Efficient Use of Energy

Mariia Aleksandrovych, Yaffa Atkins, and Olga Kliuchnik

December 19, 2022

Contents

1 Introduction

Cost of life is expensive, especially in big cities like New York. The goal of this project is to help us understand how people can spend less money on energy while using an energy source that is better for the environment.

Solar energy as an energy source is considered cheap, effective, and relatively harmless for the environment. However, varying weather conditions during the seasons may affect the efficiency of usage of solar power.

We are working to find the optimal initial investment which we give us the lowest overall cost after a year. This can help your average homeowner in New York know how much money to spend on their solar panel system while generating enough so the customers do not need to buy extra energy from the grid, but not spend too much money and over produce the energy.

Overall consumption of electrical energy goes up each year. In the United States of America in the last 10 years, consumption raised by 10% of consumption in megawatt hours [1]. Around 38% of electricity consumption comes to residential use, the second largest energy consumer in the US. The first consumer of energy is industrial use, also the electrical power plants. For example, the hydroelectric power plant's net electrical energy generation was negative for the year 2021 [1]. Only 21% of the electricity was generated by renewable energy sources, where wind sources generated 9.2%, hydropower – 6.1%, solar (small and industrial plants) – 2.8%. On top of that, about 61% of total electricity generation was from fossil fuels—coal, natural gas, petroleum, and other gases [2].

The usage of electricity for natural gas increased rapidly from 2014 to replace coal electricity generation mostly. However, natural gas has a limited amount on the planet, and with the current rate of consumption, humans will run out of natural gas in estimated 52 years [3]! Thus, will all this energy crisis we as users of the cyclization technologies have a question: "How we can affect it?". Due to this energy crisis, we as users of cyclization technologies have a question: "How we can affect it, without drastically changing our lives?". So, we have a simple answer – "Lower residents' consumption by incorporating renewable small power plants suitable for the region." Moreover, it would help lower the cost of electricity bills or fully cover it. Over the years and depending on the power plant that is installed in the household, it could become a side income.

After convincing ourselves that the energy crisis is real. Small plants for a single household or single settlement mass installation and use would have an impact on the total energy consumption. As the U.S. Energy Information Administration wrote in June 2017 *"More than half of small-scale photovoltaic generation comes from residential rooftops"* [4] This gives us an idea of scaling the usage of the small power plant, to have an impact on electrical energy usage and net generation. However, installing solar panels and batteries with it could be princely. On top of that, it is not clear which panels of efficiency certain household needs in order to cover bills, and not to overpay for unnecessarily expensive equipment. To add to this, we were focused on lowering the bills for electricity and with some investment in solar panels. Thus, we need to find the optimal solar panel amount that will minimize energy costs.

In this project, we consider the New York region area, a median household with a private house or households that have private houses, so they have space to install solar panels. Our target on the minimization of electrical energy costs with the control variable of the number of panels that we can install. Three different efficiencies of the panels were considered, the roof area was fixed, and the sunlight hours data for the preliminary analysis was taken from Global Atlas open data source [5]. Also, if we have time, we can consider the business model of the solar power plant, minimization consists of initial investment, source of income is selling carbon credits and selling electricity.

2 Problem Formulation

2.1 Model

Optimization problem: We are trying to optimize the total amount of money spent on energy in a year for the median households in the New York City. Our control variable θ is the initial investment made at the beginning of the year to buy our solar panels. Cost function is convex with θ, if we do not invest enough, you spend a lot of money on your electric bill each month, and if you invest too much, you spend a lot of money on your solar panels and over generate. Our constraints are that we have a finite amount of money to initially invest, and we have a finite amount of space to put our solar panels, but those constraints are never reached since over generating is not optimal. When you over generate you can sell back to the grid, but due to them having a monopoly on electricity, they will take a cut and your profit will not cover your additional investment causing you to ultimately lose money, so our problem ensures you do not over generate too much.

θ is initial investment of dollar for the solar panels station.

$J(\theta)$ is total cost of energy use.

$X(\theta)$ total solar energy production from the solar panel's kWh per day. It is a random variable with Weibull distribution. iid from day to day within a season, mean of distribution varies from season to season. [6].

$E[X(\theta)]$ is sigmoid function of theta, so if you invest too much, your energy generated will still plateau.

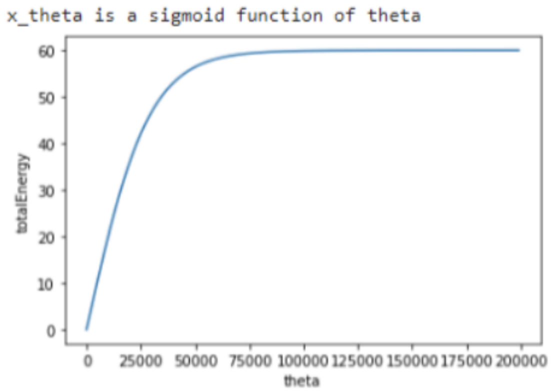


Figure 2

β - demand kWh per day. It is a random variable with Weibull distribution, iid from day to day within a season, mean of distribution varies from season to season.

Figure 2 shows he difference between b and x(theta) is high when theta is too small since we are not generating enough and when theta is too big since we are generating too much

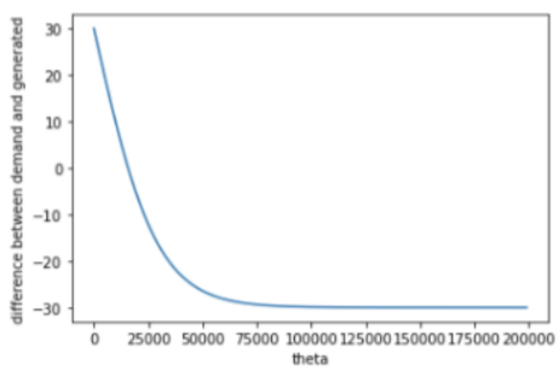


Figure 2

α is a buying price cents/kWh when purchasing electricity from the grid. It is a random variable with Weibull distribution, iid from day to day within a season, mean of distribution varies from season to season.

p is a selling price cents/kWh when selling excess generated electricity to the grid. It is calculated as 80% of the days buying price.

