Speed Maintained SVRG

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Abstract—**Stochastic gradient desent (SGD) is widely used for large-scale machine learning optimization, but has slow convergence rate due to the highly inherent variance. In recent years, the popular Stochastic Variance Reduced Gradient (SVRG) method mitigates this shortcoming, through computing the full-gradient of the entire dataset occasionally. However, conventional SVRG and its variants usually need a hyper-parameter to identify when to compute such the full gradient, which is essential to the covergene performance. Few previous studies discuss the method to identify such the hyper-parameter, which makes it hard to gain a good convergence performance in parctical machine learning tasks. In our paper, we propose a new stochastic gradient descent with variance reduction technique named SMSVRG which computes the full gradient adaptively. Moreover, we propose an improved method denoted by SMSVRG+, which is comparable to and even better than SVRG with best-tuned epoch sizes for smooth and strongly convex functions.**

I. INTRODUCTION

Many machine learning tasks such as logistic regression and linear regression can be formulated to be an optimizaton problem which is described as

$$\min F(\omega), \qquad F(\omega) = \frac{1}{n}\sum_{i=1}^{n} f_i(\omega) + R(\omega). \qquad (1)$$

where n is the size of the training data. $F(\omega)$ means the loss function or training loss of a machine model, and ω is its parameter. $R(\omega)$ is the regularizer, which is widely used to avoid overfitting. It is noting that the total number of instances, i.e. n becomes very large with the proliferation of data.

Gradient Descent (GD) is a basic method to solve such the optimization problem. The gradient of $F(\omega)$ is obtained by passing over the entire training data, which is extremely time-consuming when the size of training data, i.e. n becomes large. Besides, GD is an iterative-convergent algorithm, that is, the parameter, i.e. ω, usually needs thousands of iterations to be converged. Since GD needs to compute the gradient of $F(\omega)$ every iteration, when the volume of data is large, the computation cost increases sharply and impairs the convergent performance significantly.

Stochastic Gradient Descent (SGD) mitigates this shortcoming by replacing the calculation of $\nabla F(\omega)$ with a stochastic gradient $\nabla f_i(\omega)$ with $i \in \{1, 2, ..., n\}$. In SGD, i is selected randomly from the entire training data. Thus, SGD outperforms GD on the time efficiency significantly. Take

the expectation of i, we obtain $\mathbb{E}[\nabla f_i(\omega)] = \nabla F(\omega)$. The difference between $\nabla f_i(\omega)$ and $\nabla F(\omega)$ represents variance which makes it difficult to achieve the optimum. In order to make the loss function, i.e. $F(\omega)$ converge, a decaying learning rate is usually used to reduce the variance. However, value of the learning rate is decayed to be very small after hundreds of iterations, which impedes the loss function to converge. In a nutshell, SGD with a decaying learning rate is efficient, but difficult to gain a high quality solution due to the variance.

In recent years, variance reduced variants of SGD such as SVRG [4] is proposed to reduce the variance and gain the linear convergence performance with a constant learning rate. SVRG is organized as epochs where a full gradient is computed every epoch, and used to reduce the variance during the iterations. On the basis of SVRG, many variants have been proposed to improve its performance. SVRG-BB [12] uses the Barzilai and Borwein (BB) method proposed by Barzilai and Borwein in [2] to compute the step size before every epoch, which generally achieves the comparable convergence performance to SVRG with the best-tuned step size. CHEAPSVRG [10] and SAMPLEVR aim at reducing the expensive cost of full gradient computation through using a surrogate with a subset of the training dataset. mS2GD [6] uses mini-batch method to obtain a full gradient to reduce the variance, which shows a clear advantage for parallel computation. EMGD [14], SVR-GHT [8], Prox-SVRG [13] and SVRG with second-order information [5] modify the update rule of stochastic steps, and show advantages to SVRG in some cases. However, there are few studies to discuss how frequently should a full gradient be computed, in other words, how to set the epoch size m. Most previous researches present that the epoch size, i.e. m should be constant [4, 12, 10] or increased monotonically [6], regardless of the learning rate. It is recommended that $m = 2n$ for convex problems and $m = 5n$ for non-convex problems in SVRG, without theoretical analysis and further experimental verification.

