Linear Regression with Octave.

Introduction

In this exercise, you will implement linear regression and get to see it work on data. Before starting on this programming exercise, we strongly recommend watching the video lectures and completing the review questions for the associated topics.

To get started with the exercise, you will need to download the code folder and unzip its contents to the directory where you wish to complete the exercise. If needed, use the cd command in Octave to change to this directory before starting this exercise.

Files included in this exercise

- octaveLR.m - Octave script that will help step you through the exercise
- dataLR.txt - Dataset for linear regression with one variable
- (*) plotData.m - Function to display the dataset
- (*) computeCost.m - Function to compute the cost of linear regression
- (*) gradientDescent.m - Function to run gradient descent

⋆ indicates files you will need to complete

Throughout the exercise, you will be using the script octaveLR.m. This script sets up the dataset for the problems and makes calls to functions that you will write. You do not need to modify this file. You are only required to modify functions in other files, by following the instructions in this assignment.

1. Linear regression with one variable

In this part of this exercise, you will implement linear regression with one variable to predict profits for a food truck. Suppose you are the CEO of a restaurant franchise and are considering different cities for opening a new outlet. The chain already has trucks in various cities and you have data for profits and populations from the cities.

You would like to use this data to help you select which city to expand to next.

The file dataLR.txt contains the dataset for our linear regression problem. The first column is the population of a city and the second column is the profit of a food truck in that city. A negative value for profit indicates a loss.

The octaveLR.m script has already been set up to load this data for you.

1.1 Plotting the Data

Before starting on any task, it is often useful to understand the data by visualizing it. For this dataset, you can use a scatter plot to visualize the data, since it has only two properties to plot (profit and population). (Many other problems that you will encounter in real life are multi-dimensional and can't be plotted on a 2-d plot.)

In octaveLR.m, the dataset is loaded from the data file into the variables X and y:

```
data = load('ex1data1.txt');          % read comma separated data
X = data(:, 1); y = data(:, 2);
m = length(y);                        % number of training examples
```

Next, the script calls the plotData function to create a scatter plot of the data. Your job is to complete plotData.m to draw the plot; modify the file and fill in the following code:

```
plot(x, y, 'rx', 'MarkerSize', 10);          % Plot the data
ylabel('Profit in $10,000s');                % Set the y−axis label
xlabel('Population of City in 10,000s');     % Set the x−axis label
```

Now, when you continue to run octaveLR.m, our end result should look like Figure 1, with the same red "x" markers and axis labels.

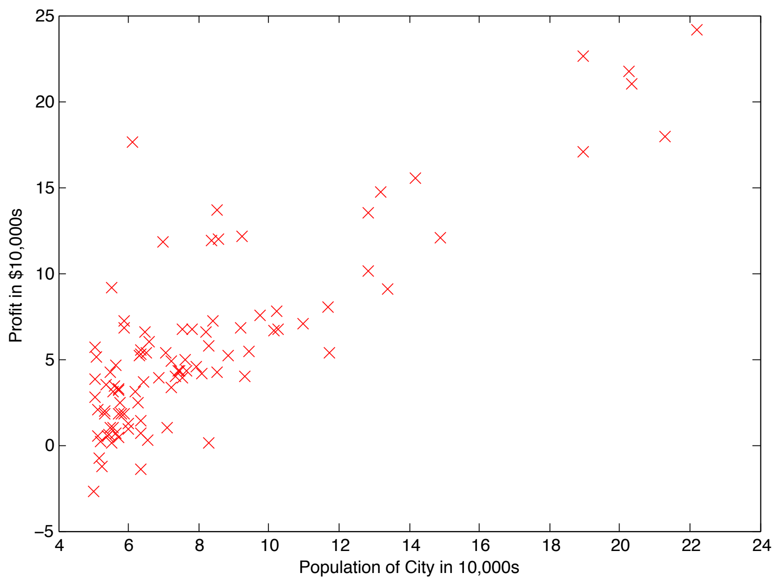

Figure 1: Scatter plot of training data

To learn more about the plot command, you can type help plot at the Octave command prompt or to search online for plotting documentation. (To change the markers to red "x", we used the option 'rx' together with the plot command, i.e., plot(..,[your options here],.., 'rx');)

1.2 Gradient Descent

In this part, you will fit the linear regression parameters θ to our dataset using gradient descent.

