Georgia Tech College of Engineering

COE 3001

MECHANICS OF DEFORMABLE BODIES

Lecture 5 – Axial loaded members

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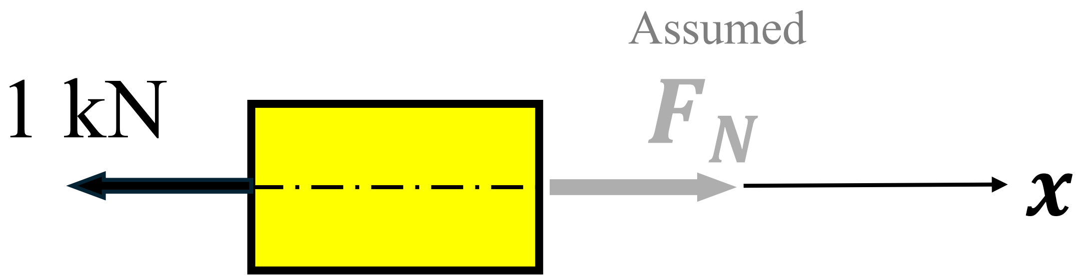
Sign Convention

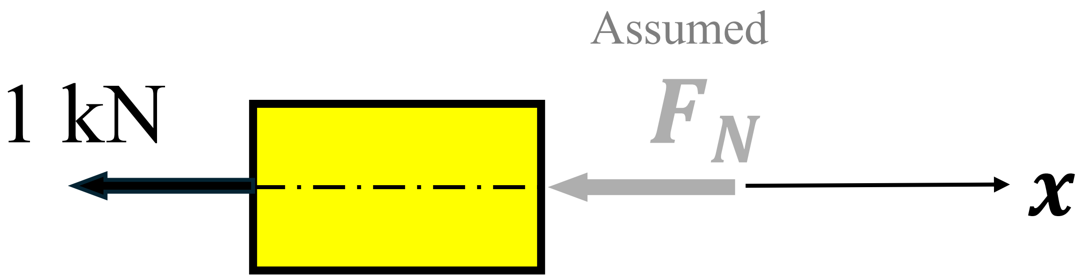
- **Force Vector** contains: 1. direction 2. magnitude
- If we draw an arrow in FBD, it means we assumed a direction



$$-1\ \mathrm{kN} + F_N = 0$$

$$F_N = 1\ \mathrm{kN}$$

- Magnitude: 1 kN,
- Direction: as assumed (tension)



$$-1\ \mathrm{kN} - F_N = 0$$

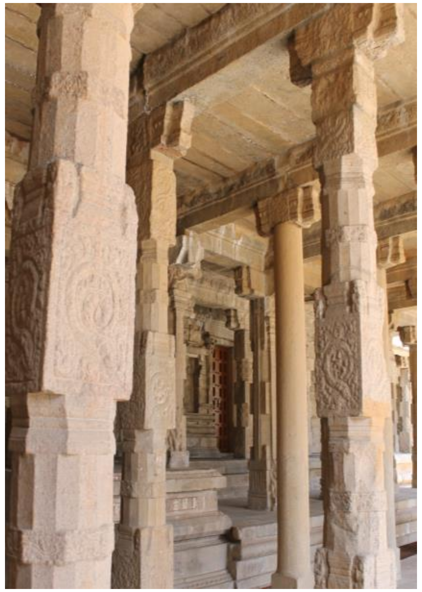
Magnitude cannot be negative

$$F_N = -1\ \mathrm{kN}$$

- Magnitude: 1 kN,
- Direction: oppose assumed direction (tension)

Axially loaded members

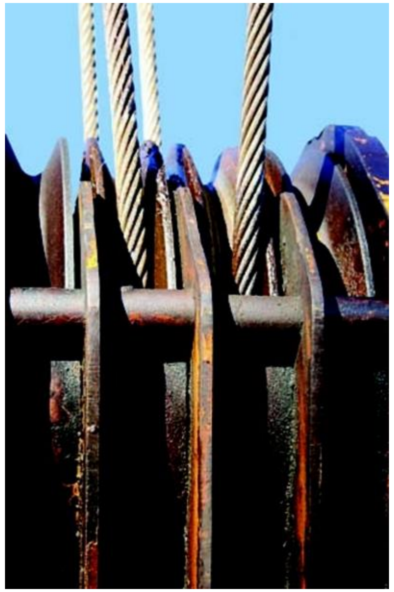
Structural components subjected **ONLY** to **tension or compression** are known as **axially loaded members.**



Columns in buildings

Bolts under preload

Hydraulic cylinder

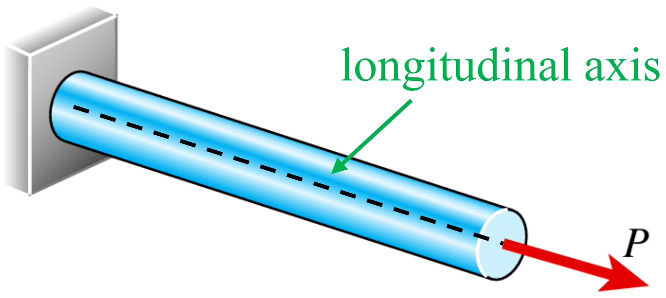


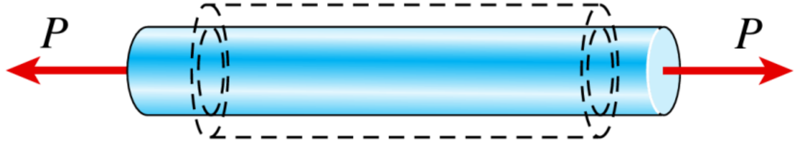
Steel cables

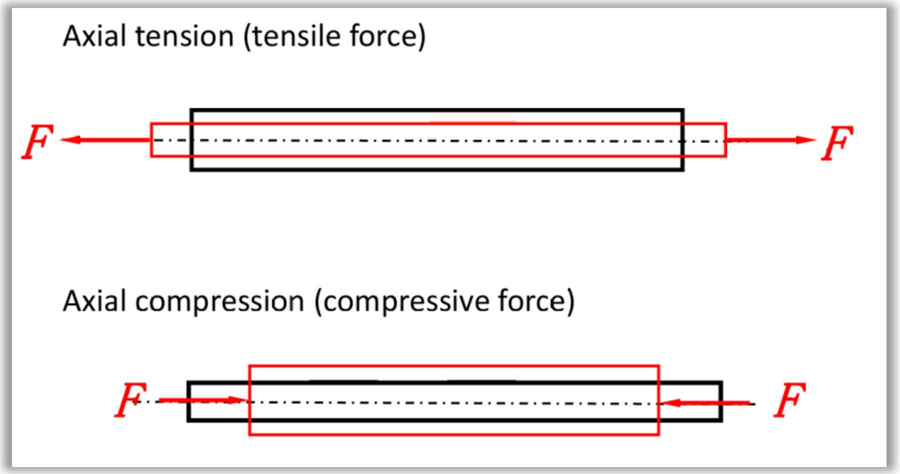
Axial Loads

Axial loads are forces applied along the **longitudinal axis** of a structural member.

