EE324 CONTROL SYSTEMS LAB

Problem Sheet 4

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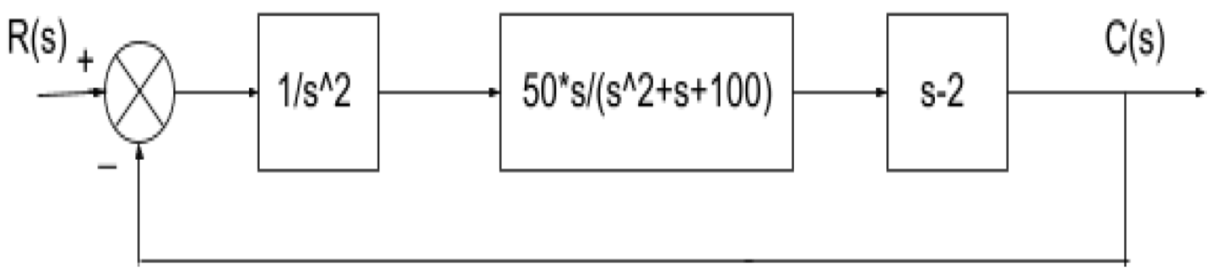
Question 1: Transfer Function for Complex Interconnected System

Part A)

In this part, we are given the following interconnected systems



The above-interconnected systems are simplified using the following Scilab Code

Scilab code for the same:

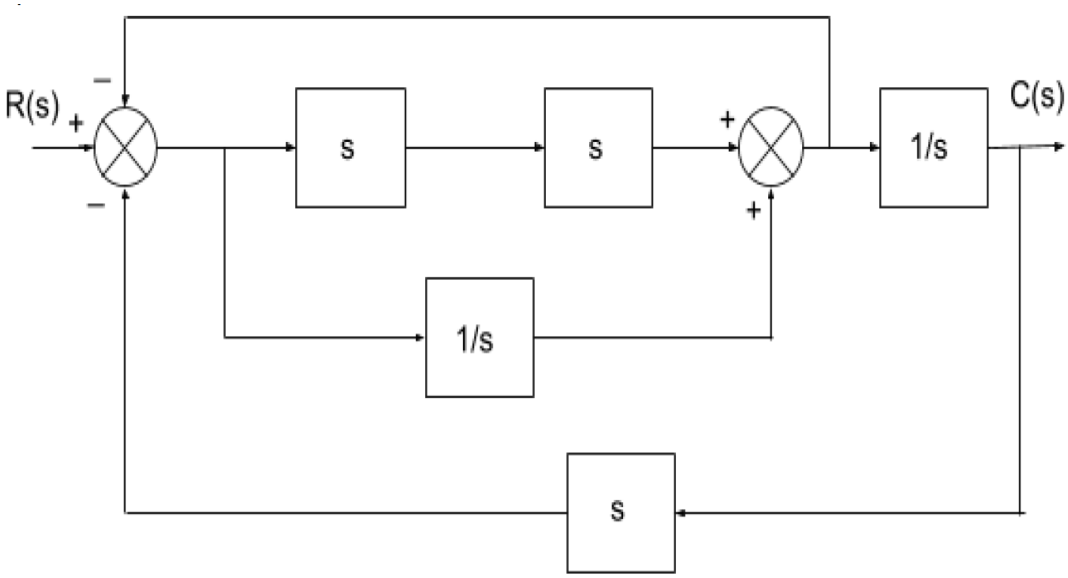
```
//Part A

s=%s;
p1=1/s^2;
p2=50*s/(s^2+s+100);
p3=s-2;
G1=p1*p2*p3 ;
Ans_a= syslin('c',G1/(1+G1));
disp(Ans_a);
```

Closed-Loop Transfer Function Obtained:

$$\frac{-100 + 50s}{-100 + 150s + 1s^2 + s^3}$$

Part B)

In this part, we are given the following interconnected system



Scilab code for the same:

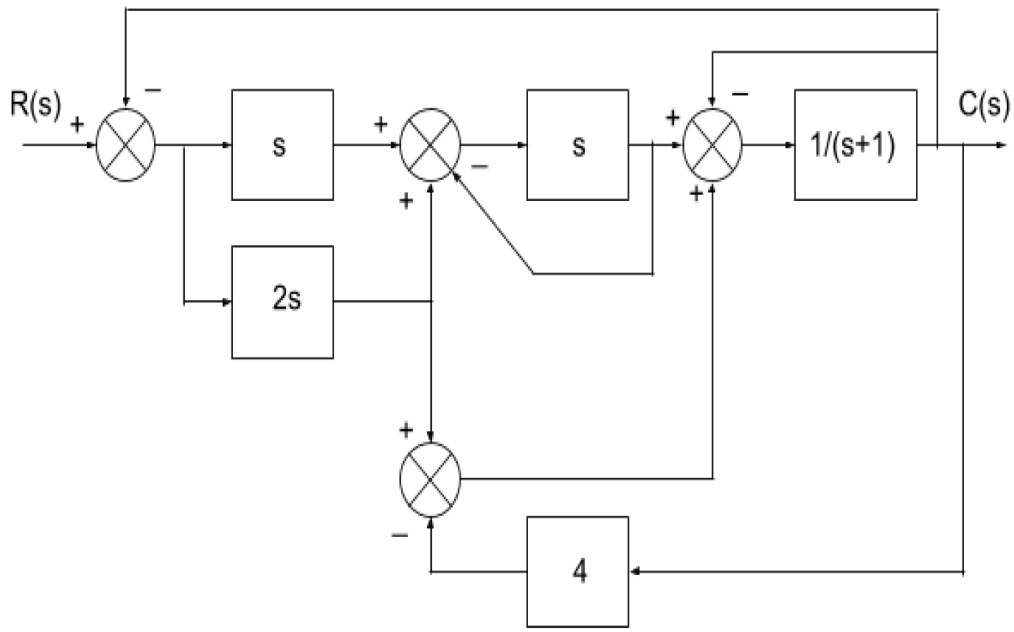
```
//Part B

s=%s;
m1=s;
m2=1/s;
G2=m1*m1+m2;
G3=2;
G4=G2/(1+G3*G2);
Ans_b= syslin('c',G4*m2);
disp(Ans_b);
```

Closed-Loop Transfer Function Obtained:

$$\frac{1 + s^3}{2s + s^2 + 2s^4}$$

Part C)

In this part, we are given the following interconnected system



Scilab code for the same:

```
//Part C

s=%s;
n1=s;
n2=2*s;
n3=1/(s+1);
G5=n1+n2;
G6=s/(s+1);
G7=G5*G6;
G8=G7+n2;
G9=n3/(1+5*n3);
G10=G8*G9;
Ans_c=syslin('c',G10/(1+G10));
disp(Ans_c);
```

Closed-Loop Transfer Function Obtained:

$$\frac{0.3333333s + 0.8333333s^2}{1 + 1.5s + s^2}$$

Question 2: Root Locus Analysis and Stability of System

Part A)

In this question, we are given the following plant Gs

$$Gs = \frac{10}{s(s+2)(s+4)}$$

This plant is connected in series with proportionality gain K. This K*Gs plant is connected in a negative feedback loop. Hence Closed Loop Transfer Function(Ts) becomes

