

CS0001

Discrete Structures 1

Module 5: Set Theory



OBJECTIVES:

By the end of this module, you will be able to:

- **Define** a set and its fundamental properties, and represent sets accurately using both roster and set-builder notation.
- **Perform core set operations**—such as union, intersection, complement, and difference—and visualize these relationships using Venn diagrams.
- **Analyze and solve** practical counting problems by modeling real-world scenarios with Venn diagrams.
- **Construct** the power set from a given set and the Cartesian product of two or more sets to create a foundation for understanding relations.
- **Prove** the equivalence of set expressions by applying the algebraic laws of set theory, such as De Morgan's laws

CS0001

Discrete Structures 1

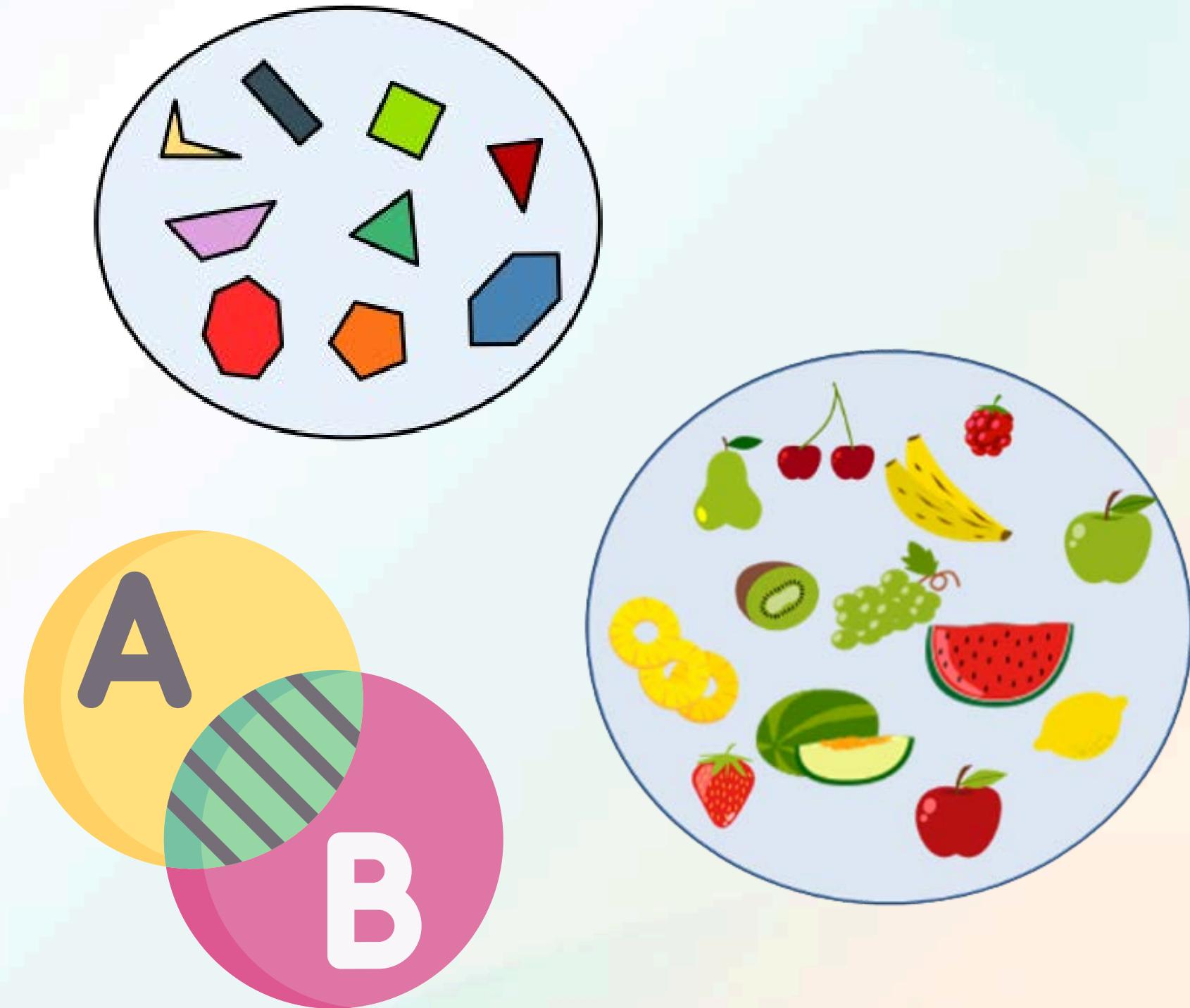
Subtopic 1: Introduction to Sets and
Terminology



What is a SET?

A **set** is a well-defined collection of distinct objects. These objects are called elements or members of the set.

Think of a set like a **grocery list** or the **players** on a **basketball team**—it's the collection that matters.



The Two Golden Rules of Sets

1. Elements Must Be Distinct

- Repeated elements in a set are only listed once.
- For example, the collection {apple, orange, banana, apple} is simply the set {apple, orange, banana}

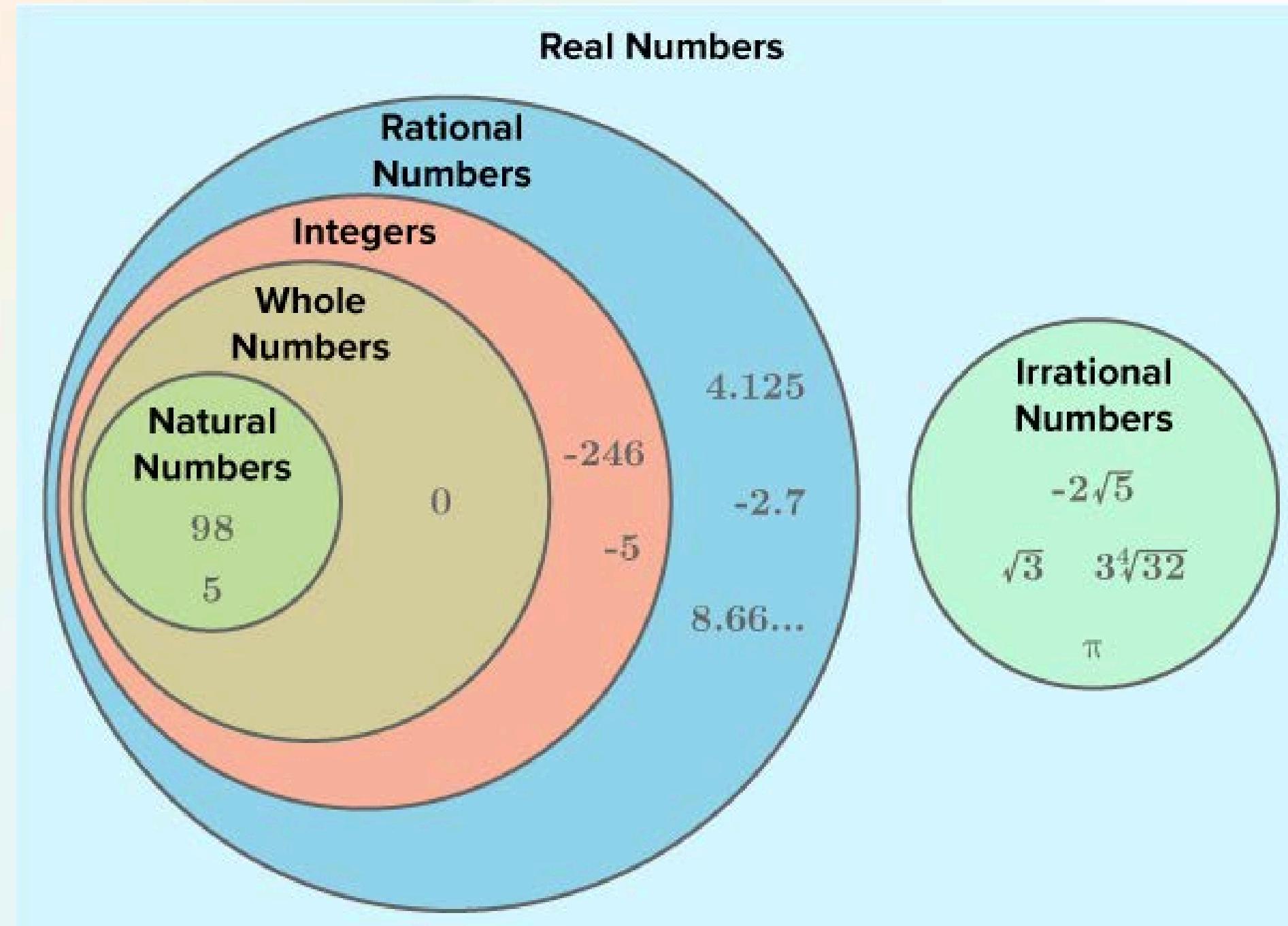
2. Order Does Not Matter

- There is no order in a set. The arrangement of elements does not change the set's identity.
- For example, {1, 2, 3} is the exact same set as {3, 1, 2}.

Examples of Sets

- A set of numbers: {1, 2, 3, 4, 7}
- A set of major universities in Manila: {FEU, UST, DLSU}
- A set of primary colors: {red, yellow, blue}
- A set of common programming languages: {Python, Java, C++, JavaScript}
- A set of logical operators: {AND, OR, NOT}
- A set of primitive data types: {integer, float, boolean, char}
- A set of programs under the College of Computer Studies: {Computer Science, Information Technology, Multimedia Arts}
- A set of the core values: {Fortitude, Excellence, Uprightness}

The Real Number System as Sets



The **Real Number System** is the **set of all numbers** that can be represented on a continuous number line.

It is comprised of two distinct and non-overlapping sets: **rational numbers** and **irrational numbers**

Types (Sets) of Numbers

Natural Numbers (*counting numbers*)

The natural numbers are the part of the number system that includes all the positive integers from 1 through infinity. They are used for the purpose of counting.

The set of natural numbers is usually represented by the letter "N".

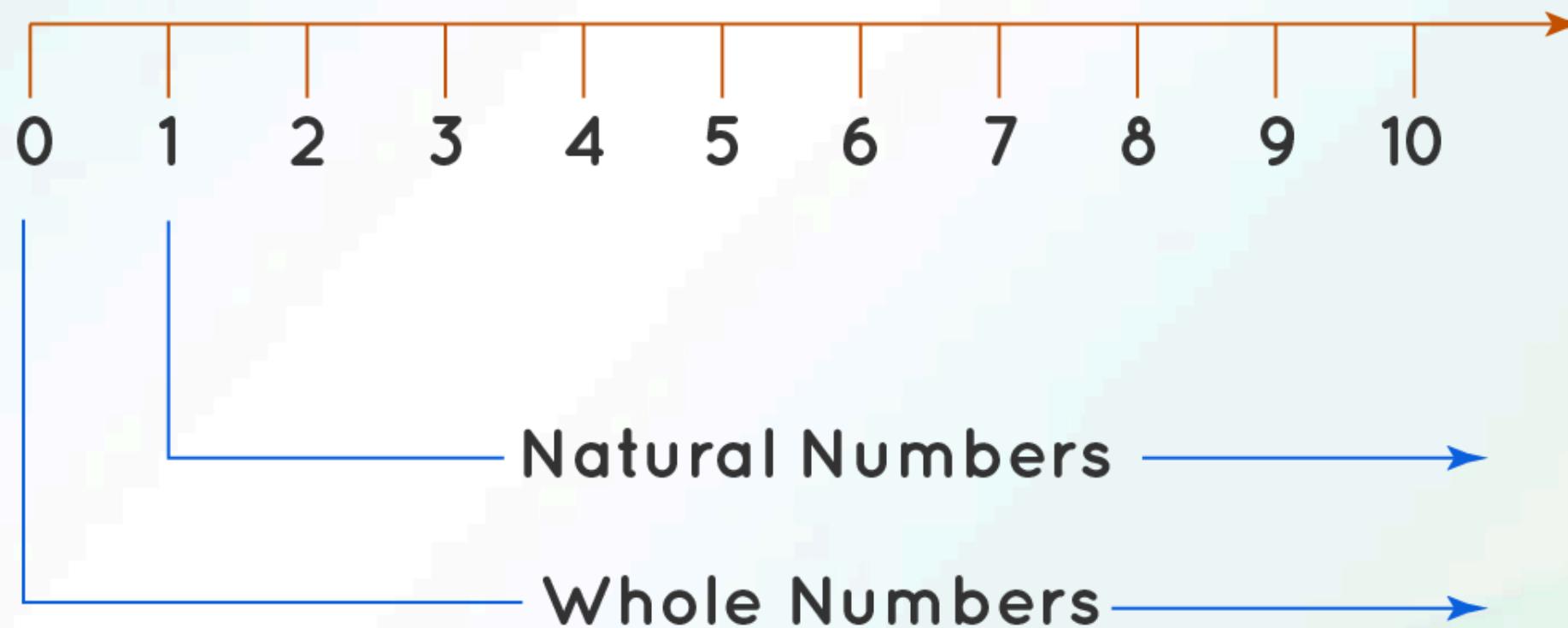
Natural Numbers



Types (Sets) of Numbers

Whole Numbers

The whole numbers are the numbers without fractions and it is a collection of positive integers and zero. It is represented by the symbol “W”. Zero as a whole represents nothing or a null value.

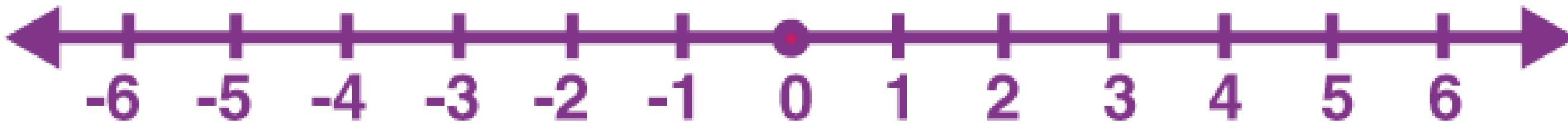


Types (Sets) of Numbers

Integers

The integers are a set of numbers consisting of the natural numbers, their additive inverses, and zero. In other words, they include all positive numbers, negative numbers, and 0.

The set of integers is usually represented by the letter "Z". It can also be represented by the letter "J".

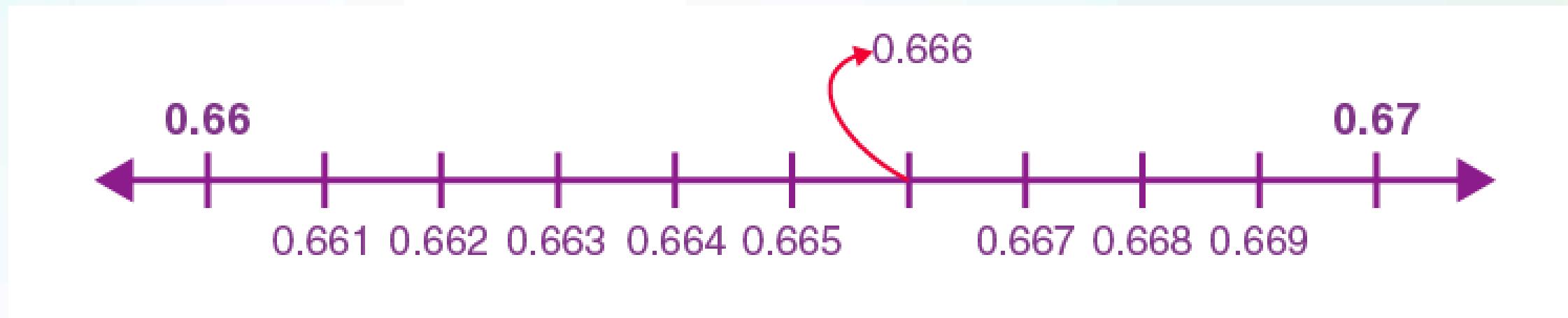


Types (Sets) of Numbers

Rational Numbers

Numbers that can be written as a fraction or as a ratio.

a divided by b where a is a whole number and b is also a whole number.



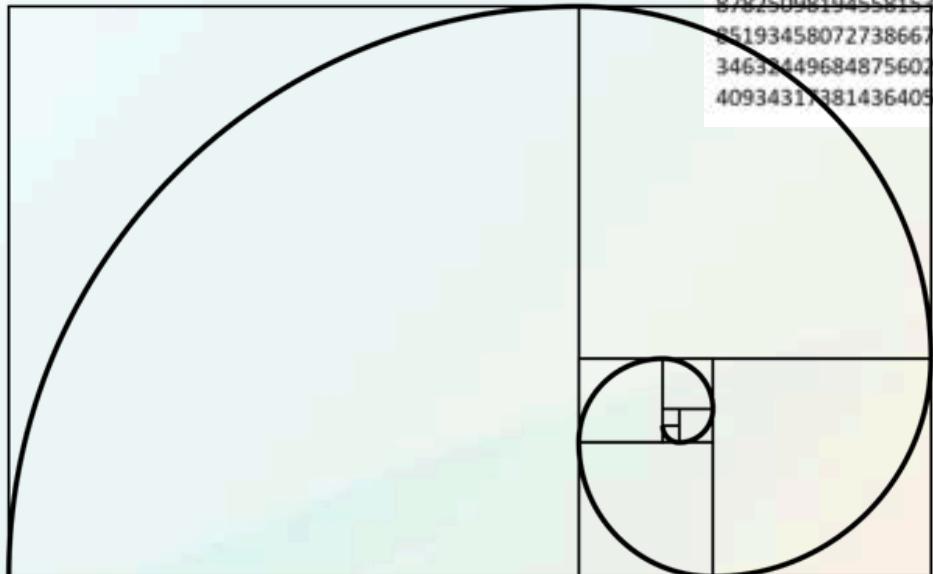
Types (Sets) of Numbers

Irrational Numbers

An irrational number is a real number that cannot be expressed as a ratio of integers; for example, $\sqrt{2}$ is an irrational number. **We cannot express any irrational number in the form of a ratio**, such as p/q , where p and q are integers, $q \neq 0$.

3.141592653589793238462643383279502
88419716939937510582097494459230781
64062862089986280348253421170679821
48086513282306647093844609550582231
72535940812848111745028410270193852
11055596446229489549303819644288109
75665933446128475648233786783165271
20190914564856692346034861045432664
82133936072602491412737245870066063
15588174881520920962829254091715364
36789259036001133053054882046652138
41469519415116094330572703657595919
53092186117381932611793105118548074
46237996274956735188575272489122793

2.71828182845904523536028747135266249775724709369995957496696762772
407663035354759457138217852516642742746639193200305992181741359662
904357290033429526059563073813232862794349076323382988075319525101
901157383418793070215408914993488416750924476146066808226480016847
741185374234544243710753907774499206955170276183860626133138458300
075204493382656029760673711320070932870912744374704723069697720931
014169283681902551510865746377211125238978442505695369677078544996
996794686445490598793163688923009879312773617821542499922957635148
220826989519366803318252886939849646510582093923982948879332036250
944311730123819706841614039701983767932068328237646480429531180232
878250981945581530175671736133206981125099618188159304169035159888
85193458072738663858942287922849989208680582574927961048419844436
346324496848756023362482704197862320900216099023530436994184914631
409343173814364054625315209618369088870701676839642437814059271456



Representing Sets

There are two primary ways to describe the elements that belong to a set: **Roster Notation** and **Set-Builder Notation**.

