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1.

①. **Parameters Setting:**

Filter Length = 21

Sampling Frequency f_s = 8000Hz

Pass Band 1800~4000Hz

Transition Band: 1600~2000 Hz

Weighting Function: W(F) = 1 for passband, W(F) = 0.8 for stop band .

Δ = 0.0001

②. **Initial Parameter:**

Pass Band Ratio = Pass Band Frequency / Sampling Frequency

Pass Band 0.225 < F < 0.5

Transition Band Ratio = Transition Band Frequency / Sampling Frequency

Transition Band 0.2 < F < 0.25

Weighting Function: W(F) = $\begin{cases} 0.8, & \text{for } 0 < \mathrm{F} < 0.2 \\ 1, & \text{for } 0.25 < \mathrm{F} < 0.5 \end{cases}$

Interval = 0.0001

③. **Step 1: Choose extreme frequencies**

Since N = 21, k = $\frac{21-1}{2}$ = 10, and choose k + 2 = 12 extreme frequencies

Extreme frequencies are divided into 12 sections, and we have to make sure if each extreme frequency point isn't into the transition band. Let the upper limit of transition band plus epsilon = 0.001 if any extreme frequency point is into the transition band.

④. **Step 2: H_d Matrix From**

$[\mathrm{R}(F_m) - H_d(F_m)]\mathrm{W}(F_m) = (-1)^{m+1}e, m = 0, 1, 2, \ldots k+1$ can be written as

$$H_d(F_m) = \sum_{n=0}^{k} s[n]\cos(2\pi n F_m) + (-1)^m W^{-1}(F_m)e$$

The original can be expressed by the matrix form$M_{i+1,j+1}\, S = H_d$, where

$M_{i+1,\ j+1} = \begin{bmatrix} \cos(2\pi(i-1)F_{j-1}) & \frac{-1^{i-1}}{W(F_{j-1})} \end{bmatrix}$, and $S = [S[i-1] \quad e]^T$. Then, using inverse of matrix can solve S, such as $S = (M_{i+1,j+1})^{-1}H_d$

⑤. **Step 3: Compute err(F) for 0≦F≦0.5**

The ideal error function of Mini-max form maximal error can be written by *Max* $|R(f) - H_d|$, and weight maximal error err(f) is written by *Max* $|W(f)(R(f) - H_d)|$

⑥. **Step 4: Find k+2 extremum (local maximal or minimal) points**

To find the extremum, we can use “islocalmax” and “islocalmin” in MATLAB package. Furthermore, the first point (F = 0) or the last point (F = 0.5) cannot be found from “islocalmax” and “islocalmin”. So, we should add the first point or the last point in the extreme point list and put it in the ascending order so that we can make sure we always have enough points and correct order.