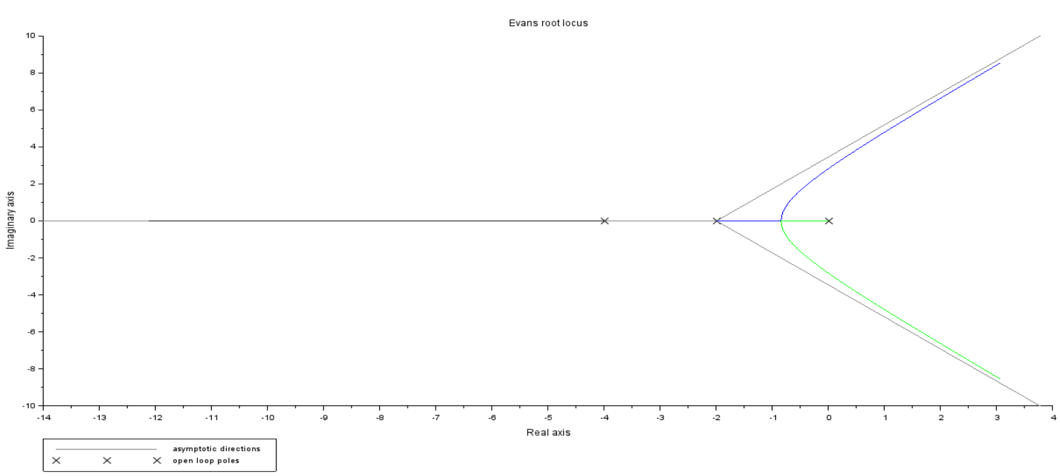
$$Ts = \frac{K * Gs}{1 + K * Gs}$$

Scilab code for the same:

```
s=%s;
Gs=10/(s*(s+2)*(s+4));
Gs=syslin('c',Gs);

k=3;                                //Random gain K
Ts=syslin('c',(k*Gs)/(1+k*Gs));
disp(Ks);
```

Part B)

This section will plot poles of a closed-loop transfer function for the gain value K from o to 100.

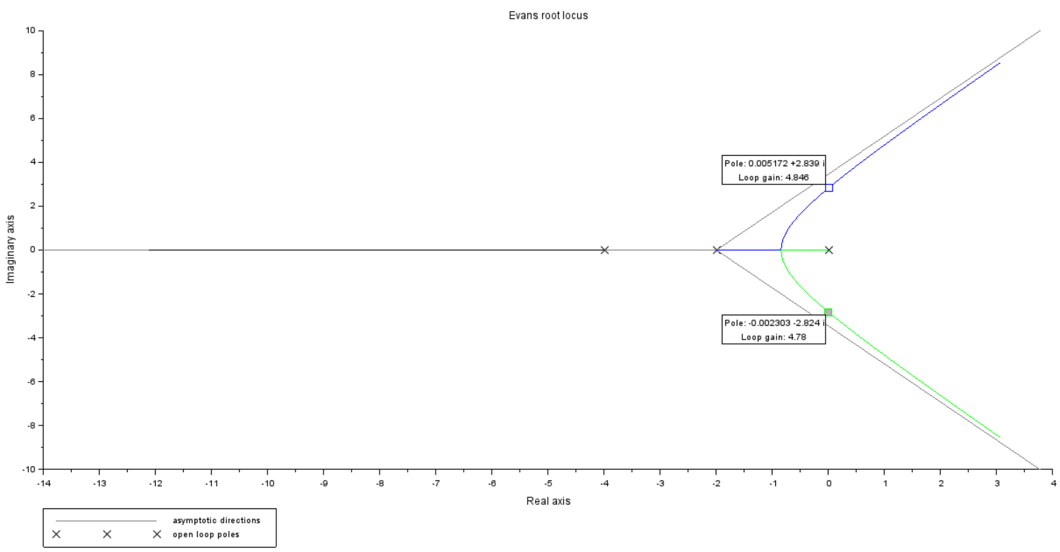


Scilab code for the same:

```
s=%s;
Gs=syslin('c',10/(s*(s+2)*(s+4)));
evans(Gs,100);
```

Part C)

This section will estimate K's value for which the closed-loop system's poles appear on an imaginary axis. From the above root locus, we can observe that pole crosses the imaginary axis at some k>0. When the poles of the system go into RHP, the system becomes unstable.



From the above figure, we can approximate that two closed-loop system poles are imaginary for k=4.8.

Part D)

We have the following closed-loop transfer function

$$\mathrm{Ts} = \frac{\mathrm{K} * \mathrm{Gs}}{1 + \mathrm{K} * \mathrm{Gs}}$$

$$\mathrm{Gs} = \frac{10}{\mathrm{s}(\mathrm{s}+2)(\mathrm{s}+4)}$$

So now Ts becomes

$$\mathrm{Ts} = \frac{10\mathrm{k}}{\mathrm{s}^3 + 6\mathrm{s}^2 + 8\mathrm{s} + 10\mathrm{k}}$$

