

Homework 4 (Due: 8th Dec.)

(1) Suppose that the STFT of $x(t)$ satisfies

$$\begin{aligned} |X(t, f)| \geq \textit{threshold} \quad & \text{when } 0 \leq t \leq 6, \quad 2t - 8 \leq f \leq 2t - 4 \\ & \text{or } -3 \leq t \leq 3, \quad t^3 + 3t^2 + 10 \leq f \leq t^3 + 3t^2 + 15 \end{aligned}$$

$$|X(t, f)| < \textit{threshold} \quad \textit{otherwise}$$

How do we sample $x(t)$ in an efficient way? (10 scores)

(2) (a) In addition to that the FT is not required, what is the main advantage of the HHT? (b) What are the similarities and differences between the sinusoid function and the intrinsic mode function? (c) Which of the following signals are IMFs?

$$\begin{array}{lll} \text{(i) } \cos(t^4+t), & \text{(ii) } t^2+\sin(2\pi t) & \text{(15 scores)} \end{array}$$

(3) Suppose that $x(t)$ is a white noise. Which of the following function is also a white noise? Why? (i) $x(3t)$, (ii) $\exp(j\pi t^2)x(t)$, (iii) $\exp(j\pi t^2) * x(t)$ where $*$ means the convolution. (iv) $\text{sinc}(t) * x(t)$. (10 scores)

(4) (a) What is the most important advantage of the Haar transform nowadays?
(b) Write the 12th row of the 32-point Haar transform. (10 scores)

(5) (a) What are the roles of the vanish moment for the wavelet transform design?
(b) What is the vanish moment of $x\exp(-|x|)$? (10 scores)

(6) Among the Gabor transform, the WDF, the HHT, and the wavelet transform, which one is better for the applications of (a) image denoising, (b) climate data analysis, (c) Tone analysis? Also illustrate the reasons. (15 scores)

(7) Write a Matlab or Python program of the Hilbert-Huang transform.

$y = \text{hht}(x, t, \text{thr})$

x: input, y: output (each row of y is one of the IMFs of x), t: samples on the t -axis, thr : the threshold used in Step 7.

In Step 8, the number of non-boundary extremes can be no more than 3.

The code should be handed out by NTUCool. (30 scores)

Example: $t = [0: 0.01: 10];$

$x = 0.2*t + \cos(2*\pi*t) + 0.4*\cos(10*\pi*t);$

$\text{thr} = 0.2;$

$y = \text{hht}(x, t, \text{thr});$

(Extra): Answer the questions according to your student ID number.

(ended with 0, 2, 3, 4, 5, 7, 8, 9)