EE324 CONTROL SYSTEMS LAB

Problem Sheet 10

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Question 1: State-Space Representation

Let's take the required state-space matrices as

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 7 & 2 & 3 \\ 2 & 9 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 1 & 6 \end{bmatrix} \quad D = \begin{bmatrix} 3 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 3 & 4 \\ 5 & 6 & 9 \\ 3 & 2 & 8 \end{bmatrix}$$

Part A)

Part1] In this part, we have to find the transfer function for the above state space representation

We have,

$$Gs = C(sI - A)^{-1}B + D$$

Using Scilab, we get,

$$Gs = \frac{3s^3 + 19s^2 - 63s - 213}{s^3 - 6s^2 - 46s + 20}$$

```
Original Gs :
   -213 -63s +19s² +3s³
   --------------------
     20 -46s -6s² +s³
```

Part2] In this part, we have to get the transfer function for transformed state-space representation as follows

$$A \rightarrow T^{-1}AT$$
$$B \rightarrow T^{-1}B$$
$$C \rightarrow CT$$

$$Gs' = C(sI - A)^{-1}B + D$$

Using Scilab, we get Gs' as

$$Gs = \frac{3s^3 + 19s^2 - 63s - 213}{s^3 - 6s^2 - 46s + 20}$$

```
Gs after Transformation :
   -213 -63s +19s² +3s³
   --------------------
     20 -46s -6s² +s³
```

As we can see that

Gs=Gs'

Both transfer functions are identical

Part B)

Now we have to verify whether the eigenvalues of A and Poles of Gs are the same or not

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 7 & 2 & 3 \\ 2 & 9 & 1 \end{bmatrix}$$

det(sI-A)=0

we get eigenvalues as

```
Eigenvalues of A are:
   10.283901 + 0.i
  -4.6978728 + 0.i
   0.4139719 + 0.i
```

Poles of Gs are

```
Poles of Gs are:
   10.283901 + 0.i
  -4.6978728 + 0.i
   0.4139719 + 0.i
```

We can observe that the poles of Gs and eigenvalues of A are the same

Part C)

Let's our two transfer functions be

$$Gs1 = \frac{s^2 + 5s + 9}{s^2 + 8s + 15}$$

$$Gs2 = \frac{s + 4}{s^2 + 8s + 15}$$

Here Gs1 is proper, and Gs2 is the strictly proper transfer function

Now we can write the proper transfer function as follows

$$Gs1 = 1 + \frac{-3s - 6}{s^2 + 8s + 15}$$

Now we can observe that there is a feedforward path in a proper transfer function

And we know that D is feedforward gain

Hence D will have a non zero value for Proper transfer function

D=0 for the strictly proper transfer function

```
Proper Transfer Function
A:
  -6.3    -0.6
   7.15   -1.7
B:
  -2.6832816
   1.3416408
C:
   1.118034   -2.220D-16
D:
   1.
```

```
Strictly Proper Transfer Function
A:
  -3.875    0.125
   7.875   -4.125
B:
  -1.5811388
   1.5811388
C:
  -0.6324555    5.551D-17
D:
   0.
```

Code for the same:

```
s=%s;
A=[3,4,1;7,2,3;2,9,1];
B=[2;3;5];
C=[2,1,6];
T=[1,3,4;5,6,9;3,2,8];
I=eye(3,3);
D=3;
Gs = C*inv(s*I-A)*B + D;
printf("Original Gs : ");
disp(Gs);

A=inv(T)*A*T;
B=inv(T)*B;
C=C*T;
Gs1 = C*inv(s*I-A)*B + D;
printf("Gs after Transformation : ");
disp(Gs1);

poles=roots(Gs.den);
```

```
printf("Poles of Gs are:");
disp(poles);

eigen_values = spec(A);
printf("Eigenvalues of A are:");
disp(eigen_values);

printf('Proper Transfer Function \n');
Gs1=(s^2+5*s+9)/(s^2+8*s+15);
[A,B,C,D]=abcd(Gs1);
printf("A:"); disp(A);
printf("B:"); disp(B);
printf("C:"); disp(C);
printf("D:"); disp(D);

printf('Strictly Proper Transfer Function \n');
Gs2=(s+4)/(s^2+8*s+15);
[A,B,C,D]=abcd(Gs2);
printf("A:"); disp(A);
printf("B:"); disp(B);
printf("C:"); disp(C);
printf("D:"); disp(D);
```

Question 2: State-Space Realization

$$\mathrm{Gs1} = \frac{s+3}{s^2+5s+4}$$

$$\mathrm{Gs2} = \frac{s+1}{s^2+5s+4}$$

Now we have required a 2x2 matrix for the state-space realization

In Gs2, we have a pole at s=-1

Hence to avoid pole-zero cancellation, we can take zero at (s+1.001)

This won't affect the matrix that much, and there will be no pole-zero cancellation

$$\mathrm{Gs2} = \frac{s+1.001}{s^2+5s+4}$$

Using Scilab, we get State-Space realization as

$$Gs1 = \frac{s+3}{s^2+5s+4} \qquad\qquad Gs2 = \frac{s+1.001}{s^2+5s+4}$$

```
Transfer function with zero at -3
A:
  -1.5384615   0.3076923
   4.3076923  -3.4615385
B:
  -1.1094004
   1.6641006
C:
  -0.9013878   5.551D-17
D:
   0.
```

```
Transfer Function with zero at -1.001
A:
  -3.9983998   0.0011991
   4.0011991  -1.0016002
B:
  -1.2649742
   0.6331196
C:
  -0.79053   5.551D-17
D:
   0.
```

