Georgia Tech
College of Engineering

COE 3001

MECHANICS OF DEFORMABLE BODIES

Lecture 4 – Tension, Compression and Shear

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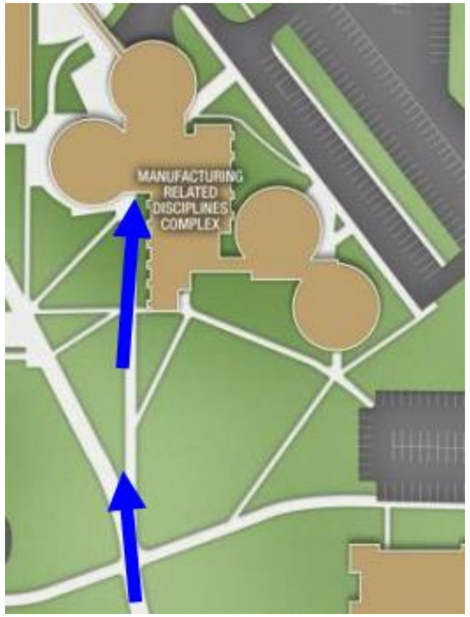
Georgia Institute of Technology

Jan. 26, 2026

Shell Tutoring

The George W. Woodruff School of Mechanical Engineering will offer **Shell Tutoring** for COE 3001 this semester. Tutoring will run from **January 19 to April 24**

