Lecture Notes: Stresses on Inclined Sections

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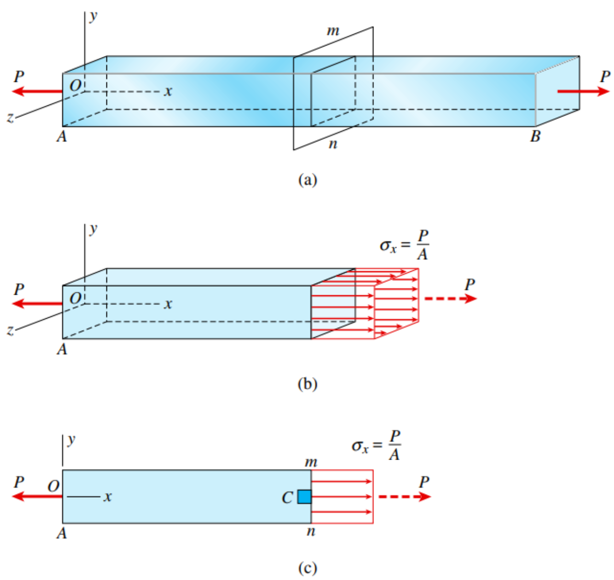
1. Introduction to Axial Loading

Axial Loading occurs when forces are applied along the longitudinal axis of a bar, generating **normal stresses** on sections perpendicular to the axis.

Normal Stress Formula:

$$\sigma_x = \frac{P}{A} \tag{1}$$



2. Stress on Inclined Sections

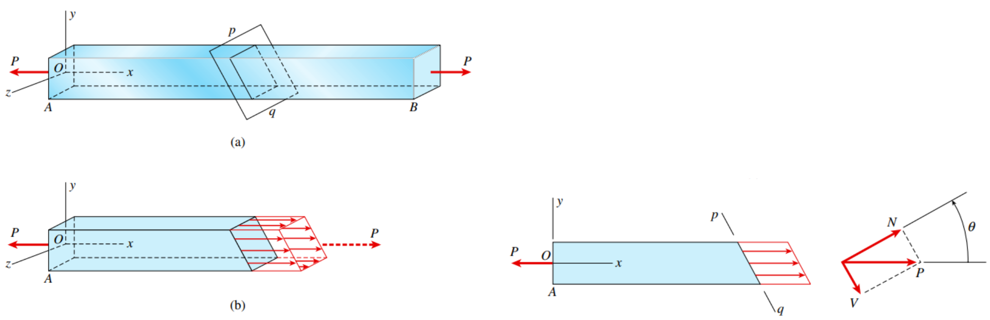
When a bar is subjected to axial force **P**, stresses also act on **inclined sections** (not just on perpendicular sections). These stresses consist of:

- **Normal force (N)** perpendicular to the inclined plane.
- **Shear force (V)** parallel to the inclined plane.

Equations of Forces:

$$N = P\cos(\theta) \tag{2}$$

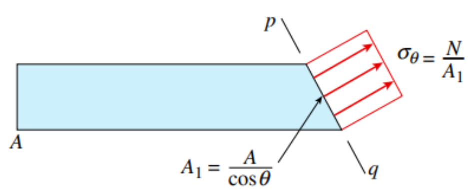
$$V = P\sin(\theta) \tag{3}$$



3. Calculating Normal and Shear Stresses

The normal and shear stresses on the inclined plane are derived from the forces acting on it. Since the inclined plane has a larger area than the cross-section perpendicular to the axis, we calculate its area as:

$$A_1 = \frac{A}{\cos(\theta)} \tag{4}$$



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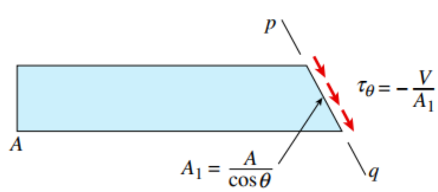


Normal and Shear Stress Equations:

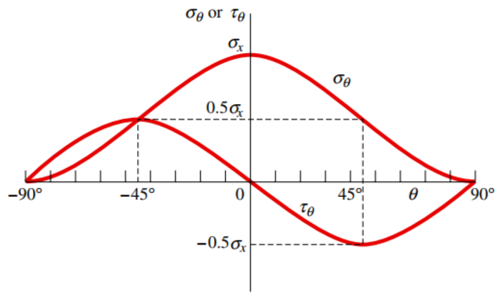
- **Normal Stress:**

$$\sigma_\theta = \sigma_x \cos^2(\theta) = \frac{1}{2}\sigma_x(1 + \cos 2\theta) \tag{5}$$

- **Shear Stress:**

$$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta) = \frac{1}{2}\sigma_x \sin 2\theta \tag{6}$$

Where θ is the inclination of the plane and σ_x is the normal stress on the perpendicular cross-section.



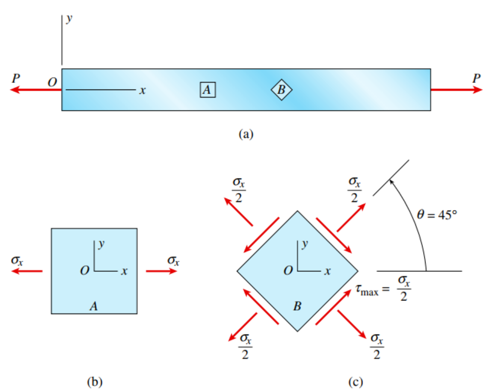
Maximum Stresses:

- **Maximum normal stress** occurs when $\theta = 0$:

$$\sigma_{\max} = \sigma_x \tag{7}$$

- **Maximum shear stress** occurs at $\theta = 45°$:

$$\tau_{\max} = \frac{\sigma_x}{2} \tag{8}$$



4. Example Problem

Example: Stress on an Inclined Plane at 30°
A bar is subjected to axial load **P**, and we want to calculate the normal and shear stresses on an inclined section at $\theta = 30°$.
Given: $\sigma_x = 50$ MPa.

- **Normal Stress:**

$$\sigma_{30} = 50 \times \cos^2(30°) = 37.5 \text{ MPa} \tag{9}$$

- **Shear Stress:**

$$\tau_{30} = 50 \times \sin(30°)\cos(30°) = 21.7 \text{ MPa} \tag{10}$$

5. Summary

- Stresses on inclined sections are a combination of **normal** and **shear stresses**.
- **Maximum normal stress** occurs when $\theta = 0°$, and **maximum shear stress** occurs when $\theta = 45°$.
- Understanding these stress distributions is essential for predicting material failure, especially in **shear failure** scenarios.