Module 9: Equilibrium and Torque

Conditions for Equilibrium

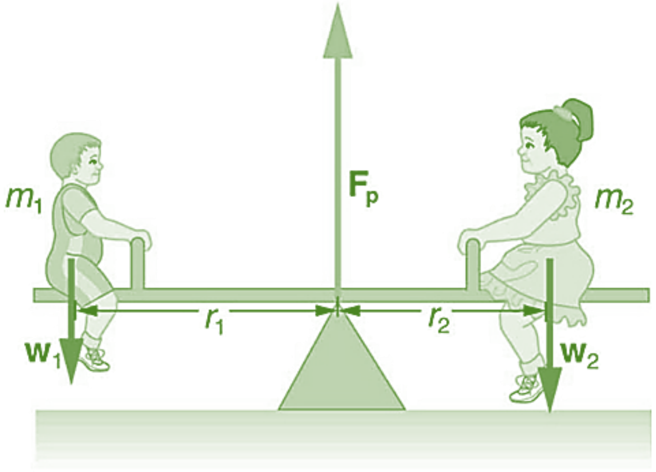
OBJECTIVES:

1. Identify the different conditions for equilibrium
2. Define torque physically and mathematically
3. Cite real life situations where torque can be seen or applied
4. Solve problems involving equilibrium and torque

What does "equilibrium" mean?

- An object at equilibrium has no net influences to cause it to move, either in translation (linear motion) or rotation.

- REMEMBER IN NEWTON'S LAWS APPLICATIONS, WE DISCUSSED "TRANSLATIONAL EQUILIBRIUM" WHICH WILL ONLY BE ATTAINED WHEN THE SUMMATIONS OF FORCES ALONG X AND Y ARE ZERO.



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Conditions for Equilibrium

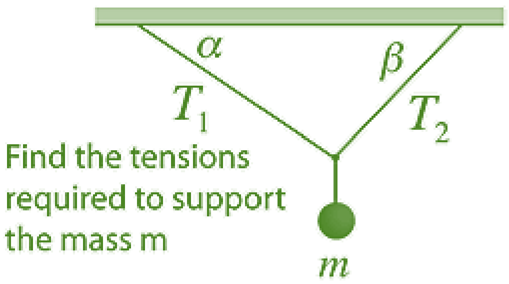
- The basic conditions for equilibrium are:

1. Net force = 0

$$\sum_i F_i = 0$$

x and y components of force may be separately set = 0.

Forces left = forces right
Forces up = forces down.



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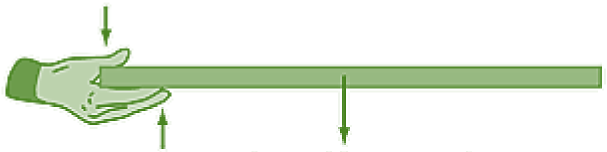
2. Net torque = 0

$$\sum_i \tau_i = 0$$

The axis may be chosen for advantage to eliminate some unknown forces..

The sum of the clockwise torques is equal to the sum of the counterclockwise torques.



Lifting a board by its end.

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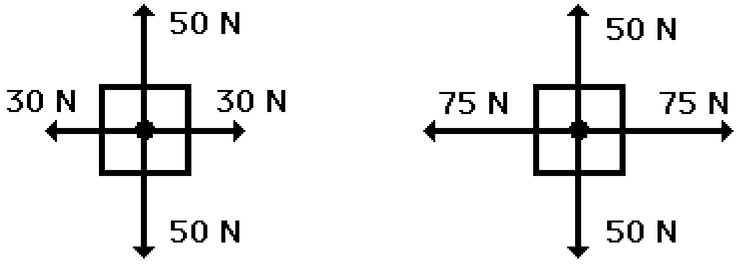
Static vs. Dynamic Equilibrium

- Dynamic equilibrium is the steady state of a reversible reaction where the rate of the forward reaction is the same as the reaction rate in the backward direction.

- Static equilibrium, also known as mechanical equilibrium, means the reaction has stopped. In other words, the system is at rest.

