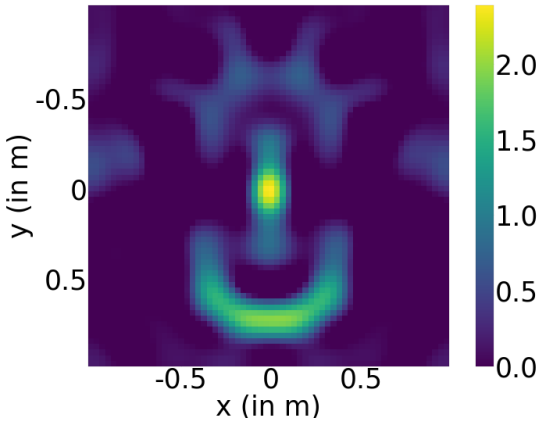
Austria Profile[1], Contrast = 4

[1] V. Berg, et al. "Contrast source inversion method: State of art." 2001

Recovering High Contrast Objects



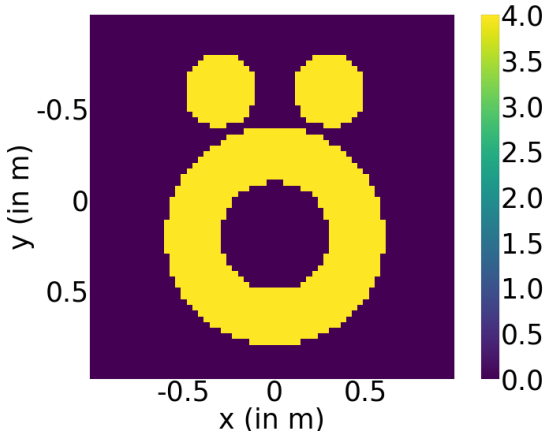
Austria Profile[1], Contrast = 4



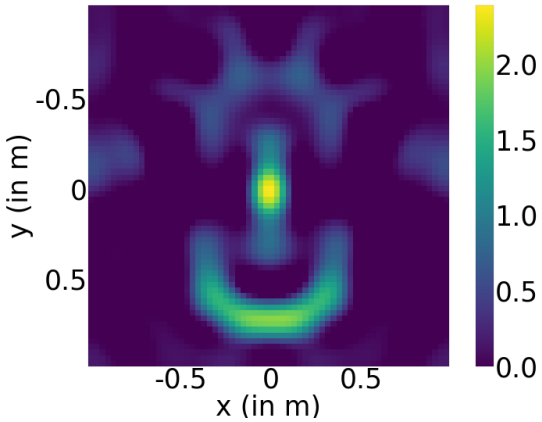
Reconstruction using SOM

[1] V. Berg, et al. "Contrast source inversion method: State of art." 2001

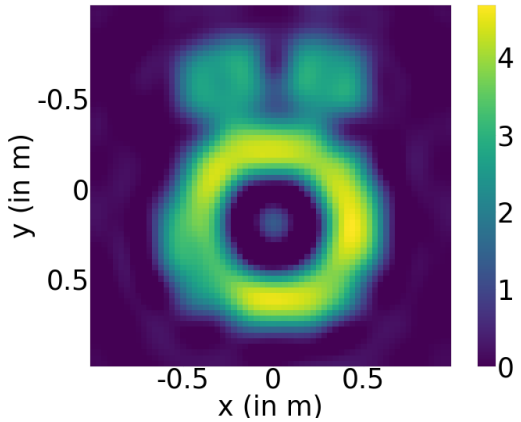
Recovering High Contrast Objects



Austria Profile[1], Contrast = 4



Reconstruction using SOM



Proposed Method

[1]V. Berg, et al. "Contrast source inversion method: State of art." 2001

Contemporary Iterative Strategies

- Contrast Source Inversion (CSI)

[2] X.Chen "Subspace-based optimization method for solving inverse-scattering problems." 2010

Contemporary Iterative Strategies

- Contrast Source Inversion (CSI)

$$\arg\min_{x, w_v} \sum_{v=1}^{V} \frac{||s_v - G_S\, w_v\,||^2}{\eta_{S,v}} + \frac{||\, w_v - x * e_v - x * (G_D\, w_v)||^2}{\eta_{D,v}}$$

$\eta_{S,v}, \eta_{D,v}$: Normalization Constants

[2] X.Chen "Subspace-based optimization method for solving inverse-scattering problems." 2010

Contemporary Iterative Strategies

- Contrast Source Inversion (CSI)

$$\arg\min_{x, w_v} \sum_{v=1}^{V} \frac{||s_v - G_S\, w_v\,||^2}{\eta_{S,v}} + \frac{||\, w_v - x * e_v - x * (G_D\, w_v)||^2}{\eta_{D,v}}$$

  $\eta_{S,v}, \eta_{D,v}$: Normalization Constants
- Subspace Optimization Method (SOM)[2]: Partition w_v to signal and noise subspace

[2] X.Chen "Subspace-based optimization method for solving inverse-scattering problems." 2010

Contemporary Iterative Strategies

- Contrast Source Inversion (CSI)

$$\arg\min_{x, w_v} \sum_{v=1}^{V} \frac{||s_v - G_S\, w_v\,||^2}{\eta_{S,v}} + \frac{||\, w_v - x * e_v - x * (G_D w_v)||^2}{\eta_{D,v}}$$

  $\eta_{S,v}, \eta_{D,v}$: Normalization Constants

- Subspace Optimization Method (SOM)[2]: Partition w_v to signal and noise subspace

$$w_v = w_v^{(s)} + w_v^{(n)}$$
$$s_v = G_s\left(w_v^{(s)} + w_v^{(n)}\right)$$

[2] X.Chen "Subspace-based optimization method for solving inverse-scattering problems." 2010

Contemporary Iterative Strategies

- Contrast Source Inversion (CSI)

$$\arg\min_{x, w_v} \sum_{v=1}^{V} \frac{||s_v - G_S\, w_v\,||^2}{\eta_{S,v}} + \frac{||\, w_v - x * e_v - x * (G_D\, w_v)||^2}{\eta_{D,v}}$$

$\eta_{S,v}, \eta_{D,v}$: Normalization Constants

- Subspace Optimization Method (SOM)[2]: Partition w_v to signal and noise subspace

$$w_v = w_v^{(s)} + w_v^{(n)}$$

$$s_v = G_s(w_v^{(s)} + w_v^{(n)})$$

[2]X.Chen "Subspace-based optimization method for solving inverse-scattering problems." 2010

Contemporary Iterative Strategies

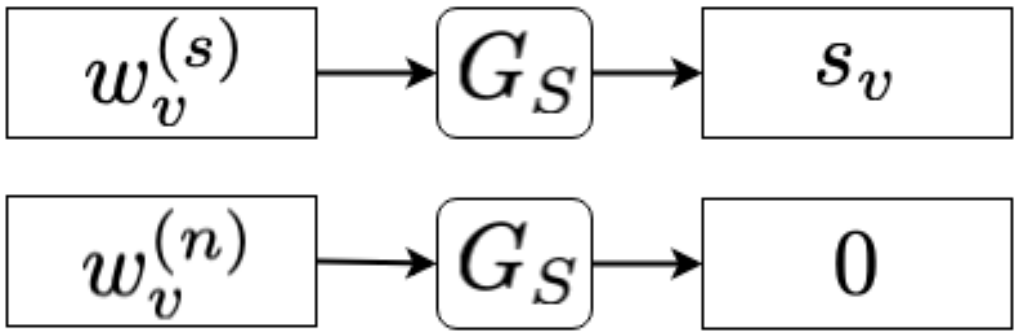
- Contrast Source Inversion (CSI)

$$\arg\min_{x, w_v} \sum_{v=1}^{V} \frac{||s_v - G_S\, w_v\,||^2}{\eta_{S,v}} + \frac{||\, w_v - x * e_v - x * (G_D\, w_v)||^2}{\eta_{D,v}}$$

$\eta_{S,v}, \eta_{D,v}$: Normalization Constants

- Subspace Optimization Method (SOM)[2]: Partition w_v to signal and noise subspace

$$w_v = w_v^{(s)} + w_v^{(n)}$$
$$s_v = G_s(w_v^{(s)} + w_v^{(n)})$$



[2]X.Chen "Subspace-based optimization method for solving inverse-scattering problems." 2010

Contemporary Iterative Strategies - II

- Signal Subspace: Recovery using Spectral Analysis (Stage-I)

$$s_v = G_s(w_v^{(s)} + w_v^{(n)})$$

Contemporary Iterative Strategies - II

- Signal Subspace: Recovery using Spectral Analysis (Stage-I)

$$s_v = G_s(w_v^{(s)} + w_v^{(n)})$$

- Noise Subspace: Recovery by Iterative Refinement (Stage-II)

$$w_v = x \cdot e_v + x \cdot (G_D w_v)$$

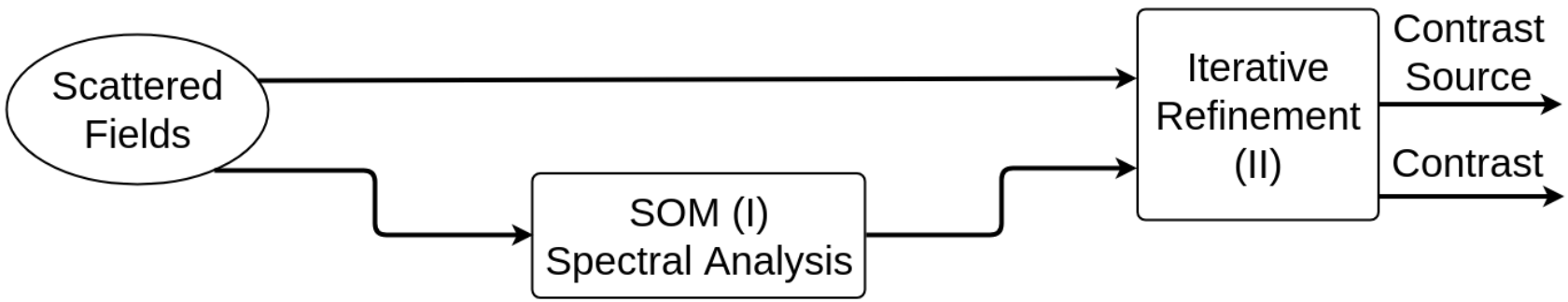
Contemporary Iterative Strategies - II

- Signal Subspace: Recovery using Spectral Analysis (Stage-I)

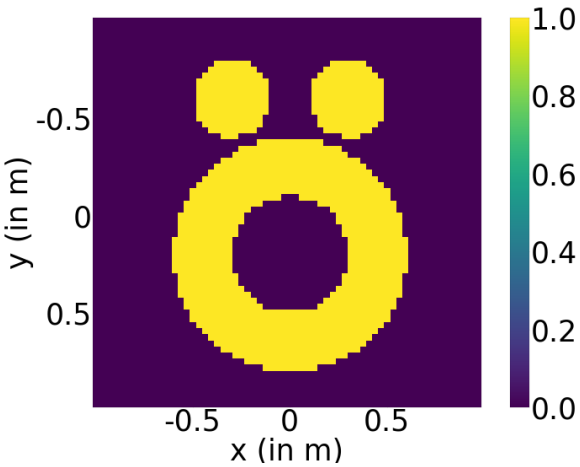
$$s_v = G_s(w_v^{(s)} + w_v^{(n)})$$

- Noise Subspace: Recovery by Iterative Refinement (Stage-II)