Roster Notation (The Listing Method)

This is the most straightforward method. You simply list all the elements of the set, separated by commas, inside curly braces

- **When to use it:** This method is best for sets with a small, finite number of elements.
- **For patterns:** If the set is large or infinite but has a clear pattern, you can use an ellipsis (...) to indicate that the pattern continues.

Roster Notation

The set of vowels in the English alphabet:

$$V = \{a, e, i, o, u\}$$

The set of positive integers from 1 to 5:

$$A = \{1, 2, 3, 4, 5\}$$

The set of positive even integers:

$$E = \{2, 4, 6, 8, \dots\}$$



Set-Builder Notation (The Rule Method)

Instead of listing elements, this method describes the set by **stating a property or rule** that its elements must satisfy to be included. The general form is $\{x \mid P(x)\}$.

Let's break down the syntax:

- x is a **variable** representing an **element**.
- \mid is a vertical bar read as "**such that**."
- $P(x)$ is the **property** or condition that x must meet.

Set-Builder Notation (The Rule Method)

Examples:

- The set of positive integers from 1 to 5:

$$A = \{x \mid x \text{ is an integer and } 1 \leq x \leq 5\}$$

- The set of all positive even integers:

$$E = \{x \mid x > 0 \text{ and } x \text{ is an even integer}\}$$

- The set of all students currently enrolled in Discrete Structures class:

$$S = \{s \mid s \text{ is a student in section CS0001}\}$$



THINK ABOUT THIS

Translate the following descriptions into both **Roster Notation** and **Set-Builder Notation**.

Be as precise as possible!

1. The set of days in a week that start with the letter 'T'.
2. The set of distinct letters in the word "STRUCTURES".
3. The set of integers between -20 and 20 that are divisible by 4.
4. The set of positive integers less than 20 that are multiples of 3.



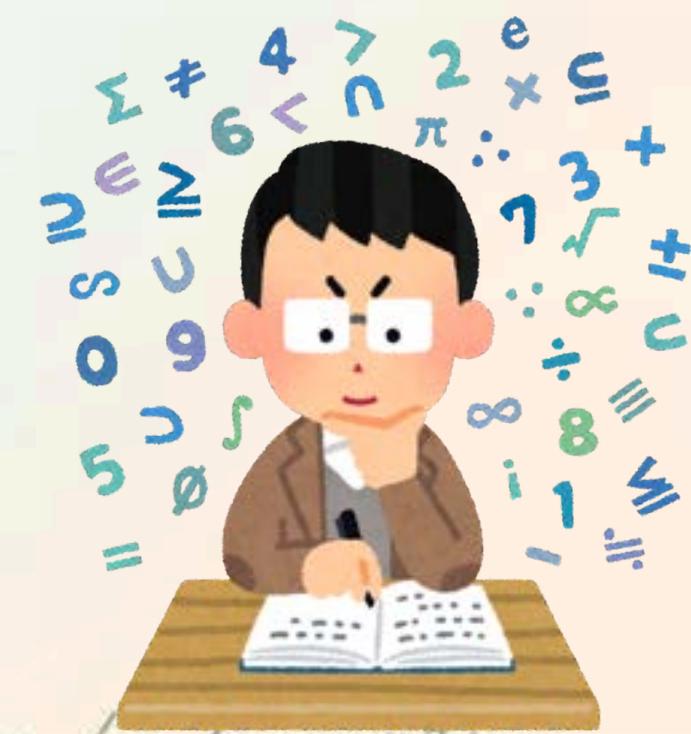
ANSWERS

The set of days in a week that start with the letter 'T'

- **Roster:** {Tuesday, Thursday}
- **Set-Builder:** { $x \mid x$ is a day of the week that begins with 'T'}

The set of distinct letters in the word "STRUCTURES"

- **Roster:** {S, T, R, U, C, E}
- **Set-Builder:** { $x \mid x$ is a letter in the word "STRUCTURES"}



ANSWERS

The set of integers between -20 and 20 that are divisible by 4

- **Roster:** $\{-16, -12, -8, -4, 0, 4, 8, 12, 16\}$
- **Set-Builder:** $\{x \mid x \text{ is an integer, } -20 < x < 20, \text{ and } x \text{ is divisible by 4}\}$

More formally: $\{x \mid x \in \mathbb{Z}, -20 < x < 20, \text{ and } x \bmod 4 = 0\}$

The set of positive integers less than 20 that are multiples of 3

- **Roster:** $\{3, 6, 9, 12, 15, 18\}$
- **Set-Builder:** $\{x \mid x \text{ is a positive integer, } x < 20, \text{ and } x \text{ is a multiple of 3}\}$



SYMBOLS

| is read as "such that" and we usually write it immediately after the variable in the set builder form and after this symbol, the condition of the set is written.

\in is read as "belongs to" and it means "is an element of".

\notin is read as "does not belong to" and it means "is not an element of".

SYMBOLS

\mathbb{N} represents natural numbers or all positive integers.

\mathbb{W} represents whole numbers.

\mathbb{Z} indicates integers.

\mathbb{Q} represents rational numbers or any number that can be expressed as a fraction of integers.

\mathbb{R} represents real numbers or any number that isn't imaginary.

COMMON SYMBOLS IN SET THEORY

Symbol	Meaning	Example
{ }	Set: a collection of elements	{1, 2, 3, 4}
$A \cup B$	Union: in A or B (or both)	$C \cup D = \{1, 2, 3, 4, 5\}$
$A \cap B$	Intersection: in both A and B	$C \cap D = \{3, 4\}$
$A \subseteq B$	Subset: every element of A is in B.	$\{3, 4, 5\} \subseteq D$
$A \subset B$	Proper Subset: every element of A is in B, but B has more elements.	$\{3, 5\} \subset D$
$A \not\subseteq B$	Not a Subset: A is not a subset of B	$\{1, 6\} \not\subseteq C$
$A \supseteq B$	Superset: A has same elements as B, or more	$\{1, 2, 3\} \supseteq \{1, 2, 3\}$
$A \supset B$	Proper Superset: A has B's elements and more	$\{1, 2, 3, 4\} \supset \{1, 2, 3\}$

COMMON SYMBOLS IN SET THEORY

Symbol	Meaning	Example
$A \not\ni B$	Not a Superset: A is not a superset of B	$\{1, 2, 6\} \not\ni \{1, 9\}$
A^c	Complement: elements not in A	$D^c = \{1, 2, 6, 7\}$ When $= \{1, 2, 3, 4, 5, 6, 7\}$
$A - B$	Difference: in A but not in B	$\{1, 2, 3, 4\} - \{3, 4\} = \{1, 2\}$
$a \in A$	Element of: a is in A	$3 \in \{1, 2, 3, 4\}$
$b \notin A$	Not element of: b is not in A	$6 \notin \{1, 2, 3, 4\}$
\emptyset	Empty set = {}	$\{1, 2\} \cap \{3, 4\} = \emptyset$
U	Universal Set: set of all possible values (in the area of interest)	

COMMON SYMBOLS IN SET THEORY

Symbol	Meaning	Example
$P(A)$	Power Set: all subsets of A	$P(\{1, 2\}) = \{ \emptyset, \{1\}, \{2\}, \{1, 2\} \}$
$A = B$	Equality: both sets have the same members	$\{3, 4, 5\} = \{5, 3, 4\}$
$A \times B$	Cartesian Product (set of ordered pairs from A and B)	$\{1, 2\} \times \{3, 4\}$ $= \{(1, 3), (1, 4), (2, 3), (2, 4)\}$
$ A $	Cardinality: the number of elements of set A	$ \{3, 4\} = 2$
$ $	Such that	$\{ n \mid n > 0 \} = \{1, 2, 3, \dots\}$
$:$	Such that	$\{ n : n > 0 \} = \{1, 2, 3, \dots\}$
\forall	For All	$\forall x > 1, x^2 > x$ <i>For all x greater than 1 x-squared is greater than x</i>

COMMON SYMBOLS IN SET THEORY

Symbol	Meaning	Example
\exists	There Exists	$\exists x \mid x^2 > x$ <i>There exists x such that x-squared is greater than x</i>
\therefore	Therefore	$a=b \therefore b=a$

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Discrete Structures 1

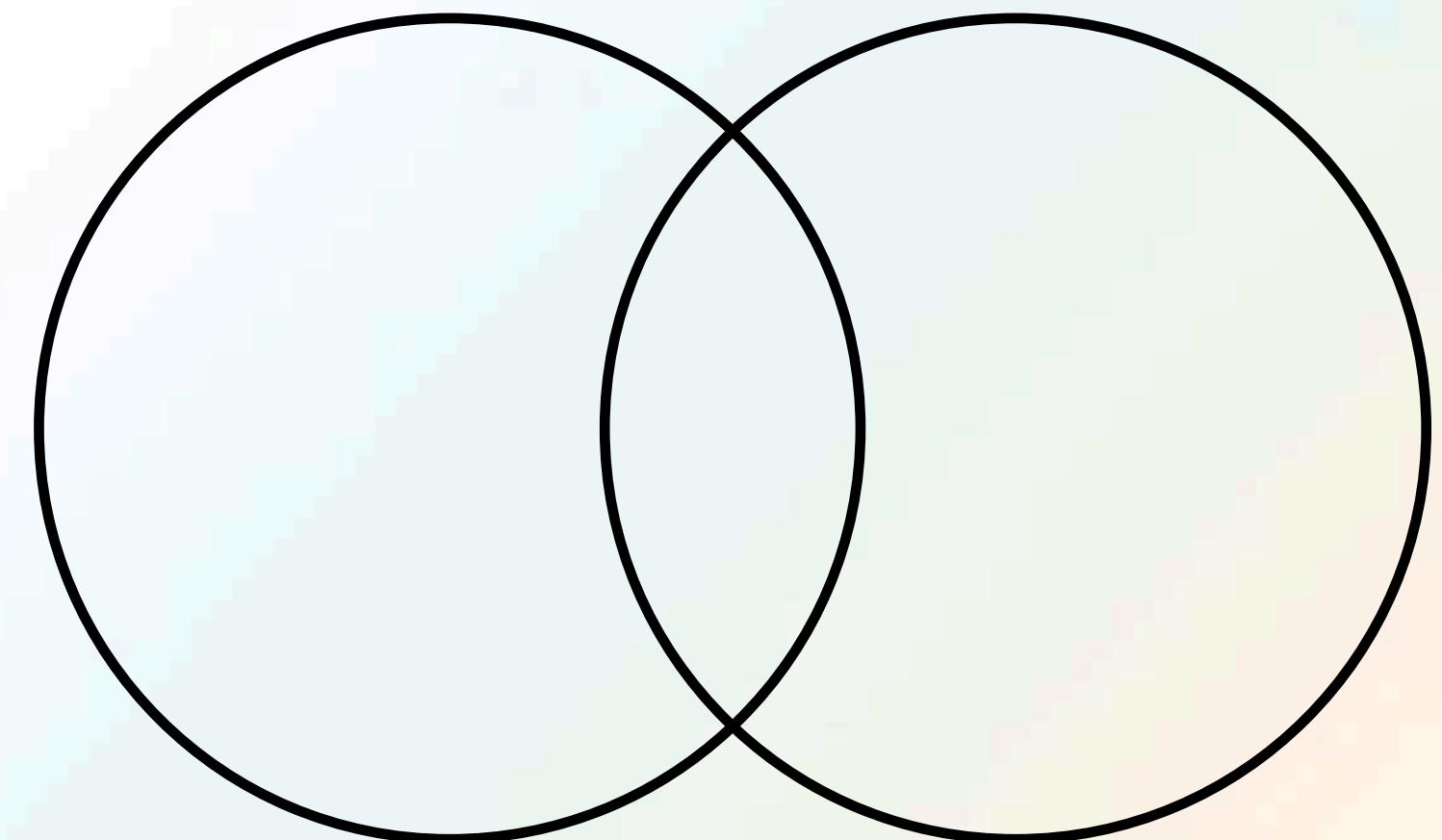
Subtopic 2: Set Operations &
Venn Diagrams



SET OPERATIONS AND VENN DIAGRAM

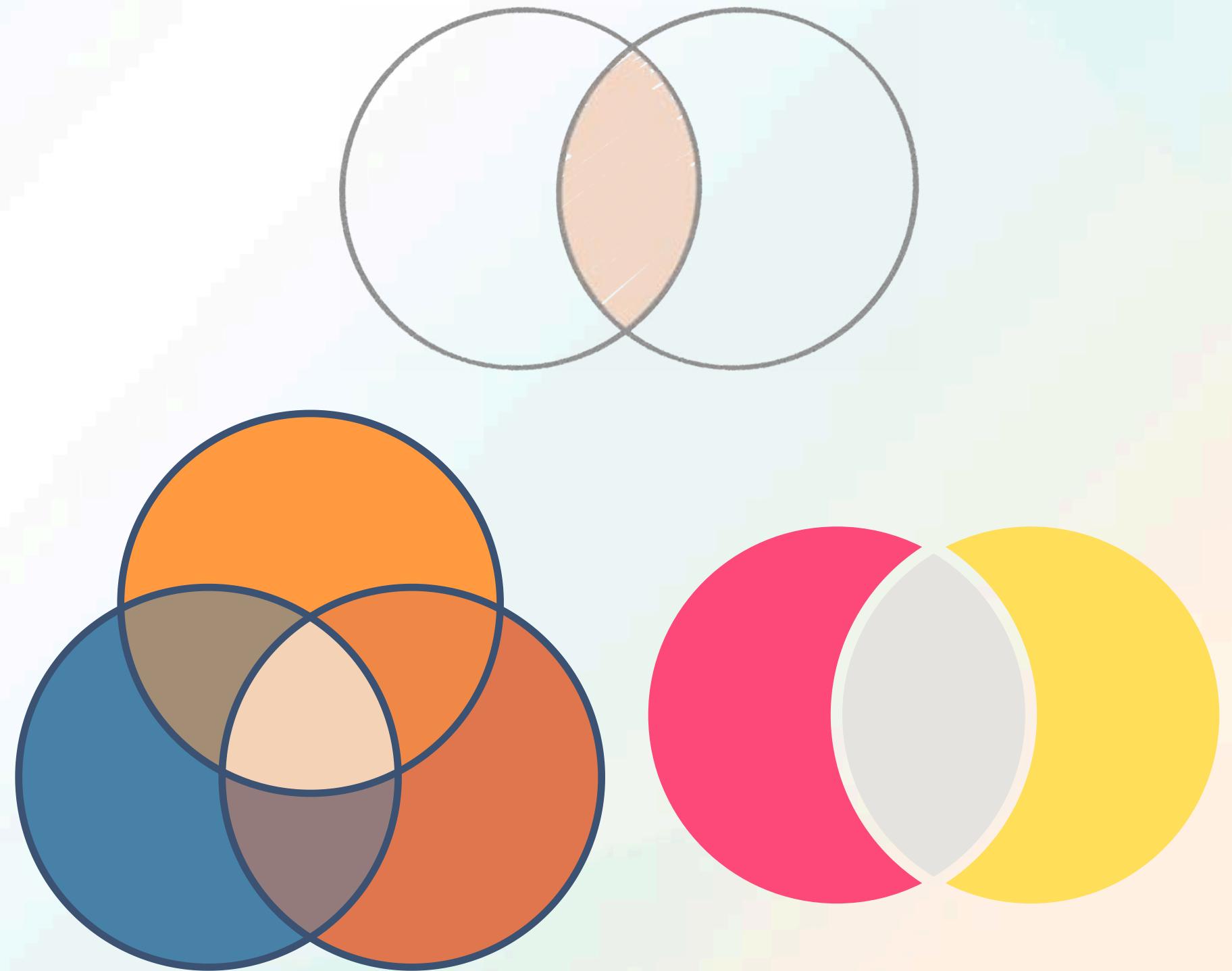
Just as we use arithmetic operations like addition and subtraction to work with numbers, we have set operations to combine, compare, and modify sets. These operations, such as **union**, **intersection**, and **complement**, allow us to create new sets from existing ones based on defined rules.

To make these abstract concepts easy to understand, we use a powerful visual tool called a **Venn diagram**.



SET OPERATIONS AND VENN DIAGRAM

A Venn diagram uses **overlapping circles** to clearly illustrate the relationships between different sets and visually represent the outcome of set operations.



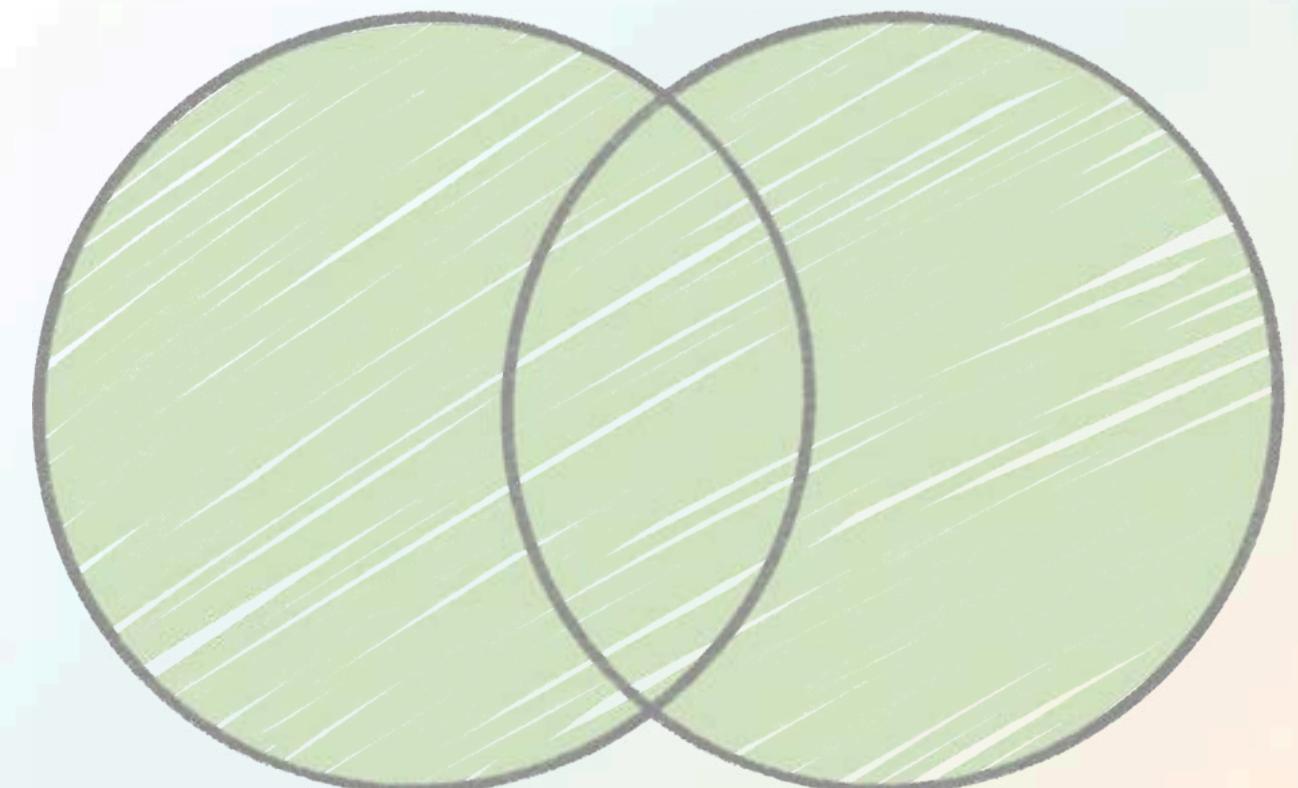
SET OPERATIONS AND VENN DIAGRAM

UNION

If 'A' and 'B' are two sets, then the union of sets A and B is a set containing all the elements present either in A or in B.

