Markowitz's Model in the Formation of an Efficient Portfolio in the German Capital Market;

The evaluation of the possible effect of the Covid-19 pandemic on the efficient portfolio, the applicability of MPT in this market, and its pertinence to predict the future.

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I. Summary

Fundamentally, it is impossible to forecast the behavior of humans' decisions about stock exchange markets. Numerous studies have been conducted that connect historical information to investors' rational judgments. In this research, the Markowitz Portfolio Optimization Model is going to be tested in the German market by diversification in uncorrelated assets as much as possible, meaning it is about to test the practicality of the MPT[1] in the German stock exchange market and offers it to investors to rely on. Based on the various names for the model in scientific studies, throughout this text, MPT or mean-variance will be the same as the Markowitz model.

Regarding the background, some publications provided a thorough analysis that is either relevant to or directly applies to the Markowitz Model. While others evaluated the model's applicability, comparability, resemblance, or modified it.

This is an applied, quantitative, and longitudinal research. From the data viewpoint, it uses external, measurable, observable, and factual data. Its mathematical tools are the rate of return, standard deviation, correlation coefficient, variance, covariance, beta coefficient, and linear regression, using an Excel data Tool pack to analyze the data which is collected from the Yahoo Finance database. The CAPM and Linear Regression are the mathematical and statistical methods to establish the project. Regarding the design, it is objectively focused, neutral, and independent. Also, it has a positivist philosophy and real ontology.

To diminish the systematic risk in the selected portfolio a group of 21[2] companies from the DAX 30 in the German stock market is selected. The criteria for sampling are market capitalization, liquidity, level of turnover, and industrially diversification. DAX companies all are with high liquidity, turnover, and market capitalization. The data has been collected from Yahoo Finance in Excel files in CSV format. Then data is examined to assess its validity by employing the correlation coefficient, heteroscedasticity, multicollinearity, and autocorrelation tests. The results of the validation process have proved data acceptability.

Some limitations and problems have taken place; database for share price and market index, the database for journal articles, the negative rate of returns in some stocks as well as one share with the unavailability of data, causing to be canceled out. However, in contrast to other common issues and limitations, the paper-writing process went off without a hitch and on schedule.

The calculation of stock variances and covariances matrix has been created by the matrix calculation in Excel, and examinations have been conducted by relevant equations. Some covariances are negative, indicating that there is an inverse link between them. While some are positive, denoting that there is a direct relationship between them. Utilizing the MS Solver, the optimization procedure has been carried out. Then, the Sharpe Ratio which is the division of the rate of return per standard deviation is the major criterion to reach the most possible amount.

[1] MPT: Modern Portfolio Theory

[2] The number 21 is based on Radović and colleagues, 2018 article.

The eventual result of diversification is to provide either more return with the same risk or less risk with the same return compared to the individual shares, keeping in mind that diversification cannot reduce the systematic risk but only can reduce the unsystematic risk.

This study aims to evaluate the model's applicability and market-prediction skills in the German stock exchange market, particularly during the Covie-19 epidemic. The final results firstly state that fundamental and economic circumstances have an impact on optimization, secondly, the Markowitz model is effective, applicable, and approved in the German market, and finally, it is unable to predict the future, based on historical data.

II. Introduction

A. The Concept of Optimization by Diversification

The portfolio selection process contains two stages, first, observation of the historical data, and second, portfolio selection based on the results from the first stage. Harry Markowitz (1952) in his famous article explained that the goal of optimization is the second stage. He explained that any behavior which does not implies the preeminence of diversification must be discarded. (Markowitz, 1952) As a major result, diversification by ignoring some imperfections of the market is the objective of the MPT; Modern Portfolio Theory.

Basically, it is impossible to forecast the behavior of humans and so to foresee the market which is a result of people's decisions based on their emotions and feelings about the market. Several studies are based on theories and simulations of past data plus the idea of rational decisions by investors. In this way, studying and finding the results to manage and decrease the inefficiencies of the markets by the use of data automation systems relying upon past data is a big challenge. (Fabozzi, et al., 2007)

In this case, this study is not using the methods to anticipate the market, but it is going to test the Markowitz model, the pandemic effects on it, and to see whether diversification can anticipate the future.

B. Main definition

1. The concept of Markowitz Model; MPT

The objective of a portfolio with diversification out of some stocks, securities, currencies, and recently cryptocurrencies is to make the best possible combination. In this area, increasing the return as well as decreasing the risk by use of diversification as an instrument developed by Harry Markowitz as the modern portfolio theory was a major and one of the most important successes in this field. In another word, Markowitz suggested that investors pay attention to both sides of return and risk, by selecting between the different mixture of this twin; risk versus return. The basic assumption of Markowitz was that an investor naturally is risk averse but likes benefit, meaning more return and lower risk is the intrinsic and initial resource of decision making. (Zanjirdar, 2020)

In detail, Harry Markowitz assumed the following assumptions, which are not always realistic in the real world.

I. People are rational, so they as investors behave in a way to maximize their wealth based on their income and investment.
II. Investors avoid the risk, so they try to minimize the risk and maximize their return.

III. All of them have the access to the information and data from markets they need and want to use.
IV. The markets are quick, efficient, and perfect in terms of gathering and issuing information.

However, the basic rule here in this theory is diversification in uncorrelated assets as much as possible, no matter how and if the pre-assumptions are practical, meaning that the result is practical and beneficial. (Kumar, 2018)

2. Developments of the MPT

After 1952, the start time of the Markowitz model (MPT: Modern Portfolio Theory or Mean-Variance Portfolio Selection Model), there has happened a lot of developments, both theoretically and empirically, in the main theory. Also, other types or versions of the theory have appeared, which resulted in maximizing the return and minimizing the risk more and more under specific conditions. Some of them are dynamic portfolio optimization, portfolio optimization with practical factors, robust portfolio optimization, and fuzzy portfolio optimization. (Zhang, et al., 2018)

C. The uniqueness of the research

The relevant papers from different databases, such as The University of Law library database, other universities, Jstor, Elsevier, Springer, and ResearchGate have been comprised to prepare a comprehensive literature review. Some specificities have not been found in the review of past articles which are very important to the author; the possible effect of the Covid-19 pandemic on the efficient portfolio, the applicability of the MPT in the German market, and its pertinence in predicting the future. Thereby, the research is unique based on the mentioned concerns.

III. Literature Review

A comprehensive review of past studies presents that some of them evaluated the applicability, comparability, or similarity of the model. Some others employed the model by changing or adding some factors and variables inside it. Additionally, as CAPM plays a major role along with the MPT in this study the literature review consists of the articles related to CAPM too and places them in the same categories.

A. The Extracted Categories

Therefore, as the literature on this famous model of portfolio optimization has been reviewed by this article, they are categorized into seven groups as follows. The first cluster contains those performed in the German market, while some of them are associated with the other categories too.

The objective of this classification is to answer the question that how and for what the MPT has been employed during its history and what are the most popular uses of it. Furthermore, it is to discern the gap of knowledge and then realize the unique question to work on.

1. Major Category. MPT German Market Research

In this section firstly the studies that have been accomplished in the German stock exchange market are going to be explained, and then they are also categorized in the following categories among the other MPT studies, based on their unique codes which begin with MC, as Major Category.

Common with category A

Assessing the applicability of CAPM in the real world versus its theoretical aspects with consideration of German share prices happened in 1988 by Hans Peter Möller, with the consequence of the extreme importance of learning how results could be changed in case the researcher had employed any modification. And a review and analysis of how CAPM has been used and what is the result of employing it in the German market showed that a domestic version, especially for large companies, functions more effectively in Germany, stated Brückner, Lehmann, and Stehle in the year of 2012. These two articles and the other one in an empirical examination which is applied in the market of the German stock exchange by Elsas, El-Shaer, and Theissen in 2003 that showed a meaningful relationship between beta and return, are connected to category A, meaning they assessed the applicability of the theory.

Common with category B

The next five papers are associated with the usage of MPT in specific conditions, category B.

Applying the mean-variance model by Schwägele (2017) to evaluate the international perspective of a German investor with home bias consideration certified the priority of international concerning regional and domestic diversification.

The application of the global CAPM provides a reliable and steady methodology compared to the traditional (local or domestic) CAPM to formulate exchange rate forecasts that can cause the extension of the forward rate approach with a risk-adjusted (risk premium) method. (Ruiz de Vargas and Breuer, 2018)

Even though the CAPM with German income tax is not pragmatic, applicable CAPM with taxation can be deduced. (Jonas, Löffler and Wiese, 2004)

While the real-world portfolio optimization mostly is far from using just the mean-variance optimal factor, as Oehler, Rummer, and Wendt (2008) explained that they found investment portfolios were affected by the home-biased effect, meaning that investors keep a higher optimal portion of domestic stocks compared with the other areas.

Common with category C

Moreover, some studies were established in different industries, like the one published by Auerswald and Leuthold (2009) in anticipation of the German renewable energy future by portfolio generation. Or the other one by Arévalo and Eduardo (2020) reached the consequences of a drastic improvement in green energy resources in Germany by 2040, having benefited from portfolio optimization models.

The results of a paper on the German real estate market among the largest institutional investors are based on open-ended funds, closed-ended funds, property companies, and pension funds of insurance companies to determine risk management by applying portfolio optimization. (Stock, 2006)

Quill in one research in 2020 examined the social dimension of business valuation employing (actor-network theory), ANT, and the capital asset pricing model (CAPM) in the judicial system of Germany. The result specified the importance of solidification and perpetuation of the arguments relevant to the system.

Studying tax CAPM empirically by Schmitt and Dausend in 2007 certified that it describes the yield generation process in the German blue chips industry between 2001 and 2004.

Common with category D

The other research in the German market tried to modify the model as follows.

In the expansion of the static mean-variance (MV) portfolio model with capital controls to be examined on quarterly data in the German private market of net wealth, Jansen (1998) demonstrated a strong link between long-term expected returns and asset holdings.

Breunig et al. (2021) believed that modifying safe assets has more effects on the behavior of investors than risky assets.

Based on the findings of one international study by Jagannathan and Wang (1996), the conditional CAPM is perfectly applicable to the North American market rather than Germany, Brazilian, and Argentinean ones. (Tambosi Filho, Garcia and Imoniana, 2009)

Resuscitating the CAPM in the market of France and Germany by Hyde, Cuthbertson, and Nitzsche (2005) by employing the parametric methodology of Hansen and Jagannathan resulted in finding a lower rate of discounts compared to the previous research which was consistent with asset return data.

Erfurth in 2015 in higher moment CAPM research discussed that a majority of several asset returns are not distributed normally and so the outcome of cross-sectional analysis disclosed that none of the variables affected returns on assets statistically.

Based on the pre-assumption that an investor follows the way to generate higher returns on the German stock market by using an active portfolio management approach rather than the passive counterpart, an active investment strategy with a long-term basis provides higher returns if avoids poor-performance months of the market (Wüsten, 2012).

Common with category E

Lastly, the articles correlated to the model's comparison which compared the Markowitz model with the other similar models for optimization are presented here in this part.

Sometimes in systematic risk addressing the time-variation is important and so the zero-intercept hypothesis cannot be rejected, meaning that portfolio returns have consistency with the Capital Asset Pricing Model (CAPM). This is the result of one research based on data collected from Europe, including Germany, written by Spyrou and Kassimatis, 2009.

CAPM extensions in the German stock market with a composition of all factors, such as two ways of regression procedure, higher co-moments, and the conditional beta method perform better in the post-recession period (Nguyen, 2018).

One paper compared the common mean-variance analysis with mean-downside risk analysis, to determine the most efficient portfolio selection (Cornu and Pintado, 1996). And the other compared the Mean-Variance with the Conditional Value at Risk in the framework of the German Stock Exchange Market. (Lawrenz, 2018)

The other one was written by Sauer and Murphy (1992) found that the CAPM is more effective in terms of an indicator for pricing the capital asset in the German market compared to the C-CAPM.

The estimations of the aversion parameter of relative risk are mostly unacceptable and wrong, besides the dividend-price ratio inclines to anticipate future consumption growth wrongly. Lund and Engsted (1996) said. And they concluded that therefore, discount rates are time-varying inconsistently with the CRRA/C-CAPM conditions.

To wrap up, it is concluded that in the German market, mostly, the CAPM has been employed in different industries, for different evaluations, and with different alterations, but there has not been found using it to assess the Markowitz model in this market.

2. Category A. Assessing the Applicability

The CAPM and Markowitz's portfolio model are theoretically great achievements. Fama and French, 2004, pointed out. They also found out that despite its seductive simplicity, the CAPM's empirical problems probably invalidate its use in applications.

The research conducted by Ivanova and Dospatliev in Bulgarian Stock Exchange Market, using 50 stocks for about 3 years based on weekly closing prices, showed that the investors can benefit from applying the Markowitz model to maximize their portfolio rate of return compared to their risk preferences. (Ivanova & Dospatliev, 2017)

In "The application of Markowitz Portfolio Theory in obtaining the best portfolio in the stock market" in Wuhan, China, The Mean-Variance Analysis was explored and applied to the concept of modern security investment. They found that the model has a specific value to use in the domestic China A-share market. Despite some shortcomings, being found in this research, its application is approved by the authors. (Chao, Tao and Zeng, 2019)

Vasile Brătian, in its paper "Portfolio Optimization, application of the Markowitz Model Using Lagrange and Profitability Forecast", running in the Romanian Stock Market, concluded that The Markowitz Model is not only useful to determine the optimal structure of the portfolio but also is great for practitioners in case of prediction the range of values of their investment return. (Brătian, 2018)

In the Serbian market study, it results that the model is not clearly and unambiguously applicable and appropriate, however, it is useful. (Radović, et al., 2018)

In "Portfolio Selection in NSE Expected Return & Risk through Markowitz Portfolio Theory" Yogesh Puri, and Abhishek Yadav discussed the aim of Harry Markowitz who first proposed the model so-called optimal portfolio theory in 1952 was to prepare a pragmatic method to maximize the return with respect their relevant risk. This article's result is that the Indian market does not track the efficient method because of the lack of pat, public, and private data. (Puri and Yadav, 2020)

Loana C. Zavera (2017) in the article on the Romanian Stock Market explains that the performance of the model to reach the optimal point of collection of stocks in the market with minimum variance and simultaneously the efficient frontier which means a high possibility of return is utilized and so the model worked appropriately.

