EE324 CONTROL SYSTEMS LAB
Problem Sheet 9
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Question 1: Nyquist Plot

$$Gs = \frac{10}{s(\frac{s}{5}+1)(\frac{s}{20}+1)}$$

Part A)

We have to introduce Lag Compensator to the transfer function Gs

$$Cs = \frac{s+3}{s+1}$$

We have

$$Ts1 = Cs * Gs = \frac{10(s+3)}{s(s+1)\left(\frac{s}{5}+1\right)\left(\frac{s}{20}+1\right)}$$

Part B)

We have to introduce Lead Compensator to the transfer function Gs

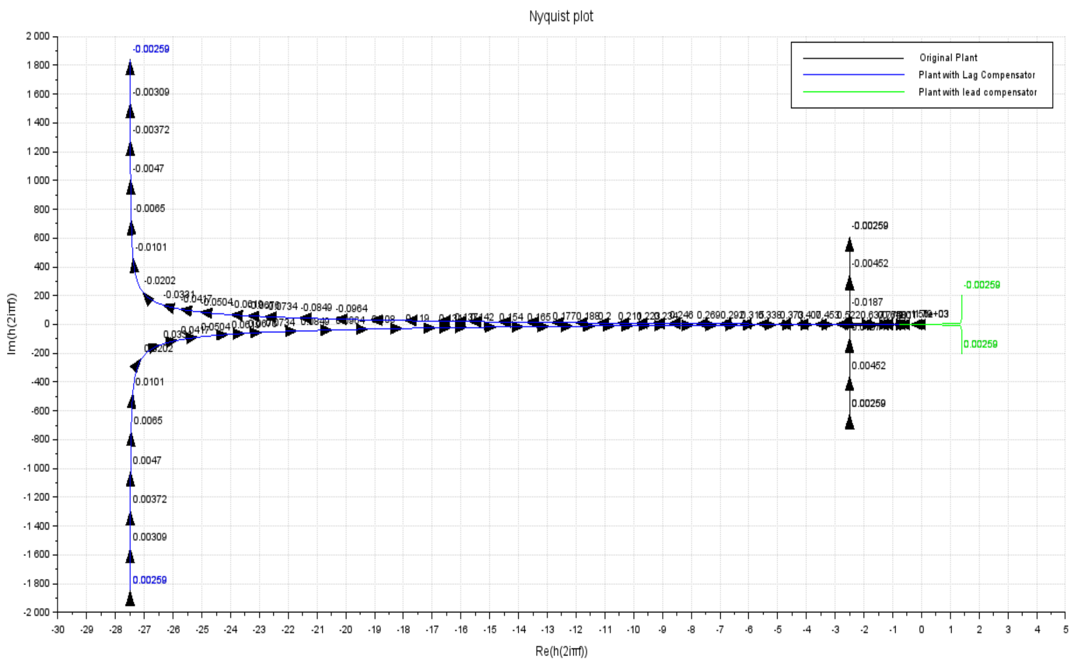
$$Cs = \frac{s+1}{s+3}$$

We have

$$Ts2 = Cs * Gs = \frac{10(s+1)}{s(s+3)\left(\frac{s}{5}+1\right)\left(\frac{s}{20}+1\right)}$$

Following are the results from Scilab

```
Gain margin for Orignal Plant = 7.958800
Gain margin for Plant with Lag Compensator = 2.076255
Gain margin for Plant with Lead Compensator = 11.759539
Phase margin for Orignal Plant = 22.535942
Phase margin for Plant with Lag Compensator = 4.024733
Phase margin for Plant with Lead Compensator= 43.173118
```



We can observe that Gain/Phase margin is highest in case of Lead Compensator and lowest for Lag Compensator

