CUNY – Data Science

Game Theory and Social Choice

Homework 7 – Common Knowledge

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Problem 1a: In the following picture explain what happens if the conversation begins with row announcing her average value. Then column speaks, then row etc.

2	3	5	2
7	8	9	10
3	2	2	5
5	4	3	2

1st: row announces his average value is 3.

2	3	5	2
~~7~~	~~8~~	~~9~~	~~10~~
3	2	2	5
~~5~~	~~4~~	~~3~~	~~2~~

2nd: Column eliminates row 2 and row 4 where the average is not 3, and column announces his average value is 2.5.

2	3	~~5~~	~~2~~
~~7~~	~~8~~	~~9~~	~~10~~
3	2	~~2~~	~~5~~
~~5~~	~~4~~	~~3~~	~~2~~

3rd: Row eliminates column 3 and column 4 where the average is not 2.5, and row announces his average is 2.5

2	3	~~5~~	~~2~~
~~7~~	~~8~~	~~9~~	~~10~~
3	2	~~2~~	~~5~~
~~5~~	~~4~~	~~3~~	~~2~~

4th: Column cannot eliminate any rows because they both have an average of 2.5, so the conversation is over and they have reached an average value of a 2.5, although the correct value is 2.

1b) Show that in any case where there are n rows and n columns, the conversation must end in consensus in at most n rounds (with each party speaking at most n times).

If all the available columns have the same average, and all the rows have the same average, then their average is the same and both players are stuck. Neither can eliminate any columns or rows so the conversation ends, and the true value is not reached. This remaining average is the average of every remaining cell, regardless of it you are dividing them by column or row. In this scenario, or if multiple rows or columns are eliminated at one, there will be less than n rounds. In the case where there are the most possible rounds, only one row or column will be eliminated each round. The conversation starts with row announcing his average, column eliminates 1^{st} row and announces, row eliminates 1^{st} column and announces, column eliminates 2^{nd} row and announces, row eliminates 2^{nd} column and announces, column eliminates 3^{rd} row and announces, row eliminates 3^{rd} column and announces, column announces his final value. Here there are 4 columns and 4 rows, both column and now announce n-1 times after they eliminate each row or column, and they announce either at the beginning or end of the conversation saying their initial average or final value.

Problem 2)

Two numbers a, b: 2 ≤ a ≤ b ≤ 100 are chosen. Let s = a + b. Let p = a × b.

There are two people, Mr. Sum and Mr. Product. Mr Sum is told the value of s in the presence of Mr. Product. Mr. Product is not told the actual valued of s. Mr Product is told the value of p in the presence of Mr. Sum. Mr. Sum is not told the actual valued of p.

The following dialogue takes place. Mr. Product: I do not know what a, b are. Mr. Sum: I knew you did not. Mr. Product: But I know them now! Mr. Sum: So do I.

Assuming the truth of these utterances

Show that a, b cannot both be prime.

A and b cannot be prime because prime numbers' only factors are 1 and itself and both a and b are greater than 1. This means if Mr. Product is told p and p is the product of two prime numbers, he would have known a and b.

Show that a + b cannot be even.

A+b cannot be even because according to Goldbach's conjecture, all even numbers can be the expressed as the sum of two primes and a,b cannot be prime because then a,b could have been those two primes and product would have been a semiprime and Mr. product would have known a,b

Show that before Mr. Sum spoke, p was consistent with a + b being even.

Before sum spoke, s could have been even because the only information we have is that product does not know a,b. This simply means p has at least 5 factors. Factors 1 and p are not possible because a,b > 1, but he does not know a,b from the other factors and its possible a+b is even. Example: if p is 36, Mr. Product does not know a,b, it could be 12 and 3 or 6 and 6, and 6+6=12 which is even.

Show that one of a, b must be a power of 2.

The sum is odd. All odd numbers can be written as the sum of an odd prime and a power of two. Since Mr. Sum knew Mr. Product does not know, there is only one possible pair of factors where a+b is odd. Therefor all of the twos going into the product, are all in b which is even, and a is odd.

Find a, b.

We eliminated any sum that can be written as the sum of two primes and since the evens are all gone, we are now eliminating all prime+2. After eliminating all possible options, using brute force calculations, the final possible product is 52, therefor the answer is 4 and 13 and the sum is 17. 52 also has 2*26 but 28 is the product of two prime numbers 5 and 23