EE324 CONTROL SYSTEMS LAB
Problem Sheet 8

Yatish Vaman Patil | 190070076

Question 1: Lag Compensator

$$\mathrm{Gs} = \frac{\mathrm{s} + \mathrm{k1}}{\mathrm{s} + \mathrm{k2}}$$

Part A)

Here we are keeping K_1/K_2 ratio constant. Assume $K_1/K_2=5$

Now we have to find the step response of Gs

Ys=Gs*Rs

Ys➔ output

Rs=1/s➔input

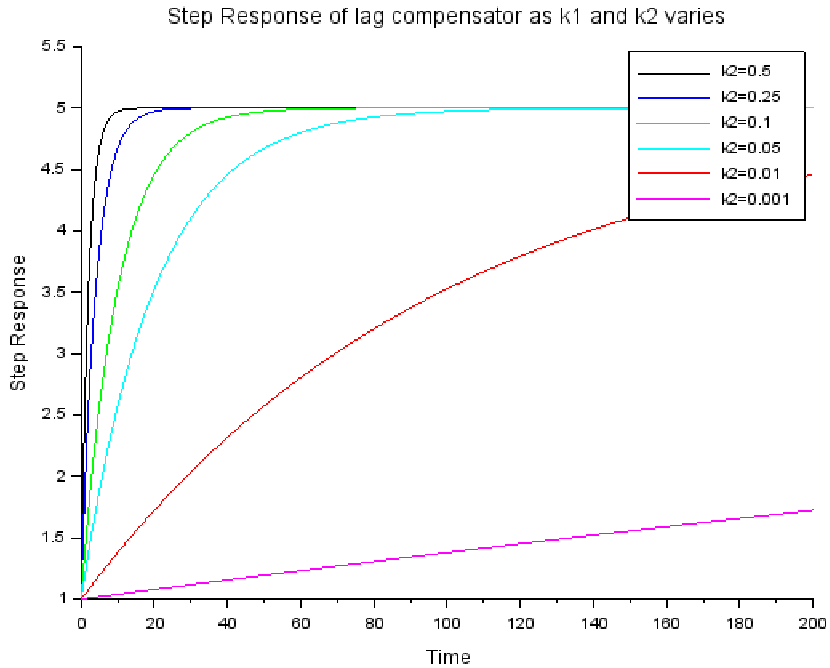
$$\mathrm{Ys} = \frac{\mathrm{s} + \mathrm{k1}}{\mathrm{s}(\mathrm{s} + \mathrm{k2})}$$

Now by taking inverse Laplace transform, we get

$$\mathbf{y(t)} = \frac{\mathbf{k1}}{\mathbf{k2}}\mathbf{u}(\mathbf{t}) + \frac{\mathbf{k2}-\mathbf{k1}}{\mathbf{k2}}\mathbf{e}^{-\mathbf{k2t}}$$

As we have taken the k1/k2 ratio constant, the first term will remain the same even if we vary the values of k1 and k2.

- When we move away from the origin, the value of k2 increases. As k2 increases, step response becomes fast and attains steady quickly because of the exponential term
- When we move towards the origin, the value of k2 decreases, so steady-state response becomes slow, and it takes a long time to attain a steady-state because of the exponential term



Code for the same:

```
s=%s;
k2=[0.5,0.25,0.1,0.05,0.01,0.001];
k1=k2.*5;
time=0:0.01:200
for i=1:length(k2)
    Gs=syslin('c',s+k1(i),s+k2(i));
    Ps=csim('step',time,Gs);
    plot2d(time,Ps,i);
end

xtitle("Step Response of lag compensator as k1 and k2 varies");
ylabel("Step Response");
xlabel("Time");
legend(["k2=0.5","k2=0.25","k2=0.1","k2=0.05","k2=0.01","k2=0.001"]);
```

Part B)