The epoch size, i.e. m has a great impact on the convergence performance of SVRG. More specifically, when m is too small, it wastes too much time to compute the full gradient frequently. When m is rather large, the variance between the stochastic gradient and the full gradient increases sharply, making the convergence of training loss extremely difficult. According

to the analysis of variance in [YaWei], both the epoch size, i.e. m and learning rate, i.e. η have a significant impact on the convergence performance. However, those previous studies do not provide a practical method to set the value of those hyper-parameters. Extensive empirical studies illustrates that the selection of good value for those hyper-parameters costs much time in real machine learning tasks.

In this paper, we propose a novel algorithm denoted by SMSVRG which can adjust the epoch size adaptively. When η is large, the training loss begins to fluctuate after merely a small number of iterations. In the other hands, the algorithm can endure far more than n iterations with a small η. Ideally, if we stop the iterations in one epoch just before the training loss begins to fluctuate, the algorithm will certainly be very efficient and outperform SVRG with a constant epoch size. A direct approach is to compute the training loss occasionally. However, the computation of training loss requires to pass over the entire dataset, which is rather time consuming. Intuitively, the change of parameters, i.e. $\Delta\omega$ is proportional to that of training loss, i.e. ΔF. Hence we can use $\Delta\omega$ instead of ΔF to detect the fluctuation. SMSVRG computes the changing amounts of parameters at the same interval, if the changing amount of current interval is greater than that of former, it finishes the current epoch and begins the next one. Besides, we analyze and give guidance of how to set the interval size. Since SMSVRG may stop iterations earlier than expectation when η is quite small, we propose an improved algorithm denoted by SMSVRG+. In a nutshell, our contributions are highlighted as follows:

- SMSVRG, an algorithm that can adjust the epoch size dynamically.
- SMSVRG+, an improvement of SMSVRG and outperforming SVRG in any case.
- Extensive empirical studies shows the effectiveness of our proposed algorithms which outperform their countparts on the convergence performance significantly.

This paper is organized as follows: Section II reviews the related work. Section IV presents the new variant of SVRG, i.e. SMSVRG. Section V demonstrates the numerical results of our algorithm. Section VI concludes this paper.

II. RELATED WORK

Several variants of SVRG focusing on epoch size has been proposed, including SVRG++ [1], S2GD [7], SVRG_Auto_Epoch [1] and so on.

SVRG++ adopts a simple strategy that epoch size m doubles between every consecutive two epochs. This method is absolutely heuristic and sometimes not justified. Our experiments show that when η is large or moderate, the exponential growth of m will incur great variance and impairs convergence.

S2GD designs a probability model of m and shows that a large epoch size is used with a high probability. However Meanwhile, the maximum of stochastic steps per epoch is also a sensitive parameter. it needs to know the lower bound on the strong convexity constant of F, which is hard to estimate in practice.

Algorithm 1 SVRG

Require: learning rate η, epoch size m, initial point $\tilde{\omega}$
1: **for** $s = 0, 1, ..., m$ **do**
2:      $\tilde{\mu} = \frac{1}{n}\sum_{i=1}^{n}\nabla f_i(\tilde{\omega}_s)$
3:      $\omega_0 = \tilde{\omega}_s$
4:      **for** $t = 0, 1, 2, ..$ **do**
5:          Randomly pick $i_t \in \{1, 2, ..., n\}$
6:          $\omega_t = \omega_{t-1} - \eta(\nabla f_{i_t} - \nabla f_{i_t}(\tilde{\omega}_s) + \tilde{\mu})$
7:      **end for**
8:      **Option I:** $\tilde{\omega}_{s+1} = \omega_m$
9:      **Option II:** $\tilde{\omega}_{s+1} = \omega_t$ for randomly chosen $t \in \{0, ..., m-1\}$
10: **end for**

SVRG_Auto_Epoch is introduced as an additional improvement of SVRG++. It determines the termination of epoch through the quality of the snapshot full gradient. It records $diff_t = \|\nabla f_i(\omega_t^s) - \nabla f_i(\tilde{\omega}^{s-1})\|$ every iteration t and uses it as a tight upper bound on the variance of the gradient estimator. Although this method is reasonable, it has too much parameters to tune. Moreover, it takes much additional computation for every iterations, which impairs performances significantly.