1.2.1 Update Equations

The objective of linear regression is to minimize the cost function

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Where the hypothesis is given by the linear model:

$$h_\theta(x) = \theta^T x = \theta_0 + \theta_1 x_1$$

Recall that the parameters of your model are the θ_j values. These are the values you will adjust to minimize cost J(θ). One way to do this is to use the batch gradient descent algorithm. In batch gradient descent, each iteration performs the update:

$$\theta_j := \theta_j - \alpha\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)} \qquad \text{(simultaneously update } \theta_j \text{ for all } j\text{).}$$

With each step of gradient descent, your parameters θ_j come closer to the optimal values that will achieve the lowest cost J(θ).

Implementation Note: We store each example as a row in the the `X` matrix in Octave. To take into account the intercept term (θ_0), we add an additional first column to `X` and set it to all ones. This allows us to treat θ_0 as simply another 'feature'.

1.2.2 Implementation

In octaveLR.m, we have already set up the data for linear regression. In the following lines, we add another dimension to our data to accommodate the θ_0 intercept term. We also initialize the initial parameters to 0 and the learning rate alpha to 0.01.

```
X = [ones(m, 1), data(:,1)]; % Add a column of ones to x
theta = zeros(2, 1); % initialize fitting parameters

iterations = 1500;
alpha = 0.01;
```

1.2.3 Computing the cost J(θ)

As you perform gradient descent to learn minimize the cost function J(θ), it is helpful to monitor the convergence by computing the cost. In this section, you will implement a function to calculate J(θ) so you can check the convergence of your gradient descent implementation.

Your next task is to complete the code in the file computeCost.m, which is a function that computes J(θ). As you are doing this, remember that the variables X and y are not scalar values, but matrices whose rows represent the examples from the training set.

Implementation Note: We store each example as a row in the the X matrix in Octave. To take into account the intercept term (θ_0), we add an additional first column to X and set it to all ones. This allows us to treat θ_0 as simply another 'feature'.

```
X = [ones(m, 1), data(:,1)]; % Add a column of ones to x
theta = zeros(2, 1); % initialize fitting parameters
iterations = 1500;
alpha = 0.01;
```

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Once you have completed the function, the next step in octaveLR.m will run computeCost once using θ initialized to zeros, and you will see the cost printed to the screen.

You should expect to see a cost of approximately 32.07.

1.2.4 Gradient descent

Next, you will implement gradient descent in the file gradientDescent.m. The loop structure has been written for you, and you only need to supply the updates to θ within each iteration.

As you program, make sure you understand what you are trying to optimize and what is being updated. Keep in mind that the cost J(θ) is parameterized by the vector θ, not X and y. That is, we minimize the value of J(θ) by changing the values of the vector θ, not by changing X or y. Refer to the equations in this handout and to the video lectures if you are uncertain.

A good way to verify that gradient descent is working correctly is to look at the value of J(θ) and check that it is decreasing with each step. The starter code for gradientDescent.m calls computeCost on every iteration and prints the cost. Assuming you have implemented gradient descent and computeCost correctly, your value of J(θ) should never increase, and should converge to a steady value by the end of the algorithm.

After you are finished, octaveLR.m will use your final parameters to plot the linear fit. The result should look something like Figure 2:

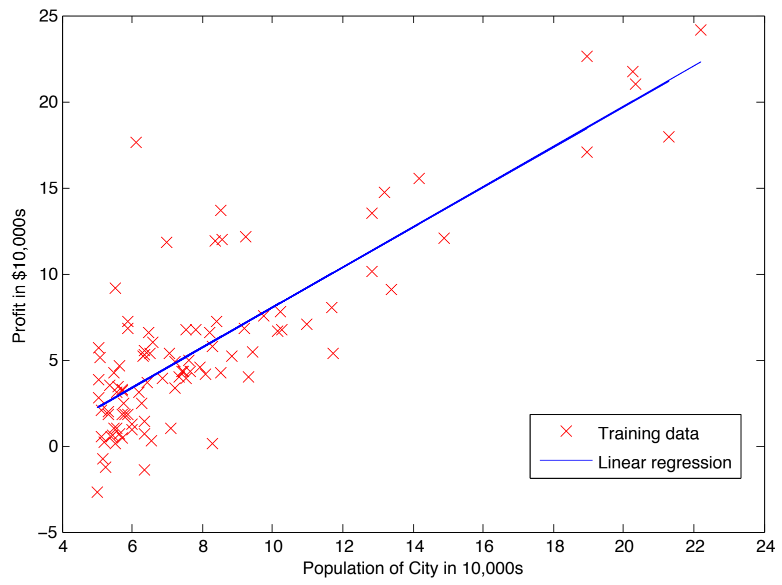


Figure 2: Training data with linear regression fit

Your final values for θ will also be used to make predictions on profits in areas of 35,000 and 70,000 people. Note the way that the following lines in octaveLR.m uses matrix multiplication, rather than explicit summation or loop- ing, to calculate the predictions. This is an example of code vectorization in Octave.

```
predict1 = [1, 3.5] * theta;
predict2 = [1, 7] * theta;
```

1.3 Debugging

Here are some things to keep in mind as you implement gradient descent:

- Octave array indices start from one, not zero. If you're storing θ_0 and θ_1 in a vector called theta, the values will be theta(1) and theta(2).

- If you are seeing many errors at runtime, inspect your matrix operations to make sure that you're adding and multiplying matrices of compat- ible dimensions. Printing the dimensions of variables with the size command will help you debug.

- By default, Octave interprets math operators to be matrix operators. This is a common source of size incompatibility errors. If you don't want matrix multiplication, you need to add the "dot" notation to specify this to Octave. For example, A*B does a matrix multiply, while A.*B does an element-wise multiplication.

1.4 Visualizing J(θ)

To understand the cost function J(θ) better, you will now plot the cost over

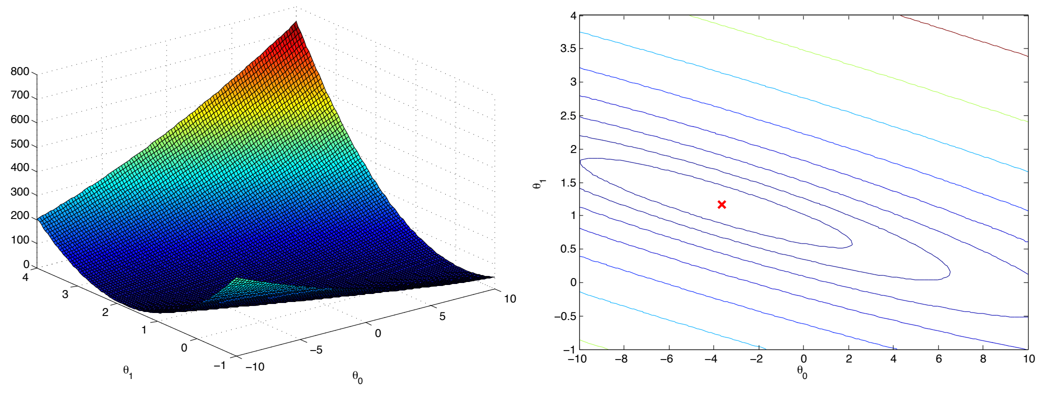
a 2-dimensional grid of θ_0 and θ_1 values. You will not need to code anything new for this part, but you should understand how the code you have written already is creating these images.

In the next step of octaveLR.m, there is code set up to calculate J(θ) over a grid of values using the computeCost function that you wrote.

```
% initialize J_vals to a matrix of 0's
J_vals = zeros(length(theta0_vals), length(theta1_vals));

% Fill out J_vals
for i = 1:length(theta0_vals)
    for j = 1:length(theta1_vals)
       t = [theta0_vals(i); theta1_vals(j)];
       J_vals(i,j) = computeCost(x, y, t);
    end
end
```

After these lines are executed, you will have a 2-D array of J(θ) values. The script octaveLR.m will then use these values to produce surface and contour plots of J(θ) using the surf and contour commands. The plots should look something like Figure 3:



(a) Surface (b) Contour, showing minimum

Figure 3: Cost function $J(\theta)$

The purpose of these graphs is to show you that how J(θ) varies with changes in θ_0 and θ_1. The cost function J(θ) is bowl-shaped and has a global mininum. (This is easier to see in the contour plot than in the 3D surface plot). This minimum is the optimal point for θ_0 and θ_1, and each step of gradient descent moves closer to this point.