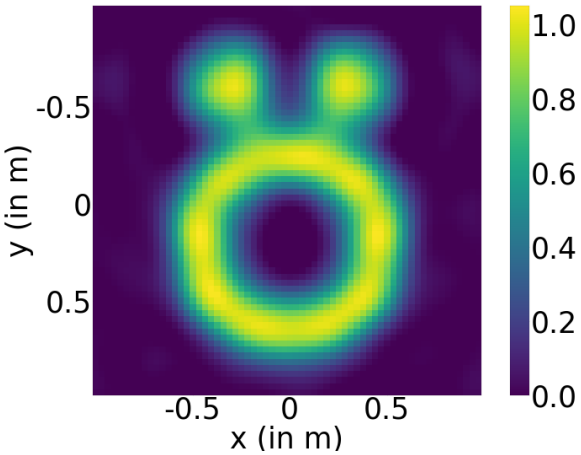
$$w_v = x \cdot e_v + x \cdot (G_D w_v)$$



Reconstruction using Conventional SOM

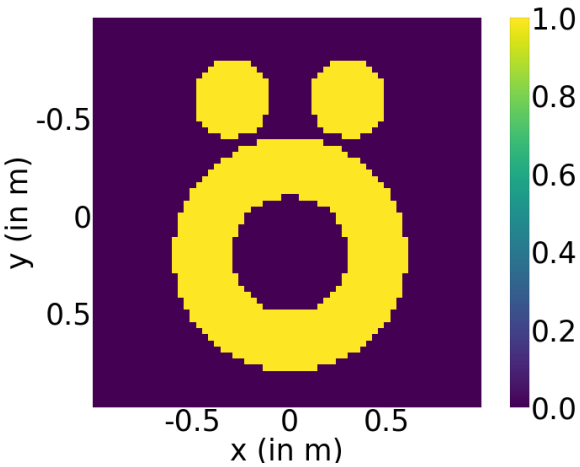


Austria, Contrast $= 1$

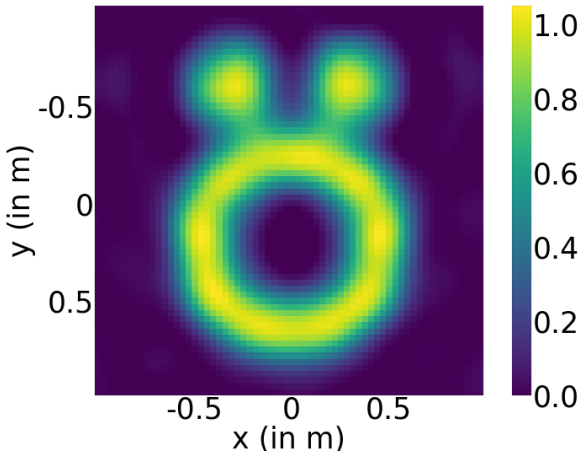


Reconstruction using SOM

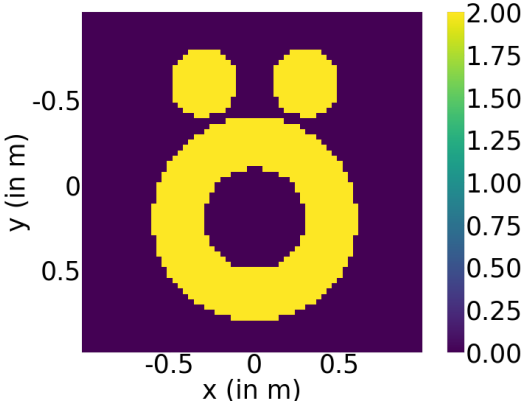
Reconstruction using Conventional SOM



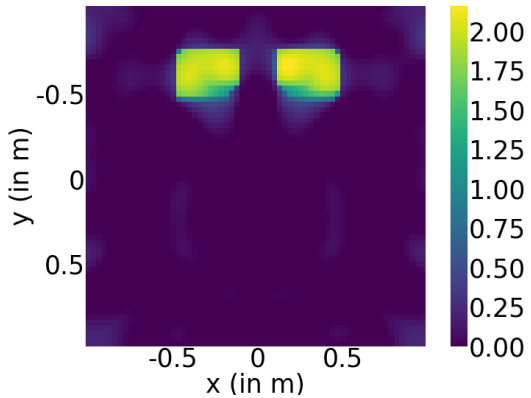
Austria, Contrast = 1



Reconstruction using SOM



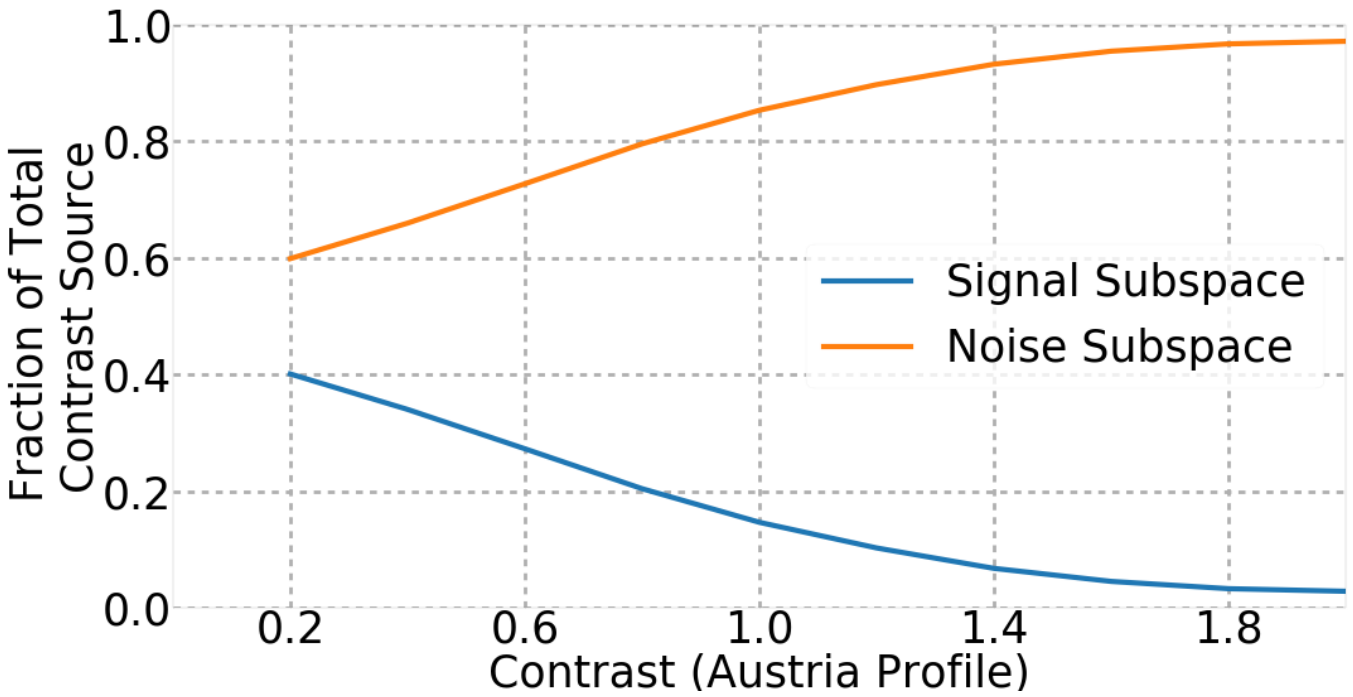
Austria, Contrast = 2



Reconstruction using SOM

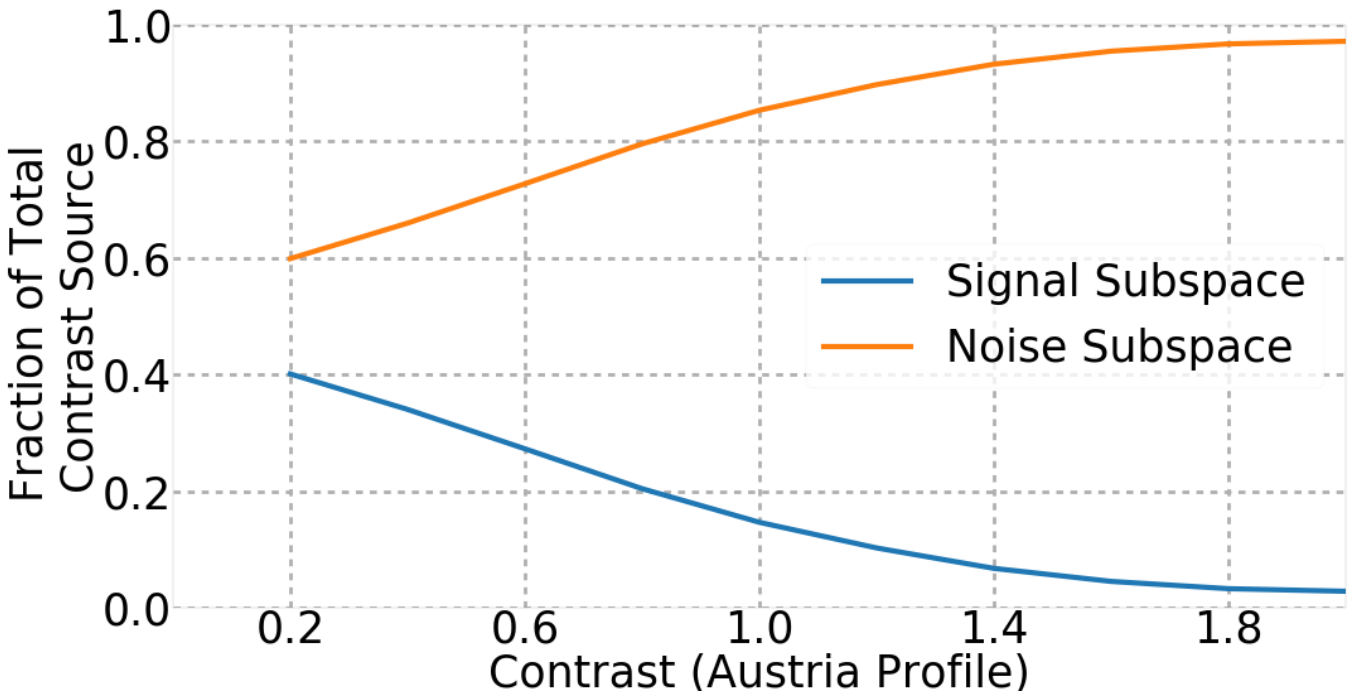
What Might Have Gone Wrong?

What Might Have Gone Wrong?



