EE324 CONTROL SYSTEMS LAB

PROBLEM SHEET 1

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Question 1 :

Part A)

$$G1(s) = \frac{10}{s^2 + 2s + 10} \qquad G2(s) = \frac{5}{s+5}$$

In this part, we are given two transfer functions connected in a cascade or series manner. The overall transfer function ($T1(s) = \frac{C(s)}{R(s)}$) is obtained by multiplying two transfer functions in the Laplace domain.

T1(s)=G1(s)*G2(s)

$$T1(s) = \frac{50}{50 + 20s + 7s^2 + s^3}$$

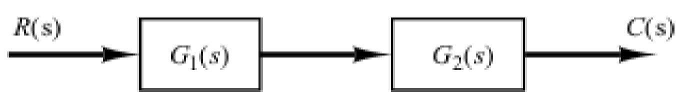

Figure 1: Cascaded Transfer functions

Scilab code for the same:

```
s = poly(0,’s’)
G1 = 10/(s^2+2*s+10);
G2 = 5/(s+5);
G1 = syslin('c',G1);
G2 = syslin('c',G2);
T1 = syslin('c',G1*G2);
```

Part B)

$$G1(s) = \frac{10}{s^2 + 2s + 10} \qquad G2(s) = \frac{5}{s + 5}$$

In this part, given two transfer functions are connected in parallel combinations. For getting the overall transfer function ($T2(s) = \frac{C(s)}{R(s)}$) of the system, we add two transfer functions in the Laplace domain.

T2(s)=G1(s)+G2(s)

$$T2(s) = \frac{100 \ + 20s \ + 5s^2}{50 \ + 20s \ + 7s^2 + s^3}$$

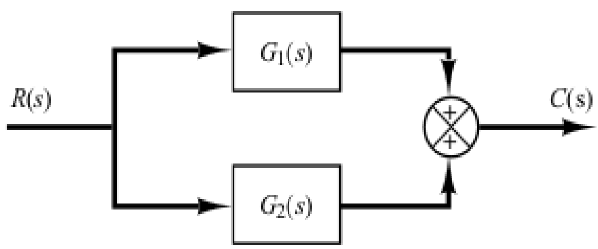

Figure 2: Parallel Transfer functions

Scilab code for the same:

```
s = poly(0,’s’)
G1 = 10/(s^2+2*s+10);
G2 = 5/(s+5);
G1 = syslin('c',G1);
G2 = syslin('c',G2);
T2 = syslin('c',G1+G2);
```

Part C)

$$G1(s) = \frac{10}{s^2 + 2s + 10} \qquad G2(s) = \frac{5}{s + 5}$$

In this part, given two transfer functions are connected as a negative feedback system. For getting overall transfer function ($T3(s) = \frac{C(s)}{R(s)}$) of the system, we can represent it as following.

$$C = G1(R - C * G2)$$

$$C * (1 + G1 * G2) = G1 * R$$

$$\frac{C(s)}{R(s)} = \frac{G1}{1 + G1 * G2}$$

$$T3(s) = \frac{G1}{1 + G1 * G2}$$

$$T3(s) = \frac{50 + 10s}{100 + 20s + 7s^2 + s^3}$$

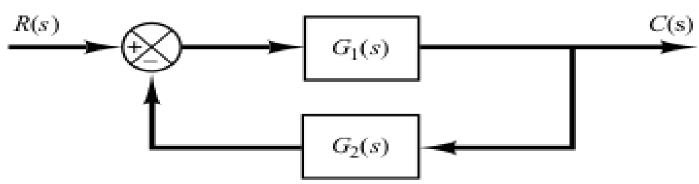

Figure 3: Closed Loop Negative Feedback System

Scilab code for the same:

```
s = poly(0,’s’)
G1 = 10/(s^2+2*s+10);
G2 = 5/(s+5);
G1 = syslin('c',G1);
G2 = syslin('c',G2);
T3 = syslin('c',G1/(1+G1*G2));
```

OR There is a direct command for negative feedback system in SCILAB

```
T3 = syslin('c',G1/.G2);
```

Part D)

In this part, we are given a second-order transfer function G1(s) and ask to find its unit step response. We can plot unit step response using the Scilab tool.