Mathematically, it is represented by the symbol 'U'. $A \cup B$ is read as A 'union' B.

If $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$



SET OPERATIONS AND VENN DIAGRAM

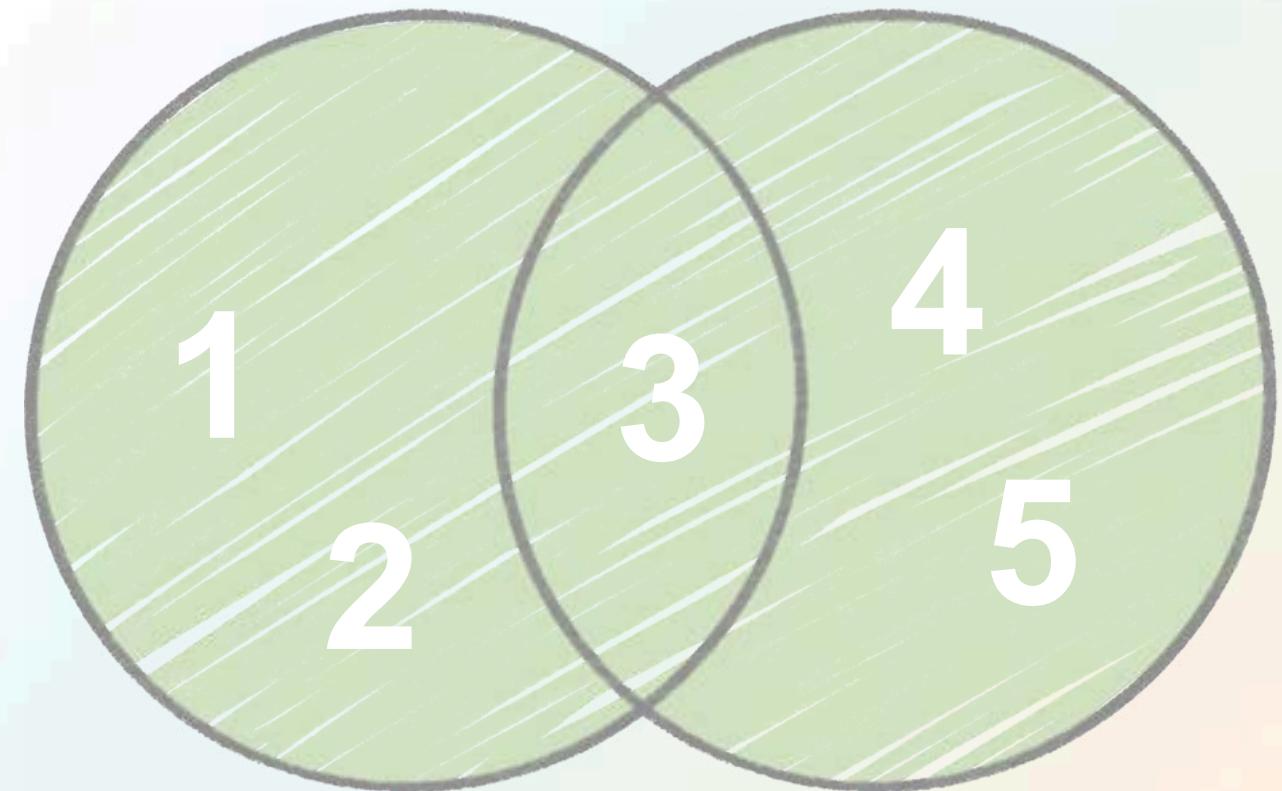
Let's consider two sets:

$$\text{Set A} = \{1, 2, 3\}$$

$$\text{Set B} = \{3, 4, 5\}$$

To find the union, we create a new set that contains all of the unique elements found in **either** Set A **or** Set B. We simply combine their elements, making sure not to repeat any.

$$\text{Result: } A \cup B = \{1, 2, 3, 4, 5\}$$



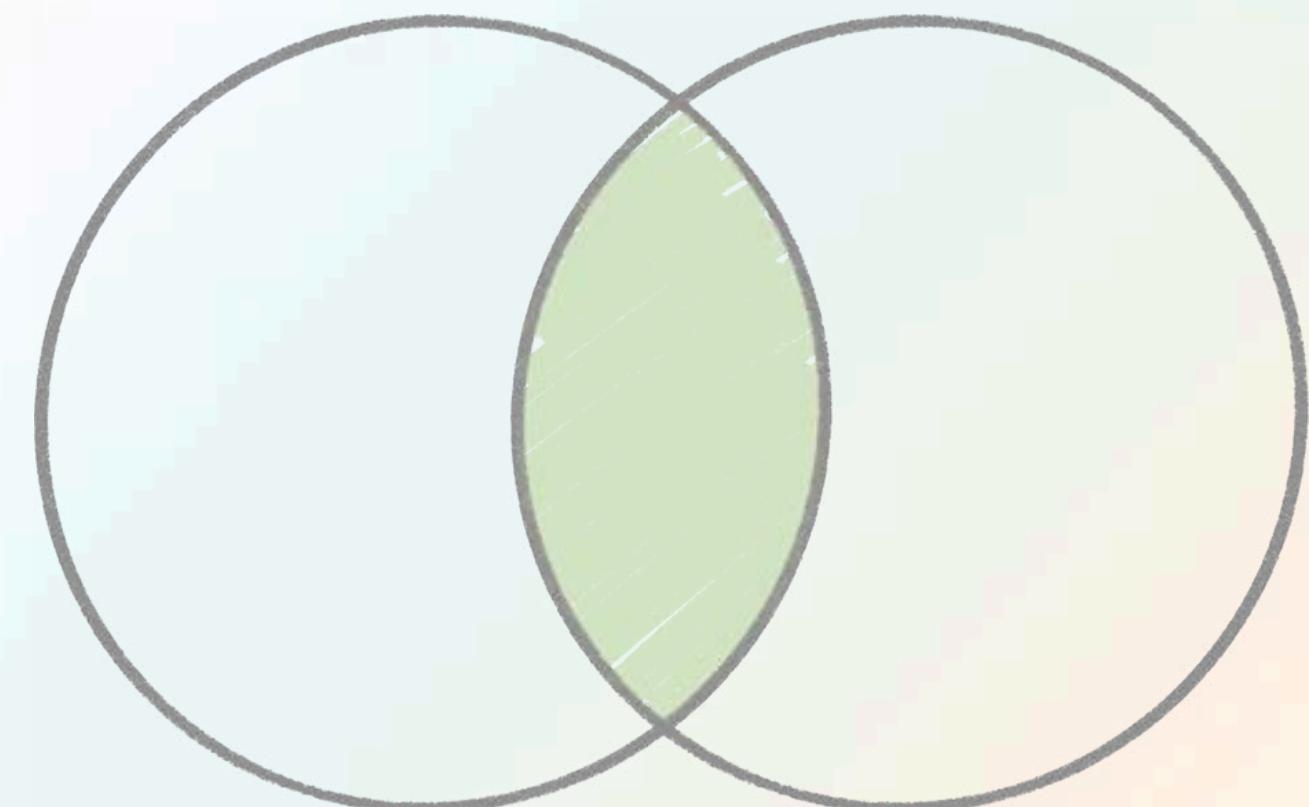
SET OPERATIONS AND VENN DIAGRAM

INTERSECTION

If 'A' and 'B' are two sets, the intersection of sets A and B is a set containing all elements **common to both** A and B.

Mathematically, it is represented by the symbol ' \cap '. $A \cap B$ is read as A 'intersection' B.

If $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$



SET OPERATIONS AND VENN DIAGRAM

Let's use the same two sets from our previous example:

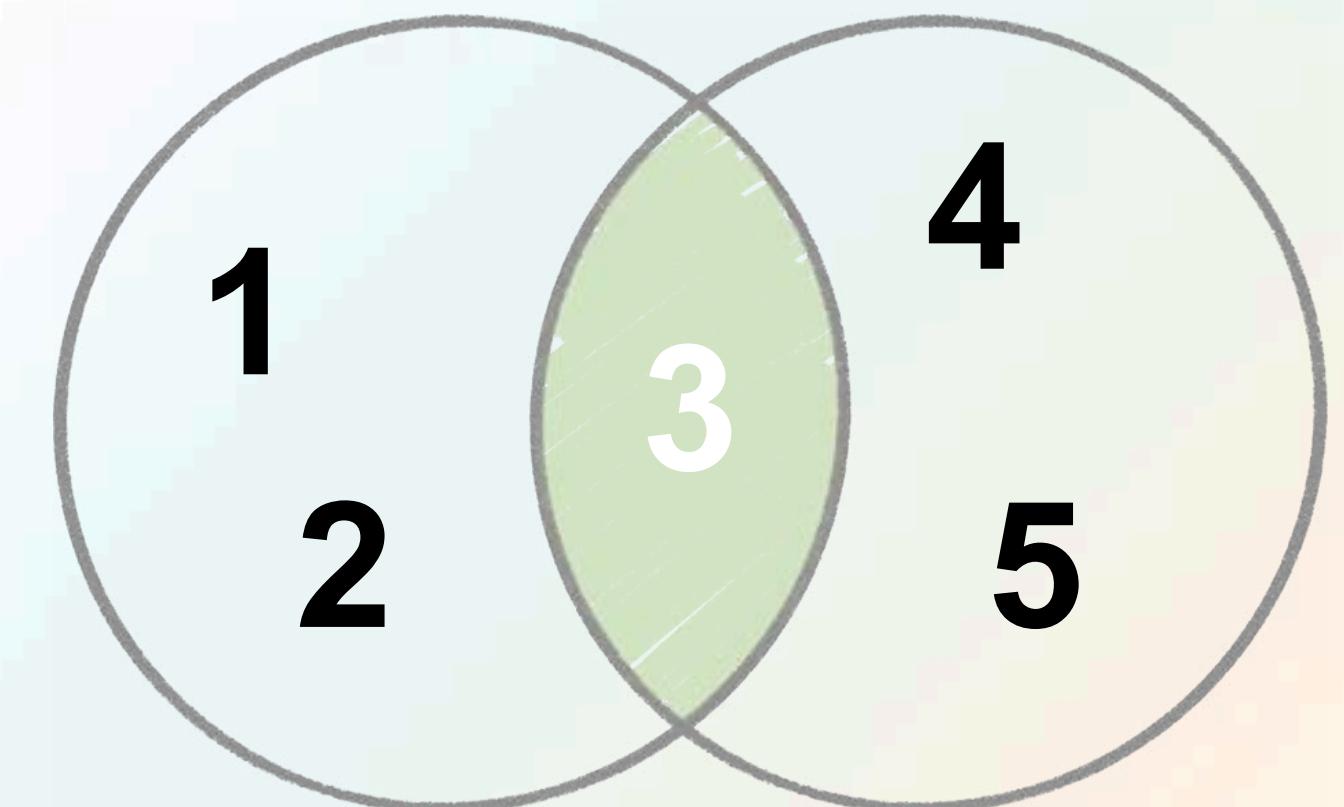
$$\text{Set A} = \{1, 2, 3\}$$

$$\text{Set B} = \{3, 4, 5\}$$

The **intersection** is a new set containing only the elements that are present in both Set A and Set B. It's the part they have in common.

By comparing the two sets, we can see that the only element they share is 3.

$$\text{Result: } A \cap B = \{3\}$$



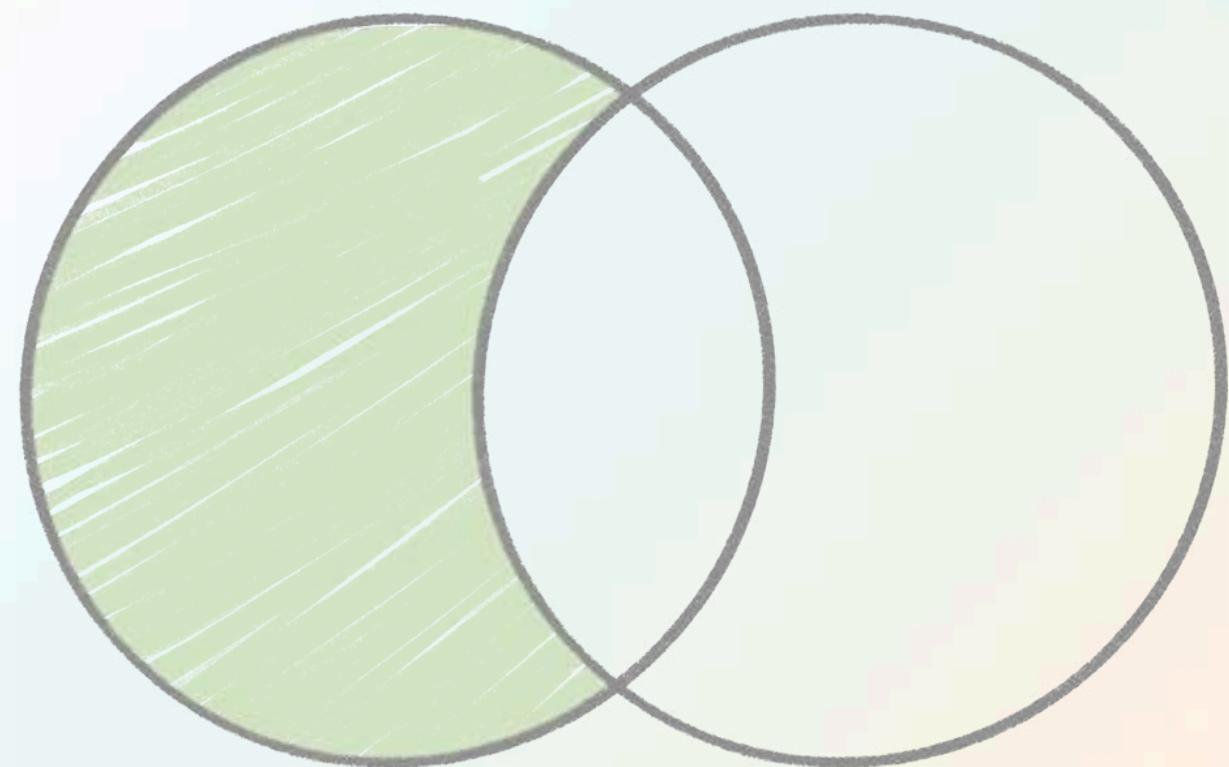
SET OPERATIONS AND VENN DIAGRAM

SET DIFFERENCE

If 'A' and 'B' are two sets, then the difference between sets A and B is a set that consists of elements present in A but not in B.

Mathematically, it is represented by the symbol '-' and is written as $A - B$.

If $A - B = \{x \mid x \in A \text{ and } x \notin B\}$



SET OPERATIONS AND VENN DIAGRAM

Let's consider two sets:

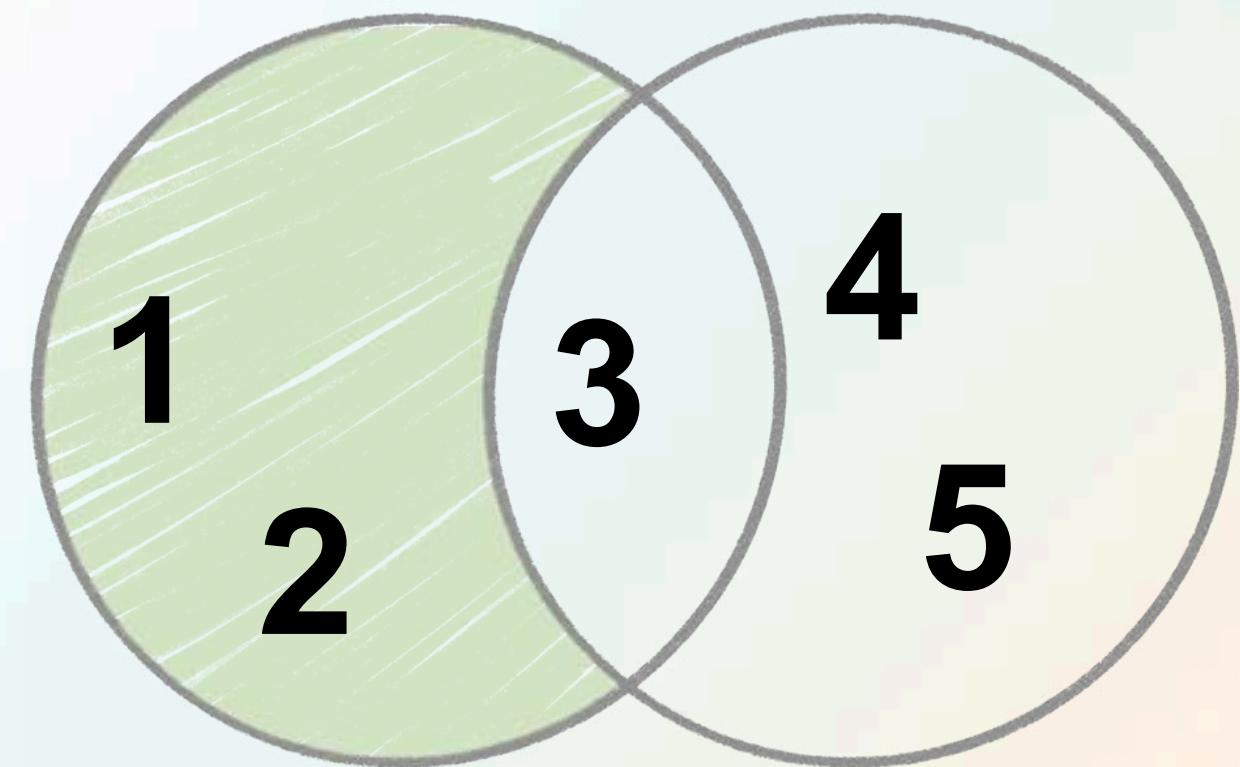
$$\text{Set A} = \{1, 2, 3\}$$

$$\text{Set B} = \{3, 4, 5\}$$

The set difference $A - B$ creates a new set that contains elements from Set A but not from Set B.