Figure 3 shows when θ is too low, $X(\theta)$ is low and cost is up since we need to purchase energy from the grid. When theta is too high, cost is in the negative since we are over generating and making a profit from selling to the grid.

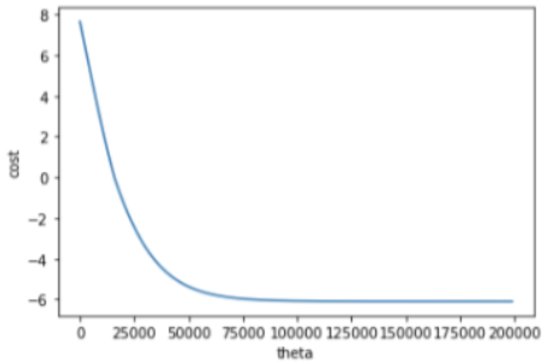


Figure 3

$$\min_\theta J(\theta)$$

$$\begin{aligned} J(\theta) = \theta + E \Bigg[& \frac{1}{122} \sum_{i=1}^{365} (\max((\beta_{smmr} - X(\theta))\alpha_{smmr}, 0) - \min((\beta_{smmr} - X(\theta))(p_{smmr}), 0)) \\ & + \frac{1}{61} \sum_{i=1}^{365} (\max((\beta_{fall} - X(\theta))\alpha_{fall}, 0) - \min((\beta_{fall} - X(\theta))(p_{fall}), 0)) \\ & + \frac{1}{90} \sum_{i=1}^{365} (\max((\beta_{winter} - X(\theta))\alpha_{winter}, 0) - \min((\beta_{winter} - X(\theta))(p_{winter}), 0)) \\ & + \frac{1}{92} \sum_{i=1}^{365} (\max((\beta_{spring} - X(\theta))\alpha_{spring}, 0) - \min((\beta_{spring} - X(\theta))(p_{spring}), 0)) \Bigg] \end{aligned}$$

$$G(\theta) = E[Y_n|\mathfrak{I}_{n-1}] \text{ also } G(\theta) = -\nabla J(\theta)$$

If we can interchange derivative and expectation value, then it will be Y_n

Max is when $\beta > X(\theta)$, cost increases, buying from grid to fill unmet need.

Min is when $\beta < X(\theta)$, cost decreases (profit increases) selling excess generated electricity to the grid.

Our optimization problem is for long term behavior.

3 Methodology and Convergence Analysis

3.1 Analysis of the Stochastic Algorithm

IPA

3.1.1 Recall the problem

We have a minimization problem:

$$\min_{\theta} J(\theta)$$

$$\begin{aligned} J(\theta) = \theta + \mathrm{E}\Bigg[& \frac{1}{122}\sum_{i=1}^{365}(max((\beta_{smmr} - X(\theta))\alpha_{smmr}, 0) - min((\beta_{smmr} - X(\theta))(p_{smmr}), 0)) \\ & + \frac{1}{61}\sum_{i=1}^{365}(max((\beta_{fall} - X(\theta))\alpha_{fall}, 0) - min((\beta_{fall} - X(\theta))(p_{fall}), 0)) \\ & + \frac{1}{90}\sum_{i=1}^{365}(max((\beta_{winter} - X(\theta))\alpha_{winter}, 0) - min((\beta_{winter} - X(\theta))(p_{winter}), 0)) \\ & + \frac{1}{92}\sum_{i=1}^{365}(max((\beta_{spring} - X(\theta))\alpha_{spring}, 0) - min((\beta_{spring} - X(\theta))(p_{spring}), 0))\Bigg] \end{aligned}$$

Stochastic approximation, we are performing gradient estimation.

$$\theta_{n+1} = \theta_n + \varepsilon Y_n(\theta)$$

Where $G(\theta)$ is an estimator of the negative derivative of the objective function:

$$G(\theta) = \mathrm{E}[Y_n|\mathfrak{J}_{n-1}] = -\nabla J(\theta) = -\frac{d}{d\theta}\mathrm{E}\left[\frac{1}{365}\sum_{i=1}^{365}(max((\beta_i - X(\theta))\alpha_i, 0) - min((\beta_i - X(\theta))(p_i), 0))\right] - 1$$

For our optimization problem, we took inspiration from the mining example 7.1.

3.1.2 Representation for random variable $X(\theta)$

We consider $X(\theta)$ as a random variable that is parameterized by θ, where $\theta \in \Theta \subset \mathbb{R}$, and a random variable is defined on common probability space $(\Omega, \mathfrak{J}, \mathbb{P})$.

Then, $X(\theta)$ Weibull distribution with shape parameter α, and scale parameter $\lambda = \sigma(\theta)$, where $\sigma(\theta) = \frac{1}{1+e^{\theta}}$ is sigmoid function, and $\frac{d}{d\theta} = \sigma(\theta)(1 - \sigma(\theta))$ is the derivative.

CDF:

$$1 - e^{-(\frac{x}{\lambda})^{\alpha}}, x \geqslant 0$$

Samples from Weibull (α, λ) can be obtained using inverse distribution method:

$$X(\theta) = \lambda(-\ln(1 - U))^{\frac{1}{\alpha}}$$

Then, we can write derivative of random variable $X(\theta)$.

Let $\lambda = \sigma(\theta)$,

$$X_{\alpha,\sigma(\theta)} = n\sigma(\theta)(-\ln(1-U))^{\frac{1}{\alpha}}$$

where n is just parameter to fit our data. In the code is numbers of the meters, that is equal to price of the solar system divided by the price per meter square.

$$\frac{d}{d\theta}X_{\alpha,\sigma(\theta)} = \sigma'(\theta)(-\ln(1-U))^{\frac{1}{\alpha}} = \sigma'(\theta)\frac{\sigma(\theta)}{\sigma(\theta)}(-\ln(1-U))^{\frac{1}{\alpha}} = \frac{\sigma(\theta)(1-\sigma(\theta))}{\sigma(\theta)}X(\theta) = (1-\sigma(\theta))X(\theta)$$

We define

$$h(x) \stackrel{\text{def}}{=} max((\beta - x)\alpha, 0) - min((\beta - x)p, 0)$$

And it is summed over some period

Derivative of the max and min functions. One thing to mentioned that these two functions are continues but not differentiable at line $(\beta - x)\alpha = 0$ and $(\beta - x)p = 0$ respectively.

