Experimental Robotics - CS225A
HW1
Yazhou Zhang

Question 1:

a. Gains: kp = 400, kv = 50
The PD gains were manually tuned to achieve critically damped motion for Joint 1. The system responded quickly (not over-damped) and exhibited no overshoot or oscillation (not under-damped).

b. The motion of the non-moving joints is not very stable. This is because the basic PD controller does not account for the full joint dynamics, only controlling based on error.

Figure 1 shows the joint trajectory for joint 1, 3, and 4 using the control law from Q1.

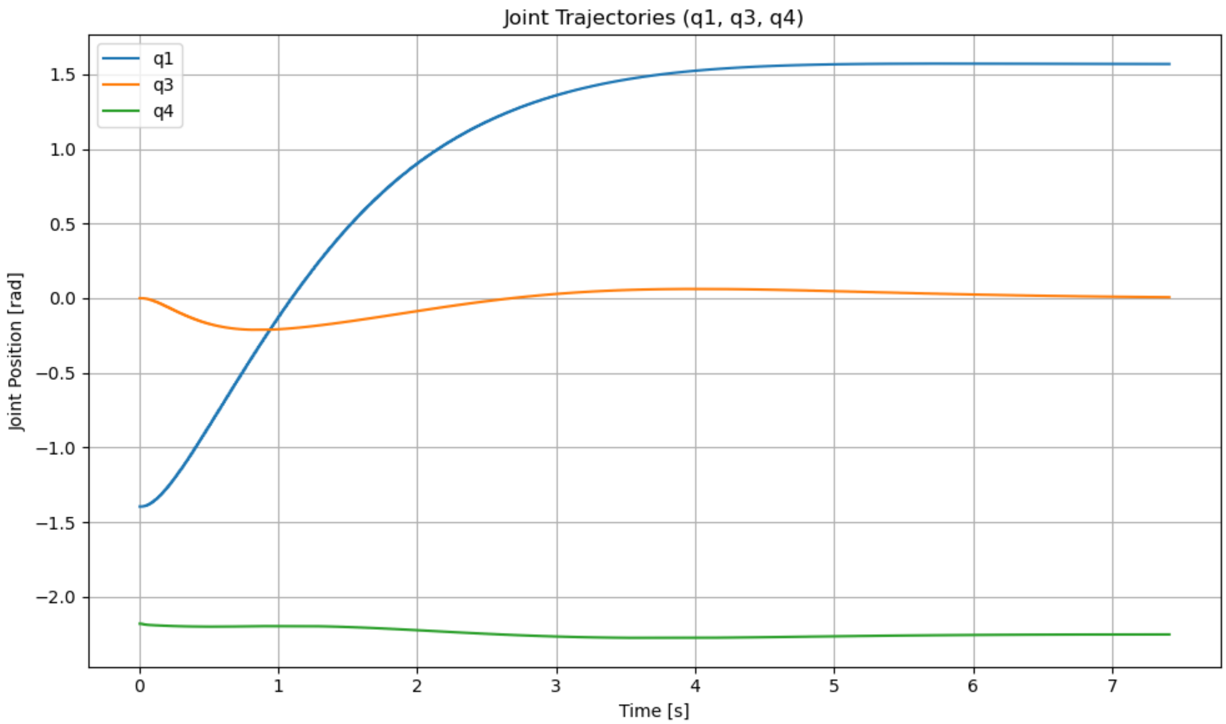

Figure 1: Q1 Control Law Joint Trajectory for Joint 1, 3, and 4

Steady-state error of each joint:

- Joint 1:

$$e_{ss,\theta_1} = |1.56956 - 1.5708| = 0.00123,$$

Since gravity acts along the axis of rotation, almost no effort is required to compensate for it.

- Joint 3:

$$e_{ss,\theta_2} = |0.00658 - 0| = 0.00658$$

The steady-state error is small because gravity has minimal influence at its final configuration.

Experimental Robotics - CS225A
HW1
Yazhou Zhang

- Joint 4:

$$e_{ss,\theta_3} = |-2.25345 - 2.18166| = 0.07179$$

Gravity induces a high torque at the final configuration, resulting in a large steady-state error.

Experimental Robotics - CS225A
HW1
Yazhou Zhang

Question 2:

a. Gains: kp = 400, kv = 50
   The damping gain remains the same as in Q1 since gravity does not affect Joint 1's trajectory, since it acts along the axis of rotation of Joint 1.

b. This controller includes gravity compensation, which improves performance by canceling out gravity-induced torques. It eliminates the steady state error at the final configuration, especially significant for Joint 4.