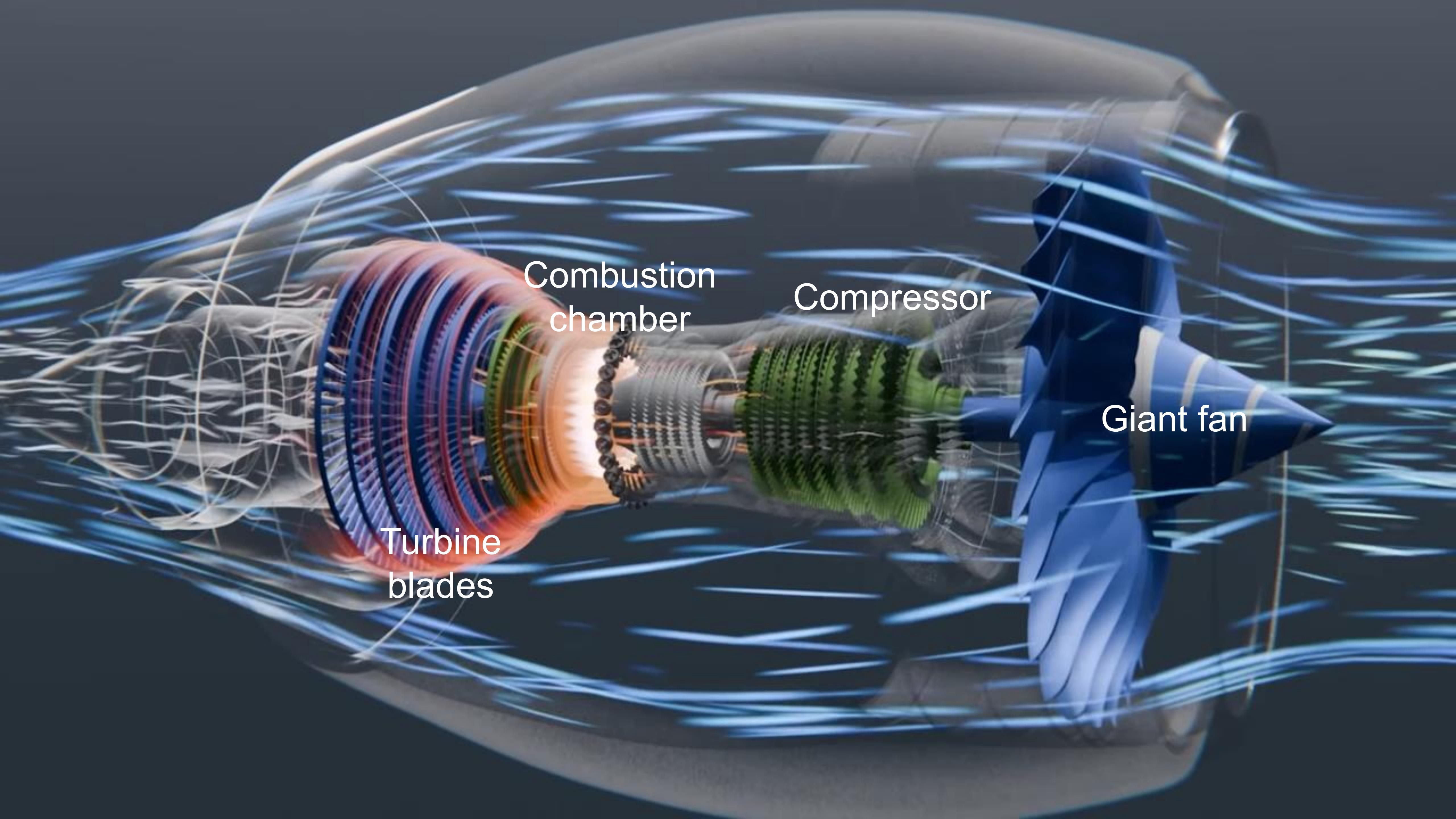


Schedule: Monday–Thursday, 6:00 PM – 8:00 PM

Location: MRDC 4th Floor (Tables near zone 1 and zone 2)

Tutors:

- Chu, Elizabeth E (echu45@gatech.edu)
- Islam, Marzanul (mislam314@gatech.edu)
- Please note that they are not TAs for this class.


Combustion chamber
Compressor
Giant fan
Turbine blades

Hooke's law: modulus of elasticity

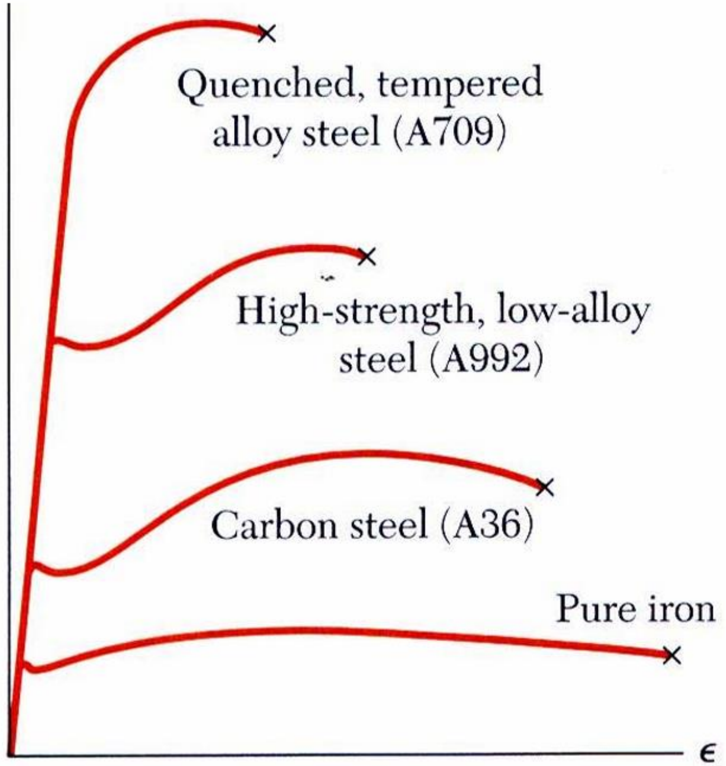


Fig. 2.16 Stress-strain diagrams for iron and different grades of steel.

- Below the yield stress

$$\sigma = E\varepsilon$$

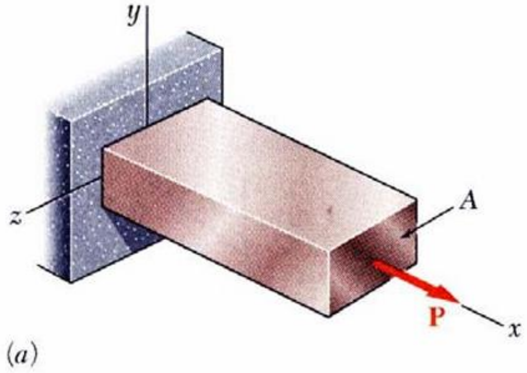
Hooke's Law

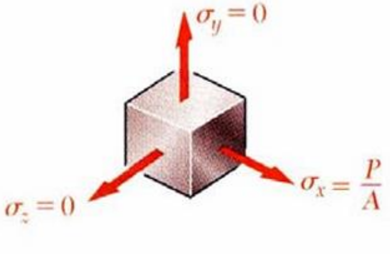
E = Youngs Modulus or Modulus of Elasticity

