1)

Suppose a random variable X has pdf as $f(x) = 2e^{-2(x-1)}, x > 1$. Which of the following represents P(0 < X < 4)? (Note: you do not need to solve for exact number).

1. $\int_0^4 2e^{-2(x-1)}dx;$
2. **$\int_1^4 2e^{-2(x-1)}dx;$**
3. $\int_0^4 x2e^{-2(x-1)}dx;$
4. $\sum_{x=0}^4 2e^{-2(x-1)};$
5. $\int_0^\infty x2e^{-2(x-1)}dx;$

2)

A random variable X has pdf

$$\boldsymbol{f(x) = \frac{2^x}{x!} e^{-2}}, \ \textbf{x= 0,1,2...}$$

Find P(X = 1).
Rscript:

```
((2)/factorial(1))*exp(-2)
```

Answer:
0.2707

Then find P(-2<X<4).
Rscript:

```
sum(((2^(0:3))/factorial(0:3))*exp(-2))
```

or

```
x_range<-c(0:3)
f.x<-function(x) ((2^x)/factorial(x))*exp(-2)
f_x<-function(x) f.x(x)*(x %in% x_range)
sum(f_x(x_range))
```

Note: I don’t know which way you prefer I don’t want my marks to be cut because I use a longer code instead of one line code. So please let me know in future which one should I use and Please don’t cut marks for writing both the scripts here as I don’t know what exactly you want here.

Answer:
0.8571

Give your answers to at least four decimal places.

3)

If two carriers of the gene for albinism marry and have children, then each of their children has a probability of 1/4 of being albino. Let the random variable Y denote the number of their albino children out of all 3 of their children. Then Y follows a binomial(n, p) distribution. Find the values for n and p.
n =___3__ p = _.25_____

Answer:
n=3
p=1/4=.25

4)

For Y following a binomial ($n = 3$, $p = 0.25$) distribution, compute the following:
$P(Y \leq 2) =$
$E(Y) =$
$Var(Y) =$
Give your answers to at least four decimal places.

Rscript:
```
sum(dbinom((0:2),size=3,p=0.25))
```

Answer:
$P(Y \leq 2) = 0.9844$

Rscript:
```
y<-c(0:3)
sum(y*dbinom(y,size=3,p=0.25))
```

Answer:
$E(Y) = 0.7500$

Rscript:
```
y<-c(0:3)
EY<-sum(y*dbinom(y,size=3,p=0.25))
sum((y-EY)^2*dbinom(y,size=3,p=0.25))
```

Answer:
$Var(Y) = 0.5625$

5)

For X following a Chi-square distribution with degree of freedom m = 3, compute the following:
P(1 < X < 4) =
E(X) =
Var(X) =
Give your answers to at least four decimal places.
Also, use a Monte Carlo simulation with sample size n=100,000 to estimate P(1 < X < 4). What is your Monte Carlo estimate? Does it agrees with the answer above?

Rscript:
```
x<-function c(0:Inf)
integrate(function(x) dchisq(x,df=3), lower=1,upper=4)
```

Answer:
P(1 < X < 4) = .5398

Rscript:
```
x<-function c(0:Inf)
integrate(function(x) x*dchisq(x,df=3), lower=0,upper=Inf)
```

Answer:
E(X) = 3 (mean is same as degree of freedom for chi-square distribution)

Rscript:
```
x<-function c(0:Inf)
EX<-integrate(function(x) x*dchisq(x,df=3), lower=0,upper=Inf)$value
integrate(function(x) ((x - EX)^2)*dchisq(x,df=3), lower=0,upper=Inf)
```

Answer:
Var(x) = 6 (Variance is twice the degree of freedom for chi-square distribution)

Rscript:
```
X<-rchisq(n=100000, df=3)
mean((1<x) & (x<4))
```
Answer:
.5378 Yes the monte carlo estimate agrees with above.