Here we have R(s)= $\frac{1}{s}$

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + 2s + 10}$$

$$C(s) = (\frac{1}{s})\ \frac{10}{s^2 + 2s + 10}$$

$$C(s) = \frac{10}{s^3 + 2s^2 + 10s}$$

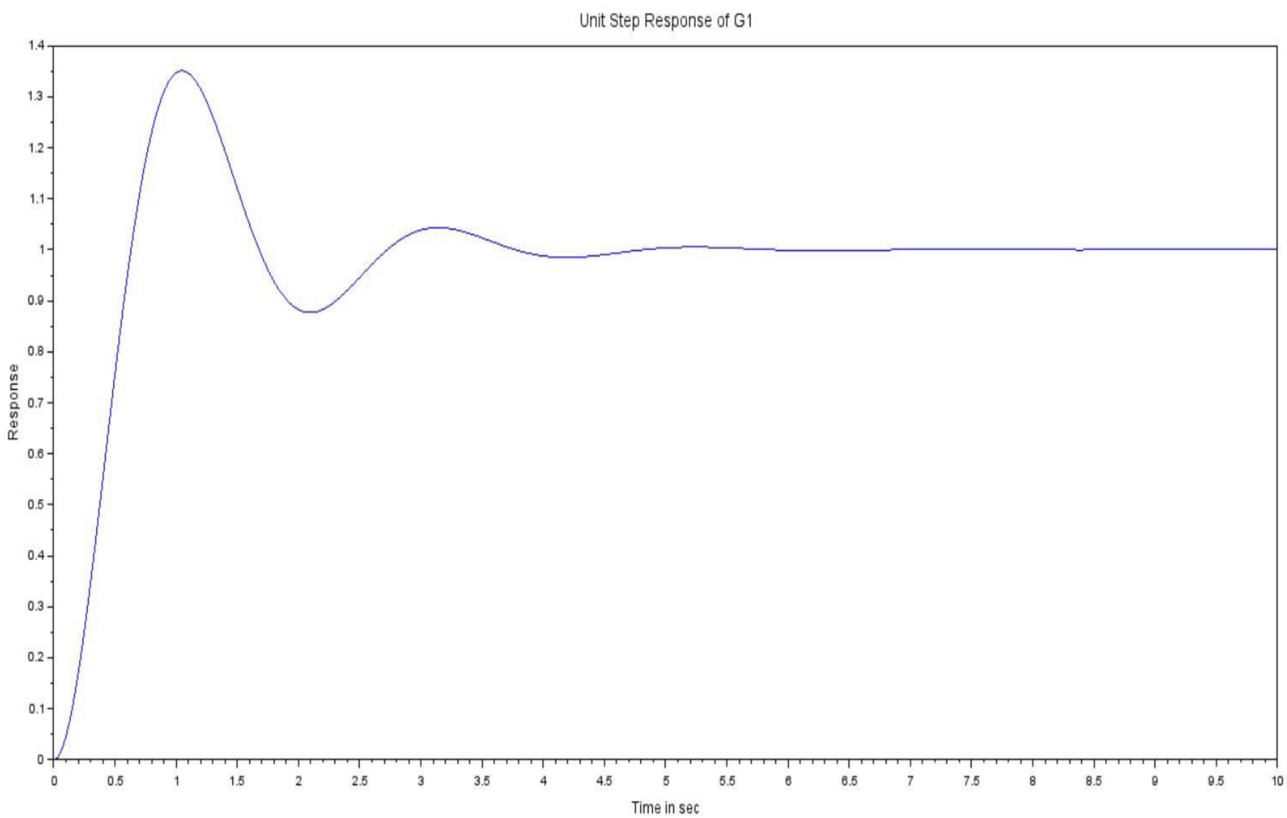

Figure 3: Unit Step Response of G1(s)

Scilab code for the same:

```
s = poly(0,’s’);
G1 = 10/(s^2+2*s+10);
G1 = syslin('c',G1);
t=0:0.01:10;
StepRes=csim('step',t,G1);
plot(t, StepRes) ;
xtitle("Unit Step Response of G1","Time in sec", "Response");
```

Question 2 :

For this question, we will be using the overall transfer functions obtained in the last question.

$$\mathrm{T1(s)} = \frac{50}{50 \ + 20\mathrm{s} \ + 7\mathrm{s}^2 + \mathrm{s}^3}$$

$$\mathrm{T2(s)} = \frac{100 \ + 20\mathrm{s} \ + 5\mathrm{s}^2}{50 \ + 20\mathrm{s} \ + 7\mathrm{s}^2 + \mathrm{s}^3}$$

$$\mathrm{T3(s)} = \frac{50 \ + 10\mathrm{s}}{100 \ + 20\mathrm{s} \ + 7\mathrm{s}^2 + \mathrm{s}^3}$$

Part A) T1(s)

