We'll begin at 7:05 am

Module 1: Vectors

1.1 Introduction to Vectors

1. Identify a vector quantity from a scalar quantity
2. Calculate the components of a vector
3. Write the vector in different notations
4. Draw the vector in the Cartesian Coordinate System

Scalars and Vectors

- When a physical quantity is described by a single number, we call it a **scalar quantity**.
- In contrast, a **vector quantity** has both magnitude and direction.

TAKE NOTE!

Combining scalar quantities use the operations of ordinary arithmetic.

Example: 6 kg + 3 kg = 9 kg
Example: 12 m – 2 m = 10 m
Example: 3 s * 3 s = 9 s^2
Example: 12 mL / 3 mL = ?

Scalar	Vectors
Distance (m)	Displacement (m)
Speed (m/s)	Velocity (m/s)
Time (s)	Acceleration (m/s^2)
Mass (kg)	Weight (N or kg*m/s^2)
Pressure (Pa)	Momentum (kg*m/s)
Volume (m^3)	Electric Field (N/C)
Resistance (Ω)	Magnetic Field (T)
Area (m^2)	Torque (N*m)
Density (kg/cm^3)	Current (A)
Entropy (J/K)	Impulse (N*s)

(1) The car is going 75 mph.

(2) You walked 4 mph toward the store.

(3) The box in the west corner of the room has a mass of 12 kilograms.

(4) The time is 12:30 pm.

(5) The pressure inside a balloon is 2 atmospheres.

Scalars and Vectors

Test Your Understanding!

(1) The car is going 75 mph. This is a scalar value because you don't know which direction the car is going.

(2) You walked 4 mph toward the store. This is a vector because you have both a magnitude and a direction.

(3) The box in the west corner of the room has a mass of 12 kilograms.

The mass of the box is a scalar quantity. Even though you know the location of the box, this fact has nothing to do with its mass.

(4) The time is 12:30 pm. This is a scalar. There is no direction.

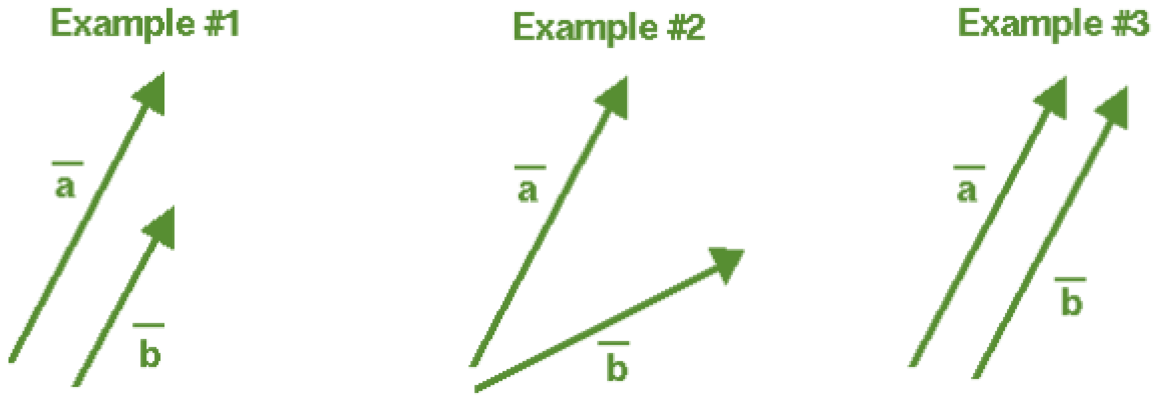
(5) The pressure inside a balloon is 2 atmospheres.

Pressure has a magnitude, but it does not have a direction. Another way of looking at it is that pressure acts in all directions at once.

Scalar and Vector Notation

- Vectors can be represented using arrows or components.
- Vectors as arrows:
  - A vector is drawn as an arrow with a head and a tail.
  - The magnitude of the vector is often described by the length of the arrow.
  - The arrow points in the direction of the vector.

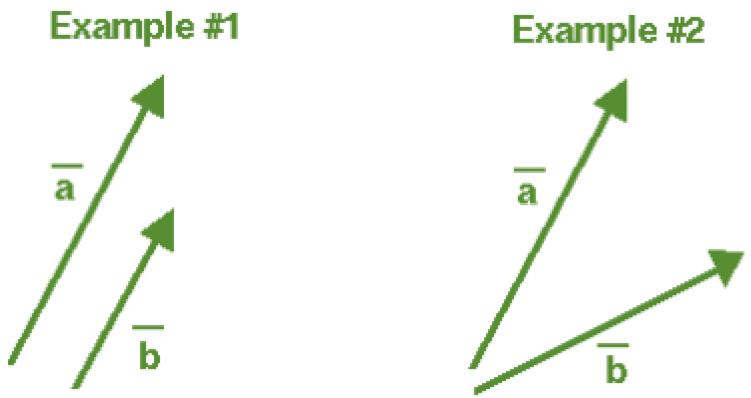
https://www.grc.nasa.gov/www/k-12/airplane/vectcomp.html



https://www.grc.nasa.gov/www/k-12/airplane/vectcomp.html

Vector as Arrow

- If two vectors have the same magnitude, they are equivalent.
- If two vectors have the same direction, they are parallel.
- If two vectors are pointing on opposite directions, they are called anti-parallel.
- If they have the same magnitude and the same direction, they are equal.