Fraction of energy in noise and signal subspace with varying contrasts

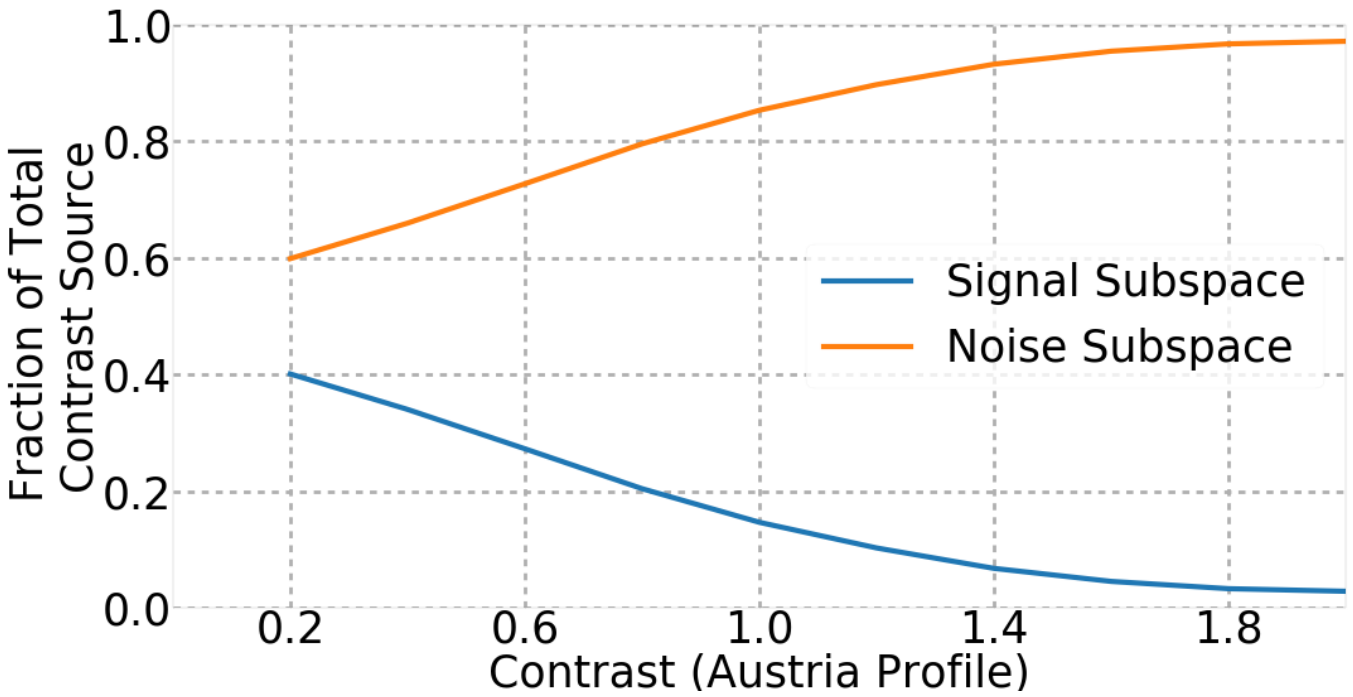
What Might Have Gone Wrong?



Fraction of energy in noise and signal subspace with varying contrasts

- As contrast increases, a lot of energy shifts to noise-subspace.

What Might Have Gone Wrong?

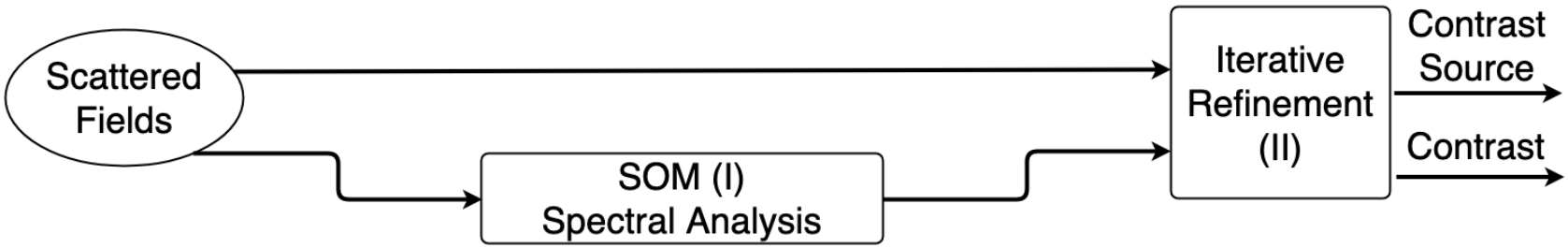


Fraction of energy in noise and signal subspace with varying contrasts

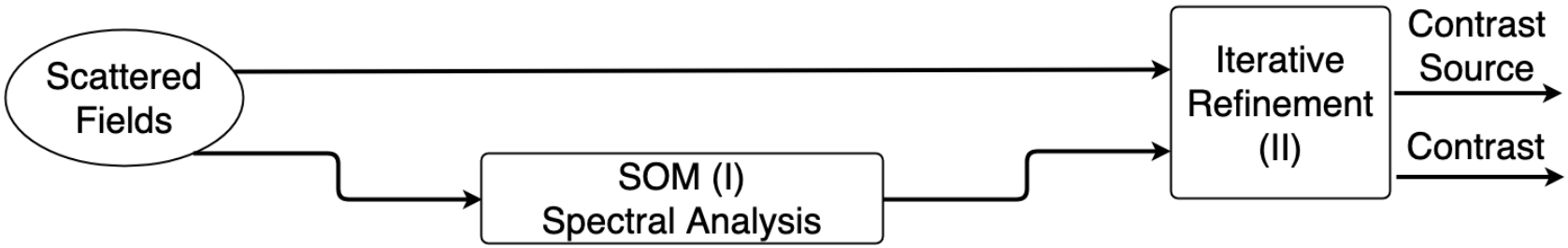
- As contrast increases, a lot of energy shifts to noise-subspace.
- Nullspace Initialization to origin in conventional SOM

Proposed Method

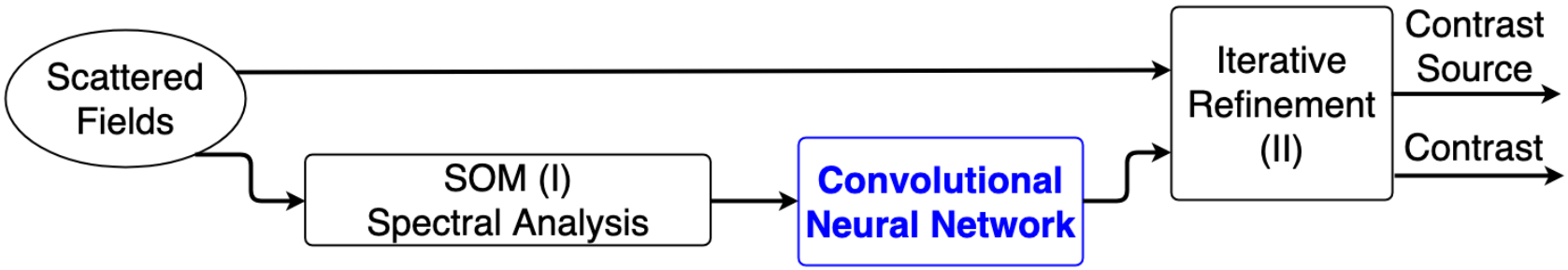
Proposed Method



Proposed Method

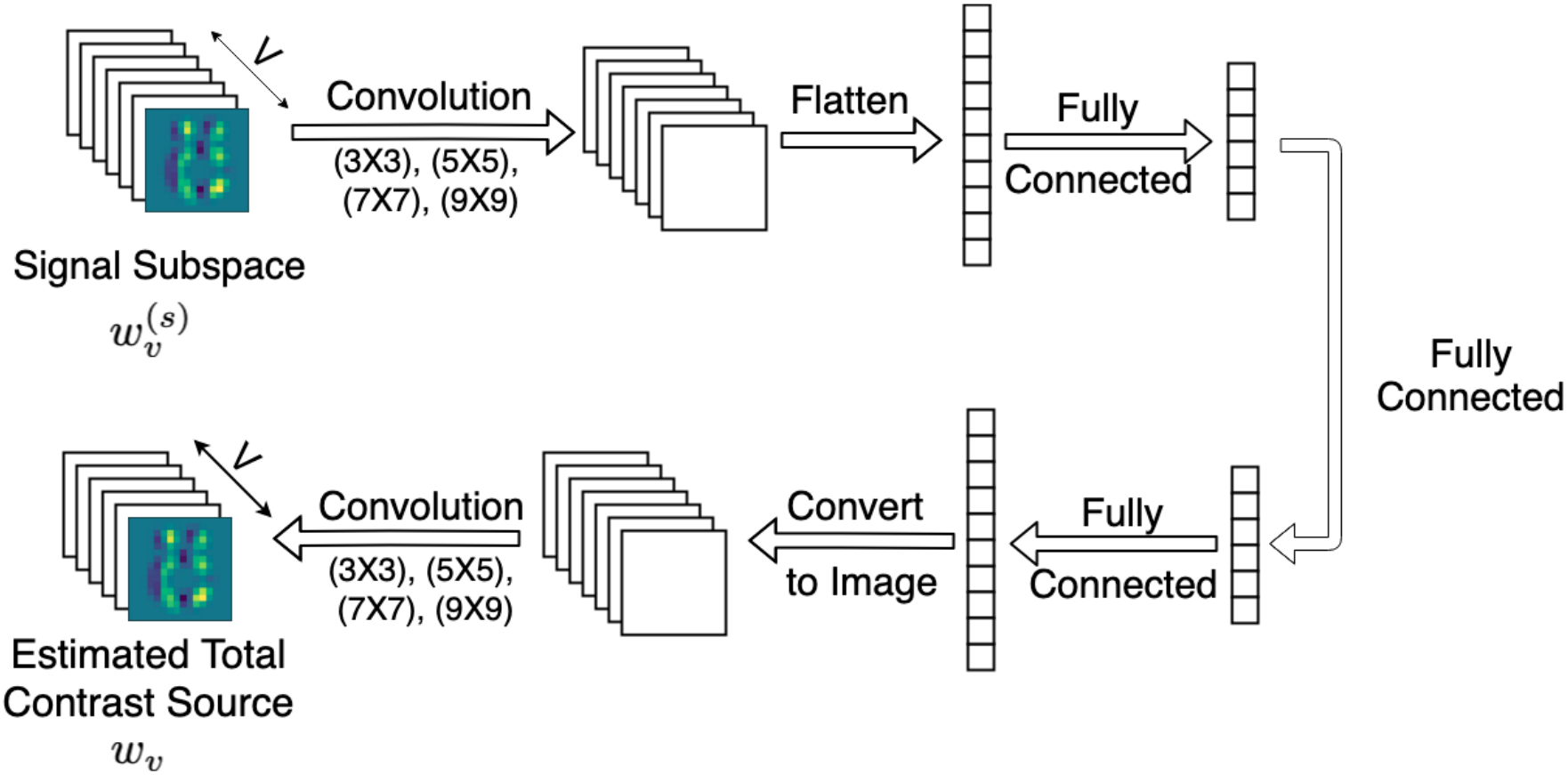


Deep Learning to the Rescue



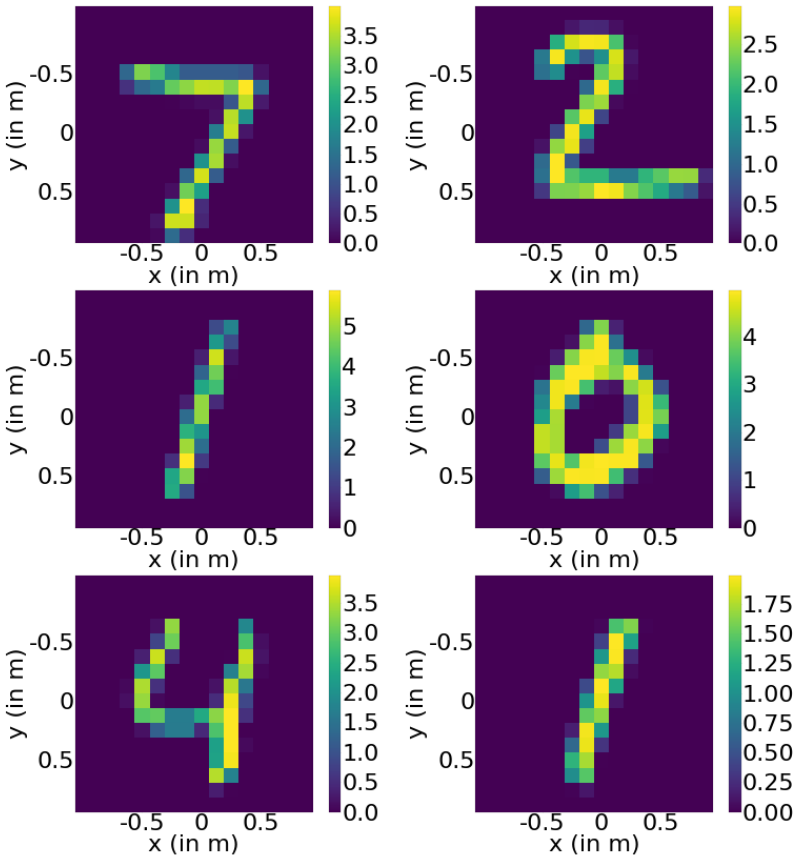
CS-Net: Novel Convolutional Neural Network

