Chromatic Dispersion Compensation (February 2017)

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***Abstract*—Chromatic Dispersion happens when pulse transmitted through optical fibers, which lead to a spreading of pulse.We compensate our proposed signal sequence which is modulated by quadrature Phase Shift Keying(QPSK), using the method of compensation fiber and FIR(Finite Impulse Response) filter by applying two methods—dispersion compensating fiber (DCF) and optical filter.**

***Index Terms*—Fiber-optic communication system, Chromatic Dispersion, material dispersion, Compensation.**

I. Introduction

CHROMATIC dispersion is considered as a key limitation of the promotion of optical communication system. The method to compensate chromatic dispersion is raised since last century. One of the widely used method is dispersion compensating fibers. However, due to the high cost and losses, dispersion compensating fibers has been replaced after the advent of optical filters.

This paper consists of 7 parts, which discusses methods to mitigate the Chromatic Dispersion (CD) in Fiber-optic communication systems caused by material dispersion. Two methods-using Dispersion Compensating Fibers (DCF) and building optical equalizing filters, will be applied to compensate CD in this article. Then a brief numerical discussion is made on the implement of compensation optical filter.

II. Signal Model

The signal model used by us will be discussed first in this section. The DQPSK method is used to modulate the signal, and the Raised-cosine filter is applied to minimize intersymbol interference (ISI).

A. DQPSK Modulation

Before the transmission in the fiber, we apply Quadrature Phase Shift Keying(QPSK) method to modulate the proposed digital signal. QPSK is a kind of Phase Shift Keying(PSK), which use for different carrier signal phase to encode two bits by once. In order to avoid phase ambiguity problems at the receiving end, we prefer Differential Quadrature Phase Shift Keying(DQPSK), as the modulation method. We assume $\{x_1, x_2, ..., x_n\}$, is the proposed sequence, each x_k is two bits of the signal. Assume $\{E_1, E_2, ..., E_n\}$ is signal after modulation. Then the modulation shift a_k for $\arg\{E_{k-1}\}$ to get E_k .Then the modulation method can be describe as

$$E_0 = x_0 \tag{1}$$

$$E_k = E_{k-1} \cdot x_k \tag{2}$$

The relationship between E_{k-1} , a_k , E_k is tabled below.

TABLE1

RELATIONSHIP BETWEEN E_{k-1}, a_k, E_k

E_{k-1} \ a_k (E_k)	00	01	11	10
00	00	01	11	10
01	01	11	10	00
11	11	10	00	01
10	10	00	01	11

Fig. 1. shows the constellation diagram of the signal after DQPSK.

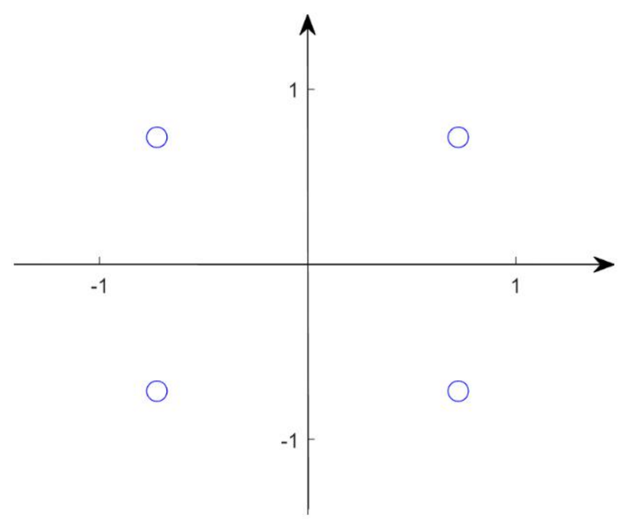

Fig. 1. the constellation diagram of the signal after DQPSK

III. Chromatic Dispersion

A. Chromatic Dispersion

Chromatic dispersion(CD) appears in the transmission of signal in fiber-optic communication system, which is always caused by a combination of waveguide and medium dispersion.[1] Fortunately, Waveguide dispersion can be controlled by careful design.[2] Hence, we only discuss the compensation of CD caused by medium dispersion which is also known as Group Velocity in this article.

