CUNY – Data Science

Game Theory and Social Choice

Final Examination

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1. State and prove the Arrow impossibility theorem (please state the theorem exactly)

Theorem: For any constitution with at least two voters and three options to vote, the only way they can reach a society-wide decision which respects completeness, transitivity, independence of irrelevant alternatives, and unanimity is through a dictatorship.

Proof: Say all voters put B at the bottom of their preference ranking, then the society puts B at the bottom. If voters put A>B and B>C, although by transitivity A>C, according to irrelevance of independent alternatives, C>A can still hold, since CA relationship does not impact AB and BC relationship.

Say one by one voters switch their ranking of B from the bottom to the top, at some point in the middle there will be a voter n = n* whose vote in pivotal in switching the society-wide ranking of b from the bottom to the top. Profile 1 can be where all voter before n* switched their ranking of B from bottom to top and Profile 2 can be where all voters after through n* switched their ranking of B from bottom to top. The only difference between profiles 1 and 2 is the vote of n* and in Profile 1, society wide ranking still had B on bottom and following Profile 2, society switched B to the top, therefore n*s vote must be a pivotal because it is his vote which made the switch

Pivotal voter n* also is a dictator over other rankings not involving B. Say pivotal voter n* puts A at the top of his ranking, so he has A>B>C but the rest of the voters leave B at the top. So other voters have B>A and B>C. By independence of irrelevant alternatives, it can hold that A>C, which is n* new ranking, and how the voters originally ranked AC in Profile 1. Since n* is a pivotal voter, society must agree with n*'s AC ranking and put A>C.

Pivotal voter n* is a dictator over any ranking between any 2 of the alternatives because there cannot be multiple dictators over multiple alternatives, otherwise there would be conflict when addressing the relationship between those alternatives. Say X is a dictator over A and Y is a dictator over B, they cannot both win when comparing AB so X must be the same as Y.

2. State and prove the Nash result on bargaining (please state the theorem precisely)

3. a, b are two integers where a = 2 and b = 5. Let S be the sum of a, b and P be the product of a, b.

There are two people, Mr. Sum and Mr. Product. They are told that 2 ≤ a ≤ b ≤ 100.

Mr sum is told the value of S (which is 7). Mr. Product is told the value of P which is 10.

Would Mr. Product be justified in saying "I don't know what a, b are?"

Would Mr. Sum be justified in saying "I don't know what a, b are?"

Would Mr. Product be justified in saying, "Sum does not know what a, b are"? Explain why.

Mr. Product would not be justified in saying "I don't know what a,b are" because the denominators of 10 are 1,10 and 2,5. A,b cannot be 1,10 because $2 \le a \le b \le 100$. Therefor Mr. Product must know a,b is 2,5.

Mr. Sum is justified in saying "I do not know what a,b are" because the intergers that add up to 7 are 1,6 2,5 3,4. While he knows a,b cannot be 1,6 because $2 \le a \le b \le 100$, Mr. Sum does know know if a,b is 2,5 or 3,4

Mr. Product is justified in saying "Sum does not know what a,b are" because Mr. Product was able to figure out that a,b are 2,5 since they are the only numbers from 2 to 100 that multiply to 10. Now Mr. Product known S=2+5=7. Mr. Product knows that Mr. Sum will not be able to figure out if a,b are 2,5 or 3,4 because both add up to 7 and Mr. Sum has no other information to narrow down his options.

4. What are the Nash equilibria of the following game? Is any equilibrium Pareto optimal?

	Bach	**Strav**
Bach	**4,2**	**0,0**
Strav	**5,1**	**2,4**

4,2 and 2,4 are the Nash equilibria of the game because the product of the gain is maximized. 2*4=8 which is greater than 5*1=5 and 0*0 = 1. Both 4,2 and 2,4 are pareto optimal because neither player can be made better off without making the other worse off. If 4,2 moves to 5,1, 5 is greater than 4, but 1 is less than 2. If 2,4 moves to 5,1, 5 is greater than 2, but 1 is less than 4.

5. State and sketch the proof of Nash's theorem on the existence of a Nash equilibrium

6. Explain the idea of a money pump showing why preferences have to satisfy transitivity

Preferences must satisfy transitivity otherwise the preferences will loop in an endless circle. Say A's preferences are vanilla > chocolate, chocolate > Strawberry, Strawberry > Vanilla. These preferences are not transitive, since if vanilla is preferred to chocolate and chocolate is preferred to strawberry, vanilla should also be preferred to strawberry. Say A has a strawberry ice-cream cone in her hand. B might offer A chocolate instead for only 1 penny. A will likely agree because A prefers chocolate to strawberry and 1 penny is not a lot of money. Now say B offers A vanilla instead of chocolate for only 1 penny. Again A will likely agree because A prefers vanilla to chocolate and 1 penny is not a lot of money. Now say B offers A strawberry instead of vanilla for only 1 penny. Again, A will likely agree because A prefers strawberry to vanilla and 1 penny is not a lot of money. Now A is back where they started with 3 less pennies to their name. This will keep happening until A realizes they are being played or until A runs out of money. Preferences must satisfy transitivity to be rational.