Comparing with the above methods, SMSVRG is apparently reasonable in intuition and does not need to tune extra parameters. Besides, it takes little additional computation cost and outperform the aforementioned three methods.

III. OVERVIEW

In this section we review the SVRG algorithm proposed by Johnson and Zhang [4]. As is shown in 1, there are two loops in SVRG. In an outer loop, a full gradient $\tilde{\mu}$ is computed at first. This snapshot of the full gradient will be used to update the local parameter, i.e. ω_t in iterations in the inner loop, which significantly reduces the variance. At the end of an epoch, the global parameter $\tilde{\omega}_{s+1}$ is initialized by using those local parameters, i.e., ω_t, which has been presented by two options. Although only the convergence of SVRG with Option I is analyzed in [4], SVRG with Option II has been confirmed numerical to perform better. We adopt Option II in our proposed methods in this paper.

It is obvious that when m is too large, SGD with the variance reduction technique will degenerate to the basic SGD, which results in huge variance. Thus our work focuses on how to set an appropriate epoch size.

IV. SPEED MAINTAINED SVRG

In this section we describe two novel algorithms: SMSVRG and SMSVRG+, which adjust the epoch size automatically and has superior convergence performance. We assume the loss function F and the component functions f_i are convex and L-smooth throughout the paper.

A. smSVRG

When we apply gradient descent to convex problem, as the ω_t will gradually approach the optimal value ω^*, thus the gradient $\nabla F(\omega_t)$ keeps decreasing. Then we have $\|\omega_{t+1} - \omega_t\| < \|\omega_t - \omega_{t-1}\|$. Considering several iterations, we can also have $\|\omega_{t+m_0} - \omega_t\| < \|\omega_t - \omega_{t-m_0}\|$. Inspired by this, SMSVRG sets $\|\omega_{t+m_0} - \omega_t\| > \|\omega_t - \omega_{t-m_0}\|$ as a stop condition in each epoch. SMSVRG just requires two parameters: the learning rate η, and the window size m_0. Note that the difference between SVRG and SMSVRG is that in the latter we adjust the epoch size dynamically, instead of using a prefixed m as in SVRG. In each epoch, SMSVRG compute the inequality

$$\|\omega_t - \omega_{t-m_0}\| > \|\omega_{t-m_0} - \omega_{t-2m_0}\|$$

every m_0 iterations, if the inequality holds the algorithm will break the inner loop and begin the next epoch.

B. Optimal Choice of the window size

In SMSVRG, we use the $\Delta\omega$ of several iterations to detect when loss function begin to fluctuate and fail to converge. However, how to choose a suitable value of window size i.e. m_0 becomes an important issue. In this section, we analyze the effects of setting different m_0 and provide guidance on setting a optimal value.

First, as the variance incurred by SGD iterations cannot be ignored, when we set m_0 to be small, the variance of $\|\omega_t - \omega_{t-m_0}\|$ is too big thus the confidence of the inequality is low. As a result, the inequality may holds because of variance when the loss function is still decreasing rapid, which wastes much time on computing full gradients. We use $C(m_0)$ to denote the confidence of inequality holds. It is obvious that $C(m_0)$ is a monotonically increasing function of m_0.

When we set m_0 to be relatively big, the variance becomes small and the confidence of inequality holds is high. However, it will be too late to detect the fluctuation of objective function and waste much time to compute iterations which make no process for optimizing the objective function. We use $D_s(m_0)$ to denote the *AbsoluteDelay* of detecting fluctuation. Furthermore, the *AbsoluteDelay* makes different effects depending on the epoch size, so it is better to consider the *RelativeDelay*:

$$D_r(m_0) = \frac{D_s(m_0)}{es + n}$$

Note that es denote the number of iterations in a specific epoch and n denotes the size of the dataset. Function $D_r(m_0)$ is also monotonically increasing with respect to m_0.