CD caused by medium dispersion can be discussed by treating fibers as a linear optical medium.[3] The propagation of an optical pulse in a linear material can be appropriately described in terms of the propagation of a superposition of plane waves with different frequencies, for the Maxwell' s equations are linear.[4] If the medium is dispersion, the phase velocity depends on the frequency. The different phase velocity of different wave components causes the chromatic dispersion.

The propagation constant in the fiber is given as follows.

$$\beta(\omega) = n(\omega)\frac{\omega}{c} \tag{4}$$

where $n(\omega)$ is the fiber reflection constant, ω is the angular frequency, and c is the speed of light. And $\beta(\omega)$ can be expended at the center by:

$$\beta(\omega) = \beta(\omega_0) + \frac{d\beta}{d\omega}\bigg|_{\omega=\omega_0}(\omega-\omega_0) + \frac{1}{2}\frac{d^2\beta}{d\omega^2}\bigg|_{\omega=\omega_0}(\omega-\omega_0)^2 \tag{5}$$

define

$$\beta_2 = \frac{d^2\beta}{d\omega^2}\bigg|_{\omega=\omega_0} \tag{6}$$

If the angular frequency is low compared with β, term after β_2 can be omitted. The effect of the third order terms is called third order dispersion. The effect of third order dispersion is sometimes necessary to be included. For instance, if the pulse wavelength nearly coincides with the zero-dispersion wavelength λ_D. [5]

CD is conventionally characterized by a dispersion parameter D, which is a measure of the pulse spreading(in unit of seconds) per unit length of the medium of transmission.[6]

$$D = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right) = -\frac{2\pi c}{\lambda^2}\beta_2 \tag{7}$$

where λ is the wavelength. c is the velocity of light and v_g is the group velocity of the pulse.

As a result of CD, the group velocity of the light depends on the frequency, which is known as group velocity dispersion (GDV). GVD will lead to a distortion on the pulse. For the different group velocity of the spectrum components, the pulse is broaden, with is called pulse spreading. If the width of the pulse is larger then the distance between two neighboring pulses, these two pulses will overlap each other, and arises aliasing. The group velocity dispersion is schematically shown in Fig. 2.

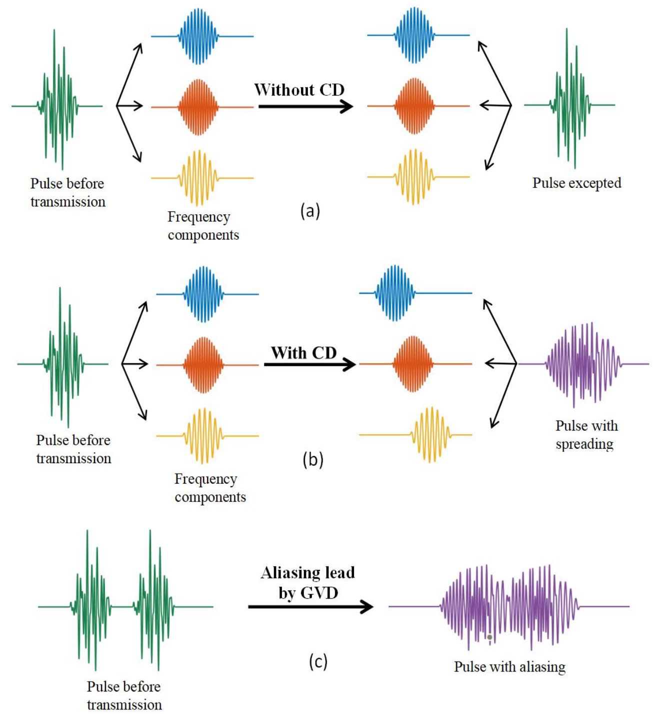

Fig. 2. (a) Pulse transmission without the effect of CD; (b) Pulse spreading due to the effect of CD; (c) The aliasing of the pulse due to the effect of GVD

B. Transfer Function

The propagation of optical pulse inside single-mode fibers can be obtained by the nonlinear Schrödinger (NLS) equation. For pulse widths>5ps, the slowly varying amplitude $A(z,t)$ of the pulse envelope can be described as follows.[7]

$$i\frac{\partial A}{\partial z} = -\frac{i\alpha}{2}A + \frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \gamma|A|^2 A \tag{8}$$

where z is the propagation distance. t is measured in a frame of reference moving with the pulse at the group velocity v_g ($T = t - z/v_g$). α is the fiber attenuation coefficient and γ is the nonlinear parameter.[8] In this equation, the three terms on the right hand refers respectively the attenuation, CD and Kerr effect.[9]. Introduce a time scale normalization to the input pulse width T_0 [10]

$$\tau = \frac{T}{T_0} = \frac{t - z/v_g}{T_0} \tag{9}$$

And then introduce a normalized amplitude $U(z,\tau)$ [11]

$$A(z,\tau) = \sqrt{P_0}exp(-\alpha z/2)U(z,\tau) \tag{10}$$

where P_0 refers to the peak power of the incident pulse and the exponential factor accounts for fiber loss.

