In [1]:

```
#importing all the libraries

import warnings
warnings.filterwarnings("ignore")
from sklearn.datasets import load_boston
from random import seed
from random import randrange
from csv import reader
from math import sqrt
from sklearn import preprocessing
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from prettytable import PrettyTable
from sklearn.linear_model import SGDRegressor
from sklearn import preprocessing
from sklearn.metrics import mean_squared_error
```

Loading the data

In [2]:
```
data = load_boston()
#data  description
print(data.DESCR)
```

```
Boston House Prices dataset
===========================

Notes
------
Data Set Characteristics:

    :Number of Instances: 506

    :Number of Attributes: 13 numeric/categorical predictive

    :Median Value (attribute 14) is usually the target

    :Attribute Information (in order):
        - CRIM     per capita crime rate by town
        - ZN       proportion of residential land zoned for lots over 25,000 sq.ft.
        - INDUS    proportion of non-retail business acres per town
        - CHAS     Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
        - NOX      nitric oxides concentration (parts per 10 million)
        - RM       average number of rooms per dwelling
        - AGE      proportion of owner-occupied units built prior to 1940
        - DIS      weighted distances to five Boston employment centres
        - RAD      index of accessibility to radial highways
        - TAX      full-value property-tax rate per $10,000
        - PTRATIO  pupil-teacher ratio by town
        - B        1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town
        - LSTAT    % lower status of the population
        - MEDV     Median value of owner-occupied homes in $1000's

    :Missing Attribute Values: None

    :Creator: Harrison, D. and Rubinfeld, D.L.

This is a copy of UCI ML housing dataset.
http://archive.ics.uci.edu/ml/datasets/Housing (http://archive.ics.uci.edu/ml/datasets/Housing)


This dataset was taken from the StatLib library which is maintained at Carnegie Mellon University.
```

```
The Boston house-price data of Harrison, D. and Rubinfeld, D.L. 'Hedonic
prices and the demand for clean air', J. Environ. Economics & Management,
vol.5, 81-102, 1978.   Used in Belsley, Kuh & Welsch, 'Regression diagnostics
...', Wiley, 1980.   N.B. Various transformations are used in the table on
pages 244-261 of the latter.

The Boston house-price data has been used in many machine learning papers that address regression
problems.

**References**

   - Belsley, Kuh & Welsch, 'Regression diagnostics: Identifying Influential Data and Sources of Collinearit
y', Wiley, 1980. 244-261.
   - Quinlan,R. (1993). Combining Instance-Based and Model-Based Learning. In Proceedings on the Tenth Inter
national Conference of Machine Learning, 236-243, University of Massachusetts, Amherst. Morgan Kaufmann.
   - many more! (see http://archive.ics.uci.edu/ml/datasets/Housing) (http://archive.ics.uci.edu/ml/dataset
s/Housing))
```

In [3]:

```python
data.feature_names#all the features in the dataset
```

Out[3]:

```
array(['CRIM', 'ZN', 'INDUS', 'CHAS', 'NOX', 'RM', 'AGE', 'DIS', 'RAD',
       'TAX', 'PTRATIO', 'B', 'LSTAT'],
      dtype='<U7')
```

In [4]:

```python
print('Shape of dataset is',data.data.shape)
print('shape of target variable is',data.target.shape)
```

```
Shape of dataset is (506, 13)
shape of target variable is (506,)
```

In [5]:
```
boston_df = pd.DataFrame(data.data,columns = data.feature_names)
boston_df.head()
```

Out[5]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33

In [6]:
```
boston_df.describe()
```

Out[6]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRAT
count	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.0000
mean	3.593761	11.363636	11.136779	0.069170	0.554695	6.284634	68.574901	3.795043	9.549407	408.237154	18.4555
std	8.596783	23.322453	6.860353	0.253994	0.115878	0.702617	28.148861	2.105710	8.707259	168.537116	2.1649
min	0.006320	0.000000	0.460000	0.000000	0.385000	3.561000	2.900000	1.129600	1.000000	187.000000	12.6000
25%	0.082045	0.000000	5.190000	0.000000	0.449000	5.885500	45.025000	2.100175	4.000000	279.000000	17.4000
50%	0.256510	0.000000	9.690000	0.000000	0.538000	6.208500	77.500000	3.207450	5.000000	330.000000	19.0500
75%	3.647423	12.500000	18.100000	0.000000	0.624000	6.623500	94.075000	5.188425	24.000000	666.000000	20.2000
max	88.976200	100.000000	27.740000	1.000000	0.871000	8.780000	100.000000	12.126500	24.000000	711.000000	22.0000

