Cyclist Model: Optimal Power Distribution during Road Races

Summary

To analyze the relationship between the rider position and power output, we need the power model and the physical model. After that we optimize the distribution of power.

In our fatigue and recovery model, we try to simulate a rider's fatigue state and recovery during one competition along the course. In this model, critical power(CP) is vital because the CP is needed for calculating anaerobic work capacity(AWC) and relation between work done and CP is helpful for recovery model. Therefore, CP is largely used here.

In the maximum power model, we try to generate the maximum power by investigating its relationship with CP and work done by the cyclist.

With these two preparing models, we come to the main models of this problem.

In the physical model, we seek to describe the dynamic movement of the cyclist. Here, we need to analyze the forces on the cyclist and apply the Newton's second law. After that, classical mechanics is used to express the power.

In the optimal control model, we split it into two cases: the model with air resistance and the model without air resistance. The latter one can be generated by the first one with some adjustment. Our goal is to minimize the time needed for the fixed power done by the cyclist in the time trial. Since optimal control is used, more mathematical analysis is required.

We first need some further detailed assumptions in order to simplify the movement of athletes. Then we specify some constraints which comes from the previous models, including velocity, acceleration, power and some other boundary conditions. These formulas contribute to the final model. With the construction of the first case, the second case is the generation of it by adding the air resistance.

As for the course routes, we use functions to approximate the elevation in order to calculate the slope. This is also helpful for the later sensitivity test.

In conclusion, we refer to some background biology power models and use physics to simulate the movement, then optimal control algorithm is applied.

Contents

1 Introduction

1.1 Background

In cycling, the success of one cyclist is influenced by the competition type, the course and the cyclist's personal strengths.

Here we focus on an individual time trial, where riders try to minimize the time needed for the course of fixed length. There are many types of riders such as time trial specialists, climbers, sprinters, rouleurs and puncheurs. Each of them have distinct strengths and power curves. For example, a time trial specialist is a rider who specializes in individual time trial events and a rouleur is a generalist and can do well in races with a wide variety of terrains.

Different riders have different power curves, which is related to the cyclist types and varies between cyclists. A power curve for a rider indicate the relationship of power and maintenance time of that power. Generally, the more power the rider applies, the shorter the maintenance time is. From another aspect, the power curve indicates the maximum power of the rider in a specific period of time, which can be exceeded slightly with a sacrifice of more time for recovery.

The road cycling is a dynamic process, so many previous factors can affect the power output including the previous power output and the accumulated fatigue of the rider.

Therefore, optimizing the distribution of power output along the course is an aim for athletes and their directeur sportif.

1.2 Restatement of the Problem

We are required to build a model to generate the relationship between the rider's instant position and the applied power. And this model should fit all types of cyclists. This model actually indicates the distribution of power along the course, which can be used to help determine the optimal distribution of power in order to best use riders' personal skills.
The tasks required are as follows:

- Build the model
- Generate the model and apply it on various courses
- Test sensitivity of the model by considering weather influence
- Test sensitivity of the model by slightly changing the key time point of changing the applied power
- Extend the model for team trials

In the modelling process, four sub-models are used:

- **Fatigue and recovery model** to indicate the fatigue and recovery state of the athletes muscle
- **Maximum power model**
- **Physical model** to simulate the movement of the athletes in a time trial
- **Optimal control model** to convert to the optimal control problem in mathematics

After that, a guidance of training for cyclists is attached.

2 Assumptions and Justifications

- **No extreme weather conditions will occur during the cycling competition.**
  The competition will be cancelled if extreme weather is going to occur like rain storm, cold wave hailstorm, typhoon and so on.
- **All cycling equipment is considered the same for all riders.**
  Professional cyclists are all well-equipped, so the influence of equipment can be ignored.
- **Bike wear is excluded and any accidents like crashing and falling are excluded.**
  Accidents in competitions can happen but the possibility is relatively small, to simplify the model we can choose the exclude these situations
- **A cyclist is viewed as a mass point.**
  This model is largely related to physics, and viewing a people as a mass point is a typical way to simplify the movement process.
- **Only sharp turns are considered as turns and others are considered to be straight.**
  Here a sharp turn means a turn of angle less than 90 degrees. If the turn is not sharp, there will not be an obvious change of velocity, which is similar to the situation on a straight road.
- **The initial velocity of riders is not zero.**
  At the beginning of the cycling, riders will speed up to a steady state. The period needed for this process is short and can be ignored.
- **No self-effect that athletes on themselves is considered.**
  Athletes can fully exert their abilities and skills regardless of self-effect such as stress since they are professional and well-trained.