- Example is an object (e.g book) lying still on the surface (e.g table). no resultant moment about a pivot, so clockwise moment equals anticlockwise moment and there is no resultant force and no motion either.



These two objects are at equilibrium since the forces are balanced. However, the forces are not equal.
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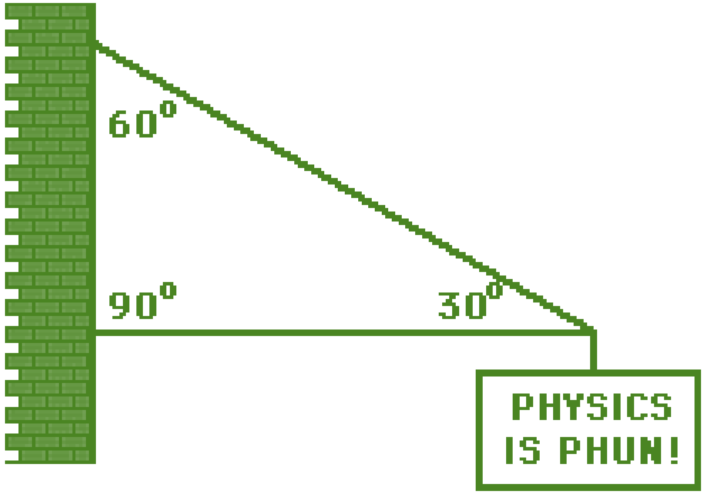
Example

The sign below hangs outside the physics classroom, advertising the most important truth to be found inside. The sign is supported by a diagonal cable and a rigid horizontal bar. If the sign has a mass of 50 kg, then determine the tension in the diagonal cable that supports its weight.



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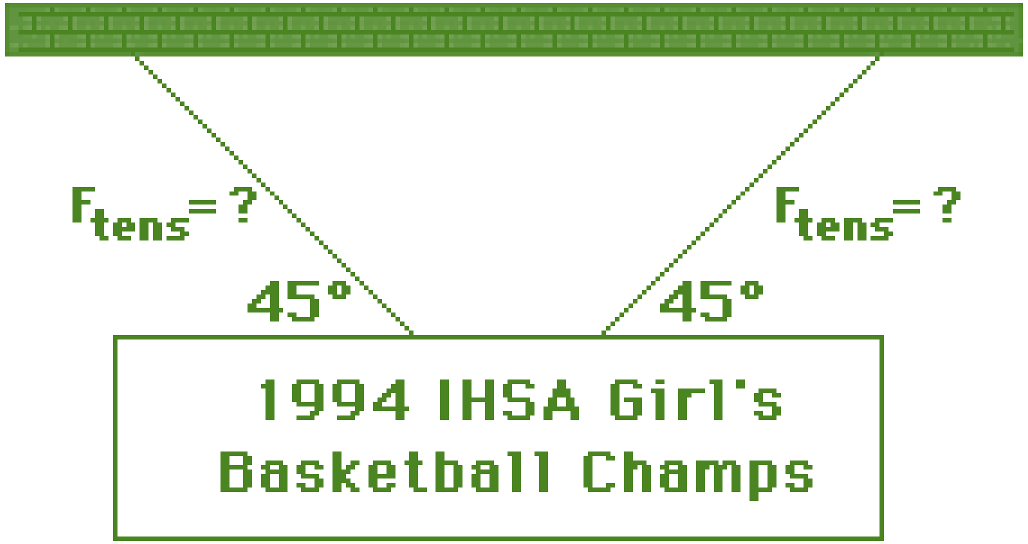
Example

A sign board below can be found hanging outside a boutique. The sign has a mass of 50 kg. Determine the tension in the cables.



Torque

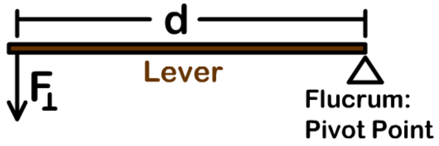
- Torque, also called moment of a force, in physics, the tendency of a force to rotate the body to which it is applied.

- The torque, specified with regard to the axis of rotation, is equal to the magnitude of the component of the force vector, multiplied by the shortest distance between the axis and the direction of the force component.