Degree of Numerator = no. of zeroes = 0

Degree of denominator = no. of poles = 3

Zeroes = None

Poles = Roots of $(50 \ + 20\mathrm{s} \ + 7\mathrm{s}^2 + \mathrm{s}^3)$ = **-5**, **-1+3i**, **-1+3i**

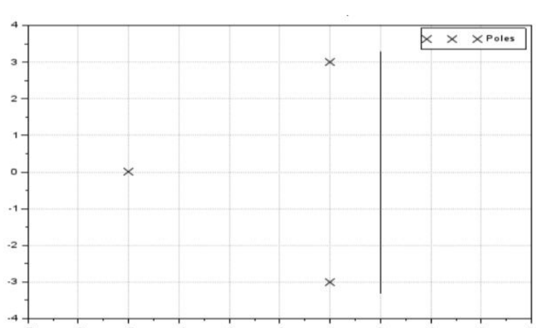

Figure 4: Poles and zeroes of T1(s)

Scilab code for the same:

```
s = poly(0,’s’);
T1 = syslin('c',T1);
num1=T1.num;
den1=T1.den;
roots(num1);
roots(den1);
plzr(T1);
```

Part B) T2(s)

Degree of Numerator = no. of zeroes = 2

Degree of denominator = no. of poles = 3

Zeroes = **-2+4i**, **-2-4i**

Poles = Roots of $(50 + 20s + 7s^2 + s^3)$ = **-5**, **-1+3i**, **-1+3i**

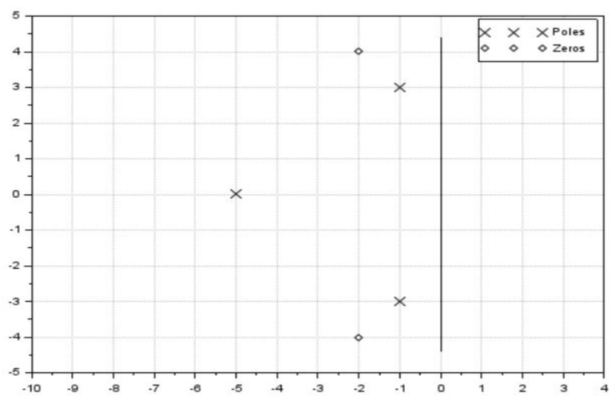

Figure 5: Poles and zeroes of T2(s)

Scilab code for the same:

```
s = poly(0,’s’);
T2 = syslin('c',T2);
num2=T2.num;
den2=T2.den;
roots(num2);
roots(den2);
plzr(T2);
```

Part C) T3(s)

Degree of Numerator = no. of zeroes = 1
Degree of denominator = no. of poles = 3

Zeroes = **-5**
Poles = Roots of $(50 + 20s + 7s^2 + s^3)$ = **-6.33, -0.332 + 3.959i**, **-0.332 - 3.959i**

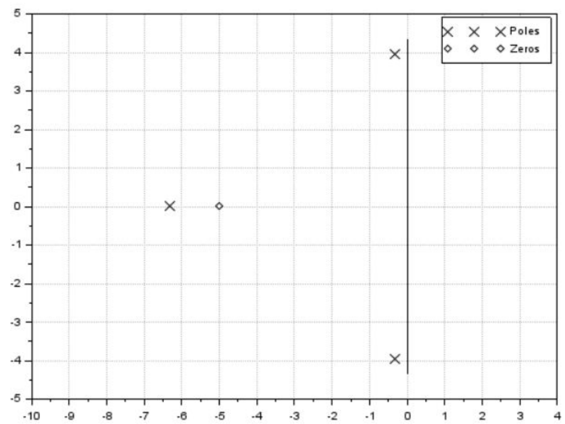

Figure 6: Poles and zeroes of T3(s)

Scilab code for the same:

```
s = poly(0,'s');
T3 = syslin('c',T3);
num3=T3.num;
den3=T3.den;
roots(num3);
roots(den3);
plzr(T3);
```

Question 3 :

In this question, following electric circuit is given. We have to perform the mesh analysis and obtain the equations in matrix vector form Z(s)I(s) = V(s).