Code for the same:

```
s=%s;

printf('Transfer function with zero at -3 \n');
Gs1=(s+3)/(s^2+5*s+4);
[A,B,C,D]=abcd(Gs1);
printf("A:"); disp(A);
printf("B:"); disp(B);
printf("C:"); disp(C);
printf("D:"); disp(D);

printf('Transfer Function with zero at -1.001 \n');
Gs2=(s+1.001)/(s^2+5*s+4);
[A,B,C,D]=abcd(Gs2);
printf("A:"); disp(A);
printf("B:"); disp(B);
printf("C:"); disp(C);
printf("D:"); disp(D);
```

Question 3: Pole-Zero Cancellation I

Let's take A, B, C as follows

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 0 & 6 \end{bmatrix}$$

There is one element 0 in C

$$(\mathrm{sI}-\mathrm{A})^{-1} = \begin{bmatrix} \frac{1}{s-2} & 0 & 0 \\ 0 & \frac{1}{s-4} & 0 \\ 0 & 0 & \frac{1}{s-7} \end{bmatrix}$$

$\mathrm{Gs} = \mathrm{C}(\mathrm{sI}-\mathrm{A})^{-1}\mathrm{B}$ (we have assumed D=0)

Now we can observe that the pole corresponding to 0 elements in C will not be there in Transfer function Gs

$$\mathrm{Gs} = \frac{36\mathrm{s}-102}{\mathrm{s}^2-9s+14} = \frac{36s-102}{(s-7)(s-2)}$$

```
Eigenvalues of A are:
   2.
   4.
   7.
Poles of Gs are:
   7. + 0.i
   2. + 0.i
```

Identical results are obtained if one of the elements in B is 0

Code for the same:

```
s=%s;
A=[2,0,0;0,4,0;0,0,7];
B=[2;4;5];
C=[3,0,6];
I=eye(3,3);
D=0;
Gs = C*inv(s*I-A)*B + D;
printf("Gs is : ");
disp(Gs);

eigen_values = spec(A);
printf("Eigenvalues of A are:");
disp(eigen_values);

poles=roots(Gs.den);
printf("Poles of Gs are:");
disp(poles);
```

Question 4: Pole-Zero Cancellation II

In this part, we have to take A such that it is an upper triangular matrix with the entry (1,3) equal to 0

$$A = \begin{bmatrix} a & b & 0 \\ 0 & c & d \\ 0 & 0 & e \end{bmatrix} \quad B = \begin{bmatrix} f \\ g \\ h \end{bmatrix} \quad C = \begin{bmatrix} i & j & k \end{bmatrix}$$

$$\mathrm{Gs} = \mathrm{C}(\mathrm{sI} - \mathrm{A})^{-1}\mathrm{B}$$

By calculating Gs, we get the expression of form

Gs = Gs1 + Gs2

Where

$$Gs1 = \frac{i(f(s-c)(s-e) + bg(s-e) + bdh)}{(s-a)(s-c)(s-e)}$$

$$Gs2 = \frac{j\big(g(s-a)(s-e) + hd(s-a)\big) + ch(s-a)(s-b)}{(s-a)(s-c)(s-e)}$$

Now there are three cases for repeated diagonal elements

- a = c : cancellation occurs when ➔ *g(c-e)+hd = 0* **OR** *b = 0*
- a = e : cancellation occurs when ➔ *b = 0* **OR** *d = 0*
- c = e : cancellation occurs when ➔ *j(c-a)+ib = 0* **OR** *d = 0*

Now following are the examples for each case

$$A_1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 5 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 3 & 4 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad A_3 = \begin{bmatrix} 9 & 7 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

```
Gs for a=c :
    -95 +40s
   ----------
   10 -7s +s^2
Eigenvalues of A1 are:
   2. + 0.i
   2. + 0.i
   5. + 0.i
Poles of Gs are:
   5. + 0.i
   2. + 0.i
```

```
Gs for a=e :
    -216 +40s
   -----------
   21 -10s +s^2
Eigenvalues of A2 are:
   3. + 0.i
   7. + 0.i
   3. + 0.i
Poles of Gs are:
   7. + 0.i
   3. + 0.i
```

```
Gs for c=e :
    -252 +40s
   -----------
   45 -14s +s^2
Eigenvalues of A3 are:
   9. + 0.i
   5. + 0.i
   5. + 0.i
Poles of Gs are:
   9. + 0.i
   5. + 0.i
```

Code for the same:

```
s=%s;
A1=[2,0,0;0,2,3;0,0,5];
A2=[3,4,0;0,7,0;0,0,3];
A3=[9,7,0;0,5,0;0,0,5];
B=[2;4;5];
C=[3,1,6];
I=eye(3,3);
D=0;
// a=c
Gs = C*inv(s*I-A1)*B + D;
printf("Gs for a=c : ");
disp(Gs);

eigen_values = spec(A1);
printf("Eigenvalues of A1 are:");
disp(eigen_values);

poles=roots(Gs.den);
printf("Poles of Gs are:");
disp(poles);
// a=e
Gs = C*inv(s*I-A2)*B + D;
printf("Gs for a=e : ");
disp(Gs);

eigen_values = spec(A2);
printf("Eigenvalues of A2 are:");
disp(eigen_values);

poles=roots(Gs.den);
printf("Poles of Gs are:");
disp(poles);
// c=e
Gs = C*inv(s*I-A3)*B + D;
printf("Gs for c=e : ");
disp(Gs);

eigen_values = spec(A3);
printf("Eigenvalues of A3 are:");
disp(eigen_values);

poles=roots(Gs.den);
printf("Poles of Gs are:");
disp(poles);
```