# Lesson 4

Differential Application

Apisit Thaweboon 65050988

## 4.1.1

**Explain the difference between an absolute minimum and a local minimum.**

In mathematics, a minimum is a value of a function that is smaller than or equal to all other values of the function within a certain domain. There are two types of minimums: absolute minimums and local minimums.

An absolute minimum is the smallest value that a function can take over its entire domain. It is the global minimum of the function and is unique, meaning there is only one absolute minimum for a given function.

A local minimum, on the other hand, is a value of a function that is smaller than its nearby values but may not be the smallest value over the entire domain. A local minimum can occur at multiple points in the domain, and there may be other points in the domain where the function takes on smaller values.

## 4.1.9

Sketch the graph of a function $f$ that is continous on $[1, 5]$ and has the given properties

1. Absolute minimum at 3
2. absolute maximum at 4
3. local maximum at 2

## 60. Find the absolute maximum and absolute minimu values of $f$ on the given interval.

$$f(x) = \frac{e^x}{1+x^2}\ ,\ [0, 3]$$

$$
\begin{aligned}
\frac{df(x)}{dx} &= \frac{d}{dx}\left(\frac{e^x}{1+x^2}\right) \\
&= \frac{d}{dx}\left((e^x)(1+x^2)^{-1}\right) \\
&= \frac{d}{dx}\left(-(e^x)(1+x^2)^{-2} + (1+x^2)^{-1}(e^x)\right) \\
&= -(e^x)(1+x^2)^{-2}\frac{d}{dx}(1+x^2) + (1+x^2)^{-1}(e^x) \\
f^{'}(x) &= \frac{2xe^x}{(1+x^2)^2} - \frac{e^x}{1+x^2}
\end{aligned}
$$

find crtical value at $f^{'}(x) = 0$

$$
\begin{aligned}
f^{'}(x) &= \frac{2xe^x}{(1+x^2)^2} - \frac{e^x}{1+x^2} = 0 \\
\frac{\cancel{e^x}}{1+x^2} &= \frac{2x\cancel{e^x}}{(1+x^2)^2} \\
\frac{\cancel{e^x}}{\cancel{1+x^2}} &= \frac{2x\cancel{e^x}}{(1+x^2)^{\cancel{2}1}} \\
1+x^2 &= 2x \\
x^2 - 2x + 1 &= 0 \\
(x+2)(x-1) &= 0 \\
x &= -2,\ 1
\end{aligned}
$$

At $x = -2$, 1 causes critical values
Now we need to check all values :

1. x = -2
2. x = 1
3. x = 0 (left bound)
4. x = 3 (right bound)

$$
f(x) = \frac{e^x}{(1+x^2)}
$$

$$
\begin{aligned}
f(-2) &= \frac{e^{-2}}{1+(-2)^2} = 0.02706 \\
f(1) &= \frac{e^1}{1+(1)^2} = 1.3591 \\
f(0) &= \frac{e^0}{1+(0)^2} = 1.0 \\
f(3) &= \frac{e^3}{1+(3)^2} = 2.0085
\end{aligned}
$$

So the **absolute minimum** is happend when $x = -2$
So the **absolute maximum** is happend when $x = 3$

### 4.2.25 Show that the equation $x^3 - 15x + c = 0$ has at most one solution in the interval $[-2, 2]$

**Use Rolle's theorm and Contradiction proof**

Let $y = x^3 - 15x + c$

By IVT, theres exists at least one real root in $x \in (-2, 2)$ such that y(x) = 0

Use method of contradiction.

$$\forall a, b \Big[ y(x_1) = 0 \ \wedge \ y(x_2) = 0 \ \wedge \ a \neq b \Big]$$

Since y is diffentialble, by Rolle's theorem, thres exists a number $a \in (x_1, x_2)$ such that $y^{'}(a) = 0$ However,

Solve inequation
y$^{'} > 0$
$3x^2 - 15 > 0$
$3(x^2 - 5) > 0$
$x^2 - 5 > 0$
$(x - \sqrt{5})(x + \sqrt{5}) > 0$

$\sqrt{5} \approx 2.2 \quad$ then
$\in (-2.2, 2.2)$

so we get that $y^{'}$ is incresing on $[-2, 2]$ then
y$^{'} \neq 0 \quad$ in this closed interval

So all of these reason we conclude that **y has only one solution on that interval** $\square$

### 4.3.3 Suppose you are given a formula for a function f.

#### A : How do you detemine where $f$ is increasing or decreasing ?

Find $\frac{d}{dx} f(x)$ then see if $f^{'}$ ¿ 0 or ¡ 0

### B : How do you determine where the graph of $f$ is concave upward or concave downward

Find $\frac{d}{dx}\frac{d}{dx}f(x) or f^{''}$ then see if $f^{''}$ ¿ 0 or ¡ 0

### C : How do you locate inflection points ?

(a) Find $f^{'}(x) Solve for \mathrm{f}^{'}(x) = 0$ -¿ get $x_i$

(b) recieved $x_i$ are inflection points corresponding to $x = c$

#### 4.3.4

##### A : State the First Derivative test

The fist derivative test is **the process of analyzing functions using theris first derivatives in order to find their extremum point**

##### A : State the Second Derivative Test. Under what circumstances is it inconclusive? What do you do if it fails.

The second derivative test stats that if $f(x) has a stattionary point when$ x $= c and if \mathrm{f}^{''}(c) = 0$ then **the test is inconclusive** and we should study the sign of the first derivative

#### 4.3.42 The graph of function $y = f(x)$ is shown. At which point(s) are the following true?

Analysis :

- from graph y is on above of x-axis so $y > 0$
- $\frac{dy}{dx}$

  at $(x, y) = B$ $\frac{dy}{dx} > 0$
  at $(x, y) = A, E$ $\frac{dy}{dx} < 0$
  at $(x, y) = C$ $\frac{dy}{dx} = 0$

##### A

$\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ are both positive Ans : B, C

##### B

$\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ are both negative Ans : A, E

**C**

$\frac{dy}{dx}$ is negative but $\frac{d^2y}{dx^2}$ is positive Ans : D

## 4.4.10 $\lim_{x \to -2} \frac{x^3+8}{x+2}$

let

$$\begin{aligned} y &= \frac{x^3+8}{x+2} \\ &= \frac{x^3+2^3}{x+2} \\ &= \frac{(x+2)(x^2-2x+4)}{x+2} \\ &= (x^2-2x+4) \end{aligned}$$

$$\begin{aligned} \lim_{x \to -2} \tfrac{x^3+8}{x+2} &= (-2)^2 - 2(-2) + 4 \\ &= 4+4+4 \\ &= 12 \end{aligned}$$

## 4.4.15 $\lim_{t \to 0} \frac{e^{2t}-1}{\sin t}$

Use L'Hospital

$$\begin{aligned} LHS &= \frac{e^{2t}(2) - 0}{-\sin t} \\ \lim_{t \to 0} \tfrac{e^{2t}-1}{\sin t} &= \frac{2e^{2*0}}{-\sin 0} \\ &= 2/1 \\ &= 2 \end{aligned}$$

## 4.4.75 Prove that $\lim_{x \to \infty} \frac{e^x}{x^n} = \infty$ for any positve interget $n$. This shows that the exponential function approaces infinity faster tahn any powe rof $x$.

if we give $x_1 = 1$ then $f^{'}(x_1) = f^{'}(1) = 3(1)^2 - 3 = 0$ or we cannot find $x_2$ by using newton's method because it will divides by 0 causes a crucial singularity
Hence Newton's method can' use to find root of this equation