CS-Net: Novel Convolutional Neural Network



Training

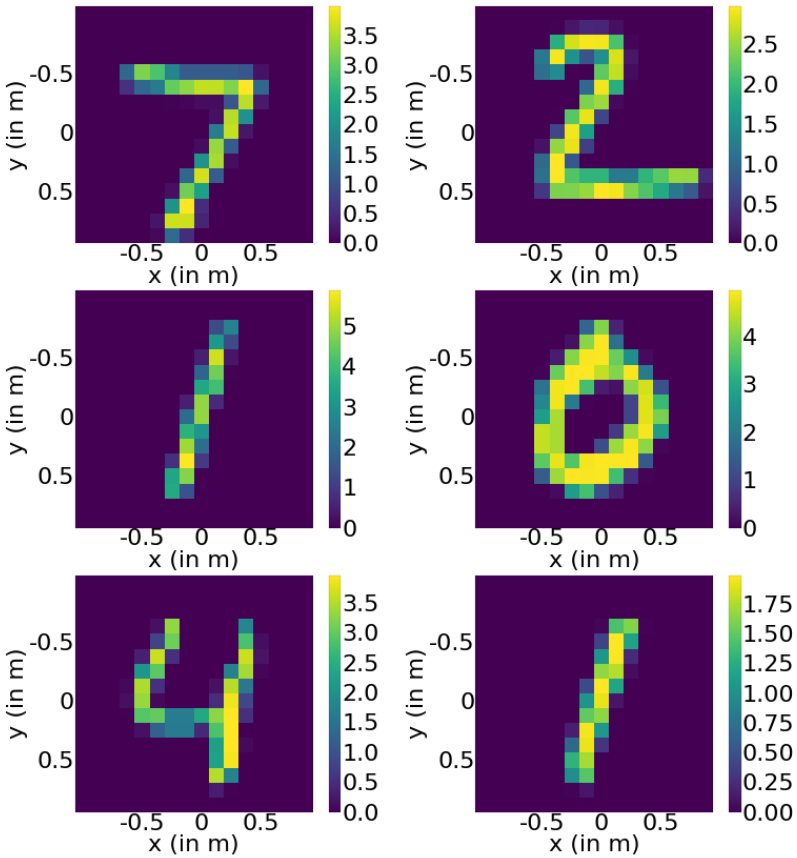
- Using publicly available MNIST dataset [3]



[3] LeCun, Y. & Cortes, C. "MNIST handwritten digit database." , 2010

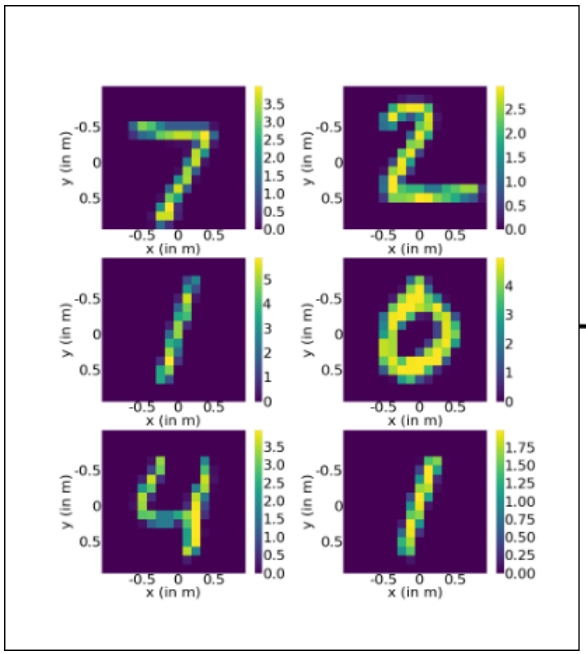
Training

- Using publicly available MNIST dataset [3]
- Convert MNIST images to dielectric objects



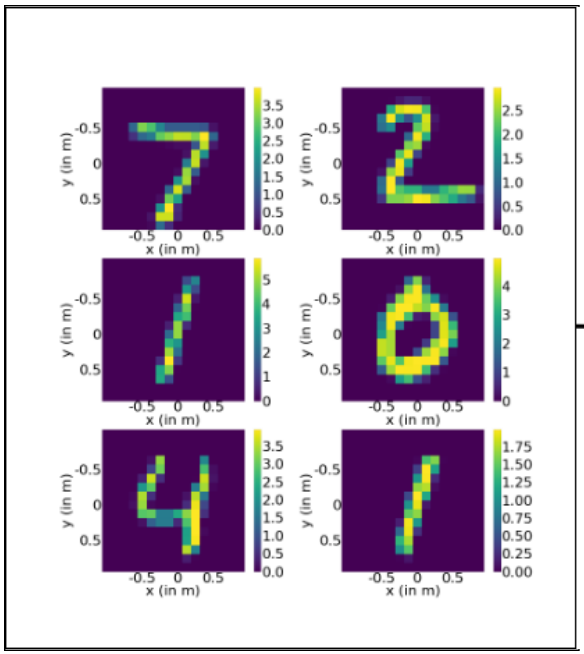
[3] LeCun, Y. & Cortes, C. "MNIST handwritten digit database.", 2010

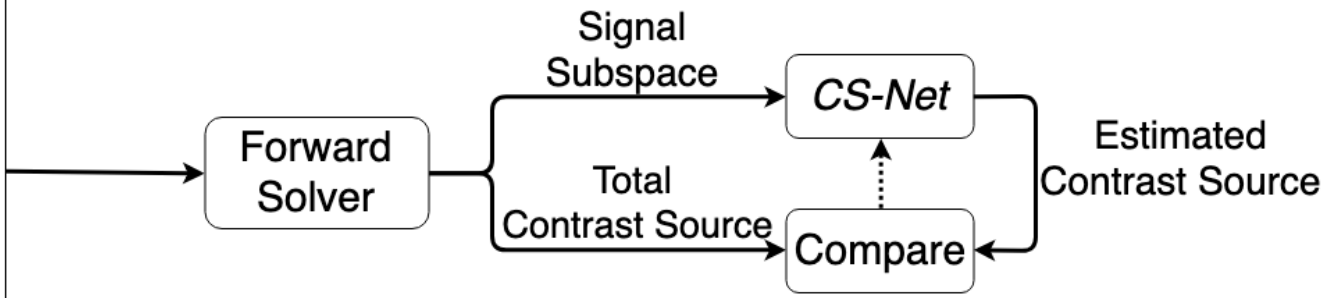
Training - II



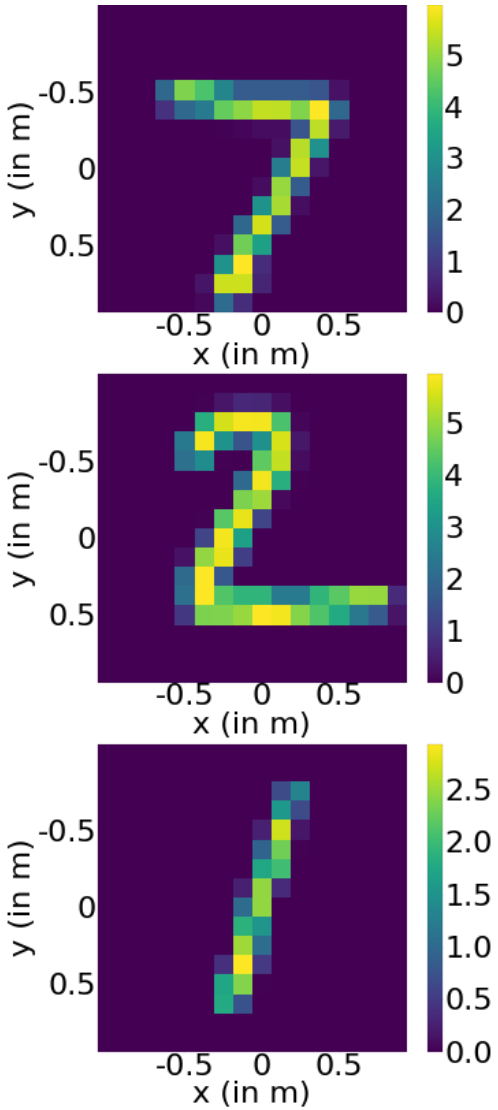
Forward Solver

Training - II


y (in m)
x (in m)

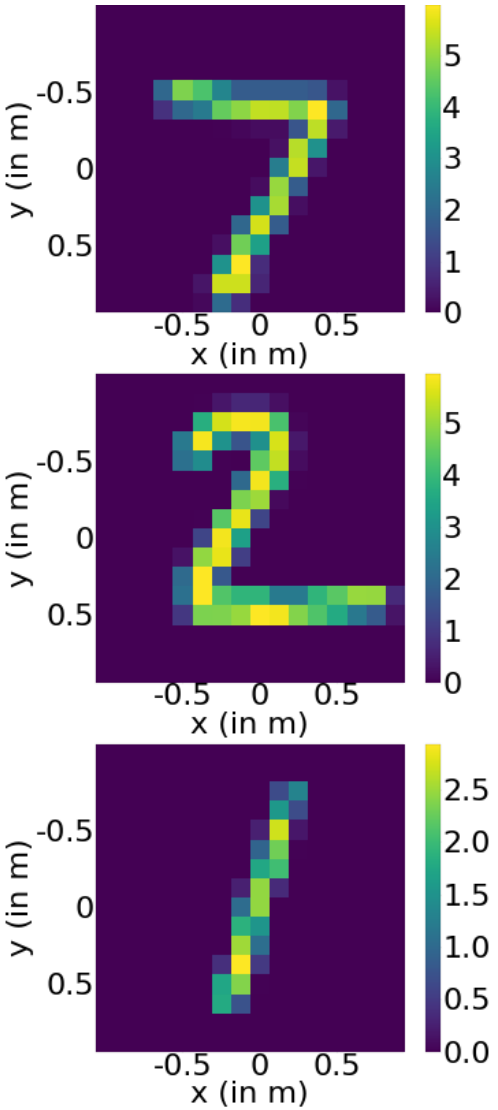

Forward Solver
Signal Subspace
Total Contrast Source
CS-Net
Compare
Estimated Contrast Source