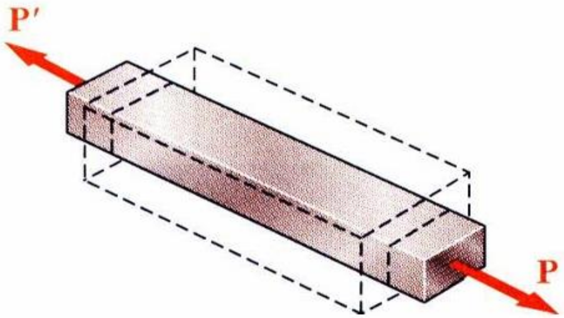
- *E* is a measure of inherent stiffness of a material.

- Strength is affected by alloying, heat treating, and manufacturing process but stiffness (Modulus of Elasticity) is not.

Poisson's ratio







For a slender bar subjected to axial loading:

$$\varepsilon_x = \frac{\sigma_x}{E} \quad \sigma_y = \sigma_z = 0$$

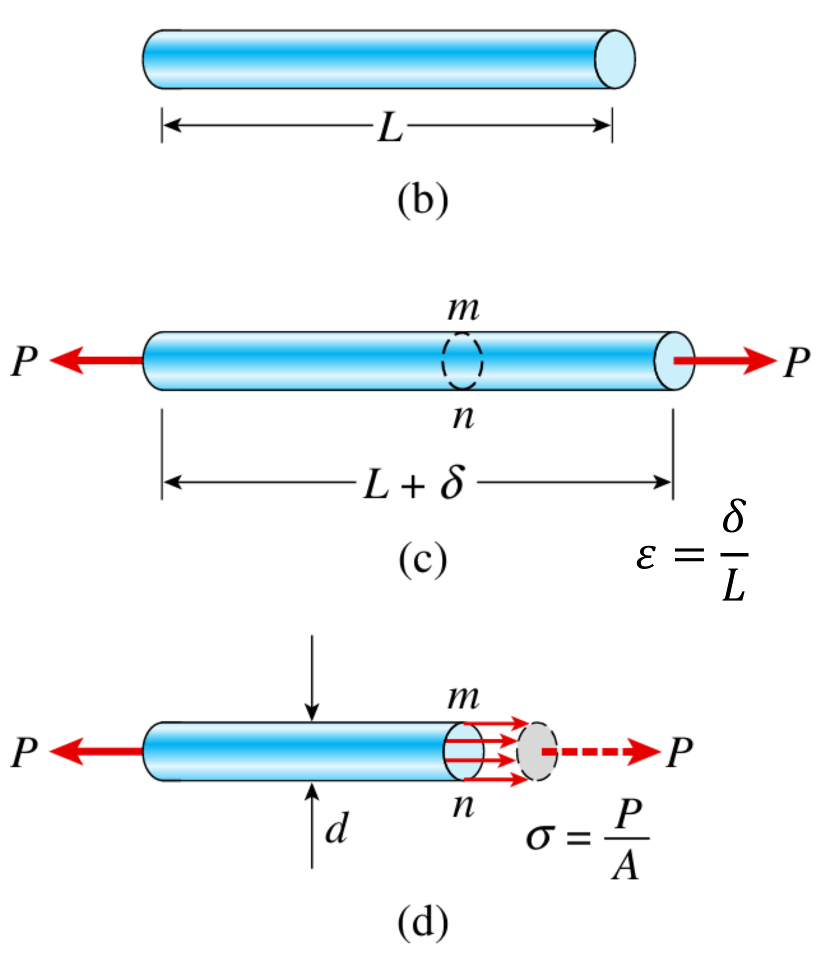
The elongation in the x-direction is accompanied by a contraction in the other directions. Assuming that the material is isotropic (no directional dependence),

$$\varepsilon_y = \varepsilon_z \neq 0$$

Poisson’s ratio is defined as

$$\nu = \left| \frac{\text{lateral strain}}{\text{axial strain}} \right| = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