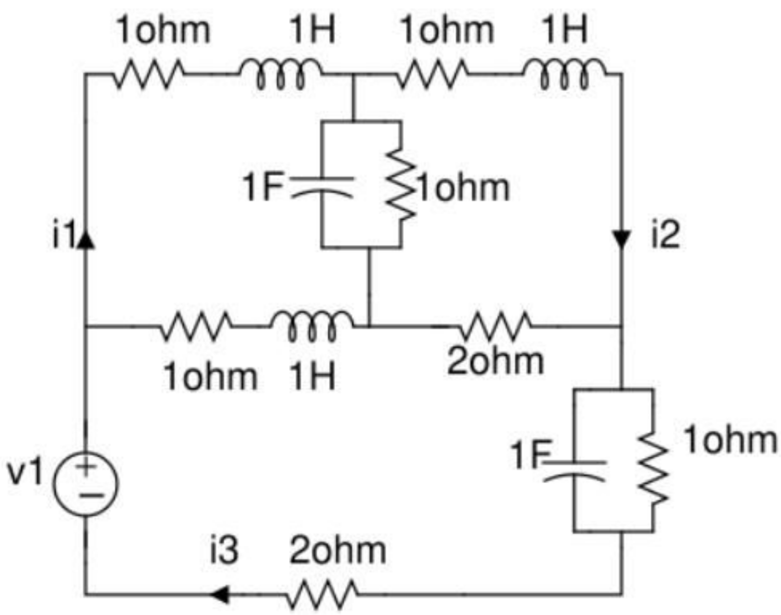

Figure 7: Electric circuit

Now we can write the impedances of different elements in laplace domain as follows

Resistor = R Capacitor = 1/sC Inductor = sL

Now using **KVL** for each of the loop we can write as follows

Loop 1: $[1+s+\frac{1}{1+s}+s+1]I_1(s) \quad - \quad [\frac{1}{1+s}]I_2(s) \quad - \quad [1+s]I_3(s) = 0$

Loop 2: $-[\frac{1}{1+s}]I_1(s) \quad + \quad [\frac{1}{1+s}+1+s+2]I_2(s) \quad - \quad [2]I_3(s) = 0$

Loop 3: $-[1+s]I_1(s) \quad - \quad [2]I_2(s) \quad + \quad [1+s+2+\frac{1}{1+s}+2+1]I_3(s) = V_1(s)$

Now we can convert above equations in vector matrix form

Z(s)I(s) = V(s)

$$\begin{bmatrix} \frac{2s^2+4s+3}{s+1} & -\frac{1}{s+1} & -(s+1) \\ -\frac{1}{s+1} & \frac{s^2+4s+4}{s+1} & -2 \\ -(s+1) & -2 & \frac{s^2+7s+7}{s+1} \end{bmatrix} \begin{bmatrix} I1(s) \\ I2(s) \\ I3(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V1(s) \end{bmatrix}$$

$$I(s)=Z^{-1}(s)V(s)$$

Now we can calculate inverse of impedance matrix Z(s)

From above voltage matrix, V1(s) can be taken to LHS

$$\begin{bmatrix} \frac{I1(s)}{V1(s)} \\ \frac{I2(s)}{V2(s)} \\ \frac{I3(s)}{V3(s)} \end{bmatrix} = Z^{-1}(s) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{I1(s)}{V1(s)} \\ \frac{I2(s)}{V2(s)} \\ \frac{I3(s)}{V3(s)} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{24+48s+35s^2+11s^3+1s^4}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{9+13s+7s^2+2s^3}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{6+14s+13s^2+6s^3+1s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \frac{9+13s+7s^2+2s^3}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{20+45s+39s^2+14s^3+1s^4}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{7+16s+13s^2+4s^3}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \frac{6+14s+13s^2+6s^3+1s^4}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{7+16s+13s^2+4s^3}{57+144s+147s^2+74s^3+17s^4+s^5} & \frac{11+28s+27s^2+12s^3+2s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{I1(s)}{V1(s)} \\ \frac{I2(s)}{V2(s)} \\ \frac{I3(s)}{V3(s)} \end{bmatrix} = \begin{bmatrix} \frac{6+14s+13s^2+6s^3+1s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \frac{7+16s+13s^2+4s^3}{57+144s+147s^2+74s^3+17s^4+s^5} \\ \frac{11+28s+27s^2+12s^3+2s^4}{57+144s+147s^2+74s^3+17s^4+s^5} \end{bmatrix}$$

Scilab code for the same:

```
s = poly(0,'s');
Zs=[(2*s^2+4*s+3)/(s+1), -1/(s+1), -s-1;
   -1/(s+1),  (s^2+4*s+4)/(s+1)     ,-2;
   -s-1,        -2,     (s^2+7*s+7)/(s+1)]
Zinv=inv(Zs);
Vs=[0;0;1] ;
IsuponVs=Zinv*Vs;
```