3 Notation

Abbreviation	*Description*
P	The power of the athlete
P_{adj}	The power after adjustment
ω	The remaining AWC
ω_{adj}	The remaining AWC of the athlete after adjustment
CP	The critical power
T_{adj}	The duration of the adjustment
F	The driving force by the athlete
F_s	The slope resistance
F_r	The rolling resistance
F_a	The air resistance
m	The mass of the athlete
v	The velocity of the athlete
a	The acceleration
C_d	The aerodynamic drag coefficient
C_r	The coefficient of rolling resistance of the road
A	The frontal area
s	The position in the whole track
y	The elevation of position s
ρ	The density of air
g	The gravitational acceleration
θ	The road slope which is positive for up hill and negative for downfill
T_f	The total time sepnt by the rider
L	The length of the route
P_{max}	The maximum power of athlete
v_{max}	The maximum velocity caused by air resistance
$P_{singular}$	The power defined by model
P^*	The optimal solution for the model

4 Model Theory

4.1 Fatigue and Recovery Model

Fatigue and recovery models of muscles indicate the accumulation and the reduction in muscles fatigue for forceful exertion tasks. For instance, the performance of an athlete of the high-intensity cycling event could be simulated by this model. Modeling the process of muscle fatigue and recovery provides a method to describe the efficiency of the time of work, workload and other parameters on muscle fatigue and recovery.[1]

When the athlete is moving at a power level upon the critical power, the rate of change of recovery energy occurs at a slower rate than expenditure. On the contrary, when the power level is below the critical power, the amount of energy recovered will be less than the area enclosed by CP and the power curve. Therefore, we have the equation

$$P_{adj} = CP - \frac{\omega_{adj}}{T_{adj}} \tag{1}$$

where w denotes the amount of anaerobic work capacity remaining during the exercise, $omega_{adj}$ denotes the recovery energy, P_{adj} denotes the power after lossing or recovering energy, T_{rec} denotes the duration of recovery and CP denotes the critical power which is the maximum power of athletes.

The researchers found that using the Moxy oxygenation sensor in the experiment could measure SmO_2 which is the balence between oxygen delivery and consumption in muscles in percentage.SmO_2 reaches at its maximum when the cyclist is fresh and decreases as the cyclist gets fatigued.[2] And the data collector from this sensor displays that SmO_2 will rise to a certain level regardless of the recovery time. Therefore, we set the model which the recovery rate of ω does not depend the duration of the recovery interval T_{rec}.

Then we can write (1) into the form of an energy consumption and recovery model as

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \begin{cases} -(P - CP) & CP \le P \\ -(P_{adj} - CP) & P < CP \end{cases} \tag{2}$$

We discover that the power after the adjustment have correlation with the power before which can be simulated as a linear relation. Then we could assume that $P_{adj} = aP + b$ where a and b are constants for the model.

4.2 Maximum Power Model

AWC is also related to the instantaneous maximum power at position. Also maximum power, force and cadence are suggested to be related in various studies. So in [2], the author indicates that

$$P_{max}(t) = \alpha\omega(t) + CP \tag{3}$$

where $P_{max}(t)$ is the maximum power, ω is the remaining work capacity and α is the parameter.

$$P = -(\frac{4P_{max}}{\omega_{max}^2})\omega^2 + (\frac{4P_{max}}{\omega_{max}}) \tag{4}$$

where (ω_{max}, P_{max}) denotes extremum point.

Therefore, maximum force and cadence is linearly related:

$$F = -(\frac{8P_{max}}{\omega_{max}^2})\omega + \frac{4P_{max}}{\omega_{max}} \tag{5}$$

Moreover, maximum cadence is influence by the fatigue state:

$$\omega_{max} = \alpha_\omega \omega + \omega_{max,f} \tag{6}$$

where α_ω is a model parameter and $\omega_{max,f}$ is the maximum cadence at AWC.
Finally, P_{max} is:

$$P_{max} = -[\frac{4(\alpha\omega + CP)}{(\alpha_\omega + \omega_{max,f})^2}]\omega^2 + [\frac{4(\alpha\omega + CP)}{(\alpha_\omega + \omega_{max,f})^2}]\omega \tag{7}$$

4.3 Physical Model

We first establishment a dynamics model to describe the movement of the athletes in the time trial course. Firstly, think of the cyclists as a particle moving on the line from start to finish in minimal time. Consider the force on the athletes, but in this case we do not take into account the effects of the weather condition or the air resistance.

- the driving force by the athletes F
- the slope resistance which is the gravity complement along the slope F_s

  $$F_s = mg\sin\theta$$

  where m is the mass of the bicycle and the athlete, g is the gravitational acceleration, θ is the road slope which is positive for up hill and negative for downfill.
- the rolling resistance F_r means that the resistance caused by rolling

  $$F_r = C_r mg\cos\theta$$

  where C_r is the coeffcient of rolling resistance of the road.
- the air resistance F_a under the air and weather situation is given by

  $$F_a = C_d \rho A v(t)^2$$

  where C_d is the aerodynamic drag coeffcient, A is the frontal area, ρ is the density of air which is assumed to be constant.

By Newton's second law, excess force of F minus the resistance will accelerate the particle, or decelerate if the excess is negative, thus we have that

$$F - F_s - F_r - F_a = ma$$

where a is the acceleration of the particle.
And again by definition in classical mechanics we have $P(t) = F(t)v(t)$, where P is the power of the particle, v is the velocity of the particle. If we substitue the expression for F into this equation, we obtain a differential equation

$$P(t) = [mg\sin\theta(t) + C_r mg\cos\theta(t) + m\frac{dv(t)}{dt}]v(t)$$

since acceleration a is the derivative of velocity v with time t.

