Background knowledge

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1 Motivation

The motivation of this document is to get a general view of how a biologically-plausible RNN is constructed, and how shall we relax the conditions in a logical and applicable way to form a spectrum from Artificial Neural Network(ANN) to biologically-plausible RNN.

2 Hodgkin -Huxley model

The Hodgkin-Huxley model is very important in Neuroscience, it provides a quantitative model that describes the dynamic of membrane potential and conductance.

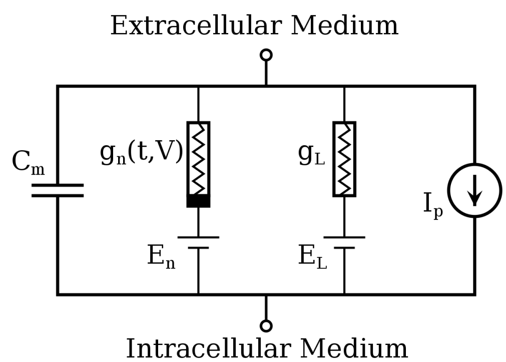

Figure 1: A diagram illustrating Hodgkin-Huxley Model

We can then come up with the following equation.

$$C^i\dot{V}^i(t) = -g_L^i + g_E(t)(\mathcal{E}_E - V^i(t)) + g_I^i(t)(\mathcal{E}_I - V^i(t))$$

g_L : Leak conductance, g_E : Conductance of excitatory gates, g_I : Conductance of inhibitory gates.

Here the upper label i represent the pre-synaptic cell, as the voltage across the membrane depends on the - voltage signal. g_L is the leakage conductance, which is a constant. The reversal potential $\mathcal{E}_E$ and $\mathcal{E}_I$ are also constants which only depends on the type of postsynaptic cell. The other terms are all time dependent, as the model proposed. we let z, the time constant term to be expressed as

$$z(t) = \frac{1}{C^i}(g_L^i + g_E^i + g_I^i) = G_b^i + \frac{g_I(t)}{C^i} + \frac{g_E(t)}{C^i}$$

Then we reorganize to get

$$\dot{V}^i(t) = -z(t)^i V^i(t) + \frac{g_E^i}{C^i}\mathcal{E}_E - \frac{g_I^i}{C^i}\mathcal{E}_i$$

As the conductance of excitatory and inhibitory neurons are both directly proportional to the firing rate of neurons, we can introduce a matrix K that interprets this relationship:

$$\frac{g_E(t)}{C^i} = \sum_{i,j} K_{ij} r_j = \sum_j A_E^i |W_{ij}|^E r_j^E$$

$$\frac{g_I(t)}{C^i} = \sum_{i,j} K_{ij} r_j = \sum_j A_I^i |W_{ij}|^I r_j^I$$

With the scaling matrix A with the following definition:

$$A^E = |W_{ij}^E|^{-1} K_{ij}^E\text{,}\;\; A^I = -|W_{ij}^I|^{-1} K_{ij}^I$$

It is automatically ensured that A is always positive. Then we have this voltage equation of:

$$\dot{\boldsymbol{V}} = -\boldsymbol{z_t} \odot \boldsymbol{V_t} + W\boldsymbol{r_t} + P\boldsymbol{x_t}$$

where

$$\boldsymbol{z_t^i} = G_b^i + A^{ij}|W_{ij}|r_j$$

We want W to reserve the feature of excitatory and inhibitory neuron, so W must satisfy the constraint that:

$$\begin{bmatrix} W_{EE} > 0 & W_{EI} < 0 \\ W_{IE} > 0 & W_{II} < 0 \end{bmatrix}$$

With the left subscript representing presynaptic cell, and the right one representing postsynaptic cell.