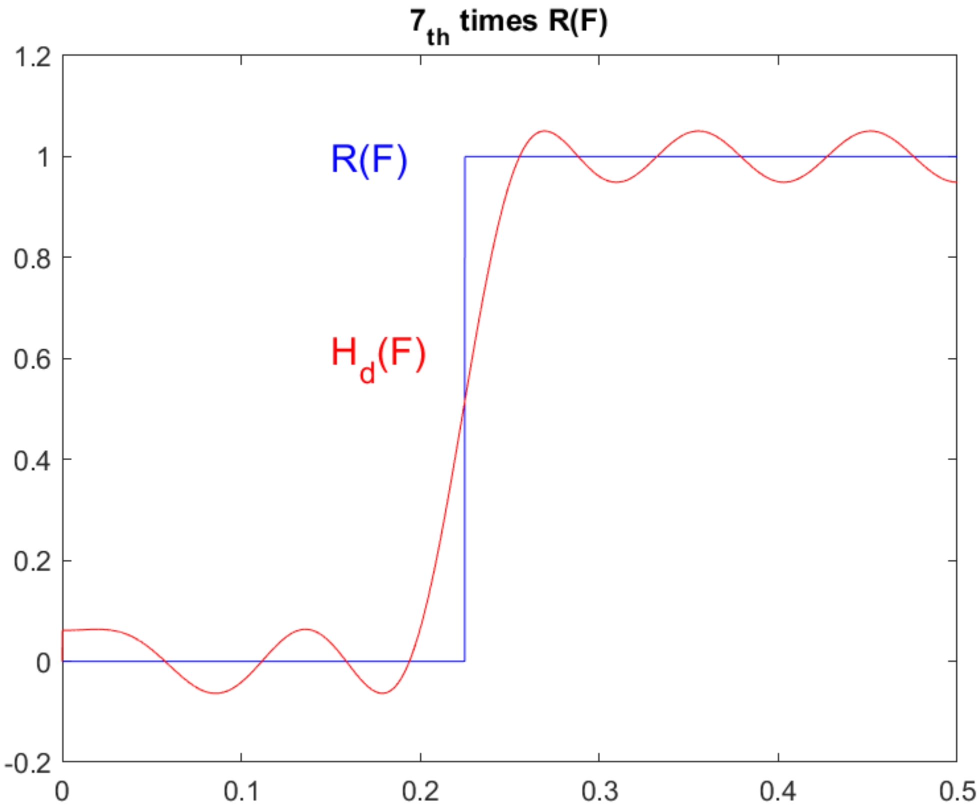
⑦. **Step 5: Minimize E_0**

Set $E_0 = Max(|err(F)|)$, if the relation between E_1 which is the E_0 of the last iteration and E_0 is satisfied by $0 < E_1 - E_0 < \Delta$, the program will stop. Otherwise, the step will continue to iterate.

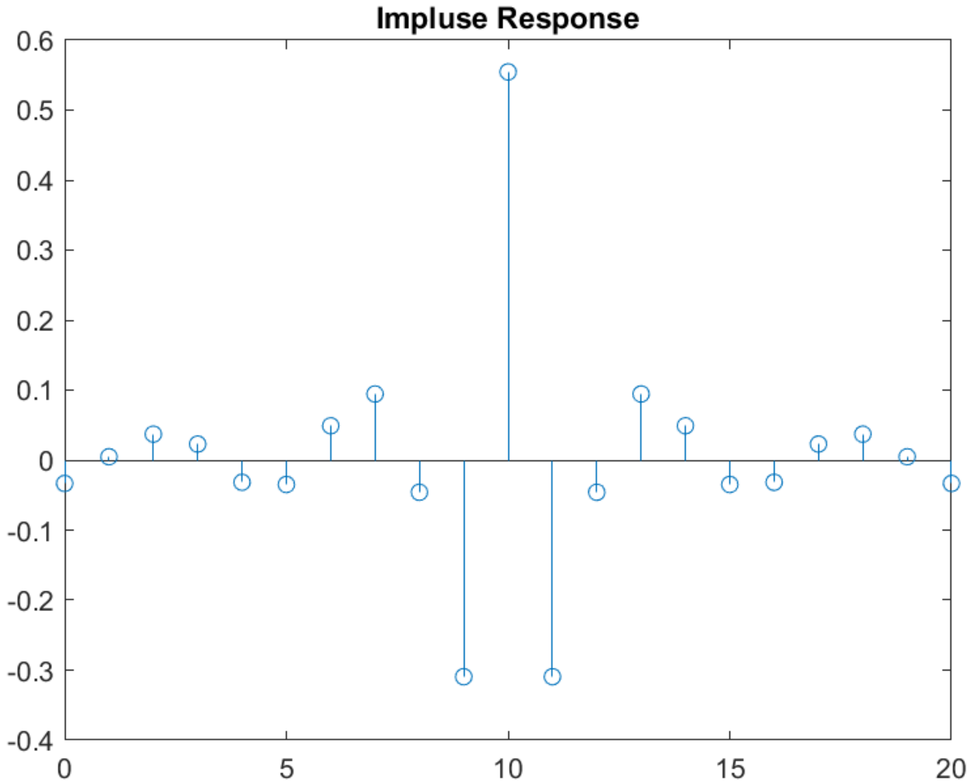
⑧. **Step 6: Find impulse response *h*[*n*]**