Think of it as starting with everything in Set A and then removing any elements that also appear in Set B.

$$\text{Result: } A - B = \{1, 2\}$$

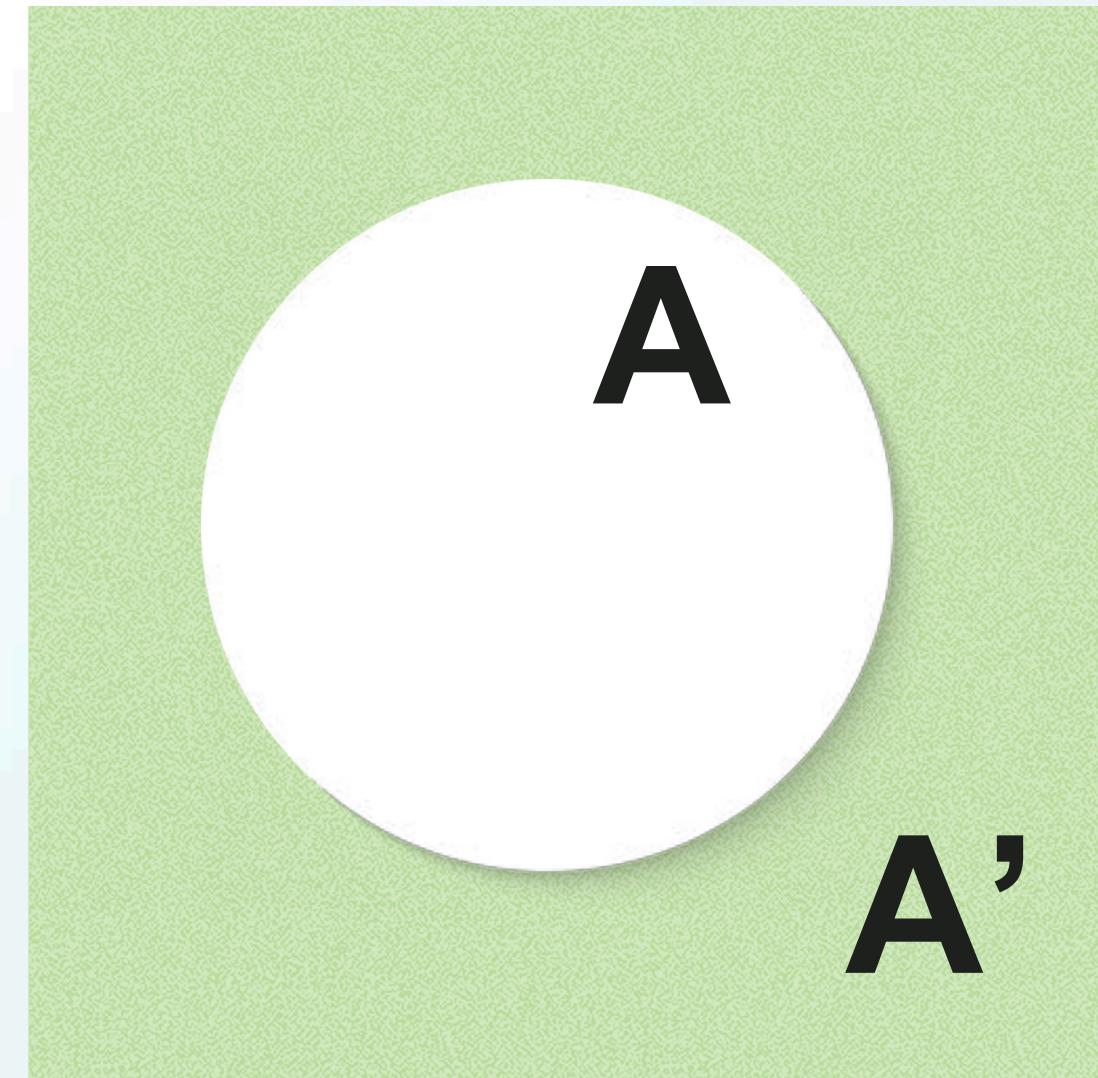


SET OPERATIONS AND VENN DIAGRAM

COMPLEMENT

If ‘U’ is a universal set and ‘A’ is any subset of ‘U,’ the complement of set A, represented by the symbol A' or A_c , is the set of all the elements in the universal set ‘U’ that are not present in set A.

Mathematically, it is written as
$$A' = \{x \mid x \in U \text{ and } x \notin A\}$$



SET OPERATIONS AND VENN DIAGRAM

Let's consider two sets:

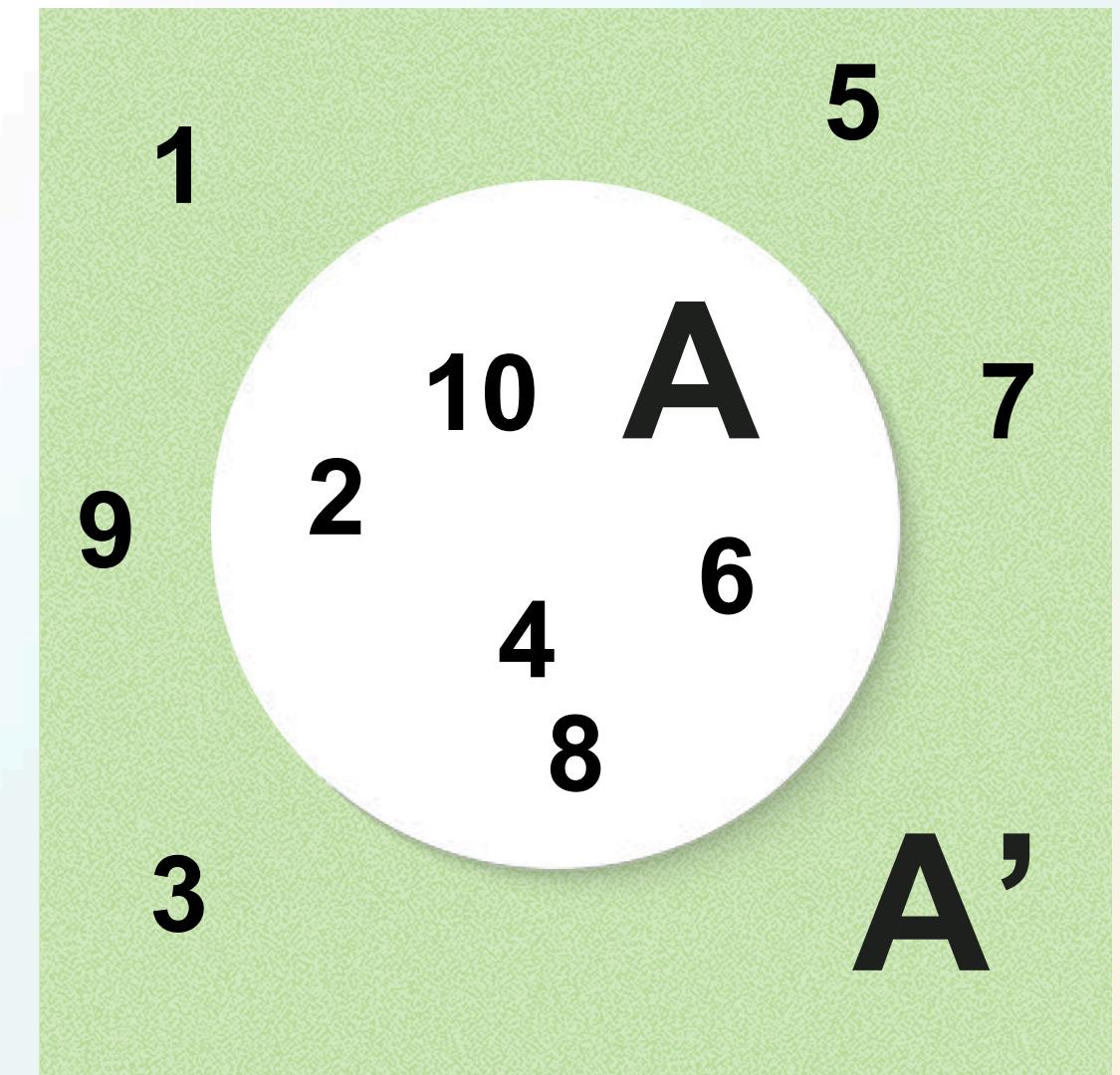
$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{2, 4, 6, 8, 10\}$$

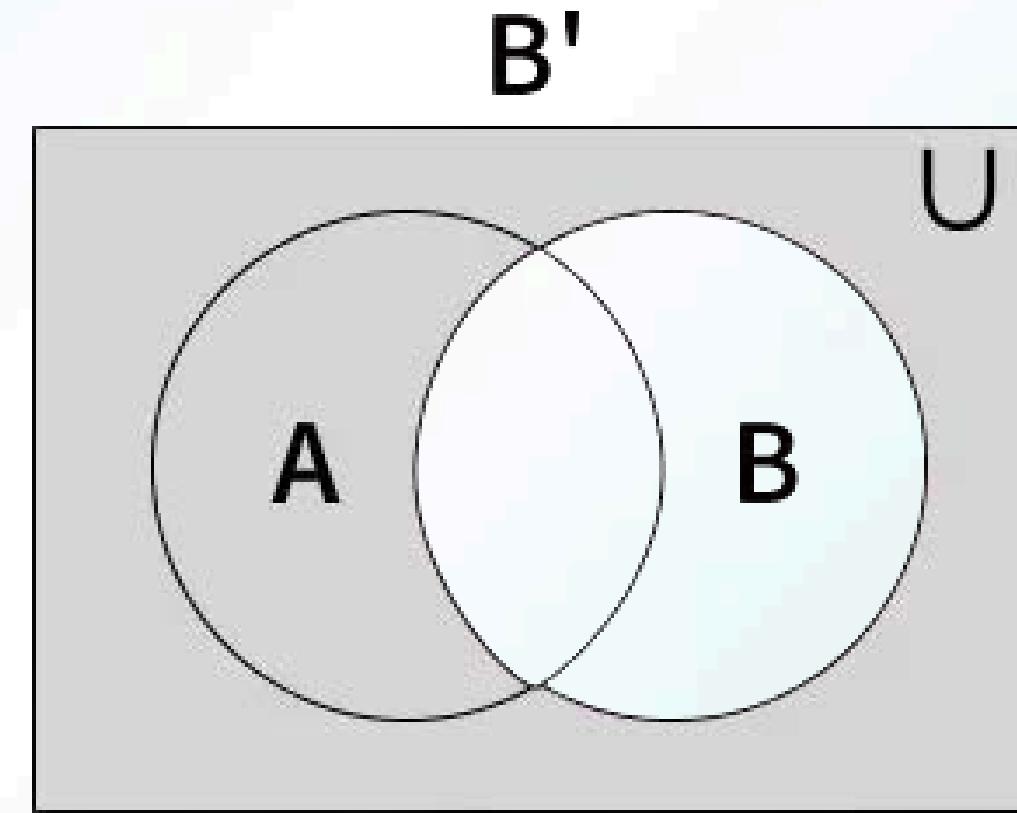
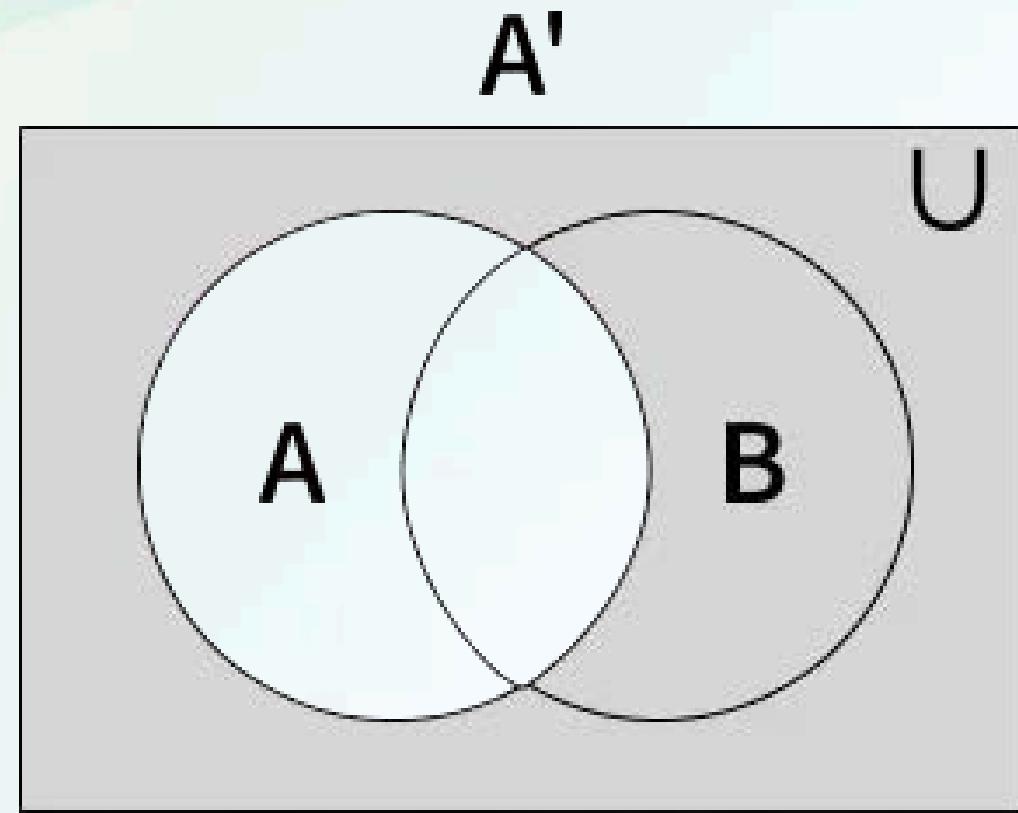
The complement of a set A, denoted as A' , is the set of all elements in the universal set U that are NOT in set A.

Then the complement of A is

$$A' = \{1, 3, 5, 7, 9\}$$

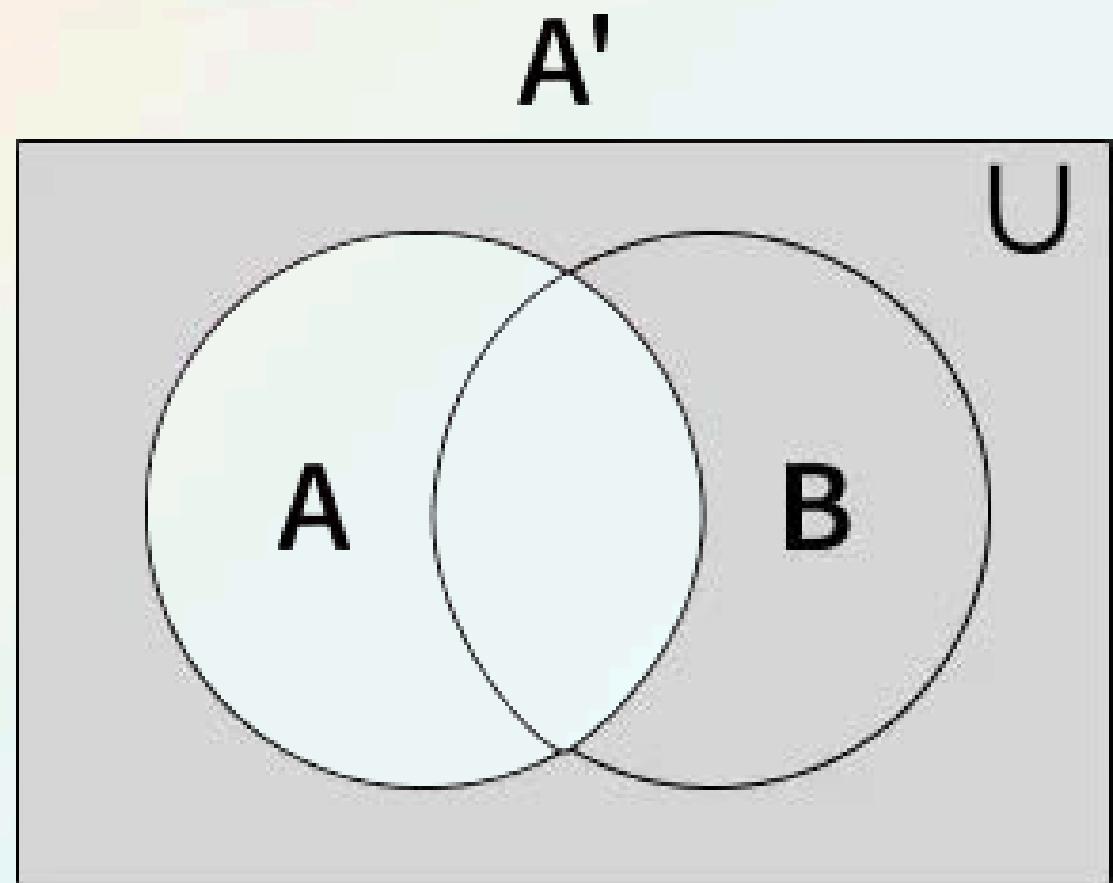


SET OPERATIONS AND VENN DIAGRAM



The concept of a complement applies even when we are working with **more than one set**. The complement is always defined as everything in the universal set (U) that is outside of the specified set.

SET OPERATIONS AND VENN DIAGRAM

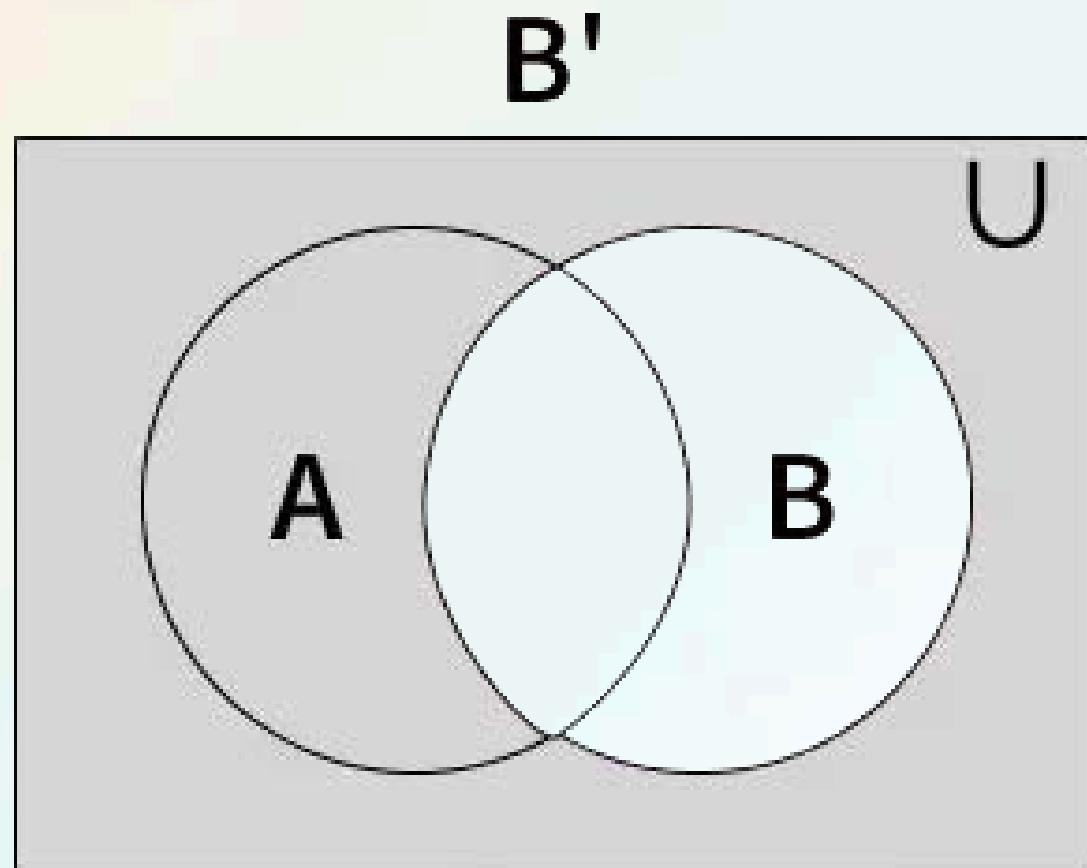


The Complement of A (A')

- This diagram shows the shaded region representing A' .
- It includes everything in the universal set U that is NOT in set A.
- Notice this means A' contains two parts:
 - a. The elements that are in B only.
 - b. The elements that are outside of both A and B.