$$f(x) = max((\beta - x)\alpha, 0) = \begin{cases} (\beta - x)\alpha, \beta - x > 0 \\ 0, \beta - x \leqslant 0 \end{cases} \quad \Rightarrow \frac{d}{d\theta}max((\beta - x)\alpha, 0) = \begin{cases} -\alpha, \beta - x > 0 \\ 0, \beta - x \leqslant 0 \end{cases}$$

$$f(x) = min((\beta - x)p, 0) = \begin{cases} (\beta - x)p, \beta - x \leqslant 0 \\ 0, \beta - x > 0 \end{cases} \quad \Rightarrow \frac{d}{d\theta}min((\beta - x)p, 0) = \begin{cases} -p, \beta - x \leqslant 0 \\ 0, \beta - x > 0 \end{cases}$$

$$\frac{d}{dx}max((\beta - x)\alpha, 0) = \begin{cases} -\alpha, \beta - x > 0 \\ 0, \beta - x \leqslant 0 \end{cases} \quad = -\alpha H((b - x)), x \neq \beta$$

$$\frac{d}{dx}min((\beta - x)p, 0) = \begin{cases} -p, \beta - x \leqslant 0 \\ 0, \beta - x > 0 \end{cases} \quad = -pH((b - x)), x \neq \beta$$

Where H is Heaviside function

$$H(x) = \begin{cases} 1, x > 0 \\ 0, x \leqslant 0 \end{cases}$$

Then

$$\frac{d}{d\theta}h(X(\theta)) = \frac{d}{d\theta}(max((\beta - X(\theta))\alpha, 0) - min((\beta - X(\theta))p, 0)) = -\alpha H((\beta - X(\theta))) + pH((X(\theta) - \beta))$$

Derivatives exist everywhere except $X(\theta) = \beta$ Then the stochastic optimization will be

$$\theta_{n+1} = \theta_n + \varepsilon_n\{-\sum\{(1 - \sigma(\theta))X(\theta)(-\alpha H(\beta - X(\theta)) - pH(X(\theta) - b))\} - 1\}$$

Where

$$Y_n = (-\sum(1 - \sigma(\theta))X(\theta)(-\alpha H(\beta - X(\theta)) - pH(X(\theta) - b))) - 1$$

3.1.3 Proof of Theorem 8.1

To use unbiased estimator Y_n, we need to show that all the conditions of Theorem 8.1 are satisfied.

From Example 8.1 we can conclude that sample path differentiation of random variable $X(\theta)$ exist with use of the Lemma 8.1. The condition *(i)* is satisfied

The mapping is $h(X(\theta))$ differentiable on $\Theta\backslash\Theta_0$, where $(\Theta_0 : X(\theta) = \beta)$ (ii) yields that condition (ii) in Theorem 8.1 is satisfied.

Using Lemma 8.3:

for $\theta_1 > \theta_0$, where $\theta_1, \theta_0 \in \Theta\backslash\Theta_0$, *it holds almost surely that*

$$|h(X(\theta_1)) - h(X(\theta_0))| \leqslant |\theta_1 - \theta_0| \sup_{\theta\in\mathbb{R}\backslash\theta=0} |\frac{d}{d(\theta}h(X(\theta))|$$

So, for any random variable K *such that* $E(K) < \infty$*, we have*

$$\sup_{\theta\in\mathbb{R}}\{|\frac{d}{d\theta}h(X(\theta))| : \theta \in \Theta\} \leqslant K$$

$h(X(\theta))$ *is a.s. Lipschitz continuous on* $\Theta\backslash\Theta_0$ *with Lipschitz modulus* K

Let's proof it for our case.

Let's consider what happens when we evaluate the function at two points $(\theta_1, \theta_2) \subset \Theta$ where $\theta_2 > \theta_1$, also $X(\theta)$ is monotonic increasing in θ, in our region of interest. Then, $h(X(\theta))$ differentiable mapping, and:

$$\sup_{\theta\in\mathbb{R}\setminus\Theta_0}\frac{d}{d\theta}h(X(\theta)) \leqslant (1-\sigma(\theta_1))X(\theta_2)h'(X(\theta_2)) \leqslant (1-\sigma(\theta_2))X(\theta_2)|h'(X(\theta_2))| \equiv K$$

Hence, we have

$$\sup_{\theta\in\mathbb{R}\setminus\Theta_0}\frac{d}{d\theta}h(X(\theta)) \leqslant K$$

$$|h(X(\theta_2)) - h(X(\theta_1))| \leqslant |\theta_2-\theta_1| \sup_{\theta\in\mathbb{R}\setminus\Theta_0}\frac{d}{d\theta}h(X(\theta)) \leqslant |\theta_2-\theta_1|K$$

Then the

$$E((1-\sigma(\theta_2))X(\theta_2)|h'(X(\theta_2))|) < \infty$$

then Lemma 8.3 yields condition (iii).

Applying Theorem 8.1 we now can interchange derivative and estimation.

$$\frac{d}{d\theta}E[h(X(\theta))] = E[(1-\sigma(\theta))X(\theta)h'(X(\theta))]$$

Relevant Code:

```
production_array = [143.825/30, 131.78/30, 103.46/30] #kWh/month/m^2 - converted to day

demand_array = [900/30, 700/30, 650/30]  #kWh/month - converted to day

price_array = [.2550, .0944, .0944] #cents/kWh

square_meter_of_panel_price = 646

alpha = 12
```

```
def h(x):
  if x > 0:
    return 1
  if x <= 0:
    return 0
```

```
def seasonCostSummation(theta, index, seasonLength):

  Yn = 0
  b = -.045

  numberOfSquareMeters = (1/(1 + np.exp(b*theta/square_meter_of_panel_price))-.5)
  generated_beta = production_array[index]/gamma(1+1/alpha)*25
  demand_beta = demand_array[index]/gamma(1+1/alpha)
  price_beta = price_array[index]/gamma(1+1/alpha)

  for i in range(0,seasonLength):

    generated = np.random.weibull(alpha)*generated_beta
    total_generated = generated*numberOfSquareMeters
    demand = np.random.weibull(alpha)*demand_beta
    buying_price = np.random.weibull(alpha)*price_beta
    selling_price = buying_price*0.8

    sigmoid = numberOfSquareMeters
    max = -buying_price * h(demand - total_generated)
    min = selling_price * h(total_generated - demand)

    Yn += (1-sigmoid)*total_generated*(max+min)

    return Yn
```