Splitting the data

In [7]:
```
X = data.data
Y = data.target
```

In [8]:

```
#performing random splitting as data is not temporal in nature

from sklearn.model_selection import train_test_split
X_train,X_test,Y_train,Y_test = train_test_split(X,Y,test_size=0.2,random_state=42)
#X_train,X_val,Y_train,Y_val = train_test_split(X_train,Y_train,test_size = 0.1,random_state = 42)
print('after splitting :')
print('Size of training data',X_train.shape)
print('Size of test data',X_test.shape)
```

```
after splitting :
Size of training data (404, 13)
Size of test data (102, 13)
```

Standardizing the data

In [9]:

```
from sklearn.preprocessing import StandardScaler

std = StandardScaler().fit(X_train)
X_train = std.transform(X_train)
X_test = std.transform(X_test)

print(X_train.shape)
print(X_test.shape)
```

```
(404, 13)
(102, 13)
```

1.1 Implementing SKlearn SGDRegressor

In [10]:
```
from sklearn.linear_model import SGDRegressor

lr_model = SGDRegressor(loss = 'squared_loss',n_iter = 5000)
lr_model.fit(X_train,Y_train)

lr_model
```

Out[10]:
```
SGDRegressor(alpha=0.0001, average=False, epsilon=0.1, eta0=0.01,
       fit_intercept=True, l1_ratio=0.15, learning_rate='invscaling',
       loss='squared_loss', n_iter=5000, penalty='l2', power_t=0.25,
       random_state=None, shuffle=True, verbose=0, warm_start=False)
```

In [11]:
```
#predicting on test data
Y_hat_skl = lr_model.predict(X_test)
print('Coefficients from this implementation are\n',lr_model.coef_)
print('=========================================\n')
print('optimal intercept is:',lr_model.intercept_)

mse_sklearn = mean_squared_error(Y_test,Y_hat_skl)
print('========================================\n')
print('mean squared error from sklearn implementation is :',mse_sklearn)
```

```
Coefficients from this implementation are
 [-0.99701126  0.69511695  0.28151959  0.71766243 -2.0261877   3.14102981
 -0.17216989 -3.08507052  2.24880257 -1.76001057 -2.0393385   1.13617232
 -3.61922561]
=========================================

optimal intercept is: [ 22.79635486]
========================================

mean squared error from sklearn implementation is : 24.3033972171
```

1.2 Implementing SGD Manually (from Scratch):

- While implementing SGD we will be doing two implementation trying with different learning rates.
  - learning rate changing with each epoch
  - learning rate constant with each epoch