https://www.grc.nasa.gov/www/k-12/airplane/vectcomp.html

Vector Notation

Polar Form

N= (N, θ)

***N* = Magnitude**
***θ* = Direction**

Ex.:
G= (89 Newton, 45°)

Magnitude
G = 89 Newton

Direction
θ = 45° North of East

(Unit Vector Form)

N= $N_x\hat{\imath} + N_y\hat{\jmath} + N_z\hat{k}$

Ex.:
E= $4\hat{\imath}$ $-8\hat{\jmath}$ $+23\hat{k}$

x-components
$E_x = +4\hat{\imath}$
y-components
$E_y = -8\hat{\jmath}$
z-components
$E_z = +23\hat{k}$

Rectangular Form Ordered Set

N= (N_x, N_y, N_z)

Components
"x, y, z"

Ex.:
M= (3, 5, -10)

x-components
$M_x = 3$
y-components
$M_y = 5$
z-components
$M_z = -10$

Matrix

N= $\begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix}$

Ex.:
P= $\begin{bmatrix} -7 \\ 11 \\ 25 \end{bmatrix}$

x-components
$P_x = -7$
y-components
$P_y = 11$
z-components
$P_z = 25$

Vector Notation

Standard Form

N= N, θ

Polar Form

N= (N, θ)

Unit Vector Form

N= $N_x\hat{\imath} + N_y\hat{\jmath} + N_z\hat{k}$

Rectangular Form Ordered Set

N= (N_x, N_y, N_z)

Matrix

N= $\begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix}$

N* = *Magnitude
θ* = *Direction

Components
"Nx, Ny, Nz"

Ex.:
G= (89 Newton, 45°)

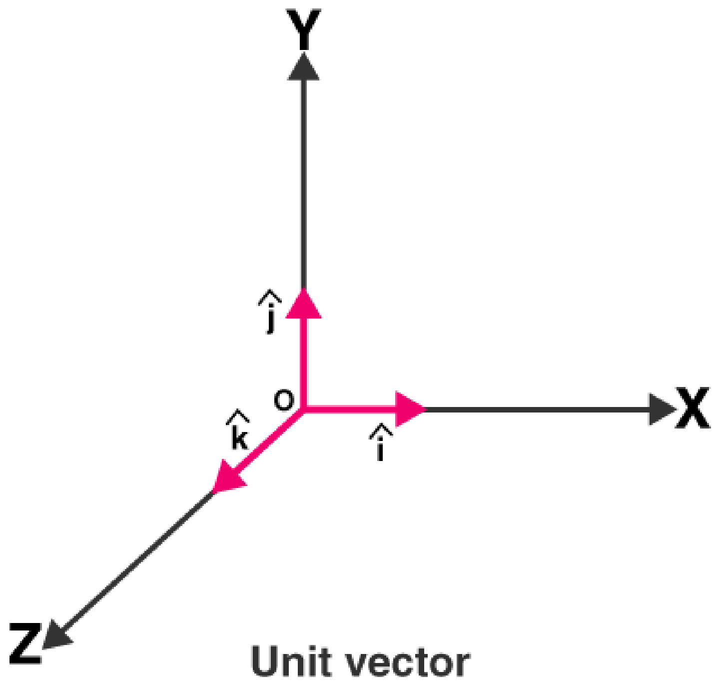
Ex.:
E= $4\hat{\imath} - 8\hat{\jmath} + 23\hat{k}$

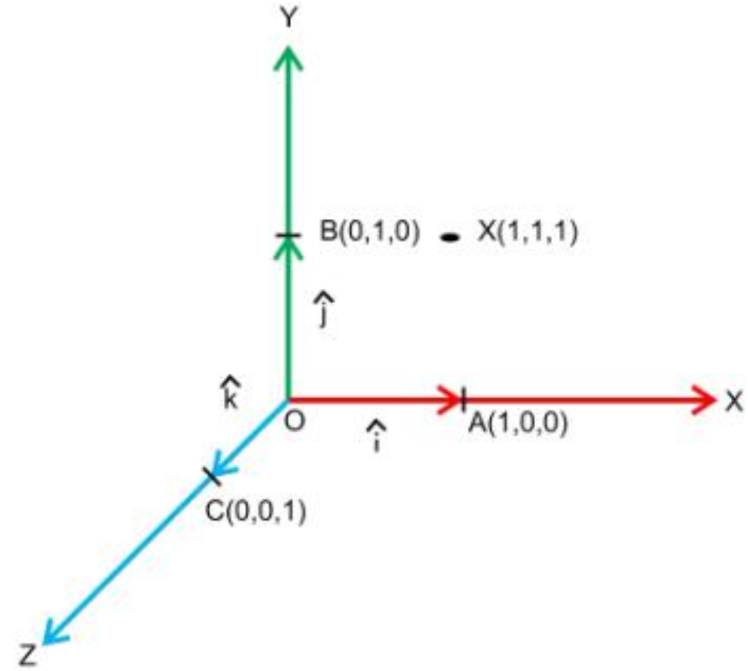
Ex.:
M= (3, 5, -10)

Ex.:
P= $\begin{bmatrix} -7 \\ 11 \\ 25 \end{bmatrix}$

Unit Vector

- A unit vector is a vector that has a **magnitude of exactly 1**.
- Unit vectors point along axes in a right-hand coordinate system.