However, we assume the medium is linear previously, hence the nonlinear parameter γ is set to zero. For a fiber with uniform loss, α is a constant which does not really distort the signal.[12]

Consider the normalized amplitude without the effect of α .It satisfy a linear partial differential equation.

$$\frac{\partial U}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 U}{\partial T^2} \tag{11}$$

Equation (11) can be conventionally solved by applying the Fourier transform method.[13]

$$\hat{U}(z,\omega) = \hat{U}(0,\omega)exp(\frac{i}{2}\beta_2\omega^2 z) \tag{12}$$

The frequency domain transfer function can be obtained from the exponential part[14]

$$G(z,\omega) = exp(i\frac{\beta_2}{2}\omega^2 z) \tag{13}$$

where ω is the angular frequency.

Equation (13) can describe the distortion of the signal after transmitted from a optical fiber quantitatively.

IV. Dispersion-Compensating Fibers

In the previous section, the slowly varying normalized amplitude can be expressed as (12)

$$\hat{U}(z,\omega)=\hat{U}(0,\omega)exp(\frac{i}{2}\beta_2\omega^2 z) \tag{12}$$

If we consider the situation in which the optical pulse propagates through two fiber segments, (12) can be written as follows.[15]

$$\hat{U}(z,\omega)=\hat{U}(0,\omega)exp\left[\frac{i}{2}(\beta_{21}L_1+\beta_{22}L_2)\omega^2\right] \tag{14}$$

where L_1, L_2 is the transmission distance in each fiber segment and β_{21}, β_{22} is the β_2 defined in (6).

If the exponential term in (14) vanishes, the CD will be compensated, for the pulse shape will not be distort at the end of the fiber. The method to eliminate the exponential term of (14) is to design

$$\beta_{21}L_1+\beta_{22}L_2=0 \tag{15}$$

consider (7) together, there is

$$D_1L_1+D_2L_2=0 \tag{16}$$

The first term refers to the normal optical fiber and the second term refers to the Dispersion Compensating Fiber(DCF).

For standard telecommunication fibers, $D_1>0$, hence D_2 is supposed to be negative. And the length have to be designed to satisfy

$$L_2=-\frac{D_1}{D_2}L_1 \tag{17}$$

The dispersion compensation fibers(DCF) can be designed by applying the waveguide dispersion of fiber. The core of the average compensating fiber is much smaller than that of standard SMF, and beams with longer wavelengths experience relatively large changes in mode size. The speed in of light in the cladding layer is lager than the fiber due to the greater propagation through the cladding of the fiber. This can lead to a negative dispersion parameter.[16]. This methods supports a single mode, by designing V parameters (which is also known as normalized frequency) as $V\approx 1$. This will result in $D\sim -100\,ps/(km-nm)$.

We simulate using a DCF with $D_2=-100\,ps/km-nm$ and $L_2=12.8km$ .The dispersion parameter of the optical fiber is $D_1=16\,ps/km-nm$ with the length of $L_1=80km$. The symbol rate is $25\times 10^9\,Bd/s$. The number of the bits per channel is 2×10^{11}, the attenuation coefficient is set as $0.2dB/km$. The duty ratio of the raised cosine is 1 with the roll-off factor set as 0.2. Arise 2048 random 01 sequence as the input signal and applying raised cosine method to avoid ISI.

Fig. 3. shows the Constellation diagram after transmission and the Constellation diagram after applying DCF of a random digital signal sequence.

The first 120 bits of input pulse before transmission, the output pulse transmitted without CD compensation and the output with CD compensation is shown in Fig. 4.

Calculate the bit error ratio of the signal which is compensated by DCF accepted by the detector. The number of the error code of I channel is 44, and 30 of Q channel. The bit error ratio of I channel is 0.0215 which of Q channel is 0.0146 , which is acceptable.

