CUNY – Data Science

Game Theory and Social Choice

Homework 2 – Utility

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1a: If both U and V represent >~, then there is a strictly monotonic function f : R -> R such that V(x) = f(U(x)).

This is not true. Even if U and V are strictly monotonic, there could be a function f that jumps around and is flat at a certain value. Say V(x) = x and U(x) = x when x ≤ 0 and U(x) = x+1 when x > 0. These are strictly monotonic because as X increases, both V(x) and U(x) increase. Then say f(x) = x when x ≤ 0 and f(x) = 0 when 0 < x ≤ 1 and f(x) = x-1 when x > 1. This is not strictly increasing. When x bigger than 0 and less than or equal to 1, f(x) is flat. When x is less than or equal to 0, f(x) =x is a straight line. when x is greater than one it's a straight line but when x is greater than zero and less than or equal to one, f(x) is flat, even though in this scenario f(x) will never receive an x greater than zero and less than or equal to one, the function is not strictly increasing because there is definition that gives a flat output.

1c: Show that in the case of X = R the preference relation that is represented by the discontinuous function u(x) = [x] (the largest integer n such that x ≥ n) is not a continuous relation

When x changes a little, U(x) can change drastically, like when exponents are used, it's not continues. A continues relation means if you prefer a to b, you prefer items like a over items like b. A discontinues utility means there is a jump in utility somewhere. A preference realization that jumps can have two items that are similar falling into different utilities (categories).

6: For any a ∈ X, aSa. For all a, b ∈ X, if aSb, then bSa. Continuity (the graph of the relation S in X × X is a closed set). Betweenness: If d ≥ c ≥ b ≥ a and dSa, then also cSb. For any a ∈ X, there is an open interval around a such that xSa for every x in the interval. Denote M(a) = max{x|xSa} and m(a) = min{x|aSx}. Then, M and m are (weakly) increasing functions and are strictly increasing whenever they do not have the values 0 or 1.

6a: These assumptions capture my intuition of "approximately the same".

6c: Let S be a binary relation that satisfies the above six properties and let ε be a strictly positive number. Show that there is a strictly increasing and continuous function H : X → R such that aSb if and only if |H(a) – H(b)| ≤ ε .

This function is continues because it does not jump around, it follows a standard method. A and b are similar if when passed through the same function, their difference is less than epsilon. Say h(x) is x+1, the relation between a and b stays the same because you are moving both values by 1 number.