Lesson 4

Differential Application

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4.1.1

Explain the difference between an absolute minimum and a local minimum.

In mathematics, a minimum is a value of a function that is smaller than or equal to all other values of the function within a certain domain. There are two types of minimums: absolute minimums and local minimums.

An absolute minimum is the smallest value that a function can take over its entire domain. It is the global minimum of the function and is unique, meaning there is only one absolute minimum for a given function.

A local minimum, on the other hand, is a value of a function that is smaller than its nearby values but may not be the smallest value over the entire domain. A local minimum can occur at multiple points in the domain, and there may be other points in the domain where the function takes on smaller values.

4.1.9

Sketch the graph of a function f that is continous on $[1, 5]$ and has the given properties

1. Absolute minimum at 3
2. absolute maximum at 4
3. local maximum at 2

60. Find the absolute maximum and absolute minimu values of f on the given interval.

$$f(x) = \frac{e^x}{1+x^2}\ ,\ [0, 3]$$

$$\begin{aligned}
\frac{df(x)}{dx} &= \frac{d}{dx}\left(\frac{e^x}{1+x^2}\right) \\
&= \frac{d}{dx}\left((e^x)(1+x^2)^{-1}\right) \\
&= \frac{d}{dx}\left(-(e^x)(1+x^2)^{-2} + (1+x^2)^{-1}(e^x)\right) \\
&= -(e^x)(1+x^2)^{-2}\frac{d}{dx}(1+x^2) + (1+x^2)^{-1}(e^x) \\
f^{'}(x) &= \frac{2xe^x}{(1+x^2)^2} - \frac{e^x}{1+x^2}
\end{aligned}$$

find crtical value at $f^{'}(x) = 0$

$$\begin{aligned}
f^{'}(x) &= \frac{2xe^x}{(1+x^2)^2} - \frac{e^x}{1+x^2} = 0 \\
\frac{\cancel{e^x}}{1+x^2} &= \frac{2x\cancel{e^x}}{(1+x^2)^2} \\
\frac{\cancel{e^x}}{\cancel{1+x^2}} &= \frac{2x\cancel{e^x}}{(1+x^2)^{\cancel{2}}} \\
1+x^2 &= 2x \\
x^2 - 2x + 1 &= 0 \\
(x+2)(x-1) &= 0 \\
x &= -2,\ 1
\end{aligned}$$

At $x = -2$, 1 causes critical values
Now we need to check all values :

1. x = -2
2. x = 1
3. x = 0 (left bound)
4. x = 3 (right bound)

$$f(x) = \frac{e^x}{(1+x^2)}$$

$$\begin{aligned}
f(-2) &= \frac{e^{-2}}{1+(-2)^2} = 0.02706 \\
f(1) &= \frac{e^1}{1+(1)^2} = 1.3591 \\
f(0) &= \frac{e^0}{1+(0)^2} = 1.0 \\
f(3) &= \frac{e^3}{1+(3)^2} = 2.0085
\end{aligned}$$

So the **absolute minimum** is happend when $x = -2$
So the **absolute maximum** is happend when $x = 3$

4.2.25 Show that the equation $x^3 - 15x + c = 0$ has at most one solution in the interval $[-2, 2]$