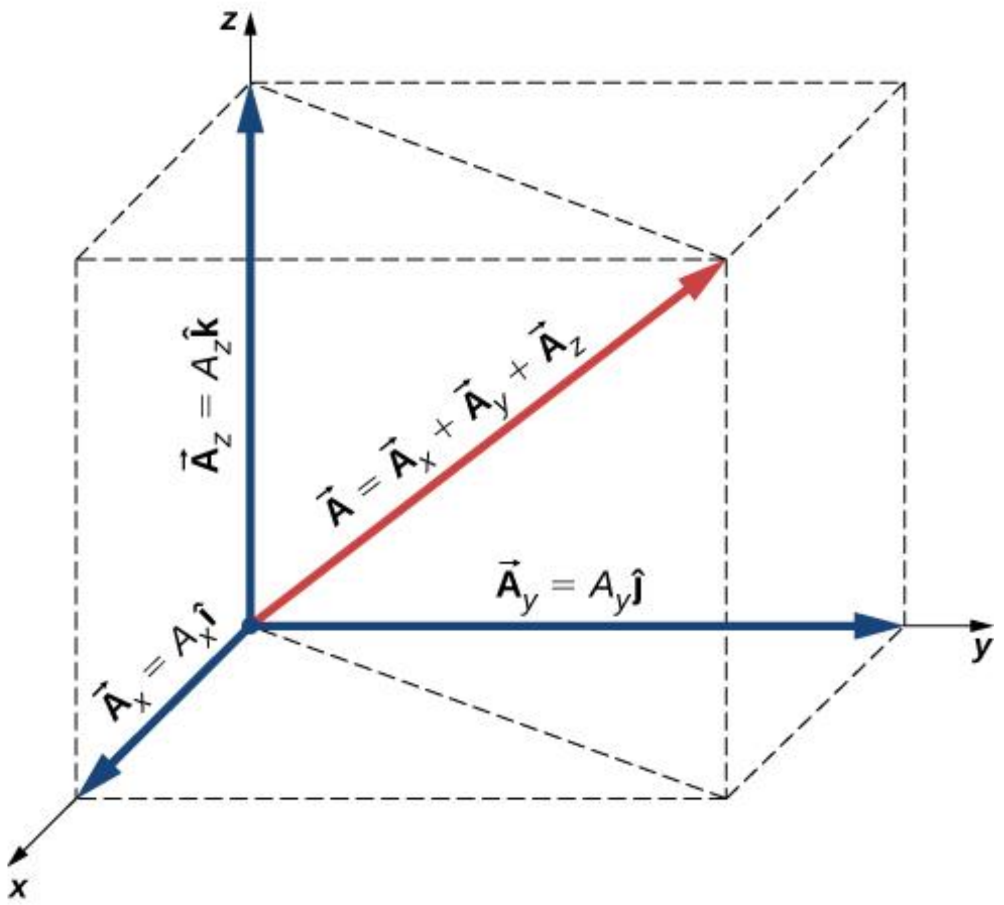
Cartesian **Vector** Representation:

$$\boldsymbol{N} = N_x\hat{\imath} + N_y\hat{\jmath} + N_z\hat{k}$$

Magnitude of Cartesian Vector:

$$\boldsymbol{N} = \sqrt{N_x^2 + N_y^2 + N_z^2}$$

Unit Vector



A vector in three-dimensional space is the vector sum of its three vector components.

// Every vector has its own **unit vector representation** described by the image on the left.

$\boldsymbol{P} = 2i + 3j + 5k$

Unit Vector Representation

To find a unit vector, $\vec{u}$, in the same direction of a vector, $\vec{A}$, we use the formula:

$$\vec{u} = \frac{\vec{A}}{|\vec{A}|}$$

// Simply get the ratio of the vector itself and its magnitude.

Example: Find the unit vector in the same direction of the vector $\vec{F} = < 3, -4 >$

Example: Find the unit vector in the same direction of the vector $\vec{V} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

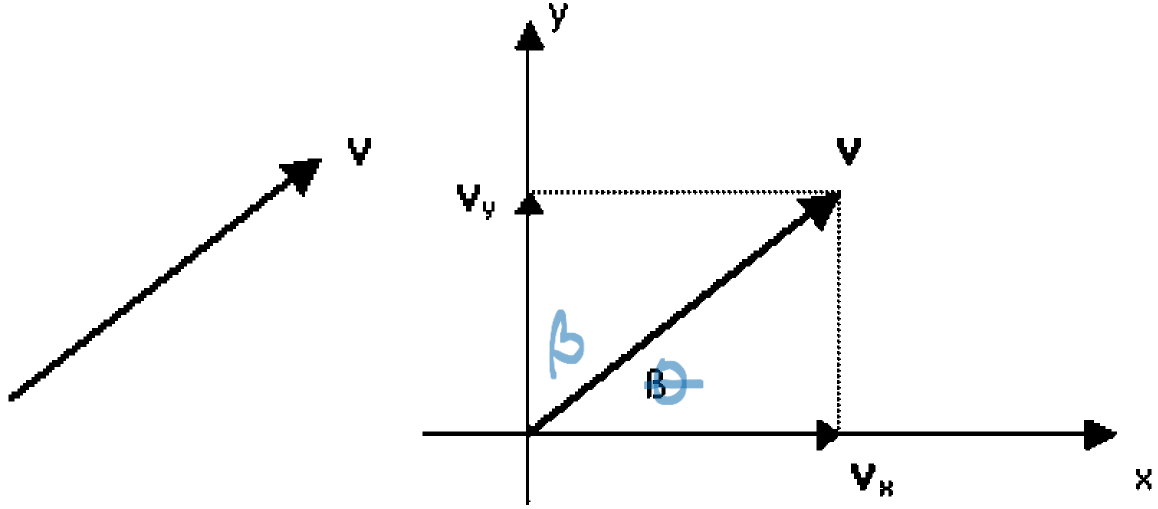
FOLLOW THESE LIFE-SAVING STEPS:

STEP 1: Get the magnitude of the vector.
STEP 2: Write the components of the vector and the magnitude as a ratio.
STEP 3: Perform the operation.
STEP 4: Write the unit vector.

Components of a Vector

Vector, **V** represents a vector quantity

- First, horizontal and vertical lines are drawn from the tail of the vector
- Second, a triangle is drawn that encloses the vector **V**
- The sides of the rectangle are the desired components, vectors **Vx** and **Vy**



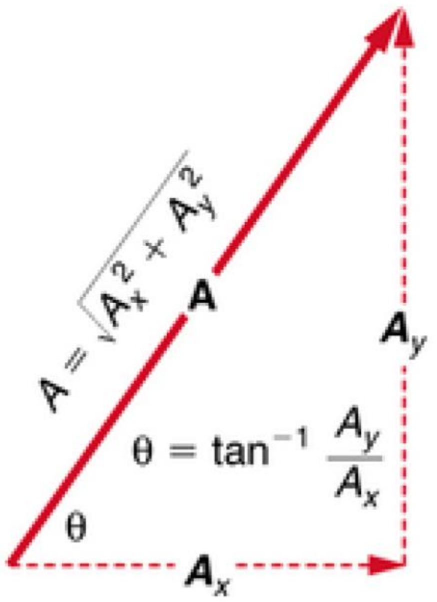
$A_x = A \sin \beta$
$A_y = A \cos \beta$

Calculation of vector components:

- x-component of Vector **A**:
  Ax = A cos θ
- y-component of Vector **A**:
  Ay = A sin θ

Note: θ makes with the horizontal

// We use the unit vectors to represent components.

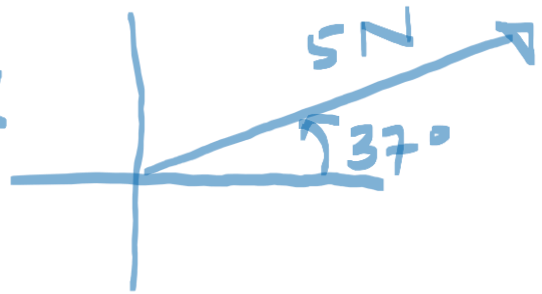


1. Resolve the vector **A** = 5 Newton, 37° N of E (find its components)

Solution:

$A_x = A\cos\theta =$ 5 cos 37° = 3.99 N

$A_y = A\sin\theta =$ 5 sin 37° = 3.01 N



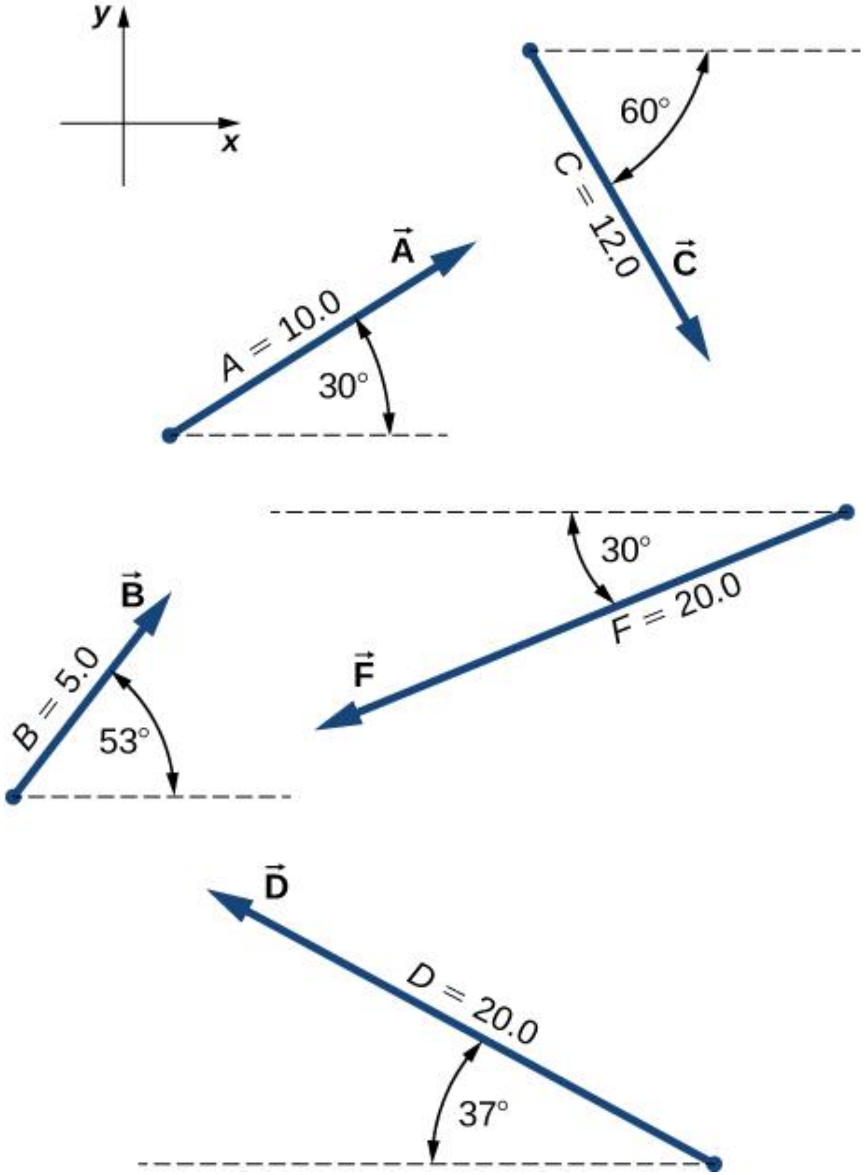
Rec (A, θ) from polar axis

Rec(5, 37°)

$R_x = X = 3.99$ N

$R_y = Y = 3.01$

2. Assuming the +x-axis is horizontal and points to the right, resolve the vectors given in the following figure to their scalar components and express them in vector component form.



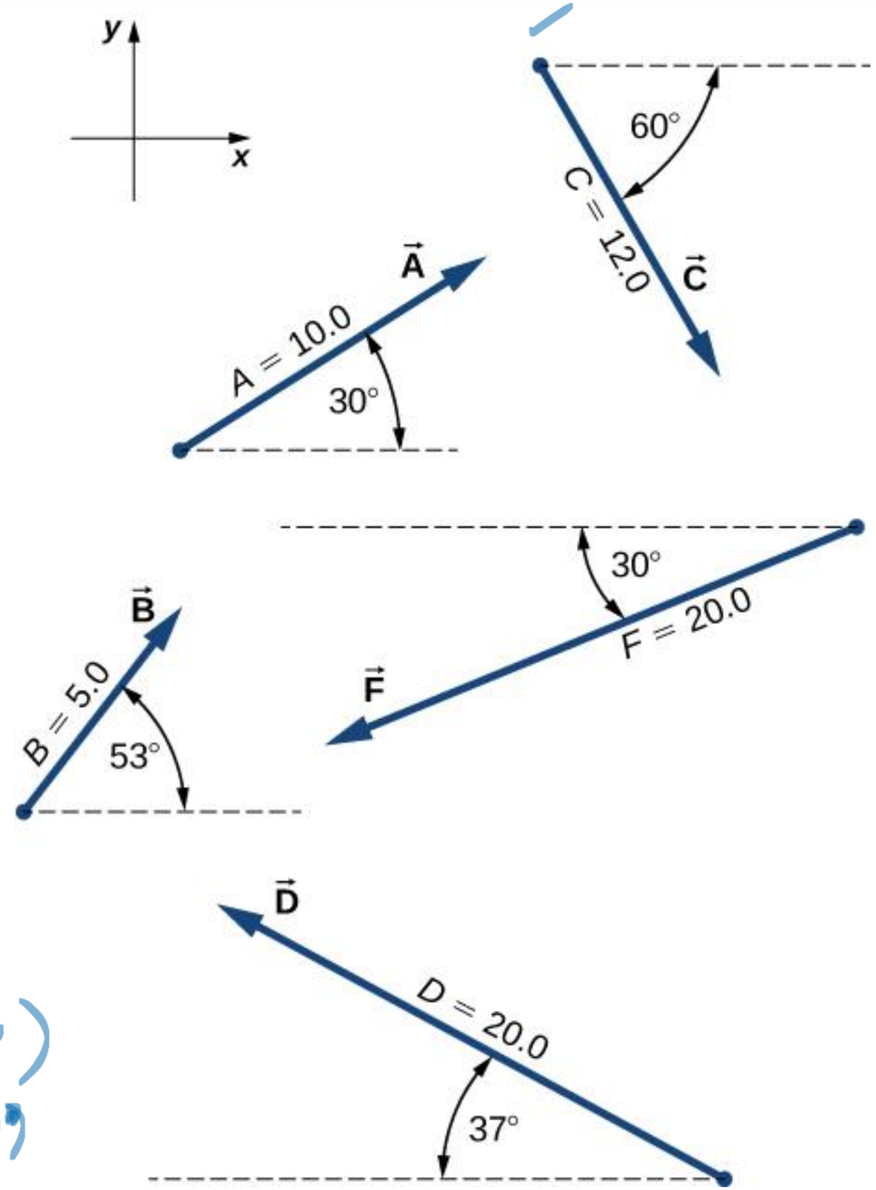
a) $\vec{A} = (10, 30°)$ → Polar form

$\vec{A} = (8.66, 5)$ → Rectangular

$= 8.66\hat{i} + 5\hat{j}$ → unit vecto i-j

$= \begin{bmatrix} 8.66 \\ 5 \end{bmatrix}$ → matrix

2. Assuming the +x-axis is horizontal and points to the right, resolve the vectors given in the following figure to their scalar components and express them in vector component form.



b.) $\vec{B} = (5, 53°)$ Polar

$\vec{B} = (3.01, 3.99)$ Rect

c) $\vec{C} = (12, -60°)$ Polar

$= (6, -10.39)$ Rect.