Wei in the article named “The Application of Markowitz Model Based Series of Companies’ Stock”, applied the model in China and the Beijing market to divide the group of investors into three different categories associated with their capability of having positions and also arbitrary constraints. The consequence is that it is possible by this categorization to employ the mean value model in a more effective way for portfolio investment to propose advice to investors. (Wei, 2022)

The articles from the major category related to the A. category are written by Möller in 1988, Brückner, Lehmann, and Stehle in 2012, and Elsas, El-Shaer, and Theissen in 2003.

In conclusion, this part shows that some articles worked on Wuhan and Beijing in China, Indian, Serbian, Romanian, and Bulgarian Stock Exchange Market to appraise the Markowitz model. Consequently, except for the Indian market and Serbian market papers that stated the market does not provide enough data and the result is not clear enough, the other projects approved the MPT applicability and feasibility in their destinations.

3. Category B. In Certain Circumstances

In these two articles the model is tested and approved in two conditions; the up or down direction of the stock market, and the liquidity of the whole market.

In research in Indonesia which took place in the LQ-45 index during 2005-2011, they found that the strength of the Markowitz model depends on the situation and the sector. The results showed that in the bearish period the portfolio was optimum in the banking and manufacturing sectors, while in the bullish period, it was the best in the commodity stocks sector. (Hamid, 2016)

The mentioned study in the last section, which was performed in the Serbian stock exchange market, assessed to perform the Modern Portfolio Theory (MPT) with shares from a very low liquid and developing market. Finally, it shows that diversification even in such a situation is pragmatic. (Radović, et al., 2018)

The articles from the major category related to the B. category are brought into the journals by Oehler, Rummer and Wendt (2008), Ruiz and Breuer (2018), Jonas, Löffler and Wiese (2004), and finally Schwägele (2017).

4. Category C. In Different Industries

The results of one research in the Tehran Stock Exchange (TSE) showed that using the Markowitz model in different industries, such as food and beverage at the first level and sugar at the second level made the portfolio more efficient. (Chizari & Vazirian, 2022)

The investigation of the US energy blockchain shows the comparatively weak performance of energy cryptos did not support the improvement of the extra return per unit of risk of each portfolio. (Gurrib, Elsharief, and Kamalov, 2020)

Fitt shows that the model is useful in the soccer industry to provide an efficient frontier rather than just for stock markets. The research suggests that it leads to new definitions which are called "efficient betting frontier" and "optimal bet portfolio" (Fitt, 2008).

The articles from the major category related to the C. category are published by Quill in 2020, Schmitt and Dausend in 2007, Auerswald and Leuthold in 2009, Stock in 2006, and the last one by Arévalo and Eduardo in 2020.

It is interesting to see that MPT is not only usable in the stock exchange market but even in a variety of industries some of which admit its high applicability. The question is whether it is beneficial in all industries or not.

5. Category D. By modifying the Methodology

In the last decade, SRI[3] had become a popular subject in both private and institutional deals. A paper by Stephan M. Gasser and Karl Weinmayer in 2017 revisited the Markowitz model and proposed a modification to enter the social responsibility measure into the process. The result implies the acceptance of the model.

Barbora Gabrikova, 2021, used the Modern Portfolio Theory by employing the Lagrange function to calculate the optimal weights of each asset. She took closing prices weekly of each stock, commodity, and mutual fund. Her research identifies that the risk associated with the portfolio was lower than the risk associated with every single asset individually.

Employing the Markowitz Model by changing the criteria for risk (using an industrial standardization measurement instead of variance) by Lwin, Qu, and MacCarthy is another modification of the model based on variable alternatives. The industry standard risk measure is called Value at Risk (VaR). Their results feature the capability of complex portfolio optimization by using the new risk benchmark of VaR without a need to simplify with attaining acceptable and reasonable solutions practically. (Lwina, et al., 2017)

[3] Socially Responsible Investment

In the research “Enhancement of the applicability of Markowitz’s portfolio optimization by utilizing random matrix theory”, Bai, Liu, and Wong (2009) claimed that they improved the model to be more practical and easier to implement by developing a new optimal return estimation to obtain the fundamental of selecting the portfolio.

The length of the portfolio timetable, so-called the length of estimation window[4], is a parameter in the research by Marcin Potrykus in 2019 in the Warsaw stock exchange market, to determine the optimal length. The result is interesting which shows 144-160 days of observation could be the best length of estimation window, while the authors state that based on the characteristics of the portfolio and the investor, it can be different.

The other article attempted to provide a model of Markowitz to be user-friendly and estimation-accurately. They pointed out that the basic problem of the Markowitz mean-variance model is that it recognized the combinations of assets with the basic characteristic of having attainable efficiency. (Wong, Leung, and Ng, 2011)

In Index 30 of the Istanbul market, Mehnet Erdas (2020) applied the MPT in certain constraints in which the risk variable is replaced by the absolute deviation instead of the standard deviation. It proved that the Markowitz model is fruitful even with the absolute deviation for the risk.

Kliber and Rutkowska-Ziarko (2021) believed that classical models for portfolio optimization are not accounted for fundamental investment companies. As a result, they extended the selection of assets by adding a new series of criteria; the expected returns, variance as risk, and economic situation of companies. They concluded that the additional criteria deliver a higher average rate of return in comparison with the classical model of Markowitz.

By using adiabatic quantum optimization inside the mean-variance model for financial portfolio management in a case security exchange market in Abu Dhabi, Elsokkary and colleagues (2017) acquired an acceptable agreement between mean-variance with this modification.

Halim and Yuliati (2020) in their article considered risk tolerance in optimization by using the Lagrangean Multiplier method to determine the weight (proportion) of stocks in the allocation of the investment portfolio. The result is successful optimization.

The articles from the major category related to the D. category are issued by the authors as follows. Hyde, Cuthbertson, and Nitzsche in 2005, Tambosi Filho, Gacia, and Imoniana in 2009, Erfurth in 2015, Wüsten in 2012, Breunig et al in 2021, and Jansen in 1998.

In this category, the complexity of working with the model is appeared by modifying it, such as employing the Lagrange function or changing the concept of return or risk. Since they are not assessing the model directly, this part cannot in any way interrupt the uniqueness of the current study.

[4] The Length of Estimation Window: The length of period to observe the statistical population.

6. Category E. Deficiencies in the MPT

Some scientists believe that the Markowitz Model has deficiencies besides its advantages. As an example, in the consequence of the research by Zavera (2017) which also is brought in category 2, the researcher showed that it cannot take macroeconomic factors into account.

By reviewing the article with Chao, Tao, and Zeng in 2019 again, in CSMAR,[5] They described that MPT has strict logic but lacks pragmatically in the real world of investment. Additionally, in the extremely rapid-changing market, it is not practical.

Markowitz’s (1952) portfolio selection needs to measure an estimator to be able to calculate the matrix of covariance of returns. To perform such a procedure Ledoit and Wolf (2017) tried to address it as a problem and they promoted a nonlinear shrinkage estimator. As a result, they claimed that their estimator was more flexible compared to the previous one.

7. Category F. Comparison with other Models

Sinha and Tripathi (2017) in their common study named “A Comparison of Markowitz and Sharpe’s Model of Portfolio Analysis,” tried to understand two models of Markowitz and Sharpe in the Indian market.

Kazemia and colleagues in their paper in 2017 worked on the multi-objective model to optimize the portfolio as a fuzzy goal programming model. They suggest selecting a portfolio based on one DSS[6] and recognizing the indices that affect the risk and return more effectively than the MPT.

Markowitz's model is not good in performance compared to the single index model, based on the idea of Komang Swara Putra and Made Dana, 2020, in a study of optimal portfolio performance in the Indonesian Stock Exchange, using the non-participant observation method.

Philipp Baumann and Norbert Trautmann in their research in 2013 compared four different portfolio optimization models one of which is MPT and took the result that all of them are suitable for optimization.

In “Portfolio selection with stable distributed returns”, the authors approved that the efficient frontier results are the same as Markowitz-Tobin’s model of mean-variance analysis, while, they used the scale parameter of the stable distributions instead of the variance as a risk parameter. (Ortobelli, Huber and Schwartz, 2002)

[5] CSMAR: China Stock Market & Accounting Research Database. The domestic A-share market in China is one part of CSMAR.

[6] DSS: Decision Support System

The articles from the major category related to the F. category are as follows: Spyrou and Kassimatis (2009), Nguyen (2018), Cornu and Pintado (1996), Lawrenz (2018), Sauer and Murphy (1992), and finally Lund and Engsted (1996).

Even though this section's articles do not reject the MPT, they offer other alternatives for optimization. It brings us the idea of modernizing the new methods as well as modifying the famous historical model of Markowitz.

B. Approved ones

By analyzing the literature review in the last 7 groups, it is noticeable that most of them either accept or approve of the Markowitz model. On the other hand, even though some do not mention the approval directly, they do not reject it. However, only two of them, Putra& Dana, 2020, and Kazemi& colleagues, 2017, ranked the Mean-Variance Model at the second level after the single index and fuzzy models respectively.

C. General Outcome

The review shows that in some non-western countries such as Iran, Bulgaria, and Serbia, the Modern Portfolio Theory is not widely employed.

Also, the mean-variance portfolio analysis as a methodology can be used to solve other problems rather than just asset portfolio optimization. On the other hand, some other optimization methods or portfolio management techniques, which are not directly related to this study, are employed to examine the process of portfolio optimization, some of which have been brought just as a reference in appendix I.

From different classes in the literature review, it is concluded that no article has been published to test the MPT directly in DAX 30 of the German exchange market, while they utilized the model to solve other problems. More specifically no one has evaluated the model of Markowitz to check the difference or applicability over time, like before and after the Covid-19 pandemic.

D. Research Questions

In conclusion from the literature review, the applicability evaluation of the Mean-Variance Analysis is not performed in the German market. Hence, The research questions are the final product of the literature review process as the gap of knowledge. Consequently, this paper finds its fundamental questions as follows:

1. Is the pandemic affects the result of the model? At this stage, the MPT is going to be employed in two consecutively periods, before and after the event of the Covid-19

pandemic to check whether the model function similarly in both times or not. Also, if there is a difference between those periods, how much is that, and what is the detail of effects from both quantitative and qualitative views?

2. Is the model in DAX 30 applicable? By this question, the author wants to know if the MPT can be applied in the market of DAX or not. Every model of optimization might work in one market, while might be rejected in another one, depending on different macro and micro-economic factors affecting the specific market. So, to revisit the model in the DAX market, it must be reviewed by using the data from the market and applying it to the model.
3. Is the model a predicting model for the future? By this question, the predictability of the model is going to be tested in the way of using the result from one period which is provided by the model to optimize the second period. If the outcome of the second period is greater than or equal to the first one then the result would be to accept the possibility of MPT to be able to anticipate the future market, otherwise, it would reject the predictability of it.

These three questions will provide the three distinct discussion hypotheses in the penultimate chapter. To assess the hypotheses, the CAPM, which is widely utilized in the past literature, is chosen.

IV. Methodology

This research wants to test the model in the German stock exchange market, as a powerful, liquid, and fully industrialized market, to see whether it is applicable and accepted by investors or not.

The following literature review's particular articles served as the foundation for the methodology.

The next two mentioned papers, combined, prepare the method. The CAPM application is illustrated by Fama& French, 2004 in “The Capital Asset Pricing Model: Theory and Evidence”. Following that, the beneficial and complementary part of the methodology is brought by Radović and colleagues, 2018, in “The Application Of The Markowitz's Model In Efficient Portfolio Forming On The Capital Market In The Republic Of Serbia” to describe two points, firstly “the expected risk of the portfolio and individual shares”, and secondly “the beta coefficient”.

A. Initial Considerations

1. Study design

This study is assumed to have applied research design as it is trying to answer the questions based on the already existing statistical method of portfolio optimization. On the other hand, it falls in the evaluative studies group. Additionally, its strategy is quantitative and longitudinal based on the data from the market for two years, going to be calculated by use of mathematical formulas; rate of return, standard deviation, correlation coefficient, variance, covariance, beta coefficient, and linear regression.

It has the positivism philosophy, as its ontology is real, external, independent, and based on true and realistic data. In terms of epistemology, it employs the scientific method, which is measurable, observable, and factual, because its elements of data have such characteristics. From the view of axiology, the researcher is totally independent of the data, neutral, and objectively focused.

2. Markowitz Model Elucidation

The optimization process takes place in a space of random possibilities that is a solution for all different problems of any kind, one of which is portfolio optimization.

In $x \in S^n$, x is any kind of valuable items such as securities, assets, stocks, shares, and so on, all as members of S which is the whole space of possibilities and n is the number of items being used in the portfolio.

Definition 1: The portfolio is efficient by minimizing the risk while the return is greater than or equal to a significant amount of μ

- Minimize the variance as the sign of risk: Var x^T R, $x \in S^n$
- $\sum_{i=1}^{n} x_i$ = 1
- Subject to the return x^T * R ≥ μ
- R is the rate of return
- 0 ≤ x ≤ 1

Definition 2: The portfolio is efficient by maximizing the return while the risk is smaller than or equal to a significant amount of q

- Maximizing the return: Var x^T * R, $x \in S^n$
- $\sum_{i=1}^{n} x_i$ = 1
- Subject to the risk x^T R ≤ q
- R is the rate of return
- 0 ≤ x ≤ 1 (Pichler, 2020)

B. CAPM

The Capital Asset Pricing Model (CAPM), being introduced by William Sharpe in 1964, and then continued by John Lintner in 1965 was a starting point of asset pricing theory which resulted in the Sharpe Nobel Prize in 1990. It is still useful and applicable widely, for example in the estimation of the cost of capital for firms and the evaluation to perform portfolios. The attractive part of this model is to measure the risk of an activity and how it is related to the return. (Fama & French, 2004)

The CAPM was built based upon a model of portfolio selection being matured by Harry Markowitz in 1959. The basic idea of Markowitz was that an investor chooses a portfolio at time t-1 and expects a return at time t. The first assumption is that the investors are averse toward risk, so their options are portfolios with a "mean-variance-efficient" characteristic. Thereby, the chosen portfolio must have one of the two conditions; either minimizing the variance as a sign of risk with a significant return or maximizing the return with a significant variance. This is the reason that The Markowitz Model is called sometimes The Mean-Variance Analysis. (Fama & French, 2004)

Brealey in 1991 discussed the contributions of the Markowitz model to economics and approved the fact that mean-variance analysis turned out to become the standard methodology to choose the normative portfolio.