4.4 Optimal Control Model

So far we can convert to the optimal control problem in mathematicas. The construction of this system needs need more analysis. In order to obtain a minimum time during the time trial course, we need some assumptions as the constraints. Of course, we should determine the objective of the optimal problem.

4.4.1 ITT Model without air resistance

The first model we will establish is a model we do not consider air resistance which is appear in the physical model.

The objective

In the individual time trial , athletes were assigned to the same course, and the athlete with the shortest time will be the champion. Thus our main objective is the time spend by athletes. In mathematicas, the model will use the integral to represented the total time of athletes to finish the individual time trial. Therefore we have that

$$\min_{P(t)} \int_0^{T_f} dt$$

where T_f is the total time spent by the rider.

The assumption

The assumption guarantee the model has the well-defined and such that the optimal problem has solutions. Thus we have assumptions as follows.

- fatigued and recovery
  Through our discussion in the previous, we assumed that the excess power $\frac{d\omega}{dt}$ is limited, and only depend on cyclist's qualities. For simplying the calculation, we also assumed that the athletes do not allow for recovery. Thus we get a constant work done by riders of the integral.

- route length
  Since the length of the route is fixed for each race or experiment, therefore the length of the circuit is a constant. Meanwhile, the trial is not too short such that it is possible to go all out and maintain peak level P_{max} for the entire trial.
- road condition
  The course is not too steep. The critical power level CP is positive such that achieve a positive velocity.
- cyclist
  The cyclist is in shape. The power by the cyclist is sufficiently high to get a velocity that maintained in the critical power.

The constraints

The constraints are come from the previous biomechanical and physical model, in this part we will convert assumptions and some qualities to mathematicas formulae.

- velocity
  By the definition of the velocity v, velocity is the distance moved per unit time. The instant velocity is derivative of the distance x with the time t, so we get

$$\frac{dx}{dt} = v(t)$$

- acceleration
  In the same way, the acceleration is meant the rate of change of velocity with time. On the other hand, by the last section we establish a physical model, and obtain representation of acceleration, which is

$$\begin{aligned}\frac{dv}{dt} &= a(t)\\ &= \frac{P(t)}{mv(t)} - g(\sin(\theta) + C_r\cos(\theta))\end{aligned}$$

- power
  If we denote W as the work done by the cyclist, then power P is the derivative of the work with the time. On the other hand, by previous section we have that

$$\frac{d\omega}{dt} = \begin{cases} -(P(t) - CP) & P \geq CP \\ -((aP(t) + b) - CP) & P \leq CP \end{cases}$$

- boundary conditions of x
  As our discussion in the assumptions, the length of the course is a constant. Thus we have that

$$x(0) = 0, x(T_f) = \int_0^{T_f} v(t)\, dt = L$$

  where L is the length of the route, it is a constant.

- boundary conditions of v
  Since all players remain still until the start of the game, the velocity of rider is 0 at beginning.
  $$v(0) = 0$$

- boundary conditions of ω
  By the assumptions above, we have that the energy of the rider is constant, thus we have that
  $$\omega(0) = 0, \omega(T_f) = \omega_f$$
  where W_f is the total energy of the athlete.

Finally, our problem can be convert into a optimal control problem that can be solved by Pontryagin's maximum principle. Thus the objective function which is

$$\min_{P(t)} \int_0^{T_f} dt$$

subject to

$$\begin{cases} \dfrac{dx}{dt} = v(t) \\ \dfrac{dv}{dt} = \dfrac{P(t)}{mv(t)} - g(\sin(\theta) + C_r \cos(\theta)) \\ \dfrac{d\omega}{dt} = \begin{cases} -(P(t) - CP) & P \geq CP \\ -((aP(t) + b) - CP) & P \leq CP \end{cases} \end{cases}$$

with boundary conditions

$$\begin{aligned} x(0) &= 0, x(T_f) = L \\ v(0) &= 0, \\ \omega(0) &= 0, \omega(T_f) = \omega_f \end{aligned}$$

4.4.2 ITT model with air resistance

Consider the normal situation which mentioned in the physical model. Recall that the formulae

$$F - F_s - F_r - F_a = ma$$

then the constraints in the model becomes

$$\frac{dv}{dt} = \frac{P(t)}{mv(t)} - g(\sin(\theta) + C_r \cos(\theta)) - \frac{1}{m} C_d \rho A v(t)^2$$

which is a first order ordinary differential equation, it is easy to solve. Furthermore, we can find that v_{max} can be attained by solve

$$\frac{dv}{dt} = 0$$

Therefore, we know that whatever the initial velocity is, it must reach a constant maximum velocity after acceleration. Then this model has a additional constraints

$$v(t) \leq v_{max}$$

And the optimal control problem becomes

$$\min_{P(t)} \int_0^{T_f} dt$$

subject to

$$\begin{cases} \dfrac{dx}{dt} = v(t) \\ \dfrac{dv}{dt} = \dfrac{P(t)}{mv(t)} - g(\sin(\theta) + C_r \cos(\theta)) \\ \dfrac{d\omega}{dt} = \begin{cases} -(P(t) - CP) & P \geq CP \\ -((aP(t) + b) - CP) & P \leq CP \end{cases} \end{cases}$$

with

$$\begin{aligned} 0 \leq & v(t) \leq v_{max} \\ 0 \leq & \omega(t) \leq AWC \\ 0 \leq & P(t) \leq P_{max} \end{aligned}$$

where AWC, P_{max} are analyzed in the previous section.