- The load vector is aligned with the centerline of the body.
- (Suggestion) Always assume the unknown force acting in outward normal direction:
  - Tensile stress: Positive
  - Tensile internal force: Positive
  - Positive direction: outward normal direction.

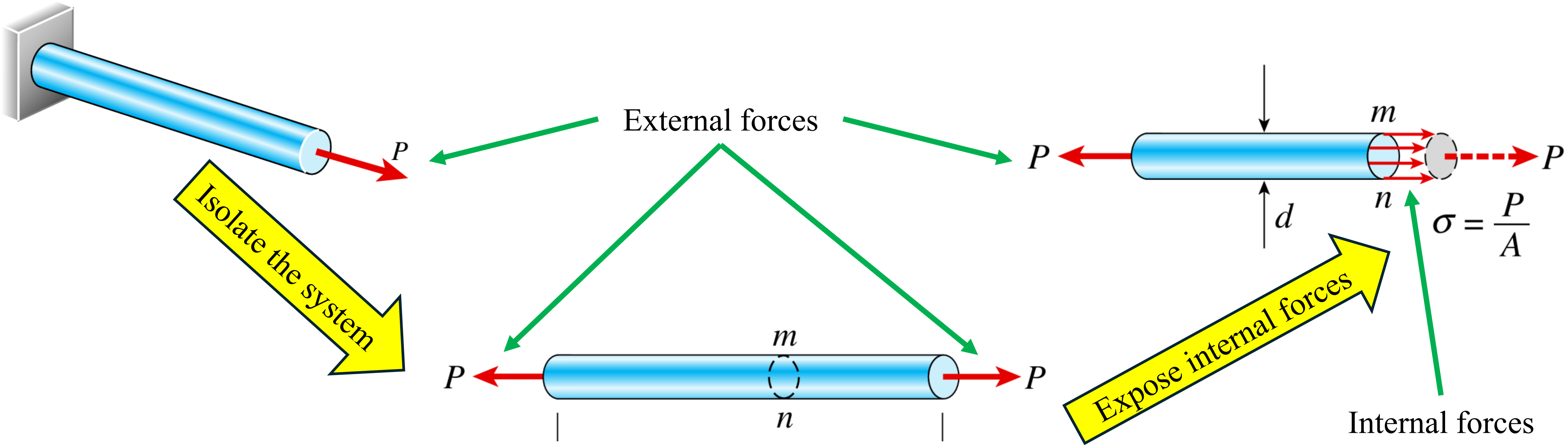






External force vs. internal force

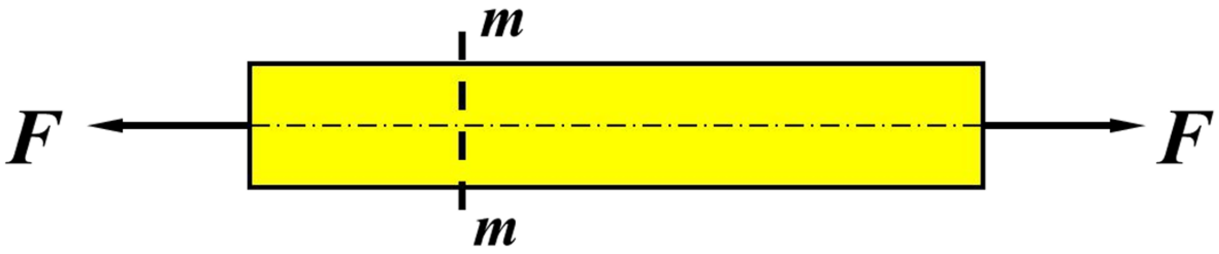
- External forces are forces acting on an object from **outside the system.**
- Internal forces are forces acting between components **within the system.**
- Internal forces exist to **balance** and counteract the external forces.



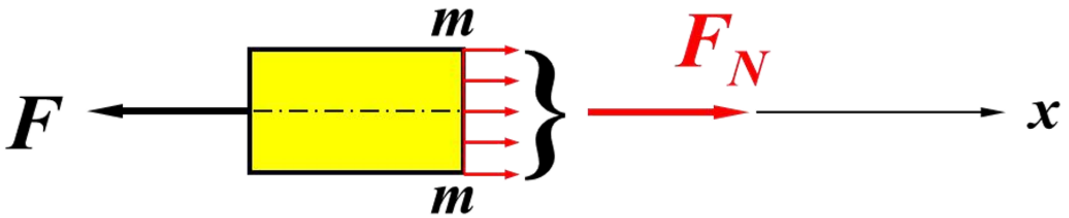
Method of sections

To determine the internal force in the cross-section ***m-m***, normally we use **method of sections**

1. Set imaginary cuts



2. Draw FBD



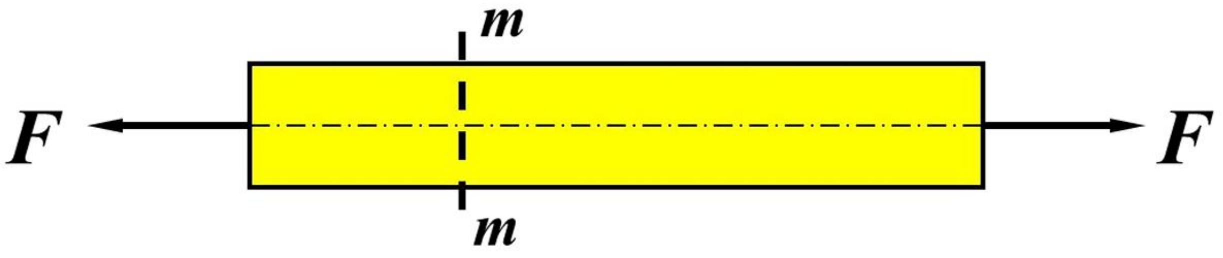
3. Apply balance equation in global coordinates.

$$\sum F_x = 0 \Rightarrow -F + F_N = 0 \Rightarrow F_N = F$$

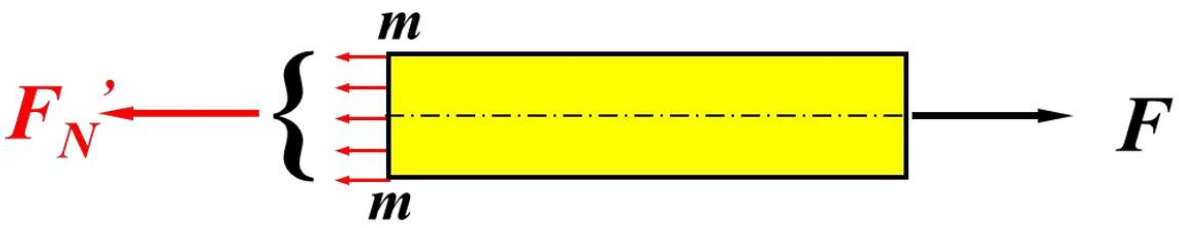
Similarly

To determine the internal force in the cross-section ***m-m***, normally we use **method of sections**

