CUNY – Data Science

Midterm Examination, Games and Social Choice

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1. Let >~ be a preference relation on a set X. Define I(x) to be the set of all y ∈ X for which y ∼ x. Show that the set (of sets!) {I(x)|x ∈ X} is a partition of X, that is,

For all x and y, either I(x) = I(y) or I(x) ∩ I(y) = ∅

For every x ∈ X, there is y ∈ X such that x ∈ I(y)

Is the y unique in general?

2. State and prove the Arrow impossibility theorem

Theorem: Arrows theorem says that when voters have at least three options, no ranked voting electoral system can transform individuals' preferences into community-wide preferences that is complete, transitive, and respects irrelevance of independent alternatives. Only a dictatorship.

Proof: If all voters put B at the bottom of their ranking, B will be at the bottom of the community wide ranking. Additionally, if every voter puts A>B, B>C, although it is not rational and by transitivity A>C, C>A can still hold because according to irrelevance of independent alternatives, CA does not affect AB and BC. If one by one voters 1 through n move B from the bottom to the top of their ranking, there will be one point where a specific voter n = n* who after moving his ranking of B from the bottom to the top causes the community wide ranking of B to go from the bottom to the top. Compare profile 1 where all voters before n* moved B from the bottom to the top with profile 2 where the only difference is voter n* also moved his ranking of B from the bottom to the top. Following profile one, the community wide ranking still has B at the bottom and following profile 2 the community wide ranking now has B at the top and the only difference between the votes in the profiles is n*'s vote so n* must be a pivotal voter.

Pivotal voter n* is also a dictator over rankings not involving B. Say pivotal voter n* changes his ranking and puts A at the top, so his ranking is A>B>C and the other voters leave B at the top, so B>A and B>C according to all other voters, by independence of irrelevant alternatives it can still hold that A>C, since BA and BC do not affect the AC relationship. A>C is how the voters originally ranked AC and since n* put A>C, social preference must agree with n*'s AC ranking.

Pivotal voter n* is a dictator over any ranking including any 2 out of the three alternatives because there cannot be multiple dictators over multiple alternatives, otherwise there would be conflict when addressing the relationship between those two options. Say X is a dictator over A and Y is a dictator over B, they cannot both win when comparing AB so X must be the same as Y.

3. Give an informal account of the result which shows why (under certain circumstances) a candidate running for election is best off being explicit. Be sure to include some mathematical details

A candidates average satisfaction is alpha = average of s(v, w) with v being the voters and w being the states of the world based on statements the candidate has made. Z is the set of w. If a candidate reveals more, the set of w compatible with one voter can increase while it can decrease for another. Say S is the candidate's best possible statement. If he leaves anything unaddressed in S, leaving it incomplete, there is room for both A and its negation in S. There exists both S with A and S its negation, either A or its negation will benefit him or at least leave him no worse off. So the best possible Statement S is where everything including A is addressed.

A pessimistic voter looks at the worst possible outcome, v(Z) = min{s(v,w) for all w in Z}

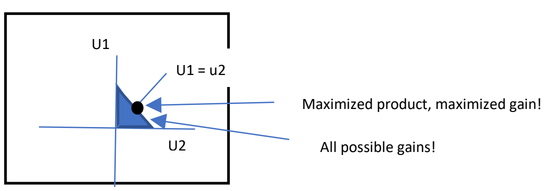
An expected value voter looks at the average v(Z) = avg{s(v,w) for all w in Z} (possibly also taking probabilities into account)

For the pessimistic voter, being as explicit as possible is always a better option because they either way already assume the worst so you can never make it worse for yourself with them. For the expected value voter, either A or its negation will help you get their vote or at least leave you no worse off with them. So be explicit!

4. State and sketch the proof of the Nash result on bargaining.

When Ann is given a choice among vanilla, chocolate and strawberry she chooses vanilla. But when she is given a choice between vanilla and chocolate, she chooses chocolate. Which condition of Nash is she not following?

There is a maximum available to be gained and according to Nash the maximized solution is the point in the first quadrant where u1*u2 is maximized. Make a graph where y = u1 and x = u2 and draw a line enclosing the set of possible alternatives, the solution is the point where the product of the gains is maximized.



She is not following completeness, if when choosing between vanilla, chocolate and strawberry, she chooses vanilla, her preferences are V>C and V>S. When only given the option between V and C, her preferences should still be V>C but they were C>V. Pareto Optimality: do not choose an alternative in the presence of a better alternative.

5. Ann weakly prefers A to B, weakly prefers B to C and strongly prefers C to A. Show how Ann is vulnerable to the "money pump" argument.

Say Ann's preferences are A>~B>~C>A then she may fall victim to the money pump because say she originally has C, then she is offered B for a penny, she will agree and take B, then say she is offered A for a pennies, she will agree and take A, then say she is offered C for 5 pennies, since she strongly prefers C to A, she will agree. Now she is back where she started with C and has lost 7 pennies. This can keep happening until she runs out of money or realizes she is being played. This is happening because her preferences do not follow transitivity which would say if she preferred A to B and B to C, even if the preference is a weak one, she should prefer A to C, not C to A.

6. a, b are two integers satisfying 2 ≤ a ≤ b ≤ 100 Let S be the sum of a, b and P be the product of a, b. There are two people, Mr. Sum and Mr. Product Mr. Sum is told the value of S. Mr. Product knows this but not what S is. Mr. Product is told the value of P. Mr. Sum knows this but not what P is. The following conversation then takes place. The Conversation

Mr. Product: I don't know what a, b are.

Mr. Sum: I knew you did not.

Mr. Product: But I know them now.

Mr. Sum: And so do I.

Why can't a, b be 3 and 5? Why can't they be 2 and 6? Show that one of a, b is odd and the other is even.

A, B cannot be 3 and 5 because those are both prime numbers and if A and B are both prime numbers then Mr. Product would be able to figure out A B right away since the only factors of prime numbers are 1 and itself and A,B are both greater than 1, so the only possible factors of two prime numbers multiplied together are those prime numbers.

A, B cannot be 2 and 6 because one of the numbers must be odd and one must be even. This is because a+b cannot be even because all even numbers can be written as the sum of two primes and we already addressed that A, B cannot both be prime