It is natural that we want to choice a suitable m_0 which can achieve a big $C(m_0)$ and a small $D_r(m_0)$. In order to obtain a trade-off between $C(m_0)$ and $D_r(m_0)$, we convert the parameter choosing problem to the following maximization problem:

$$\begin{aligned} m_0 &= \max_{m_0} C(m_0) - D_r(m_0) \\ &= \max_{m_0} C(m_0) - \frac{D_s(m_0)}{es + n} \\ &\text{s.t.} \, 0 < m_0 < n \end{aligned} \quad (2)$$

Algorithm 2 SMSVRG

Require: The learning rate η, The window size m_0, and the initial point $\tilde{\omega}_0$
1: **for** $s = 0, 1, \ldots$ **do**
2:      $\tilde{\mu} = \frac{1}{n} \sum_{i=1}^{n} \nabla f_i(\tilde{\omega}_s)$
3:      $\omega_0 = \tilde{\omega}_s$
4:      **for** $t = 0, 1, 2, ..$ **do**
5:          **if** $t \geqslant 2m_0$ and $t\%m_0 = 0$ and $\|\omega_t - \omega_{t-m_0}\| > \|\omega_{t-m_0} - \omega_{t-2m_0}\|$ **then**
6:             break
7:          **end if**
8:          Randomly pick $i_t \in \{1, 2, ..., n\}$
9:          $\omega_t = \omega_{t-1} - \eta(\nabla f_{i_t} - \nabla f_{i_t}(\tilde{\omega}_s) + \tilde{\mu})$
10:      **end for**
11:      $\tilde{\omega}_{s+1} = \omega_t$
12: **end for**
13: **return** $\tilde{\omega}_{s+1}$

Note that we do not have the exact expression of C and D_s, instead, we only know their monotonicity. Nonetheless, it is enough to guide us to set the parameter m_0. From (2) we can obtain the following conclusions:

1. When epoch size es is small, the D_s tends to be more important than C. Hence we should set m_0 to be relatively small to maximize the objective function. On the contrary, it is recommended to set m_0 to be big. In spirit of this, we can set m_0 to be proportional to the iteration number of previous epoch.

2. According to our experiment on SVRG, when the learning rate η is large, the loss function begin to fluctuate after merely $n/10$ iterations, so we should set the initial m_0 to the same order of magnitude as $10^{-1} \times n$

C. SMSVRG+

On the basis of the analysis of section IV-B, we improve our algorithm SMSVRG through dynamically adjusting the window size, instead of a constant value. According to the two conclusions, we initialize the m_0 to be $0.1n$ and set it to be $(int(es/n) + 1) * (0.1n)$ in each epoch, where es denote the iteration number of previous epoch. With this strategy, when each epoch seems to be capable of enduring more iterations, SMSVRG+ will expand the window size to improve the confidence, avoiding stopping the epoch ahead of time due to variance.

V. Numerical Experiments

In this section, we conduct some experiments to demonstrate the efficiency of our proposed algorithm. We evaluate our algorithm on four training datasets, which are public on the LIBSVM website[1]. In our experiments, SMSVRG is applied for two standard machine learning tasks: $l2$-regularized logistic regression and $l2$-regularized ridge regression.
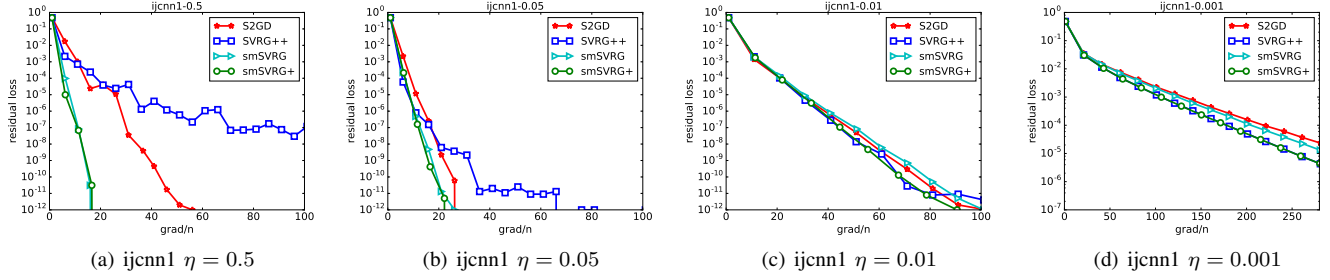
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[1] http://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/