We can approximately state the stability order as

Plant with Lead Compensator > Original Plant > Plant with Lag Compensator

Code for the same:

```
s=%s;
Gs=syslin('c',10,s*(s/5+1)*(s/20+1));
Gs_lag=syslin('c',10*(s+3),s*(s+1)*(s/5+1)*(s/20+1));
Gs_lead=syslin('c',10*(s+1),s*(s+3)*(s/5+1)*(s/20+1));

nyquist(Gs);
nyquist(Gs_lag);
nyquist(Gs_lead);

nyquist([Gs;Gs_lag;Gs_lead],["Original Plant";"Plant with Lag
Compensator";"Plant with lead compensator"]);

[gm,w_pcf]=g_margin(Gs);
[gm_lag,w_pcf1]=g_margin(Gs_lag);
[gm_lead,w_pcf2]=g_margin(Gs_lead);
```

```
[pm,w_gcf]=p_margin(Gs);
[pm_lag,w_gcf1]=p_margin(Gs_lag);
[pm_lead,w_gcf2]=p_margin(Gs_lead);
printf('Gain margin for Orignal Plant = %f',gm);
printf('\nGain margin for Plant with Lag Compensator = %f',gm_lag);
printf('\nGain margin for Plant with Lead Compensator = %f',gm_lead);
printf('\nPhase margin for Orignal Plant = %f',pm);
printf('\nPhase margin for Plant with Lag Compensator = %f',pm_lag);
printf('\nPhase margin for Plant with Lead Compensator= %f',pm_lead);
```

Question 2: Notch Filter

We have to design the notch filter that rejects/attenuates the frequency 50Hz
As we make our transfer function in 's' and
S=jw
w=2πf
We will convert 50Hz into rad/sec
w' = 314.16 rad/sec

We know that the magnitude bode plot of the notch filter has a constant slope everywhere except at we (frequency which has to be rejected)

Now, if we consider the Numerator of Notch filter, it must have minima at 50 Hz and constant before 50Hz

So we can take numerator as
Nr = $s^2 + w'^2$
Nr = $s^2 + 314.16^2$

For f << 50Hz, we can approximate this as 314.16^2, which is constant

Now we must choose a denominator such that the magnitude bode plot is constant everywhere except at 50Hz

So for f << 50Hz, it can be approximated to 314.16^2
And for f >> 50Hz, it can be approximated to s^2

This tells us that denominator is the second-order equation in s

Dr = $s^2 + \sigma s + 314.16^2$

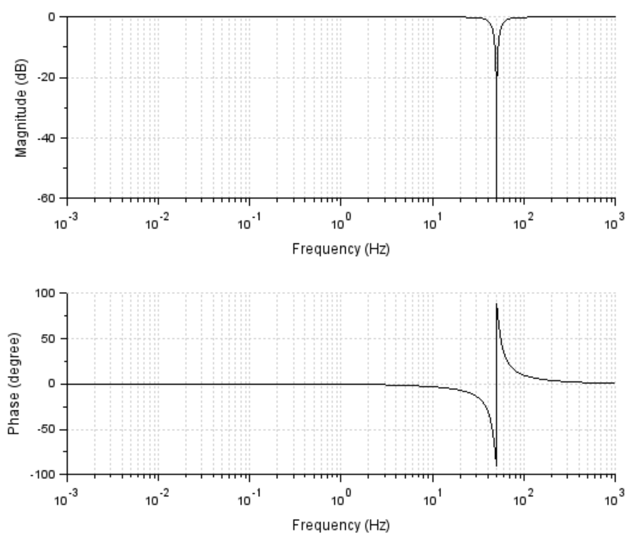
Notch filter is

$$Gs = \frac{s^2 + 314.16^2}{s^2 + \sigma s + 314.16^2}$$

Let's take σ=80

$$Gs = \frac{s^2 + 314.16^2}{s^2 + 80s + 314.16^2}$$

Bode plot of Gs



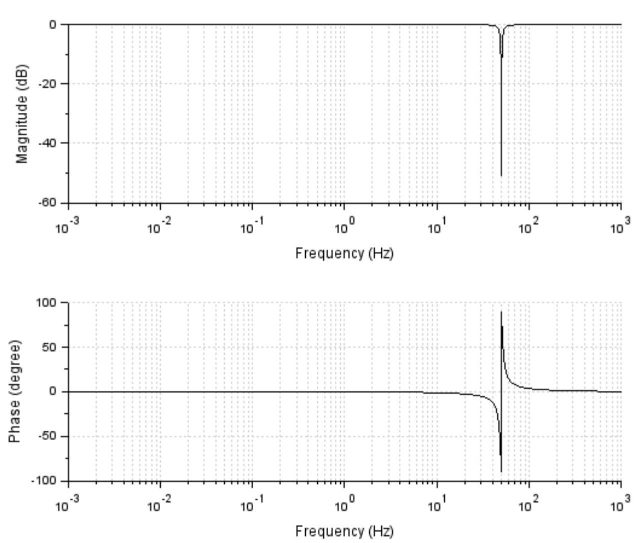
As we decrease the value of σ, the slope becomes steeper for the magnitude bode plot