Set h[k] = s[0], and h[k+n] = h[k-n] = $\frac{s[n]}{2}$, fir n = 1, 2, 3, … , k

(a)



(b)



(c)

Iteration	1	2	3	4	5	6	7
Max[|err(F)|]	0.20976	0.094389	0.066435	0.05275	0.051288	0.051288	0.051288

2.

(a). Linear time invariant (LTI) can be implemented by convolution.

(b). After Fourier transformation, convolution can be converted to multiplication, and then taking the logarithm, multiplication can be converted to addition.

3.

(a).

①. The output is not infinite length.

②. The impulse response is relatively short, so the computation loading is smaller.

③. Because the function is of finite length, the filter is stable.

(b).

$$y[0] = x[0] * (0.7^0 u[0] + 0.2^0 u[0])$$
$$= 0.7^0 \sum_{t=-\infty}^{0} x[t] + 0.2^0 \sum_{t=-\infty}^{0} x[t]$$

$$y[1] = x[1] * (0.7^1 u[1] + 0.2^1 u[1])$$
$$= 0.7^1 \sum_{t=-\infty}^{1} x[t] + 0.2^1 \sum_{t=-\infty}^{1} x[t]$$
$$= 0.7^1 (y[0] + x[1]) + 0.2^1 (y[0] + x[1])$$

...

$$y[n] = x[n] * (0.7^n u[n] + 0.2^n u[n])$$
$$= 0.7^n \sum_{t=-\infty}^{n} x[t] + 0.2^n \sum_{t=-\infty}^{n} x[t]$$
$$= 0.7(y[n-1] + x[n]) + 0.2(y[n-1] + x[n])$$
$$= 0.9(y[n-1] + x[n])$$

4.

(a). The transition band can affect the performance of the filter in terms of the amount of attenuation or ripple in the passband and stopband, and the steepness of the transition. A wider transition band usually results in a smoother filter response, but also a wider transition region and a slower roll-off. Conversely, a narrower transition band leads to a steeper transition and a faster roll-off, but may also result in more passband ripple or stopband attenuation.

(b). The weight function is designed according to which band is more important, and the error in the transition band can be adjusted by the weight function.

5.

The sampling frequency $f_s = \frac{1}{0.001} = 1000$ Hz, and the signal length N = 6000

(a). $f = m\frac{f_s}{N}$, $\mathrm{m} = \mathrm{N}\frac{f}{f_s} = 6000 \times \frac{200}{1000} = 1200Hz$

(b). f = -100 Hz is the negative frequency, so frequency should be written by $f = m\frac{f_s}{N} - f_s$. Then we have $\mathrm{m} = \mathrm{N}\frac{f+f_s}{f_s} = 6000 \times \frac{-100+1000}{1000} =$

$5400\ Hz$

6.

7-point → N = 7, k = 3, -3 $\leqq$ n $\leqq$ 3

$$H_d(F) = \begin{cases} 1, & 0.1 < |F| < 0.4 \\ 0, & |F| > 0.4\ or\ |F| < 0.1 \end{cases}$$

$$\frac{\partial MSE}{\partial s[n]} = 2\sum_{\tau=0}^{k} s[\tau]\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi nF)\cos(2\pi\tau F)\,dF$$

$$-2\int_{-\frac{1}{2}}^{\frac{1}{2}} H_d(F)\cos(2\pi nF)\,dF \quad = 0$$

s[0] = $\int_{-\frac{1}{2}}^{\frac{1}{2}} H_d(F)dF$, s[n] = $2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi nF)H_d(F)dF$

s[0] = $\int_{-\frac{1}{2}}^{\frac{1}{2}} H_d(F)dF \;=\; \int_{-0.4}^{-0.1} H_d(F)dF \;+\; \int_{0.1}^{0.4} H_d(F)dF$