Characteristic Polynomial is

$$Ps = s^3 + 6s^2 + 8s + 10k$$

We will now use Routh-Hurwitz Criterion

Routh Table for the above Polynomial is

s^3	1	8	0
s^2	6	10k	0
s^1	$\frac{48-10k}{6}$	0	0
s^0	10k	0	0

For all the poles of the system to lie in LHP or Imaginary axis

$$\frac{48-10k}{6} \geq 0 \text{ and } 10k>0$$

$$k \leq 4.8$$

Hence for k=4.8, poles of the CL system are the Imaginary axis. i.e. system is on the verge of instability

Question 3: Routh-Hurwitz Criterion

In this part, we have to use the special command in Scilab called "routh_t" for obtaining routh tables of given polynomials

Scilab code for the same:

```
s=%s;
P1=s^5 + 3*s^4 + 5*s^3 + 4*s^2 + s + 3;
[r1,num1] = routh_t(P1);

P2=s^5 + 6*s^3 + 5*s^2 + 8*s + 20;
[r2,num2] = routh_t(P2);

P3=s^5 - 2*s^4 + 3*s^3 - 6*s^2 + 2*s - 4;
[r3,num3] = routh_t(P3);
```

```
P4=s^6 + s^5 - 6*s^4 + s^2 + s - 6;
[r4,num4] = routh_t(P4);

mprintf("For Polynomial P1=s^5+3*s^4+5*s^3+4*s^2+s+3   ==> Sign
Changes=%g",num1);
disp(r1);
mprintf("\nFor Polynomial P2=s^5+6*s^3+5*s^2+8*s+20      ==> Sign
Changes=%g",num2);
disp(r2);
mprintf("\nFor Polynomial P3=s^5-2*s^4+3*s^3-6*s^2+2*s-4 ==> Sign
Changes=%g",num3);
disp(r3);
mprintf("\nFor Polynomial P4=s^6+s^5-6*s^4+s^2+s-6      ==> Sign
Changes=%g",num4);
disp(r4);
```

Routh Tables:

1) P1=s^5+3*s^4+5*s^3+4*s^2+s+3

	1	2	3
1	1	5	1
2	3	4	3
3	3.6667	0	0
4	4	3	0
5	-2.75	0	0
6	3	0	0

```
For Polynomial P1=s^5+3*s^4+5*s^3+4*s^2+s+3   ==> Sign Changes=2
```

2) P2=s^5+6*s^3+5*s^2+8*s+20

```
              1                          6        8
              -                          -        -
              1                          1        1

             eps                         5       20
             ---                         -       --
              1                          1        1

         -5 +6eps                   -20 +8eps    0
         --------                   ---------    -
             eps                        eps      1

     -25 +50eps -8eps²                  20       0
     -----------------                  --       -
          -5 +6eps                      1        1

 -2.274D-13 -160eps -64eps²              0        0
 --------------------------              -        -
     -25 +50eps -8eps²                   1        1

              20                         0        0
              --                         -        -
              1                          1        1
```

```
For Polynomial P2=s^5+6*s^3+5*s^2+8*s+20      ==> Sign Changes=2
```

3)P3=s^5-2*s^4+3*s^3-6*s^2+2*s-4

	1	2	3
1	1	3	2
2	-2	-6	-4
3	-8	-12	-0
4	-3	-4	0
5	-1.3333	0	0
6	-4	0	0

```
For Polynomial P3=s^5-2*s^4+3*s^3-6*s^2+2*s-4 ==> Sign Changes=1
```

4)P4=s^6+s^5-6*s^4+s^2+s-6

```
 1     -6   1    -6
 -     --   -    --
 1      1   1     1

 1      0   1     0
 -      -   -     -
 1      1   1     1

 -6     0   -6    0
 --     -   --    -
 1      1   1     1

-24     0   0     0
---     -   -     -
 1      1   1     1

eps    -6   0     0
---    --   -     -
 1      1   1     1

-144    0   0     0
----    -   -     -
eps     1   1     1

864     0   0     0
----    -   -     -
-144    1   1     1
```

```
For Polynomial P4=s^6+s^5-6*s^4+s^2+s-6        ==> Sign Changes=3
```

Question 4: Routh Table

Part A)

In this part, we have to construct a six degree polynomial such that the entire corresponding to s^3 of its Routh Table should be zero.

If even Polynomial is a factor of the original Polynomial, there exists a row in Routh Table with all entries as zero.