We can see it in the following bode plot

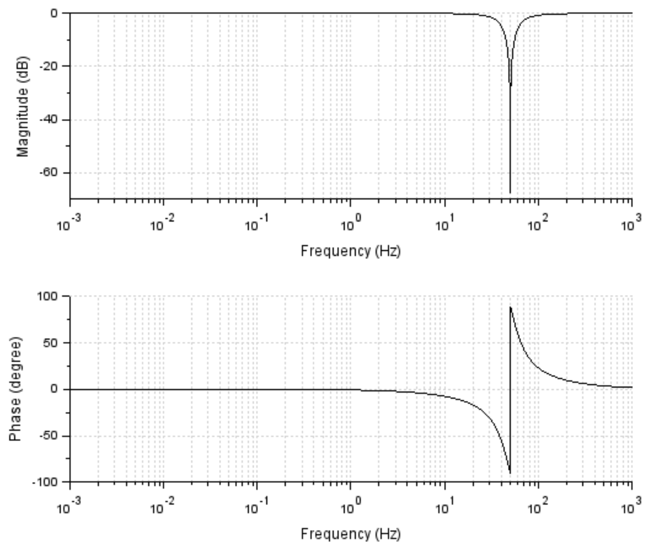
$$Gs1 = \frac{s^2 + 314.16^2}{s^2 + 30s + 314.16^2}$$

$$Gs2 = \frac{s^2 + 314.16^2}{s^2 + 200s + 314.16^2}$$

For Gs1(σ=30)



For Gs2(σ=200)



Code for the same:

```
s=%s;
Gs=syslin('c',s^2+314.16^2,s^2+80*s+314.16^2);
bode(Gs);
scf(2);
Gs1=syslin('c',s^2+314.16^2,s^2+30*s+314.16^2);
bode(Gs1);
scf(3);
Gs2=syslin('c',s^2+314.16^2,s^2+200*s+314.16^2);
bode(Gs2);
```

Question 3: Delay

We have transfer function

$$\mathrm{Gs} = \frac{100}{\mathrm{s}+30}$$

Ideally, delay of time T is represented in Laplace domain as

$$\mathrm{e}^{-\mathrm{sT}}$$

Ideally, the minimum delay required for instability of system is

$$\mathrm{td} = \frac{\mathrm{PM}}{\mathrm{wgcf}}$$

PM and w_{gcf} of Gs are obtained from scilab

$$PM = 105.45^{o} = 1.84 \text{ rad}$$

$$W_{gcf} = 15.18 \text{ Hz}$$

Ideal td=0.019 sec

But we don't have an exponential function in Scilab. Hence we will approximate it using pade