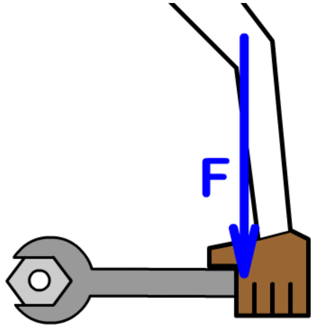
- Mathematically, torque is given by:

Torque = Force applied x lever arm

The lever arm is defined as the perpendicular distance from the axis of rotation to the line of action of the force.



CCW: Counterclockwise

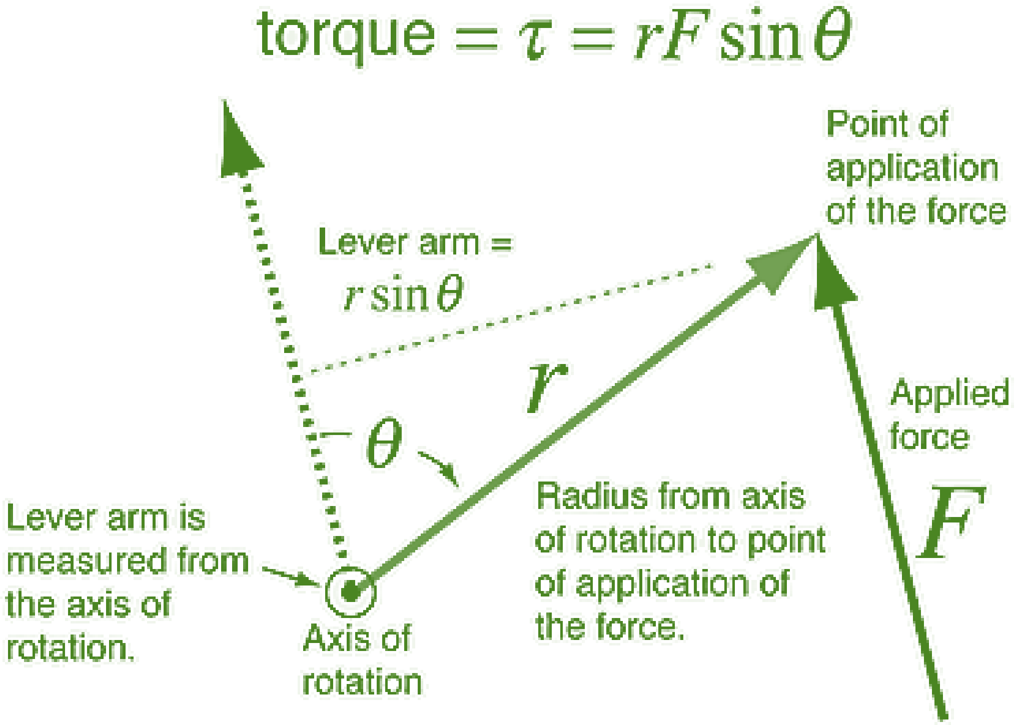


Most torque

Torque

- Note that this is "cross product", meaning:

$$\underset{\text{Torque}}{\tau} = \vec{r} \times \vec{F} = r\,F\,\sin\theta$$



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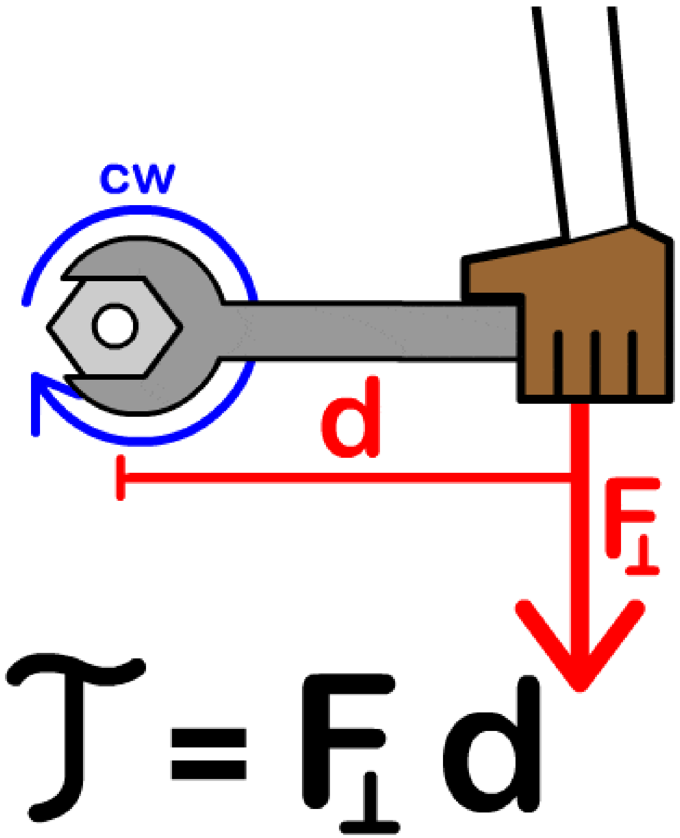
- Torque always points in the direction of the object's angular acceleration.

Torque

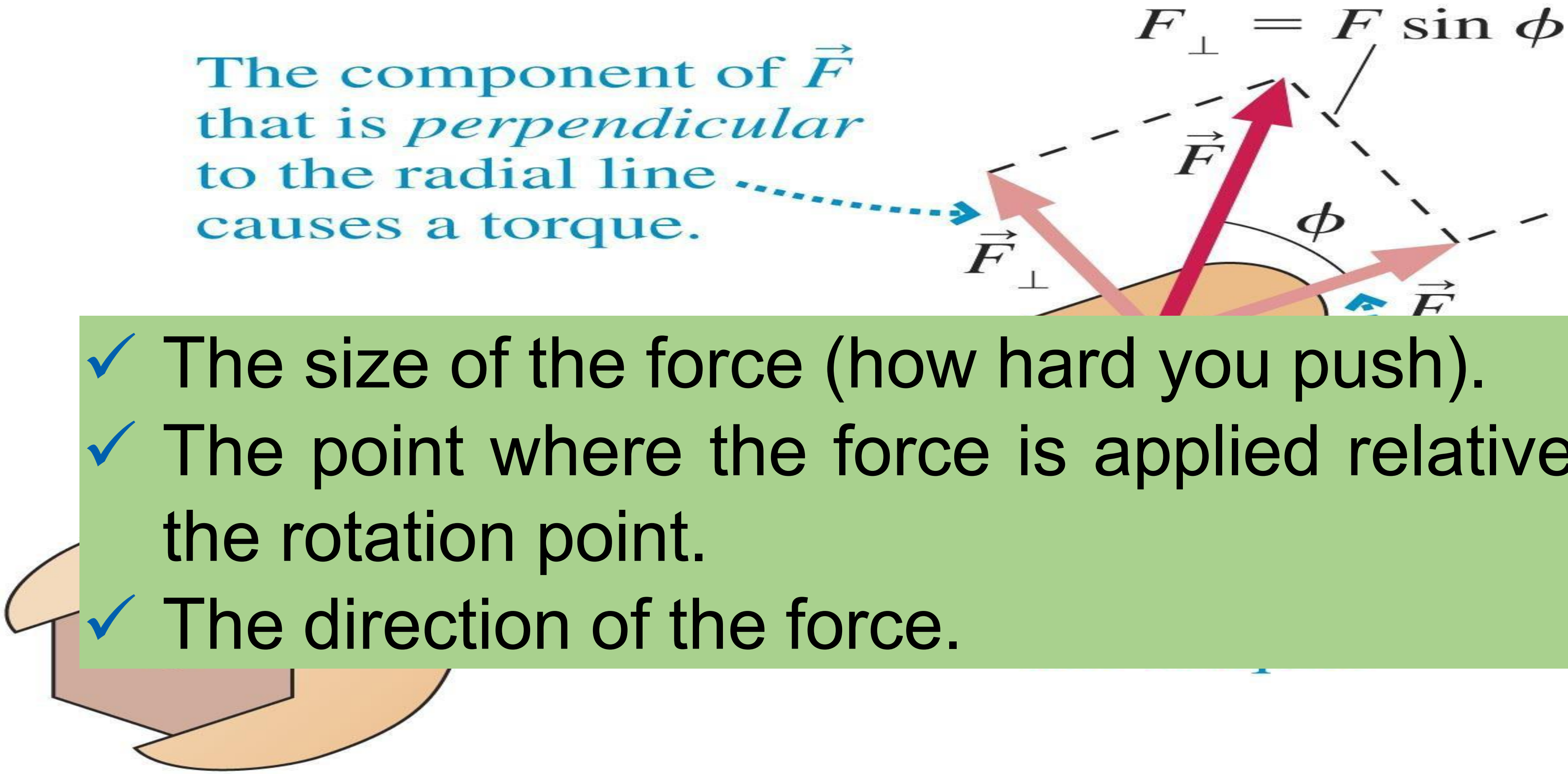
- **Static torque** does not produce angular acceleration.
  Ex. pushing a closed door is a static torque.