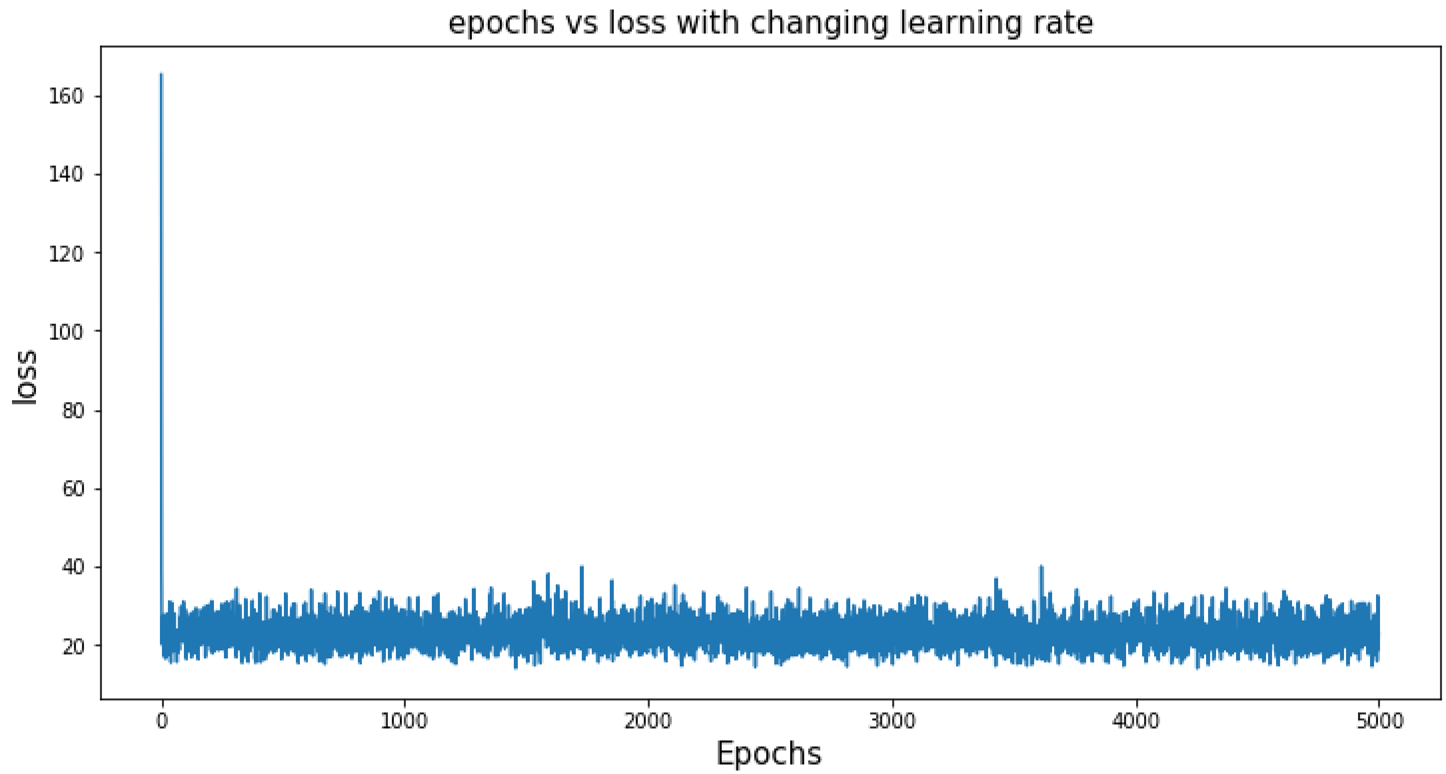
1.2.1 Changing learning rate with each epoch

In [12]:
```
Weights = np.random.randn(X_train.shape[1],1)#initializing the weights
Coeff = np.random.randn(1,1)#initializing the coefficient values

n_epochs = 5000#number of times we want all of the data to be given as input
m = X_train.shape[0]#total number of data points in the training data
ep_loss = [] #list for loss at the end of each epoch
lr = 1 #learning rate

for ep in range(1,n_epochs+1):
    loss = 0 #the mean squred error


    for i in range(m):
        #here we are looping over the total number of data points once,for completion of one epoch

        b = np.random.randint(0,m)#the batch size ,as we want one random data point per iteration

        #reshaping the one training sample and corresponding y
        X = X_train[b,:].reshape(1,X_train.shape[1])
        Y = Y_train[b].reshape(1,1)


        Y_pred = np.dot(X,Weights) + Coeff #finding the predicted values on training data
        loss = loss + (Y_pred - Y)**2 #computing the sum of mean squared errors in every iteration


        Weights = Weights - (2/m)*(lr)*(X.T.dot(Y_pred - Y))#updating the weights
        Coeff = Coeff - (2/m)*(lr)*(Y_pred - Y) #updating the bias term

    lr = lr/2 # learning rate gets reduced to half with every epoch


    ep_loss.append(loss[0][0]/m)