(C_x, C_y)

Polar

$C_x = 12\cos(-60°)$

$= +12\cos 60°$

$C_y = 12\sin(-60°)$

$= -12\sin(60°)$

Components of a Vector

Example

2. Assuming the +x-axis is horizontal and points to the right, resolve the vectors given in the following figure to their scalar components and express them in vector component form.

d) $\vec{D} = (20, 143^\circ)$ Polar

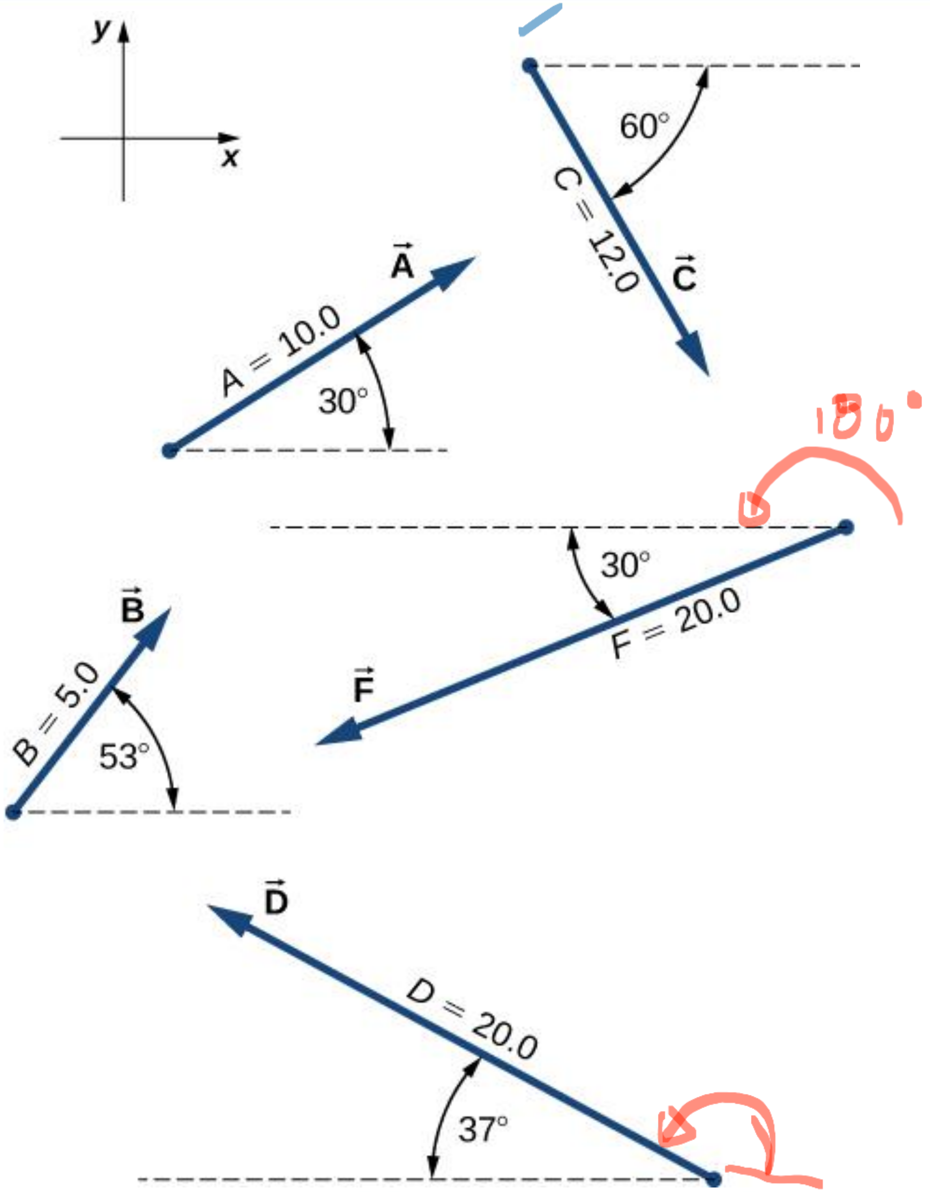
$\vec{D} = (20, 143^\circ)$ Rect

$D_x = 20\cos 143^\circ$

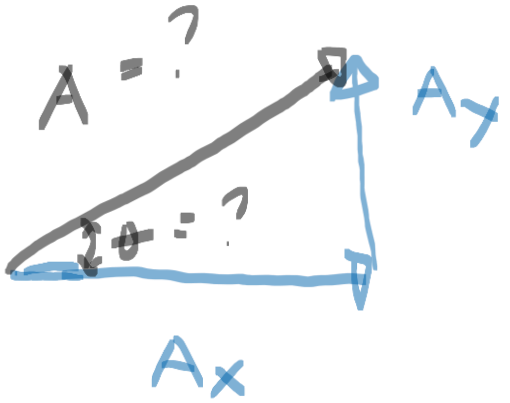
$D_y = 20\sin 143^\circ$

e) $\vec{F} = (20, 210^\circ)$ Polar

$\vec{F} = (-17.32, -10)$ Rect.