Here we are keeping K_1/K_2 ratio constant. Assume $K_1/K_2=5$

Now we have to find the impulse response of Gs

Ys=Gs*Rs

Ys→ output

Rs=1➔input

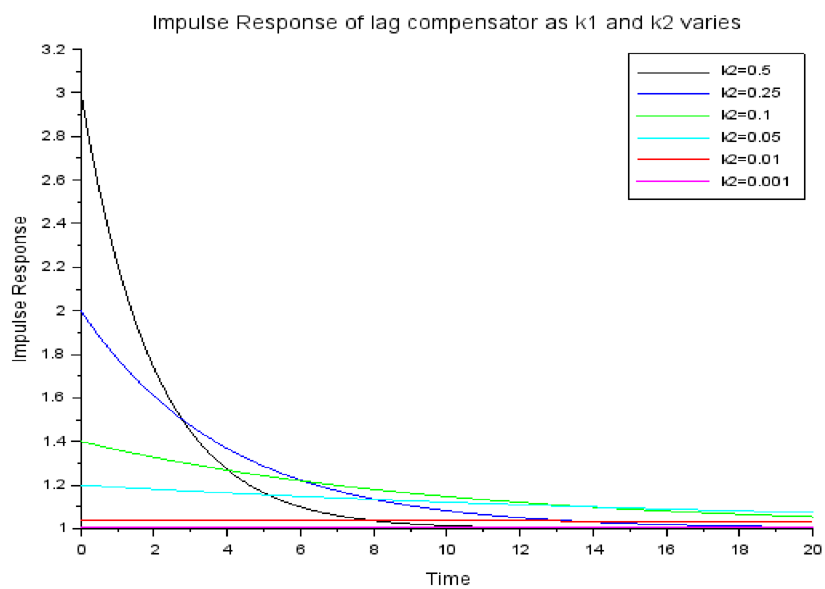
$$Ys = \frac{s + k1}{s + k2}$$

Now by taking inverse Laplace transform, we get

y(t) = $\boldsymbol{\delta(t) + (k1 - k2)e^{-k2t}}$

The first term will remain the same even if we vary the values of k1 and k2.

- If we take k2 farther from the origin, the exponential term will decay quickly and reach a steady-state value of 0 quickly



Code for the same:

```
s=%s;
k2=[0.5,0.25,0.1,0.05,0.01,0.001];
k1=k2.*5;
time=0:0.01:20
for i=1:length(k2)
    Gs=syslin('c',s+k1(i),s+k2(i));
    Ps=csim('impulse',time,Gs);
    plot2d(time,Ps,i);
end

xtitle("Impulse Response of lag compensator as k1 and k2 varies");
ylabel("Impulse Response");
xlabel("Time");
legend(["k2=0.5","k2=0.25","k2=0.1","k2=0.05","k2=0.01","k2=0.001"]);
```

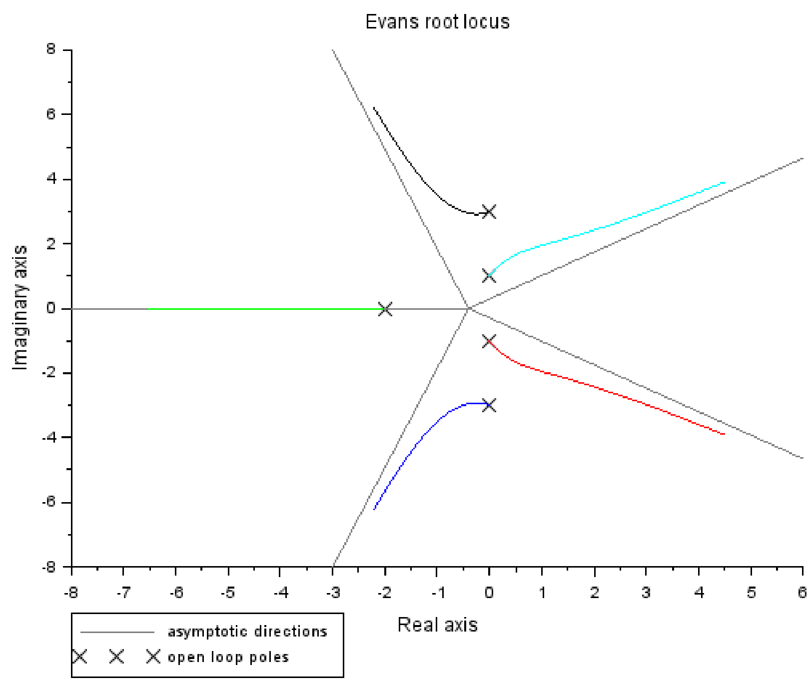
Question 2: Multiple phase Cross-over frequencies