```

In [13]:

```
weights_vlr = Weights#value of weight vector at the end of 5000th epoch
Coeff_vlr = Coeff
print('Weights for learning rate changing with every epoch are:',Weights)
print('Constant term for learning rate changing with every epoch is:',Coeff)

%matplotlib inline

#plotting the epochs vs loss
plt.figure(figsize = (12,6))
plt.plot([i for i in range(0,n_epochs)],ep_loss)
plt.xlabel('Epochs',fontsize = 15)
plt.ylabel('loss',fontsize = 15)
plt.title('epochs vs loss with changing learning rate',fontsize = 15)

Y_hat = ((Weights.T).dot(X_test.T) + Coeff ).reshape(102,1)#computing the predicted values on test data
Y_test = np.array(Y_test)
mse_sgd = mean_squared_error(Y_test,Y_hat)#computing the mean squared error
print('mean_squared_error after implementing sgd manually with changing leaarning rate is :',mse_sgd)

```

```
Weights for learning rate changing with every epoch are: [[-0.78454833]
 [ 0.3468268 ]
 [-0.03658473]
 [ 1.04243126]
 [-0.97777929]
 [ 3.50226501]
 [-0.21894241]
 [-1.78534528]
 [ 1.04669725]
 [-1.24923678]
 [-1.58278309]
 [ 1.10452903]
 [-3.10811876]]
Constant term for learning rate changing with every epoch is: [[ 22.42254032]]
mean_squared_error after implementing sgd manually with changing leaarning rate is : 26.6180348135
```



1.2.2 Constant learning rate with each epoch

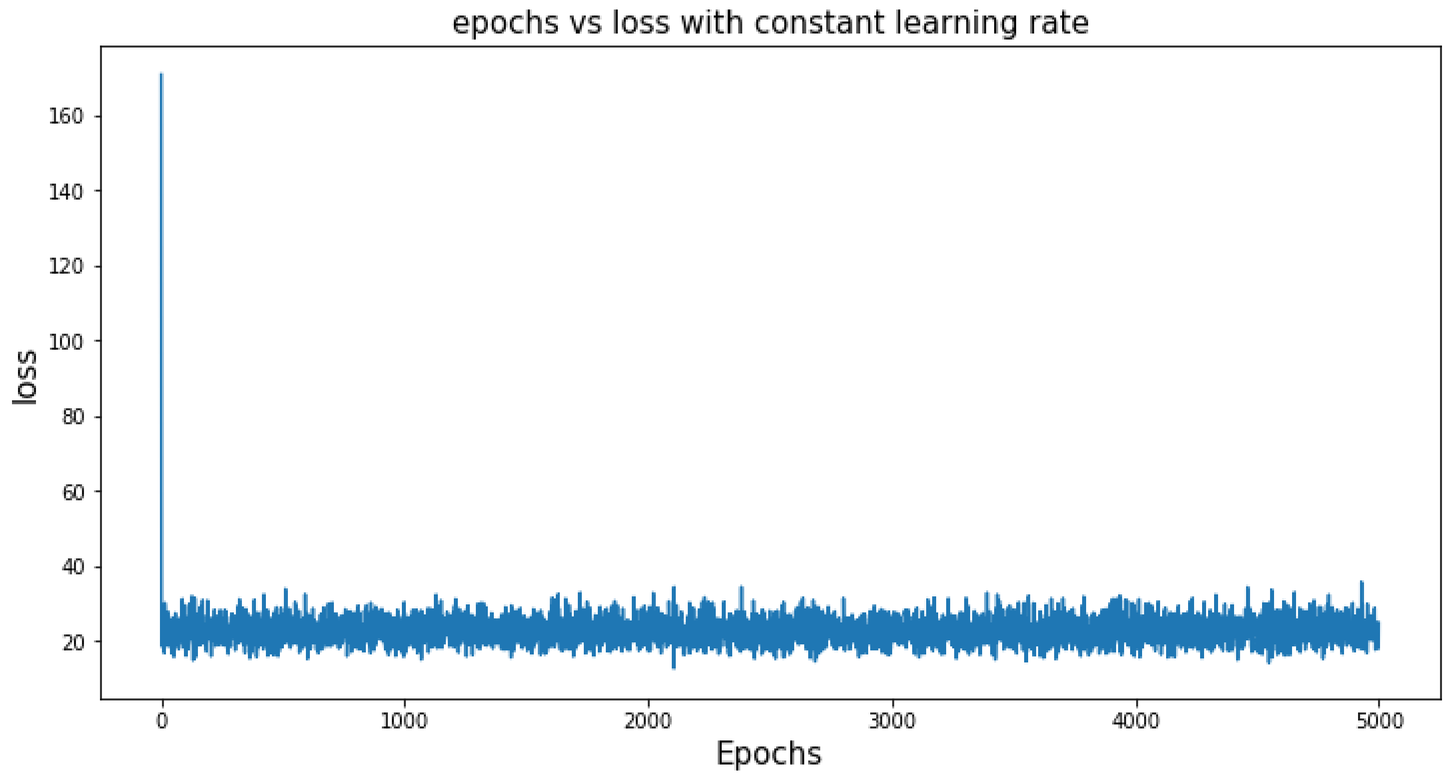
In [14]:

```
Weights = np.random.randn(X_train.shape[1],1)#initializing the weights
Coeff = np.random.randn(1,1)#initializing the coefficient values

n_epochs = 5000#number of times we want all of the data to be given as input
m = X_train.shape[0]#total number of data points in the training data
ep_loss = [] #list for loss at the end of each epoch
lr = 1 #learning rate

for ep in range(1,n_epochs+1):
    loss = 0 #the mean squred error


    for i in range(m):
        #here we are looping over the total number of data points once,for completion of one epoch

        b = np.random.randint(0,m)#the batch size ,as we want one random data point per iteration

        #reshaping the one training sample and corresponding y
        X = X_train[b,:].reshape(1,X_train.shape[1])
        Y = Y_train[b].reshape(1,1)


        Y_pred = np.dot(X,Weights) + Coeff #finding the predicted values on training data
        loss = loss + (Y_pred - Y)**2 #computing the sum of mean squared errors in every iteration


        Weights = Weights - (2/m)*(lr)*(X.T.dot(Y_pred - Y))#updating the weights
        Coeff = Coeff - (2/m)*(lr)*(Y_pred - Y) #updating the bias term

    #lr = lr/2


    ep_loss.append(loss[0][0]/m)