$\vec{A} = (A_x, A_y)$ or $\vec{A} = A_x\hat{i} + A_y\hat{j}$



$A = \sqrt{A_x^2 + A_y^2}$ (2-D)

$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ (3-D)

$\tan\theta = \frac{A_y}{A_x}$

$\theta = \tan^{-1}\frac{A_y}{A_x} + 180^\circ$ (II, III)

$+ 360^\circ$ (IV)

Polar. directi

if tan

(−, +)

Magnitude of a Vector

For a vector $\vec{A}$ in a 2-dimensional space, the magnitude can be calculated as:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

For a vector $\vec{A}$ in a 3-dimensional space, the magnitude can be calculated as:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Try these!

Find the magnitude of the following vectors:

1. <3,5,2> $= \vec{a}$ $\quad |\vec{a}| = \sqrt{3^2+5^2+2^2} = \sqrt{38} = 6.16$
2. $i - j + 2k$ $= \vec{b}$ $\quad |\vec{b}| = \sqrt{1^2+(-1)^2+2^2} = \sqrt{6} = 2.45$
3. <0,0,9> $= \vec{c}$ $\quad |\vec{c}| = \sqrt{9^2} = 9$

// Just think of it this way: to get the magnitude of a vector, just get the square root of the sum of the squares of the components ☺

Direction of a Vector

For a vector $\vec{A}$ in a 2-dimensional space, the direction can be calculated as:

$$\theta = \tan^{-1}\frac{A_y}{A_x} + 180^\circ \text{ (QII or QIII)}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x} + 360^\circ \text{ (QIV)}$$

polar form
ccw

Try these!

Find the angle of inclination of the following vectors:

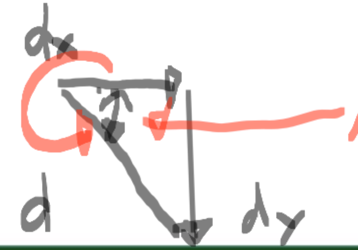
1. <1.13, -3.22> $= \vec{d}$
2. <15.02, 29.16>
3. 4*i* + 7*j*
4. <5,0>

(QIV)

$\theta_d = \tan^{-1}\left(\frac{-3.22}{1.13}\right) + 360^\circ$

$= 289.34^\circ$

$\theta_d \approx 289^\circ$



$\beta = -70.66^\circ$

TAKE NOTE!
The angle theta, θ is actually the angle of inclination of a vector with respect to the positive x-axis. For a vector in 3D space, the angle theta does not completely describe its direction.

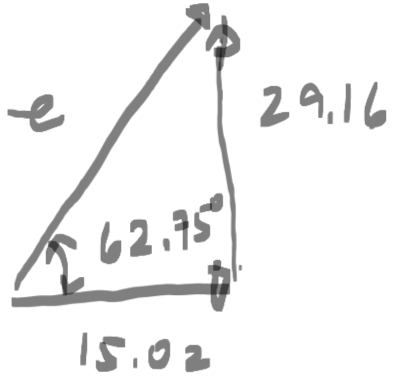
Direction of a Vector

For a vector $\vec{A}$ in a 2-dimensional space, the direction can be calculated as:

$$\theta = \tan^{-1}\frac{A_y}{A_x} + 180^{\circ} \text{ (QII or QIII)}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x} + 360^{\circ} \text{ (QIV)}$$

polar form
ccw



Try these!

Find the angle of inclination of the following vectors:

1. <1.13, -3.22> -
2. <15.02, 29.16> = $\vec{e}$
3. 4*i* + 7*j*
4. <5,0>

$$\theta_e = \tan^{-1}\frac{29.16}{15.02} = 62.75^{\circ}$$

$$\mathrm{Pol}(A_x, A_y) \Rightarrow \mathrm{Pol}(15.02, 29.16)$$

$$R = e = 32.8$$

$$\theta = \theta_e = 62.75^{\circ}$$

Direction of a Vector

For a vector $\vec{A}$ in a 2-dimensional space, the direction can be calculated as:

$$\theta = \tan^{-1}\frac{A_y}{A_x} + 180^{\circ} \text{ (QII or QIII)}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x} + 360^{\circ} \text{ (QIV)}$$

polar form
ccw

Try these!

Find the angle of inclination of the following vectors:

1. <1.13, -3.22>
2. <15.02, 29.16>
3. $4i + 7j$ = f
4. <5,0> = g

5. (0, -5) = h

$\theta_f = \tan^{-1}\frac{7}{4} = 60.26^{\circ}$

Pol(4, 7)

