CUNY – Data Science

Game Theory and Social Choice

Homework 6 – Social Choice

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Problem 1. Prove that any Borda rule does not satisfy IIA for any case in which there are at least three alternatives and at least two individuals.

The Borda rule says that alternative x gets a ranking of k if it is in the kth place in the ranking order of all alternatives. To find societies choice, add the total ranking given by each person for each alternative and the one with the highest number is societies choice. The Borda rule does not satisfy independence of irrelevant alternatives if there are at least three alternatives and two individuals because if an individual changes his preference relation between two of the alternatives, it can change the social choice. For example, say there are alternatives a,b,c and voters 1 and 2. If voter 1 has preference a>b>c and voter 2 has preference b>c>a. a gets a total of 2, b gets a total of 3, and c gets a total of 1, so b is societies choice. If voter 2 switches his preferences and now prefers b the least, making his preference c>b>a, a gets a total of 3, b gets a total of 1, and c has a total of 2 making a societies choice.

Problem 3. Let X = [0, 1] and assume that each individual's preference is single-peaked, that is, for each i there is an alternative a∗ i such that if a∗ i ≥ b > c or c > b ≥ a∗ i , then b i c. Show that for any odd n, if we restrict the domain of preferences to single peaked preferences, then the majority rule induces a "well-behaved" SWF.

If everyone's preferences are single peaked, that means everyone has a most preferred alternative a*i that is not tied with any other alternative, weather a*i is the highest (they are trying to maximize the result) or lowest (they are trying to minimize the result). If we consider the single peaked most preferred alternatives of each individual, to my understanding, if there is a majority alternative or if everyone agrees on one alternative, it is trivial and that alternative is the social choice resulting in a well behaved SWF, but there can be a tie regardless of there being an odd or even amount of people. Say there are 3 individuals and 3 alternatives. If everyone's single peaked preference is a different alternative, there is a tie and no majority, so there is no one peaked choice. Say there are three people whose preferences are a > b > c, c > a > b, and b > c > a. 2 people prefer a over b, 2 people prefer b over c, and 2 people prefer c over a, so it is a tie and there is no majority. There can still be a social welfare function consisting of indifferent relations, such as a ≥ b, b ≥ c, and c ≥ a, but I do not see how the majority rule applies in this example. Another example is say there are 7 alternatives and 5 individuals, if 2 individuals peaked preference is alternative 1 and 2 individuals peaked preference is alternative 2, and the last individuals most preferred alternative is alternative 3, then there is a tie between alternative 1 and 2 and no single peaked choice.

Problem 4. Each of N individuals chooses a single object from among a set X, interpreted as his recommendation for the social action. We are interested in functions F:XN → X that aggregate the individuals' recommendations (not preferences, just recommendations!) into a social decision. Social Choice 129 Discuss the following axioms:

- **Par: If all individuals recommend x∗, then society chooses x∗.**

• I: If the same individuals support an alternative x ∈ X in two profiles of recommendations, then x is chosen in one profile if and only if it is chosen in the other.

a. Show that if X includes at least three elements, then the only aggregation method that satisfies P and I is a dictatorship.

Since it is their preference, we are only receiving one alternative as a response from everyone rather than a preference relation of all the alternatives. If x includes at least 3 elements, the only aggregation method that satisfies p and I is a dictatorship because when there is a dictatorship, alternative a is only chosen if the dictator recommends it. According to axiom I, this happens either in both or neither profile. If the same individuals recommend x in two separate recommendation profiles, either the dictator is in that group of individuals, so x is chosen both times, or the dictator is not, and x is not chosen either time. If there are at least three elements, it can be proven that this is a dictatorship because if the same 40% recommend a both times and a is chosen both times, even though the first time, b was recommended by 30% and c by 30% and the second time b was recommended by 50% and c by 10%, it means it is a dictatorship and the dictator chose a which happened to be the majority in the first profile, while the second time b got the majority but a still won because of the dictator. In short, if the same people recommend the first option both times, and the first option is chosen both times, you can prove it's a dictatorship because while people recommending the first option stays the same, since there are three options, enough people that voted for the third option can switch their vote to the second option for the second round causing the second option to receive majority of the votes, but the first option is still chosen because the dictator chose it. I is also satisfied because if everyone recommends a, then the dictator also recommended a because he is part of everyone so a is chosen.

b. Show the necessity of the three conditions P, I, and |X| ≥ 3 for this conclusion.

Without p, the theoretical dictator would not always get what he wants, p is saying that since everyone prefers the same thing, they are all part of the majority, and they all get what they want including the theoretical dictator, but if they do not all have the same recommendation and societies choice is chosen based on the majority, the theoretical dictator could be in the minority and not get what he wants, or he could happen to be in the majority group and his preferred choice is the selected one, but this would not prove that it is a dictatorship. Without I, the theoretical dictator would not necessarily get what he wants because if different individuals recommend x in each recommendation profile, then it could always turn out that the chosen recommendation is also the majority's recommendation, and it does not prove that it is a dictatorship. There needs to be at least three options because otherwise, if a gets the same number of recommendations both times, then b also gets the same number of recommendations both times and while the theoretical dictator chose a, it can simply turn out that a gets the majority and is chosen, this does not prove that it needs to be a dictatorship.