```

In [16]:

```
weights_c = Weights#value of weight vector at the end of 5000th epoch
Coeff_c = Coeff
print('Weights for learning rate changing with every epoch are:',Weights)
print('Constant term for learning rate changing with every epoch is:',Coeff)

%matplotlib inline

#plotting the epochs vs loss
plt.figure(figsize = (12,6))
plt.plot([i for i in range(0,n_epochs)],ep_loss)
plt.xlabel('Epochs',fontsize = 15)
plt.ylabel('loss',fontsize = 15)
plt.title('epochs vs loss with constant learning rate',fontsize = 15)

Y_hat_c = ((Weights.T).dot(X_test.T) + Coeff ).reshape(102,1)#computing the predicted values on test data
Y_test = np.array(Y_test)
mse_sgd_c = mean_squared_error(Y_test,Y_hat_c)#computing the mean squared error
print('mean_squared_error after implementing sgd manually with constant learning rate is :',mse_sgd_c)

```

```
Weights for learning rate changing with every epoch are: [[-0.9872763 ]
 [ 0.85009917]
 [ 0.06636749]
 [ 0.87204787]
 [-2.11683567]
 [ 3.15769378]
 [-0.52586822]
 [-3.37107063]
 [ 2.55775089]
 [-1.71615317]
 [-1.83617658]
 [ 0.72633639]
 [-3.26997752]]
Constant term for learning rate changing with every epoch is: [[ 23.21874236]]
mean_squared_error after implementing sgd manually with constant learning rate is : 22.8566063721
```