5 Model Implementation and Results

5.1 Solution for Optimal Problem

We can now apply the Pontryagin's minimum principle to obatin the solution for the optimal control system in the 1.2.2. Since the model in the 1.2.1 is a simplying of the model 1.2.2. Here is an introduction to the Pontragin's minimum principle from "Mathworld"

Theorem 1. *Define*

$$H(\psi, x, u) = (\psi, f(x, u)) = \sum_{a=0}^{n} \psi_a f^a(x, u).$$

Then in order for a control $u(t)$ and a trajectory $x(t)$ to be optimal, it is necessary that there exist nonzero absolutely continuous vector function $\psi(t) = (\psi_0(t), \psi_1(t), ..., \psi_n(t))$ corresponding to the functions $u(t)$ and $x(t)$ such that
1. The function $H(\psi(t), x(t), u)$ attains its maximum at the point $u = u(t)$ almost everywhere in the interval $t_0 <= t <= t_1$,

$$H(\psi(t), x(t), u(t)) = \max_{u \in U} H(\psi(t), x(t), u).$$

2. At the terminal time t_1, the relations $\psi_0(t_1) <= 0$ and $H(\psi(t_1), x(t_1), u(t_1)) = 0$ are satisfied.

The proof of this theorem can be find from the book of control theory, thus we do not proof here.
Then our optimal control system convert into a Hamiltonian function

$$H(\psi(t), v(t), P(t)) = v(t) + \psi_1(t)v(t) + \psi_2(t)\left(\frac{P(t)}{mv(t)} - \frac{C_d \rho A v(t)^2}{m} - g(\sin\theta + C_r \cos\theta)\right) + \psi_3(t)(P(t) - CP)$$

We can denote $h(s)$ as the $g(\sin\theta + C_r \cos\theta)$. According to [wenxian1], he solved this problem with answer

$$P^* = P_{max} \qquad or \qquad P_{singular} \qquad or \qquad CP \qquad or \qquad 0$$

where $P_{singular}$ is calculated by

$$\frac{\psi_2}{v(t)} = -\frac{\psi_3}{m}$$

noitce that ψ_3 are constants. Finally [2] give a formulae to $P_{singular}$

$$P_{singular} = mv(h(s) + \frac{1}{2m}(C_d \rho A)v^2)$$

Thus we see that P^* must be one of these 4 types. But we do not know how many times when the switching between these types. According to [3], we got a theorem to solve this question.

Theorem 2. *In an optimal pacing strategy, the rider switches back in power.*

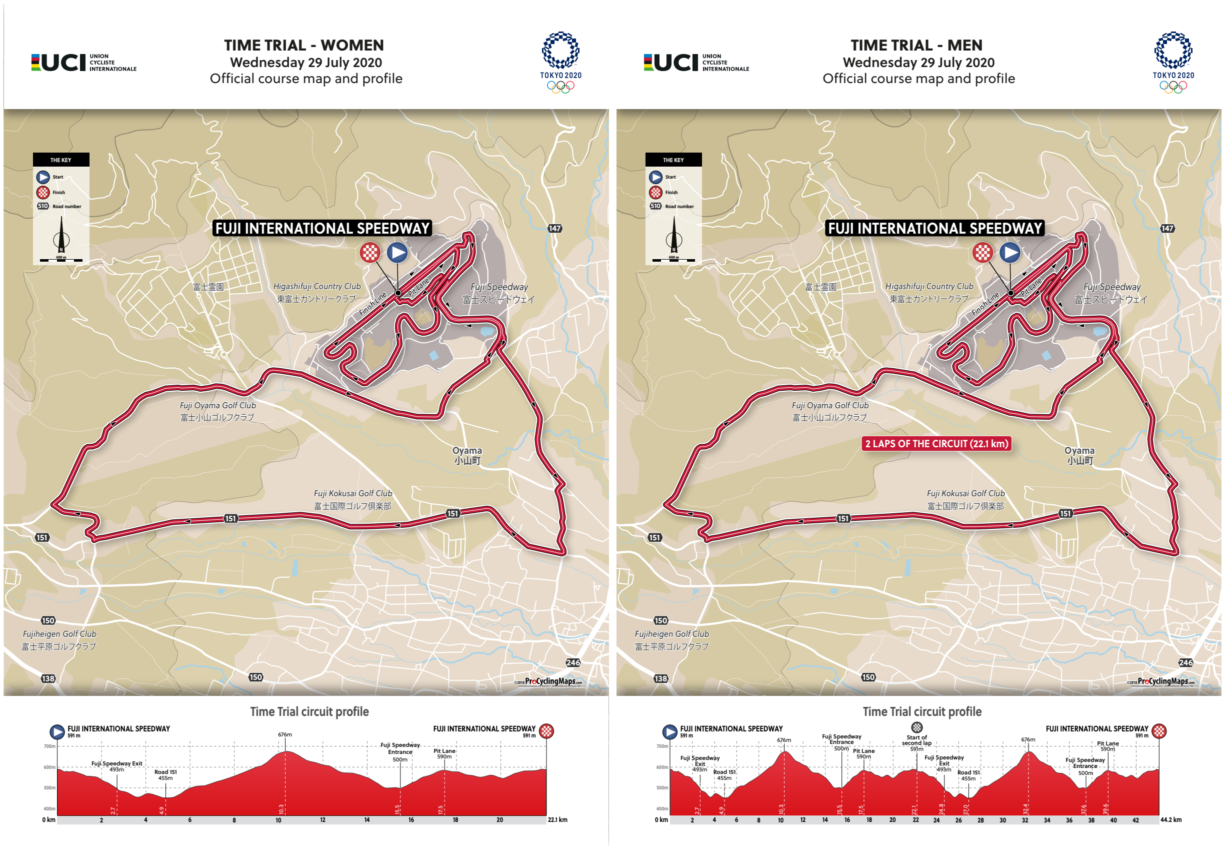
The proof of the theorem can be found in [3]. Thus the cyclist must apply P_{max} and then apply $P_{singular}$. Therefore, the power switches only once. In the other words, switches must the less frequent the better.