CS-Net: After Training

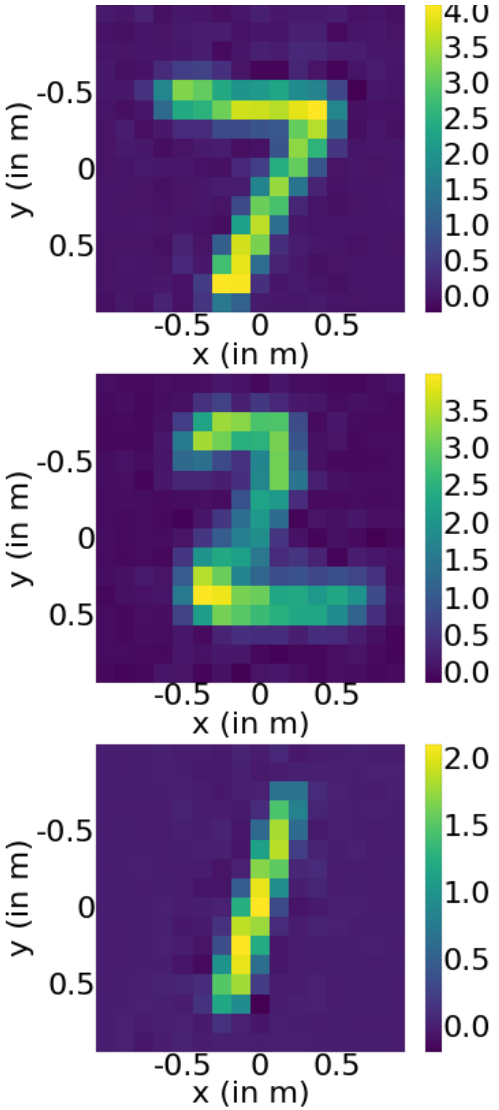


True Contrasts

CS-Net: After Training



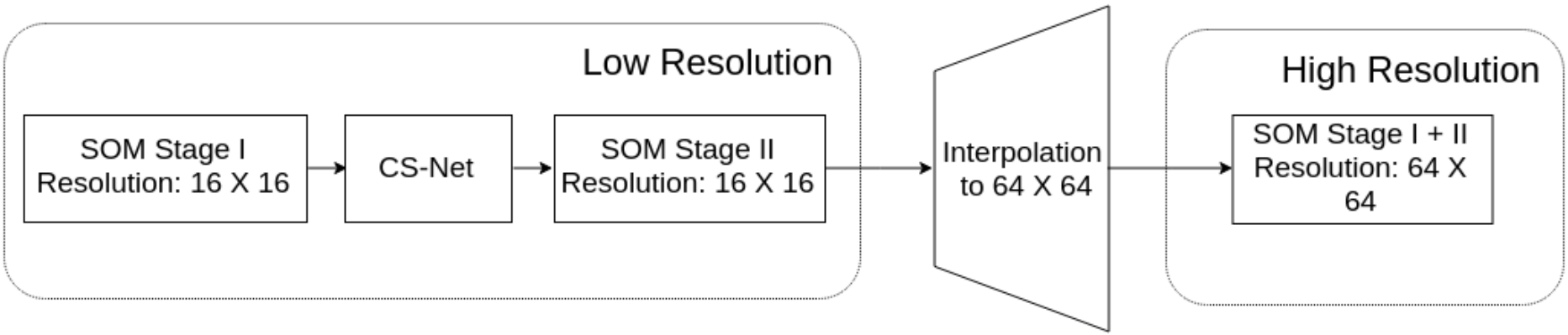
True Contrasts



Retrieved Contrasts

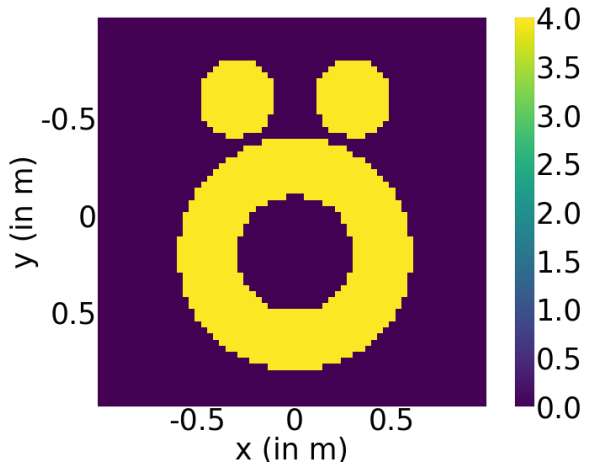
Multi-Resolution Strategy

- Multi-resolution strategy

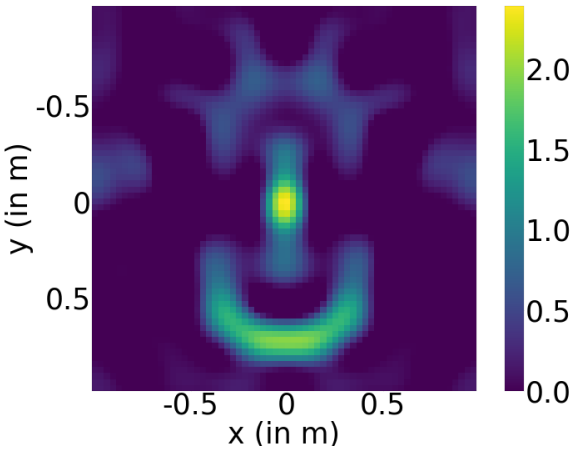


Multi-Resolution Strategy: Different Stages

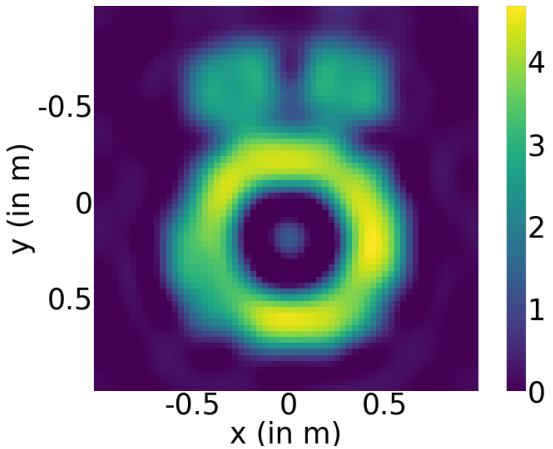
High Contrast Reconstructions



Austria Profile, Contrast = 4

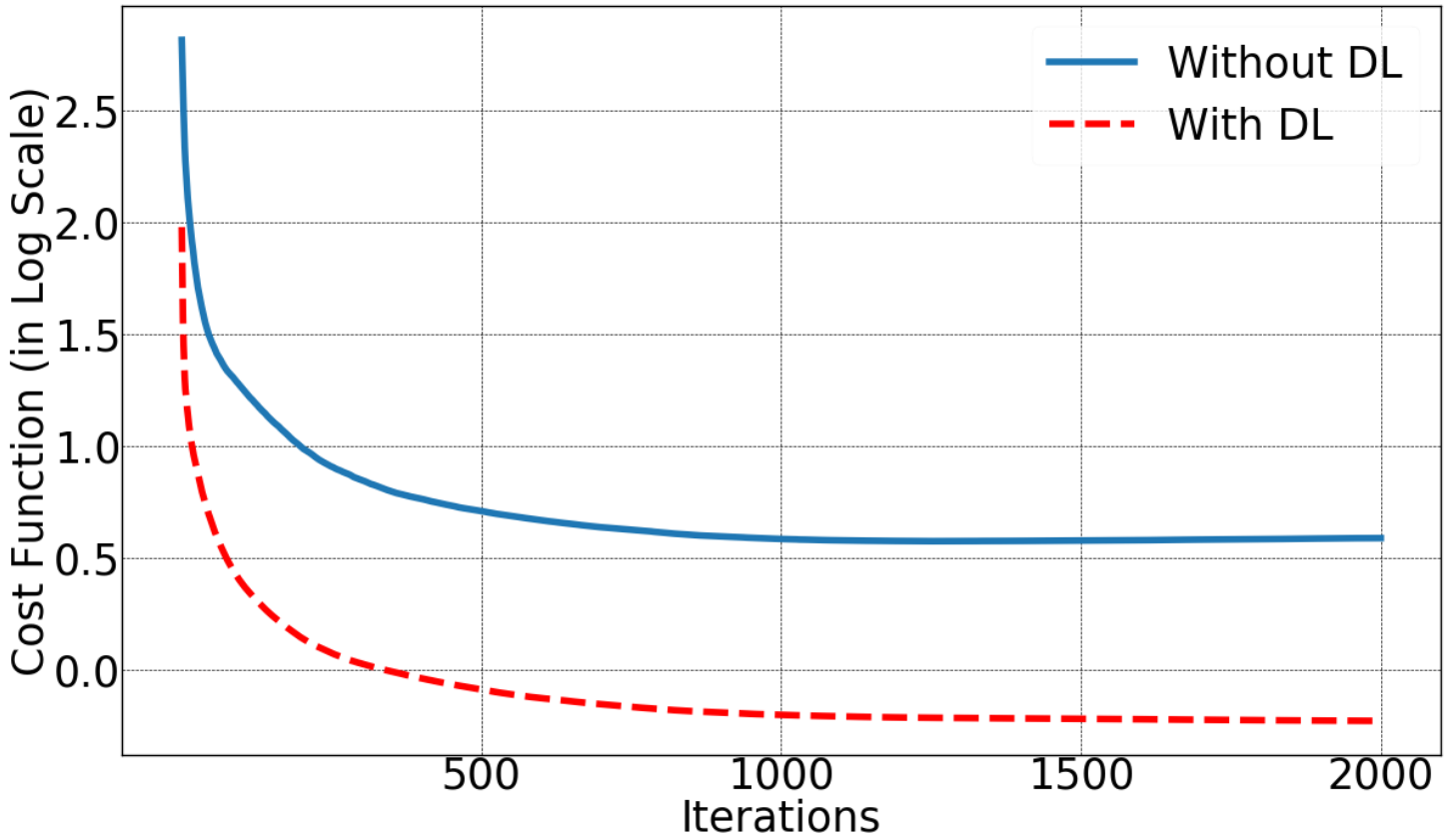


Reconstruction using SOM



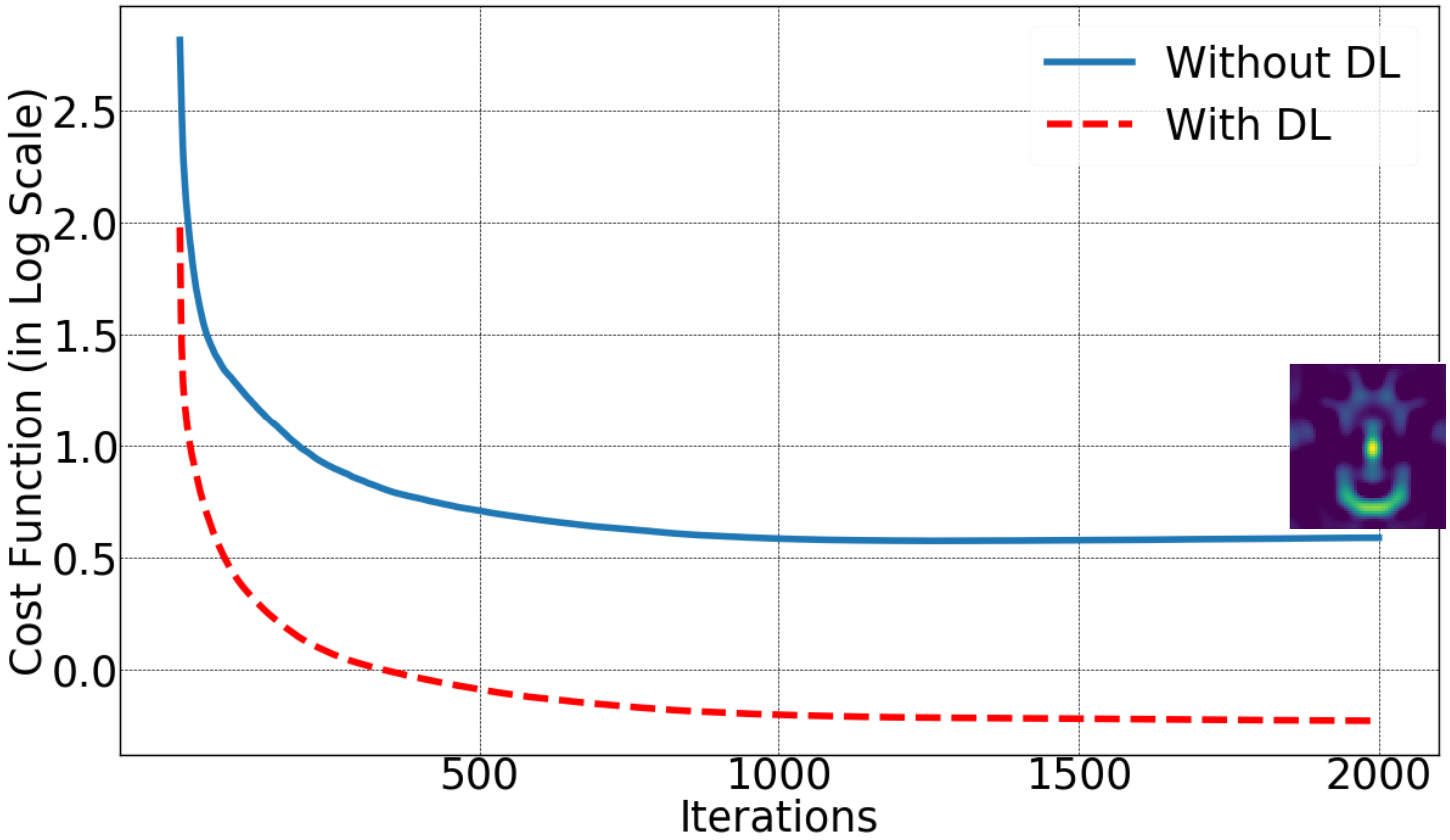
Proposed Method