Part A)

Let's take four non-repeating purely imaginary poles, and one real pole be

-2, $\pm 3j$, $\pm j$

$$\mathrm{Gs} = \frac{1}{(s+2)(s^2+9)(s^2+1)}$$



Code for the same:

```
s=%s;
Gs=syslin('c',1,(s+2)*(s^2+9)*(s^2+1));
evans(Gs,10000);
```

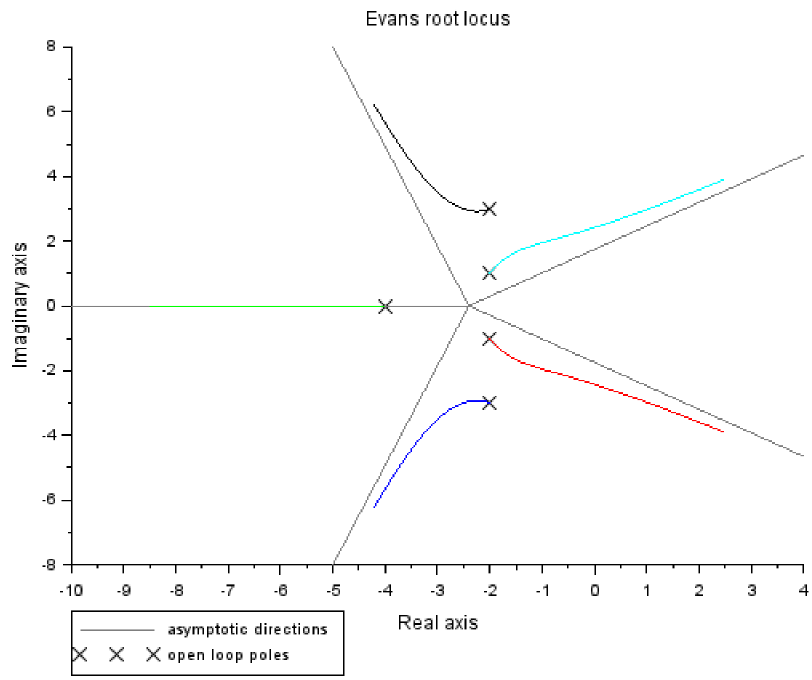
Part B)

We can shift the root locus in the left half-plane by two if we replace 's' with 's+2.'