5.2 Applications and Results

5.2.1 2021 Olympic Time Trial course in Tokyo, Japan

According to the lack of dataset of the athlete in the Olympicswe use the experimentally determined parameters for the six objects[2] and the data sets from the Cycling Road Results Book to obtain the data we could have. In the 2021(originally 2020) Olympic Time trial course in Tokyo, the 22.1km time trial is with the following parameters: initial velocity v=16m/s[4] for men and the initial velocity v=12m/s for women, the critical power CP=400W for men and the critical power CP=300W for women. The starting and ending of the track is almost at the same place so we assume the energy the athlete consume in climbing will be recovered in the downhill road. The man cyclist and cycle weight 90kg in total(The man athlete weights 80 kilograms and the bicycle weight 10 kilograms).The woman cyclist and cycle weight 75kg in total(The athlete weights 65 kilograms and the bicycle weight 10 kilograms).The rolling fraction coefficient is suggested in the interval $[0.018, 0.020]$ so we pick$C_r = 0.019$ And the $C_d A$ is equal to 0.32 according to recommendation of Wilson and Papadopoulus[5].

These results are similar to the average power of a data from a professional road cycling athlete.



Here we use points to find a function to simulate the topographic relief: Then we have the function as follows:

$$y = \begin{cases} (s-5.2)^2 + 555 & 0 \le s \le 10.3 \\ (s-14.2)^2 + 660 & 10.3 < s \le 17.5 \\ 590 & 17.5 < s \le 22.1 \end{cases} \tag{8}$$

$$\frac{\mathrm{d}y}{\mathrm{d}s} = \begin{cases} 2(s-5.2) & 0 \le s \le 10.3 \\ 2(s-14.2) & 10.3 < s \le 17.5 \\ 0 & 17.5 < s \le 22.1 \end{cases} \tag{9}$$

where y denotes the elevation height at position s.And the derivative of this equation $\frac{\mathrm{d}y}{\mathrm{d}s}$ gives the value of the slope. We have $\frac{\mathrm{d}y}{\mathrm{d}s} = tan(\theta)$. Then:

$$\theta = arctan(\frac{\mathrm{d}y}{\mathrm{d}s}) = \begin{cases} arctan(2(s-5.2)) & 0 \le s \le 10.3 \\ arctan(2(s-14.2)) & 10.3 < s \le 17.5 \\ 0 & 17.5 < s \le 22.1 \end{cases} \tag{10}$$

Then we use the P_{man} to denote the $P_{singular}$ for man:

$$P_{man} = mv(h(s) + 1/2m(C_d A)v^2)$$

and also the P_{woman} denotes the $P_{singular}$ for women:

$$P_{woman} = mv(h(s) + 1/2m(C_d A)v^2)$$

where each parameter has it unique value and $h(s) = g(\sin\theta + C_r \cos\theta)$ which is a continue function about θ. If we substitute the position s in the equation, we can get the value P_{man} or P_{woman}.

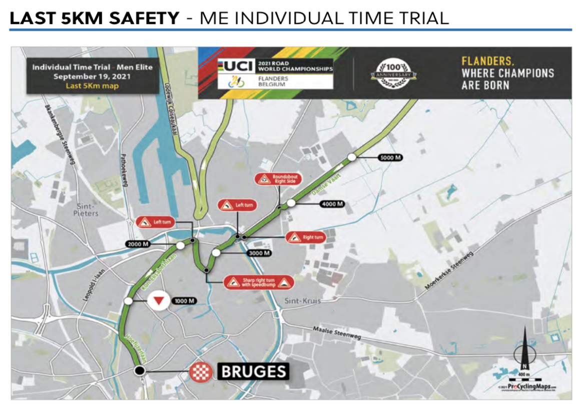
5.2.2 2021 UCI World Championship time trial course in Flanders, Belgium

