Lecture Notes: Torsion in Circular Shafts

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1 Introduction to Torsion

Definition: Torsion refers to the twisting of a straight bar when it is loaded by moments (torques) about its longitudinal axis.

Examples:

- Turning a screwdriver
- Drive shafts in automobiles
- Axles, propeller shafts, and drill bits

2 Torque and Moment of a Couple

Torque (T): The moment produced by two equal and opposite forces acting at a distance.

Formula:

$$T = P \times d$$

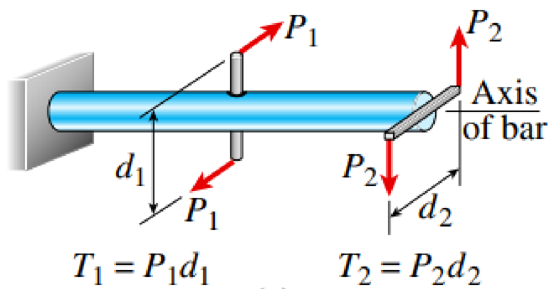
where:

- P is the applied force
- d is the perpendicular distance between forces.

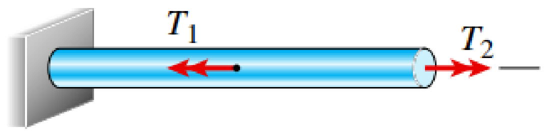
Units of Torque:

- SI: Newton-meter (N·m)
- USCS: Pound-foot (lb·ft), Pound-inch (lb·in)

Representation: Torques can be represented as vectors or curved arrows depending on preference.



(a)



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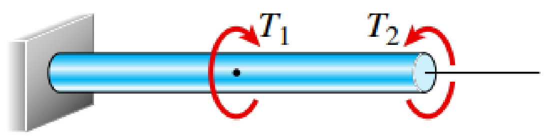

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3 Distinguishing Torque and Bending Moment

Torque (Torsion): A moment that twists an object around its longitudinal axis. It produces **shear stresses** and results in **twisting deformation**.

Bending Moment: A moment that bends an object, creating **tensile and compressive stresses** across the cross-section. It results in **bending deformation**.

Key Point: Torque causes twisting, while bending moments cause flexure. Both involve moments but act in fundamentally different ways on materials.

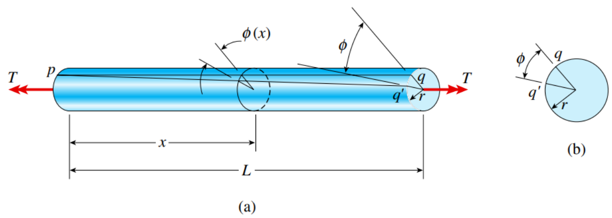
4 Torsional Deformation of Circular Bars

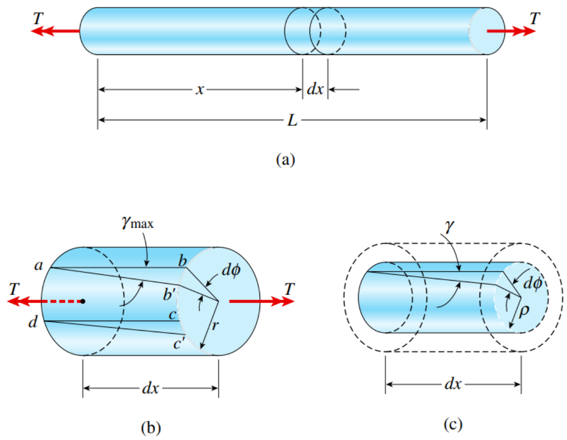
Deformation: Consider a circular bar subjected to a torque T applied at both ends.

- Each cross-section rotates relative to the fixed end.
- Cross sections remain plane, circular, all radii remain straight.
- If the angle of rotation is small, the length of the bar and its radius remain unchanged.

Angle of Twist ϕ: The angular displacement between two sections.

Shear Strain γ:





At the Surface: Maximum shear strain occurs at the outer surface. The shear strain on the surface of the bar is related to the angle of twist as:

$$\gamma_{\max} = \frac{bb'}{ab} = \frac{r d\phi}{dx}$$

where:

- r is the radius of the bar
- $\frac{d\phi}{dx}$ is the rate of change of twist along the bar's length

If $\theta = \frac{d\phi}{dx}$ is constant, then:

$$\gamma_{\max} = r\theta = r\frac{\phi}{L}$$

where:

- L is the length of the bar

Strain varies linearly with radial distance from the center, reaching zero at the center.

$$\gamma = \rho\frac{d\phi}{dx} = \frac{\rho}{r}\gamma_{\max}$$

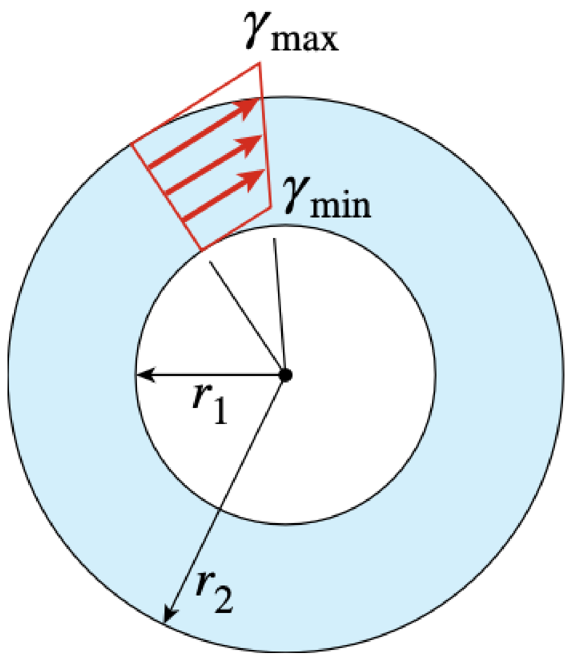
5 Shear Strain in Circular Tubes

For a **circular tube** with inner radius r_1 and outer radius r_2, shear strain also varies linearly from the inner to the outer surface.

Shear strain at inner and outer surfaces:

$$\gamma_{\max} = \frac{r_2}{L} \cdot \phi$$

$$\gamma_{\min} = \frac{r_1}{r_2}\gamma_{\max} = \frac{r_1}{L} \cdot \phi$$



6 Conclusion

- Torsion creates a state of pure shear.
- Shear strain and stress vary linearly with radial distance from the center of a circular bar, reaching a maximum at the outer surface.
- The equations for shear strain are applicable to both solid bars and hollow tubes.
- The concept of shear strain is a **geometric concept**, valid for any material but ϕ and θ should be **small**.