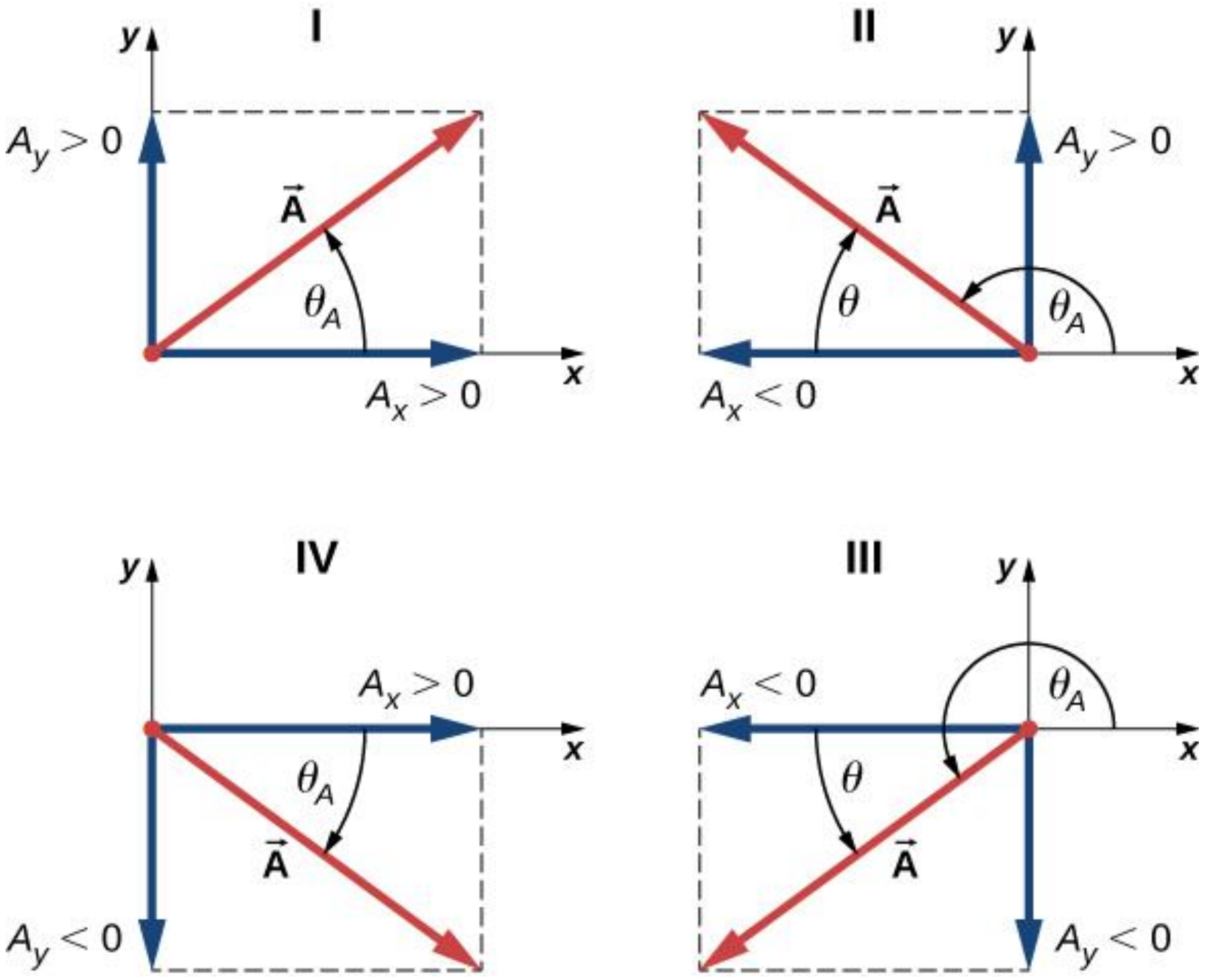
$\theta_f = 60.26$

$\theta_g = 0^{\circ}$

$\theta_h = 270^{\circ}$ or -90°

$\theta_h = \tan^{-1}\frac{-5}{0}$ undefined

Graphical Representation of a Vector



- Scalar components of a vector may be positive or negative.
- Vectors in the first quadrant (I) have both scalar components positive and vectors in the (III) quadrant have both scalar components negative.
- For vectors in quadrants II and IV, the direction angle of a vector is $\theta_A = \theta + 180°$.

Practice Problem

Identify 3 vectors that are parallel with the vector $\vec{V} =< -3, -2,5 >$

Given that $\vec{Y} =< 2, -3 >$ and $\vec{Z} =< 8, y >$ are parallel, what must be the value of y?

In general, a vector in a 3-dimensional space given by $\vec{A} = a < A_x, A_y, A_z >$ is parallel with any vector with what values of “a”?

What are the magnitudes of the vectors given below:

$\vec{A} =< 2, -1, -6 >$

$\vec{B} =< 0,0,9 >$

$\vec{C} =< -7, -5, -1 >$

$\vec{D} = -11\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$

Find the unit vector in the same direction of the vectors:

$\vec{E} =< 3,3,3 >$

$\vec{F} =< 3,0,4 >$

$\vec{G} = 8\hat{\imath} - \hat{k}$

Module 1: Vectors

Got Any Questions????

References:

- Ling, Samuel J., et.al (2018). University Physics Volume 1.OpenStax Rice University. Texas. Accessed at https://openstax.org/details/books/universityphysicsvolume1
- Mathonline Standard Unit Vectors. Accessed at http://mathonline.wikidot.com/standardunitvectors
- Math and Physics Homework by Email (2020). Accessed at https://www.jfinternational.com/ph/vectorsscalarsanalyticmethod.html
- https://openstax.org/books/university-physics-volume-1/pages/2-4-products-of-vectors
- https://www.cuemath.com/geometry/cross-product/