6)

Suppose X follows a Chi-square distribution with degree of freedom m = 5 so that E(X) = 5 and Var(X) = 10. Also, let Y = 4X - 10. Find E(Y) and Var (Y). Does Y follow a Chi-square distribution with degree of freedom m=10?

E(Y) =

Var(Y) =

Does Y follow a Chi-square distribution with degree of freedom m = 10?

Answer:
Y = 4X-10
Using the formula E(aX+b)= aE(X) + b
E(Y) = 4*E(X) – 10
E(Y) = 20-10 = 10

Using the formula Var (aX+b) = a^2Var(X)
Var(Y) = 4^2Var(X)
Var(Y) = 16*10= 160

For Y to follow chi-square distribution with degree of freedom m=10 then it should follow 2 conditions namely;

1. E(Y) = 10 as mean for chi square distribution is same as degree of freedom
2. Var(Y) = 20 as variance for chi square distribution is twice the degree of freedom.

E(Y) is 10 but Var(Y) is 160 so Y doesn't follow a chi square distribution.

7)

The Zyxin gene expression values are distributed according to $N(\mu = 1.6, \sigma = 0.4)$

(a) What is the probability that a randomly chosen patient have the Zyxin gene expression values between 1 and 1.6?

Rscript:

```
x<-c(-Inf,Inf)
integrate(function(x) dnorm(x,mean=1.6,sd=0.4), lower=1,upper=1.6)$value
```

Answer:

0.4332

(b) Use a Monte Carlo simulation of sample size n=500,000 to estimate the probability in part (a). Give your R code, and show the value of your estimate.

Rscript:

```
x<-rnorm(n=500000, mean=1.6, sd=.4)
mean((1<x) & (x<1.6))
```

Answer:

0.4334

(c) What is the probability that exactly 2 out of 5 patients have the Zyxin gene expression values between 1 and 1.6?
Please show your work on how to arrive at the answer. Give your answer to at least four decimal places.

Rscript:

```
dbinom(2,size =5, prob=.4332)
```

Answer:

Now this normal distribution become binomial distribution with set =5 and probability = 0.4332.
We have to find P(X=2) which we can calculate by dbinom(2,size=5, prob=.4332)
Which gives us the answer - .3417
.3417 is the probability of exactly 2 patients out of 5 having the zyxin gen expression values between 1 and 1.6.

8)

(a) Hand in a R script that calculates the mean and variance of two random variables X~F(m=2,n=5) and Y~F(m=10,n=5) from their density functions.

Rscript:

Mean of X:

```
x<-c(0,Inf)
integrate(function(x) x*df(x,df1=2,df2=5), lower=0,upper=Inf)$value
```

Answer:

1.6667

Variance of X:

```
x<-c(0,Inf)
EX<-integrate(function(x) x*df(x,df1=2,df2=5), lower=0,upper=Inf)$value
integrate(function(x) (x-EX)^2*df(x,df1=2,df2=5), lower=0,upper=Inf)
```

Answer:

13.8889

Mean of Y:

```
y<-c(0,Inf)
integrate(function(y) y*df(y,df1=10,df2=5), lower=0,upper=Inf)$value
```

Answer:

1.6667

Variance of Y:

```
y<-c(0,Inf)
EY<-integrate(function(y) y*df(y,df1=10,df2=5), lower=0,upper=Inf)$value
integrate(function(y) (y-EY)^2*df(y,df1=10,df2=5), lower=0,upper=Inf)
```

Answer:

7.2222

(b) Use the formula in Table 3.4.1 to calculate the means and variances directly.

Mean of X:

$\frac{n}{n-2}$ = 5/3 = 1.6667

Variance of X:

$\frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$ = 50*5/18 = 13.8889

Mean of Y:

$\frac{n}{n-2}$ = 5/3 = 1.6667

Variance of Y:

$\frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$ = 50*13/90 = 7.2222

(c) Run your script in (a), and check that your answers agree with those from part (b).

Yes, my scripts agrees with the answers from part (b)