Fama & French (2004) clarified that Sharpe and Lintner had enhanced the Markowitz model by adding two new assumptions. The first one is called "complete agreement" which states that based on market clearing asset prices at time t-1, all investors agree on the joint distribution of their stock returns during t-1 and t. The second one is called "borrowing and lending at a risk-free rate" which means all investors have the same opportunities to access the risk-free rate resource of both investing to take interest or borrowing to provide interest for the resource from which they have borrowed firstly, not depending on the amount of lending or borrowing.

In figure 1, the CAPM and portfolio prospects are shown graphically. The vertical axis and horizontal one, respectively, are the rate of return and standard deviation, while the latter refers to the risk of the portfolio. The "minimum variance frontier", the curve *abc*, indicates a sequence of expected risks and returns in a risky-asset portfolio that minimize the return variance. This minimization happens at many different levels of expected return. These kinds of portfolios are not using the risk-free amounts of borrowing and lending money. Every point of the curve is suitable for every investor related to their specific risk attitude. For example, a highly risk-retention person chooses the point a, as it has almost the highest possible of both risk and return. On the other hand, the opposite side is point c with again the nearly highest possible rate of volatility but about the lowest return. To see the optimum possibility, we must consider point T as the best opportunity when it comes to having the highest possible yield with the lowest probability of instability. Therefore, the section between b and a is the mean-variance-efficient part as it maximizes the return as relevant to their point-to-point risk applicable, while it does not consist of a risk-free asset. (Fama & French, 2004)

The other line of figure 1 is the straight line from R_f to g, which uses the risk-free lending/borrowing asset too. It means that by adding the risk-free asset, the curve efficient set turns into the straight one, having a combined x percentage of risk-free plus 1-x percentage of other assets with intrinsic risk, g. The R_f point is the point that all of the investment is allocated in the risk-free advantage, which is a completely stable portfolio, with no variance at all, with a fixed rate of return. The right section of the g point is the portfolio consisting of borrowing from risk-free resources to invest in risky assets. (Fama & French, 2004)

The formulas for the straight line of R_f – g are as follows: Risk-Free Portfolio Straight Line Formula:

$R_p = xR_f + (1\text{-}x)R_g$ *IV-I*

$E(R_p) = xR_f + (1\text{-}x)E(R_g)$ *IV-II*

$\sigma(R_p) = (1\text{-}x)\sigma R_g,\ x \leq 1{,}0,$ *IV-III* *(Fama & French, 2004)*

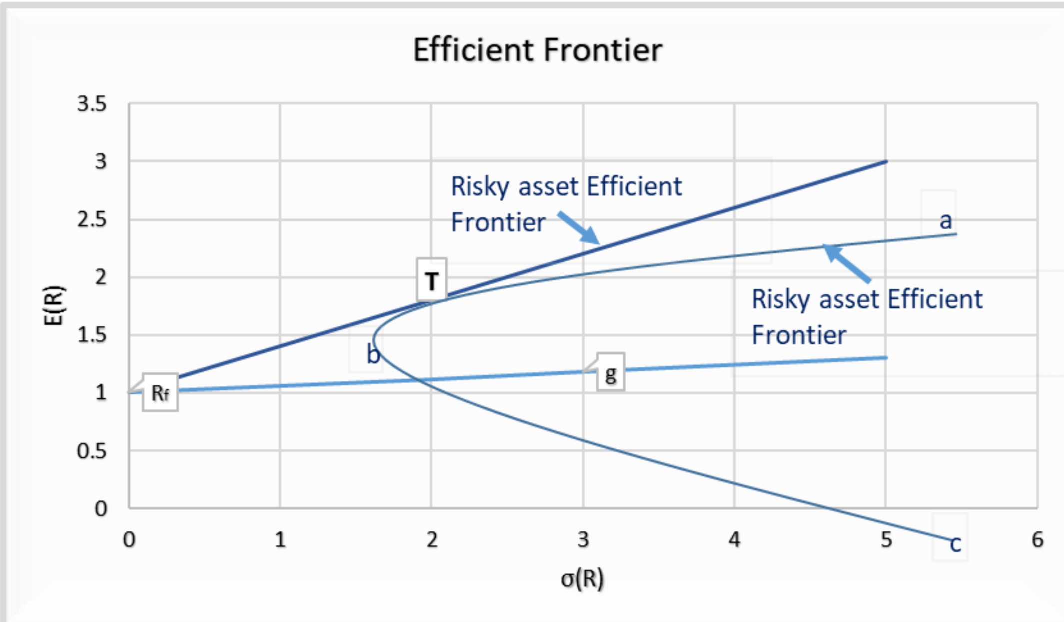

Figure 1: Efficient Frontier (Provided by the Author)

C. Expected Return on one Share

The expected return on a portfolio is the weighted average of total stocks by using the probability of them in our calculation as follows:

- Expected return on portfolio with two shares = $r_1p_1 + r_2p_2$
- Expected return on the share *E(Ri)*, for a sample period of ownership of the share, is the mean of the total returns in that period of T:

  $E(R_i) = \frac{1}{T}\sum_{t=1}^{T} R_{it}$ IV-IV

  - T: Time, e.g. the number of days
  - R_{it}: The return of asset i in the period of t, which can be sometimes an average itself, e.g. if T is the number of months, then the average of every month separately is used for R_{it}
- Expected Portfolio Return: R_p

  $E(R_p) = \sum_{i=1}^{n} x_i R_i$ IV-V

  - x_i: percentage or probability of a share in a portfolio
  - n: number of shares in the portfolio (Radović, et al., 2018)

D. The Rate of Return

The rate of return in stock investment is defined as the difference between the price of a share two consecutive times divided by the first price. The share price might have some adjustments related to the paid dividend and share splits (Biktimirov and Barnes, 2003). The data for the current research is downloaded from Yahoo Finance, including both dividend and splits adjustments.

E. The Expected Risk of Portfolio and Individual Shares

As risk is considered to be related to volatility and uncertainty, it is calculated as the standard deviation of a security or an individual share. Standard deviation is the root of variance and variance is the square deviation of the expected return as in the following formulas are shown. (Radović, et al., 2018)

- Portfolio and share Variance for a multi-stocks portfolio
  - The variance of a share is the square deviation of its actual rate of expected return
    - $V_i = \frac{1}{T}\sum_{t=1}^{T}(\mathrm{E}(\mathrm{R}_{it}) - (\mathrm{E}(\mathrm{R}_i))^2$ IV-VI
  - The variance of a portfolio equals the combination of the square of the percentage multiplying the variance of each share plus the multiplication of every two shares' percentages and their covariance.
    - $V_{port} = \sum_{i=1}^{n} x_i^2 V_i + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j Cov_{ij}$ IV-VII
    - V_{port} is the variance of a portfolio
    - x_i and x_j are percentages share i and j in a portfolio.
    - V_i is the variance of one share
  - Covariance or return covariance of two shares is the correlation measurement between two values. This correlation is calculated by the weighted products of three elements; the standard deviation of each two stocks and the correlation coefficient between them.
    - Covij = σiσjρij IV-VIII
    - σ_i is the standard deviation of one share I,
    - σ_j is the standard deviation of another share j,
    - ρ_{ij} is the correlation coefficient of two shares which is the changing relationship between them, meaning when one share changes then how much

the other one in return for the first one is going to change. In other words, the correlation coefficient, mathematically, is a statistical linear tool to measure the linear connection between two variables. The range of it is from -1 to 1. The two variables are perfectly and negatively correlated when ρ_{ij} = -1 and so when one is going up then the other simultaneously in the opposite way going down, while +1 means those two are perfectly and positively correlated, resulting to change in the same direction and the same amount. In the middle, zero means the variables are not correlated at all. The other amounts on both minus and plus are showing the negative and positive relationship respectively but not perfectly.

- The standard deviation of a portfolio
  - $\delta_{port} = \sqrt{V_{port}}$ IV-IX
  - Standard deviation is in general the root of the variance of any kind of variable. So, for portfolios, it is the root of the variance of the portfolio.
- Efficient Portfolio
  - How to optimize a portfolio? In the Markowitz model, one way is to minimize the risk in terms of portfolio variance, keeping the return constant. So, the formula of portfolio variance is useful for minimizing the result by changing only the weights of the shares.
  - Min $V_{port} = \sum_{i=1}^{n} x_i^2 V_i + \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j Cov_{ij}$ IV-X
  - Min V_{port} is the minimum variance of a portfolio between all different states.
  - x_i and x_j are percentages of assets i and j in a portfolio which take an important role here by changing and finally making the whole result of portfolio variance minimum.
  - V_i is the variance of a share that historically is calculated and so at the time of measuring the portfolio variance and providing the minimum amount of its result must be constant.
  - $E(R_p) = \sum_{i=1}^{n} x_i \mathrm{R_I}$ = Const. IV-XI
  - The return of the portfolio is constant, in this case, because we are calculating to minimize the variance for the portfolio as a symbol of risk.
  - $\sum_{i=1}^{n} x_i = 1$
  - x_i which is the percentage or probability of a share in a portfolio must be in total equal to 1.

- n is the number of shares. It begins with 0 up to the number of assets in a portfolio
- $0 \leq x_i \leq 1$, i&j = 1, 2, 3, …, n (Radović, et al., 2018)

F. The Beta Coefficient

This coefficient represents the relationship between the return rate of an asset and that of the portfolio or systematic (market) risk.

Total risk = Diversifiable risk + Non-diversifiable risk (Omisore, 2012)

It quantifies the riskiness rate of an asset associated with the market risk, but it does not express the amount of expected return of the asset itself. The change in returns of both an asset and the portfolio consisting of a combination of assets is a grouping of the market return change and the return change in every single asset intrinsically. The expression below shows how to calculate the return of an asset using the beta coefficient, alpha coefficient, and statistical residual. (Radović, et al., 2018)

- $R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$(SLC Formula) IV-XII
  - This key formula is a type of security characteristic line (SCL) formula to estimate the regression equation. (Srivastava, 2006)
  - The SLC is one of the SIMs (Single Index Model), by which it summarizes the effects of the explanatory variables X1,..., Xd within a single variable called the index. Here in the SLC formula, every X is R as the return of assets and the index is the market index. (Härdle et al., 2004)
  - The basic statistical of Beta, as a prime linear regression, is $Y = X\beta + \varepsilon$, where in this case Y is a vector describing the return of assets, X is a known matrix of market return, β is the vector of the correlation of the asset with the market and ε is the residue vector. (Manea Todose et al., 2019)
- R_{it} is the return rate of the share i in the period of t.
- α_i, the so-called alpha coefficient, the constant figure which is a part of linear regression, indicates the specific return of every individual asset independent of systematic character related to the market.
- β_i is the coefficient for every single share i.
- R_{mt} is the market return or the index of the market in the period of t

- ε_{it} is the statistical residual which expresses an inexplicable variation with a specific standard deviation of every single stock with the assumption of normal distribution. Its total equals zero ($E(\varepsilon_{it}) = 0$).
    - The process of minimizing the ε_{it}, squared residual, is the basic point of OLS regression. The process is called "ordinary least squares regression" and it is to provide the best fit as a result for the SLC formula. (Troeger, 2012)
- Calculation of the Beta Coefficient:
    - To calculate the beta coefficient, it is applicable to use the regression analysis of the rate of share and market return in a specific period. Beta is the coefficient of a linear equation that has two variables; the independent variable which represents the market index and the dependent variable which is the return of a share. So it is calculated based on the participation of values of the shares in a portfolio in the way that it is a weighted summing up of all shares' beta.
    - $\beta_p = \sum_{i=1}^{n} x_i \beta_i$ IV-XIII
    - β_p is the beta coefficient of the portfolio
    - β_i is the beta coefficient of a single stock
    - x_i is the percentage of every single stock in the portfolio
    - n is the number of members of the portfolio (Radović, et al., 2018)

G. Variables

As per the last sections, in the efficient portfolio optimization process, the portfolio beta coefficient is an independent variable that represents the market systematic risk, and the rate of return is the dependent variable.

Overall, in the initial procedure of optimization, there are beta coefficients, alpha coefficients, correlation coefficients, volatilities including variances, covariances, and standard deviations, liquidities, rate of return, and the market index from which the last three are primary data and the others are computed based on the primary data.

H. Portfolio Optimization, Non-Linearity

The problem is diminishing transparency. Representing that, e.g., in trading strategy, the process of constructing the portfolio from multiple signals and combining them, investors cannot expect the portfolio to be shaped exactly in a simple linear system. Consequently, Tricker and colleagues

(2019) concluded that mean-variance portfolio optimization is a type of multitude of non-linearity.

However, the MPT performs in a way to optimize the profit by keeping the risk fixed and it is widely acceptable and beneficial.

I. Hypotheses

1. Hypothesis One

Can a fundamental macroeconomic factor like a pandemic affect portfolio optimization? If yes, how much and in what way?

Having said that, *H0* for the first discussion as the null hypothesis is:

There is a relationship between Covid-19 and portfolio optimization.

And, *H1* for this discussion as the alternative hypothesis rejects it and is:

There is no relationship of any kind between them.

2. Hypothesis Two

The second discussion is to check the approval of MPT in the German market by looking at the linear dependency between variables.

Thereby the null hypothesis *H0,* for the second discussion is:

There is a linear dependency between the rate of returns, the dependent variable, and the beta coefficient, the independent variable.

And *H1* as the alternative one says*:*

There is no dependency between them.

3. Hypothesis Three

The weaknesses of the Markowitz model bring us to the third question.

The third discussion's null hypothesis *H0* is:

The MPT can predict the market based on historical data from the past.

And its alternative hypothesis H1 states:

The predictability of MPT is rejected.