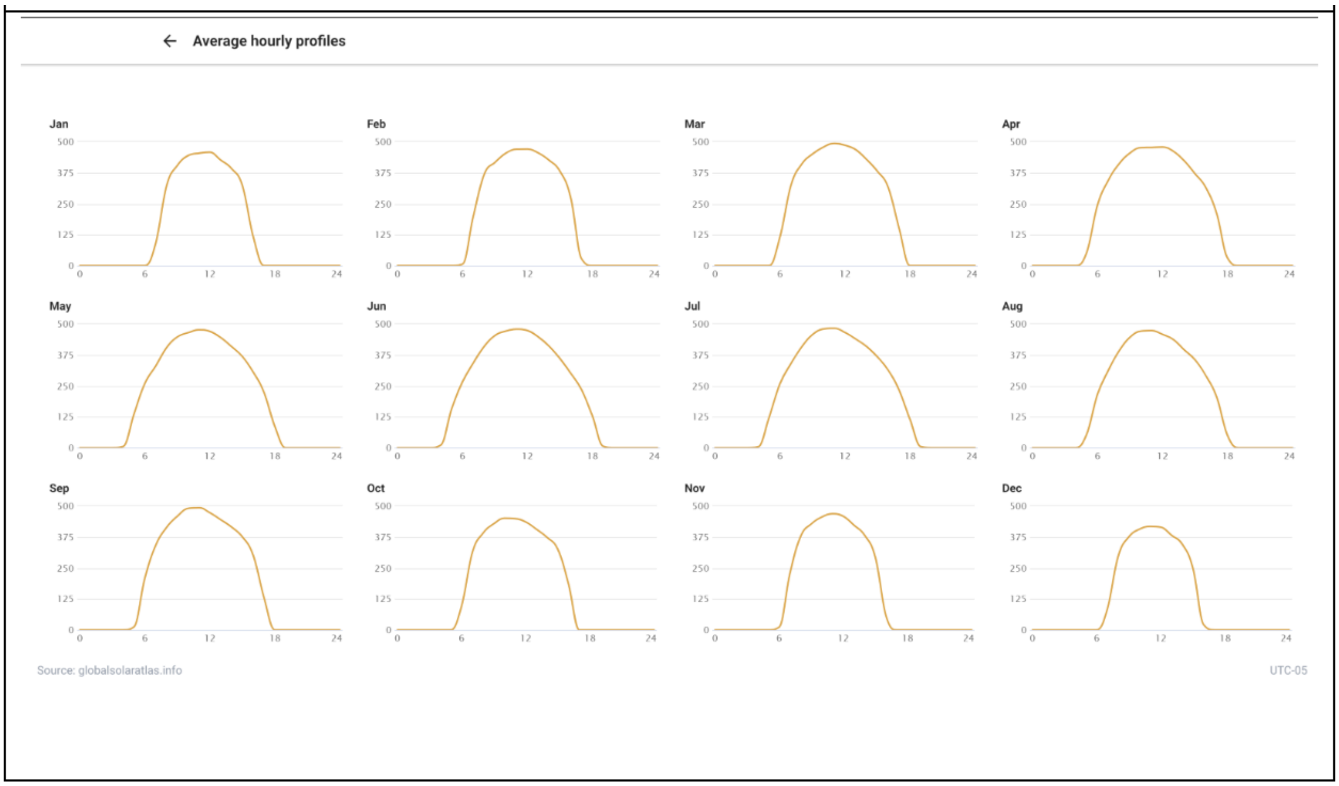
4 Input data

Table 1: Refers to Median household in New York City

	June 1 – September 30	October 1 – November 30	December 1 - February 28	April 1 – May 30
Electricity usage per month	900 kWh	700	650	700
Price* cents/kWh	25.50 cents/kWh	9.44 cents/kWh		

*Subject to change as usage could increase or decrease over decades

Table 2. Average hourly subpeak profiles throughout year



Our seasons are:

Summer = June 1 to September 30

Fall = October 1 to November 30 x

Winter = December 1 to February 28

Spring = March 1 to May 31

Table 3 Monthly average energy production in kWh/m^2

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
106.5	111.2	139.5	142.9	149.6	147.8	150.9	140.2	136.4	119.5	107.4	92.7

A typical solar panel for residential use takes up about 15 sq. feet. With convention where 1 sq. feet = 0.092903 m^2 and 15 sq. feet = 1.39355 m^2, then for 16.66 panels it will be 250 sq feet or 23 sq meters.

According to this webpage [7] Cost of one 200-Watt solar panels with installation $1000. The total area of one 200-Watt panel is 1.65 x 0.67 = 1.1055 m^2 (Table 4). So price per square meter is $\$\frac{1000}{1.1055}$ m^2, which is $904\m^2. To get 1 kWh system we would need 5 panels that is in total 5.5275 m^2. If you live in New York, a 1 kW solar system will only produce 3.5 kWh/day, where is consumption is 30 kWh/day on average. Thus, we need 8 kWh system which would take around 47.4 m^2.

Table 4. One kW Solar Panel Price

Solar Panel Wattage	Amount of Solar Panels	Total Wattage	Size of Each Panel	Total area for 1kw solar panel	Price Renogy	Price/Watt
50 Watts	x 20	1kw	558 x 508 x 25 mm	5.67 sqm	$1,499.98	**$1.49**
100 Watts	x 10	1kw	1044 x 508 x 35 mm	5.3 sqm	$999	**$0.99**
200 Watts	x 5	1kw	1650 x 670 x 35 mm	5.53 sqm	$1399.95	**$1.39**
320 Watts	x 4	1,2kw	1666 x 1002 x 35 mm	5.2 sqm	$1299.99	**$1.01**
450 Watts	x 2	0.9kw	1909 x 1134 x 35 mm	4.81 sqm	$899.99	**$0.99**
550 Watts	x 2	1.1kw	2279 x 1134 x 35 mm	4.7 sqm	$1,099.99	**$0.99**

5 Experiments

We experimented with different initial theta and with different parameters.