   Figure 2 shows the joint trajectory for joint 1, 3, and 4 using the control law from Q2.

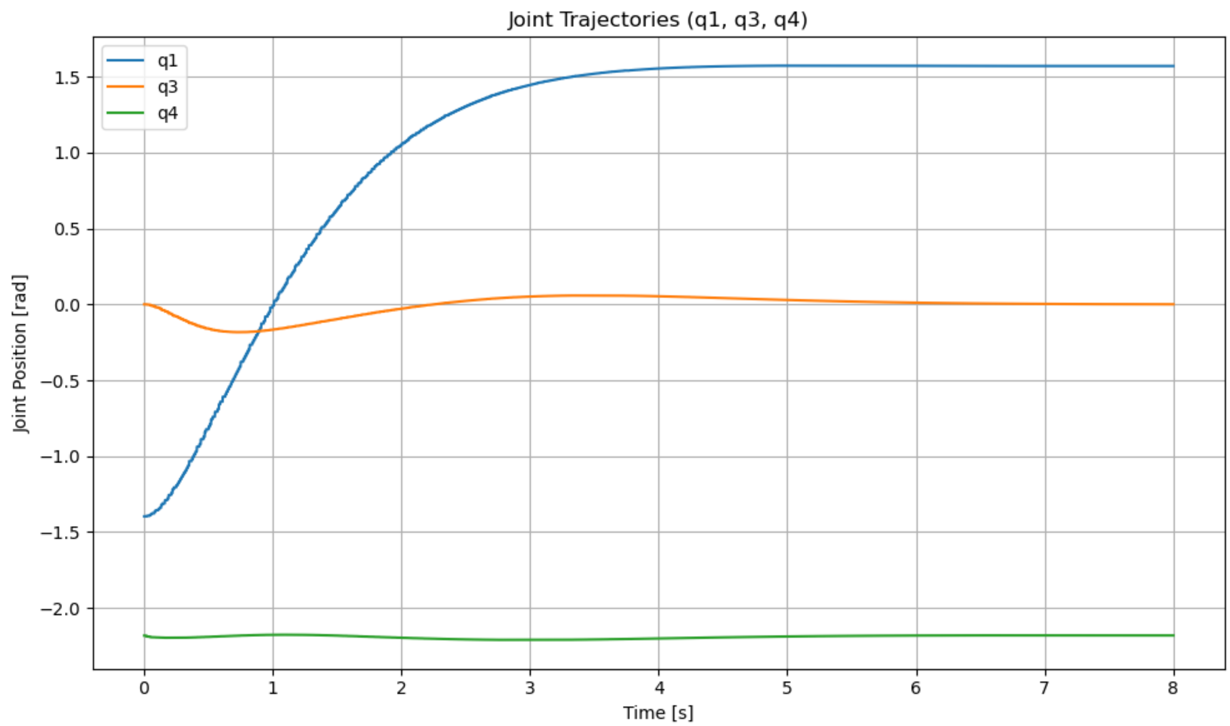

Figure 2: Q2 Control Law Joint Trajectory for Joint 1, 3, and 4

Experimental Robotics - CS225A
HW1
Yazhou Zhang

Question 3:

a. Gains: kp = 400, kv = 38

b. By introducing the mass matrix, A(q) into the control law, we linearize the system dynamics and compensate for the inertial effects of the robot. This allows each joint to behave as if it were a unit-mass system. It decouples the system, reducing the influence of the inertia from other joints. A smaller kv is therefore required to achieve critical damping in Joint 1. Moreover, the improved dynamic modeling leads to greater overall stability, especially for Joint 3 which becomes much more stable. Incorporating A(q) results in a better approximation of the true system behavior. However, the controller does not account for Coriolis and Centrifugal effects, causing joint 4 to oscillate.

   Figure 3 shows the joint trajectory for joint 1, 3, and 4 using the control law from Q3.