Fig. 1. Comparison of SMSVRG, SMSVRG+, SVRG++, S2GD

Algorithm 3 SMSVRG+

Require: learning rate η, window size m_0, initial point $\tilde{\omega}$
Initialize: $m_0 = 0.1n$
1: $\tilde{\mu} = \frac{1}{n}\sum_{i=1}^{n} \nabla f_i(\tilde{\omega})$
2: **for** $s = 0, 1, ...$ **do**
3: $\quad \tilde{\mu} = \frac{1}{n}\sum_{i=1}^{n} \nabla f_i(\tilde{\omega}_s)$
4: $\quad \omega_0 = \tilde{\omega}_s$
5: $\quad$ **for** $t = 0, 1, 2, ..$ **do**
6: $\quad\quad$ **if** $t > m_0$ and $t\%m_0 = 0$ and $\|\omega_t - \omega_{t-m_0}\| > \|\omega_{t-m_0} - \omega_{t-2m_0}\|$ **then**
7: $\quad\quad\quad$ break
8: $\quad\quad$ **end if**
9: $\quad\quad$ Randomly pick $i_t \in \{1, 2, ..., n\}$
10: $\quad\quad$ $\omega_t = \omega_{t-1} - \eta(\nabla f_{i_t} - \nabla f_{i_t}(\tilde{\omega}_s) + \tilde{\mu})$
11: $\quad$ **end for**
12: $\quad es = t$
13: $\quad m_0 = (int(es/n) + 1) * (0.1n)$
14: $\quad \tilde{\omega}_{s+1} = \omega_m$
15: **end for**
16: **return** $\tilde{\omega}_{s+1}$

The $l2$-regularized logistic regression task is conducted on the two datasets: ijcnn1, a9a. Since the label of each instance in these datasets is set to be 1 or -1, the loss function of $l2$-regularized logistic regression task is:

$$\min_{\omega} \frac{1}{n}\sum_{i=1}^{n} \log(1 + e^{-y_i\omega^{\mathrm{T}}x_i}) + \lambda \parallel \omega \parallel^2 . \quad (3)$$

The $l2$-regularized ridge regression task is conducted on the four datasets: YearPredictMSD, and cadata. The loss function of $l2$-regularized ridge regression task is:

$$\min_{\omega} \frac{1}{n}\sum_{i=1}^{n} \left(\omega^{\mathrm{T}}x_i - y_i\right)^2 + \lambda \parallel \omega \parallel^2 . \quad (4)$$

We scale the value of all features to $[-1, 1]$ and set the weighting parameter λ to 10^{-4} for all evaluations. In all figures, the x-axis denotes the computational cost, which is measured by the number of gradient computation divided by the size of training data, i.e. n. The y-axis denotes training loss residual, i.e. $F(\tilde{\omega}_s) - F(\omega^*)$. Note that the optimum ω^* is estimated by running the gradient descent for a long

TABLE I
DETAIL INFORMATION OF DATASETS AND MODELS

Dataset	size	dimension	model	λ
ijcnn1	49990	22	logistic	10^{-4}
a9a	32561	123	logistic	10^{-4}
YearPredictionMSD	463715	90	linear	10^{-4}
cadata	20640	8	linear	10^{-4}

time. Our numerical experiments include two parts: comparing with existing related methods and comparing with SVRG of different epoch sizes. Both experiments show the superior performance of our methods.