SET OPERATIONS AND VENN DIAGRAM

The Complement of B (B')



- Similarly, this diagram shows the shaded region representing B' .
- It includes everything in the universal set U that is NOT in set B .
- This means B' contains two parts:
 - a. The elements that are in A only.
 - b. The elements that are outside of both A and B .

SET OPERATIONS AND VENN DIAGRAM

Let's define our sets:

- Universal Set, $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- Set A = {1, 2, 3, 4}
- Set B = {3, 4, 5, 6}

Finding the Complement of A (A')

To find A' , we list all the elements from the universal set U that are NOT in A.

- We start with $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- We remove the elements of A: {1, 2, 3, 4}
- Result: $A' = \{5, 6, 7, 8\}$

SET OPERATIONS AND VENN DIAGRAM

Let's define our sets:

- Universal Set, $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- Set A = {1, 2, 3, 4}
- Set B = {3, 4, 5, 6}

Finding the Complement of B (B')

Similarly, to find B' , we list all the elements from U that are NOT in B.

- We start with $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- We remove the elements of B: {3, 4, 5, 6}
- Result: $B' = \{1, 2, 7, 8\}$

SET OPERATIONS

Using the seven sets defined on the left (A through G), perform the required set operation for each problem listed on the right.

$$A = \{X, Y, Z\}$$

$$B = \{W, V, M, N\}$$

$$C = \{P, Q, R\}$$

$$D = \{M, N, W, V, F, G, Y, Z\}$$

$$E = \{F, G, H, Y, N, Z\}$$

$$F = \{X, A, B\}$$

$$G = \{W, Q, P, R, X\}$$

a. $A \cup F$

b. $B \cup C$

c. $D \cap F$

d. $D - E$

e. $D \cap G$



SET OPERATIONS

a. $A \cup F = \{X, Y, Z, A, B\}$

- This is the union, so we combine all the unique elements from set A and set F.

b. $B \cup C = \{W, V, M, N, P, Q, R\}$

- This is the union of two sets that have no common elements, so the result is simply all of their elements combined.

c. $D \cap F = \{\} \text{ or } \emptyset$

- This is the intersection. Since sets D and F have no elements in common, their intersection is the empty set.

d. $D - E = \{M, W, V\}$

- This is the set difference. We take all elements from set D and remove any that are also found in set E ($\{F, G, Y, N, Z\}$).

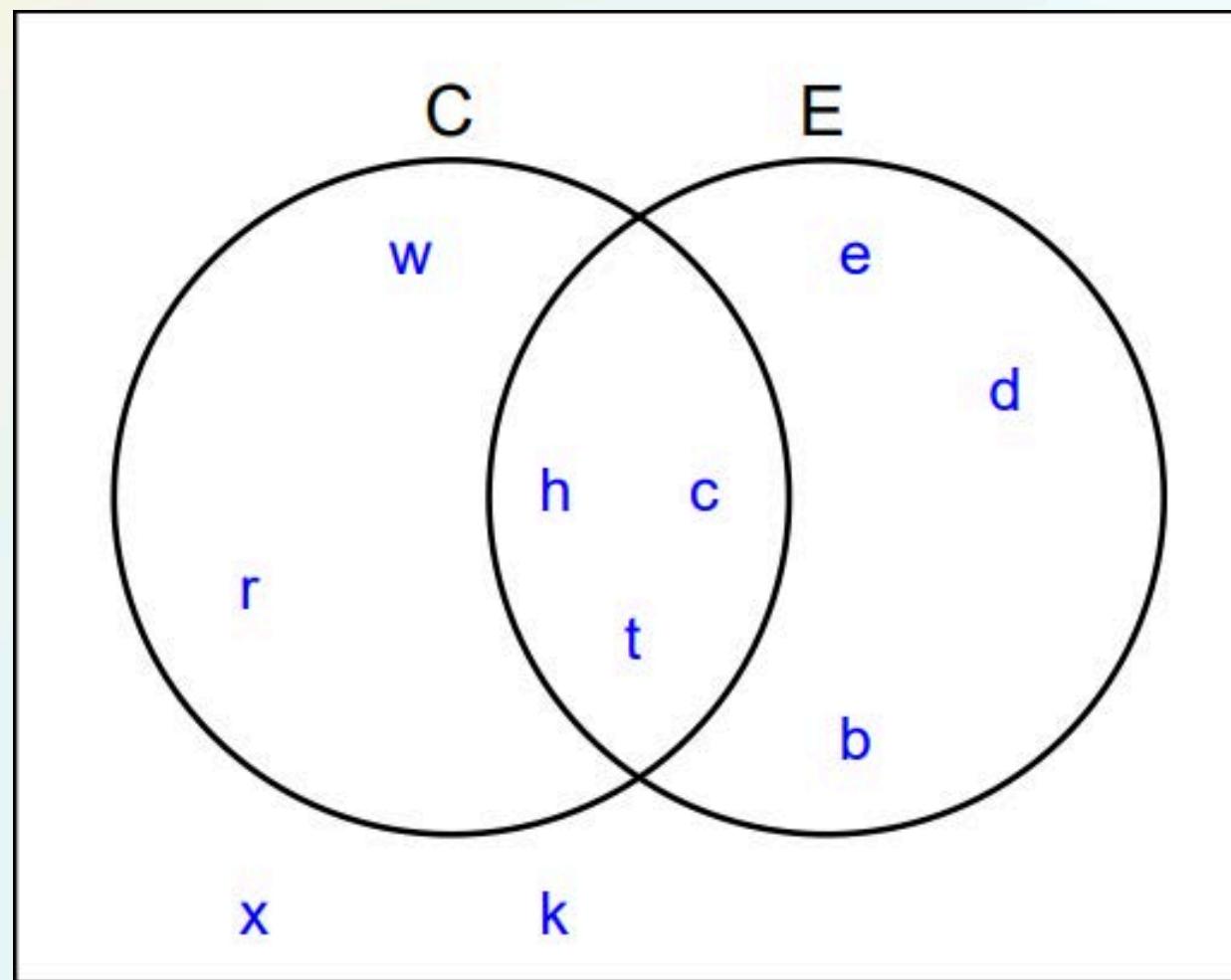
e. $D \cap G = \{W\}$

- This is the intersection. By comparing sets D and G, we find that the only element they have in common is 'W'.



Advanced Operations: Combining the Basics

Now that you understand the individual set operations, let's solve problems that combine them. These questions require you to perform more than one step to find the final answer.



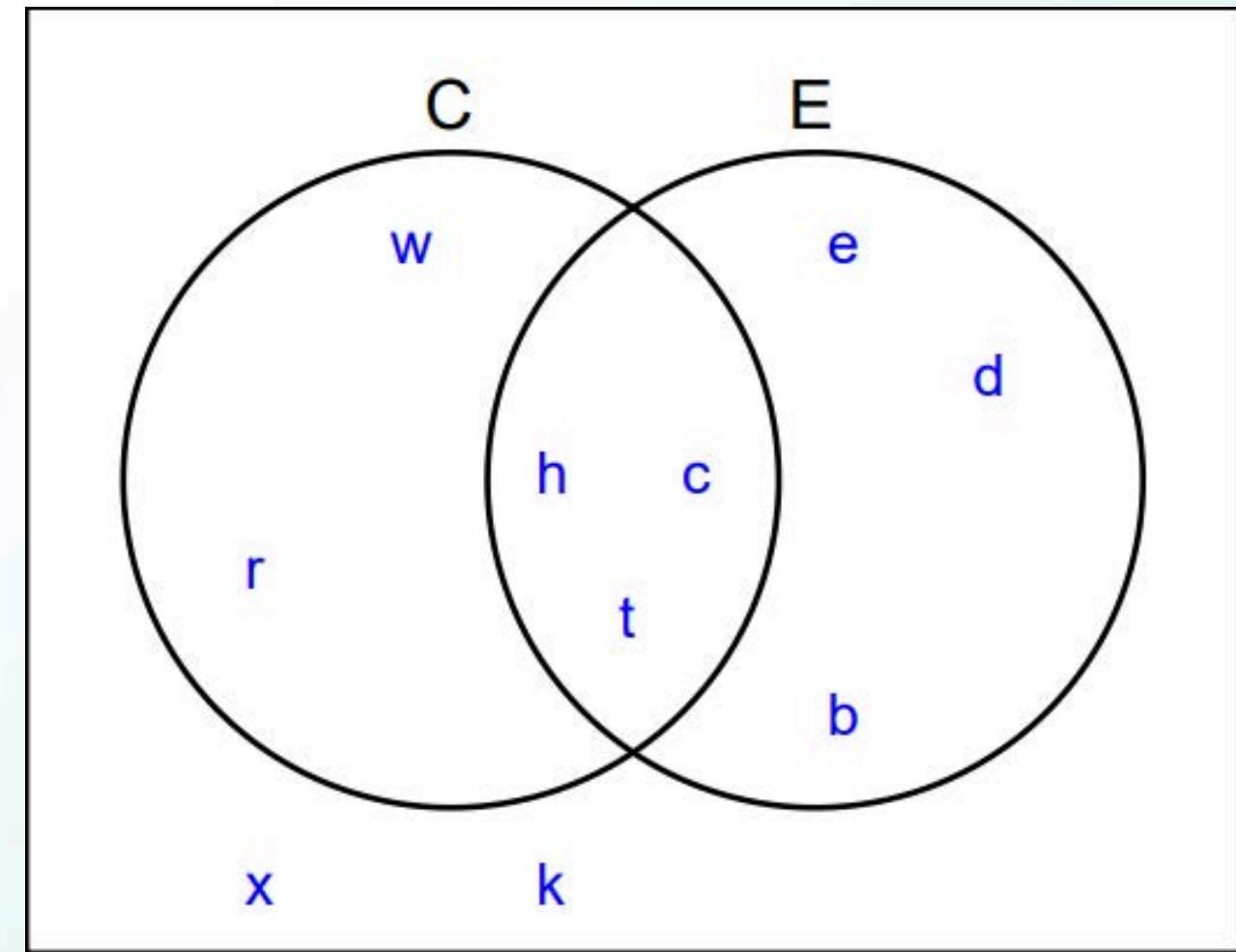
$$E' - C' = \underline{\hspace{10cm}}$$
$$C \cup E = \underline{\hspace{10cm}}$$
$$C \cap E' = \underline{\hspace{10cm}}$$
$$C - E' = \underline{\hspace{10cm}}$$

Advanced Operations: Combining the Basics

The key is to handle the operations in the correct order. **Always find the complement (')** of a set first before using it in another operation like difference or intersection.

Follow these steps:

1. **List the elements** for each set shown in the Venn diagram (Set C, Set E, and the Universal Set U).
2. Find any necessary **complements** first. For a problem like $E' - C'$, you must first determine the elements of E' and C' separately.
3. Perform the **final operation** (union, intersection, or difference) on the resulting sets.



$$E' - C' = \underline{\hspace{2cm}}$$

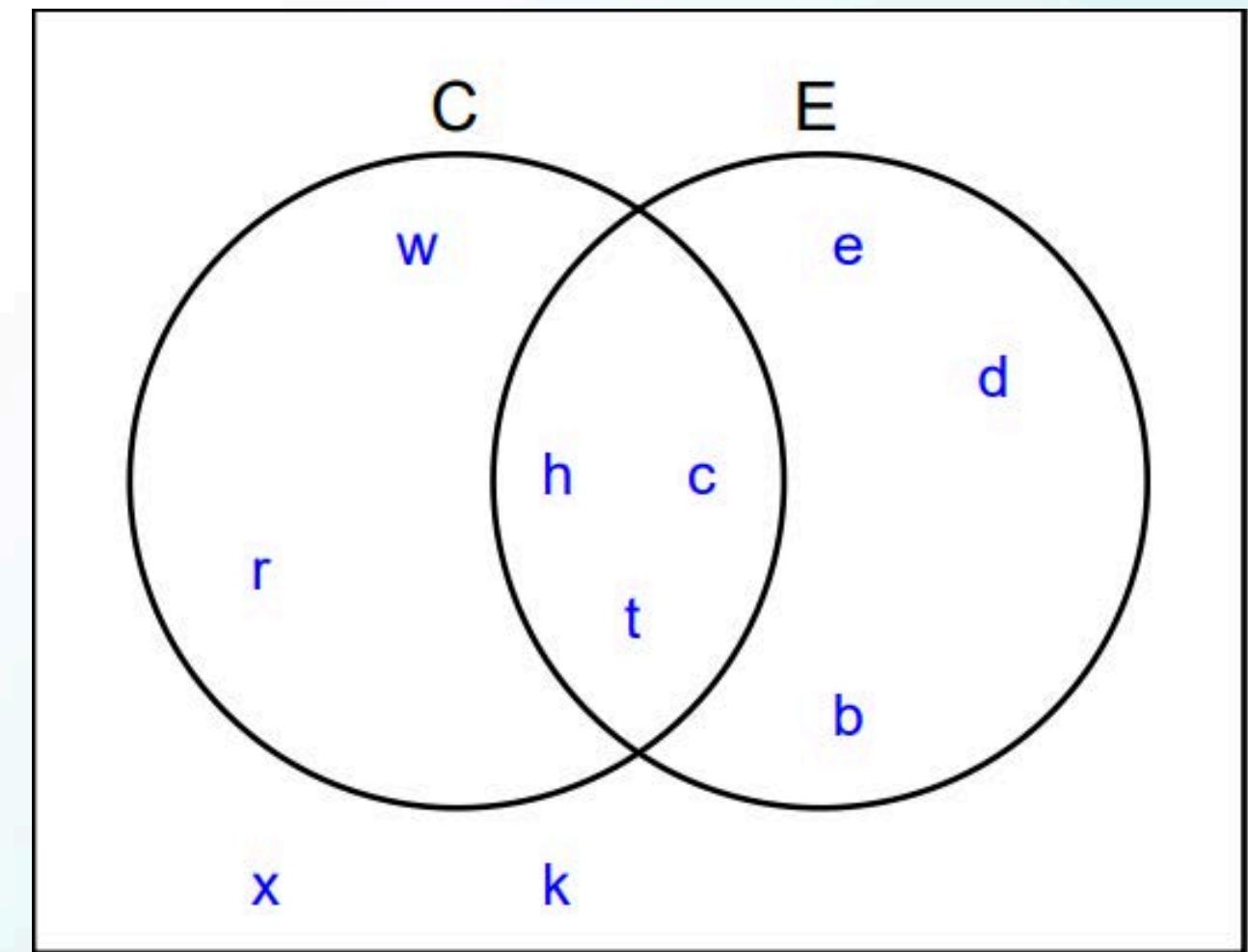
Advanced Operations: Combining the Basics

Let's do the first part together: From the diagram:

- Universal Set is $U = \{w, r, h, c, t, e, d, b, x, k\}$
- $E = \{h, c, t, e, d, b\}$.
- E' (everything in U that is NOT in E) = $\{w, r, x, k\}$.
- $C = \{w, r, h, c, t\}$
- $C' = \{e, d, b, x, k\}$

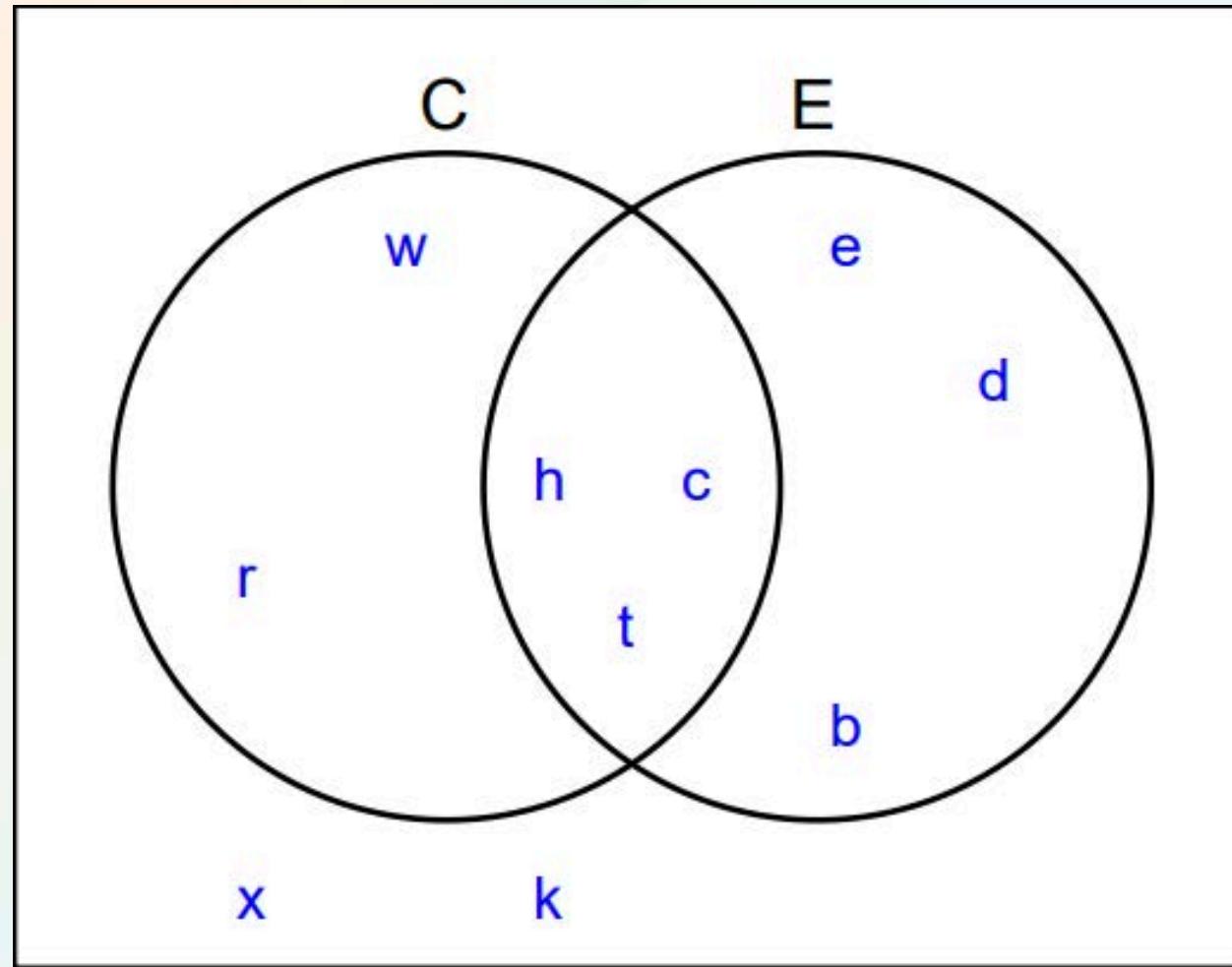
$E' - C'$

- We take set $E' = \{w, r, x, k\}$ and remove the elements it shares with $C' = \{e, d, b, x, k\}$. The common elements are $\{x, k\}$.
- Result: $\{w, r\}$