V. The Data Analysis

A. The market and the package

Deutsche Börse Group’s index offers a range of indices that are recognized for maximum reliability, transparency, and precision, having more than 12,000 indices. DAX 30[7] is one of the most famous ones that proposes the stockholders' opportunities to deal with specific situations and prepare powerful tools as well as simple systems for them to make the best decisions. (Deutsche Börse AG., 2022c)

Since the financial and operational situation of companies during the longer term can be drastically volatile, the shorter period for MPT is more suitable and effective. It is contrary to the popular stereotype for very long periods, which caused many misunderstandings and wrongly applied the mean-variance optimization in many articles. (Keller, Butler and Kipnis, 2015). So this paper extends the timetable only for one year, 12 months.

Endogeneity occurs when a variable, observed or unobserved, that is not included in our models is related to a variable that is incorporated in our model. (Antonakis et al., 2014) To avoid this problem, restriction of the period is acceptable since it limits the economic, especially macroeconomic, and fundamental irrelated factors.

Knowing this key information, the index table of DAX during 2019 and 2020 is used to calculate the beta coefficient.

[7] It was changed to DAX 40 in 2020, however, as this article is going to perform from 2019, DAX 30 is acceptable.

Date	Open	High	Low	Close	Adj Close	Volume
01.01.2019	10477.77	11321.62	10386.97	11173.10	11173.10	1869330700
01.02.2019	11198.46	11556.87	10863.56	11515.64	11515.64	1606225800
01.03.2019	11584.24	11823.29	11299.80	11526.04	11526.04	1867513900
01.04.2019	11617.18	12376.06	11612.70	12344.08	12344.08	1670784200
01.05.2019	12349.10	12435.67	11662.07	11726.84	11726.84	2168473300
01.06.2019	11661.12	12438.37	11620.64	12398.80	12398.80	1852946200
01.07.2019	12616.34	12656.05	12115.28	12189.04	12189.04	1874735700
01.08.2019	12134.71	12254.03	11266.48	11939.28	11939.28	1995691100
01.09.2019	11939.99	12494.25	11869.28	12428.08	12428.08	1753331600
01.10.2019	12469.67	12986.49	11878.98	12866.79	12866.79	1835726200
01.11.2019	12912.09	13374.27	12896.72	13236.38	13236.38	1598514500
01.12.2019	13264.93	13425.85	12886.55	13249.01	13249.01	1372865900
01.01.2020	13233.71	13640.06	12948.17	12981.97	12981.97	1737147300
01.02.2020	13033.17	13795.24	11724.12	11890.35	11890.35	2231747600
01.03.2020	12030.27	12272.99	8255.65	9935.84	9935.84	4826748300
01.04.2020	9610.67	11235.57	9337.02	10861.64	10861.64	2507299800
01.05.2020	10543.36	11813.14	10160.89	11586.85	11586.85	2221913600
01.06.2020	11586.85	12913.13	11586.85	12310.93	12310.93	2754888800
01.07.2020	12391.72	13313.90	12095.11	12313.36	12313.36	1712733000
01.08.2020	12374.46	13221.82	12365.61	12945.38	12945.38	1260622200
01.09.2020	13037.20	13460.46	12341.58	12760.73	12760.73	1617666600
01.10.2020	12812.08	13151.80	11450.08	11556.48	11556.48	1555195300
01.11.2020	11602.91	13445.11	11551.36	13291.16	13291.16	1828909000
01.12.2020	13371.66	13903.11	13009.48	13718.78	13718.78	1360090900

Table 1: DAX indices, 2019&2020, (Prepared by the Author, Source: Yahoo Finance., 2022c)

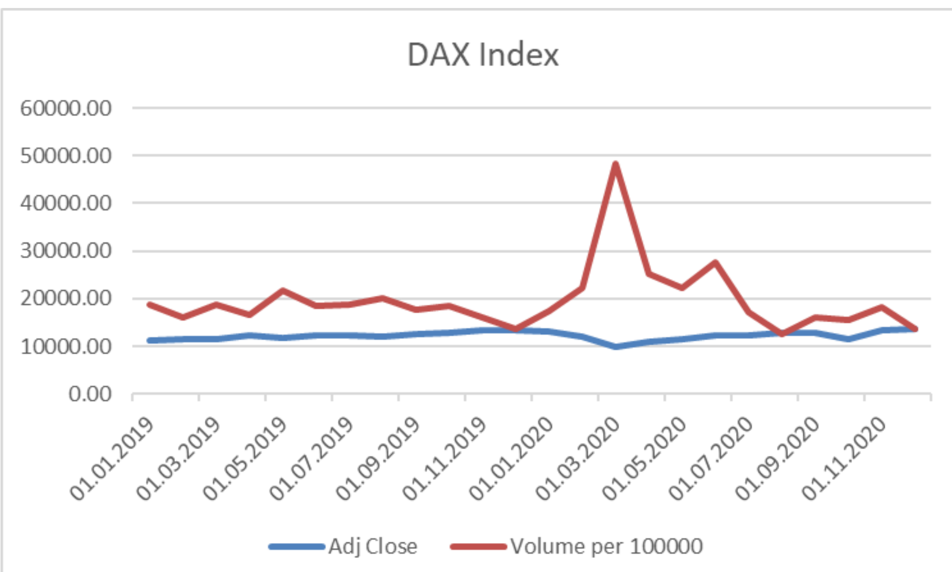

Figure 2: DAX Index, data from Yahoo Finance (Provided by the Author).

The diagram's blue line, the adjusted close demonstrates the stability of the DAX 30 index which in fact is its strength of consistency and safety for investors. On the other hand, the red line, volume, is the expression of its power of liquidity which is one of the best in the world. The volume is divided by 100,000 just to be able to fit and to be able to compare into the same diagram with the adjusted close line. The interesting point happened in March 2020 in which the volume is extremely more than other times yet there is a moderate fall in the adjusted close, representing that at the time the investors made an enormously huge number of deals but with significantly lower amounts. This event happened in the third month of officially stating the Covid-19 start point, identifying the probable excitement of more people to make deals with very lower amounts. However, this event does not disturb the current study.

B. Share selection on the capital market

1. Data collection methods and instruments

The selection process of shares can be based on the fundamental factors of companies. It is important to decrease the systematic risk by considering the fundamental factors, such as revenue, size, and some financial ratios. That is because of the deficiency of MPT, as Chandra (2003) suggested, in increasing the systematic risk of the portfolio while decreasing the unsystematic risk.

Grounded on the MPT's basic concept of lessening the unsystematic risk, the systematic risk which fundamentally stems from the characteristics of assets themselves and their market dependencies is not recognized and solved by MPT. Therefore, to diminish the systematic risk in the selected portfolio a group of 21 companies from the DAX 30 in the German stock market is established. The criteria for sampling are market capitalization, liquidity, level of turnover, and industrially diversification.

Associated with the data sampling and the intrinsical type of data which is financial numbers and the fact that it has been formed in the past, longitudinal data, this paper started firstly to collect it from some databases like Financial Times, Bloomberg, Yahoo Finance, QuantShare, Börse Berlin, Foretrade, and Marketbeat. Finally, the data has been collected from Yahoo Finance in Excel files in CSV format.

2. The criteria for selecting the shares

The criteria of the shares of the portfolio composition are market capitalization, liquidity, and level of turnover. Stemming from the last sections, all DAX companies are with high liquidity, turnover, and market capitalization. But the most important point of view is sector diversification. The reason for that is the original reason for diversification, meaning the portfolio shares must come from various sectors of industries. From 30 firms, as in table 2, five of them in red are eliminated, being explained in table 10 based on negative annual return in 2019. Then to reach the number of 21, two companies from "consumer discretionary", one from "Multi-Sector", and one from the "chemical industry" are canceled out, because we have 5 from the first group, 3 from the second one, and 2 from the last one. Therefore, in this way, the diversification of our portfolio is the highest possibility among the 25 stocks.

Symbols	Sector	Company Name
1COV.DE	Multi-Sector	Covestro AG
ADS.DE	Consumer Discretionary	adidas AG
AIR.DE	Aerospace	Airbus SE
ALV.DE	Finance	Allianz SE
BAS.DE	Chemical Industry	BASF SE
BAYN.DE	Health and Agriculture	Bayer Aktiengesellschaft
BEI.DE	Skin Products	Beiersdorf Aktiengesellschaft
BMW.DE	Consumer Discretionary	Bayerische Motoren Werke Aktiengesellschaft
CON.DE	Automative	Continental Aktiengesellschaft
DB1.DE	Exchange	Deutsche Börse AG
DBK.DE	Banking	Deutsche Bank Aktiengesellschaft
DPW.DE	Post and Delivery	Deutsche Post AG
DTE.DE	Telecommunication	Deutsche Telekom AG
DTG.DE	Automobile	Daimler Truck Holding AG
FME.DE	Health-Care Manufacturing	Fresenius Medical Care AG & Co. KGaA
FRE.DE	Health Equipment	Fresenius SE & Co. KGaA
HEI.DE	Cement and Aggregate	HeidelbergCement AG
HFG.DE	Food Services	HelloFresh SE
HNR1.DE	Finance and Insurance	Hannover Rück SE
IFX.DE	Semi-conductors	Infineon Technologies AG
LIN.DE	Gas and Engine	Linde plc
MRK.DE	Multi-Sector	MERCK Kommanditgesellschaft auf Aktien
MTX.DE	Engine Manufacturing	MTU Aero Engines AG
PUM.DE	Consumer Discretionary	PUMA SE
RWE.DE	Multi-Sector	RWE Aktiengesellschaft
SHL.DE	Health Care	Siemens Healthineers AG
SIE.DE	Multi-Sector	Siemens Aktiengesellschaft
SY1.DE	Chemical Industry	Symrise AG
VOW3.DE	Consumer Discretionary	Volkswagen AG
ZAL.DE	Consumer Discretionary	Zalando SE

Table 2: DAX Shares Chosen List (Prepared by the Author, Source: Yahoo Finance)

Referring to table one displays the high liquidity for all of the shares without any problem. Among them, DBK.DE has the highest liquidity as well as volatility in 2019, but it has the lowest return. On the other hand, AIR.DE in 2020 has the lowest return which is minus, and the lowest liquidity as well, but the penultimate highest volatility in terms of standard deviation. Back to the volatility issue, when a share like DBK.DE has the highest liquidity, it does not mean essentially a good position by paying attention to its high volatility.

There are some unexpected stocks like ADS.DE, 2019, with high return and beta as a favorite aspect for an investor, but high volatility, and low liquidity as avoiding aspects. Concerning the beta as the systematic risk which is not diversifiable, some shares are in 2019 above the red line of one in their beta coefficient, and then in the next year, this position is changed. Anyway, some of them are risky in this concern in both periods; BAS.DE, BAYN.DE, BMW.DE, CON.DE, DBK.DE, IFX.DE.

Normally, it is happening that when one share has high systematic risk, its intrinsic unsystematic risk is high too, but there appear also some exceptions such as HFG.DE that in both years stayed at below than one for its beta whereas its volatility rang the bell of risk for stockholders.

C. Data Validation

1. Correlation Coefficient Test

Predictive Validity is usually employed in evaluation, especially for applied research. As an analysis, it is constructed to predict some future events. The higher the correlation the greater the validity of the predictor, while the correlation is measured between the variables. In the perfect case which is not real, the correlation coefficient is 1, but for most real situations, the modest possibilities are something between 0.3 and 0.6 (Mohajan, 2017). The coefficient of determination, R_2, is illustrated as the definition of the total squares related to the regression and then divided by the sum of total squares. Generally, the interpretation of R_2 is the percentage of variation in the dependent variable explained by variation in the independent variables. (Figueiredo, Júnior and Rocha, 2011)

To execute it, the Regression Data Analysis from Tool Pack in Excel is used. The results of R-squared for both years 2019 and 2020 are 0.640458633 and 0.781829367 respectively. Both are bigger than 0.6, while the independent and dependent variables are weighted Beta and Return for the portfolio respectively. Consequently, there is a highly acceptable relationship between the two variables, meaning the data collection and analysis is valid from this view.

It is noticeable that beta is weighted here to calculate the R-square or correlation coefficient, based on the idea of average for both variables, as stated by Chong, Jennings, and Phillips (2018). To provide it, after finding betas for all assets, they must be multiplied by the possibility or weight of every stock represented in the portfolio.

2. Heteroscedasticity Test

Based on the different types of plotted heteroscedasticities, explained in “Interpreting Heteroscedasticity by Downs and Rocke (1979)”, the graphs of residuals as dependent variables per weighted rate of return as an independent variable for both years show no problematic heteroscedasticity as a barrier to continue the analysis.