$X(\theta)$ is a sigmoid function of theta

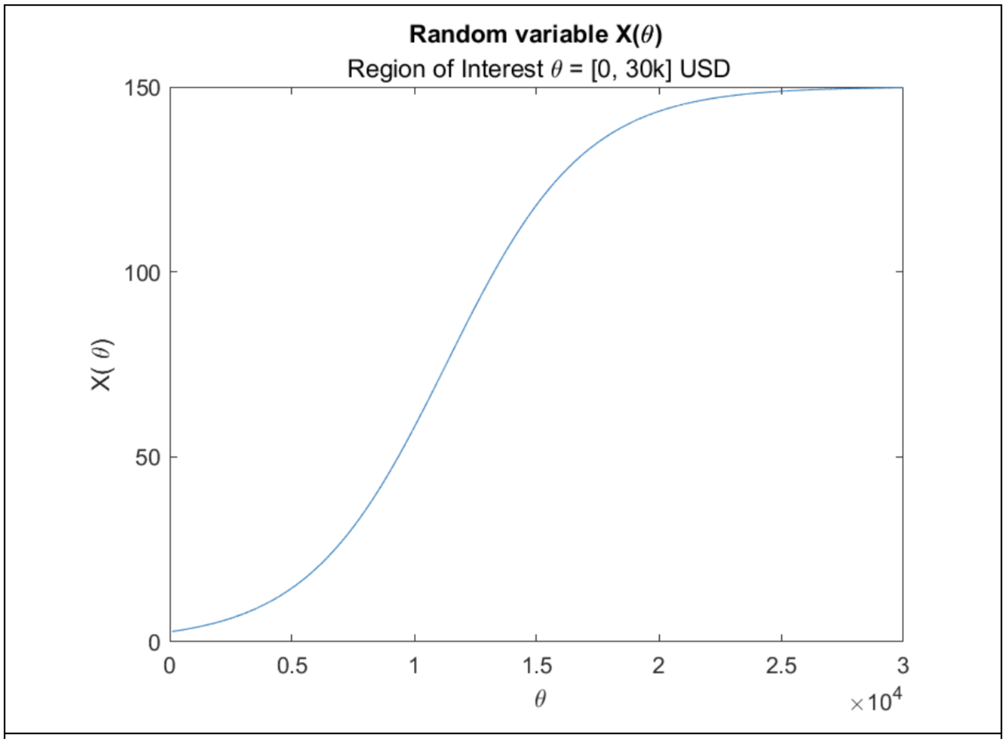


Figure 1. $X(\theta)$ **dependence of** θ**. For the Summer data.**

IPA

Experiment 1:

Consider constant step size $\varepsilon = 10$.

For this experiment we used different random variable representation, sigmoid multiplied by θ:

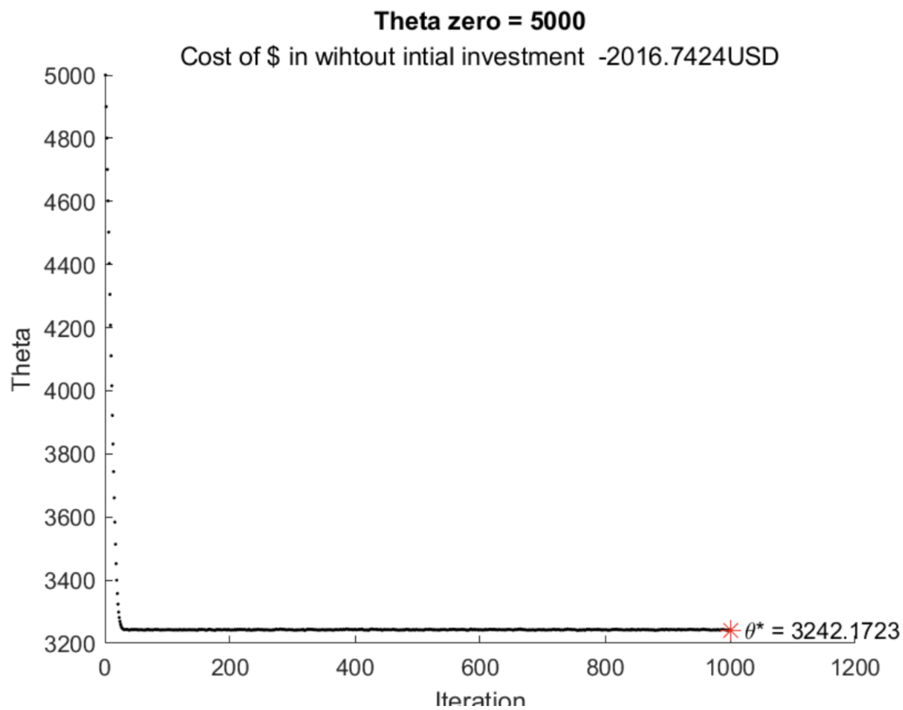
$$X_{\alpha,\sigma(\theta)} = n \cdot \theta \cdot \sigma(\theta)(-\ln(1-U))^{\frac{1}{\alpha}}$$

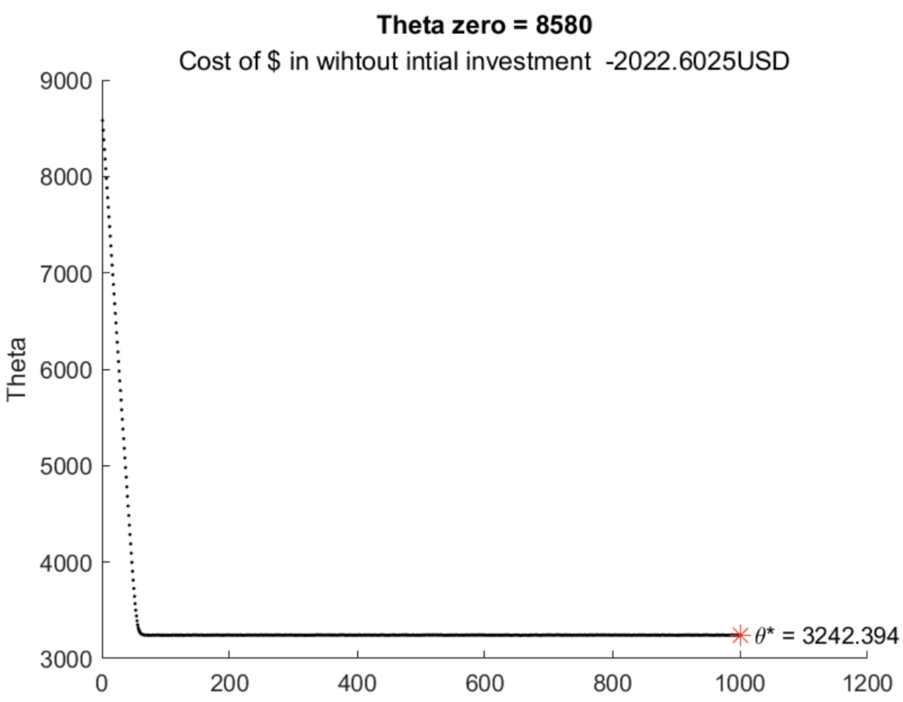
Then derivative:

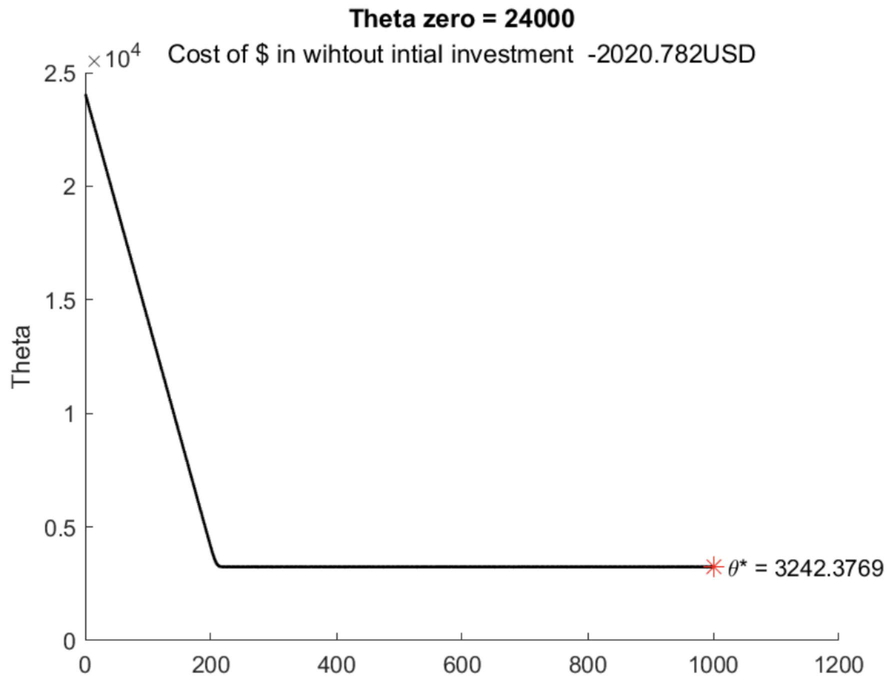
$$\begin{aligned}
\frac{d}{d\theta}X_{\alpha,\sigma(\theta)} &= n \cdot \sigma(\theta)(-\ln(1-U))^{\frac{1}{\alpha}} + n \cdot \sigma'(\theta)(-\ln(1-U))^{\frac{1}{\alpha}} \\
&= n \cdot \sigma(\theta)(-\ln(1-U))^{\frac{1}{\alpha}} + n \cdot \sigma'(\theta)\frac{\sigma(\theta)}{\sigma(\theta)}(-\ln(1-U))^{\frac{1}{\alpha}} \\
&= (1 + \frac{\sigma'(\theta)}{\sigma(\theta)})X(\theta) = (1 + (1 - \sigma(\theta)))X(\theta) = (2 - \sigma(\theta))X(\theta)
\end{aligned}$$

And

$$Y_n = -(\sum (2 - \sigma(\theta))X(\theta)(-\alpha \cdot H(\beta - X(\theta)) + p \cdot H(X(\theta) - \beta))) - 1$$







For this experiment, θ converges at an optimal value of 3242 USD. However, at the end of the year, additional cost paid to the electricity provider is still around $2k.

Experiment 2

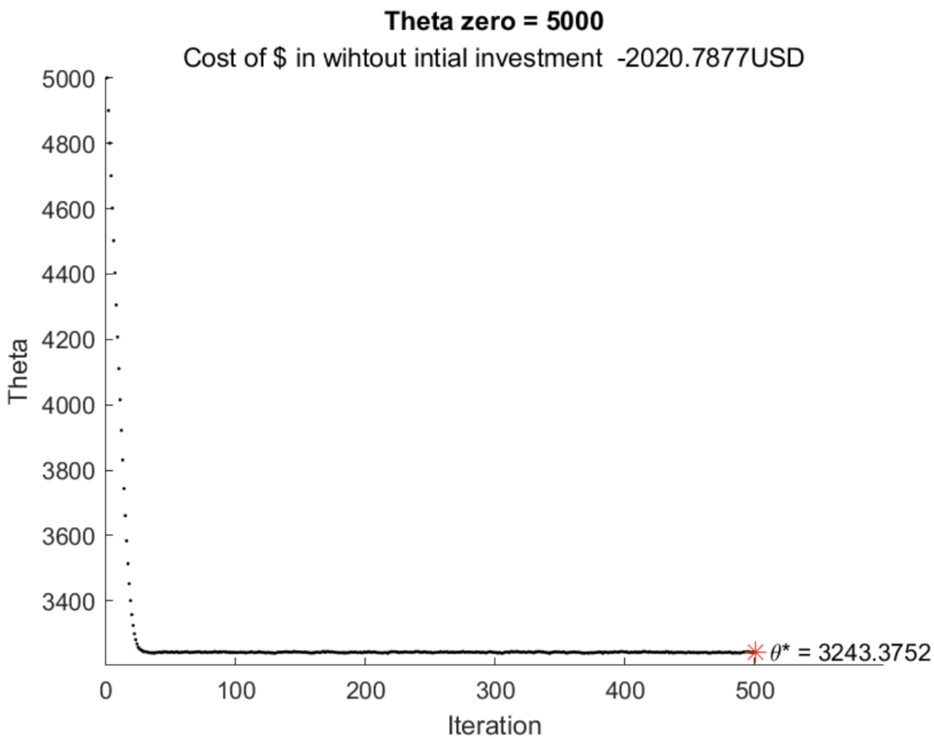
Uses original random variable representation with fixed numbers of meter square at 50 m^2

