CUNY – Data Science

Game Theory and Social Choice

Homework 4 – Choice

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Problem 1. (Easy) The following are descriptions of decision-making procedures. Discuss whether the procedures can be described in the framework of the choice model presented in this lecture and whether they are compatible with the rational man paradigm.

b. The decision maker asks his two children to rank the alternatives (1,2,3...) and then chooses the alternative that gets the lowest average score.

This decision-making procedure can be described in the framework of the choice model presented because the decision maker must make a choice from a set of alternatives, the set contains alternative options that he himself determines, and a single element is assigned to each choice problem, dependent on the ranking given to the alternatives by his sons, different rankings provide different scenarios. According to the choice function, in each scenario he chooses the option with the lowest average ranking. This is not compatible with the rational man paradigm because he is not determining feasibility of the options and he is choosing the option with the lowest average ranking rather than the highest, he is asking the kids their preferences and choosing what they want the least, so he is not maximizing their gain and he is not even considering his personal preferences, so he is not maximizing his own gain.

d. The decision maker looks for the alternative that appears most often in the choice set.

This decision-making procedure cannot be described in the framework of the choice model presented because in the choice model he is considering a set of alternatives, ignoring the order in which they are presented and the number of times each item is presented, but here the agent is deciding solely based on the number of times each alternative is presented. This is not compatible with the rational man diagram because the agent is not considering which alternatives he prefers and is therefore not maximizing his personal gain.

e. The decision maker has in mind an ordering of the alternatives in X and always chooses the median element.

This decision-making procedure can be described in the framework of the choice model presented because the agent is making a choice solely based on the alternative options and regardless of the way they are presented. This is not compatible with the rational man diagram because the rational man would be aggressive and maximize his gains while this agent is being moderate and only getting partial gain.

Problem 2. (Moderately difficult) A choice correspondence C satisfies the path independence property if for every set A and a partition of A into A1 and A2 (A1, A2 6= Ø, A = A1 ∪ A2 and A1 ∩ A2 = Ø) we have C(A) = C(C(A1) ∪ C(A2)). (The definition applies also to choice functions.)

a. Show that the rational decision maker satisfies path independence

Path independence maximizes personal gain, so it satisfies the rational decision maker. In path independence, the alternatives are split into two groups and the most preferred option is chosen from each of those sub-groups, next the two chosen alternatives are compared and the more preferred one is the final choice. The best between those two options is the best from the whole original group, so in the end, path independence selects the most preferred option out of all alternatives maximizing the gain. Say x is the most preferred option in A, all options in A are then split into subsets A1 and A2, next the most preferred option is selected from A1 and A2. Say x ended up in A1, it will be the most preferred option in A1, when the most preferred options from both A1 and A2 are compared, x will be chosen. This is because it is the overall preferred option in A so it will be preferred over the most preferred option in A2.

c. Show that if a choice function satisfies path independence, then it satisfies condition α.

Condition alpha says if the most preferred option of the whole group is in a subset, it's the most preferred of the subset and the most preferred of the whole group, so the choice from the subset and from the whole group will be the same. In path independence, both A1 and A2 are subsets of A and if the most preferred option of A is in A1, it will be chosen from A1 and will ultimately be chosen between the most preferred option of A1 and the most preferred option of A2, so if a choice function satisfies path independence, then condition alpha is satisfied. In both, if the most preferred option of a group, option X, is in a subset of the group, A1, then X will be the most preferred option of both the subset and the whole group A.