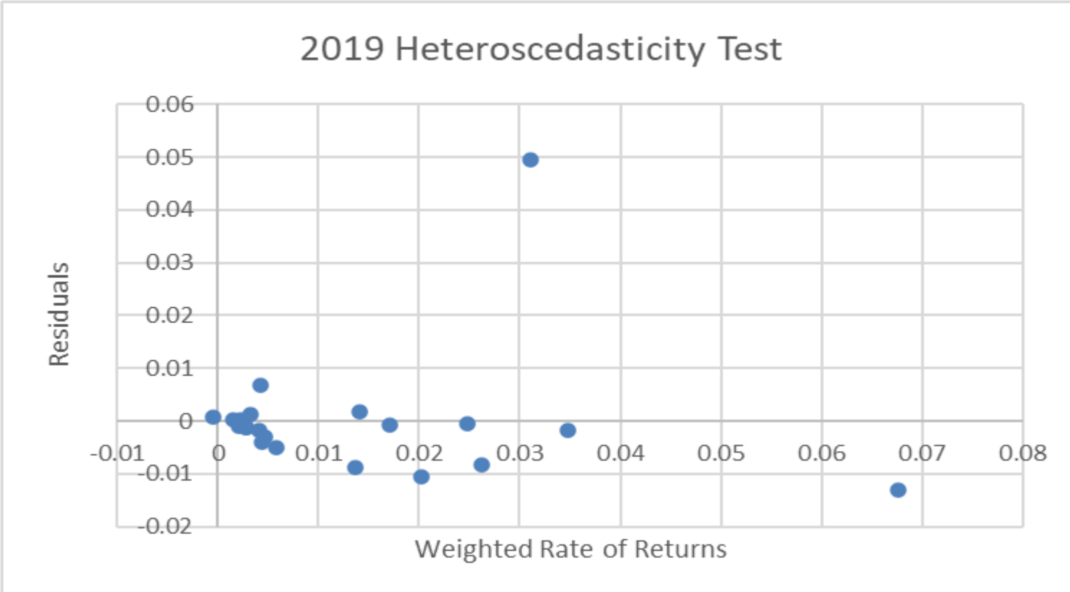

Figure 3: Heteroscedasticity Test, 2019 (Provided by the Author)

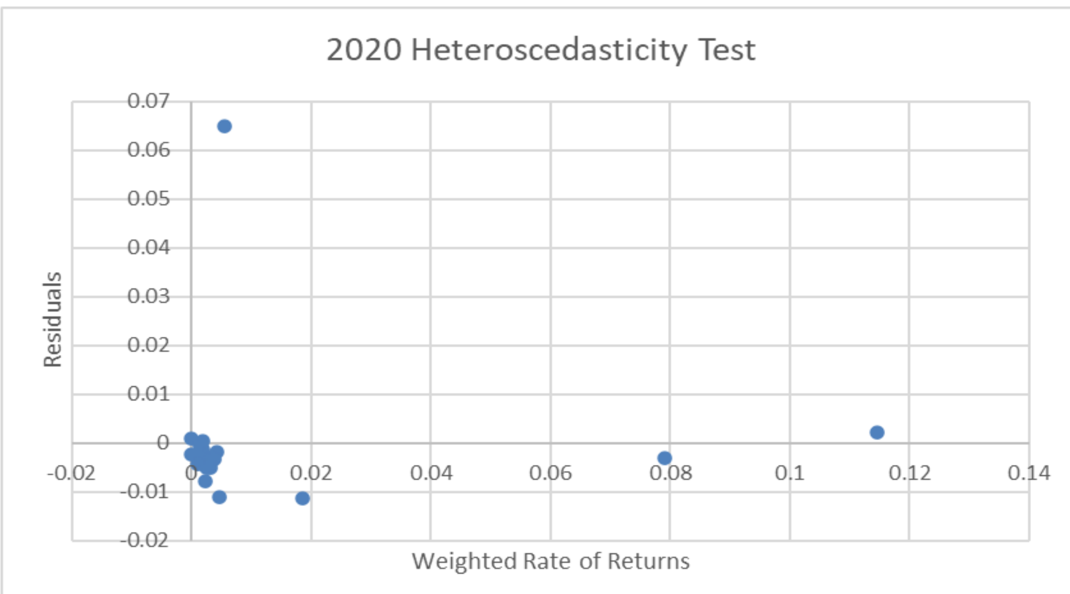

Figure 4: Heteroscedasticity Test, 2020 (Provided by the Author)

Compared to the graphs from the resource, Downs & Rocke, which are either highly scattered or widening alongside the x-axis in the cases of heteroscedasticity, this paper's graphs show no noticeable problems.

3. Multicollinearity Test

Variance Inflation Factor (VIF) is widely applied to measure whether the variance of the estimated regression correlation coefficient is inflated or not, and if it does how much. Also, whether the independent variables are correlated or not.

The formula is

VIF = (1 – R^^{2})^$^{-1}$ = 1/ Tolerance *V-I*

Where the inverse of the VIF is simply the tolerance. The higher the tolerance the less possibility might be of multicollinearity amid the variables.

- VIF =1 represents the not-being correlated for independent variables to each other.
- 1< VIF < 5 indicates the moderate correlation between independent variables.
- 5< VIF < 10 shows the highly correlated independent variables meaning multicollinearity among the predictors
- VIF ≥ 10 denotes the weakness of regression coefficients estimation with the presence of multicollinearity

(Shrestha, 2020)

Multicollinearity Test for 2019: 1/(1-R Square) = 2.781321129

Multicollinearity Test for 2020: 1/(1-R Square) = 4.583568312

The results are between 1 and 5 which states a moderate correlation, justifying the approval.

4. Autocorrelation Test

OLS is the popular ordinary least square which is an estimator. Because the linear regression model is very important in econometric modeling and the data will be inconsistent when the model's residuals are autocorrelated, the data must be confirmed not to have autocorrelation. (King and David, 1995) To do that, this paper performs the Durbin-Watson statistical significance test. The figures for this test have a range from 0 to 4 as follows:

- A value close to 2 means non-autocorrelation
- A value toward 0 specifies positive autocorrelation
- A value near 4 suggests negative autocorrelation. (Evans, 2020)

To be familiar more deeply with this matter, as Anderson et al. in 2012 explain for stock return autocorrelation. There are some different categories by their suggestions, from which the partial price adjustment (PPA) is the main resource of autocorrelation. As a result and because in a highly hyperactive and rich market like the German stock exchange the adjustments are happening usually, the results for the data of this study are completely acceptable and normal, particularly, because the DWS around 2 means no autocorrelation.

The outcome of the current study indicates the DWS as:

Durbin-Watson Test 2019: 2.091231961

Durbin-Watson Test 2020: 2.138610314

The results are near 2 which means no autocorrelation

VI. Discussion

The *H0* and alternatively *H1* are discussing whether there is any or no relationship between two variables or two events. (Helwig, 2020) This study provides three discussions from which three outcomes are expected and in which three *H0* hypotheses are evaluated.

But before them, the matrix of variances and covariances presents the essential parameters to form the set of optimal portfolios. This matrix is a symmetrical square matrix with dimensions of 21 x 21, which is the number of shares that form the portfolio. The calculation of the matrix of variances and covariances of shares has been done by matrix calculation in Excel, and examinations have been conducted based on the expression III-VI & III-VII, described in the methodology section. Its diagonal members are equal to variances of individual shares, and other elements are covariances of couples of shares. The matrices of variances and covariances for both years have been provided in appendix III.

Some of the covariances are negative like BMW and RWE in 2019, representing the opposite correlation between them, while the same factor in the next year is a very small positive amount, signifying approximately no correlation between them.

In total, by looking at both tables of these matrices, it is understandable that the whole covariances are either minus or extremely small due to a perfectly uncorrelated stock in DAX 30, indicating an outstandingly great opportunity for diversification.

The basic idea of forming the optimized portfolio is the percentage of participation of every individual stock in it and the parameters are important to be considered; vector of returns of stocks, vector of variance, standard deviation, and covariances between them.

This process of optimization has been performed based on the formulas mentioned in the methodology section and using the MS Solver. In MS Solver, the formula that is used to estimate the efficient portfolio has to multiply the daily return and volatility by 251 for the year 2019 and 254 for the next year, associated with the working days of the German stock exchange market to annualize the figures. This is based on the idea of making the estimator more accurate, such as one that Andrew Lo (2002) clarifies in his article "The Statistics of Sharpe Ratios", by using time aggregation instead of performing the annual data directly, meaning by multiplying the daily data with the number of weekdays in the year.

Then, the Sharpe Ratio which is the division of the rate of return per standard deviation is the major criterion to be the most possible amount. (Sharpe, 1994) This is the final part of computing the best optimum for the portfolio.

In figures 5 and 6, possible portfolios are brought, being connected to draw the line of the efficient frontier. Some portfolios with lower variance are the set of efficient ones and perform the efficient frontier. By analyzing and comparing them, it is clear that the best-optimized portfolio is in the front of the curved line, which has the most Sharpe ratio. The figures for beta in both time phases are a little more than 62 percent which is less risky since they are noticeably less than 1. Having said that, it is important to know that 62% means the portfolio has been affected by 62 percent of the systematic risk of the market. In this case, when the index market for example rises by 1 degree then the portfolio increases by 62 percent of that unit and vice versa.

Because of the optimization calculation, there are no other possibilities for portfolios rather than the most optimized. To understand it more clearer, it is noticeable that all individual shares have been plotted under or inside the efficient frontier. Notice that diversification cannot reduce the systematic risk but only can reduce the unsystematic risk.

A. Analysis of the First Hypothesis

Already, we know that the Covid-19 pandemic has affected the markets in many aspects, usually negatively, causing a widespread strong recession. However, there is a question of whether this disaster can influence portfolio optimization or not, and if yes, how much and in what way.

In the following tables, the calculations for the rate of return, the average rate of return, percentage or weight of share, weighted rate of return, liquidity of share, volatility in both standard deviation and variance forms, the coefficient of beta, weighted beta, and finally alpha are presented for both years of 2019 with 251 working days and 2020 with 254 working days for 21 stocks from DAX 30, out of the German stock exchange market.

The outcome is the most possible fitted portfolio, computing by the Excel Solver, based on the Sharpe ratio to be the maximum which equals the dividing of portfolio return on standard deviation.

n: number of shares in the portfolio
Expected Portfolio Return: E(Rp) = Σ(xiRi)
xi: percentage, weight or probability of a share in a portfolio

Ri, The return of every single asset is the average of the returns from the whole period, in 2019 with 252

Symbol	Ri: Retrun	Average Return	xi: Percentage	Ri*xi	Liquidity	Volatility: STD	Volatility: Var	Beta	Weighted Beta	Alpha
RWE.De	0.3218778	0.0012824	0.03	0.011034357	3355290.9	0.013883798	0.00019276	0.383051405	0.013131463	0.001024002
ADS.DE	0.4410920	0.0017573	0.12	0.054429299	641412.3147	0.014184698	0.000201206	1.018350353	0.12566109	0.001076987
AIR.DE	0.2596651	0.0010345	0.01	0.00259665	72783.85259	0.013206208	0.000174404	0.957053356	0.009570534	0.000397572
ALV.DE	0.1926868	0.0007677	0.01	0.001926868	1023460.825	0.007865969	6.18735E-05	0.834260018	0.0083426	0.000212981
BAS.DE	0.0433854	0.0001729	0.01	0.000433854	2908993.247	0.012654128	0.000160127	1.329398333	0.013293983	-0.000706892
BAYN.DE	0.2274263	0.0009061	0.01	0.002274263	3203740.952	0.015379241	0.000236521	1.295759976	0.0129576	4.56722E-05
BEI.DE	0.2960244	0.0011794	0.27	0.080720271	438988.8526	0.009455485	8.94062E-05	0.222854379	0.060768181	0.001027339
BMW.DE	0.1662389	0.0006623	0.01	0.001662389	1690380.215	0.01135332	0.000128898	1.066068916	0.010660689	-4.52531E-05
CON.DE	0.2041144	0.0008132	0.02	0.004925333	658092.0956	0.016540024	0.000273572	1.239639528	0.029912814	-9.72595E-06
DB1.DE	0.2626467	0.0010464	0.06	0.016262905	579953.9681	0.010711058	0.000114727	0.580890547	0.035968341	0.000658139
DBK.DE	0.0723764	0.0002884	0.01	0.000723764	15015869.7	0.021488088	0.000461738	1.598428847	0.015984288	-0.000769743
DPW.DE	0.2371860	0.0009450	0.04	0.009770019	3122399.956	0.010237047	0.000104797	1.010282999	0.041614958	0.000273171
DTE.DE	0.0262936	0.0001048	0.01	0.000262936	10155726.68	0.007086163	5.02137E-05	0.471470501	0.004714705	-0.000207417
SHL.DE	0.3242811	0.0012920	0.10	0.033136774	522064.004	0.012694829	0.000161159	0.660262891	0.067469191	0.000850235
MTX.DE	0.3477283	0.0013854	0.05	0.015881121	146387.9482	0.012881489	0.000165933	0.667535003	0.030487033	0.000938471
PUM.DE	0.1147637	0.0004572	0.01	0.001147637	389310.1673	0.016316682	0.000266234	0.926946143	0.009269461	-0.000157526
HFG.DE	0.2801105	0.0011160	0.02	0.004429373	396117.5378	0.025623298	0.000656553	0.71926983	0.011373777	0.000635937
HNR1.DE	0.2085644	0.0008309	0.09	0.017872299	152834.8526	0.00889216	7.90705E-05	0.60831604	0.052127803	0.000425392
IFX.DE	0.1660526	0.0006616	0.01	0.001660526	6861870.08	0.017207087	0.000296084	1.380819001	0.01380819	-0.000254119
LIN.DE	0.1515345	0.0006037	0.01	0.001515345	792591.2032	0.011565556	0.000133762	1.048091828	0.010480918	-9.17166E-05
MRK.DE	0.2616462	0.0010424	0.09	0.02434322	503144.7809	0.010561216	0.000111539	0.532854645	0.049576095	0.000685932
Totals	4.6056952	0.0183494	100%	29%	52631414.13	0.2797875	0.0041206	18.5516045	0.6271737	0.0060094
				E(Rp) = Σ(xiRi)		Optimal STD				
Sharpe Ratio	282%					10%				