1. Set imaginary cuts



2. Draw FBD

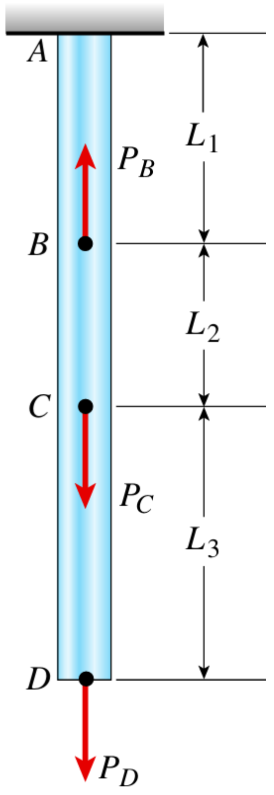


3. Apply balance equation in global coordinates.

$$\sum F_x = 0 \Rightarrow -F_N' + F = 0 \Rightarrow F_N' = F$$

Bars with intermediate axial loads

Example: Determine the internal forces in prismatic bar AD:



Idea:

- Apply method of sections in each segments.

Recall:

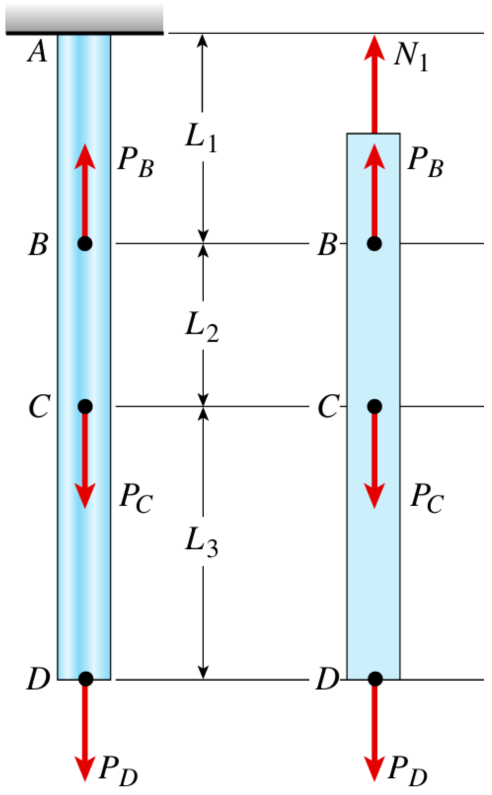
Method of sections:

1. Set imaginary cut
2. Draw FBD
3. Balance equation

Bars with intermediate axial loads

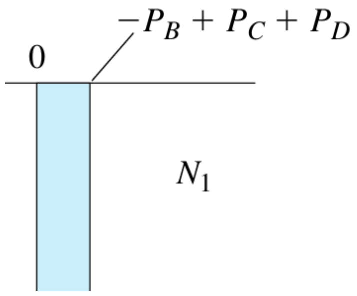
Method of sections:
1. Set imaginary cut
2. Draw FBD
3. Balance equation

- Segment AB:



$$\sum F_y = 0 \implies N_1 + P_B - P_C - P_D = 0$$

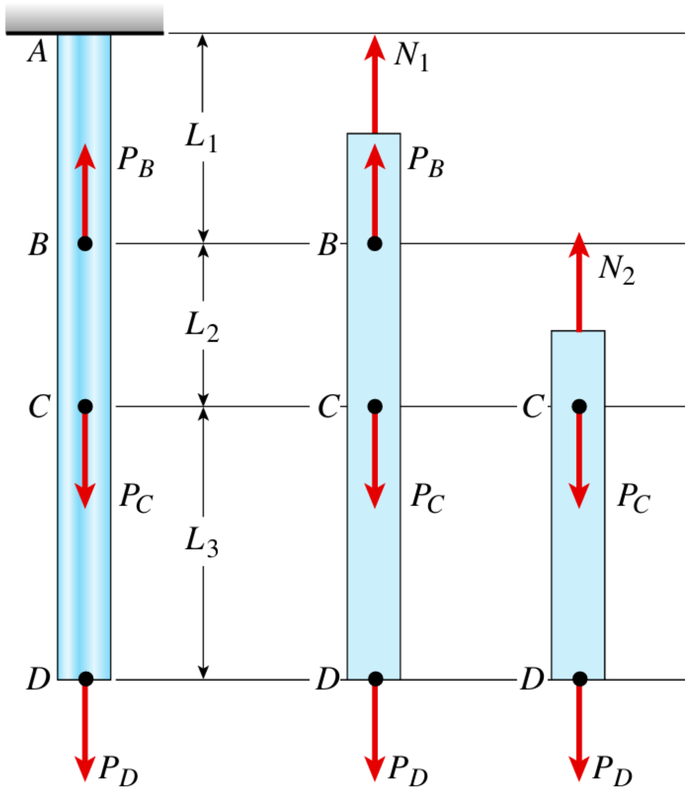
$$\implies N_1 = -P_B + P_C + P_D$$



Bars with intermediate axial loads

Method of sections:
1. Set imaginary cut
2. Draw FBD
3. Balance equation

- Segment BC:

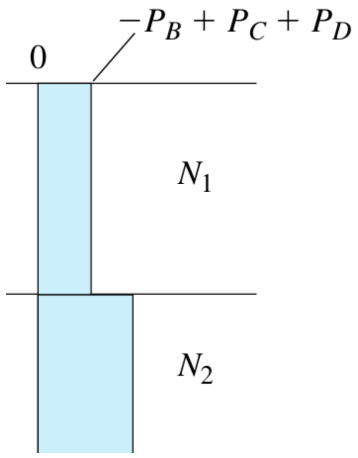


$$\sum F_y = 0 \Rightarrow N_1 + P_B - P_C - P_D = 0$$

$$\Rightarrow N_1 = -P_B + P_C + P_D$$

$$\sum F_y = 0 \Rightarrow N_2 - P_C - P_D = 0$$

$$\Rightarrow N_2 = P_C + P_D$$

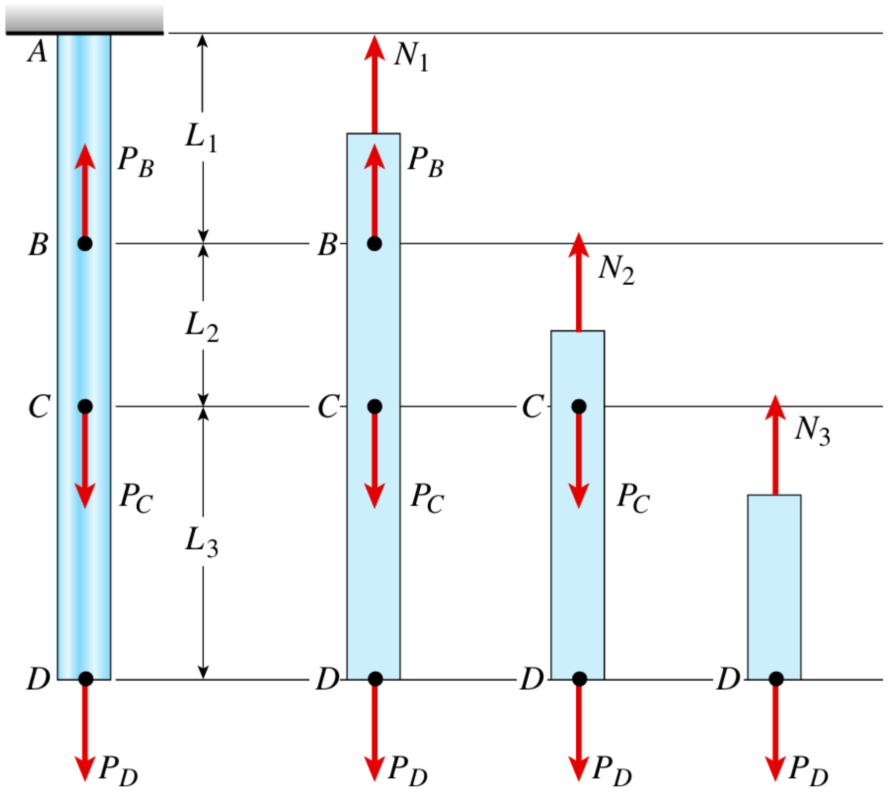


Bars with intermediate axial loads

Method of sections:

1. Set imaginary cut
2. Draw FBD
3. Balance equation

- Segment CD:



$$\sum F_y = 0 \implies N_1 + P_B - P_C - P_D = 0$$

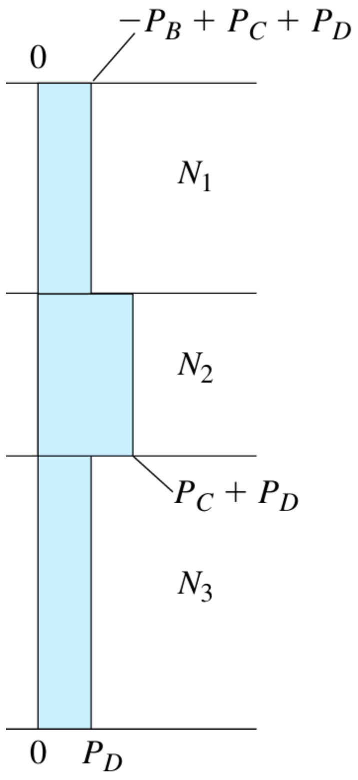
$$\implies N_1 = -P_B + P_C + P_D$$

$$\sum F_y = 0 \implies N_2 - P_C - P_D = 0$$

$$\implies N_2 = P_C + P_D$$

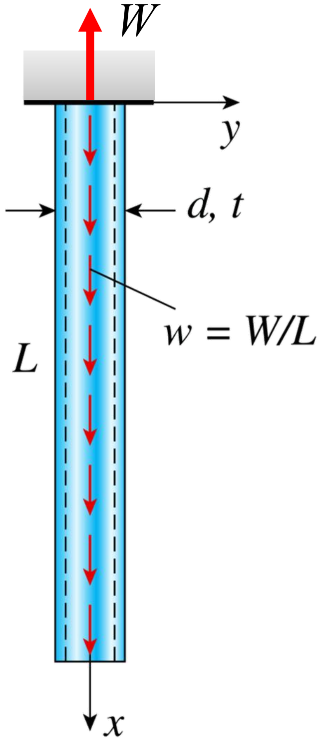
$$\sum F_y = 0 \implies N_3 - P_D = 0$$

$$\implies N_3 = P_D$$



Bars with distributed axial loads

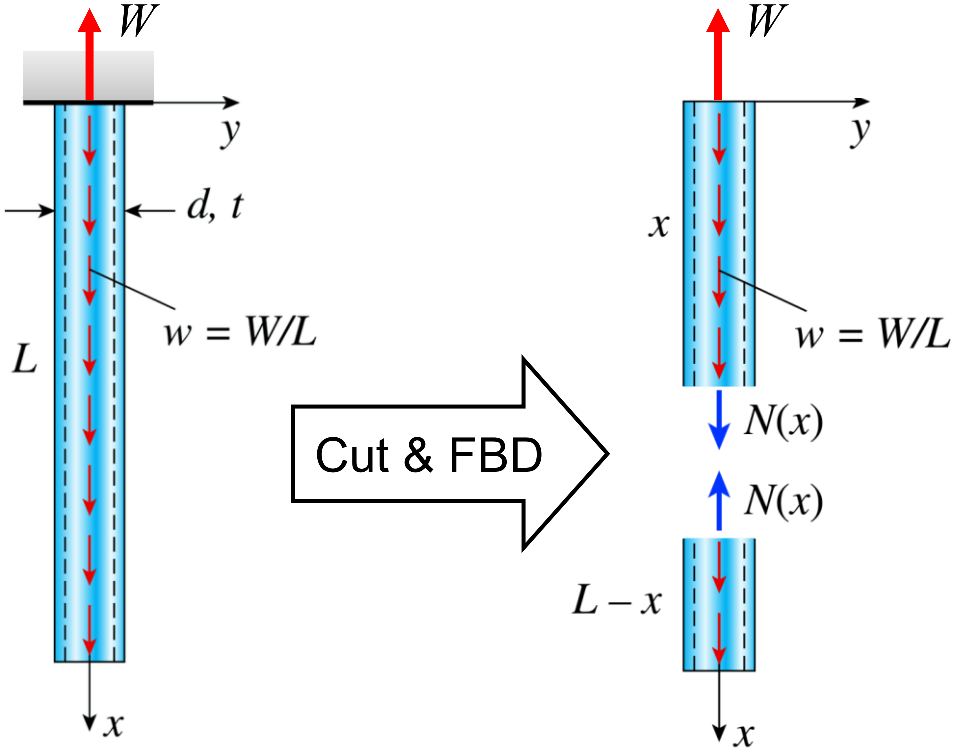
Example: Determine the internal forces in prismatic pipe hanging from drill rig



Bars with distributed axial loads

Method of sections:
1. Set imaginary cut
2. Draw FBD
3. Balance equation

Example: Determine the internal forces in prismatic pipe hanging from drill rig



Upper half:

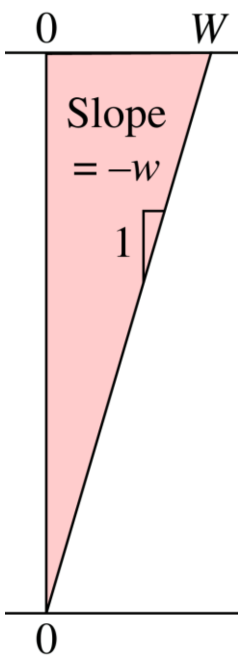
$$\sum F_x = 0 \Rightarrow -W + wx + N(x) = 0$$

$$\Rightarrow N(x) = W - wx = w(L - x)$$

Lower half:

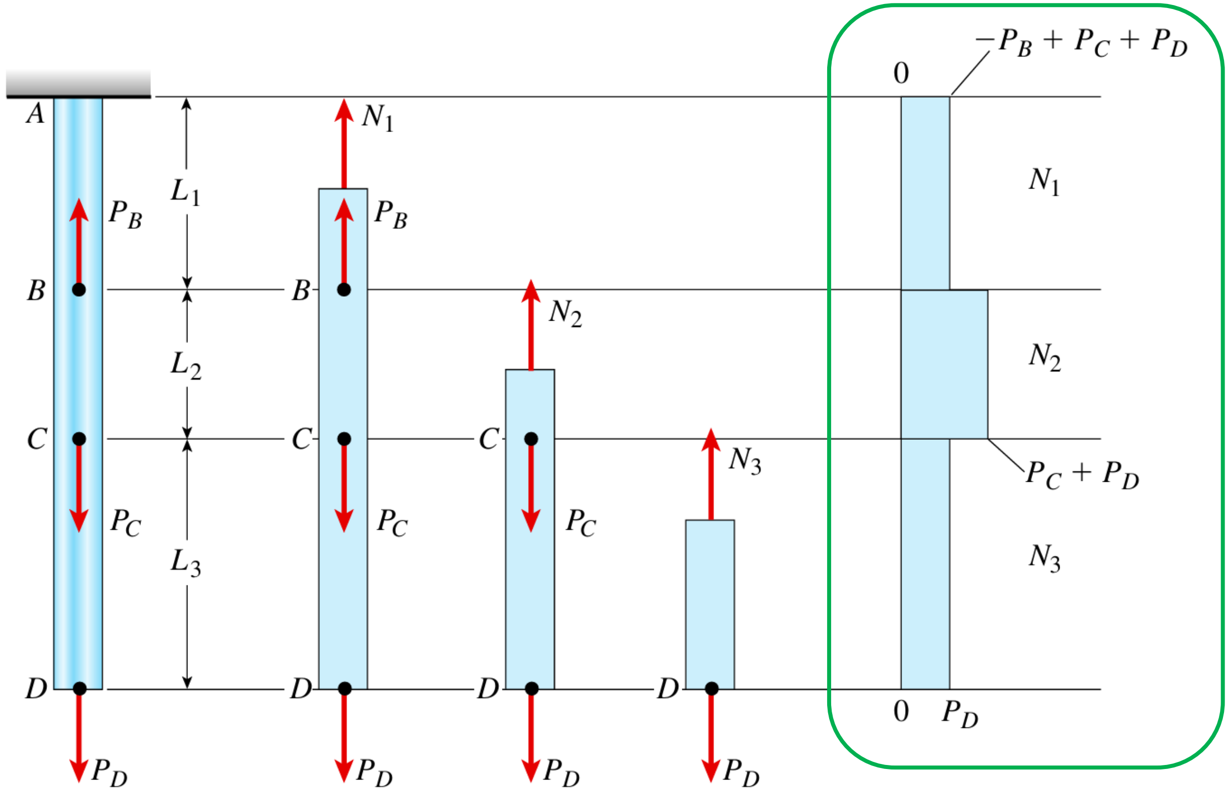
$$\sum F_x = 0 \Rightarrow -N(x) + w(L - x) = 0$$

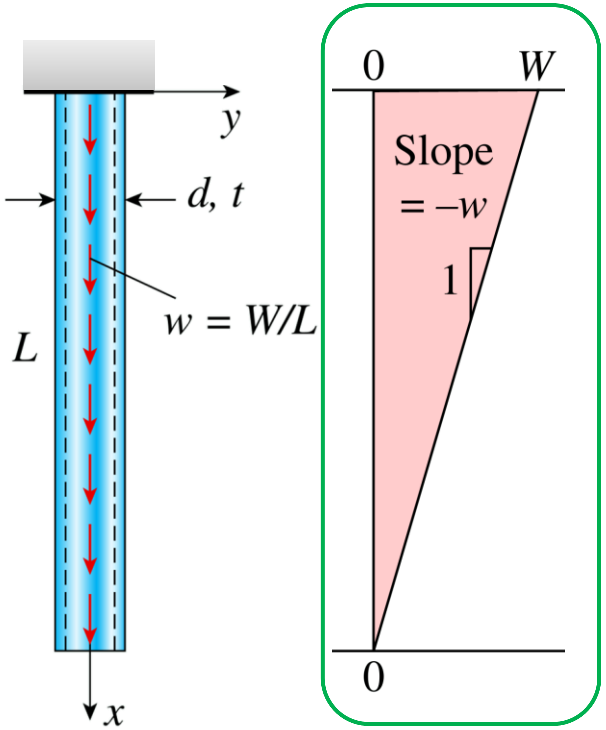
$$\Rightarrow N(x) = w(L - x)$$



Axial force diagram (AFD)

An axial force diagram (AFD) graphically represents the internal axial forces (tension or compression) along a structural member.

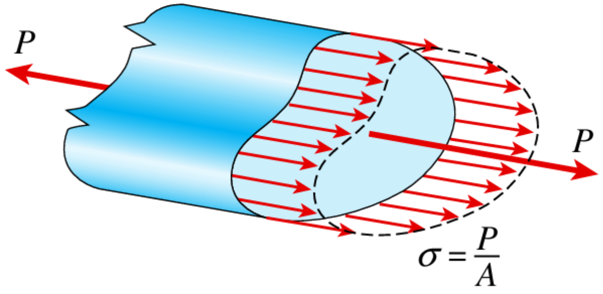




Stress in an axial loaded bar

Previously, we have

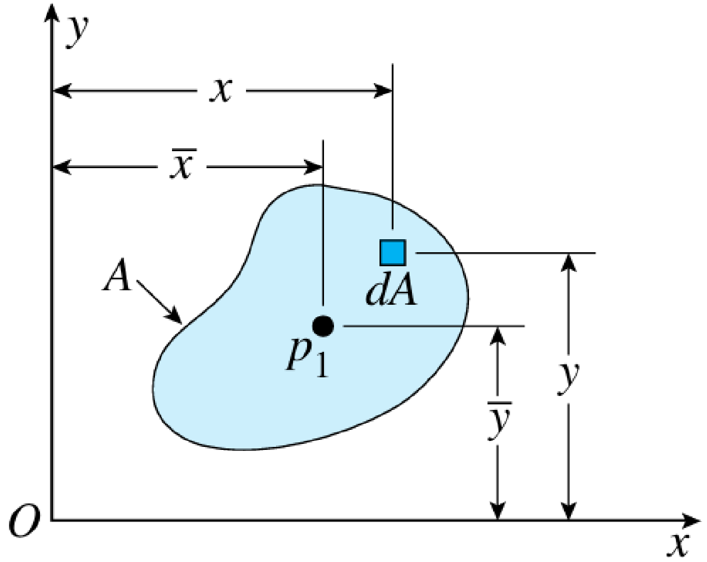
$$\sigma = \frac{P}{A}$$



The assumption behind:

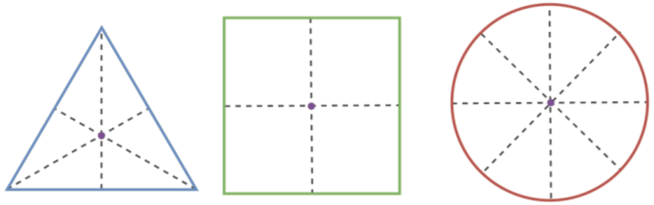
Force uniformly distributed on the entire cross-section.