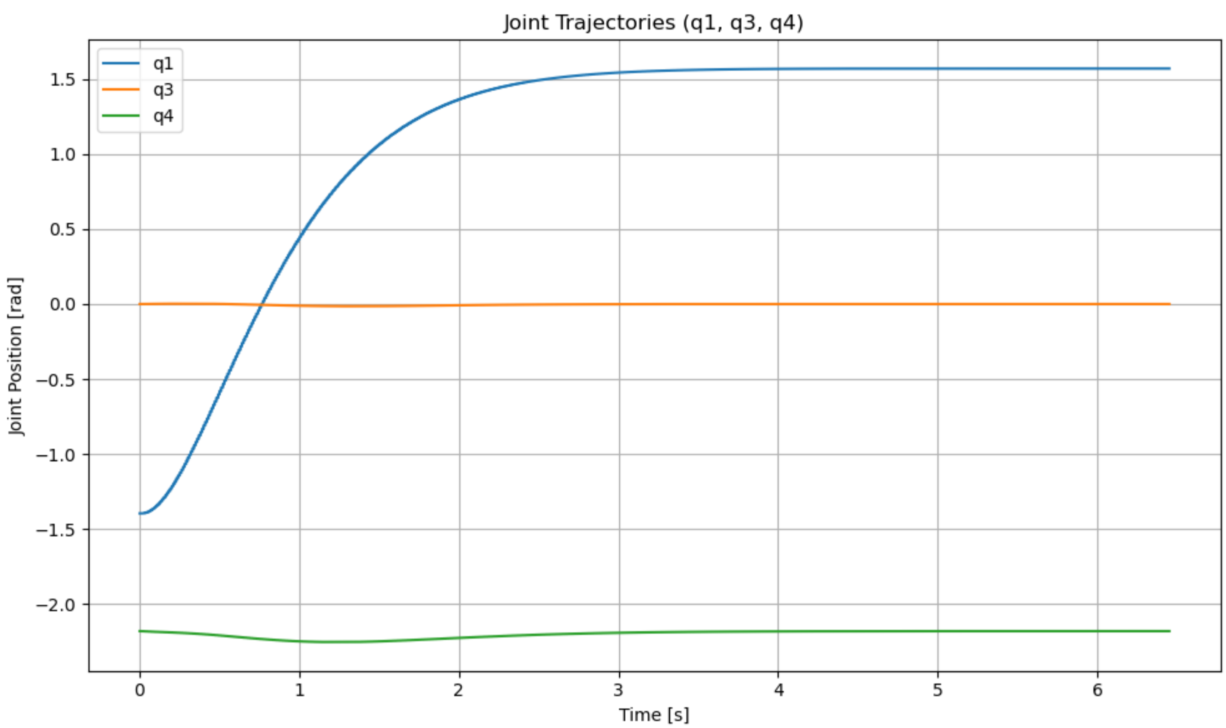

Figure 3: Q3 Control Law Joint Trajectory for Joint 1, 3, and 4

Experimental Robotics - CS225A
HW1
Yazhou Zhang

Question 4:

a. Gains: kp = 400, kv = 38

b. A full dynamic model is now used in the controller including the inertia, Coriolis and gravity term. Since the controller perfectly matches the model, the motion of all joints is well-tracked with minimal overshoot.

   The difference is especially noticeable for Joint 4 when comparing this controller to that in Q3. This is because Joint 4, under the given configuration, experiences stronger velocity-induced torques resulting from the motion of Joint 1 due to dynamic coupling.

   Figure 4 shows the joint trajectory for joint 1, 3, and 4 using the control law from Q4.