Table 3: Empirical Results, 2019 (Calculated by the author)

n: number of shares in the portfolio Expected Portfolio Return: E(Rp) = Σ(xiRi) xi: percentage, weight or probability of a share in a portfolio Ri, The return of every single asset is the average of the returns from the whole period, in 2020 with 254										
Symbol	**Ri: Retrun**	**Average Return**	**xi: Percentage**	**Ri*xi**	**Liquidity**	**Volatility: STD**	**Volatility: Var**	**Beta**	**Weighted Beta**	**Alpha**
RWE.De	0.0953955	0.0003756	0.01	0.000953955	3098460.78	0.018605894	0.000346179	0.835406351	0.008354064	0.000592045
ADS.DE	0.2280836	0.0008980	0.01	0.002280836	804349.6102	0.019030019	0.000362142	0.93271725	0.009327172	0.001139654
AIR.DE	-0.6174235	-0.0024308	0.01	-0.00617424	305035.4449	0.037174822	0.001381967	1.558040547	0.015580405	-0.002027079
ALV.DE	-0.1892476	-0.0007451	0.01	-0.001892476	1683857.055	0.0197272	0.000389162	1.233047022	0.01233047	-0.00042556
BAS.DE	-0.0837582	-0.0003298	0.01	-0.000837582	4127895.236	0.019447263	0.000378196	1.229729262	0.012297293	-1.11066E-05
BAYN.DE	-0.5491414	-0.0021620	0.01	-0.005491414	3949866.5	0.018745783	0.000351404	1.02893973	0.010289397	-0.001895353
BEI.DE	0.0886404	0.0003490	0.01	0.000886404	447913.4173	0.015198093	0.000230982	0.502598173	0.005025982	0.000479212
BMW.DE	-0.0323996	-0.0001276	0.01	-0.000323996	2053916.961	0.019860091	0.000394423	1.071047982	0.01071048	0.000149975
CON.DE	0.2273891	0.0008952	0.03	0.007333354	783941.626	0.026354335	0.000694551	1.410482626	0.045488405	0.00126072
DB1.DE	-0.0067752	-0.0000267	0.01	-6.77525E-05	664958.2008	0.01882916	0.000354537	0.879000496	0.008790005	0.000201094
DBK.DE	0.2451654	0.0009652	0.01	0.002451654	17328372	0.028890548	0.000834664	1.470985719	0.014709857	0.001346383
DPW.DE	0.0640282	0.0002521	0.01	0.000640282	4178902.039	0.01829395	0.000334669	0.918801062	0.009188011	0.000490161
DTE.DE	-0.3358069	-0.0013221	0.01	-0.003358069	12764402.66	0.013513647	0.000182619	0.730648872	0.007306489	-0.001132747
SHL.DE	-0.2259301	-0.0008895	0.01	-0.002259301	853813.4213	0.018679966	0.000348941	0.519015275	0.005190153	-0.000755
MTX.DE	0.0559865	0.0002204	0.01	0.000559865	393251.9646	0.034832091	0.001213275	1.378517565	0.013785176	0.000577624
PUM.DE	0.3639046	0.0014327	0.21	0.076019475	422628.8937	0.024157124	0.000583567	0.847326081	0.17700596	0.001652256
HFG.DE	0.4501051	0.0017721	0.16	0.070510268	1114228.193	0.039903045	0.001592253	0.110773707	0.017353023	0.001800771
HNR1.DE	-0.2431747	-0.0009574	0.01	-0.002431747	180406.2874	0.020254471	0.000410244	1.075756981	0.01075757	-0.000678628
IFX.DE	0.0254118	0.0001000	0.01	0.000254118	7219915.173	0.025053419	0.000627674	1.262592693	0.012625927	0.000427212
LIN.DE	0.1146440	0.0004514	0.01	0.00114644	1129551.976	0.015602883	0.00024345	0.855363879	0.008553639	0.000672998
MRK.DE	0.2700418	0.0010632	0.43	0.116711345	561443.6575	0.015481512	0.000239677	0.587230363	0.253799434	0.001215321
Totals	-0.0548614	-0.0002160	100%	26% E(Rp) = Σ(xiRi)	64067111.10	0.4676353 Optimal STD 21%	0.0114946	20.4380216	0.6684689	0.0050800
Sharpe Ratio	121%									

Table 4: Empirical Results, 2020 (Calculated by the author)

The first official news about the Covid-19 virus has been issued on the last day of 2019 by WHO *(WHO., 2022c)*, so the last and the first year before and after this time is subject to be covered by the paper. Coming from the tables, it is obvious that the pandemic raised the volatility in terms of the STD[8] by 11 percent, making it more than double compared to its figure the year before. On the other hand, the portfolio return falls not too much, only by 3 percent from 29% to 26%, resulting in a dramatic drop in the Sharpe ratio by about 160%, from 282% to 121%. It means that the Shape ratio is lost its power by more than half. Thereby, the effectiveness of the same portfolio but with new weights in the pandemic period is extremely lower due to the number of stocks that present a negative rate of return in the second year, whereas they brought positive yield in the first time.

Therefore, *H0* approved that MPT is significantly under the influence of the economic situation and so fundamental alteration can alter the portfolio results, meaning that investors cannot rely on MPT independently. The question for further research is whether other optimization methods can ever be independently effective without being influenced by fundamental reforms.

By looking at the weighted portfolio beta coefficients for both years, it is considerable that this figure climbed by 4% approximately. The amount of difference between the two years is not very important. But the point here is that both figures in both years (62 and 66 percent for 2019 and 2020 respectively) show a positive and strong relationship of dependency from the portfolio on the DAX market. Consequently, it clarifies and consolidates the result of *H0*, denoting that the market's impact on the portfolio plays extensively a key role.

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[8] STD: Standard Deviation

B. Analysis of the Second Hypothesis

As one of the common statistical approaches to testing the relationship between two variables, the statistical T-test is often performed to analyze the mean value of the normal distribution between an independent variable and a dependent one. (Kyaw, Lin, and Thu, 2021) Testing the hypotheses in these types of statistical problems like portfolios are being performed with the threshold of 0.05. (Kim, 2015) The slope of the statistical p-value is the criteria for that.

based on the results of tables 5 and 6, as the p-values for both years are meaningfully less than the threshold (0.01E-7 and 0.025 for the years of 2019 and 2020 respectively) the *H0* hypothesis is approved and so there is statistically a linear dependency between the rate of return and beta coefficient, denoting the connection between the rate of return of the portfolio and the market systematic risk. Therefore MPT is approved by the current research.

Results of regression analysis with threshold of significance of 0.05 for 2019						
coef. of regression	**value**	**t-stat**	**p-value**	**R2**	**cor. R2**	**significance**
intercept	-0.003144796	-0.788854885	0.439929799	0.64045863	0.6215354	
slope	0.562922048	5.817654291	1.04297E-09			Yes

Table 5: Regression Analysis for 2019, (Calculated by the Author)

Results of regression analysis with threshold of significance of 0.05 for 2020						
coef. of regression	**value**	**t-stat**	**p-value**	**R2**	**cor. R2**	**significance**
intercept	-0.00243303	-0.631223012	0.535415404	0.781829367	0.7703467	
slope	0.460761971	8.251533065	0.025236467			Yes

Table 6: Regression Analysis for 2020, (Calculated by the Author)

Symbol 2019	Ri: Retrun	Volatility: STD
RWE.De	0.3219	0.0139
ADS.DE	0.4411	0.0142
AIR.DE	0.2597	0.0132
ALV.DE	0.1927	0.0079
BAS.DE	0.0434	0.0127
BAYN.DE	0.2274	0.0154
BEI.DE	0.2960	0.0095
BMW.DE	0.1662	0.0114
CON.DE	0.2041	0.0165
DB1.DE	0.2626	0.0107
DBK.DE	0.0724	0.0215
DPW.DE	0.2372	0.0102
DTE.DE	0.0263	0.0071
SHL.DE	0.3243	0.0127
MTX.DE	0.3477	0.0129
PUM.DE	0.1148	0.0163
HFG.DE	0.2801	0.0256
HNR1.DE	0.2086	0.0089
IFX.DE	0.1661	0.0172
LIN.DE	0.1515	0.0116
MRK.DE	0.2616	0.0106
EP	0.2870	0.0064
P (1)	0.2800	0.0063
P (2)	0.2400	0.0061
P (3)	0.2200	0.0066
P (4)	0.2000	0.0075
P (5)	0.3100	0.0071
P (6)	0.3300	0.0079
P (7)	0.3500	0.0092
P (8)	0.3700	0.0101
P (9)	0.3900	0.0117
P (10)	0.3945	0.0121

In, 2019, the rate of return for the portfolio is a little less than 30 percent with a very low daily volatility of about 0.6 percent.

Grounded on the model and by looking at the 2019 efficient portfolio diagram, it is noticeable that the efficient portfolio has the optimum position in terms of the conditions of a portfolio, identifying the lowest volatility compared to the stocks.

If an investor chooses any other stocks alone with higher profitability, the risk would be higher than the portfolio. Therefore, from the results of this diagram and the fact of approving the *H0* hypothesis of this section, it is acceptable that portfolio diversification perfectly works in the German stock exchange market.

Above all, the Sharpe ratio is the most among all portfolios, verifying strongly the point that the Markowitz model of optimization applies in this market. The number of portfolios to be able to draw the curved efficient frontier is chosen to be 11 as this number does not matter practically as long as the line is applicable. Another point is that nowadays the optimum of the Sharpe ratio is simply able to be computed by modern programming, meaning that even without calculating the other portfolios, it is pragmatic to reach the point of diversification just with the optimum one easily and confidently.

Table 7: Efficient Frontier Data, 2019, (Calculated by the Author)

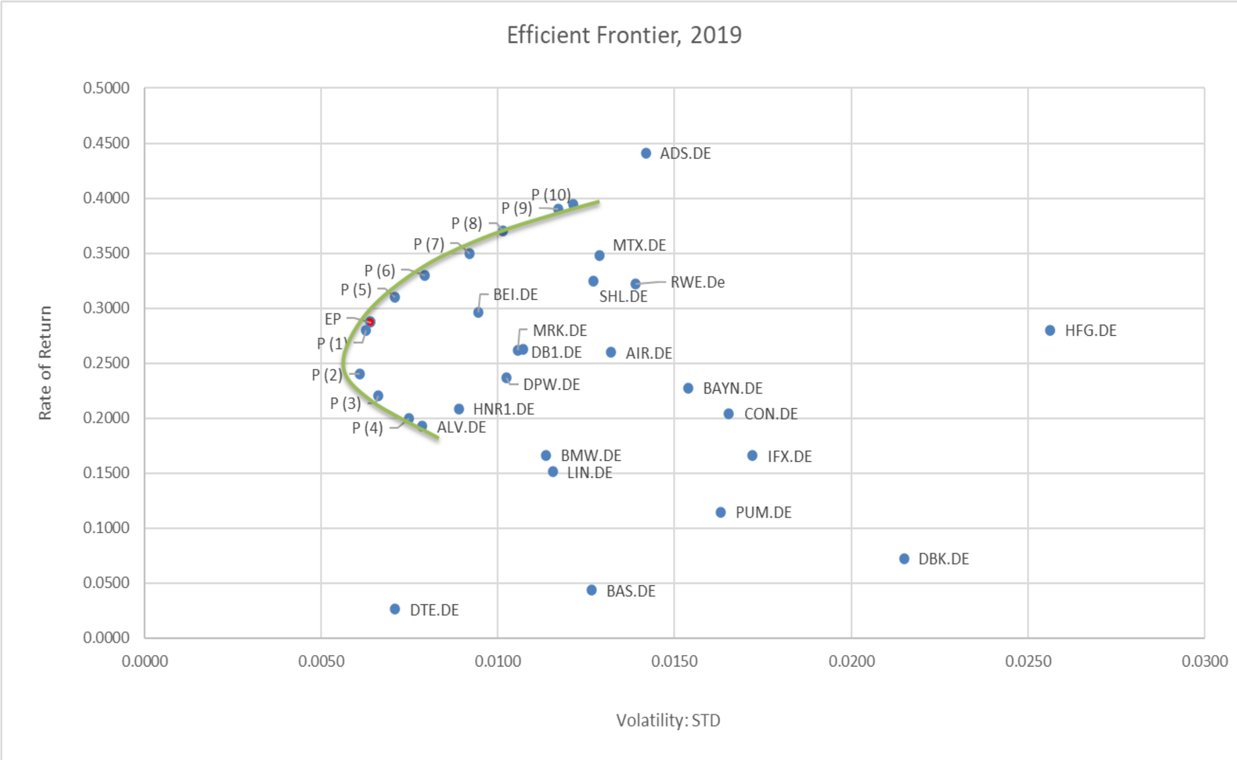

Figure 5: Efficient Frontier diagram, 2019 (Provided by the Author)

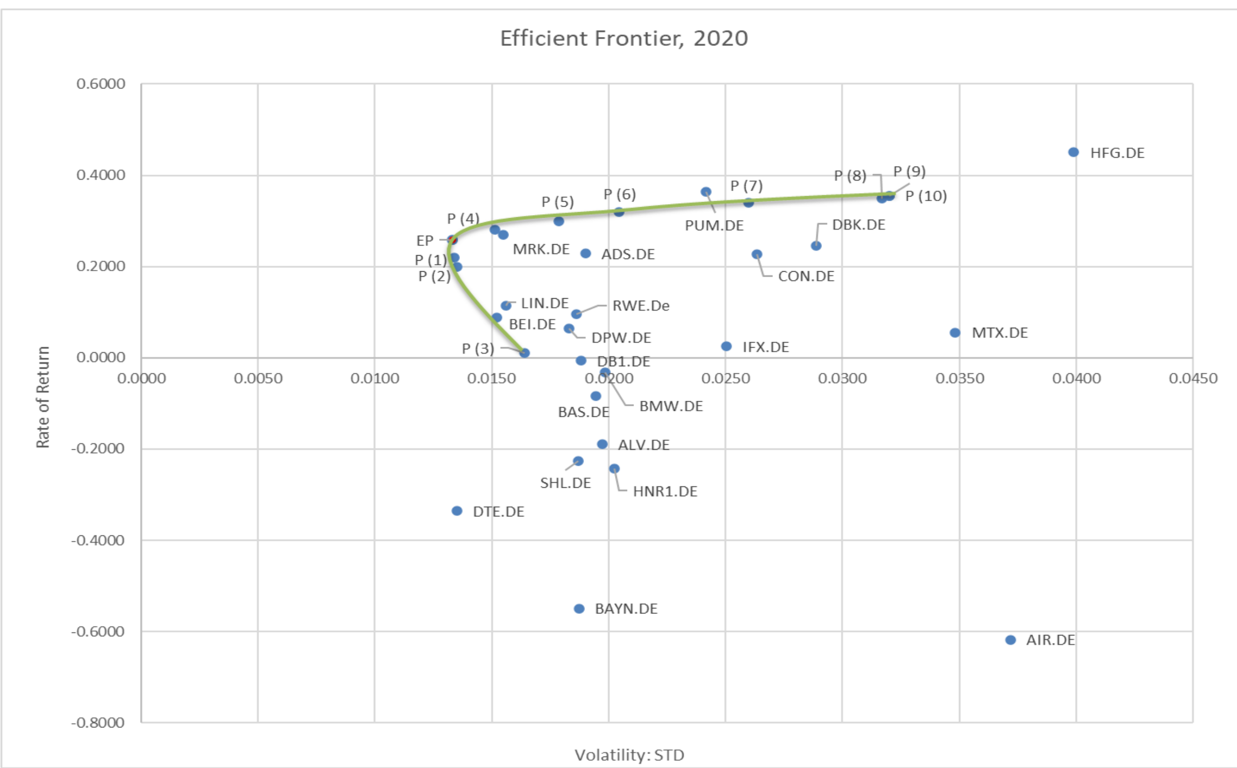

Figure 6: Efficient Frontier diagram, 2020 (Provided by the Author)

Symbol 2020	Ri: Retrun	Volatility: STD
RWE.De	0.0954	0.0186
ADS.DE	0.2281	0.0190
AIR.DE	-0.6174	0.0372
ALV.DE	-0.1892	0.0197
BAS.DE	-0.0838	0.0194
BAYN.DE	-0.5491	0.0187
BEI.DE	0.0886	0.0152
BMW.DE	-0.0324	0.0199
CON.DE	0.2274	0.0264
DB1.DE	-0.0068	0.0188
DBK.DE	0.2452	0.0289
DPW.DE	0.0640	0.0183
DTE.DE	-0.3358	0.0135
SHL.DE	-0.2259	0.0187
MTX.DE	0.0560	0.0348
PUM.DE	0.3639	0.0242
HFG.DE	0.4501	0.0399
HNR1.DE	-0.2432	0.0203
IFX.DE	0.0254	0.0251
LIN.DE	0.1146	0.0156
MRK.DE	0.2700	0.0155
EP	0.2569	0.0133
P (1)	0.2200	0.0134
P (2)	0.2000	0.0135
P (3)	0.0100	0.0164
P (4)	0.2800	0.0151
P (5)	0.3000	0.0179
P (6)	0.3200	0.0204
P (7)	0.3400	0.0260
P (8)	0.3500	0.0317
P (9)	0.3550	0.0320
P (10)	0.3550	0.0320

In 2020, the rate of return for the portfolio is more than 25 percent, less than the same amount in 2019, with low daily volatility of about 1 percent.