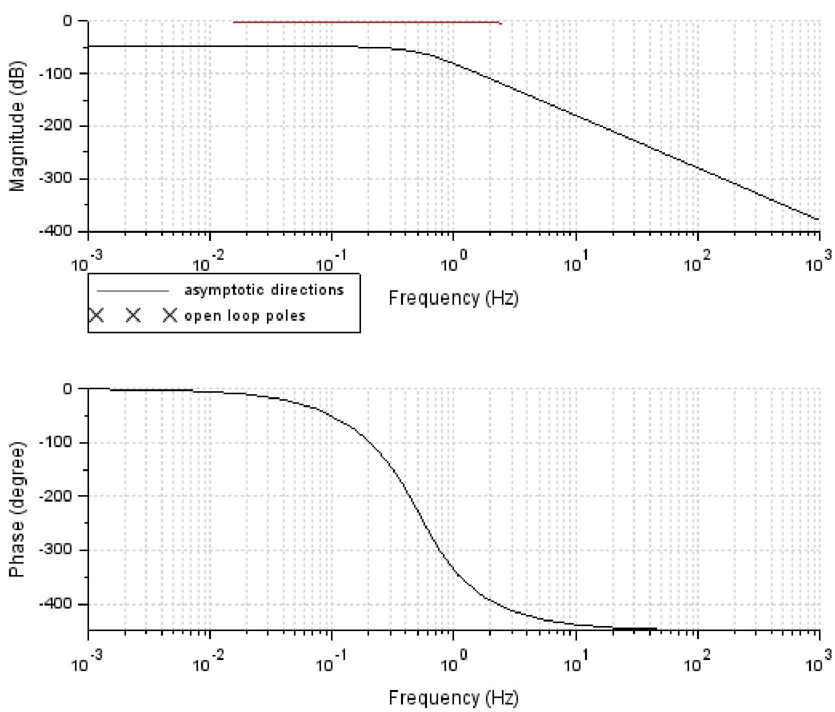
Gs becomes

$$\mathrm{Gs} = \frac{1}{((s+2)+2)((s+2)^2+9)((s+2)^2+1)}$$

Now the Gs is open-loop stable as all of its poles are in LHP



Bode plot for the given transfer function is



Code for the same:

```
s=%s;
Gs=syslin('c',1,(s+4)*((s+2)^2+9)*((s+2)^2+1));
evans(Gs,10000);
bode(Gs);
```

Part C)

In the above bode plot, we can see that the total phase change of the system is -450^0 as there are five poles. For getting two-phase crossover frequencies, a phase plot should first cross the -180^0 mark and then again cross $-180^{0,}$ so that the final value is above -180^0. We can take a net change in phase as -90^0, which indicate➔ **no of poles – no of zeros =1**

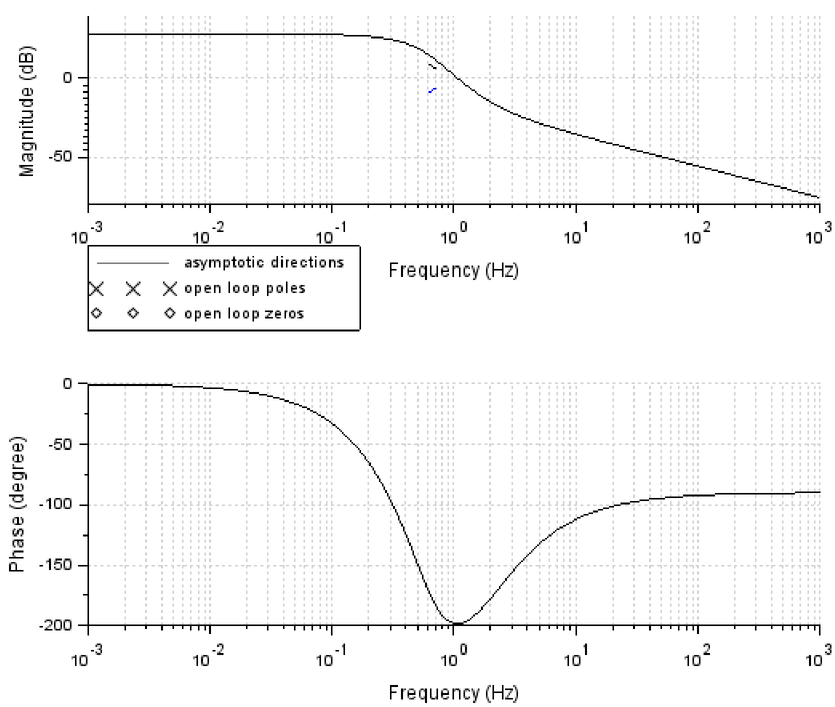
Hence we introduce four zeros such that there are minima of phase plot with a minimum value less than -180^0

This implies that zeros are away from origin as compared to poles. Hence I can take repeated zeros at, let's say, -9

