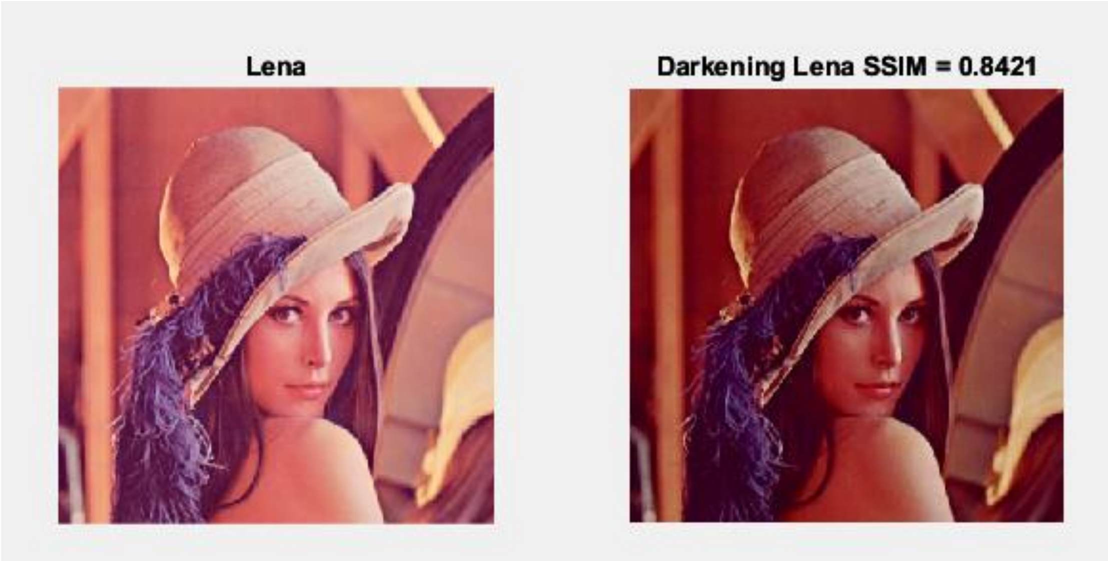
D11942011 林政均

1.



2.

(a).

假設有兩複數(a + bi)及(c + di)做相乘，得到

e + fi = (a + bi) (c + di) = (ac – bd) + (ad + bc)i

可改寫為

$$\begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} c & -d \\ d & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} c & c \\ c & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 0 & -(d+c) \\ d-c & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

①. $g_1 = c(a+b)$ $h_1 = g_1$

②. $g_2 = -b(c+d)$ $h_2 = \mathrm{a}(\mathrm{d}-\mathrm{c})$

③. $e = g_1 + g_2$ $\mathrm{f} = h_1 + h_2$

其中使用了$c(a+b)$、$-b(c+d)$和$\mathrm{a}(\mathrm{d}-\mathrm{c})$，3 個乘法

(b).

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} b_1 & -b_2 & -b_3 & -b_4 \\ b_2 & b_1 & b_4 & -b_3 \\ b_3 & b_4 & b_1 & -b_2 \\ -b_4 & b_3 & b_2 & b_1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} b_1 & -b_2 & -b_3 & -b_4 \\ b_2 & b_1 & b_4 & -b_3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$= \begin{bmatrix} b_1 & -b_2 \\ b_2 & b_1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} -b_3 & -b_4 \\ b_4 & -b_3 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \end{bmatrix}$$

$$= \begin{bmatrix} b_1 & b_1 \\ b_1 & b_1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} 0 & -b_2 - b_1 \\ b_2 - b_1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$+ \begin{bmatrix} -b_3 & -b_3 \\ -b_3 & -b_3 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \end{bmatrix} + \begin{bmatrix} 0 & -b_4 + b_3 \\ b_4 + b_3 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\begin{bmatrix} c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} b_3 & b_4 & b_1 & -b_2 \\ -b_4 & b_3 & b_2 & b_1 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \end{bmatrix}$$

$$= \begin{bmatrix} b_3 & b_4 \\ -b_4 & b_3 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \end{bmatrix} + \begin{bmatrix} b_1 & -b_2 \\ b_2 & b_1 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \end{bmatrix}$$

$$= \begin{bmatrix} b_3 & b_3 \\ b_3 & b_3 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \end{bmatrix} + \begin{bmatrix} 0 & b_4 - b_3 \\ -b_4 - b_3 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \end{bmatrix}$$

$$+ \begin{bmatrix} b_1 & b_1 \\ b_1 & b_1 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \end{bmatrix} + \begin{bmatrix} 0 & -b_2 - b_1 \\ b_2 - b_1 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \end{bmatrix}$$

共使用了 12 個乘法。

3.

假設 N-point DFT，其中 N＝ P_1P_2，其中P_1, P_2彼此互質，P_m-point DFT 的乘法量為B_m，則 N-point DFT 總乘法量可寫作P_1B_1 ＋ P_2B_2

又如果 N＝ P_1P_2，其中P_1, P_2最大公因數不為 1，P_m-point DFT 的乘法量為B_m，則 N-point DFT 總乘法量可寫作P_1B_1 ＋ P_2B_2 ＋3D_1 ＋ 2D_2

(a).