$$Cs = e^{-sT} = \frac{1 - \frac{sT}{2}}{1 + \frac{sT}{2}}$$

Now the above approximation has magnitude one and phase change equal to

$$\text{Phase change} = -2 \tan^{-1}(sT/2)$$

Now for calculation of delay T,

Total phase change required = Phase margin

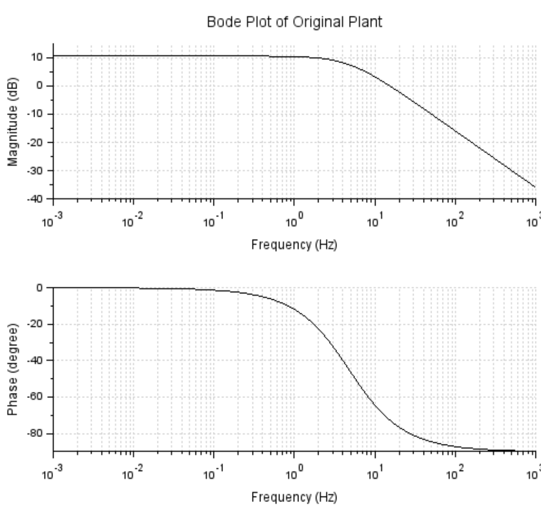
$$1.84 = 2 \tan^{-1}(sT/2)$$

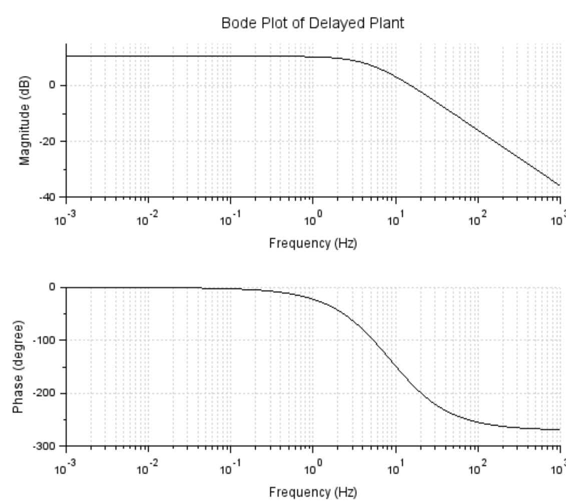
$$sT = 2.62$$

$$T = 2.62/ W_{gcf}$$

$$T = 0.0286 \text{ sec}$$

Following are the bode plots of Gs and Cs*Gs






```
Phase margin for Orignal Plant = 107.457603
Wgcf for Orignal Plant = 15.182414
Phase margin for delayed Plant = -0.000000
Wgcf for delayed plant = 15.182414
```

Code for the same:

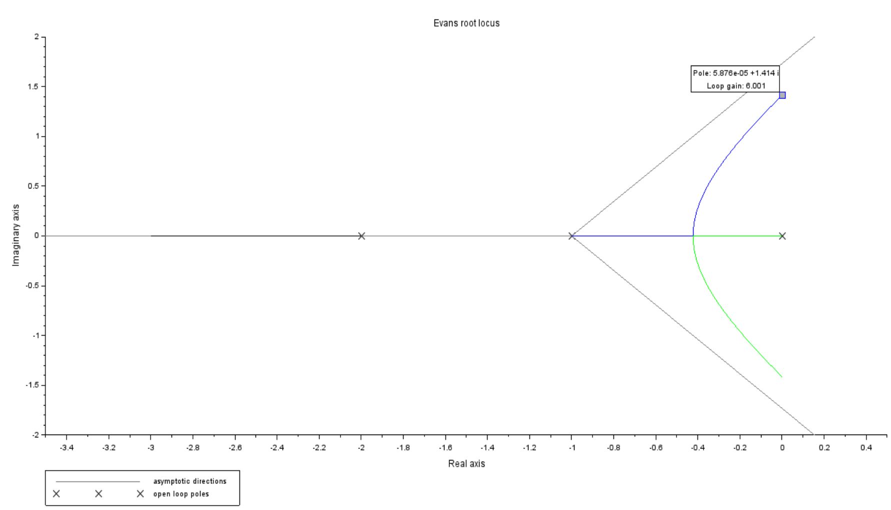
```
s=%s;
T=0.0286;
Gs=syslin('c',100,s+30);
Gs1=syslin('c',100*(1-s*T/2),(s+30)*(1+s*T/2));
[pm,w_gcf]=p_margin(Gs);
printf('\nPhase margin for Orignal Plant = %f',pm);
printf('\nWgcf for Orignal Plant = %f',w_gcf);
[pm1,w_gcf1]=p_margin(Gs1);
printf('\nPhase margin for delayed Plant = %f',pm1);
printf('\nWgcf for delayed plant = %f',w_gcf1);
bode(Gs);
xtitle("Bode Plot of Original Plant")
scf(2);
bode(Gs1);
xtitle("Bode Plot of Delayed Plant");
```