$$E' - C' = \underline{\hspace{2cm}}$$

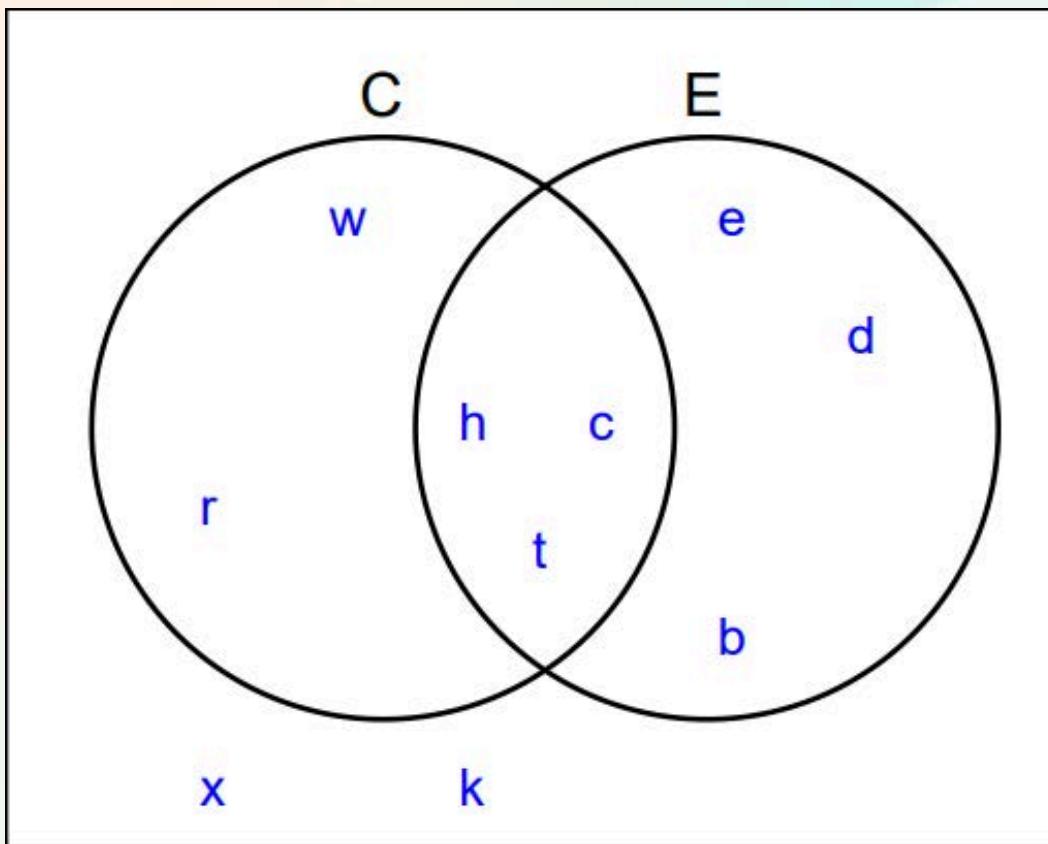
Advanced Operations: Combining the Basics



$$E' - C' = \underline{\hspace{10cm}}$$
$$C \cup E = \underline{\hspace{10cm}}$$
$$C \cap E' = \underline{\hspace{10cm}}$$
$$C - E' = \underline{\hspace{10cm}}$$



Advanced Operations: Combining the Basics



C ∪ E

We combine all the unique elements from $C = \{w, r, h, c, t\}$ and $E = \{h, c, t, e, d, b\}$.

Result: $\{w, r, h, c, t, e, d, b\}$

C ∩ E'

We find the common elements between $C = \{w, r, h, c, t\}$ and $E' = \{w, r, x, k\}$.

$$E' - C' = \underline{\hspace{2cm}}$$

Result: $\{w, r\}$

$$C \cup E = \underline{\hspace{2cm}}$$

C - E'

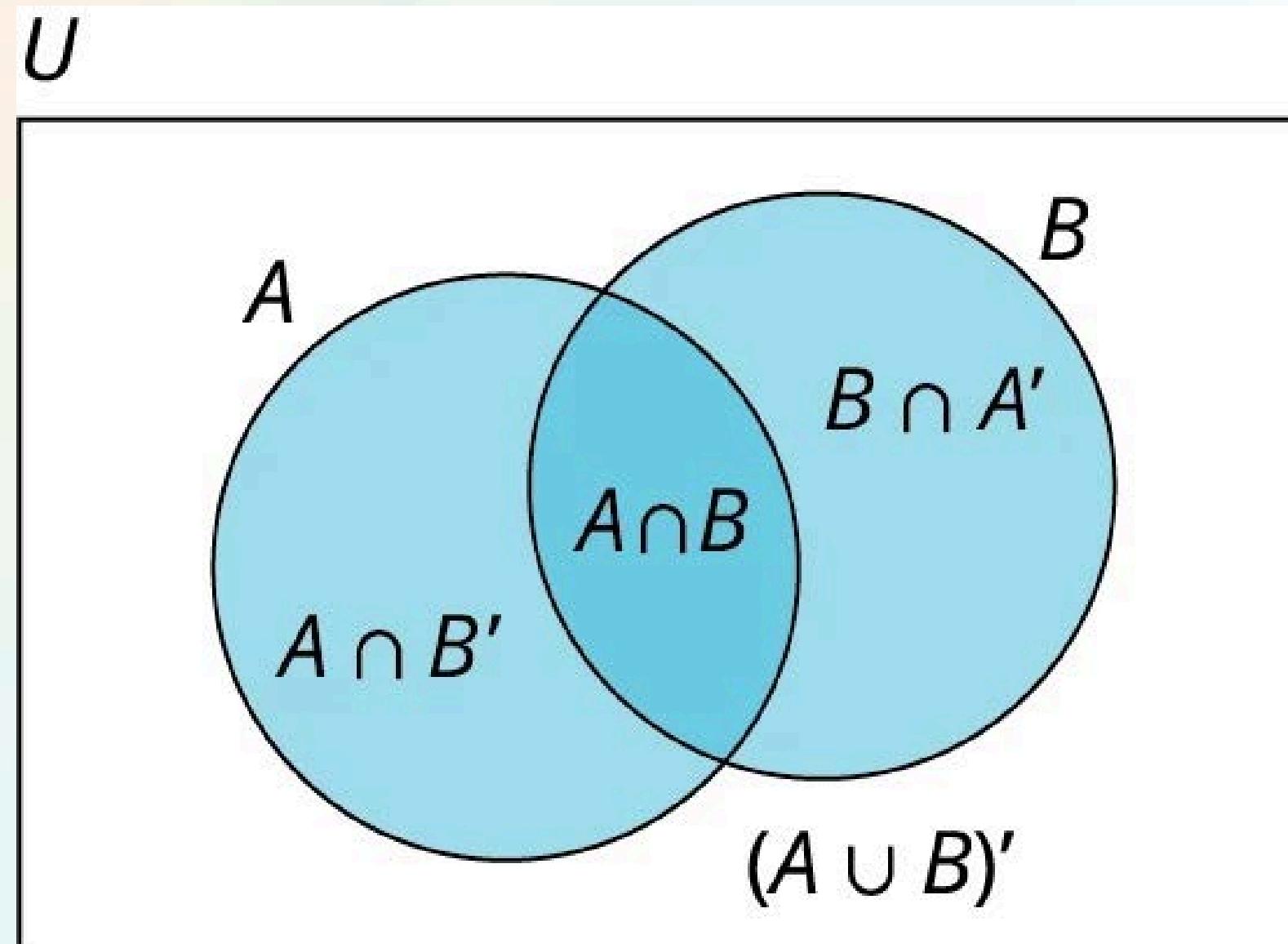
$$C \cap E' = \underline{\hspace{2cm}}$$

We take set $C = \{w, r, h, c, t\}$ and remove the elements it shares with $E' = \{w, r, x, k\}$. The common elements are $\{w, r\}$.

$$C - E' = \underline{\hspace{2cm}}$$

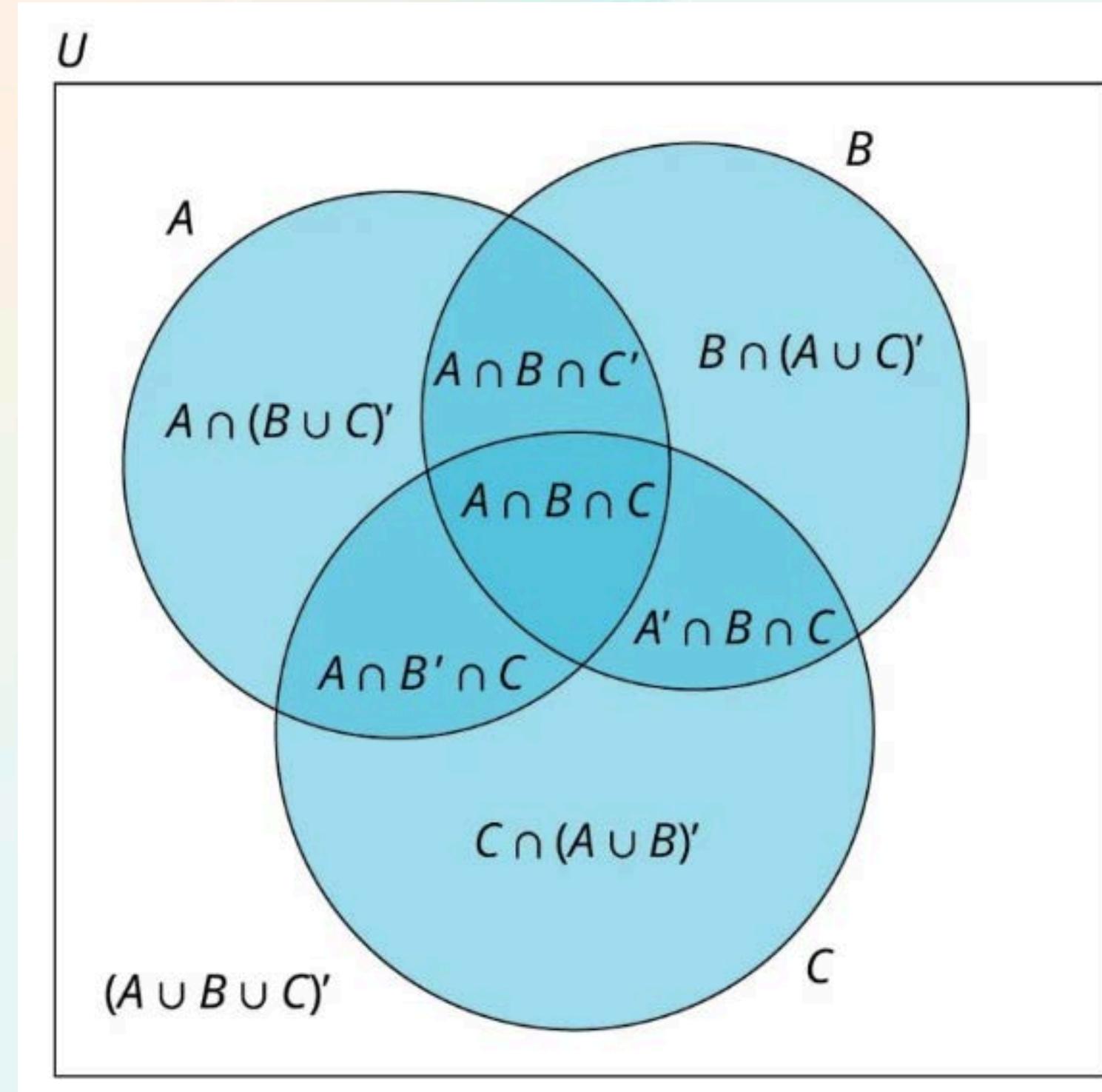
Result: $\{h, c, t\}$

VENN DIAGRAMS WITH TWO SETS



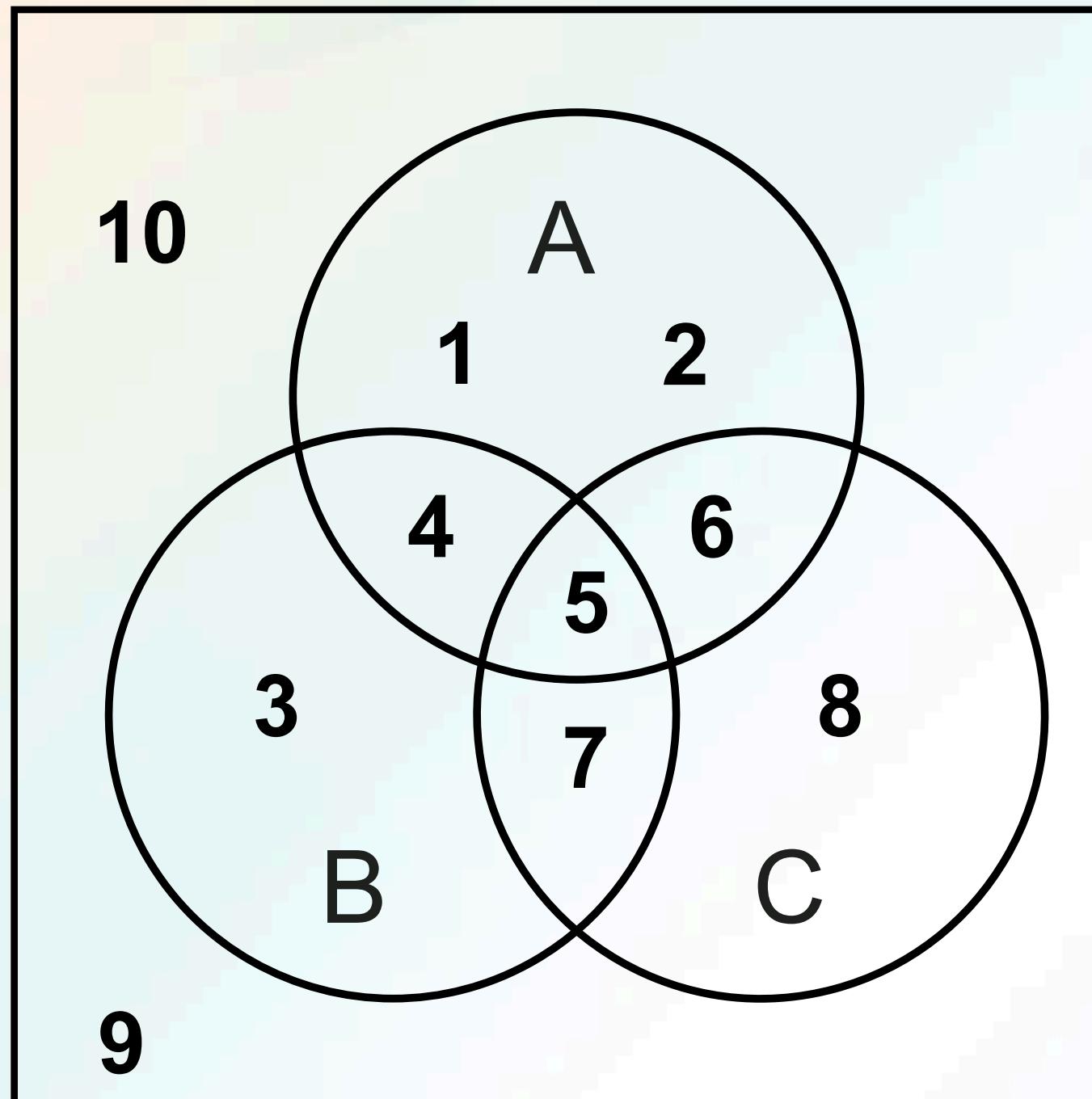
This diagram illustrates how two overlapping sets, A and B , partition the universal set (U) into four distinct and mutually exclusive regions.

VENN DIAGRAMS WITH THREE SETS



Next, we see a Venn diagram with three intersecting sets, which breaks up the universal set into eight distinct regions.

VENN DIAGRAMS WITH THREE SETS



Example:

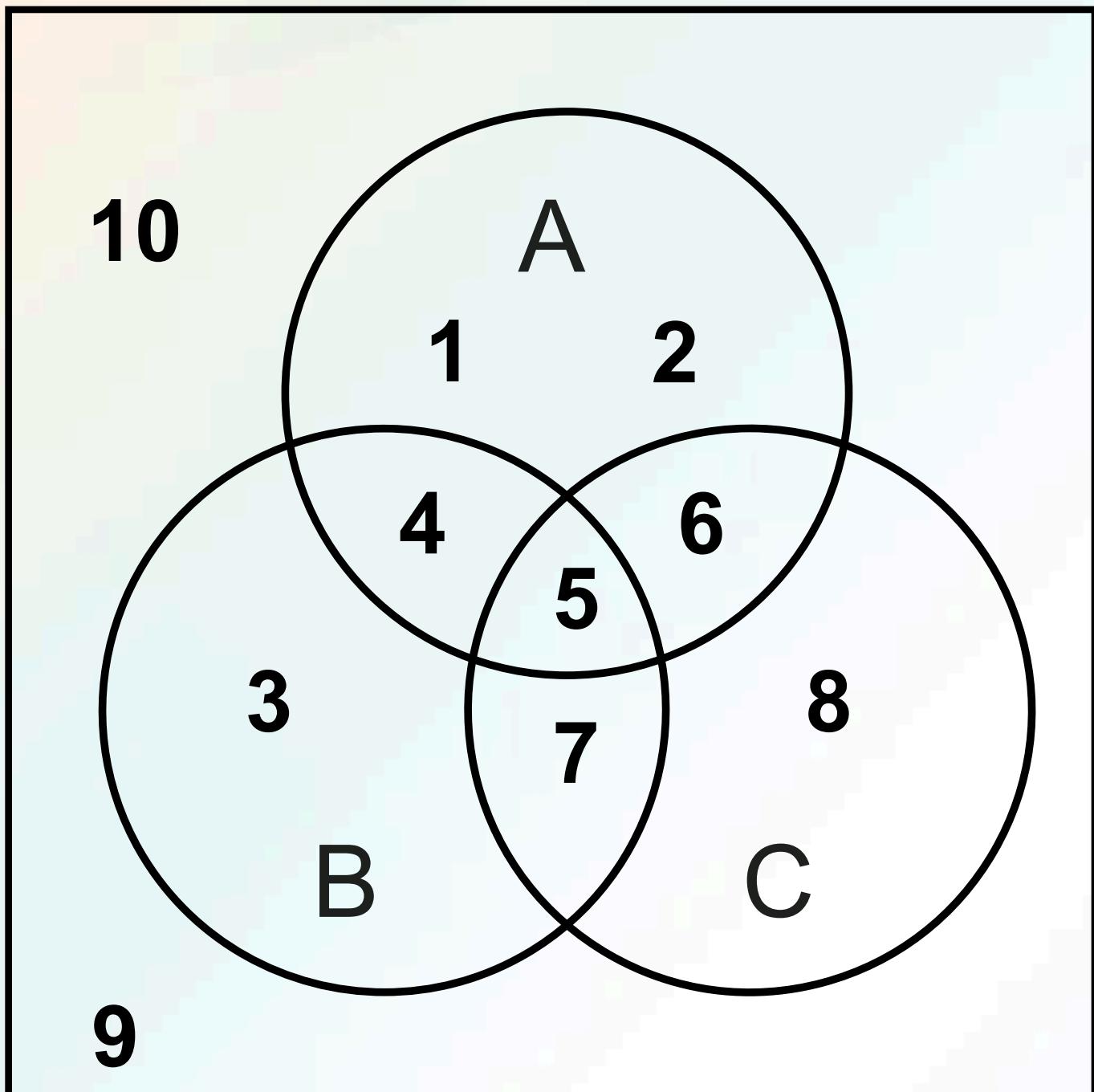
$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 2, 4, 5, 6\}$$