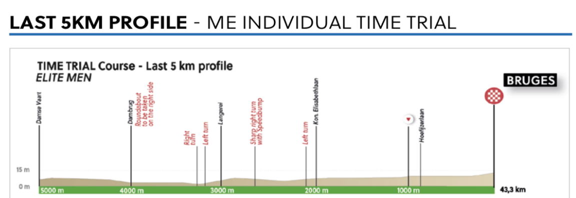
The parameters in this 43.3km track are as follows: initial velocity v=16m/s for men and the initial velocity v=12m/s for women. We find that the track is very smooth from observing the elevation profile. Although there are some uphills and downhills, along these long race road, we can consider this track as flat. So the $h(s)$=0. The man cyclist and cycle weight 90kg in total(The man athlete weights 80 kilograms and the bicycle weight 10 kilograms).The woman cyclist and cycle weight 75kg in total(The athlete weights 65 kilograms and the bicycle weight 10 kilograms). And the $C_dA = 0.32$.Then we have the P_{man} to denote the $P_{singular}$ for man:

$$P_{man} = mv(h(s) + 1/2m(C_dA)v^2) = 803.34$$

and also the P_{women} denotes the $P_{singular}$ for women:

$$P_{women} = mv(h(s) + 1/2m(C_dA)v^2) = 338.91$$





5.2.3 Our design

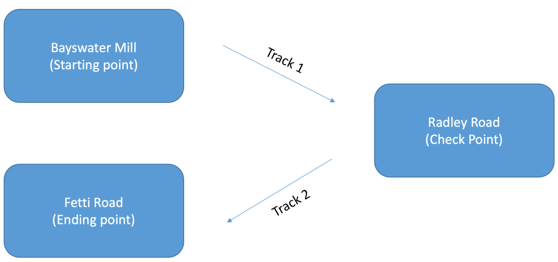
Cycling routes designing process is as follows:

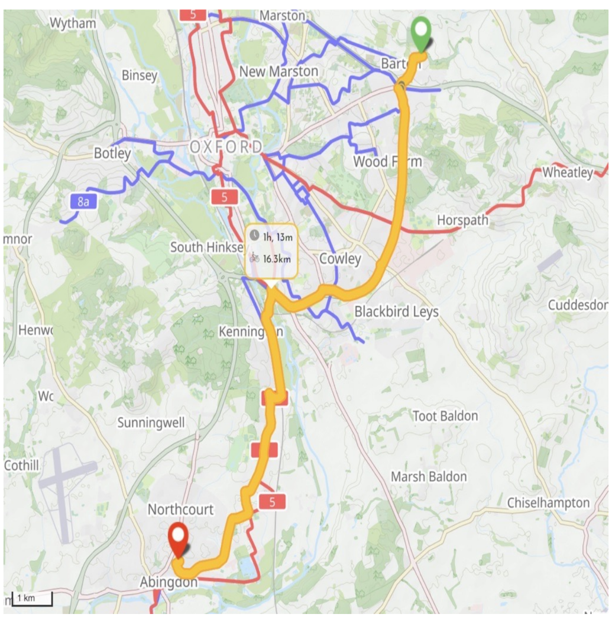
- Try to find a plain city with normal elevation of, and Oxford, UK is ideal.

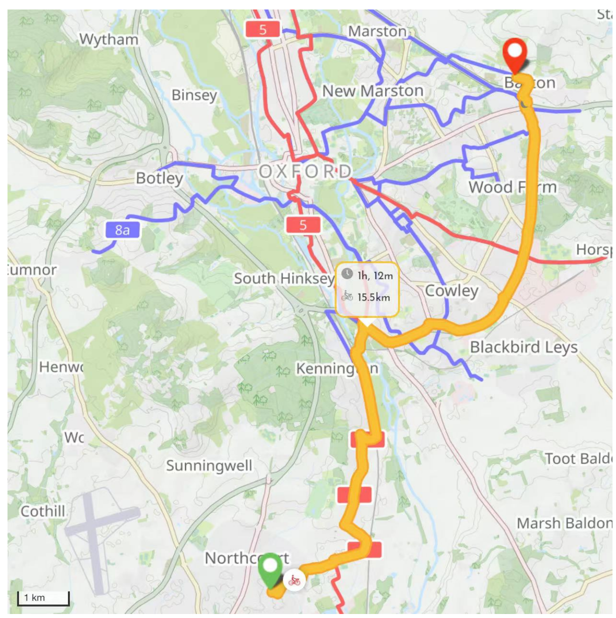
- Pick starting and ending points. We need starting and ending points to be close, so we split it by two routes, and adding an intermediate check point. Cyclists need to go from the starting along track 1 to the check point and cycle from the check point to the ending along track 2:

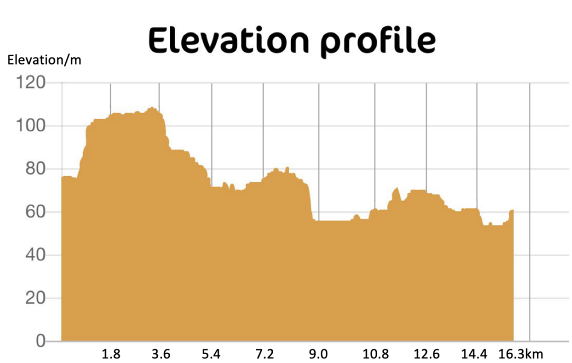
- The total distance is approximately 16.3+15.5=31.8 km. Both tracks contain sharp turns.