125-point DFT⇒$125 = 5 \times 25$

$MUL_{125} = 5MUL_{25} + 25MUL_5 + 3D_1 + 2D_2 = 5 \times 148 + 25 \times$

$10 + 3 \times 62 + 2 \times 0 = 1176$

(b).

147-point DFT⇒$147 = 3 \times 49$

$MUL_{49} = 7MUL_7 + 7MUL_7 + 3D_1 = 7 \times 16 \times 2 + 3 \times 26 =$

302

$MUL_{147} = 49MUL_3 + 3MUL_{49} = 49 \times 2 + 3 \times 302 = 1004$

(c).

385-point DFT⇒$385 = 11 \times 35$

$MUL_{385} = 11MUL_{35} + 35MUL_{11} = 11 \times 150 + 35 \times 40 = 3050$

4.

1D DFT 的複雜度為 $O(N\log_2 N)$

3D DFT 的複雜度可寫作

$MN \times K\log_2 K + NK \times M\log_2 M + KM \times N\log_2 N$

$= MNK \times \log_2(MNK)$

$\Rightarrow O(MNK \log_2(MNK))$

5.

$\text{Precision} = \frac{TP}{TP+FP} = \frac{900}{900+100} = 0.9$

$\text{Recall} = \frac{TP}{TP+FN} = \frac{900}{900+300} = 0.75$

$\text{F-score} = 2 \times \frac{\text{Precision}\times\text{Recall}}{\text{Precision}+\text{Recall}} = 2 \times \frac{0.9\times 0.75}{0.9+0.75} = 0.82$

6.

(a).

length(y[n]) = 500

N = 1100, M = 500

①. Direct:

FFT 點數為 500

實數乘法量 3MN = 3 × 1100 × 500 = 1650000

②. Sectioned convolution:

$L_0 = 550$, $P_0 = L_0 + M - 1 = 949$

取 P = 840，則 L = P – M + 1 = 341，$MUL_{840} = 4580$

$S = [\frac{1100}{341}] = 3$

FFT 點數為 840

實數乘法量 $2 \times 3 \times MUL_{840} + 3 \times 3 \times 840 = 35040$

③. Non-sectioned convolution

$P \geqq 1599$

取 P = 1680, $MUL_{1680} = 10420$

FFT 點數為 1680

實數乘法量 $2 \times MUL_{1680} + 3 \times 1680 = 25880$

Non-sectioned convolution 實數乘法量最少，因此最適合(a) (b).

length(y[n]) = 40

N = 1100, M = 40

①. Direct:

FFT 點數為 40

實數乘法量 3MN = 3 × 1100 × 40 = 132000

②. Sectioned convolution:

$L_0 = 248$, $P_0 = L_0 + M - 1 = 287$

取 P = 28，則 L = P – M + 1 = 249，$MUL_{288} = 1160$

$S = [\frac{1100}{249}] = 4$

FFT 點數為 288

實數乘法量 $2 \times 4 \times MUL_{288} + 3 \times 4 \times 288 = 12736$

③. Non-sectioned convolution

P ≧ 1139

取 P = 1152, $MUL_{1152} = 7088$

FFT 點數為 1152

實數乘法量 $2 \times MUL_{1152} + 3 \times 1152 = 52159$

Sectioned convolution 實數乘法量最少，因此最適合(b)

(c).

length(y[n]) = 6

N = 1100, M = 6

①. Direct:

FFT 點數為 6

實數乘法量 3MN = 3 × 1100 × 6 = 19800

②. Sectioned convolution:

$L_0 = 19$, $P_0 = L_0 + M - 1 = 24$

取 P = 24，則 L = P – M + 1 = 19，$MUL_{24} = 28$

$S = [\frac{1100}{19}] = 57$

FFT 點數為 24

實數乘法量 2 × 57 × MUL_{24} + 3 × 57 × 24= 7296

③. Non-sectioned convolution

P ≧ 1105

取 P = 1152, $MUL_{1152} = 7088$

FFT 點數為 1152

實數乘法量 2 × MUL_{1152} + 3 × 1152 = 52159

Sectioned convolution 實數乘法量最少，因此最適合(c)

(d).

length(y[n]) = 2

N = 1100, M = 2

①. Direct:

FFT 點數為 2

實數乘法量 3MN = 3 × 1100 × 2 = 6600

②. Sectioned convolution:

$L_0 = 2,\ P_0 = L_0 + \mathrm{M} - 1 = 3$

取 P = 4，則 L = P – M + 1 = 3，$\mathrm{MUL}_4 = 0$

$\mathrm{S} = [\frac{1100}{3}] = 366$

FFT 點數為 4

實數乘法量 2 × 366 × MUL_4 + 3 × 366 × 4= 4392

③. Non-sectioned convolution

P ≧ 1101

取 P = 1152, $\mathrm{MUL}_{1152} = 7088$

FFT 點數為 1152

實數乘法量 2 × MUL_{1152} + 3 × 1152 = 52159

Sectioned convolution 實數乘法量最少，因此最適合(d)