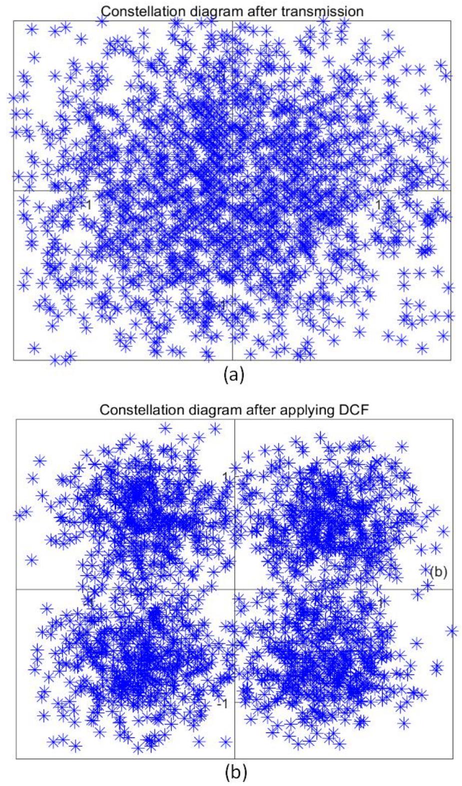

Fig. 3. (a) The constellation diagram of the signal after transmission through the fiber without compensating CD; (b) The constellation diagram after compensating CD by applying DCF

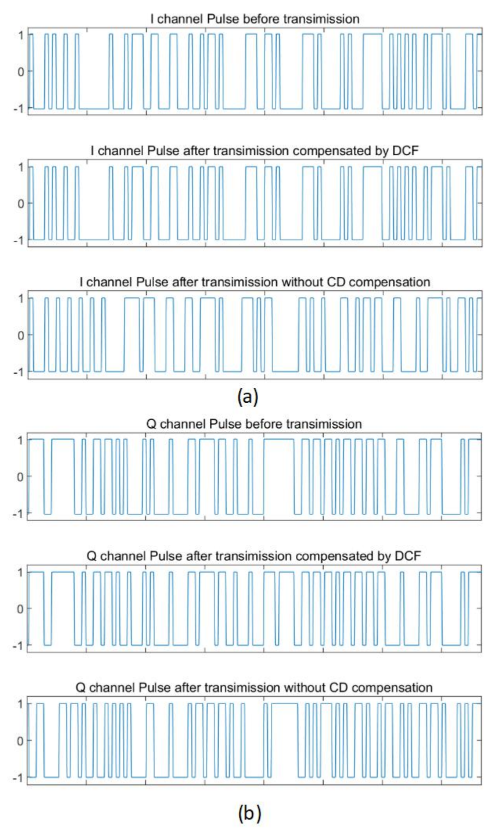

Fig. 4. (a) The input pulse of I channel before transmission, the output pulse of I channel transmitted without CD compensation and the output of I channel compensated by DCF; (b)The input pulse of Q channel before transmission, the output of Q channel pulse transmitted without CD compensation and the output of Q channel compensated by DCF

Though DCF is an available approach to compensating. It's mentioned previously that the radius of the core is smaller than standard single mode fiber(SMF). The size of DCF's core is typically $19\mu m^2$,and that of SMF is $85\mu m^2$. [17] This leads to a higher nonlinearity and a higher losses because of increase in bending losses($\alpha = 0.4 - 0.6 dB/km$) [18]. Furthermore, DCF is limited to a fixed compensation value. Last, if design the DCF with $D_1 = 16 ps/nm-km$ and $D_2 = -100 ps/nm-km$, the length of DCF have to reach 1/6 of the SMF, this will lead to losses.

Because of the losses, an amplifier is needed in the compensation module. So although the use of DCF has already existed around 1980, it is not been promoted until the optical amplifier is invented.[19]

V. Optical Filter

A more practical method to compensate CD is design an optical equalizer filer.

As discussed in (13), the transfer function through the optical fiber of each spectral component of the_pulse is

$$G(z,\omega) = exp(i\frac{\beta_2}{2}\omega^2 z) \tag{13}$$

Then the dispersion compensating filter can be designed by consider $1/G(z,\omega)$. We can obtain

$$C(\omega) = exp(-i\frac{\beta_2}{2}\omega^2 L) \tag{19}$$

where C refers to $1/G(z,\omega)$, and L refers to the length of the fiber. This introduces a all-pass filter, which can be approximated using non-recursive or recursive digital filters. However, it is impossible to design such kinds of filters with the desired phase response.