The resultant force passes through the **centroid** (p_1) of the cross section.



$$\bar{x} = \frac{\int x dA}{\int dA}, \qquad \bar{y} = \frac{\int y dA}{\int dA}$$

Centroid in Geometry



Stress in an axial loaded bar

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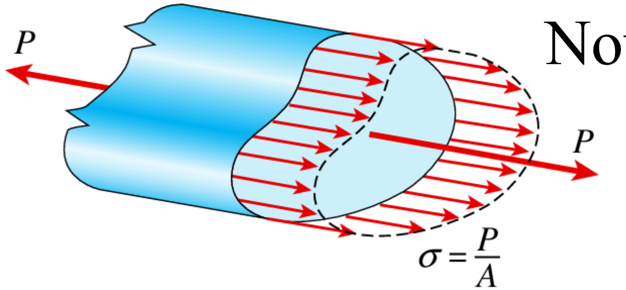


The assumption behind:

Force uniformly distributed on the entire cross-section.

↓

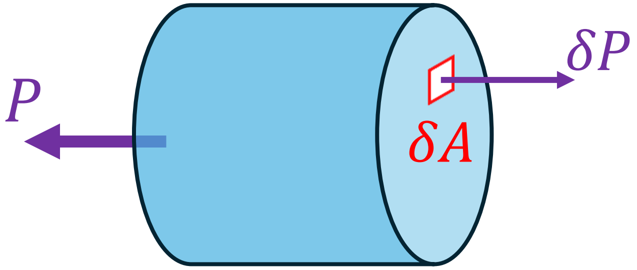
The resultant force passes through the **centroid** (p_1) of the cross section.

Now, we can update our definition:

$$\sigma = \lim_{\delta A \to 0} \frac{\delta P}{\delta A} = \frac{dP}{dA}$$

Then,

$$dP = \sigma dA \Longrightarrow P = \int_A \sigma dA$$



Why do we need limit-based definitions?

- In practice, **real boundary forces are never uniformly distributed** across a surface.
- They are applied **over finite, sometimes irregular areas**, often with unknown or complex distributions.
- But mostly, we still use the **average stress**:

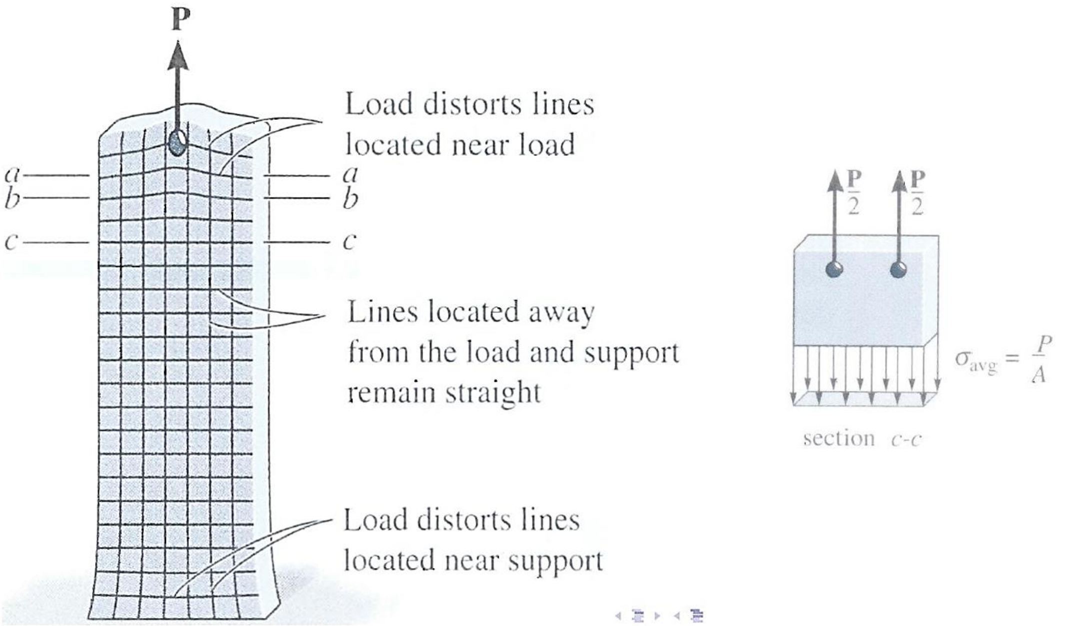
$$\sigma = \frac{P}{A}$$

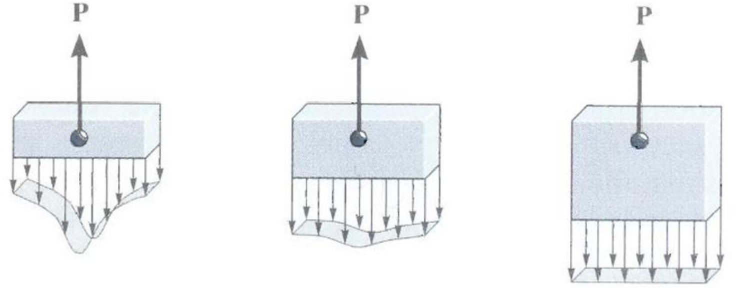
Oversimplified?

The answer lies in
Saint-Venant's Principle

Saint-Venant's Principle

- "*... the difference between the effects of two different but **statically equivalent loads** becomes very small at **sufficiently large distances** from load.*"
- If two different loadings have the **same resultant force and moment**, then they produce **nearly identical** stress and strain fields at points sufficiently far from the loaded region.





section $a-a$ **section** $b-b$ **section** $c-c$, $\sigma_{avg} = \frac{P}{A}$

- Details of load distribution have little effect on the stress distribution on sections far away enough from the loading area
- Loading system can be freely replaced by an equivalent one.

Force-displacement relations

In real engineering design, we don’t measure stress and strain directly.
Instead, we ask:

- How much **load** can I apply?
- How much **deformation** will it cause?

You care more about *force* and *displacement*, not stress and strain per se.