Convergence with and without *CS-Net*



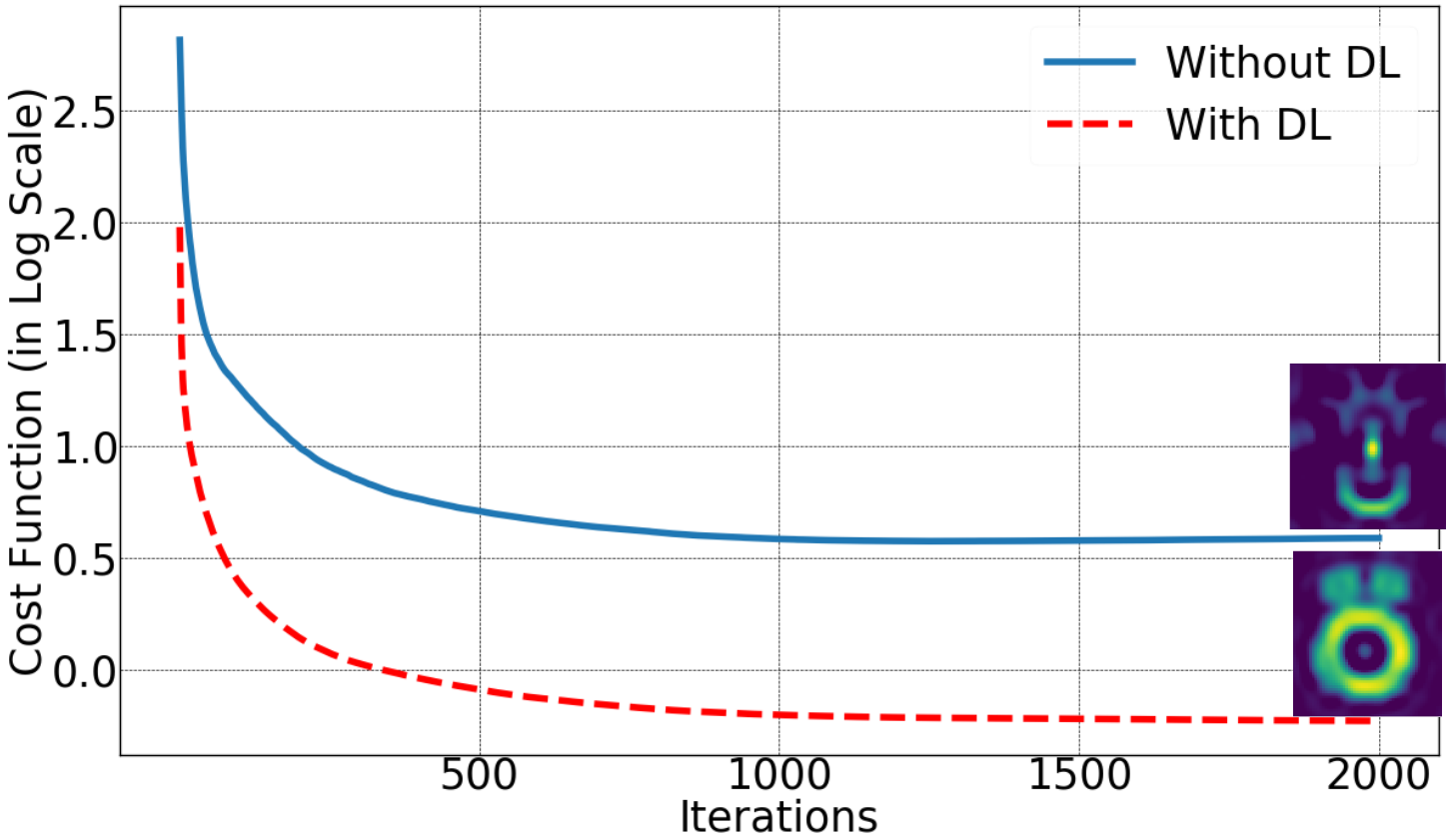
Cost function vs. Iterations for plain SOM and proposed method

Convergence with and without *CS-Net*



Cost function vs. Iterations for plain SOM and proposed method

Convergence with and without *CS-Net*



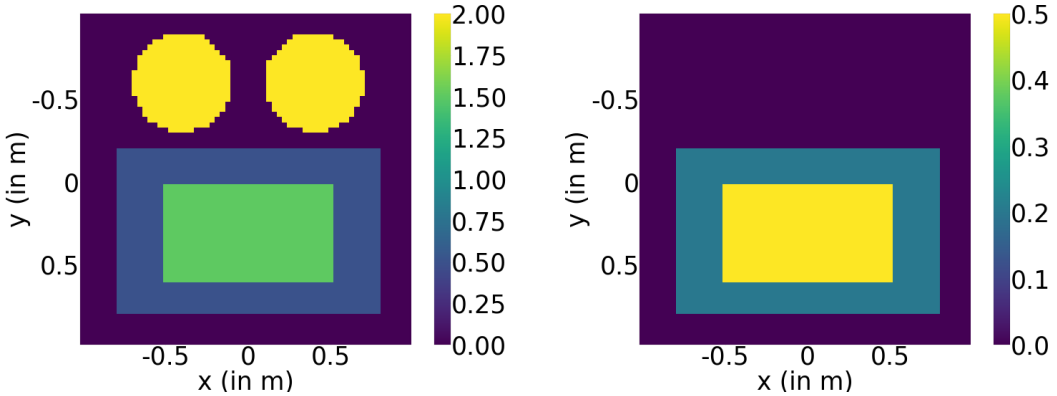
Cost function vs. Iterations for plain SOM and proposed method

Inversion Times

Time (s) $\rightarrow$ Experiment $\downarrow$	Total Time
$x = 1$,DL	199
$x = 1$,no DL	194
$x = 4$,DL	4697
$x = 4$,no DL	2791

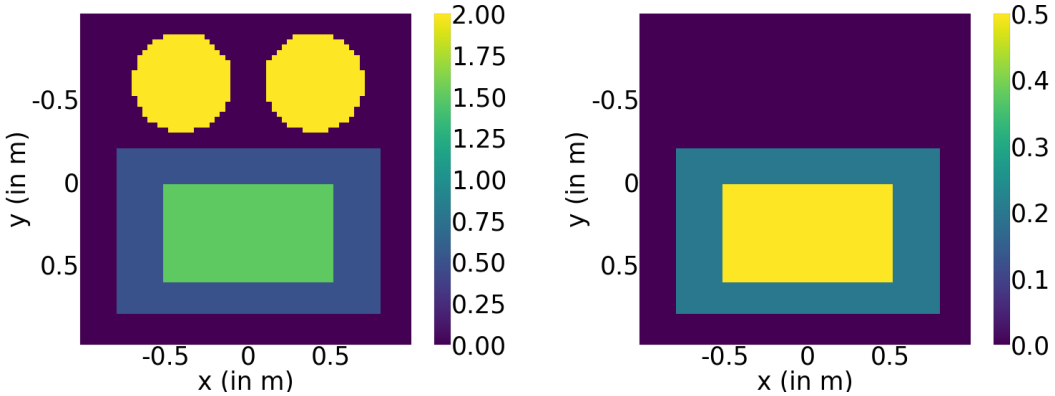
Computational times for different experiments