$$B = \{3, 4, 5, 7\}$$

$$C = \{5, 6, 7, 8\}$$

VENN DIAGRAMS WITH THREE SETS



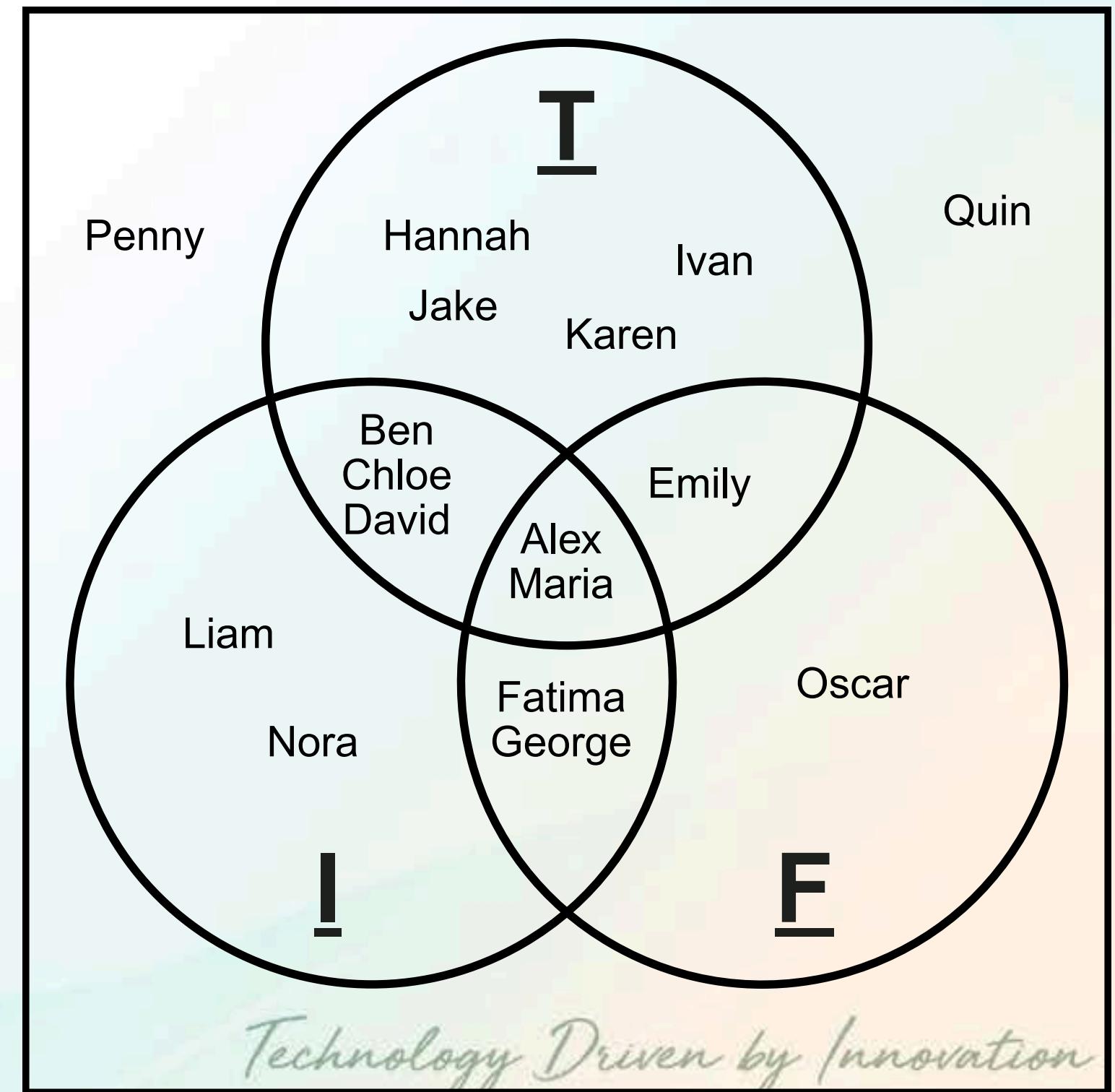
The diagram on the left visually places the numbers from $U = \{1, 2, \dots, 10\}$ into the **eight distinct regions** created by sets A, B, and C.

1. A and B and C (Center): $A \cap B \cap C = \{5\}$
2. A and B, but not C: $A \cap B \cap C' = \{4\}$
3. A and C, but not B: $A \cap B' \cap C = \{6\}$
4. B and C, but not A: $A' \cap B \cap C = \{7\}$
5. A only: $A \cap (B \cup C)' = \{1, 2\}$
6. B only: $B \cap (A \cup C)' = \{3\}$
7. C only: $C \cap (A \cup B)' = \{8\}$
8. Neither A, B, nor C (Outside): $(A \cup B \cup C)' = \{9, 10\}$

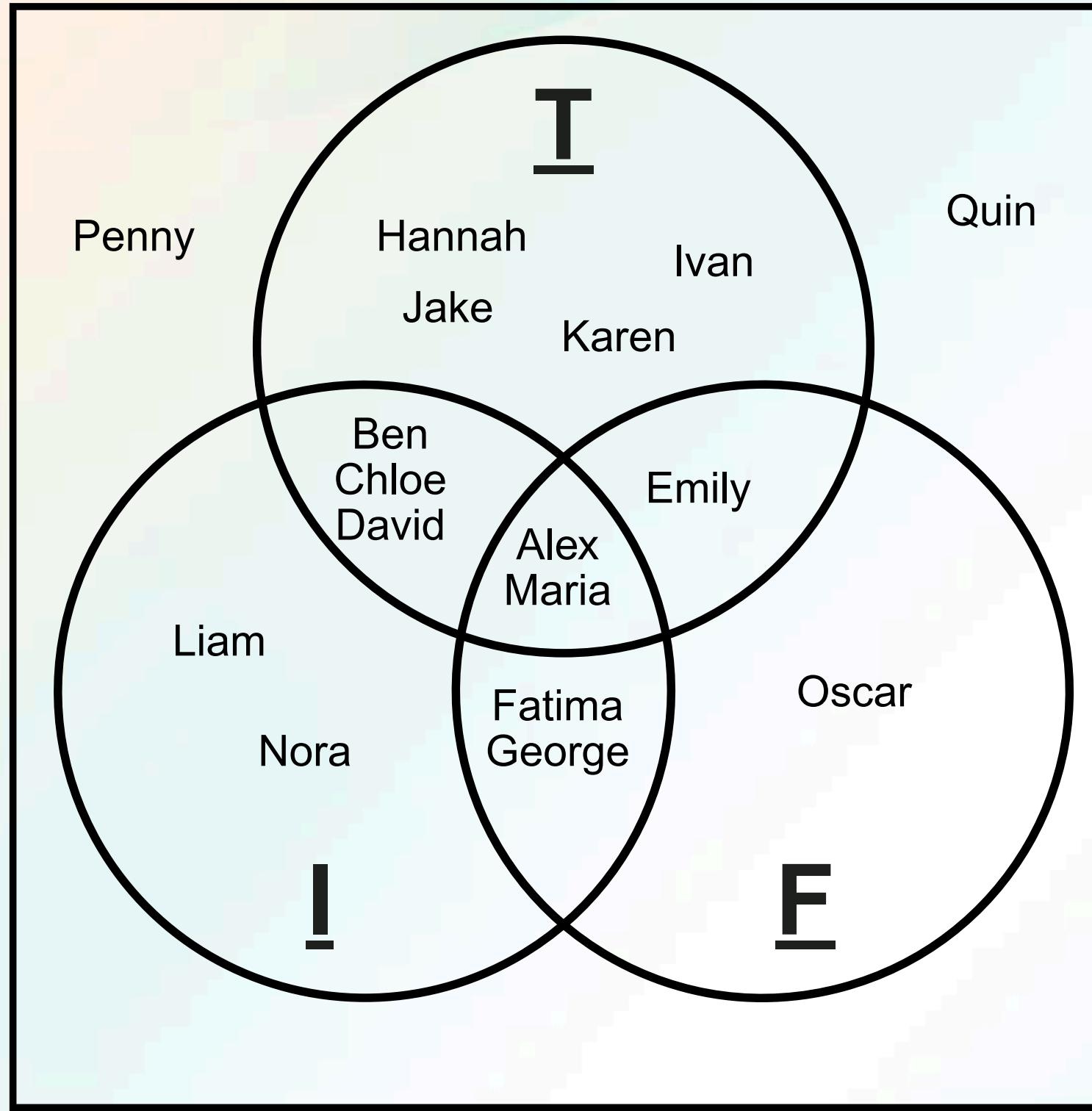
VENN DIAGRAMS WITH THREE SETS

Let's consider a survey of students about the social media platforms they use.

- Universal Set, U: All students in the class.
- T = {Alex, Maria, Ben, Chloe, David, Emily, Hannah, Ivan, Jake, Karen}
- I = {Alex, Maria, Ben, Chloe, David, Fatima, George, Liam, Nora}
- F = {Alex, Maria, Emily, Fatima, George, Oscar}



VENN DIAGRAMS WITH THREE SETS



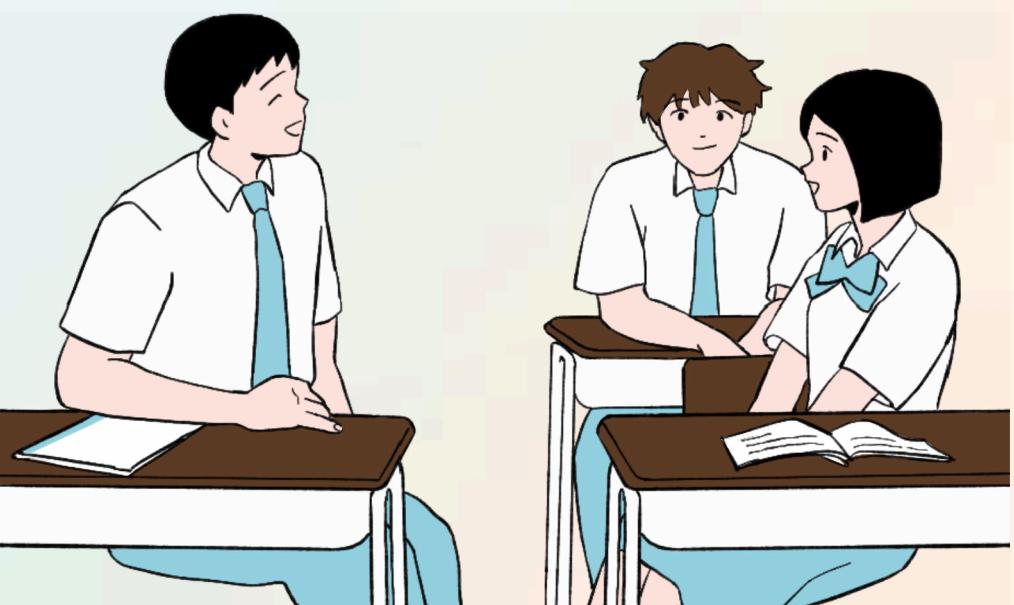
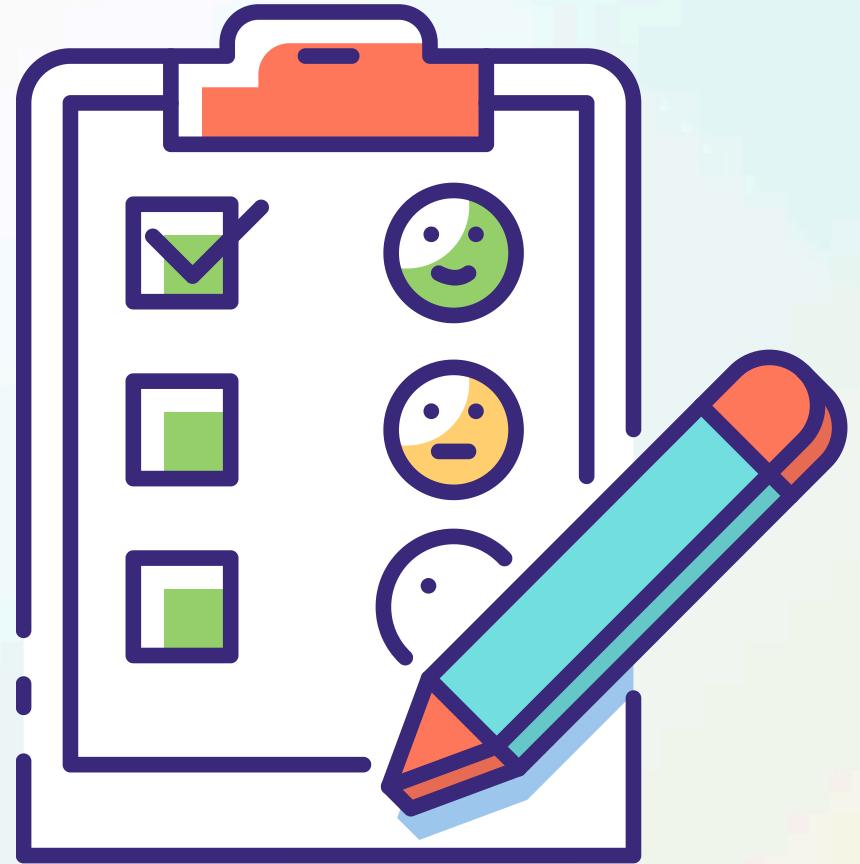
Analysis:

1. Uses all three platforms
 $T \cap I \cap F = \{Alex, Maria\}$
2. TikTok and Instagram, but not Facebook
 $(T \cap I \cap F') = \{Ben, Chloe, David\}$
3. TikTok and Facebook, but not Instagram
 $(T \cap I' \cap F) = \{Emily\}$
4. Instagram and Facebook, but not TikTok
 $(T' \cap I \cap F) = \{Fatima, George\}$
5. Tiktok only
 $(T \cap (I \cup F))' = \{Hannah, Jake, Karen\}$
6. Instagram only
 $(I \cap (T \cup F))' = \{Liam, Nora\}$
7. Facebook only
 $(F \cap (T \cup I))' = \{Oscar\}$
8. Use none of these platforms
 $((T \cup I \cup F)') = \{Penny, Quinn\}$

CLASSROOM ACTIVITY

To collect data from your classmates and use it to create and analyze a three-set Venn diagram.

Today, you'll become a data analyst! Your mission is to conduct a survey, organize the results, and display your findings in a Venn diagram.



CLASSROOM ACTIVITY

Step 1: Choose Your Three Sets (Topics)

With your group, decide on three related topics to ask your classmates about. The best topics are things that people might like more than one of.

Examples:

- Favorite Foods: Pizza, Burgers, Tacos
- Streaming Services: Netflix, YouTube, Disney+
- Hobbies: Playing Video Games, Playing Sports, Reading Books
- Pets: Dogs, Cats, Fish

CLASSROOM ACTIVITY

Step 2: Survey Your Classmates

You need to survey at least 15 classmates. Ask each person which of the three options they like.

Step 3: Organize Your Data

Now, use your survey table to count how many students fall into each of the eight regions. Remember to start from the middle and work your way out!

CLASSROOM ACTIVITY

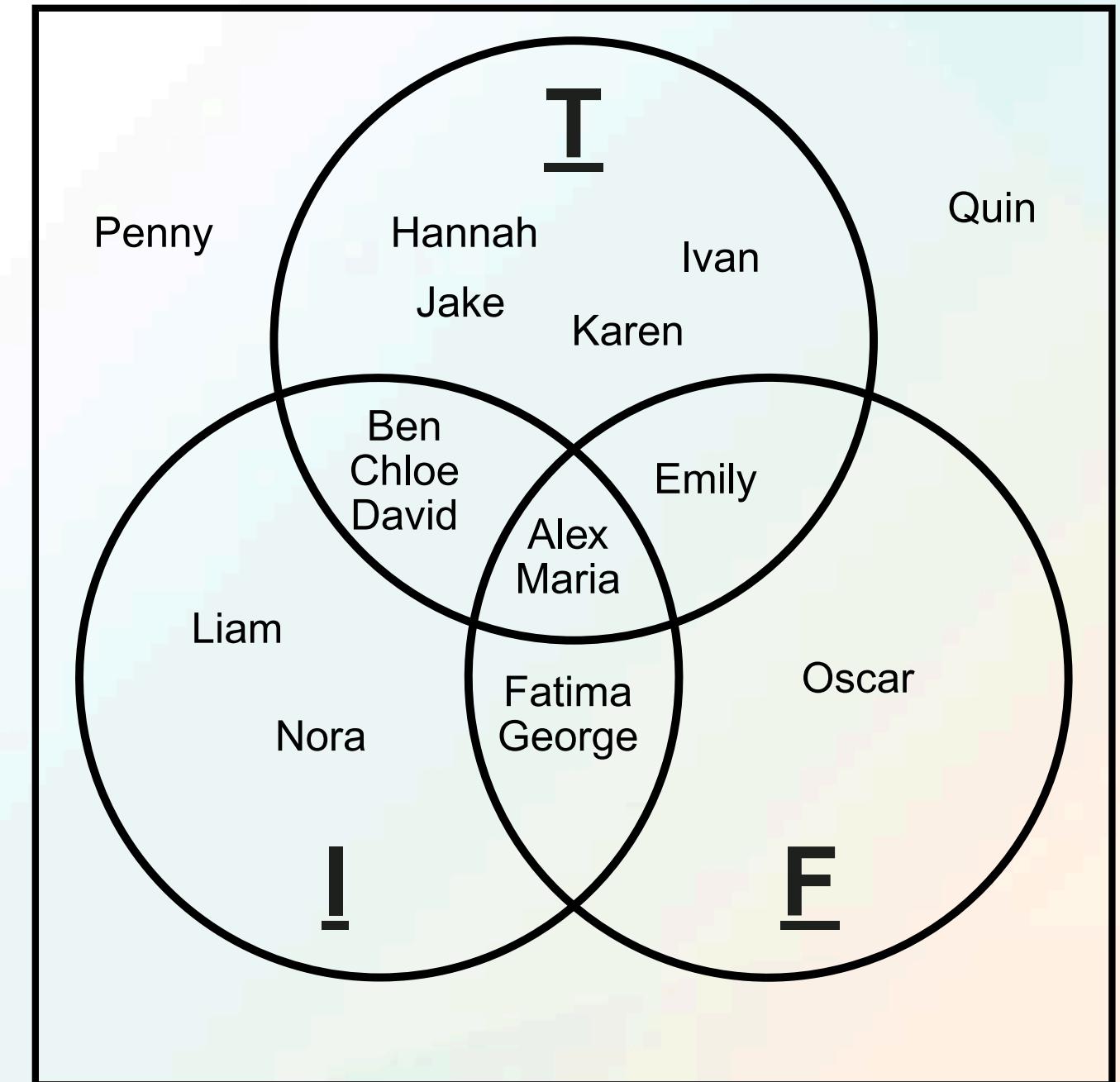
Step 3: Organize Your Data

- How many students like A, B, AND C? _____
- How many students like A and B, but not C? _____
- How many students like A and C, but not B? _____
- How many students like B and C, but not A? _____
- How many students like ONLY A? _____
- How many students like ONLY B? _____
- How many students like ONLY C? _____
- How many students like NONE of the three? _____

CLASSROOM ACTIVITY

Step 4: Create Your Venn Diagram

Draw a large three-set Venn diagram below. Label each circle as Set A, Set B, and Set C. Write the number you calculated for each of the eight regions in the correct place on the diagram.