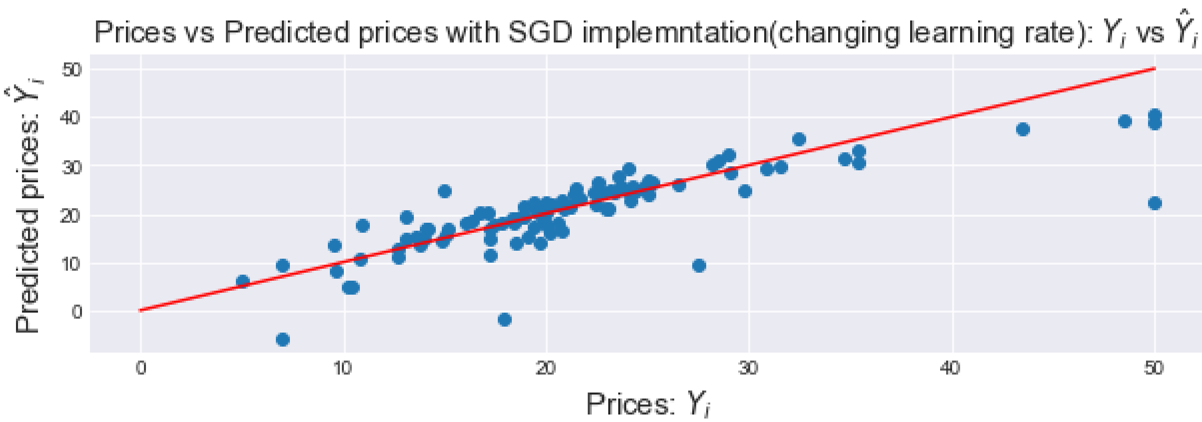
We see that the loss converges very fast between 0 to 10 number of epochs and then it oscillates between a certain minimum and maximum ,as the function is trying to find a global minima for convergence.

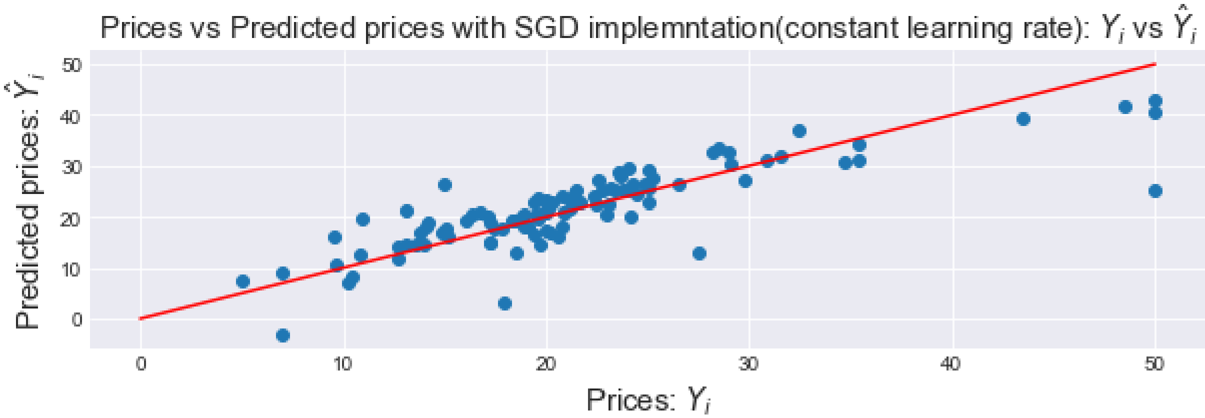
Comparison between Sklearn and Manual Implementation

In [17]:

```
import seaborn as sns


sns.set_style('darkgrid')
plt.figure(figsize = (10,6))
plt.subplot(211)
plt.scatter(Y_test,Y_hat_skl)
plt.plot(np.linspace(0,50,10),np.linspace(0,50,10),'r')
plt.xlabel("Prices: $Y_i$",fontsize = 15)
plt.ylabel("Predicted prices: $\hat{Y}_i$",fontsize = 15)
plt.title("Prices vs Predicted prices with Sklearn implementation: $Y_i$ vs $\hat{Y}_i$",fontsize = 15)
#plt.show()


plt.figure(figsize = (10,6))
plt.subplot(212)
plt.scatter(Y_test,Y_hat)
plt.plot(np.linspace(0,50,10),np.linspace(0,50,10),'r')
plt.xlabel("Prices: $Y_i$",fontsize = 15)
plt.ylabel("Predicted prices: $\hat{Y}_i$",fontsize = 15)
plt.title("Prices vs Predicted prices with SGD implementation(changing learning rate): $Y_i$ vs $\hat{Y}_i$"
plt.show()


plt.figure(figsize = (10,6))
plt.subplot(212)
plt.scatter(Y_test,Y_hat_c)
plt.plot(np.linspace(0,50,10),np.linspace(0,50,10),'r')
plt.xlabel("Prices: $Y_i$",fontsize = 15)
plt.ylabel("Predicted prices: $\hat{Y}_i$",fontsize = 15)
plt.title("Prices vs Predicted prices with SGD implementation(constant learning rate): $Y_i$ vs $\hat{Y}_i$"
plt.show()
```