Question 4: Gain Margin

$$Gs = \frac{1}{s^3 + 3s^2 + 2s} = \frac{1}{s(s+1)(s+2)}$$

Part A) Root Locus

Using Scilab, we get the root locus of the system



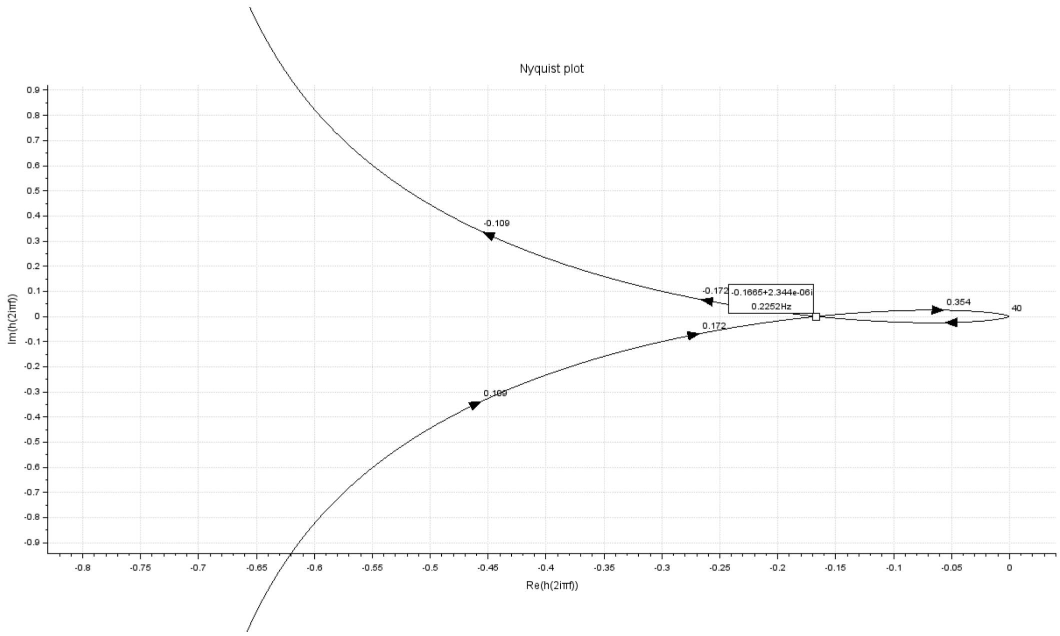
We can see that

Max value of k=6.001

Gain Margin = 20 log(6) = 15.56 dB

Part B) Nyquist Plot

Using Scilab, we get the Nyquist Plot of the system



We can see that, Nyquist plot cuts the horizontal axis, i.e., 180^{o} phase line at x = -0.17