Lossy Profile

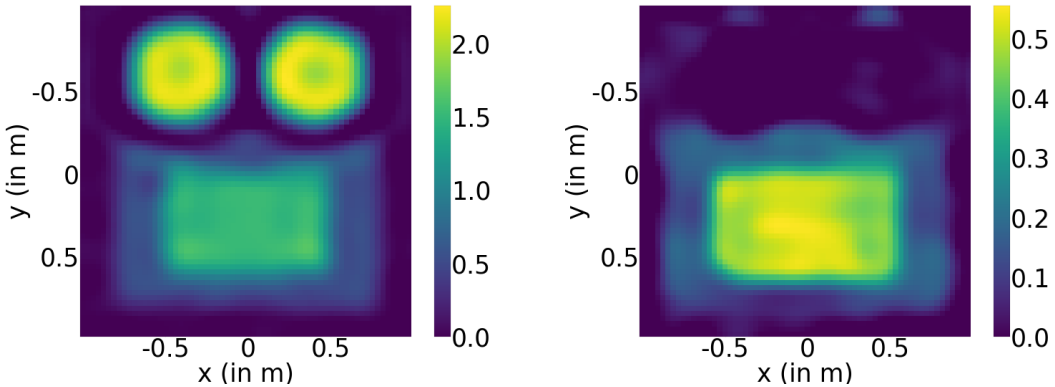


Lossy Profile (Left) Real (Right) Imaginary part of contrast

Lossy Profile

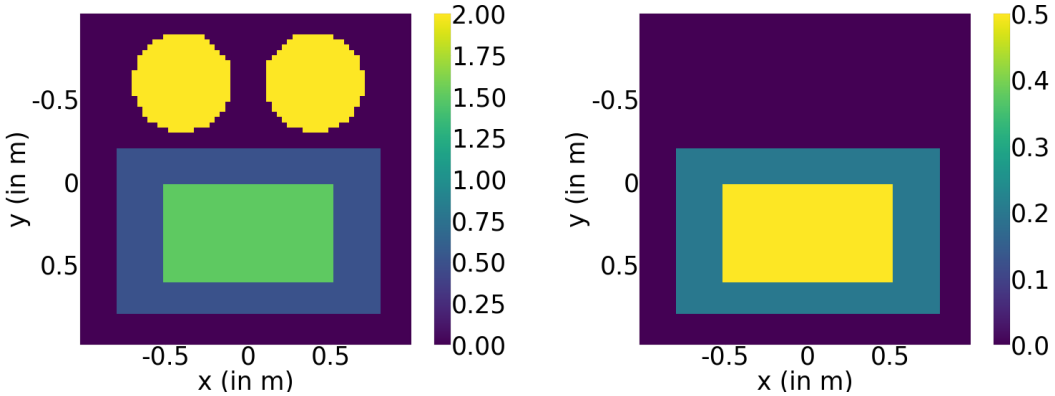


Lossy Profile (Left) Real (Right) Imaginary part of contrast

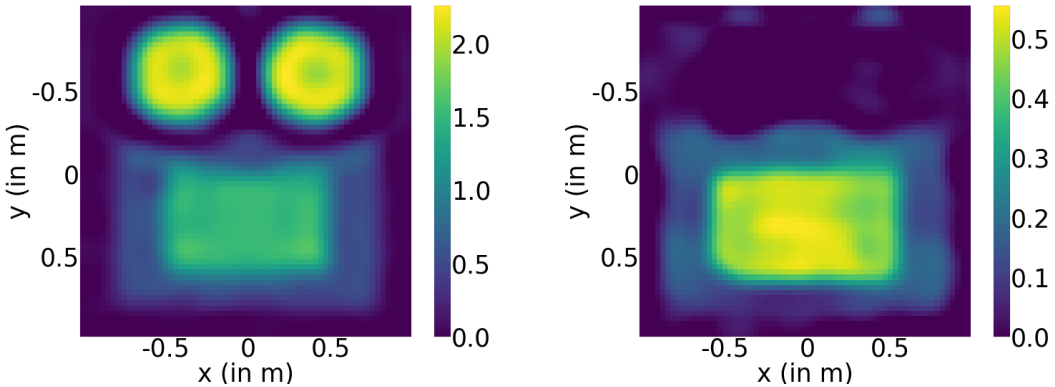


Reconstruction with DL (Left) Real (Right) Imaginary part of contrast

Lossy Profile



Lossy Profile (Left) Real (Right) Imaginary part of contrast



Reconstruction with DL (Left) Real (Right) Imaginary part of contrast

Proposed Method (with CS-Net) : Not a Black Box.

Experimental Data Validation

- Validation using the Fresnel dataset[4]

[4] Geffrin et. al. "Free space experimental scattering database continuation: experimental set-up and measurement precision." 2005

Experimental Data Validation

- Validation using the Fresnel dataset[4]
- Reconstruction using scattered fields at 6GHz

[4] Geffrin et. al. "Free space experimental scattering database continuation: experimental set-up and measurement precision." 2005

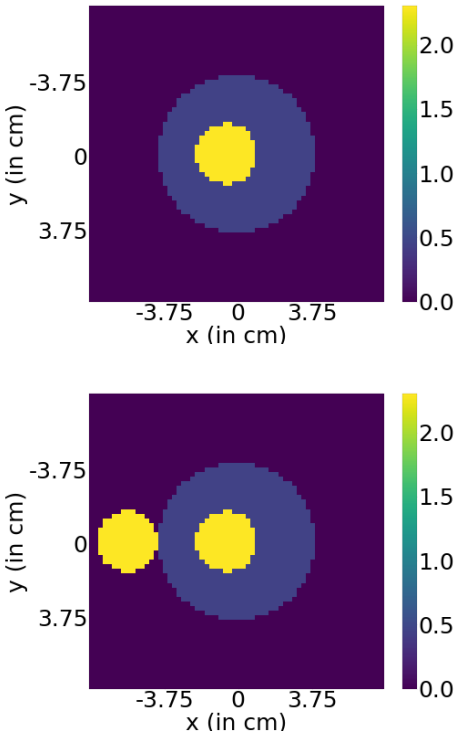
Experimental Data Validation

- Validation using the Fresnel dataset[4]
- Reconstruction using scattered fields at 6GHz
- Very different imaging configuration, retrained entire *CS-Net*

[4] Geffrin et. al. "Free space experimental scattering database continuation: experimental set-up and measurement precision." 2005

Experimental Data Validation

- Validation using the Fresnel dataset[4]
- Reconstruction using scattered fields at 6GHz
- Very different imaging configuration, retrained entire *CS-Net*

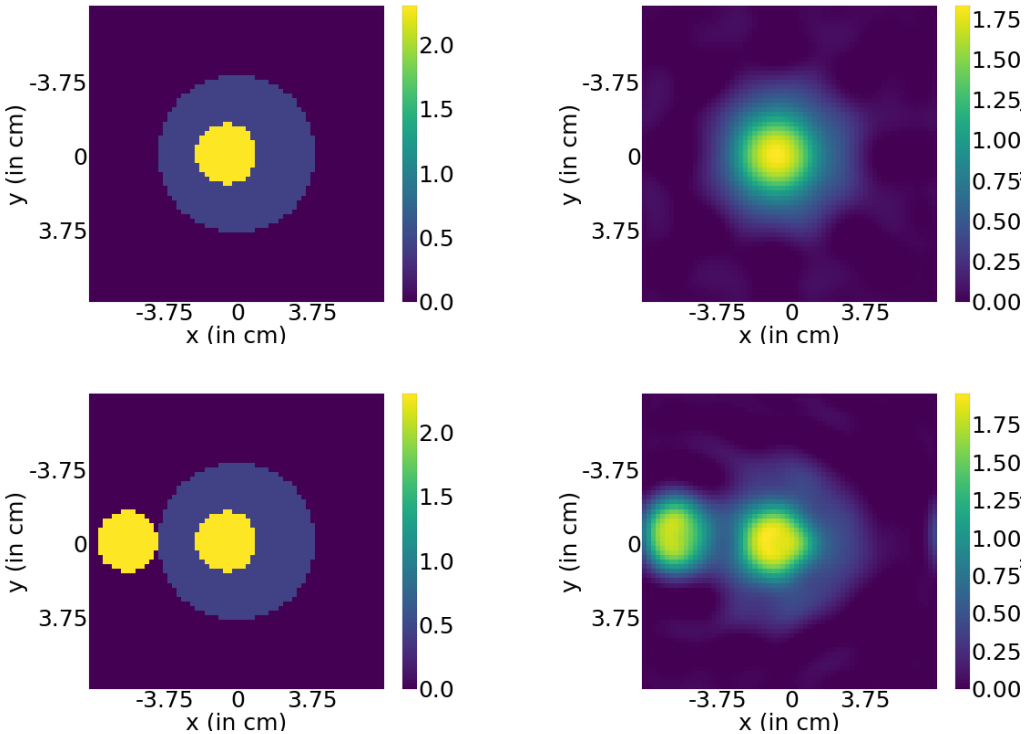


(Left) True Profile

[4] Geffrin et. al. "Free space experimental scattering database continuation: experimental set-up and measurement precision." 2005

Experimental Data Validation

- Validation using the Fresnel dataset[4]
- Reconstruction using scattered fields at 6GHz
- Very different imaging configuration, retrained entire *CS-Net*

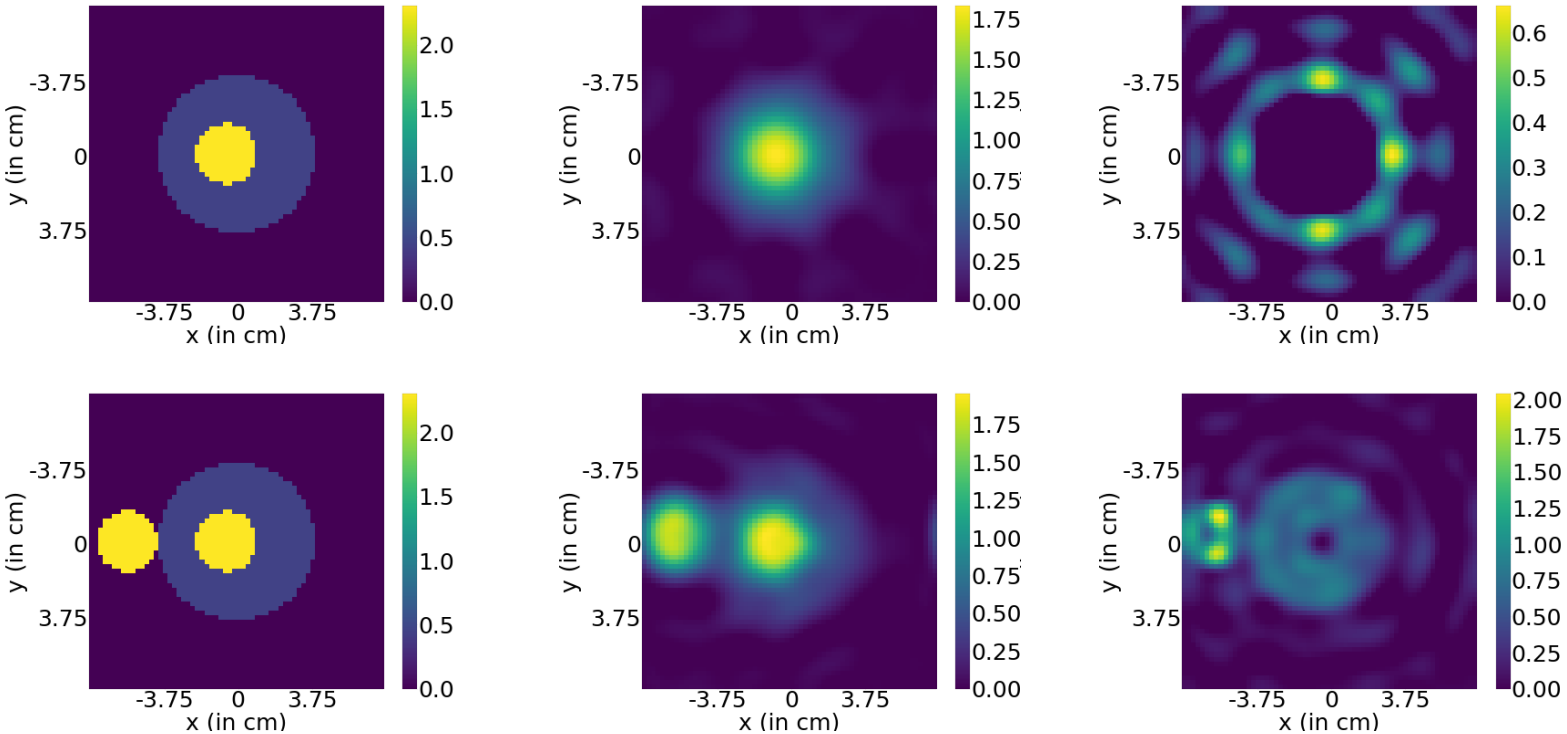


(Left) True Profile

[4] Geffrin et. al. "Free space experimental scattering database continuation: experimental set-up and measurement precision." 2005

Experimental Data Validation

- Validation using the Fresnel dataset[4]
- Reconstruction using scattered fields at 6GHz
- Very different imaging configuration, retrained entire *CS-Net*