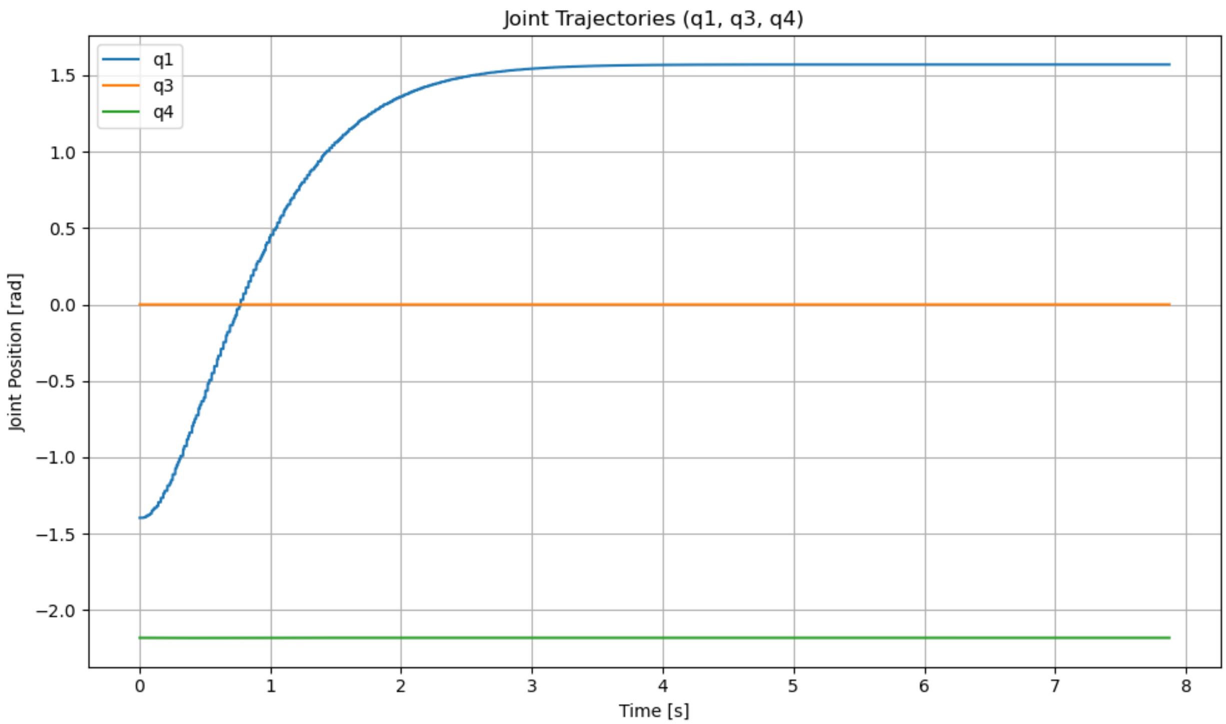

Figure 4: Q4 Control Law Joint Trajectory for Joint 1, 3, and 4

Experimental Robotics - CS225A
HW1
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Question 5:

a. Gains: kp = 400, kv = 38

b. The model controller is not updated to reflect the increased mass in Link 7, so the estimated dynamics (inertia, Coriolis, and gravity terms) are no longer accurate. This mismatch introduces instabilities. Trajectory of joint 3 and 4 are no longer kept smooth and Joint 4 had a large steady state error.

   Figure 5 shows the joint trajectory for joint 1, 3, and 4 using the control law from Q5.