Transfer function becomes

$$\mathrm{Gs} = \frac{(s+9)^4}{((s+2)+2)((s+2)^2+9)((s+2)^2+1)}$$

The Bode Plot of the new system is



We can see from the bode plot that there are two-phase crossover frequencies

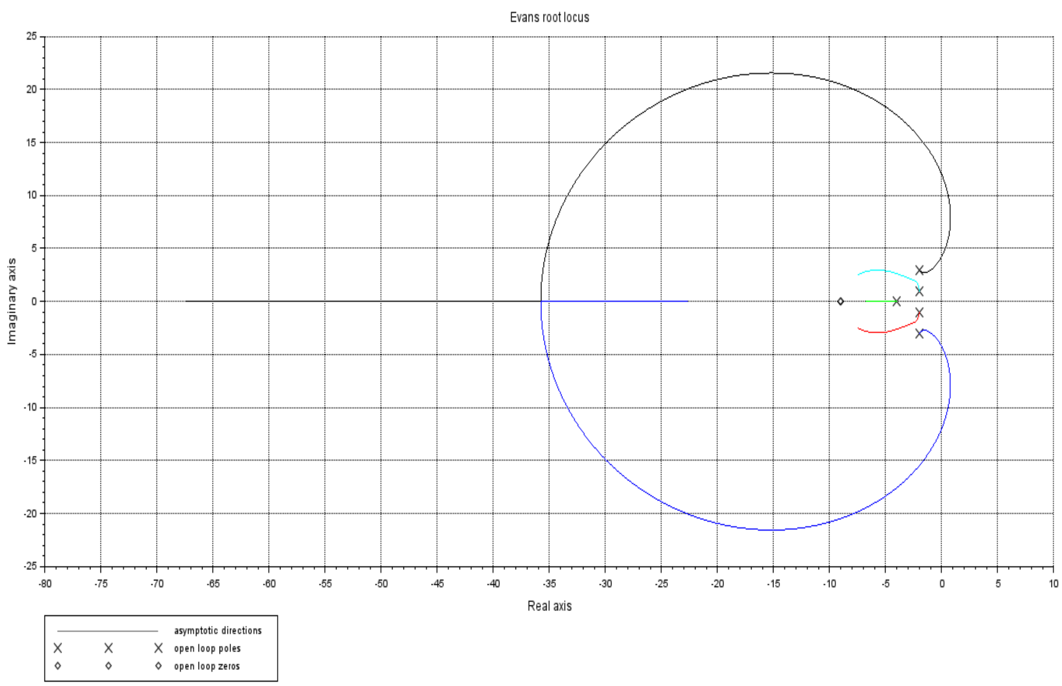
Code for the same:

```
s=%s;
Gs=syslin('c',(s+9)^4,(s+4)*((s+2)^2+9)*((s+2)^2+1));
bode(Gs);
```

Part D)

Root locus of

$$Gs = \frac{(s+9)^4}{((s+2)+2)((s+2)^2+9)((s+2)^2+1)}$$



We can see that the root locus of the new system crosses the imaginary axis at two points, i.e. two values of phase crossover frequencies

Code for the same:

```
s=%s;
Gs=syslin('c',(s+9)^4,(s+4)*((s+2)^2+9)*((s+2)^2+1));
xgrid();
evans(Gs,100);
```