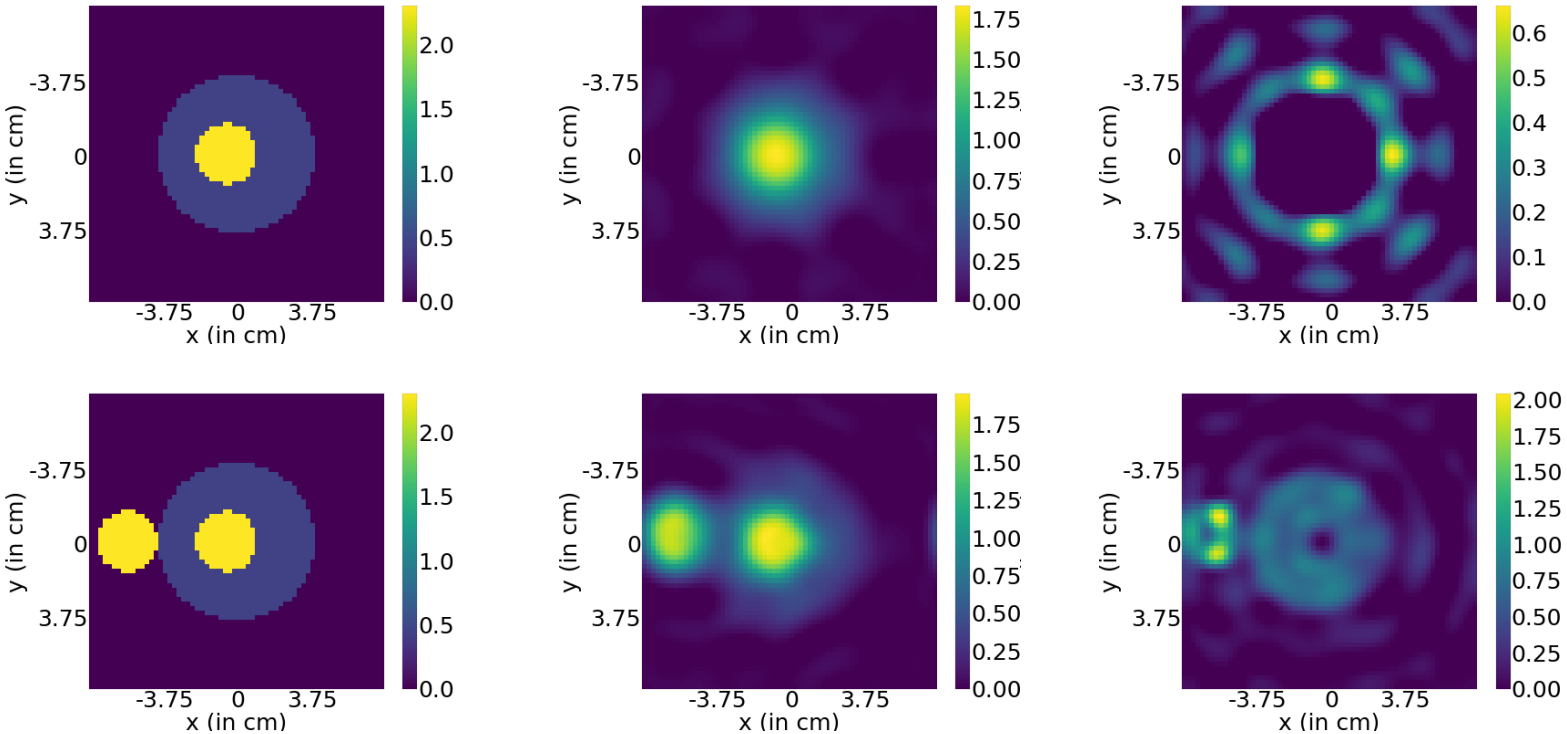


(Left) True Profile (Middle) Using DL

[4] Geffrin et. al. "Free space experimental scattering database continuation: experimental set-up and measurement precision." 2005

Experimental Data Validation

- Validation using the Fresnel dataset[4]
- Reconstruction using scattered fields at 6GHz
- Very different imaging configuration, retrained entire *CS-Net*



(Left) True Profile (Middle) Using DL (Right) Without DL

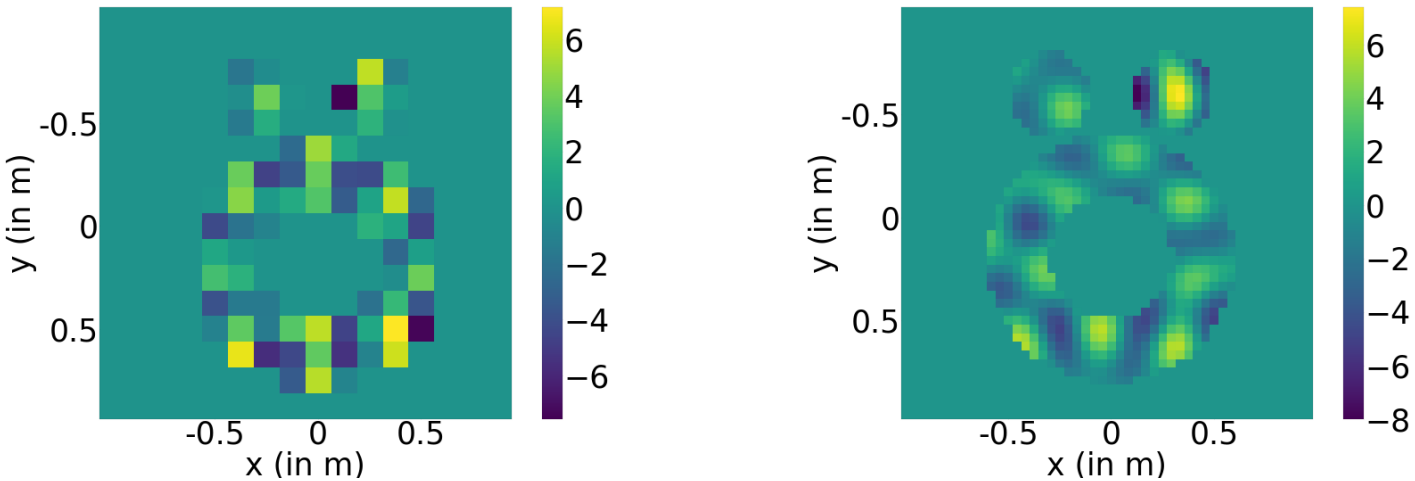
[4] Geffrin et. al. "Free space experimental scattering database continuation: experimental set-up and measurement precision." 2005

Conclusion and Future Work

- *CS-Net* for higher contrasts

Conclusion and Future Work

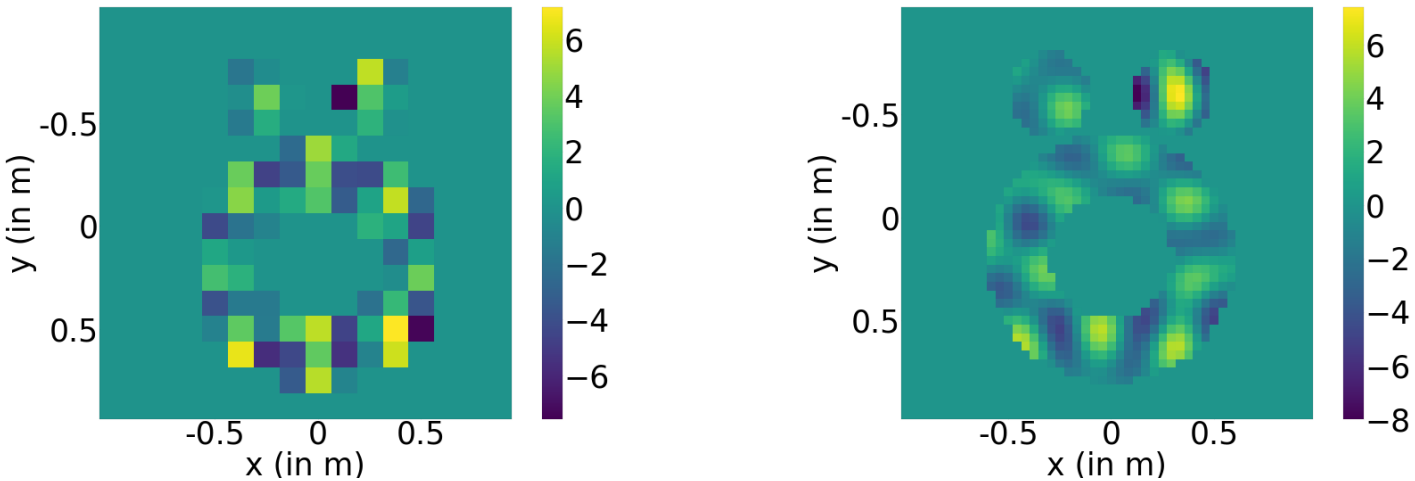
- *CS-Net* for higher contrasts
- Higher resolution input for *CS-Net*



Contrast Source for Resolution (Left) 16×16 (Right) 48×48

Conclusion and Future Work

- *CS-Net* for higher contrasts
- Higher resolution input for *CS-Net*

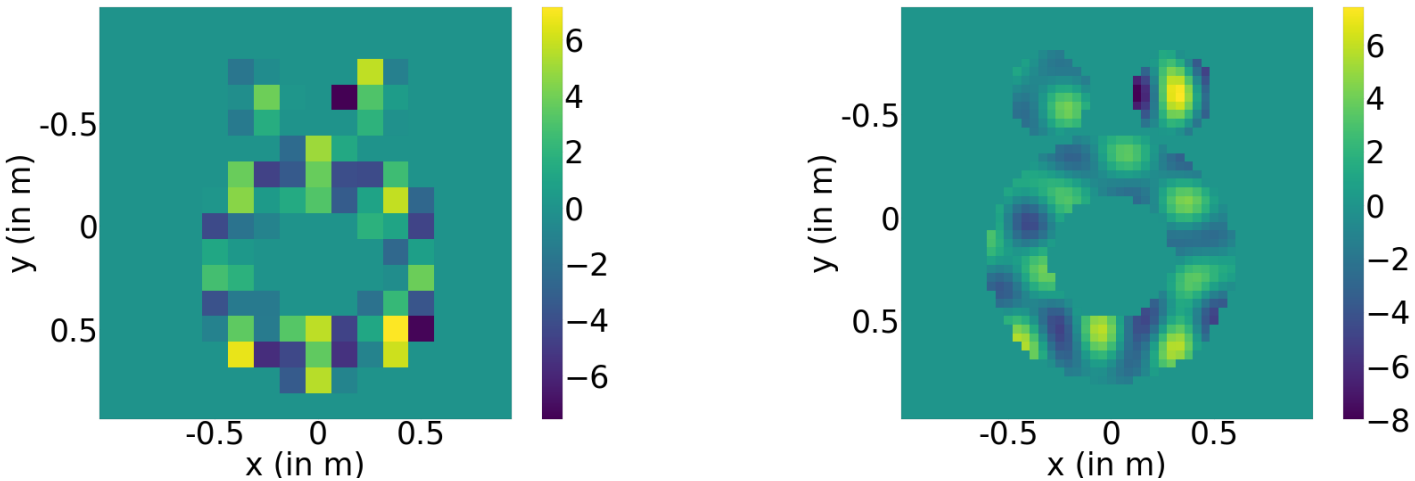


Contrast Source for Resolution (Left) 16×16 (Right) 48×48

- Less data-intensive variations, applications for 3D inverse scattering.

Conclusion and Future Work

- *CS-Net* for higher contrasts
- Higher resolution input for *CS-Net*



Contrast Source for Resolution (Left) 16×16 (Right) 48×48

- Less data-intensive variations, applications for 3D inverse scattering.
- Presented work under revision as "Y. Sanghvi, Y. Kalepu, U.Khankhoje, *Embedding Deep Learning in Inverse Scattering Problems*, 2019".