A. Comparing with existing related methods

In this section, we compare our SMSVRG and SMSVRG+ with two aforementioned existing methods: SVRG++ and S2GD. We do not compare with SVRG_Auto_Epoch in that we find its termination condition of epoch is never satisfied and keeps doing SGD iteration, resulting in nonconvengence. For SVRG++, we initialize $m = n$. For S2GD, we set the maximum of m to be $4n$. For both SMSVRG and SMSVRG+, we set the window size, i.e. m_0 to be $n/10$. We evaluate these methods by running logistic regression on dataset ijcnn1. As illustrated in Figure V-A, we can see that SVRG++ fluctuates violently and fails to converge due to the large variance caused by too large epoch size. It only performs well when η is quite small. As shown in Figures 1(a), 1(b), both SMSVRG and SMSVRG+ converge rapidly with the relatively large η because they can constrain m in a small range. However, it can be seen from Figures 1(c), 1(d), when η decreases to a small value, the performance of SMSVRG drops gradually and even becomes inferior to others, while SMSVRG+ always performs better than its countparts. The main reason is that when η is small, more iterations can be computed in one epoch, thus checking the inequality frequently will resulting algorithm to stop epochs too early. The strategy of increase of window size applied by SMSVRG+ can reduce the variance. Hence it always outperforms other methods.

B. Comparing with SVRG by varying learning rate

Since SMSVRG+ performs better than SMSVRG, SMSVRG+ is used to conduct the comparison with SVRG. For SVRG, we increase the epoch size, i.e. m as four different values: $n, 2n, 4n, 10n$. The first three values are chosen for
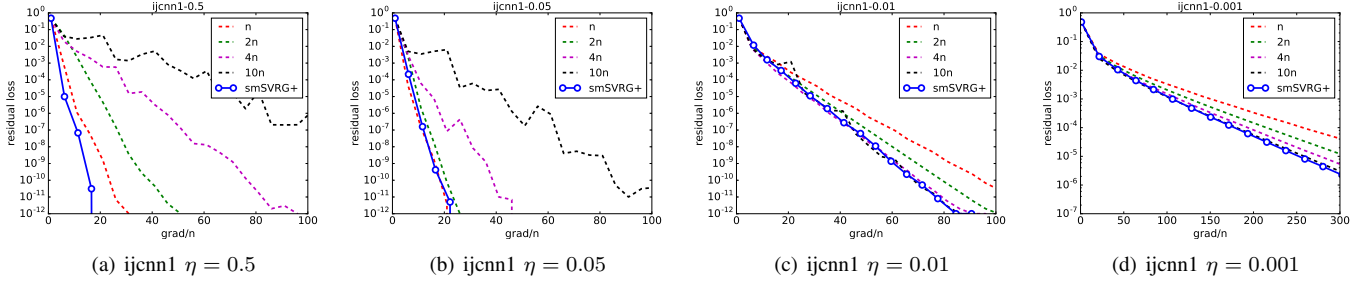
Fig. 2. Generally, SMSVRG can automatically set an appropriate m with different learning rates for the $l2$-regularized logistic regression
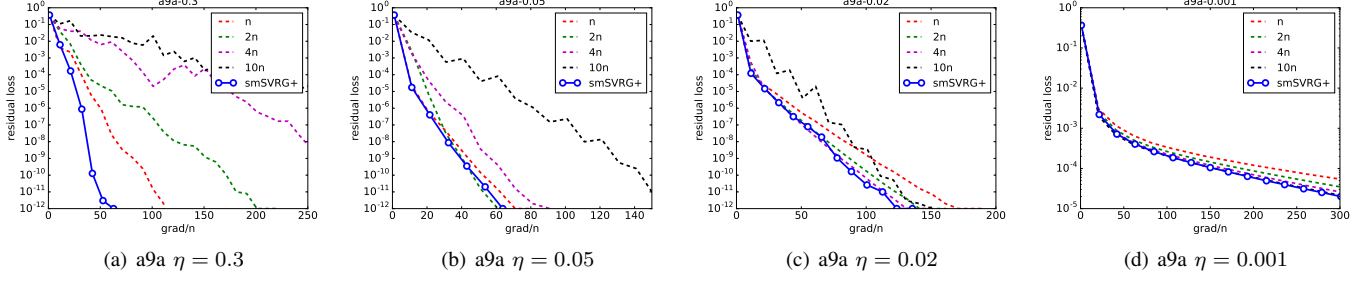