Question 2: Bode Plot

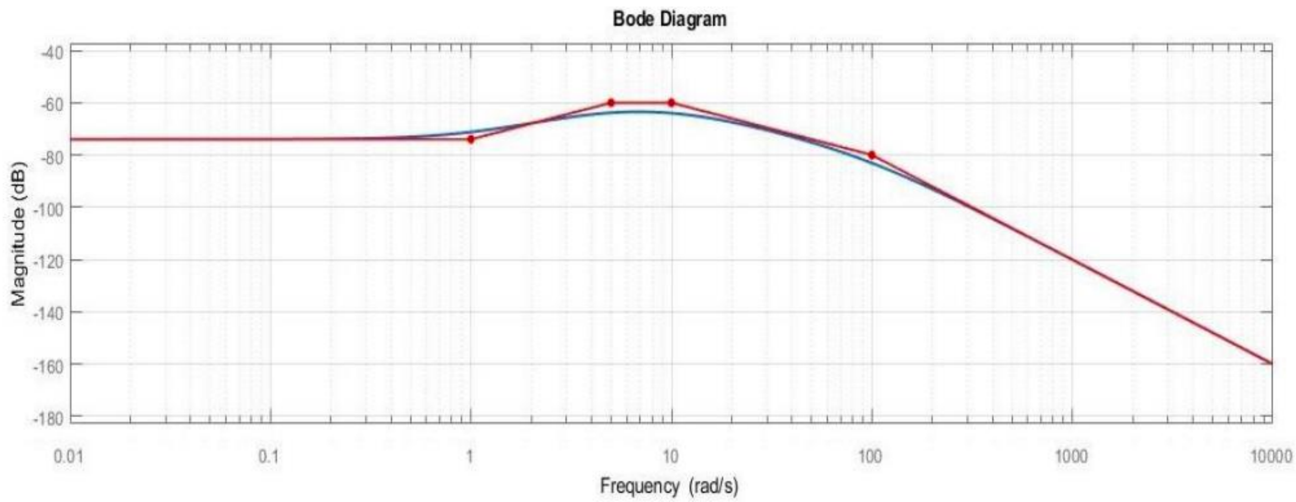


Fig: Magnitude plot of the transfer function

From the given magnitude bode plot, we will focus on asymptotic plot

We know that the slope of magnitude bode plot changes as

- For Poles → Slope decreases by 20 dB/decade for single pole
- For zeros → Slope increases by 20 dB/decade for single zero

By observing the above asymptotic bode plot, we can say that

- Slope decreases by 20 dB/decade at 5, 10, 100
- Slope increases by 20 dB/decade at 1

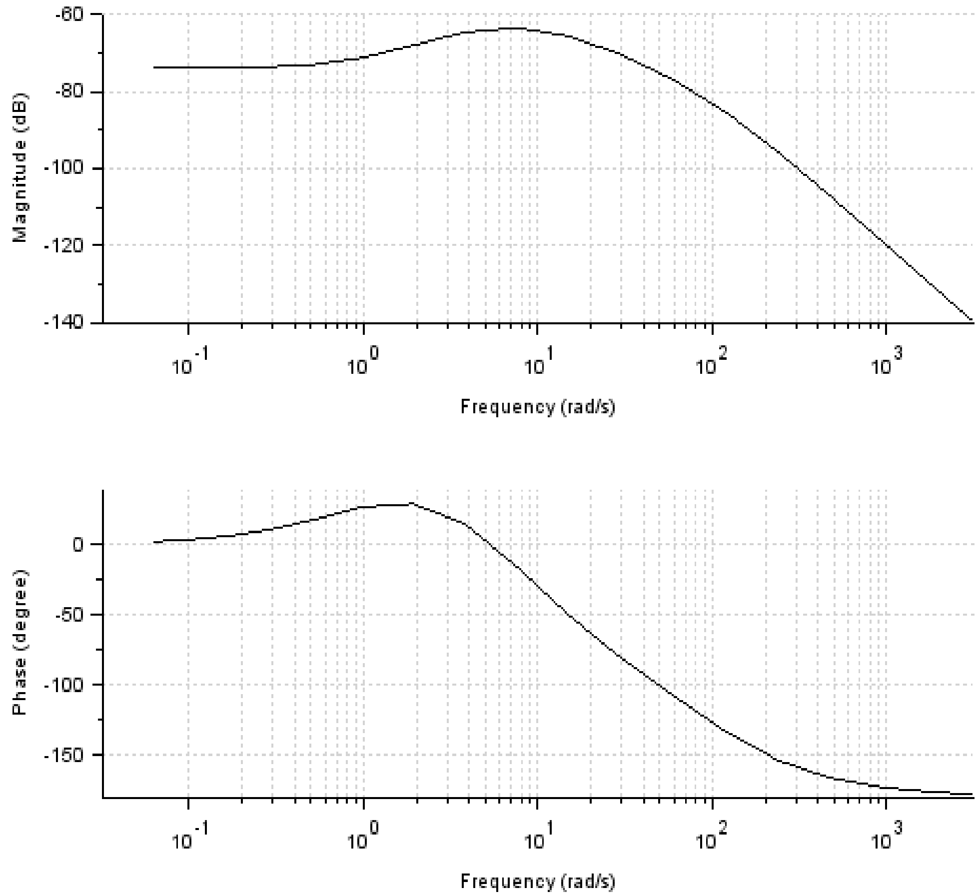
Hence we have,

- Poles → 5, 10, 100
- Zero → 1

$$\mathrm{Gs} = \frac{s+1}{(s+5)(s+10)(s+100)}$$

Gs also satisfy the initial value of -73 dB

Following is the bode phase and magnitude plot for Gs



Code for the same:

```
s=%s;
Gs=syslin('c',(s+1),(s+5)*(s+10)*(s+100));
bode(Gs,"rad");
```