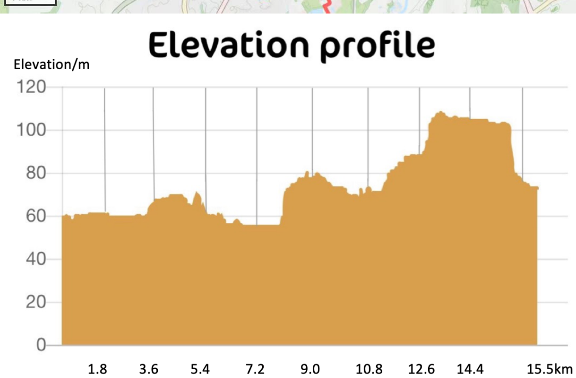
- Each track map indicates the starting with a green mark and ending with a red mark. Corresponding elevation profiles are attached.











The parameters in this 31.8km track, initial velocity v=12m/s for men and the initial velocity v=10m/s for women, The man cyclist and cycle weight 90kg in total(The man athlete weights 80 kilograms and the bicycle weight 10 kilograms).The woman cyclist and cycle weight 75kg in total(The athlete weights 65 kilograms and the cycle weight 10 kilograms).

Here we use points to find a function to simulate the topographic relief. Obviously, the two elevation profile are almost symmetric, so we can just calculate the first one .Then we have the function as follows:

$$y = \begin{cases} 24x + 78 & 0 \le s \le 1 \\ 102 & 1 < s \le 3.6 \\ -17.8x + 166 & 3.6 < s \le 5.4 \\ 16.3 & 5.4 < s \le 16.8 \end{cases} \tag{11}$$

$$\frac{dy}{ds} = \begin{cases} 24 & 0 \le s \le 1 \\ 0 & 1 < s \le 3.6 \\ -17.8 & 3.6 < s \le 5.4 \\ 0 & 5.4 < s \le 16.8 \end{cases} \tag{12}$$

where y denotes the elevation height at position s.And the derivative of this equation$\frac{dy}{ds}$ gives the value of the slope. We have $\frac{dy}{ds} = tan(\theta)$. Then:

$$\theta = arctan(\frac{dy}{ds}) = \begin{cases} arctan(24) & 0 \le s \le 1 \\ 0 & 1 < s \le 3.6 \\ -arctan(17.8) & 3.6 < s \le 5.4 \\ 0 & 5.4 < s \le 16.8 \end{cases} \tag{13}$$

Then we use the P_{man} to denote the $P_{singular}$ for man:

$$P_{man} = mv(h(s) + 1/2m(C_d A)v^2)$$

and also the P_{woman} denotes the $P_{singular}$ for women:

$$P_{woman} = mv(h(s) + 1/2m(C_d A)v^2)$$

where each parameter has it unique value and $h(s) = g(\sin\theta + C_r \cos\theta)$ which is a constant. Then we can get the value P_{man} or P_{woman}.

6 Sensitivity Test

In our model, we introduce comprehensive formulas, which are effected by various parameters such as the resistance of the air, the mass of the athletes, the velocity of the athletes and the elevation(the position of the athlete in the entire competition). It is necessary for us to test their sensitivity.

We firstly analyse the probably unstable factors. For example, on account of different physical conditions and different weight of equipment, the total weight m of athlete and its equipment have diverse value. Also, the velocity of each athlete could cause the changes of $P_{singular}$.Therefore, we define a range for our model:(The unit for mass is kg and the unit for the velocity is m/s)

$$m \in [65, 95], v \in [9, 17]$$

And we test the sensitivity of these intervals.The results shows that the value in them provides a power close to reality. The bias in these two interval and small bias in the different coefficient are exceptable.

7 Further Discussion

Team time trial is different from the individual time trial, the game played by six athletes and finish at the forth athlete. Thus our final time is depend on the forth rider. In the other words, the objective of the model become T_4

$$\min T_4$$

Supposed that the final time of six riders is sequence $T_1, T_2, T_3, T_4, T_5, T_6$ with $T_1 \le T_2 \le T_3 \le T_4 \le T_5 \le T_6$.
We assumed that each rider in the Team Time Trial as a rider in our Individual Time Trial. Thus we have that

$$\min T_4 \tag{14}$$

subject to

$$\begin{cases} \dfrac{dx}{dt} = v(t) \\ \dfrac{dv}{dt} = \dfrac{P(t)}{mv(t)} - g(\sin(\theta) + C_r \cos(\theta)) \\ \dfrac{d\omega}{dt} = \begin{cases} -(P(t) - CP) & P \ge CP \\ -((aP(t) + b) - CP) & P \le CP \end{cases} \end{cases}$$

with

$$\begin{aligned} 0 \le & v(t) \le v_{max} \\ 0 \le & \omega(t) \le AWC \\ 0 \le & P(t) \le P_{max} \end{aligned}$$