Fig. 3. Generally, SMSVRG can automatically set an appropriate m with different learning rates for the $l2$-regularized logistic regression
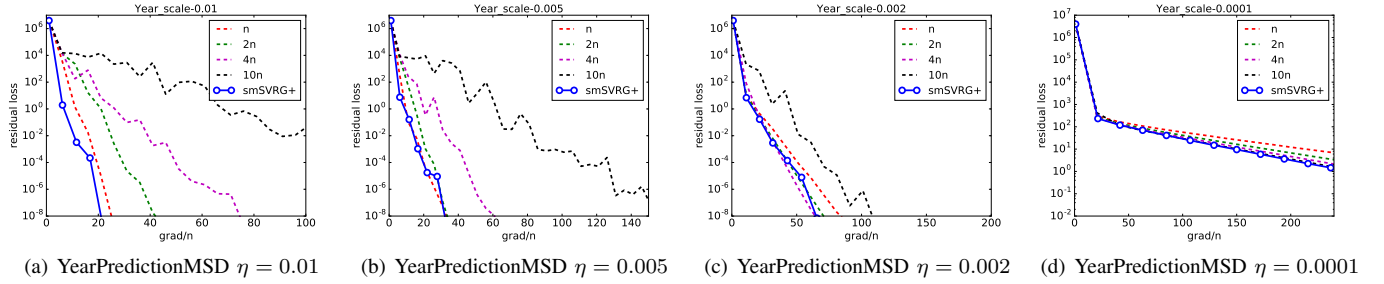

Fig. 4. Generally, SMSVRG can automatically set an appropriate m with different learning rates for the $l2$-regularized linear regression

the reason that they are commonly used in most previous researches [? ?]. Besides, our experiments show that epoch size which is large than $10n$ leads to much variance and slows the convergence of the training loss. It is easy to comprehend that the full gradient computation becomes less important when epoch size is rather large. In other words, SVRG with an extremely large epoch size degenerates to SGD. In all figures, the dashed lines represents SVRG with fixed epoch size; while the green solid lines stands for SMSVRG+.

As illustrated in Figures V-A and V-A, SMSVRG+ can always have the similar performance as SVRG with most best tuning epoch size. We observe that when η is large, and m is set to be a small value, e.g. n, can achieve best performance. The main reason is that when η is large, the variance becomes significant simultaneously, so m must be set to be small in order to bound the variance. As η decays, the optimal value of m increases, which means that the algorithm can tolerate more variance induced by extra iterations. As illustrated in Figures, our method is comparable to and sometimes even better than

SVRG with best-tuned epoch sizes when learning rate is large or medium. However, if η is set to be too small, SMSVRG+ performs slightly inferior to SVRG with large epoch sizes, but outperforms SVRG with recommended epoch sizes, i.e. n and $2n$. It is noting that setting η to be too small is not a practical approach when using SVRG or its variants, because the convergence rate will be extremely low. Therefore, the sub-optimal performance of SMSVRG+ with very small η is acceptable.