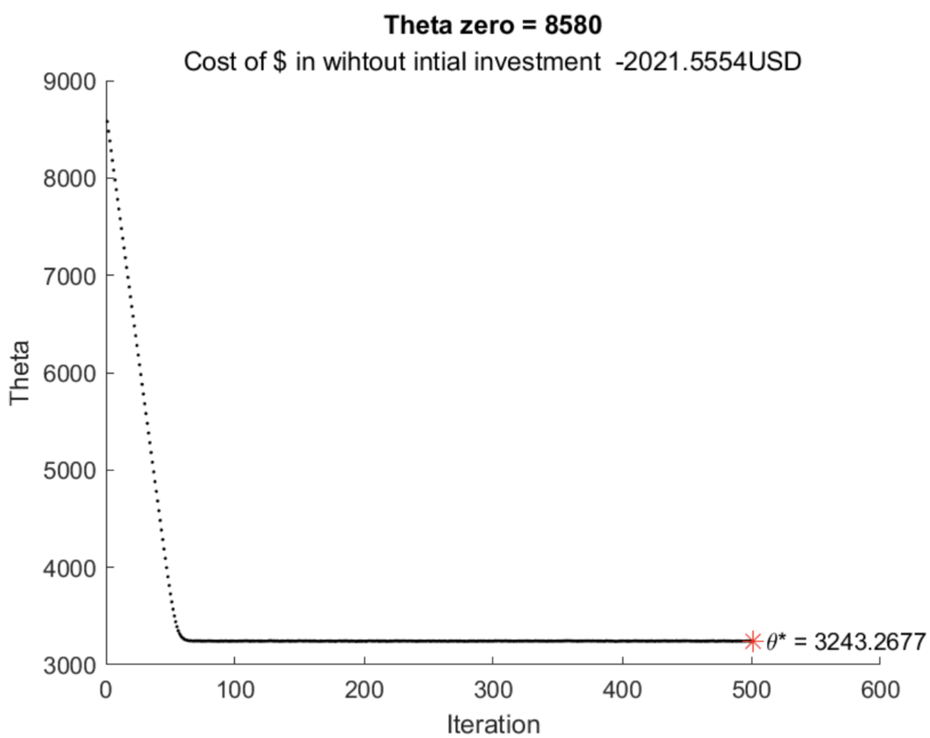
$$X_{\alpha,\sigma(\theta)} = n \cdot \sigma(\theta)(-\ln(1-U))^{\frac{1}{\alpha}}$$

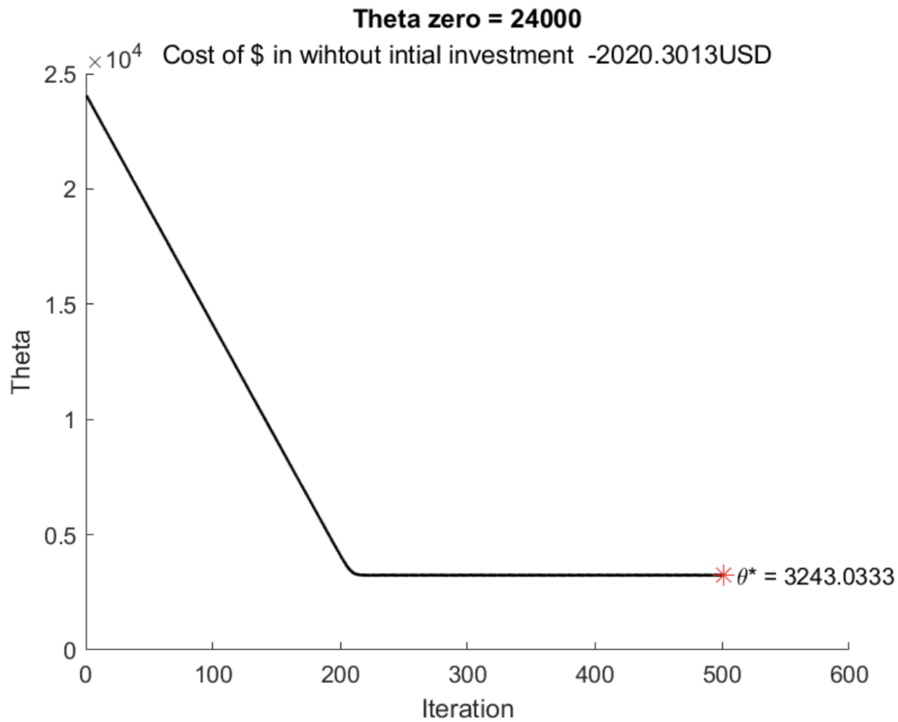
From this experiment, we recorded the average X_θ, and average demand per day for each season. These generated averages correlate with the real data that we showed in table 1 and 3

Simulated generated solar energy kWh per day during different seasons			
summer	fall	winter	spring
4.7	4.36	3.4	4.36
Simulated household demand kWh per day during different seasons			
summer	fall	winter	spring
29.9	25	21	23

In experiment 2 we obtain the same optimal theta as experiment 1, around 3242 USD, and we still pay an additional cost around $2k to the electricity provider at the end of the year


Theta zero = 5000
Cost of $ in wihtout intial investment -2020.7877USD
Theta
Iteration
θ* = 3243.3752
5000
4800
4600
4400
4200
4000
3800
3600
3400
0
100
200
300
400
500


Theta zero = 8580
Cost of $ in wihtout intial investment -2021.5554USD
Theta
Iteration
θ* = 3243.2677
9000
8000
7000
6000
5000
4000
3000
0
100
200
300
400
500
600


Theta zero = 24000
Cost of $ in wihtout intial investment -2020.3013USD
$\times 10^4$
2.5
2
1.5
1
0.5
0
Theta
θ* = 3243.0333
0
100
200
300
400
500
600
Iteration

6 Final Results

Parameters:

We set price per square meter of solar panel at $646

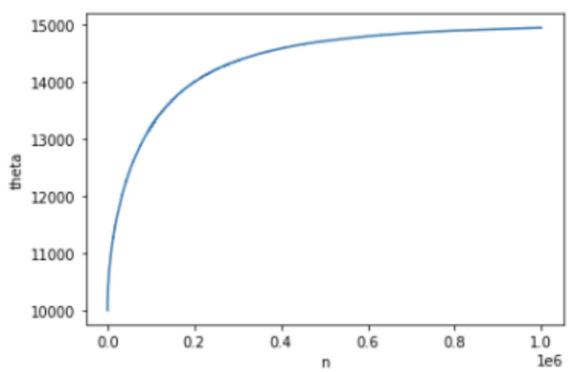
To generate demand and buying price, we used Weibull distribution with shape $\alpha = 12$ and scale beta dependent on the desired mean found in table 1 and 3

Selling price is calculated as 80% of buying price

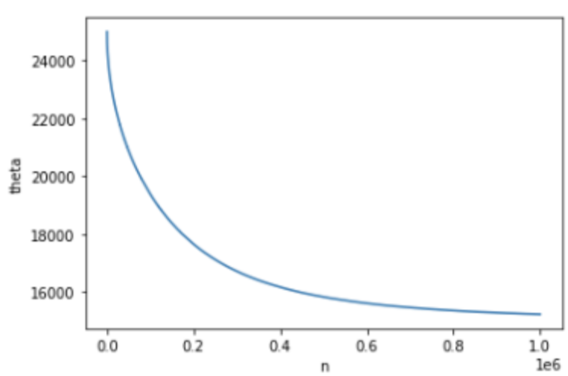
$X(\theta)$, the total amount generated, was generated using Weibull distribution with shape $\alpha = 12$ and scale beta dependent on the desired mean found in table 3, multiplied by the number of square meters which is a sigmoid function dependent on $\frac{\theta}{646}$ scaled by $\beta = -.045$ and shifted by -0.5

We simulated a years' worth of energy demand, production, and cost to calculate Yn and find optimal theta using gradient optimization $\theta_n + 1 = \theta_n + \epsilon Y_n$. We used step size $\epsilon_n = \frac{1}{\sqrt{n}}$

With initial theta of 10,000, optimization converges to about 15,000



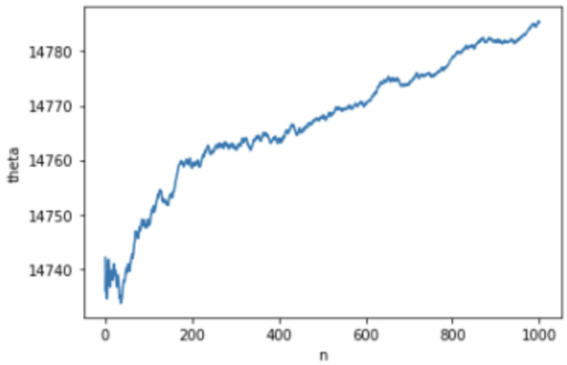
With the initial theta of 25,000, optimization also converges to about 15,000



There is room for variance in convergence due to all the randomness.

Since we have a million iterations, stochastic behavior is not clear, it is very zoomed out.

Here is an image zoomed in towards the end of convergence to show stochastic behavior.



At the end of the year, cost is 15106.61 for optimal θ 15103.89. This means cost is always approximatly θ. This means that the optimal amount of the investment it is to buy the solar panels that will cover up the electricity demand.

We calculated a confidence interval of the gradient estimation in this way. We have bounded variance, so we can apply the CLT. Then :

$$\overline{Y}(\theta)_n \pm 1.96\sqrt{Var(Y_n(\theta))}$$

The confidence interval of the gradient estimator. Hence, we have mean $\overline{Y}(\theta)_n = 0.256$ then interval is [-0.067 0.579]

7 Conclusion

In this work, we presented the stochastic optimization of the total cost of installation of solar panel systems in the New York City metropolitan area. As the amount of the kWh generated by solar panels per day, demand of the median household, and price per kWh are stochastic variables, we used an IPA method, to estimate stochastic derivative. This method can be applied to the proposed cost function as it is convex and has a minimum. Using the results of our optimization, we can conclude the optimal investment of solar panel systems will be 15 thousand UDS with 95% confidence. The confidence interval the confidence interval of the gradient estimator is [-0.067 0.579]. With such an initial investment, the total cost of the solar power plant will be $15000 in one year. This means that optimal theta is a theta that generates approximately the amount of solar energy needed to meet your demand, and it is not worth the extra investment to over generate and sell excess electricity to the grid for a cut. This estimation will show that even with an investment of $15k buyers should be aware it may take a while to see a return on their investment. The research was done to encourage people to consider installing solar panels as a long-term investment and contribution to reducing their carbon footprint.

8 Code

The project code is located in a folder.

9 Bibliography

1. Electricity Data Browser. (n.d.). https://www.eia.gov/electricity/data/browser/

2. Frequently Asked Questions (FAQs) - U.S. Energy Information Administration (EIA). (n.d.). https://www.eia.gov/tools/faqs/faq.php?id=427&t=3U. S. Energy Information Administration. Frequently Asked Questions

3. World Natural Gas Statistics - Worldometer. (n.d.). https://www.worldometers.info/gas/

4.More than half of small-scale photovoltaic generation comes from residential rooftops. (n.d.). https://www.eia.gov/todayinenergy/detail.php?id=31452

5. Global Solar Atlas. (n.d.). The World Bank Group. https://globalsolaratlas.info/mapc

6. How To Calculate Solar Panel Output? (n.d.).https://www.sunbasedata.com/how-to-calculat

7. 2022 400-Watt Solar Panel Price | Cost to Install a 400-Watt Solar Panel. (2021, October 20). Fixr.com. https://www.fixr.com/costs/400-watt-solar-panel

8. Felisa V´azquez-Abad, Bernd Heidergott. 2022. "Theory of Stochastic Optimization and Learning"

9. Gómez Sánchez, Mariam, Yunesky Masip Macia, Alejandro Fernández Gil, Carlos Castro, Suleivys M. Nuñez González, and Jacqueline Pedrera Yanes. 2021. "A Mathematical Model for the Optimization of Renewable Energy Systems" Mathematics 9, no. 1: 39.

10. Pullinger, Martin et al. “The IDEAL household energy dataset, electricity, gas, contextual sensor data and survey data for 255 UK homes.” Scientific data vol. 8,1 146. 28 May. 2021, doi:10.1038/s41597-021-00921-y