- **Dynamic torque** produces angular acceleration.
  Ex. The drive shaft of a racing car produces angular acceleration allowing the race car to drive on its track.
  A wrench is activated by force applied at one end, creating an unbalanced torque on the tightened element.



- Torque is also called **moment of force,** r is called the **length of the moment arm or lever arm**. Notice that the longer the lever arm, the higher the torque.

What does torque depend on?



- ✓ The size of the force (how hard you push).
- ✓ The point where the force is applied relative to the rotation point.
- ✓ The direction of the force.

Example

A 4 m beam of negligible weight balances in equilibrium with a fulcrum placed 1.5 m from its left end. If a force of 60 N is applied on its right end, how much force must be applied to the left?

Solution:

Given:

$F_1 = ?$

$r_1 = 1.5$ m

$F_2 = 60$ N

$r_2 = 4$ m – 1.5 m

$= 2.5$ m

Torque on the left lever arm $(\tau) = r_1 \, x \, F_1 = 1.5 \, m \, x \, F_1$

Torque on the right lever arm $(\tau) = r_2 \, x \, F_2$

$$= 2.5 \, m \, x \, 60 \, N = 150 \, N{\cdot}m$$

Since the beam is balanced, the two torques cancel out. Therefore,

$$\tau_1 = \tau_2$$

$$1.5 \, m \, x \, F_1 = 150 \, N{\cdot}m$$

$$F_1 = 150 \, N{\cdot}m \, / 1.5 \, m = 160 \, N$$

Example

One side of a seesaw carries a 32 kg mass four meters from the fulcrum and a 36.5 kg mass two meters from the fulcrum. To balance the seesaw, what mass should be placed eight meters from the fulcrum on the side opposite the first two masses?

Solution

Given: $m_1 = 32\,kg, r_1 = 4\,m, m_2 = 36.5\,kg, r_2 = 2\,m, m_3 = ?, r_3 = 8\,kg$

Using the torque balance equation

$$r_1 \cdot F_1 + r_2 \cdot F_2 + r_3 \cdot F_3 = 0$$

$$4\,m \cdot 32\,kg \cdot g + 2\,m \cdot 36.5\,kg \cdot g - 8\,m \cdot m_3 \cdot g = 0$$

$$m_3 = (4\,m \cdot 32\,kg + 2\,m \cdot 36.5\,kg)/8\,m = 25.1\,kg$$

Module 9: Equilibrium and Torque

Got any questions???

REFERENCES:

Ling, Samuel J., et.al (2018). University Physics Volume 1. OpenStax Rice University. Texas. Accessed at https://openstax.org/details/books/universityphysics- volume-1

The Physics Classroom (2020). Accessed at https://www.physicsclassroom.com

Professor Dave Explains (Mar 27, 2017). Work and Energy. Accessed at https://www.youtube.com/watch?v=zVRH9d5PW8g

The Physics Classroom. Work Concept Builder. Accessed at https://www.physicsclassroom.com/Concept-Builders/Work-and-Energy/Work/Concept-Builder

PhETInteractive Simulations. Hooke's Law. Accessed at https://phet.colorado.edu/sims/html/hookes-law/latest/hookes-law_en.html