Force (load) ⟵ – – – – – ⟶ Displacement (deformation)

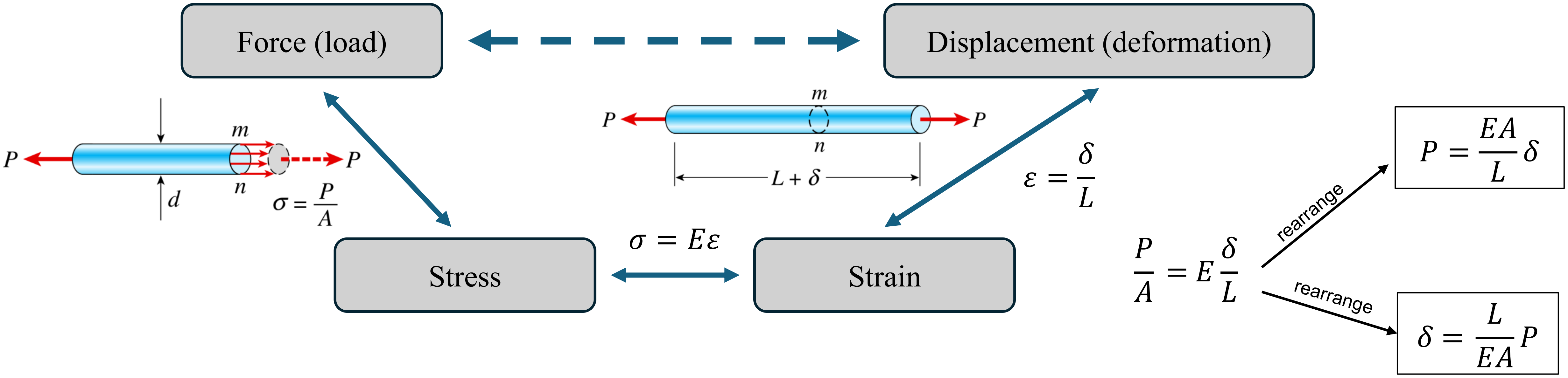
- Geometry
- Material behavior
- Boundary conditions
- …

The force–displacement relation is **not explicit** and **often too complex to derive directly**. Instead of solving it all at once, we take a **systematic route.**

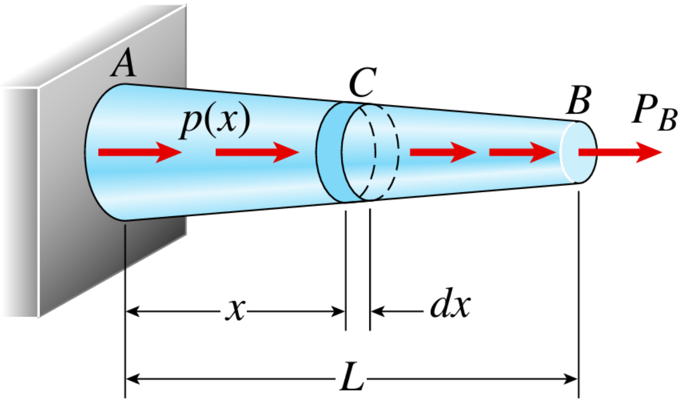
Force-displacement relations

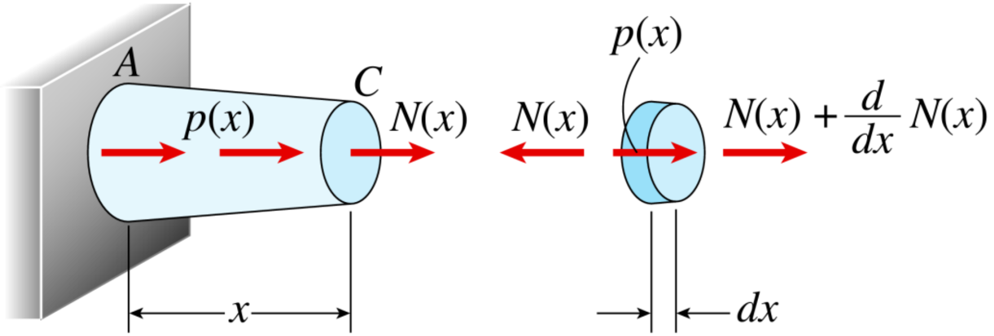
Bypass Strategy:

- **Force** ⇔ **Stress** via equilibrium and definition of stress.
- **Stress** ⇔ **Strain** via constitutive laws (e.g. Hooke’s Law).
- **Strain** ⇔ **Displacement** via compatibility or definition of strain.



More generally





Force (load)

Displacement (deformation)

$$\sigma(x) = \frac{N(x)}{A(x)}$$

$$\varepsilon(x) = \frac{d\delta(x)}{dx}$$

$$\sigma(x) = E\varepsilon(x)$$

Stress

Strain

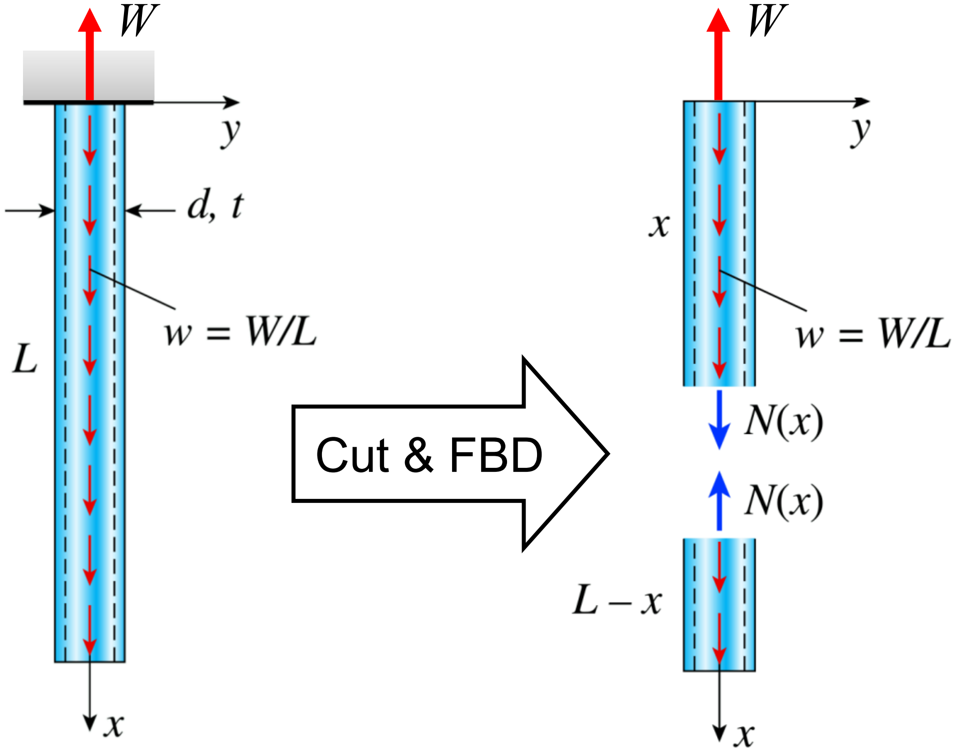
$$d\delta(x) = \frac{N(x)dx}{EA(x)}$$

$$\Rightarrow \delta(x) = \int_0^x \frac{N(\xi)}{EA(\xi)} d\xi$$

$$\Rightarrow \delta = \int_0^L \frac{N(\xi)}{EA(\xi)} d\xi$$

Axial Displacement Diagram (ADD)