where AWC, P_{max} are analyzed in the previous section.
Notice that in our physical model, we considered the air resistance by the wind. Thus we can distribution the position of these six cyclists to reduce the air resistance for a better result. We can complete our discussion by fatigue recovery model. Roughly speaking, the first athlete to start the race apply the P_{max} and the other fives should apply less power to follow the leader. The rider at the end of the team needs to accelerate to the head of team to lead team when the leader deplete the AWC, and then the old leader apply the CP to follow in the end of the team to regain strength. Repeat the above description until the end of the race.
In addition, we notice that only the final time is valid which is of the fourth athlete to finish the race, thus at the end of the course, the fifth and sixth athlete in the team who finish the course should ideally return to the end of the team to rest.

8 Conclusion

8.1 Strengths and Weaknesses

8.1.1 Strengths

- **Steadiness**
  Our model can fit a wild range of parameters, which is justified by the two sensitivity tests.

- **Flexibility**
  Our model can fit all types of riders with different skills. Also, we can apply the model to various official courses to simulate the power distribution of the riders. Therefore, it can help to guide the cyclists for many competitions.

- **Authoritative competition data**
  Our map and course data are extracted from official websites such as Union Cycliste Internationale(UCI) and Olympics, which are reliable.

- **Reduced Calculation**
  In our model, the use PMP is vital for simplifying the calculation process, which will be otherwise time-consuming. The solution method of optimal controlling also helps with the reducing of calculation.

8.1.2 Weaknesses

- **Self-effect**
  We assume that athletes can fully exert their abilities and skills in different competitions and therefore we ignore the self-effect that athletes on themselves, such as health conditions, body temperature and inner energy change.

- **Supply**
  In real-world competitions, supply of water is necessary and is often delivered by supplying cars or supply stations. The location distribution of supply stations can influence the power distribution and velocity. And supply cars can affect the concentration of athletes who do not need supply.

- **Limited data size**
  Since the data for athletes Critical Power is private information, the data collection is difficult. To ensure the authenticity, the size of the dataset is limited. For this reason, the final graphs can be underfitting.

- **Weather effect**
  In our model, we only consider the simple impact of wind. In real competitions, more complex and combined weather conditions can occur. For instance, courses on rainy days are harder to simulate and rain can also influence visibility and velocity.

8.2 Guidance

8.2.1 General Training Suggestions

- **Training with bigger gear ratio**
  Riding with a larger gear ratio at the same tread frequency allows for a stable higher power output. In normal riding, you can gradually improve the gear ratio. For instance, if you want to ride a local 5km mountain trail using a 34x19 gear ratio, you can increase the gear ratio to 34x17 for 3 minutes and we assume you use a 50/34 gear and 11/28 flywheel). Next time, you can ride 34x17 for 4 minutes, then 5 minutes. Until you can ride it all the way on the same pedal with a 34x17 gear ratio. This type of training will significantly increase your power which fits the idea of our model.

- **Climbing training**
  Climbing is a great way to build muscle endurance and improve critical power,allowing you to ride for longer periods of time with a relatively high tooth ratio and the right pace. It works because climbing a hill lowers the pedal frequency and increases the force of the pedal. One way to significantly increase your power output is to gradually increase the distance you climb. For example, start climbing 400 meters and slowly increase your height until you can climb 900 meters at a time. Another way is to do short, intense interval training on steep hills. Sprints should be at least 60-90 seconds, with a simple downhill recovery followed by a sprint uphill, 12 repetitions per session.

- **Wind training**
  In our models, we consider the air resistance, so cycling against the wind will be helpful. Since the wind force matters, different levels of wind can be helpful. From another aspect, the wind in the lab or gym can be robust, so cycling in the nature wind can help cyclists get used to wind changing in the real competitions. Also, cycling against the wind is beneficial for improving critical power.

- **Block training**
  Block training consists of two or three days of intense training followed by an equal amount of recovery (rest days or very light recovery). Because your muscles and cardiovascular system are under heavy strain, block training is an effective way to promote physiological adaptation and significantly boost power. The key is to make sure you give your body enough time to recover. For example, you can create a 4-day training block that includes uphill intervals (day 1), sprint intervals (day 2), rest days (day 3), and an easy ride (day 4).

8.2.2 Specific Suggestions

- **Step 1- Collect data for athletes such as CP**
  Collect the critical power data for the athletes and record it.

- **Step2- Apply Power-based training and simulation**
  Use the critical power as a measurement to determine the corresponding training level for distinct riders.

- **Step3- Determine strengths and weakness with road cycling simulation**
  Hold road cycling simulation competition and apply the model for the power profiles of the cyclists. With power profiles, we can determine the strengths and weaknesses of the riders and also the types of the riders.

- **Step4- Train according strengths and record data**
  We can group cyclists by types and do distinct training for distinct groups. Meanwhile, training for critical power is necessary and data should be recorded.

- **Step5- Repeat the steps**
  With the updated data for cyclists, we can see the progress and decide whether to change the training levels and whether the training methods are suitable. After the analysis, we can repeat the above steps.

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