Fortunately, A method designing a FIR filter in time domain is available. Compared with frequency domain, a time domain approach gives a estimation to the taps required by the filter for a particular dispersion as well as gives a simple closed form solution.[20]

Applying inverse Fourier transfer on (13) and (19) there's

$$g(t) = \sqrt{\frac{i}{2\beta_2 L\pi}} exp(-\frac{i}{2\beta_2 L}t^2) = \sqrt{-\frac{ic}{D\lambda^2 L}} exp(\frac{i\pi c}{D\lambda^2 L}t^2)$$

$$C(t) = \sqrt{-\frac{i}{2\beta_2 L\pi}} exp(\frac{i}{2\beta_2 L}t^2) = \sqrt{\frac{ic}{D\lambda^2 L}} exp(-\frac{i\pi c}{D\lambda^2 L}t^2) \tag{20}$$

where $g(t)$ is the impulse response of the chromatic dispersion fiber. For an arbitrary input, the output can be obtained by applying convolution on the impulse response and the input pulse. Hence, if convolving a inverse of $g(t)$,which is given by $c(t)$, the effect of chromatic dispersion can be eliminated. The impulse response of the chromatic dispersion fiber in time domain is given by $c(t)$.

However, it's not easy to implement a filter with such a impulse response digitally, for it is infinite in duration and is non-causal. And since it passes all frequency, for a finite sampling frequency, aliasing will occur.[21]

The truncation to the impulse response to limit the duration into a finite region is vital to implement the filter.

Define the sampling frequency as f_s , then the sampling periodic is $T_s = 1/f_s$. If we consider the component of CD as a angular rotate factor, which subjects to normal distribution, we can consider the power of the exponential part

$$-\frac{i\pi c}{D\lambda^2 L}t^2$$

as the amount of the angular varying with time. And the speed of the variation is the angular frequency of the angular change. Hence, there is

$$\omega = \frac{d}{dt}(-\frac{i\pi c}{D\lambda^2 L}t^2) = -\frac{2i\pi c}{D\lambda^2 L}t \tag{21}$$

To avoid aliasing, we have to limit the ω , according to Nyquist –Shannon sampling theorem, $|\omega| < 2\pi f_s$. We can get the scope of t:

$$-\frac{DL\lambda^2 f_s}{c} \leqslant t \leqslant \frac{DL\lambda^2 f_s}{c} \tag{22}$$

Then since this impulse response is in an infinite duration, sample in the frequency domain, there is

$$c_s(t) = \frac{1}{f_s}\sum_{n=-N/2}^{N/2} c(t - nT_s) \tag{23}$$

Then we can get the tap weight as follows

$$c_k = \sqrt{\frac{icT_s^2}{D\lambda^2 L}} exp(-\frac{i\pi c T_s^2}{D\lambda^2 L}k^2) \tag{24}$$

where

$$-\left\lfloor \frac{N}{2} \right\rfloor \leqslant k \leqslant \left\lfloor \frac{N}{2} \right\rfloor \tag{25}$$

and

$$N = 2\left\lfloor \frac{|D|L\lambda^2 f_s}{cT_s} \right\rfloor + 1 = 2\left\lfloor \frac{|D|L\lambda^2}{cT_s^2} \right\rfloor + 1 \tag{26}$$

Here N refers to the total number of taps.[22] And $\lfloor x \rfloor$ refers to the integer part of x rounded towards minus infinity.

It can be simulated by using MATLAB.

We simulate using a optical filter with the dispersion parameter of the optical fiber being $D = 16 ps/km-nm$ with the length of $L = 80km$. The symbol rate is $25\times10^9 Bd/s$. The number of the bits per channel is 2×10^{11} , the attenuation coefficient is set as $0.2dB/km$. The duty ratio of the raised cosine is 1 with the roll-off factor set as 0.2. Arise 2048 random 01 sequence as the input signal and applying raised cosine method to avoid ISI.

The Constellation diagram after transmission and the Constellation diagram after applying DCF of a random digital signal sequence is shown below in Fig. 5.