= $\mathrm{F}|_{-0.4}^{-0.1} \;+\; \mathrm{F}|_{0.1}^{0.4}$

= 0.6

s[1] = $2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi \times 1 \times F)H_d(F)dF$

= $2(\int_{-0.4}^{-0.1} \cos(2\pi F)H_d(F)dF \;+\; \int_{0.1}^{0.4} \cos(2\pi F)H_d(F)dF)$

= $2(\frac{\sin(2\pi F)}{2\pi}|_{-0.4}^{-0.1} + \frac{\sin(2\pi F)}{2\pi}|_{0.1}^{0.4})$

= $\frac{1}{\pi}\{[\sin(-0.2\pi) - \sin(-0.8\pi)] \;+\; [\sin(0.8\pi) - \sin(0.2\pi)]\}$

= $\frac{1}{\pi}\{[\sin(0.8\pi) - \sin(0.2\pi)] \;+\; [\sin(0.8\pi) - \sin(0.2\pi)]\}$

= $\frac{2}{\pi}[\sin(0.8\pi) - \sin(0.2\pi)]$

= 7.0679×10^{-17}

s[2] = $2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi \times 2 \times F)H_d(F)dF$

= $2(\int_{-0.4}^{-0.1} \cos(4\pi F)H_d(F)dF \;+\; \int_{0.1}^{0.4} \cos(4\pi F)H_d(F)dF)$

= $2(\frac{\sin(4\pi F)}{4\pi}|_{-0.4}^{-0.1} + \frac{\sin(4\pi F)}{4\pi}|_{0.1}^{0.4})$

= $\frac{1}{2\pi}\{[\sin(-0.4\pi) - \sin(-1.6\pi)] \;+\; [\sin(1.6\pi) - \sin(0.4\pi)]\}$

$= \frac{1}{\pi}[\sin(1.6\pi) - \sin(0.4\pi)]$

= -0.6055

s[3] = $2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi \times 3 \times F) H_d(F) dF$

$= 2(\int_{-0.4}^{-0.1} \cos(6\pi F) H_d(F) dF + \int_{0.1}^{0.4} \cos(6\pi F) H_d(F) dF)$

$= 2(\frac{\sin(6\pi F)}{6\pi} |_{-0.4}^{-0.1} + \frac{\sin(6\pi F)}{6\pi} |_{0.1}^{0.4})$

$= \frac{1}{3\pi}\{[\sin(-0.6\pi) - \sin(-2.4\pi)] + [\sin(2.4\pi) - \sin(0.6\pi)]\}$

$= \frac{2}{3\pi}[\sin(2.4\pi) - \sin(0.6\pi)]$

$= 2.3560 \times 10^{-17}$

7.

The passband ripple and the stopband ripple are smaller than 0.01, so δ_1 and δ_2 are equal to 0.01.

$\Delta F = \frac{f_1 - f_2}{f_s} = (f_1 - f_2)T = (3300 - 3000) \times 0.0001 = 0.03$

Then, the estimated length N can be

$N = \frac{2}{3}\frac{1}{\Delta F}\log_{10}\frac{1}{10\delta_1\delta_2} = \frac{2}{3} \times \frac{1}{0.03} \times \log_{10}\frac{1}{10 \times 0.01 \times 0.01} = 66.67$

Therefore, the estimated length of the digital filter is 67.

Extra (1):

Given sample frequency $f_s = 8000\ Hz$ and signal length N = 120000, what value is f if m = 96000?

Since m = 96000 is larger than $\frac{N}{2} = 60000$, Then $f = m\frac{f_s}{N} - f_s = 96000 \times \frac{8000}{120000} - 8000 = -1600\ Hz$