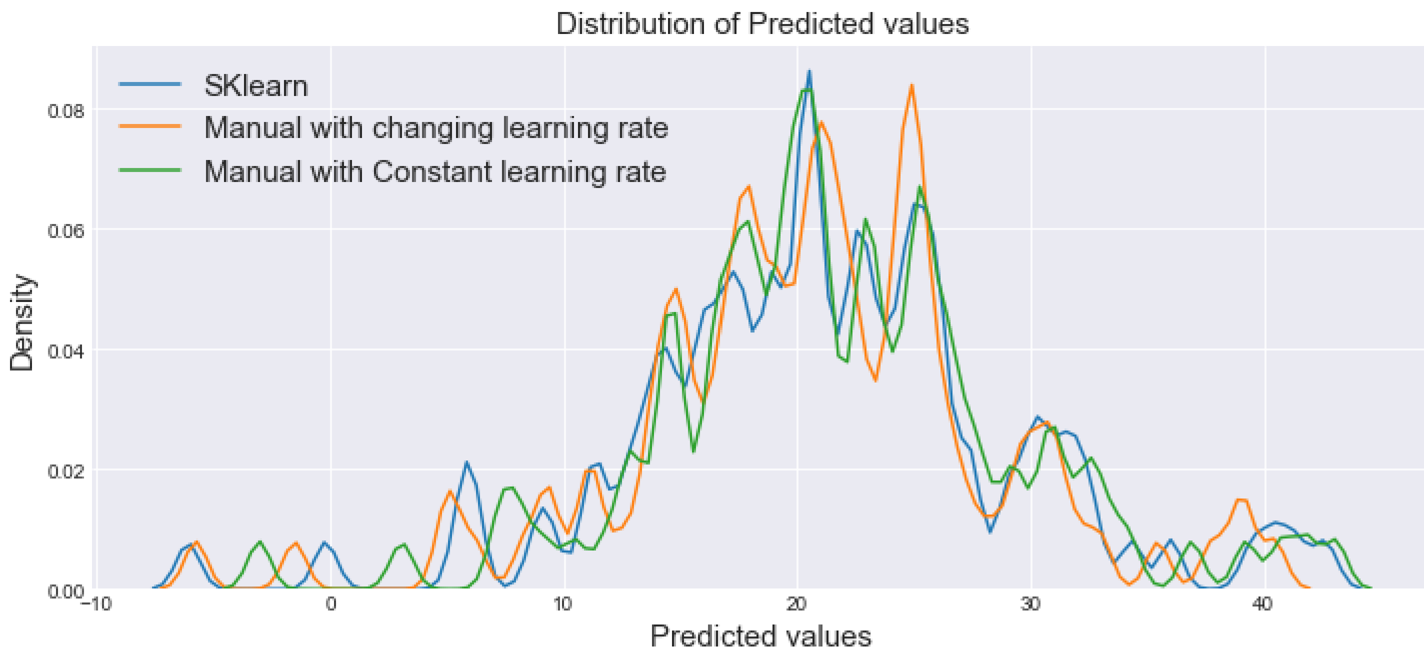

Prices vs Predicted prices with Sklearn: Y_i vs $\hat{Y}_i$
Predicted prices: $\hat{Y}_i$
Prices: Y_i
0
10
20
30
40
50


Prices vs Predicted prices with SGD implemntation(changing learning rate): Y_i vs $\hat{Y}_i$
Predicted prices: $\hat{Y}_i$
Prices: Y_i
0
10
20
30
40
50


