CUNY – Data Science

Game Theory and Social Choice

Homework 1 - Preferences

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Problem 1: Let >~ be a preference relation on a set X. Define I(x) to be the set of all y in X for which y ~ x. Show that the set (of sets) {I(x)|x in X} is a partition of X.

Show that for all x and y, either I(x) = I(y) or I(x) n I(y) = 0:

Everything within I(x) is tied with x, and everything within I(y) is tied with y. Say the intersection is not empty and there is some b that is tied with both x and y, then all values within I(x) are tied with b and all values within I(y) are tied with b, and since I(x) and I(y) are defined the same way, they are identical. Alternatively, there is no b that is tied with both x and y, then no values within I(x) and I(y) are tied, and no values are in common, and they are distinct sets.

Show that for every x of X, there is y of X such that x is in I(y):

The set I(X) is all y that is tied to x. Every value is tied to itself, therefor every x has itself in I(X)

Problem 4: Let > be an asymmetric binary relation on a finite set X that does not have a cycle, that is there is no finite sequence of elements x1, x2,. . ., xk, where K > 2, such that x1 > x2 > . . . > xk > x1. Show (by induction on the size of X) that > can be extended to a complete ordering (i.e., a complete, asymmetric, and transitive binary relation).

Since the set is asymmetric there are no duplicates and since there is no cycle, the set cannot loop back on itself. Therefor the set can be placed in a finite line going in one direction. If there are two elements in the set, one if bigger than the other so put the smaller number first and the set will have complete ordering. Add another value to the set X, and find its place in the line, if it is the smallest, it will come first, if it is largest, it will come last, if one value is larger and one is smaller it will be placed in the middle and there will be complete ordering. Add a fourth element, find the values larger and smaller than it and put it in its correct spot in the line. As elements are added to the set, as long as they are not duplicates and as long as they are placed in their correct spot, the set will extend to complete ordering.

Problem 5: It is assumed that a decision maker who is sometimes unable to compare between alternatives uses the following procedure: he has n criteria in mind, each represented by an ordering > i (i = 1. . . n)(which is asymmetric, transitive, and complete binary relation). He demonstrates that x > y if and only if x > iy for every i.

Part 1: Verify that the relation > generated by this procedure is asymmetric and transitive. Bring a real-life example where it is reasonable that a decision maker uses this procedure.

Since the criteria has an ordering which is asymmetric, transitive, and complete binary relation, this procedure generates an asymmetric and transitive relationship. It is asymmetric because greater than and less than are used, and it is transitive because if x is preferred over y, that means x is preferred over all criteria of y, and if y is preferred over z, it means y is preferred over all criteria of z, from this we can derive that x is preferred over all criteria of z. A real-life example would be buying a bottle of ketchup. Say there are two options, if one bottle is bigger and cheaper it is preferred, it passes both criteria.

Part 2: It is claimed that this assumption is vacuous: given any asymmetric and transitive relation, >, one can find a set of complete orderings > l, . . ., > n such that x > y iff x > iy for every i. Demonstrate this claim for the binary relation on the set X = {a,b,c} according to which only a > b and the comparisons between [b and c] and [a and c] are not determined.

Bottle A of orange juice is 20oz and $10, Bottle B of orange juice is 40 oz and $5. Bottle C of orange juice has more vitamins it is. According to this information: a > b, but it is difficult to compare a and b with c, so their relationships are not determined.

Part 3: Prove this claim for the general case.

There are always other factors that need to be considered when ordering items, different items excel in different criteria, so it is hard to accurately order the items.