VI. CONCLUSION

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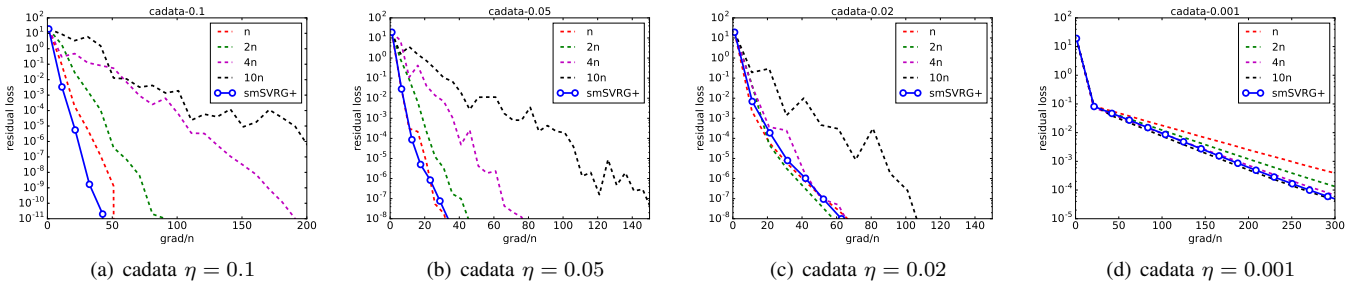| (a) cadata $\eta = 0.1$ | (b) cadata $\eta = 0.05$ | (c) cadata $\eta = 0.02$ | (d) cadata $\eta = 0.001$ |

Fig. 5. Generally, SMSVRG can automatically set an appropriate m with different learning rates for the $l2$-regularized linear regression

REFERENCES

[1] Zeyuan Allen-Zhu and Yang Yuan. Improved SVRG for non-strongly-convex or sum-of-non-convex objectives. In International Conference on Machine Learning, New York, USA, June 2016.

[2] Jonathan Barzilai and Jonathan M. Borwein. Two-point step size gradient methods. IMA Journal of Numerical Analysis, 8(1):141–148, 1988.

[3] Aaron Defazio, Francis Bach, and Simon Lacoste-Julien. Saga: A fast incremental gradient method with support for non-strongly convex composite objectives. In Advances in Neural Information Processing Systems, pages 1646–1654, Montréal, Canda, December 2014.

[4] R Johnson and T Zhang. Accelerating stochastic gradient descent using predictive variance reduction. Advances in Neural Information Processing Systems, pages 315–323, December 2013.

[5] Ritesh Kolte and Murat Erdogdu. Accelerating svrg via second-order information. 2016.

[6] Jakub Konečnỳ, Jie Liu, Peter Richtárik, and Martin Takáč. Mini-batch semi-stochastic gradient descent in the proximal setting. IEEE Journal of Selected Topics in Signal Processing, 10(2):242–255, 2016.

[7] Jakub Konečnỳ and Peter Richtárik. Semi-stochastic gradient descent methods. arXiv preprint arXiv:1312.1666, 2013.

[8] Xingguo Li, Tuo Zhao, Raman Arora, Han Liu, and Jarvis Haupt. Stochastic variance reduced optimization for nonconvex sparse learning. In International Conference on Machine Learning, New York, USA, June 2016.

[9] Mark Schmidt, Nicolas Le Roux, and Francis Bach. Minimizing finite sums with the stochastic average gradient. Mathematical Programming, pages 1–30, 2016.

[10] Vatsal Shah, Megasthenis Asteris, Anastasios Kyrillidis, and Sujay Sanghavi. Trading-off variance and complexity in stochastic gradient descent. arXiv preprint arXiv:1603.06861, 2016.

[11] Shai Shalev-Shwartz. SDCA without duality, regularization, and individual convexity. In International Conference on Machine Learning, New York, USA, June 2016.

[12] Conghui Tan, Shiqian Ma, Yu Hong Dai, and Yuqiu Qian.

Barzilai-borwein step size for stochastic gradient descent. arXiv preprint arXiv:1605.04131, 2016.

[13] Lin Xiao and Tong Zhang. A proximal stochastic gradient method with progressive variance reduction. SIAM Journal on Optimization, 24(4):2057–2075, 2014.

[14] Lijun Zhang, Mehrdad Mahdavi, and Rong Jin. Linear convergence with condition number independent access of full gradients. In Advances in Neural Information Processing Systems, pages 980–988, Lake Tahoe, USA, December 2013.