CLASSROOM ACTIVITY

Step 5: Analyze Your Results

Answer the following questions based on the Venn diagram you created.

1. How many students like Set A in total? (Add all numbers in the 'A' circle)
 - Answer: _____
2. How many students like Set B or Set C? (Add all numbers that appear in either the 'B' or 'C' circle)
 - Answer: _____
3. How many students like exactly two of the options?
 - Answer: _____
4. How many students did you survey in total? (Add all eight numbers together). Does this match the number of people you surveyed in Step 2?
 - Answer: _____

CS0001

Discrete Structures 1

Subtopic 3: Principle of Inclusion-Exclusion

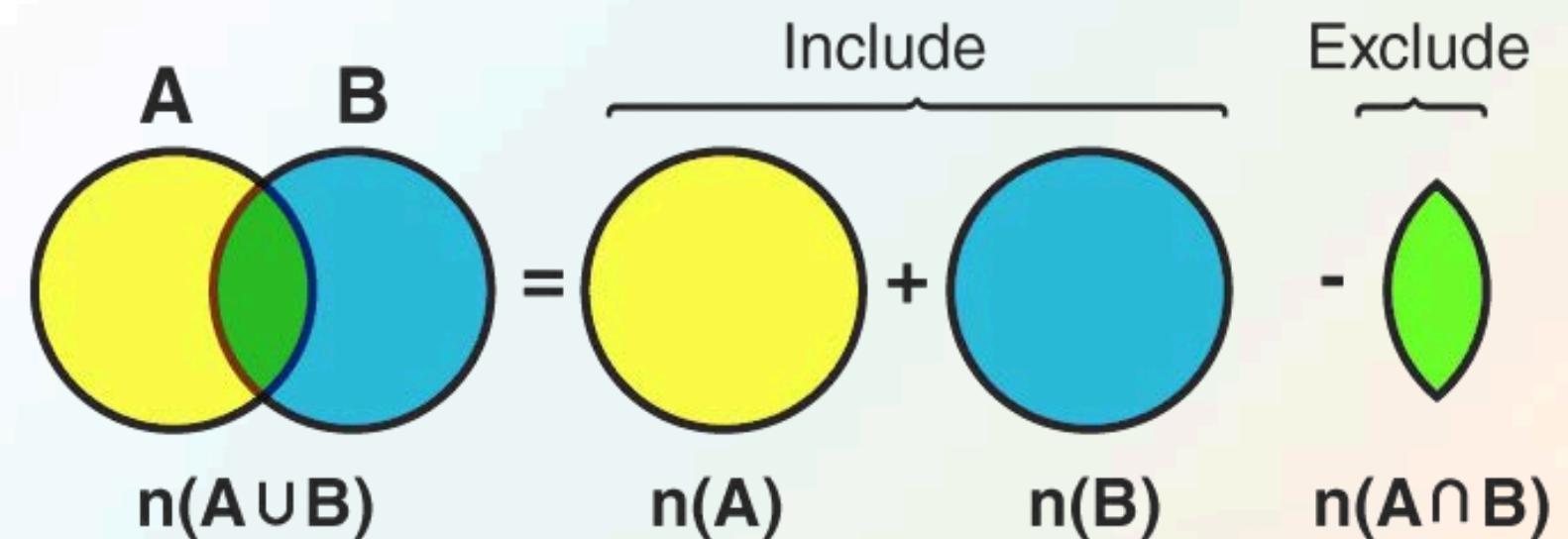


PRINCIPLE OF INCLUSION-EXCLUSION

The **Principle of Inclusion-Exclusion** is a counting method that finds the size of a union of sets by adding the sizes of the individual sets, subtracting the sizes of their pairwise intersections, adding back the size of their three-way intersection, and so on, **to avoid double-counting elements**.

Consider two finite sets, A and B. We can denote the Principle of Inclusion and Exclusion formula as follows:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



PRINCIPLE OF INCLUSION-EXCLUSION

Music and Arts Club

In a survey of **50 students**, it was found that:

- 30 students are in the Music Club.
- 25 students are in the Arts Club.
- 12 students are in both the Music Club and the Arts Club.

Questions:

1. How many students are in the Music Club or the Arts Club?
2. How many students are in neither club?



PRINCIPLE OF INCLUSION-EXCLUSION

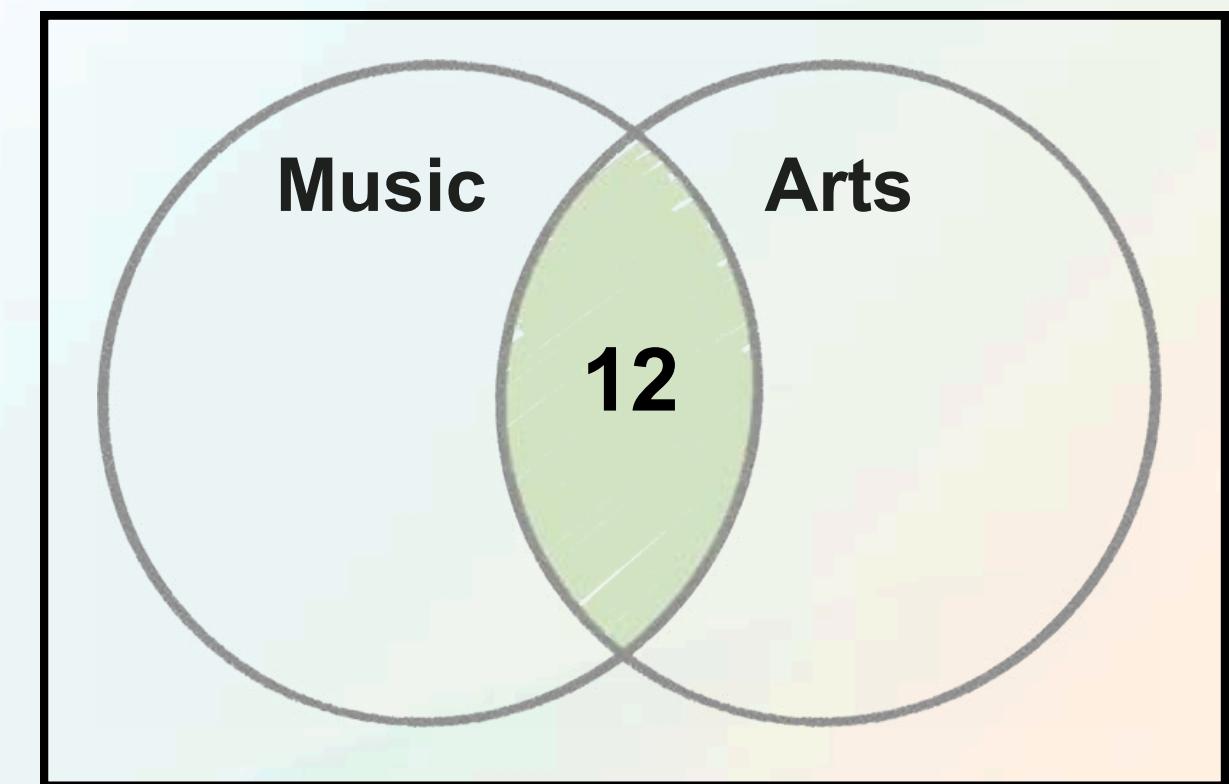
The "Inside-Out" Strategy

The most important rule for solving these problems is to start from the center and work your way outwards. The center region is the only piece of information you can place directly without any calculations.

Find the center:

In a survey of 50 students, it was found that:

- 30 students are in the Music Club.
- 25 students are in the Arts Club.
- **12 students are in both the Music Club and the Arts Club.**



PRINCIPLE OF INCLUSION-EXCLUSION

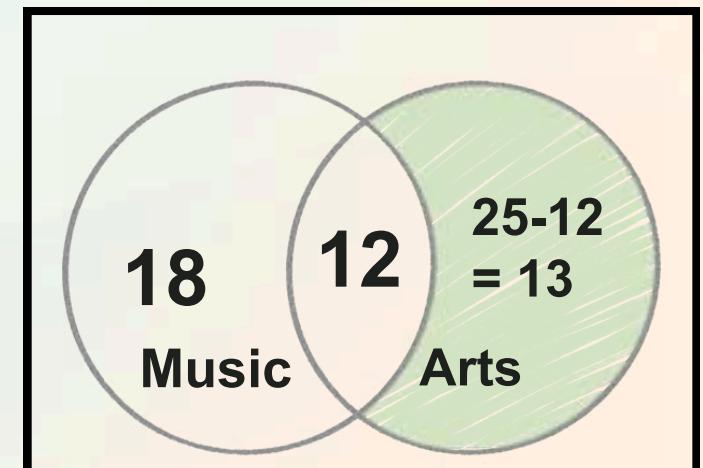
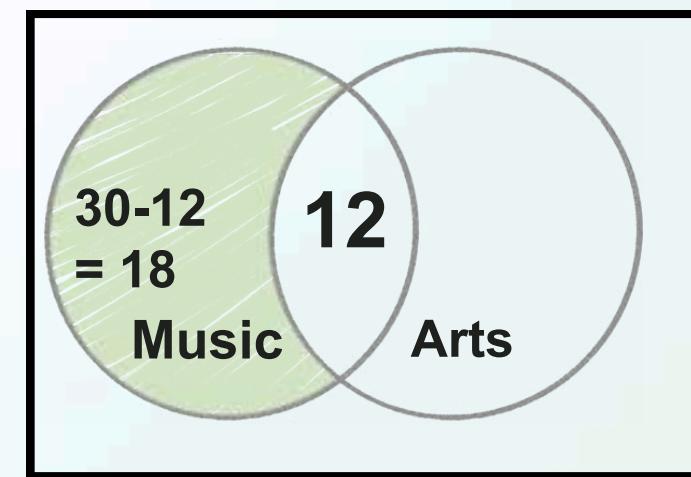
Calculate the "Only" Regions

Now, find the values for the parts of each circle that don't overlap with any other. The problem gives you the total for the entire circle, so you must subtract the three inner values you've already placed.

Find the center:

In a survey of 50 students, it was found that:

- 30 students are in the Music Club.
- 25 students are in the Arts Club.
- **12 students are in both the Music Club and the Arts Club.**



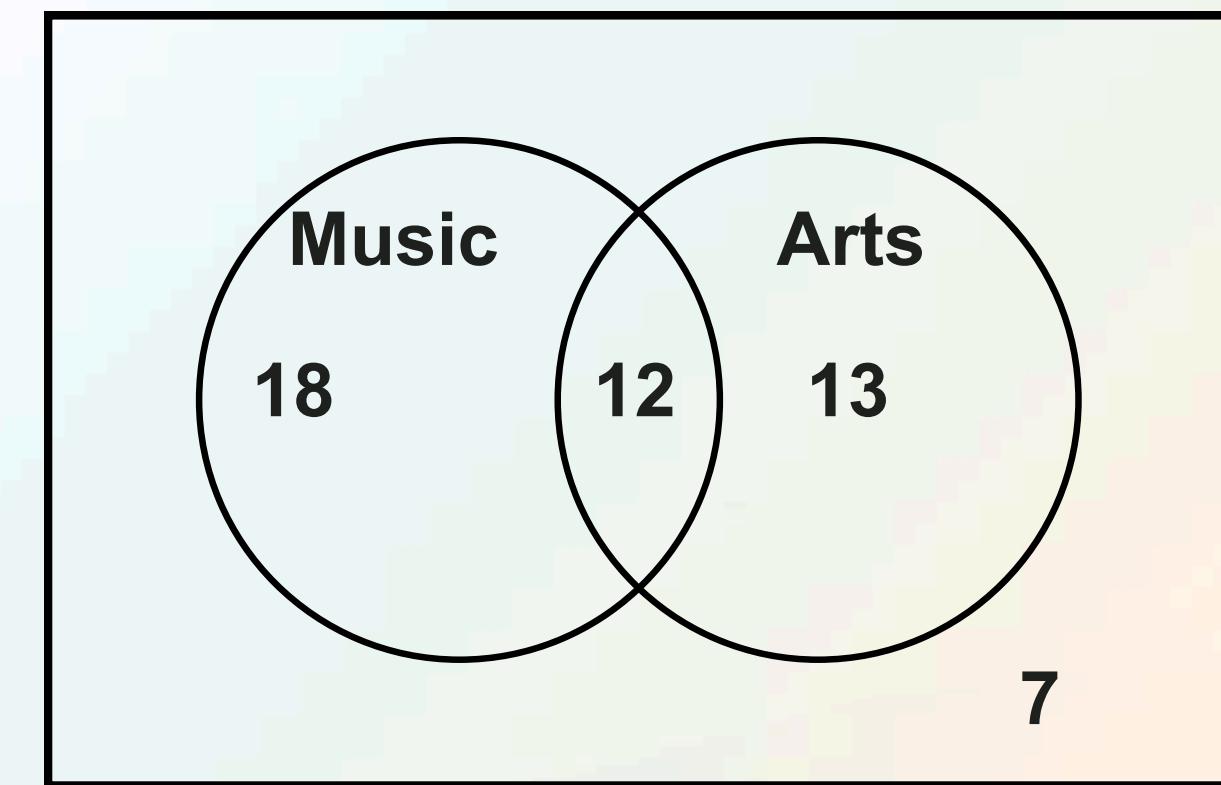
PRINCIPLE OF INCLUSION-EXCLUSION

Find the Outside Region (None of the Sets)

Finally, find out how many elements are in none of the sets. Add up all seven numbers you've placed inside the circles and subtract that sum from the total number of players.

- **Total Inside:** $12 + 18 + 13 = 43$
- **Total Players:** 50
- **Calculation:** $50 - 43 = 7$

Place 7 outside the circles but inside the main box.



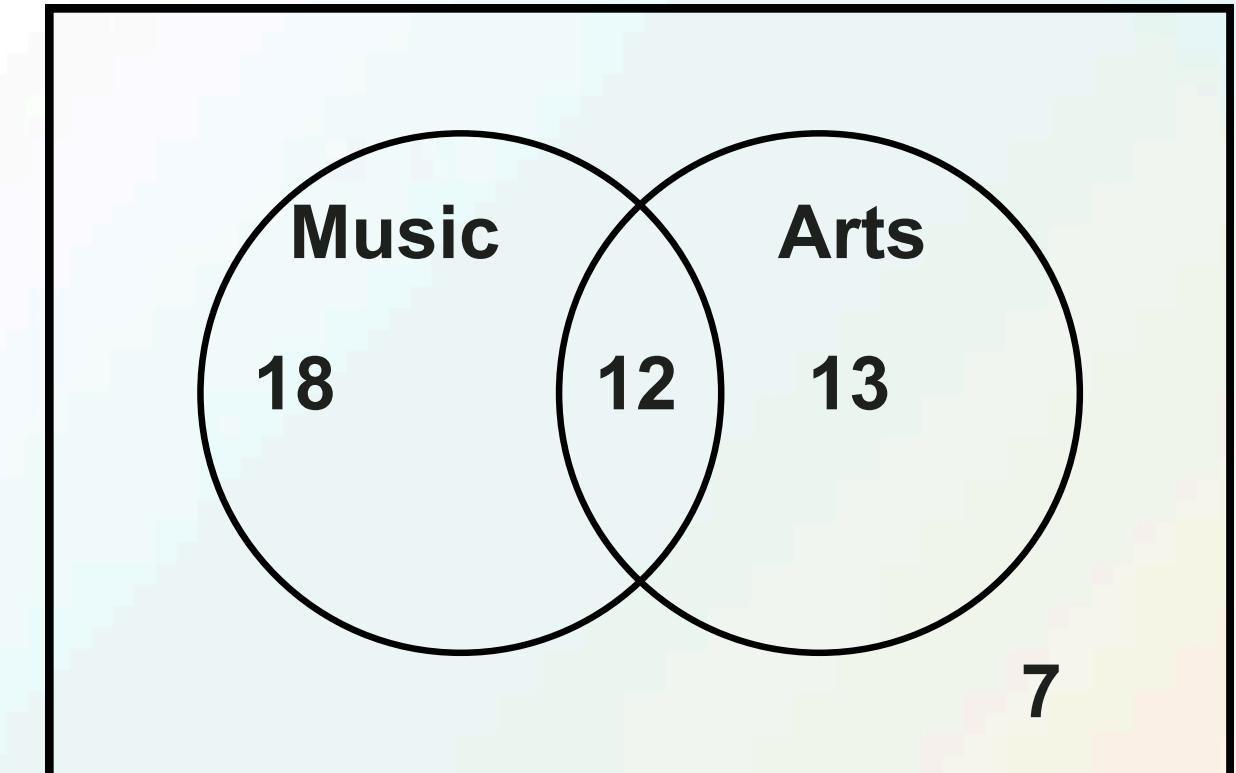
PRINCIPLE OF INCLUSION-EXCLUSION

Questions:

How many students are in the Music Club or the Arts Club?

(Students in Music Club) + (Students in Arts Club) - (Students in BOTH)

$$30 + 25 - 12 = 43$$



How many students are in neither club?

(Total Students) - (Students in Music or Arts)

$$50 - 43 = 7$$

TRY IN 3-SET

Coach Alpha offered to buy hot dogs for players on his team.

Of the 44 players:

- 6 wanted all three condiments
- 11 wanted ketchup and relish
- 10 wanted ketchup and mustard
- 8 wanted mustard and relish
- 28 wanted ketchup
- 20 wanted mustard
- 14 wanted relish

Create a Venn diagram to model the data and answer the following.

How many players wanted?

- a. Ketchup only
- b. Mustard but not relish
- c. Relish but not mustard
- d. Ketchup and mustard but not relish

TRY IN 3-SET

In a survey of 75 people, Rona found that:

- 9 liked all the three products
- 24 people liked both products A and B
- 20 people liked products C and A
- 18 people liked product B and C
- 38 people liked product A
- 36 liked product B
- 39 liked product C

QUESTIONS:

- a) How many liked product A or B?
- b) How many liked product both A and B, but not C?
- c) How many didn't like any of the three products?

TRY IN 3-SET

105 adults were asked whether they had studied Hiligaynon, Ilokano or Kapampangan in school. Here are the results of the survey:

- 5 are taking all three
- 24 have studied both Hiligaynon and Ilokano
- 9 have studied both Kapampangan and Hiligaynon
- 11 have studied both Kapampangan and Ilokano
- 52 have studied Hiligaynon
- 63 have studied Ilokano
- 25 have studied Kapampangan

Questions:

- a) How many have studied Ilokano but not Hiligaynon?
- b) How many have studied Kapampangan but not Hiligaynon?
- c) How many have studied both Hiligaynon and Ilokano?
- d) How many have studied Hiligaynon or Ilokano?
- e) How many have studied both Hiligaynon and Ilokano but not Kapampangan?

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Discrete Structures 1

END OF MODULE 5



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