Again, by looking at the 2020 efficient portfolio diagram, the result is the same as the last year that the efficient portfolio has the optimum position identifying the lowest volatility compared to the stocks and other portfolios.

The result would be the same for the stockholder similar to the prior year.

Founded on diversification and the Sharpe ratio similar to the past year, this outcome is that theoretically and practically the MPT worked on the German exchange market DAX 30.

Table 8: Efficient Frontier Data, 2020, (Calculated by the Author)

C. Analysis of the Third Hypothesis

The third hypothesis is based on the part of the literature review that considers the issue of weaknesses of the Markowitz model, one of which is not considering the factual and fundamental aspects of the real market.

If looking at the 2019 portfolio stocks and comparing them with 2020, it is noticeable that many of them have a negative outcome in the second year. Consequently, the case of using the shares' weights of 2019 to establish the new portfolio in 2020 does not end in profit.

If the percentages of the portfolio had been used from the prediction of 2019 in the 2020 portfolio, then the outcome would have been 4 percent of return in total, compared with the total return in the portfolio of 2020 by its own percentages which is 26 percent. It is conspicuous that the great difference between 26 and 4 percent suggests that H1 is accepted and so H0 is rejected. So, MPT cannot predict the future.

The Prediction Application of 2019 in 2020			
Symbols	Percentages	from 2020	Weighted
RWE.De	3%	0.095395506	0.003270273
ADS.DE	12%	0.228083552	0.028144761
AIR.DE	1%	-0.617423543	-0.006174235
ALV.DE	1%	-0.189247612	-0.001892476
BAS.DE	1%	-0.083758205	-0.000837582
BAYN.DE	1%	-0.549141425	-0.005491414
BEI.DE	27%	0.088640397	0.024170562
BMW.DE	1%	-0.032399591	-0.000323996
CON.DE	2%	0.227389122	0.005486957
DB1.DE	6%	-0.006775245	-0.000419519
DBK.DE	1%	0.245165381	0.002451654
DPW.DE	4%	0.064028242	0.002637412
DTE.DE	1%	-0.335806891	-0.003358069
SHL.DE	10%	-0.225930136	-0.023086749
MTX.DE	5%	0.055986485	0.002556962
PUM.DE	1%	0.363904604	0.003639046
HFG.DE	2%	0.450105083	0.007117488
HNR1.DE	9%	-0.243174709	-0.020838121
IFX.DE	1%	0.025411815	0.000254118
LIN.DE	1%	0.114644001	0.00114644
MRK.DE	9%	0.270041758	0.02512433
	100%	Total of Return	4%

Table 9: The 2020 Portfolio based on 2019 Weights (Calculated by the Author)

Once more like one of the byproducts of the analysis in the first discussion, the outcome of the current discussion demonstrates the key role of fundamental events in the economic system in the optimization of portfolios based on historical data.

To clarify the argument, referring to table 9, basically and logically the negative stocks should not play any role in the portfolio which would not happen in the case of pre-knowledge. However, because we do not know what is going to come true in the future and we cannot predict it if stockholders arrange their portfolio in this way, then some largely negative returns would come to the portfolio, such as DB1.DE with 6%, SHL.DE with 10%, and particularly HNR1.DE with 9%. The final return turns into 4% in total.

VII. Conclusion

Established from the study results, it is noticeable that the German stock exchange market, particularly, DAX 30, is sufficiently stable and liquid for shareholders. Even when the amount of transactions is lower in one specific period of March 2020, the volume which is a sign of liquidity is exceptionally high. However, its portfolio with profitable shares cannot guarantee the future or predict it.

Regarding the first discussion's question, whether Covid-19 could influence portfolio optimization or not, the outcome reveals that H1 is rejected and *H0* is accepted. Diversification is, definitely, supportive and effective, yet it can not exclude fundamental factors like a pandemic.

Concerning the linear dependency or correlation between the dependent variable which is the rate of returns and the independent variable as the beta coefficient utilizing the T-test, the *H0* proves the linear dependency, and the alternative hypothesis is rejected. This fact defines that the DAX 30 had a considerable effect on the market index. Additionally, figures 5 and 6 and tables 7 and 8 suggest a shareholder can not find any of the individual stocks to satisfy the criteria of optimization by diversification equal to or better than that of the efficient portfolio, neither lower risk with the same rate of return nor the higher return with the same volatility. Moreover, none of the other portfolios have the same or higher Sharpe ratio as the ratio in the efficient one. It concludes that diversification is exactly and helpfully applicable in the DAX market in terms of reducing the unsystematic, diversifiable risk with the optimum return.

Considering the third discussion and its hypothesis accordingly, the MPT cannot predict the market based on historical data from the past. it represents that in the specific case of using the analysis and shares' weights of 2019 to build up a portfolio in the next year, not a desirable ending to profit would occur. If the portfolio structure is built based on the percentages of the last year, the total return would be 4 percent. Having compared the total return in the portfolio of 2020 by its weights which has a 26 percent return, it is obvious that the diversification of the Markowitz model can not anticipate the future.

In the conclusion, this paper approves the applicability of the Markowitz (mean-variance) model on the German stock exchange market, however, states that it is not predominant to predict the future of investment, suggesting that investors cannot rely on MPT independently.

Eventually, there remained one issue to need more research for others as a gap that current research is not able to follow. The question for further research is whether other optimization methods can ever be independently effective without being influenced by fundamental reforms.

VIII. Problems, Limitations, and Analysis

A. Problems and Limitations

Not all of the databases are free and easy to access, but after many searches, all of the data have been found in Yahoo Finance. (Appendix II)

Some of the stocks in the first year because of the annual negative rate of return and despite the past attractiveness to the investors, high liquidity, or any specific features must be canceled out.

Relating to some market problems like liquidity and the percentage of the trading days of one single share, fortunately, the German stock exchange DAX 30 does not have such a problem.

In the case of multiple markets, the problem could happen because of the different number of workdays, however, this study does not have such a problem.

Nevertheless, none of the problems or limitations influenced the final data, analysis process, or the ending results, but only took more time and effort.

B. Corrections

Dividends are not only important because of the payment on which the investor count, but also because they play a key role to support the firm from which the shares are being purchased. To justify this argument here are some examples of that; Zaremba and Konieczka (2014) proved that high dividend stocks substantially execute better on a risk-adjusted basis, particularly, in the big size markets, meaning that more dividend causes more market capital. Sander in the article in 2016 states that earnings yield with dividend together has a strong effect as predictors of dividend growth. Erik Börjesson and Harald Lindström (2019) admit the decline in risk to an increase in dividend exposure. And the last but not least, Erdaş (2020) explained that related to dividends, to have the calculations exactly realistic, the reflection of them to the return calculation of the share price is important.

Thereby to adjust the share price by dividend to take the effects of dividends into account, this paper uses Yahoo's adjustment method, which is:

Adjustment = 1 – (Dividend / Share Price) VIII-I *(Felton and Jain, 2018)*

All share prices before the date of dividend are multiplied by this adjustment. In this way, the reduction of the share price after the dividend payment had been prepared beforehand.

On the other hand, if a split on the share happens, then all the share prices before the date of the event are divided by the rate of the split. For example, if the split is 2:1 then the old prices are divided by 2. (Felton and Jain, 2018)

Yahoo Finance prepares the data after adjustment based on the above methods. (Yahoo Finance., 2022f) Consequently, the data for share price from this resource does not need any adjustment.

However, due to the problem of data after downloading from Yahoo Finance which is related to the difference between comma and dot as decimal or thousand separators, a correction had to be made to achieve the correct value of the final return rate. It was done by multiplying the figures by 10^(-6). It, also, could be done based on the settings in Excel as Excel Tech Community, and Support. Microsoft, 2022a, explained.

The companies listed in the following table have been eliminated owing to the problems that are stated.

Company Name and their Symbols	The Reasons for Elimination
Daimler Truck Holding AG (DTG.DE)	The data is not available for 2019 and 2020. (Yahoo Finance., 2022b)
HeidelbergCement AG (HEI.DE)	The average adjusted return is negative. The figure equals approximately -0.066
Fresenius SE & Co. KGaA (FRE.DE)	The average adjusted return is negative. The figure equals approximately -0.063
Covestro AG (1COV.DE)	The average adjusted return is negative. The figure equals approximately -1.753
Fresenius Medical Care AG & Co. KGaA (FME.DE)	The average adjusted return is negative. The figure equals approximately -5.568

Table 10: Removing Some Stocks (Provided by the Author)

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X. Appendices

Appendix I

Some additional optimization methods or portfolio management techniques:

- Development of cogeneration in Germany: a mean-variance portfolio analysis of individual technologies prospects given the new regulatory framework (Westner and Madlener, 2011)
- On-line portfolio selection strategy with prediction in the presence of transaction costs. (Albeverio, Lao and Zhao, 2001)
- Optimal portfolio selection when stock prices follow a jump-diffusion process. (Wenjing Guo and Chengming Xu, 2004)
- Portfolio optimization under transaction costs in the CRR model. (Sass, 2005)
- A unified approach to portfolio optimization with linear transaction costs. (Zakamouline, 2005)
- Elasticity approach to portfolio optimization. (KRAFT, 2003)
- Risk-sensitive portfolio optimization. (Stettner, 1999)
- Optimal portfolio policies under bounded expected loss and partial information. (Sass and Wunderlich, 2010)
- Mean-variance portfolio selection for a non-life insurance company. (Delong and Gerrard, 2007)
- Risk-efficient portfolio crop choice with amended water and irrigation policies in northern Germany (Buchholz and Musshoff, 2013)
- Portfolio optimization of power futures market: evidence from France and Germany (Fianu, 2018)
- The Optimal Portfolio of PAYG Benefits and Funded Pensions in Germany (Anders and Groneck, 2022)

Appendix II

Not all the articles were accessible directly and simply through the university database. So, I tried to use a variety of resources, in particular, the library of The University of Law, London, to fetch them from other resources to which they had access.

Finding the data from databases took a long time firstly because I had not known at the first step that Yahoo Finance provide them for free and easily. Because of that, initially, I emailed other providers and request them to help me.

- Hargreaves Lansdown Service, Brian.Leung@hl.co.uk
- Deutsche-Boerse, info@deutsche-boerse.com
- Financial Times, inside the website contact
- Börse Berlin AG, petra.greif@boerse-berlin.de

Appendix III

Tables of variances and covariances.

