CUNY – Data Science

Game Theory and Social Choice

Homework 5 – Expected Utility

Yaffa Tziporah Atkins – 1235

1) Consider two preference relations that were described in the text: "the size of the support" and "comparing the most likely prize".

a) Check carefully whether they satisfy axioms I and C.

- Axiom I says that if subset r is added to both p and q with the same probability, the preference relation between p and q does not change. Axiom C says that if p is preferred to q, p' which is similar to p is preferred to q' which is similar to q.

According to "the size of the support", the support is the number of possible outcomes, and the decision maker prefer the set with less possible outcomes.

- Axiom I is not satisfied because if r contains elements that are in q but not in p, although before r was added there were less possibilities in p and p was preferred, now when r is added, rp can contain more options than rq since there are overlaps in r and q, and rq will be preferred. Example: Say p is quarters and q is nickels and dimes. P has less options, so p is preferred. r contains pennies, nickels, and dimes, add it to both p and q. now rp is pennies, nickels, dimes, and quarters, and rq is nickels, dimes, and quarters. Rq has less options than rp so rq is preferred and now the preference has changed. In summary, if r overlaps with q, then the preference relation can change.
- Axiom C is not satisfied. P has support 1 because only quarters is possible, (both nickles and dimes have probability 0) so p is preferred to q because it has a smaller support, support of q is two because both nickels and dimes are possible. Say that in p, the probability of nickels and dimes is switched to epsilon which is very close to 0, now the support of p is 3 which is bigger than the support of q which is 2 so q is preferred, switching the preference relation. In summary, since size of the support only counts items with probability over zero, slightly changing the probability of an item to epsilon, changes the support of the set.

According to "comparing the most likely prize" the decision maker considers the prize in each lottery that is most likely and chooses the lottery where the most likely prize is more preferred.

- Axiom I is not satisfied because the most likely option in p can be better than the most likely option in q, but when r is added to both, if r contains an option that is more likely than all options in q, but not more likely than all option in p, and the most likely option in r is better than the most likely option in p, rq is preferred to rp, switching the preference relation. Example: say the most likely item in p is dimes at 25% and the most likely item in q is nickels at 15%, when comparing the most likely outcomes, since dimes are preferred to nickels, p is preferred. Now add r to the equation, r contains quarters at 20% likelihood. Now the most likely item in pr is dimes and the most likely item in qr is quarters. Since quarters are preferred to dimes, qr is preferred to pr so it is seen that the preference relation switched. In summary, if r contains an option more likely than everything in q, but not in p, and preferred to the most likely option in p, then the preference relation will change.

- Axiom C is not satisfied because say p and q both have nickel and dime with 50% probability each, their preference relation is p is preferred to or just as preferred as q. is the probabilities are slightly changed in q to dime having 50% plus epsilon probability, and nickel having 50% minus epsilon probability, then the preference relation is changed, and q is preferred to p.

b) These preference relations are not immune to a certain "framing problem". Explain.

The framing problem is when a decision makers choice is influenced by the way in which the options are presented. When comparing the size of the support, the decision maker is simply counting the number of alternatives and choosing the set with less alternatives. Maybe this can be affected by the framing problem if one alternative is mentioned twice but worded differently and so the decision maker counts them as two separate alternatives instead of one, even though they have the same outcome. Comparing the most likely can be affected by the framing problem because of the two most likely from each lottery, the decision maker can say he prefers 70% survive, to 30% do not survive because 30% do not survive sounds worse since it is written in the negative format, even though they produce the same end results.

2) A decision maker has a preference relation >~ over the space of lotteries L(Z) with a set of prizes Z. On Sunday he learns that on Monday he will be told whether he has to choose between L1 and L2 (probability 1 > α > 0) or between L3 and L4 (probability 1 − α). He will make his choice at that time. Consider two possible approaches the decision maker can take: Approach 1 : He delays his decision to Monday ("why bother with the decision now when I can make up my mind tomorrow . . ."). Approach 2 : He makes a contingent decision on Sunday regarding what he will do on Monday, that is, he decides what to do if he faces the choice between L1 and L2 and what to do if he faces the choice between L3 and L4 ("On Monday morning I will be too busy . . .").

a) Formulate Approach 2 as a choice between lotteries.

Say C(x) is choice function when choosing between a set of lotteries. The decision maker will be making a choice on Monday between A = {L1, L2} or B = {L3,L4}. In approach 2, the decision maker makes both choices C(A) and C(B) on Sunday. When x is chosen on Monday to be either a or b, he choses C(x) which he already decided upon on Sunday. Ultimately, choice function C yields αC(A) ⊕ (1 − α)C(B). Example: Say from A, he prefers L2, and from B he prefers L3, then on Sunday he decided C(A) = L2 and C(B) = L3. On Monday, it is decided that x = A, meaning he is to choose from set A, he chooses C(A) which he already determined on Sunday to be L2.

b) Show that if the preferences of the decision maker satisfy the independence axiom, then his choice under Approach 2 will always be the same as under Approach 1.

It does not matter how his decisions are split up, he will make the same decision in the end. If L2 is the most preferred from A and L3 is the most preferred from B, C(A) = L2 and C(B) = L3. Following the independence axiom, since L2 and L3 are the most preferred, αL2 ⊕ (1 − α)L3 is preferred to any other alternative. αL2 ⊕ (1 − α)L3 is preferred to αL1 ⊕ (1 − α)L3 which is preferred to αL1 ⊕ (1 − α)L4 and αL2 ⊕ (1 − α)L3 is preferred to αL2 ⊕ (1 − α)L4, which is preferred to αL1 ⊕ (1 − α)L4. Either on Sunday he will chose C(A) and C(B) and then on Monday he will chose between C(A) and C(B) depending on the set he is given, or on Monday he will chose either C(A) or C(B) depending on the set he is given.