The first 120 bits of input pulse before transmission, the output pulse transmitted without CD compensation and the output with optical filter compensation is shown in Fig. 6. It is can be observed obviously that, in the diagram of the pulse without compensation, the pulse is spread. The width of the pulse accepted by the coherent is larger than that pass the compensation optical filter.

Calculate the bit error ratio of the signal which is compensated by optical filter accepted by the detector. The number of the error code of I channel is 59 per 2048 bits, and 57 of Q channel. The bit error ratio of I channel is 0.0288 which of Q channel is 0.0278 , which is acceptable.

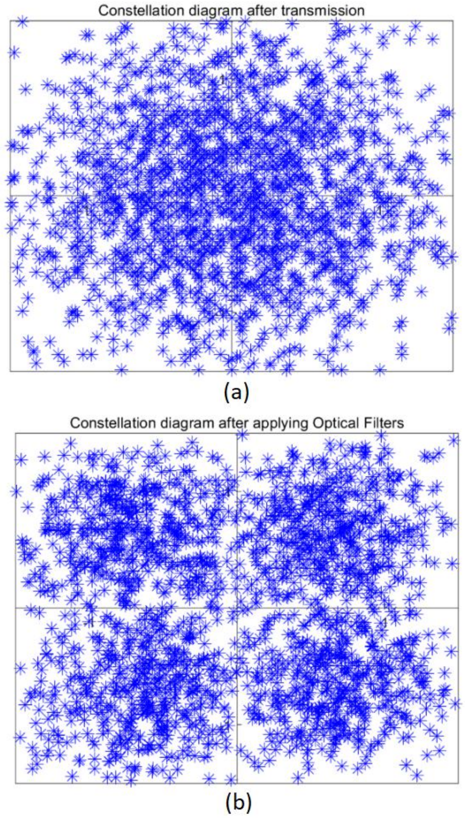

Fig. 5. (a) The constellation diagram of the signal after transmission through the fiber without compensating CD; (b) The constellation diagram after compensating CD by applying OF.

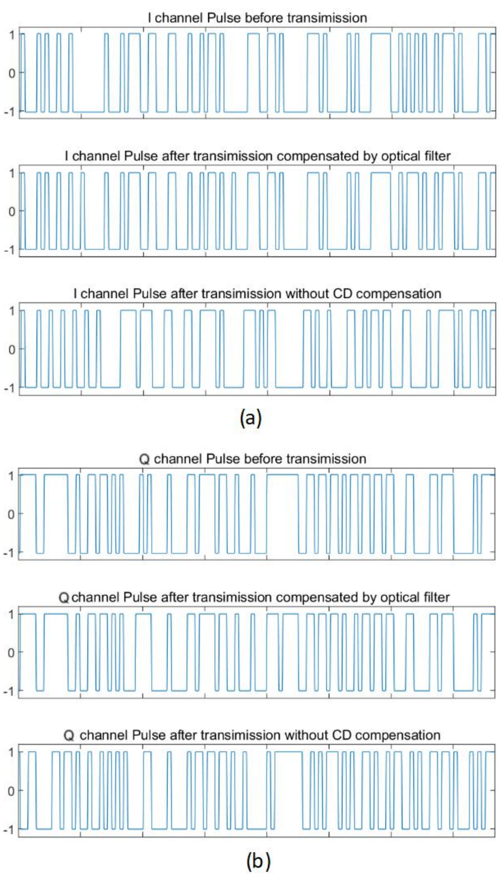

Fig. 6. (a) The constellation diagram of the signal after transmission through the fiber without compensating CD; (b) The constellation diagram after compensating CD by applying OF

The spectral diagram of the input pulse before transmission, spectral diagram of the output pulse transmitted without CD compensation and the spectral diagram of output with optical filter compensation is shown in Fig. 7. The spectral diagram does not change by the optical filter largely. This means, the compensation do not change the spectral characteristic of the pulse, it is reasonable for both the dispersion and the compensation are happened in time domain.