If the entire corresponding to s^3 is zero, then the degree of that even Polynomial is 4.

Let the even Polynomial with degree 4 be

$$Ps1=s^4 + 4s^2 + 5$$

Now we require polynomial with degree 6. We can get this Polynomial by multiplying non-even 2-degree Polynomial to the above Polynomial.

$$Ps2=s^2 + 2s + 3$$

So our 6 degrees required Polynomial is

Ps=Ps1*Ps2

$$Ps= s^6 + 2s^5 + 7s^4 + 8s^3 + 17s^2 + 10s + 15$$

We can verify the above results using Scilab

Routh Table:

	1	2	3	4
1	1	7	17	15
2	2	8	10	0
3	3	12	15	0
4	12	24	0	0
5	6	15	0	0
6	-6	0	0	0
7	15	0	0	0

Scilab code for the same:

```
s=%s;
P1=s^6 + 2*s^5 + 7*s^4 + 8*s^3 + 17*s^2 + 10*s + 15;
[r1,num1] = routh_t(P1);
```

Part B)

In this part, we have to find an 8-degree polynomial whose s^3 row in Routh Table is zero. We will follow the same process as in Part A.

If the entire corresponding to s^3 is zero, then the degree of that even Polynomial is 4.

Let the even Polynomial with degree 4 be

Ps1=$s^4 + s^2 + 1$

Now we require polynomial with degree 8. We can get this Polynomial by multiplying non-even 4-degree Polynomial to the above Polynomial.

Ps2=$s^4 + 4s^3 + 3s^2 + 5s + 8$

So our 6 degrees required Polynomial is

Ps=Ps1*Ps2

Ps= $s^8 + 4s^7 + 4s^6 + 9s^5 + 12s^4 + 9s^3 + 11s^2 + 5s + 8$

We can verify the above results using Scilab

Routh Table:

	1	2	3	4	5
1	1	4	12	11	8
2	4	9	9	5	0
3	1.75	9.75	9.75	8	0
4	-13.2857	-13.2857	-13.2857	0	0
5	8	8	8	0	0
6	32	16	0	0	0
7	4	8	0	0	0
8	-48	0	0	0	0
9	8	0	0	0	0

Scilab code for the same:

```
s=%s;
P2=(s^4+s^2+1)*(s^4+4*s^3+3*s^2+5*s+8);
[r2,num2] = routh_t(P2);
```

Part C)

In this part, we have to find a 6-degree polynomial such that the first entry of the s^3 row of its Ruth Table is zero.

Here we have to build the Routh Table from bottom to top according to specifications

Step 1] Assume entries of s^3 rows be

s^3	0	2	0

Step 2] We can assume entries in s^4 rows at random with the constraint of no more than the first three entries should be non zero

s^4	2	3	5
s^3	0	2	0

Step 3] Now, we have to choose s^5 row such that the first entry of s^3 is 0 and the second entry is 2

s^5	4	6	12	0
s^4	2	3	5	0
s^3	0	2	0	0

Step 4] Same iteration for s^6 row now

s^6	1	3.5	6	5	0
s^5	4	6	12	0	0
s^4	2	3	5	0	0
s^3	0	2	0	0	0

Hence our required equation is

$$\text{Ps}= s^6 + 4s^5 + 3.5s^4 + 6s^3 + 6s^2 + 12s + 5$$

We can verify this result using Scilab also

Scilab code for the same:

```
s=%s;
P3=s^6 + 4*s^5 + 3.5*s^4 + 6*s^3 + 6*s^2 + 12*s + 5;
[r3,num3] = routh_t(P3);
```

Routh Table:

```
        1          3.5   6    5
        -          ---   -    -
        1           1    1    1

        4           6    12   0
        -           -    --   -
        1           1    1    1

        2           3    5    0
        -           -    -    -
        1           1    1    1

       eps          2    0    0
       ---          -    -    -
        1           1    1    1

   -4 +3eps         5    0    0
   --------         -    -    -
      eps           1    1    1

-8 +6eps -5eps²     0    0    0
---------------     -    -    -
   -4 +3eps         1    1    1

        5           0    0    0
        -           -    -    -
        1           1    1    1
```