Hence we get k equal to

$$k = -\frac{1}{-0.17} = 5.88$$

GM = 15.38 dB

Part C) Asymptotic Bode Plot

Using asymptotic bode plot, we get -180^{o} phase between 1 and 2 rad/sec

Lets take 1.5 rad/sec(avg. of 1 and 2) as phase crossover frequency

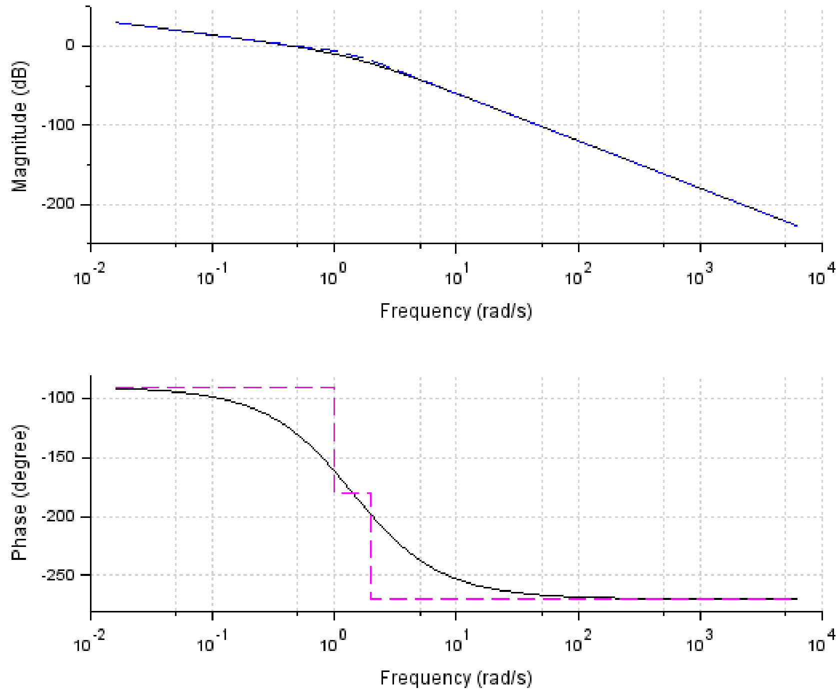
Magnitude of Gs at w=1.5 rad/sec is

= -12.94 dB

Hence

Gain Margin = 12.94 dB

Part D) Actual Bode Plot



Actual Gain margin = 15.56

Gain Margin = 15.563025

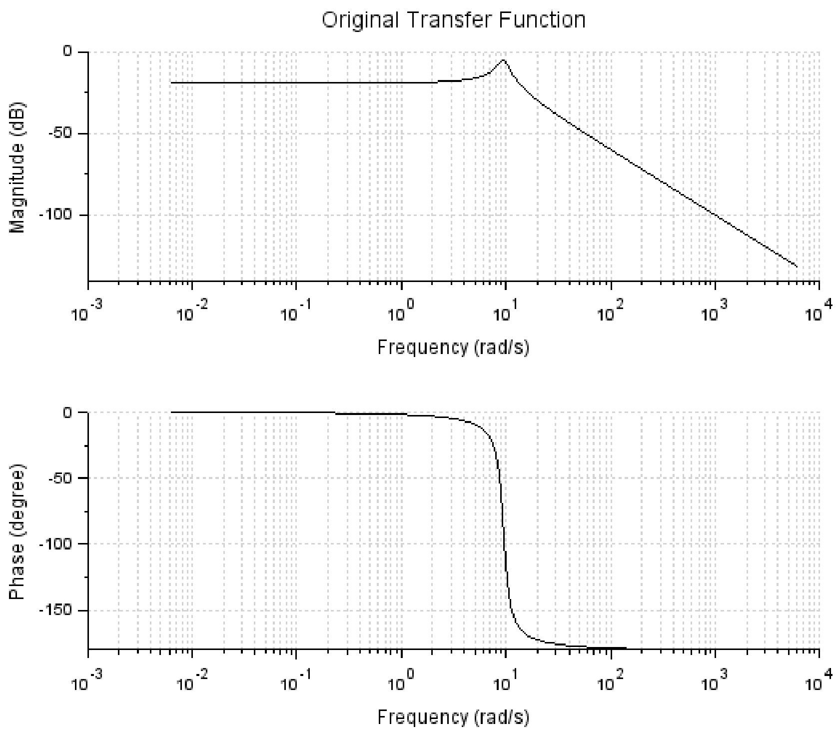
Code for the same:

```
s=%s;
Gs=syslin('c',1,s^3+3*s^2+2*s);

//Part A
evans(Gs,kpure(Gs));
scf(1);
//Part B
nyquist(Gs,0.05,40);
scf(2);

//Part D
bode(Gs,'rad');
bode_asymp(Gs);
[gm,wpcf]=g_margin(Gs);
printf("Gain Margin = %f",gm)
```

Question 5: Steady-State Error Improvement

$$Gs = \frac{10s + 2000}{s^3 + 202s^2 + 490s + 18001}$$

Part A)



Here Magnitude plot is always below 0 dB line, Hence infinite Phase margin

Phase plot never crosses -180° line, hence infinite Gain Margin

Part B)

Steady-state error for step input for the original function is

$$e(\infty) = \frac{1}{1 + G(0)}$$

$$e(\infty) = \frac{1}{1 + \frac{2000}{18001}} = \frac{1}{1 + \frac{1}{9}} = \frac{9}{10}$$

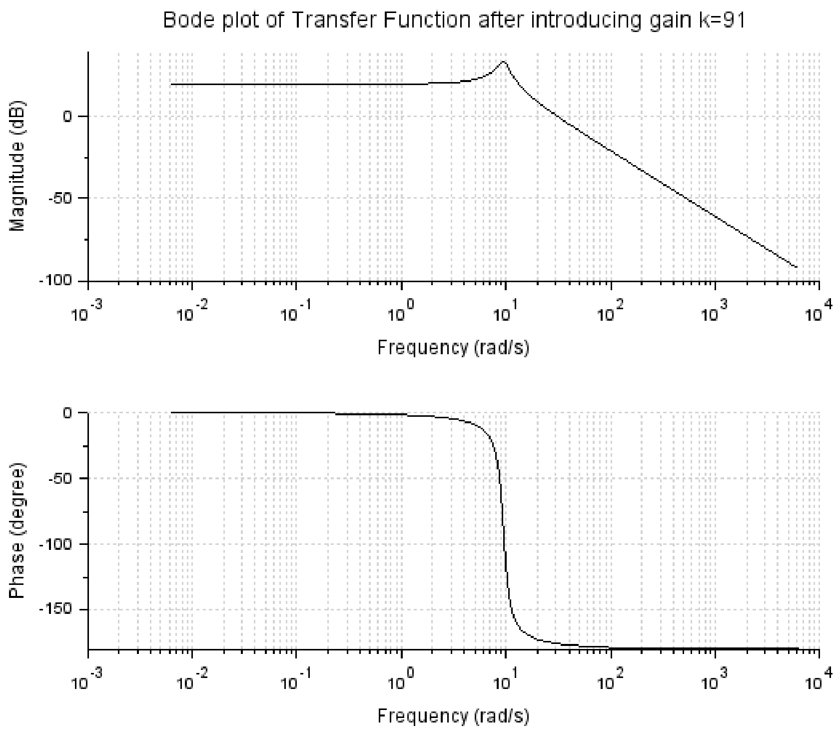
new steady-state error be e' (∞)

$$e'(\infty) = \frac{9}{100}$$

After introducing gain k,

$$e'^{(\infty)} = \frac{1}{1 + k * G(0)} = \frac{9}{100} = \frac{1}{\frac{100}{9}} = \frac{1}{1 + 91 * G(0)}$$

