26/07/2022 1-N

$$V_0(x) = \mathbb{1}_{[0,+\infty)}(x)$$

$$V(t,x) = \frac{H(t,x)}{B(t,x)}$$

$$H(t,x) = \int_0^{+\infty} \exp\left(-\frac{1}{\sigma^2}\left[\frac{(x-y)^2}{2t} + y\right]\right) dy$$

$$B(t,x) = \int_{-\infty}^{+\infty} \exp\left(-\frac{1}{\sigma^2}\left[\frac{(x-y)^2}{2t} + y\mathbb{1}_{\{y\geq 0\}}\right]\right) dy$$

$$= \int_{-\infty}^{0} \exp\left(-\frac{1}{\sigma^2}\left[\frac{(x-y)^2}{2t}\right]\right) dy + H(t,x)$$

$$\ell(t,x,y) = \exp\left(-\frac{1}{\sigma^2}\left[\frac{(x-y)^2}{2t} + y\right]\right) =: \text{integrand}$$

$$d\ell(t,x,y) = -\frac{(x-y)}{\sigma^2 t} \exp\left(-\frac{1}{\sigma^2}\left[\frac{(x-y)^2}{2t} + y\right]\right) = \frac{\partial \ell(t,x,y)}{\partial x}$$

$$dH(t,x) = \frac{\partial H(t,x)}{\partial x} = \int_0^{+\infty} d\ell(t,x,y)\, dy$$

$$g(t,x,y) = \exp\left(-\frac{1}{\sigma^2}\left(\frac{(x-y)^2}{2t}\right)\right)$$

$$dg(t,x,y) = \frac{\partial g(t,x,y)}{\partial x} = -\exp\left(-\frac{1}{\sigma^2}\frac{(x-y)^2}{2t}\right) \cdot \frac{x-y}{\sigma^2 t}$$

$$dB(t,x) = \frac{\partial B(t,x)}{\partial x} = \int_{-\infty}^{0} dg(t,x,y)\, dy + dH(t,x)$$

$$u(t,x) := \frac{\partial V(t,x)}{\partial x} = \frac{1}{(B(t,x))^2}\Big(B(t,x) \cdot dH(t,x) - H(t,x)\, dB(t,x)\Big)$$