Cov Matrix	RWE.De	ADS.DE	AIR.DE	ALV.DE	BAS.DE	BAYN.DE	BEI.DE	BMW.DE	CON.DE	DB1.DE	DBK.DE	DPW.DE	DTE.DE	SHL.DE	MTX.DE	PUM.DE	HFG.DE	HNR1.DE	IFX.DE	LIN.DE	MRK.DE
RWE.De	1.93E-04	6.06E-05	9.74E-06	1.60E-05	2.12E-06	1.29E-05	4.49E-05	-6.19E-06	-2.03E-05	4.17E-05	-1.70E-05	1.65E-05	2.99E-05	3.75E-05	4.61E-05	6.52E-05	6.12E-05	3.60E-05	-1.64E-06	2.15E-05	4.30E-05
ADS.DE	6.06E-05	2.02E-04	5.68E-05	4.45E-05	4.18E-05	5.87E-05	3.63E-05	4.23E-05	4.92E-05	5.37E-05	4.27E-05	4.92E-05	3.06E-05	3.19E-05	5.82E-05	1.28E-04	2.78E-05	3.30E-05	5.97E-05	5.46E-05	3.57E-05
AIR.DE	9.74E-06	5.68E-05	1.75E-04	4.80E-05	6.32E-05	6.94E-05	2.76E-05	5.26E-05	6.04E-05	3.12E-05	5.46E-05	5.31E-05	1.84E-05	4.27E-05	7.38E-05	7.53E-05	4.90E-05	3.70E-05	7.63E-05	5.81E-05	3.79E-05
ALV.DE	1.60E-05	4.45E-05	4.80E-05	6.15E-05	5.98E-05	4.94E-05	8.82E-06	5.15E-05	5.11E-05	3.05E-05	6.93E-05	4.98E-05	2.57E-05	3.02E-05	2.53E-05	4.08E-05	2.48E-05	4.03E-05	5.53E-05	4.81E-05	1.75E-05
BAS.DE	2.12E-06	4.18E-05	6.32E-05	5.98E-05	1.60E-04	1.06E-04	-1.49E-06	9.22E-05	1.16E-04	1.88E-05	1.36E-04	7.62E-05	2.37E-05	3.69E-05	3.36E-05	3.73E-05	5.57E-05	3.67E-05	1.25E-04	7.01E-05	3.08E-05
BAYN.DE	1.29E-05	5.87E-05	6.94E-05	4.94E-05	1.06E-04	2.37E-04	1.95E-05	6.69E-05	8.38E-05	3.63E-05	1.15E-04	6.72E-05	2.83E-05	5.41E-05	5.30E-05	6.52E-05	6.98E-05	4.09E-05	8.89E-05	5.52E-05	3.21E-05
BEI.DE	4.49E-05	3.63E-05	2.76E-05	8.82E-06	-1.49E-06	1.95E-05	8.98E-05	-8.18E-06	-2.46E-05	2.36E-05	-4.39E-05	9.77E-06	2.31E-05	2.57E-05	4.29E-05	4.47E-05	1.14E-05	1.20E-05	-2.38E-05	2.67E-05	2.63E-05
BMW.DE	-6.19E-06	4.23E-05	5.26E-05	5.15E-05	9.22E-05	6.69E-05	-8.18E-06	1.29E-04	1.21E-04	2.09E-05	1.22E-04	6.42E-05	1.98E-05	3.39E-05	1.95E-05	4.18E-05	3.93E-05	3.63E-05	1.07E-04	4.74E-05	1.90E-05
CON.DE	-2.03E-05	4.92E-05	6.04E-05	5.11E-05	1.16E-04	8.38E-05	-2.46E-05	1.21E-04	2.74E-04	4.80E-06	1.62E-04	7.56E-05	1.44E-05	4.87E-05	1.87E-05	3.75E-05	3.53E-05	3.13E-05	1.57E-04	3.64E-05	1.53E-05
DB1.DE	4.17E-05	5.37E-05	3.12E-05	3.05E-05	1.88E-05	3.63E-05	2.36E-05	2.09E-05	4.80E-06	1.15E-04	2.41E-05	2.71E-05	2.29E-05	2.08E-05	4.45E-05	5.93E-05	4.87E-05	3.19E-05	1.51E-05	3.49E-05	2.70E-05
DBK.DE	-1.70E-05	4.27E-05	5.46E-05	6.93E-05	1.36E-04	1.15E-04	-4.39E-05	1.22E-04	1.62E-04	2.41E-05	4.53E-04	9.69E-05	2.06E-05	3.65E-05	1.39E-05	4.13E-05	8.27E-05	4.66E-05	1.36E-04	6.33E-05	2.13E-05
DPW.DE	1.65E-05	4.92E-05	5.31E-05	4.98E-05	7.62E-05	6.72E-05	9.77E-06	6.42E-05	7.56E-05	2.71E-05	9.69E-05	1.05E-04	2.65E-05	2.71E-05	2.96E-05	5.12E-05	3.81E-05	3.01E-05	7.34E-05	5.32E-05	1.90E-05
DTE.DE	2.99E-05	3.06E-05	1.84E-05	2.57E-05	2.37E-05	2.83E-05	2.31E-05	1.98E-05	1.44E-05	2.29E-05	2.06E-05	2.65E-05	5.04E-05	2.48E-05	2.17E-05	2.61E-05	3.89E-05	2.28E-05	6.33E-07	3.41E-05	1.98E-05
SHL.DE	3.75E-05	3.19E-05	4.27E-05	3.02E-05	3.69E-05	5.41E-05	2.57E-05	3.39E-05	4.87E-05	2.08E-05	3.65E-05	2.71E-05	2.48E-05	1.61E-04	4.56E-05	3.28E-05	3.56E-05	2.55E-05	3.21E-05	4.26E-05	4.34E-05
MTX.DE	4.61E-05	5.82E-05	7.38E-05	2.53E-05	3.36E-05	5.30E-05	4.29E-05	1.95E-05	1.87E-05	4.45E-05	1.39E-05	2.96E-05	2.17E-05	4.56E-05	1.66E-04	6.94E-05	5.71E-05	3.71E-05	2.90E-05	4.76E-05	3.30E-05
PUM.DE	6.52E-05	1.28E-04	7.53E-05	4.08E-05	3.73E-05	6.52E-05	4.47E-05	4.18E-05	3.75E-05	5.93E-05	4.13E-05	5.12E-05	2.61E-05	3.28E-05	6.94E-05	2.67E-04	5.82E-05	4.71E-05	4.03E-05	5.78E-05	4.24E-05
HFG.DE	6.12E-05	2.78E-05	4.90E-05	2.48E-05	5.57E-05	6.98E-05	1.14E-05	3.93E-05	3.53E-05	4.87E-05	8.27E-05	3.81E-05	3.89E-05	3.56E-05	5.71E-05	5.82E-05	6.57E-04	1.45E-05	5.40E-05	5.04E-05	1.36E-05
HNR1.DE	3.60E-05	3.30E-05	3.70E-05	4.03E-05	3.67E-05	4.09E-05	1.20E-05	3.63E-05	3.13E-05	3.19E-05	4.66E-05	3.01E-05	2.28E-05	2.55E-05	3.71E-05	4.71E-05	1.45E-05	7.87E-05	3.21E-05	3.14E-05	7.06E-06
IFX.DE	-1.64E-06	5.97E-05	7.63E-05	5.53E-05	1.25E-04	8.89E-05	-2.38E-05	1.07E-04	1.57E-04	1.51E-05	1.36E-04	7.34E-05	6.33E-07	3.21E-05	2.90E-05	4.03E-05	5.40E-05	3.21E-05	2.95E-04	6.04E-05	1.50E-05
LIN.DE	2.15E-05	5.46E-05	5.81E-05	4.81E-05	7.01E-05	5.52E-05	2.67E-05	4.74E-05	3.64E-05	3.49E-05	6.33E-05	5.32E-05	3.41E-05	4.26E-05	4.76E-05	5.78E-05	5.04E-05	3.14E-05	6.04E-05	1.34E-04	3.76E-05
MRK.DE	4.30E-05	3.57E-05	3.79E-05	1.75E-05	3.08E-05	3.21E-05	2.63E-05	1.90E-05	1.53E-05	2.70E-05	2.13E-05	1.90E-05	1.98E-05	4.34E-05	3.30E-05	4.24E-05	1.36E-05	7.06E-06	1.50E-05	3.76E-05	1.12E-04

Table 11: Covariance Matrix 2019 (Calculated by the Author)

Cov Matrix	RWE.De	ADS.DE	AIR.DE	ALV.DE	BAS.DE	BAYN.DE	BEI.DE	BMW.DE	CON.DE	DB1.DE	DBK.DE	DPW.DE	DTE.DE	SHL.DE	MTX.DE	PUM.DE	HFG.DE	HNR1.DE	IFX.DE	LIN.DE	MRK.DE
RWE.De	3.46E-04	1.08E-04	1.55E-04	1.23E-04	1.32E-04	1.38E-04	1.07E-04	1.14E-04	1.29E-04	1.77E-04	1.64E-04	1.35E-04	1.20E-04	8.13E-05	1.17E-04	1.28E-04	1.62E-04	1.65E-04	1.62E-04	1.31E-04	1.37E-04
ADS.DE	1.08E-04	3.62E-04	3.56E-04	1.66E-04	1.68E-04	1.63E-04	7.39E-05	1.29E-04	2.15E-04	1.19E-04	2.27E-04	1.11E-04	1.09E-04	1.05E-04	3.31E-04	3.20E-04	7.40E-05	1.53E-04	2.18E-04	1.38E-04	1.03E-04
AIR.DE	1.55E-04	3.56E-04	1.38E-03	3.84E-04	3.66E-04	2.37E-04	2.74E-05	3.05E-04	4.36E-04	1.28E-04	4.51E-04	1.70E-04	1.89E-04	1.26E-04	9.62E-04	3.94E-04	-2.47E-04	2.22E-04	4.25E-04	1.90E-04	1.47E-04
ALV.DE	1.23E-04	1.66E-04	3.84E-04	3.89E-04	2.74E-04	1.97E-04	7.31E-05	2.73E-04	3.23E-04	1.33E-04	3.16E-04	1.73E-04	1.55E-04	6.63E-05	3.68E-04	1.61E-04	-1.28E-04	2.81E-04	2.48E-04	1.50E-04	8.40E-05
BAS.DE	1.32E-04	1.68E-04	3.66E-04	2.74E-04	3.78E-04	2.11E-04	9.96E-05	2.72E-04	3.54E-04	1.25E-04	3.54E-04	1.84E-04	1.55E-04	9.75E-05	3.52E-04	1.27E-04	-8.91E-05	2.39E-04	2.17E-04	1.78E-04	7.49E-05
BAYN.DE	1.38E-04	1.63E-04	2.37E-04	1.97E-04	2.11E-04	3.51E-04	1.12E-04	1.66E-04	2.12E-04	1.47E-04	2.53E-04	1.51E-04	1.46E-04	9.15E-05	2.38E-04	1.26E-04	5.79E-05	1.90E-04	1.81E-04	1.33E-04	1.02E-04
BEI.DE	1.07E-04	7.39E-05	2.74E-05	7.31E-05	9.96E-05	1.12E-04	2.31E-04	8.01E-05	8.53E-05	9.45E-05	8.48E-05	5.63E-05	6.80E-05	6.29E-05	3.48E-05	1.09E-05	1.04E-04	1.28E-04	3.55E-05	1.07E-04	2.98E-05
BMW.DE	1.14E-04	1.29E-04	3.05E-04	2.73E-04	2.72E-04	1.66E-04	8.01E-05	3.94E-04	3.92E-04	8.19E-05	3.19E-04	1.42E-04	1.18E-04	2.75E-05	2.95E-04	9.29E-05	-1.11E-04	2.03E-04	1.81E-04	1.32E-04	5.52E-05
CON.DE	1.29E-04	2.15E-04	4.36E-04	3.23E-04	3.54E-04	2.12E-04	8.53E-05	3.92E-04	6.95E-04	1.08E-04	4.52E-04	1.81E-04	1.49E-04	5.38E-05	3.83E-04	2.09E-04	-1.21E-04	2.70E-04	3.05E-04	1.63E-04	7.62E-05
DB1.DE	1.77E-04	1.19E-04	1.28E-04	1.33E-04	1.25E-04	1.47E-04	9.45E-05	8.19E-05	1.08E-04	3.55E-04	1.43E-04	1.73E-04	1.07E-04	1.51E-04	6.36E-05	1.25E-04	1.68E-04	1.27E-04	2.21E-04	1.35E-04	1.33E-04
DBK.DE	1.64E-04	2.27E-04	4.51E-04	3.16E-04	3.54E-04	2.53E-04	8.48E-05	3.19E-04	4.52E-04	1.43E-04	8.35E-04	2.21E-04	1.83E-04	1.11E-04	4.76E-04	2.00E-04	-5.86E-05	3.05E-04	3.21E-04	1.79E-04	1.14E-04
DPW.DE	1.35E-04	1.11E-04	1.70E-04	1.73E-04	1.84E-04	1.51E-04	5.63E-05	1.42E-04	1.81E-04	1.73E-04	2.21E-04	3.35E-04	1.10E-04	1.30E-04	1.38E-04	1.36E-04	3.88E-05	1.30E-04	2.36E-04	1.22E-04	1.11E-04
DTE.DE	1.20E-04	1.09E-04	1.89E-04	1.55E-04	1.55E-04	1.46E-04	6.80E-05	1.18E-04	1.49E-04	1.07E-04	1.83E-04	1.10E-04	1.83E-04	7.20E-05	1.75E-04	1.14E-04	1.66E-05	1.39E-04	1.44E-04	1.00E-04	6.26E-05
SHL.DE	8.13E-05	1.05E-04	1.26E-04	6.63E-05	9.75E-05	9.15E-05	6.29E-05	2.75E-05	5.38E-05	1.51E-04	1.11E-04	1.30E-04	7.20E-05	3.49E-04	1.58E-04	1.06E-04	1.70E-04	6.61E-05	1.19E-04	8.62E-05	5.70E-05
MTX.DE	1.17E-04	3.31E-04	9.62E-04	3.68E-04	3.52E-04	2.38E-04	3.48E-05	2.95E-04	3.83E-04	6.36E-05	4.76E-04	1.38E-04	1.75E-04	1.58E-04	1.21E-03	3.05E-04	-3.22E-04	3.01E-04	2.71E-04	1.79E-04	1.07E-04
PUM.DE	1.28E-04	3.20E-04	3.94E-04	1.61E-04	1.27E-04	1.26E-04	1.09E-05	9.29E-05	2.09E-04	1.25E-04	2.00E-04	1.36E-04	1.14E-04	1.06E-04	3.05E-04	5.84E-04	7.49E-05	1.22E-04	2.42E-04	9.66E-05	1.12E-04
HFG.DE	1.62E-04	7.40E-05	-2.47E-04	-1.28E-04	-8.91E-05	5.79E-05	1.04E-04	-1.11E-04	-1.21E-04	1.68E-04	-5.86E-05	3.88E-05	1.66E-05	1.70E-04	-3.22E-04	7.49E-05	1.59E-03	-1.50E-05	2.25E-05	4.01E-05	3.80E-05
HNR1.DE	1.65E-04	1.53E-04	2.22E-04	2.81E-04	2.39E-04	1.90E-04	1.28E-04	2.03E-04	2.70E-04	1.27E-04	3.05E-04	1.30E-04	1.39E-04	6.61E-05	3.01E-04	1.22E-04	-1.50E-05	4.10E-04	1.61E-04	1.34E-04	7.54E-05
IFX.DE	1.62E-04	2.18E-04	4.25E-04	2.48E-04	2.17E-04	1.81E-04	3.55E-05	1.81E-04	3.05E-04	2.21E-04	3.21E-04	2.36E-04	1.44E-04	1.19E-04	2.71E-04	2.42E-04	2.25E-05	1.61E-04	6.28E-04	1.55E-04	1.54E-04
LIN.DE	1.31E-04	1.38E-04	1.90E-04	1.50E-04	1.78E-04	1.33E-04	1.07E-04	1.32E-04	1.63E-04	1.35E-04	1.79E-04	1.22E-04	1.00E-04	8.62E-05	1.79E-04	9.66E-05	4.01E-05	1.34E-04	1.55E-04	2.43E-04	9.37E-05
MRK.DE	1.37E-04	1.03E-04	1.47E-04	8.40E-05	7.49E-05	1.02E-04	2.98E-05	5.52E-05	7.62E-05	1.33E-04	1.14E-04	1.11E-04	6.26E-05	5.70E-05	1.07E-04	1.12E-04	3.80E-05	7.54E-05	1.54E-04	9.37E-05	2.40E-04

Table 12: Covariance Matrix 2020 (Calculated by the Author)