K=91



Part C)

As we can see that the magnitude plot of the Transfer function is crossing the 0 dB line; hence we have a Gain crossover frequency

Therefore we have a finite Phase Margin

Phase plot is still not crossing -180^{o} line

Therefore infinite Gain Margin

Part D)

For phase margin to be greater than 90^{o}, our phase plot must be above -90^{o} line at w_{gcf}

To be on the safe side, if we introduce a zero, total phase change will be 90^{o}, which ensures our requirement of $>=90^{o}$ PM

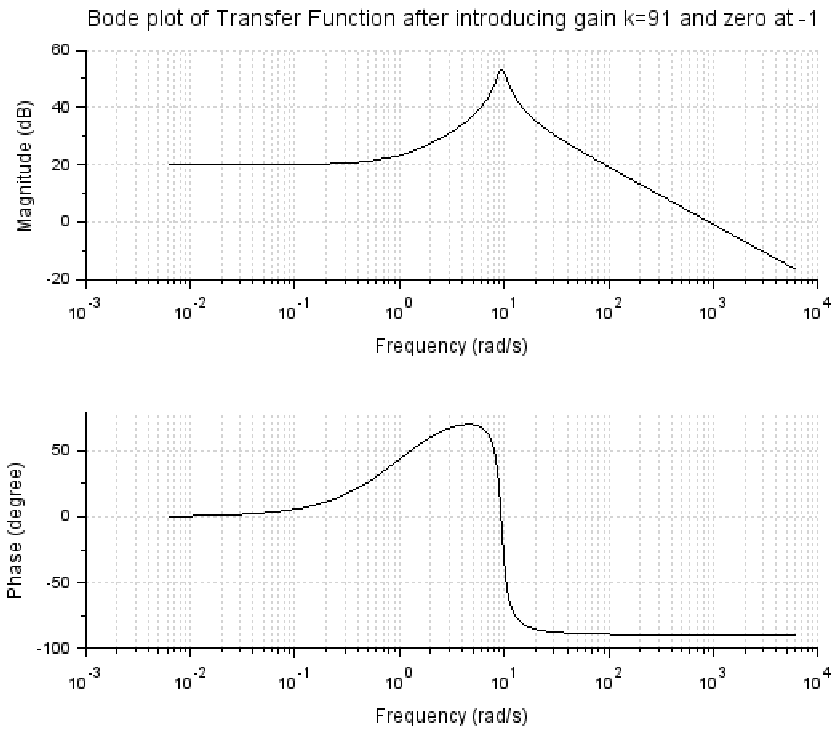
In the transfer function, the pole and zero at -200, gets cancelled

So two imaginary poles with real part -1 are remaining

Hence we have to add a zero before or at -1

Let's take a zero at -1; our new Gs becomes

$$Gs = \frac{(s+1)(10s+2000)}{s^3 + 202s^2 + 490s + 18001}$$



Part E)

The above closed Loop system has no Phase Crossover frequency, i.e. infinite Gain Margin

And the Phase margin is > 90°

From both the observations, it is clear that all the poles of the system are always in Open Left Half Plane

Therefore the system is always stable

Code for the same:

```
s=%s;
Gs=syslin('c',10*s+2000,s^3+202*s^2+490*s+18001);
bode(Gs,'rad');
xtitle("Original Transfer Function");
```

```
scf(1);
k=91;
Gs1=syslin('c',k*(10*s+2000),s^3+202*s^2+490*s+18001);
bode(Gs1,'rad');
xtitle("Bode plot of Transfer Function after introducing gain k=91");
scf(2);
Gs2=syslin('c',k*(10*s+2000)*(s+1),s^3+202*s^2+490*s+18001);
bode(Gs2,'rad');
xtitle("Bode plot of Transfer Function after introducing gain k=91 and
zero at -1");
```