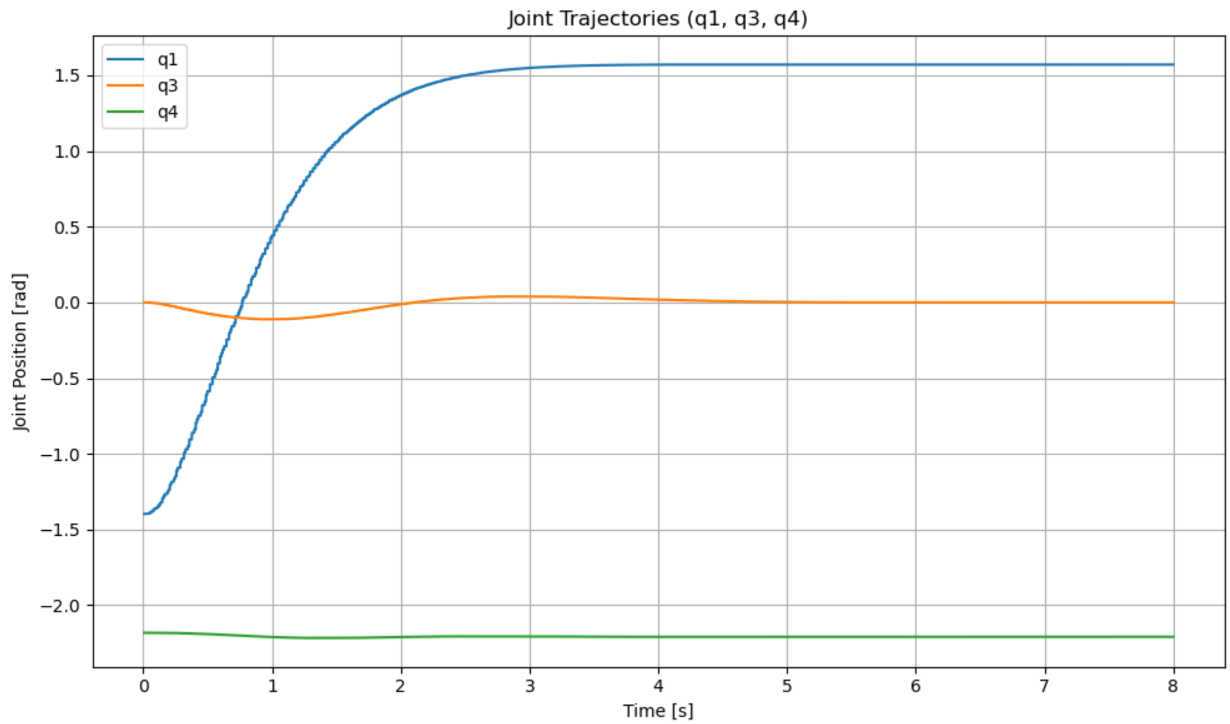

Figure 5: Q5 Control Law Joint Trajectory for Joint 1, 3, and 4

Experimental Robotics - CS225A
HW1
Yazhou Zhang

Question 6:

a. Gains: kp = 400, kv = 38

b. To accurately control the robot with the added payload, the controller must account for the extra mass at link 7. The control law is updated with the augmented terms.

The augmented mass matrix and gravity vector are given as following with J_v^T of link 7:

$$A_{aug}(q) = A(q) + m_{paylaod} J_v^T(q) J_v(q)$$

$$g_{aug} = g(q) + J_v^T(q)(m_{paylaod} g)$$

The resulting control law becomes:

$$\tau = A_{aug}(q)\big(-k_p(q - q_d) - k_v\dot{q}\big) + b(q, \dot{q}) + g_{aug}(q)$$

The augmented mass matrix compensates for the inertia introduced by the payload. The augmented gravity term reduces steady-state error to balance the added weight, allowing the joints to settle at the desired configuration.

Although the controller performs better than it in Q5, some residual instability remains. This is because the Coriolis and centrifugal effects have not been updated to reflect the payload.

Figure 6 shows the joint trajectory for joint 1, 3, and 4 using the control law from Q6.

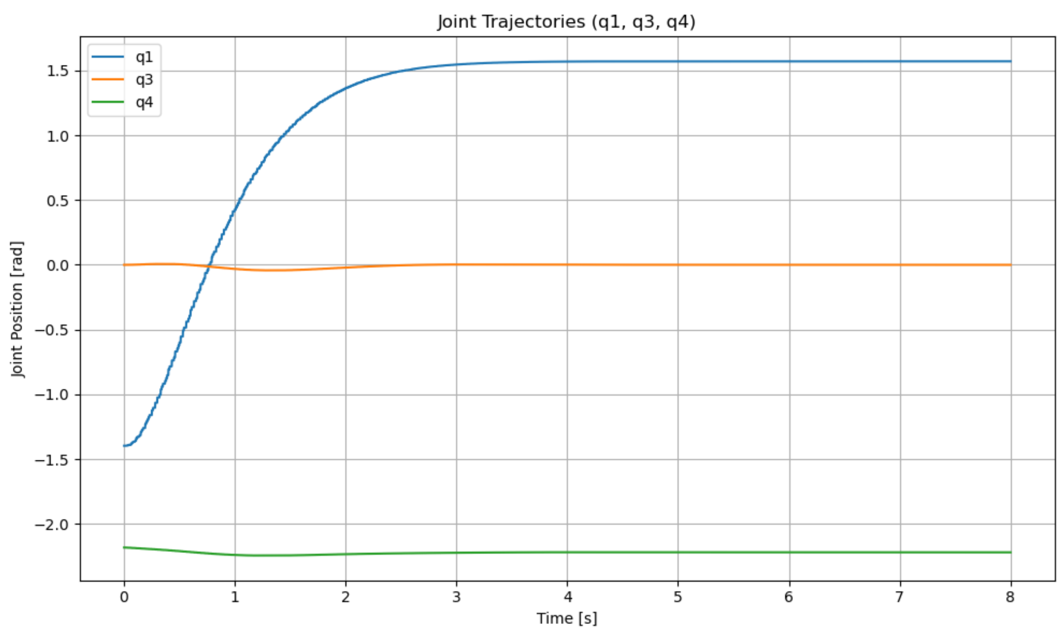

Figure 6: Q6 Control Law Joint Trajectory for Joint 1, 3, and 4