Prices vs Predicted prices with SGD implemntation(constant learning rate): Y_i vs $\hat{Y}_i$
Predicted prices: $\hat{Y}_i$
Prices: Y_i
0
10
20
30
40
50
0
10
20
30
40
50

In [54]:
```
"""WE will plot the kernel density estimation plots of error and predicted values"""

#differenc between actual and predicted values for all three cases
delta_y1 = Y_test - Y_hat_skl #sklearn
delta_y2 = np.asmatrix(Y_test) - Y_hat #changing learning rate
delta_y3 = np.asmatrix(Y_test) - Y_hat_c #constant learning rate

sns.set_style('darkgrid')
plt.figure(figsize = (12,5))
sns.kdeplot(np.array(delta_y1), bw=0.5,label = 'SKlearn')
sns.kdeplot(np.asarray(delta_y2)[0], bw=0.5,label = 'Manual with changing learning rate')
sns.kdeplot(np.asarray(delta_y3)[0], bw=0.5,label = 'Manual with Constant learning rate')
plt.title('Distribution of error',fontsize = 15)
plt.xlabel('Error',fontsize = 15)
plt.ylabel('Density',fontsize = 15)
plt.legend(loc = 'best',fontsize = 15)
plt.show()



#distribution density of predicted values

plt.figure(figsize = (12,5))
sns.kdeplot(np.asarray(Y_hat_skl), bw=0.5,label = 'SKlearn')
sns.kdeplot(np.asarray(Y_hat).T[0], bw=0.5,label = 'Manual with changing learning rate')
sns.kdeplot(np.asarray(Y_hat_c).T[0], bw=0.5,label = 'Manual with Constant learning rate')
plt.title('Distribution of Predicted values',fontsize = 15)
plt.xlabel('Predicted values',fontsize = 15)
plt.ylabel('Density',fontsize = 15)
plt.legend(loc = 'best',fontsize = 15)
plt.show()
```


Distribution of error
SKlearn
Manual with changing learning rate
Manual with Constant learning rate
Density
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
−20
−10
0
10
20
Error


Distribution of Predicted values
SKlearn
Manual with changing learning rate
Manual with Constant learning rate
Density
0.00
0.02
0.04
0.06
0.08
−10
0
10
20
30
40
Predicted values

```
- The distribution of error for Manual implemetation with changing learning rate has more variance than one with constant learning rate which can be understood with non constant lr.

-
```

In [76]:

```python
# Comparing the final weights for each of the three implenetations
weights_dict = {'SKlearn_implementation':lr_model.coef_,'Manual with Changing lr':np.array(weights_vlr).T[0]
                'Manual with constant LR':np.array(weights_c).T[0]}
Weights_df = pd.DataFrame(weights_dict,index = None)
Weights_df
```

Out[76]:

	SKlearn_implementation	Manual with Changing lr	Manual with constant LR
0	-0.997011	-0.784548	-0.987276
1	0.695117	0.346827	0.850099
2	0.281520	-0.036585	0.066367
3	0.717662	1.042431	0.872048
4	-2.026188	-0.977779	-2.116836
5	3.141030	3.502265	3.157694
6	-0.172170	-0.218942	-0.525868
7	-3.085071	-1.785345	-3.371071
8	2.248803	1.046697	2.557751
9	-1.760011	-1.249237	-1.716153
10	-2.039338	-1.582783	-1.836177
11	1.136172	1.104529	0.726336
12	-3.619226	-3.108119	-3.269978

In [78]:

```python
print('MSE with SKlearn: ',mse_sklearn)
print('MSE with Manual Implementation with Changing lr:',mse_sgd)
print('MSE with Manual Implemetation with Constant lr:',mse_sgd_c)
```

```
MSE with SKlearn:  24.3033972171
MSE with Manual Implementation with Changing lr: 26.6180348135
MSE with Manual Implemetation with Constant lr: 22.8566063721
```

Conclusion:

- We get the best Mean Squared Error value with manual implementation of SGD keeping the learning rate constant for every epoch.
- The Sklearn implementation fared well than manual sgd where we changed the learning rate with every epoch.This can be attributed to the regularization terms which were used for avoiding overfitting in the data.
- Though None of the above model is good enough for prediction of regression values