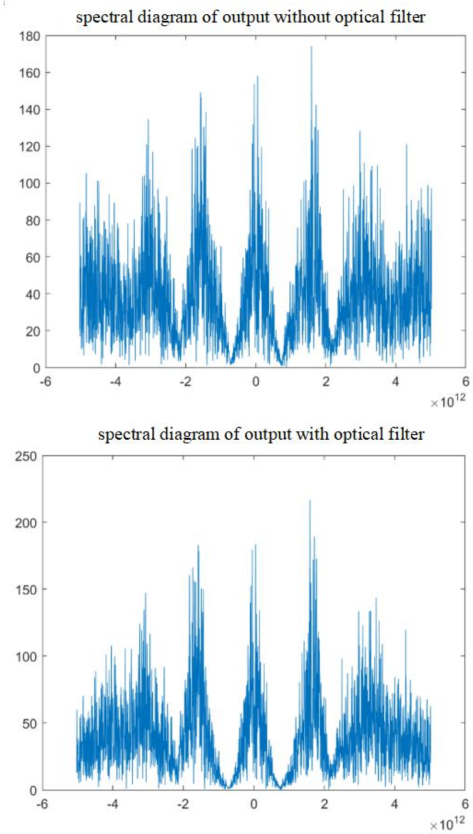

Fig. 7. (a) The constellation diagram of the signal after transmission through the fiber without compensating CD; (b) The constellation diagram after compensating CD by applying OF

VI. Numerical Investigation

In the last section, the upper bound of the number of taps is discussed. However, to simplify the implement of FIR filter in practice, the number of the total taps of the optical filter is supposed to be reduced to a floor value.

From (24), we use filters with tap numbers from the upper bound to 0. Simulate using a optical filter with the dispersion parameter of the optical fiber being $D = 16 ps/km-nm$ with the length of $L = 80km$. The symbol rate is $25\times10^{9} Bd/s$. The number of the bits per channel is 2×10^{11}, the attenuation coefficient is set as $0.2dB/km$. The duty ratio of the raised cosine is 1 with the roll-off factor set as 0.2. Arise 2048 random 01 sequence as the input signal and applying raised cosine method to avoid ISI.

The relationship between the total number of taps and the bit error rate is shown in Fig. 8.

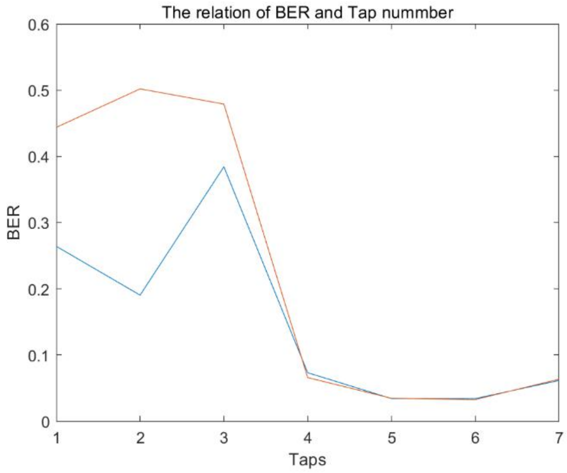

Fig. 8. The relationship between Tap number and Bit error rate

From Fig. 8. , we can get that: When the number of taps is larger than 4, the bit error rate is not decreased sharply. If the tap number reduced lower than 4, the bit error rate increases rapidly. Hence, we can draw the conclusion that, the bit error code was better to be maintained between 4 and 7. The best value is 5 which can no only have a excellent bit error rate but can also assure a low order of FIR.

VII. Conclusions and Summary

A. Chromatic Dispersion

Chromatic dispersion is always caused by a combination of waveguide and medium dispersion. And the latter is always happens because of the dependence of the group velocity on the frequency of the wave. This may lead to the spreading of the pulse, with may cause bit error and aliasing. Chromatic dispersion plays the limitation of the development of optical communication for years. The compensation is meaningful.

B. DCF

Dispersion compensation fibers are considered as a significant method to compensate chromatic dispersion for years. It also performs better in the simulation. However, in practice the cost of the DCF is larger, for it introduce larger losses and the weakness on fixing compensation values. So, it ha been eliminated for years by optical filter.

C. Optical Filters

Optical filters are the most prevalent method to compensate chromatic dispersion. It is designed by the transfer function of the dispersion fiber. A all-pass filter can be designed by the transfer function. And a FIR filter can be designed by truncating the all-pass filter and sampling it.

The implement of the FIR filter request a lower taps. Hence the best value of the tap number can be obtained by calculating the bit error rate of the pulse passed through filters with different taps. There is a lowest number, less than which the bit error rate will increase rapidly.

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