Recall exercise: prismatic pipe hanging from drill rig



Upper half:

$$\sum F_x = 0 \Longrightarrow -W + wx + N(x) = 0$$

$$\Longrightarrow N(x) = W - wx = w(L - x)$$

Lower half:

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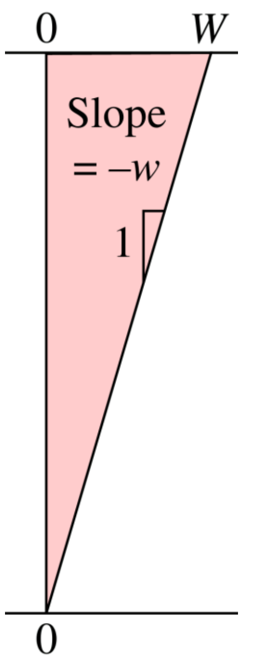

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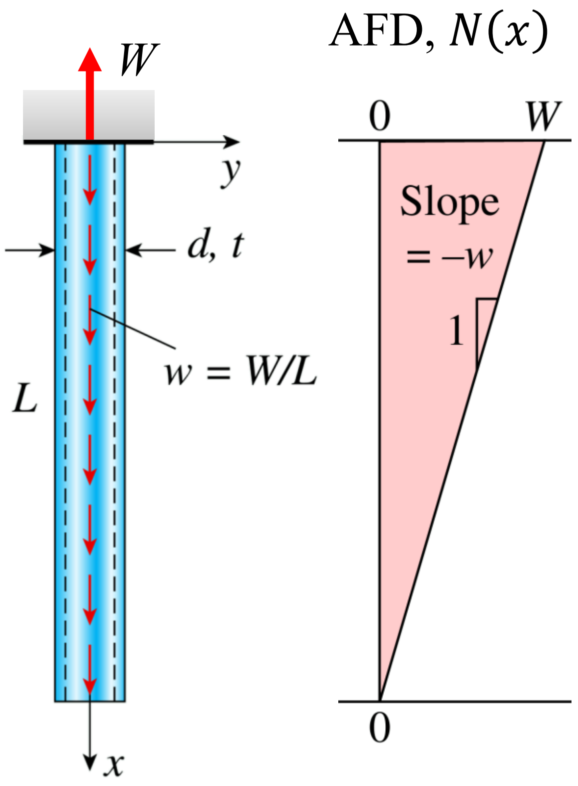


$$N(x) = w(L - x)$$

The force-displacement relation:

$$\delta(x) = \int_0^x \frac{N(\xi)}{EA} d\xi = \int_0^x \frac{w(L-\xi)}{EA} d\xi = \frac{wL^2}{2EA}\left[\frac{2x}{L} - \left(\frac{x}{L}\right)^2\right]$$

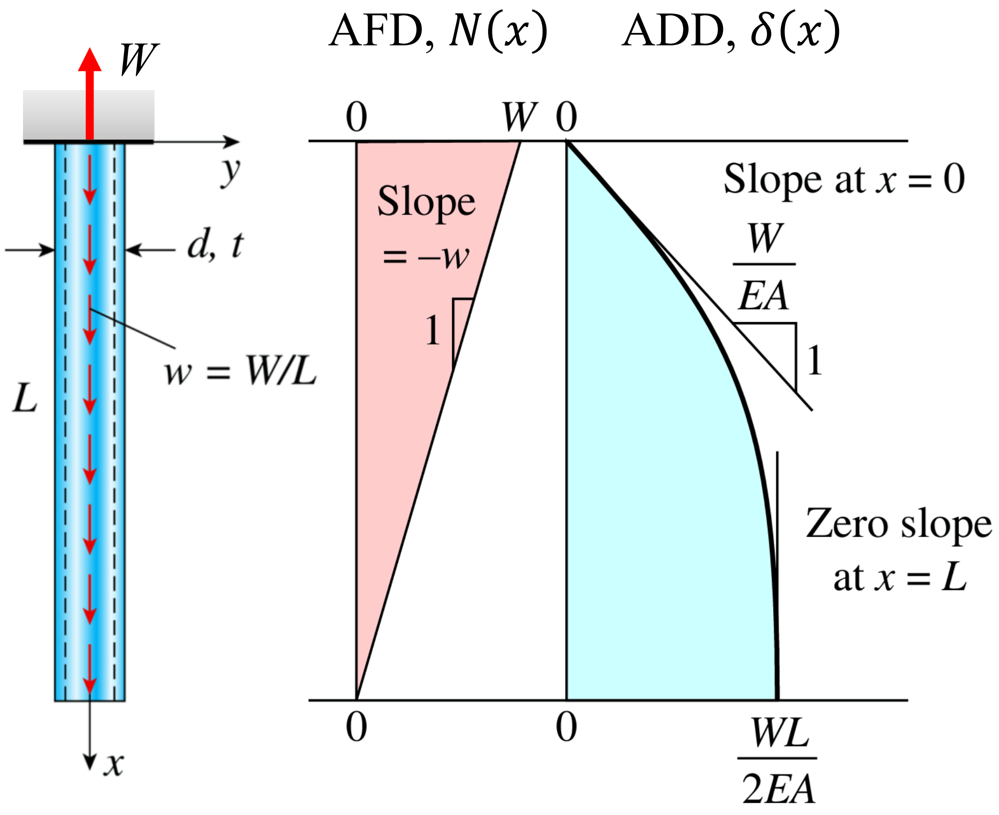
which is a quadratic function.

$$\delta(0) = 0$$

$$\delta_{\max} = \delta(L) = \frac{wL^2}{2EA}$$

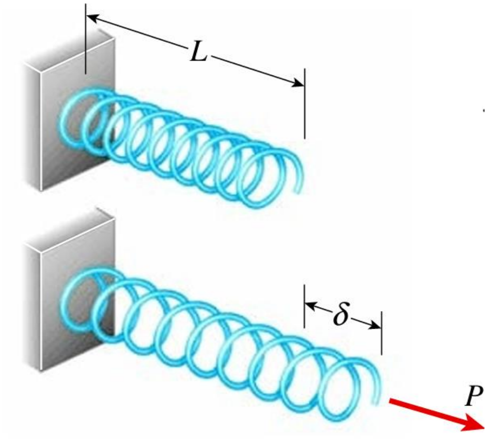
Axial Displacement Diagram (ADD)

Recall exercise: prismatic pipe hanging from drill rig



Rigidity, stiffness and flexibility

$$\delta = \frac{L}{EA} P \Leftrightarrow P = \frac{EA}{L} \delta$$



Following the definition of spring:

$$P = k\delta, \qquad \delta = fP$$

Stiffness:

$$k = \frac{EA}{L}$$

Force required to produce a unit elongation

Flexibility:

$$f = \frac{L}{EA}$$

Elongation due to a unit load

Axial rigidity:

$$EA$$