Normal stress and strain

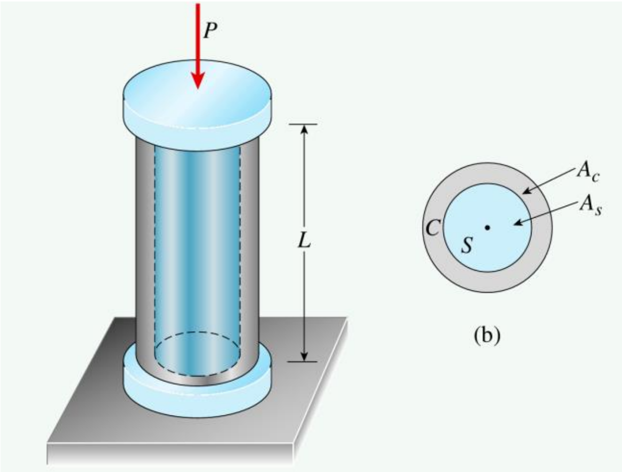


- Normal stresses act in a direction **perpendicular** to the cut surface.
- Normal stresses may be either **tensile** or **compressive**.
- **Sign convention**: tensile stress is considered as positive while compressive stress is defined as negative.

- Normal strain is related to **elongation**.
- Normal strain may be either **tensile** (stretching) or **compressive** (shortening).
- Normal strain is associated with normal stresses.

Exercise

A short post constructed from a hollow circular tube of aluminum supports a compressive load of 240 kN. The inner and outer diameters of the tube are d_1 = 90 mm and d_2 = 130 mm, respectively, and its length is 1 m. The shortening of the post due to the load is measured as 0.55 mm.
Determine the compressive stress and strain in the post. (Disregard the weight of the post itself, and assume that the post does not buckle under the load.)



Exercise

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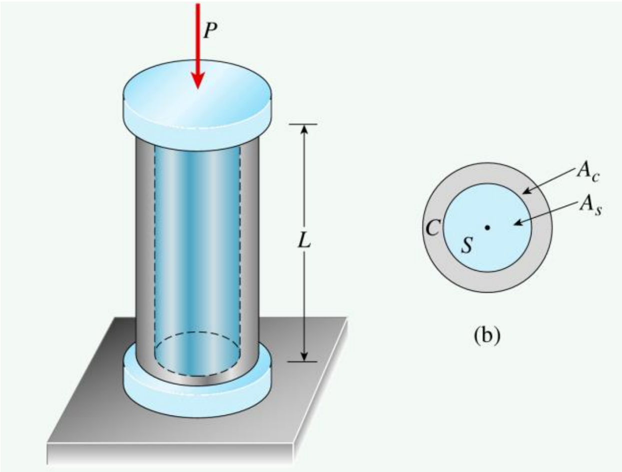


(b)

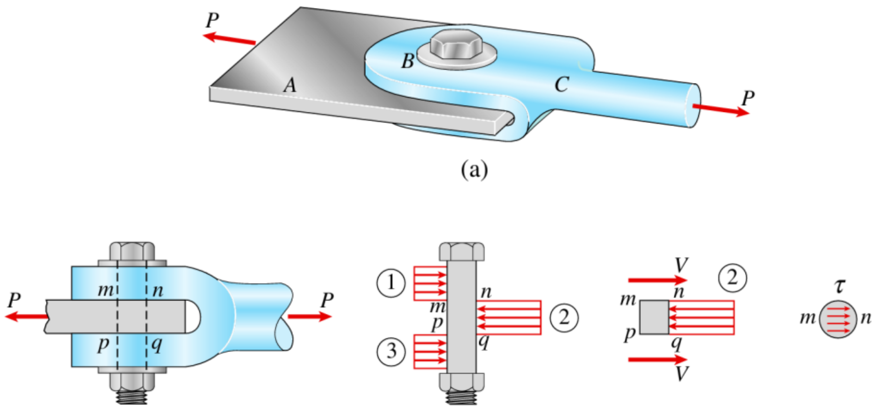
Solution:

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 6912 \text{ mm}^2$$

$$\text{stress: } \sigma = \frac{P}{A} = 34.7 \text{ MPa}$$

$$\text{strain: } \epsilon = \frac{\delta}{L} = 550 \times 10^{-6}$$

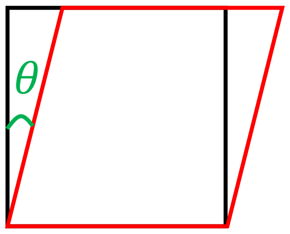
Shear stress and strain



- Shear stresses arise from **sliding forces** and act in a direction **tangential** to the cut surface.
- Shear stresses are denoted by τ, and calculated by

$$\tau = \frac{V}{A}$$

- Shear strain is related to **distortion** or **angular change** (rather than elongation).
- Shear strain is defined as the **change in angle** between two originally orthogonal line segments.



$$\gamma = \tan\theta \approx \theta$$

Sign convention: To be discussed later in **Torsion section.**

Hooke's Law in Shear

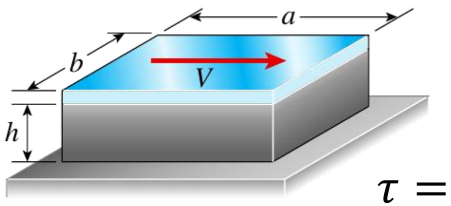
- For many materials, the initial part of the shear stress-strain diagram is a straight line through the origin, just as it is in tension.
- In the **linear elastic region**, shear stress and shear strain are related by:

$$\tau = G\gamma$$

where G is the **shear modulus**, analogous to Young's modulus in tension/compression.

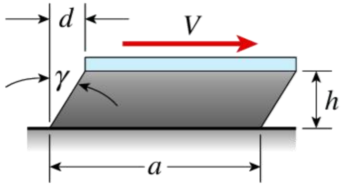
- The Young's modulus and shear modulus are related by

$$G = \frac{E}{2(1+\nu)}$$

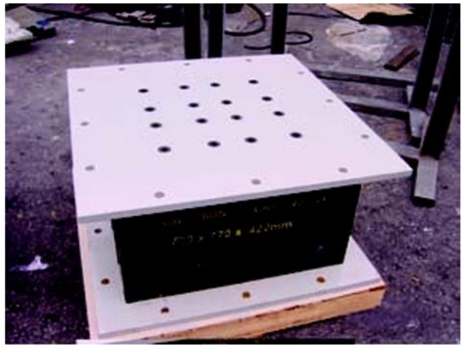


(a)

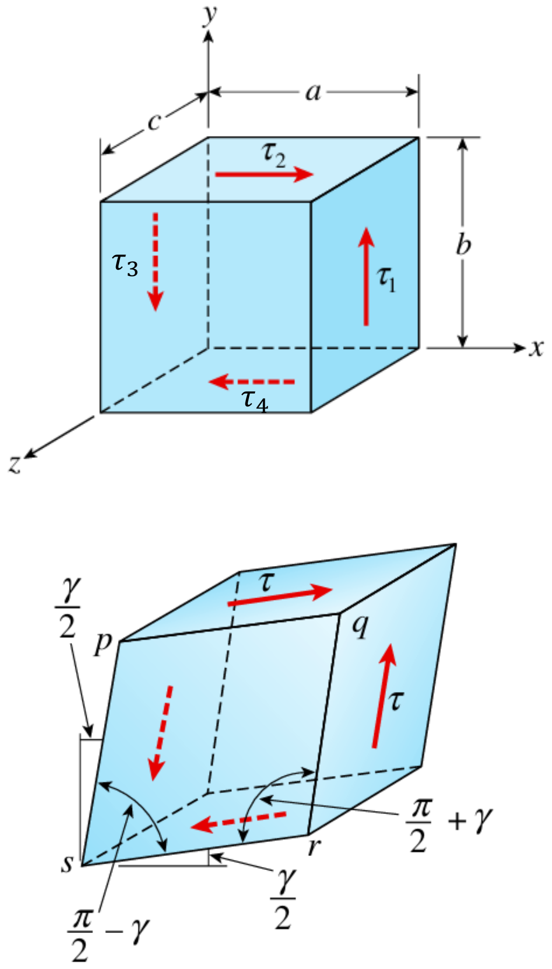
$$\tau = \frac{V}{ab}$$



(b)



Equality of Shear Stresses



Equilibrium of the element:

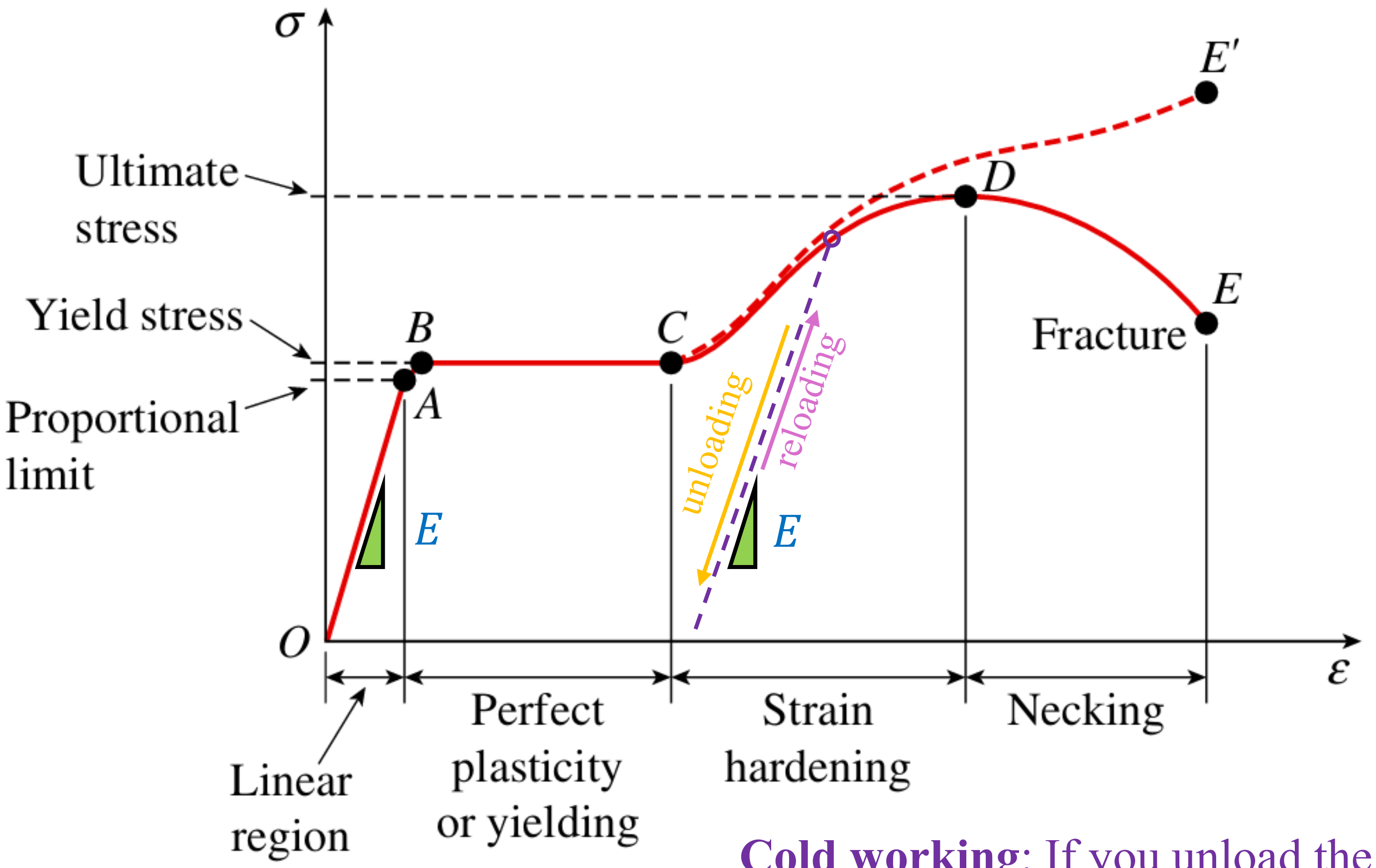
$$\sum F_x = 0 \Longrightarrow \tau_2 ac - \tau_4 ac = 0 \Longrightarrow \tau_2 = \tau_4$$

$$\sum F_y = 0 \Longrightarrow \tau_1 bc - \tau_3 bc = 0 \Longrightarrow \tau_1 = \tau_3$$

$$\sum M_z = 0 \Longrightarrow (\tau_1 bc)a - (\tau_2 ac)b = 0 \Longrightarrow \tau_1 = \tau_2$$

- Shear stresses on opposite (and parallel) faces of an element are **equal in magnitude** and **opposite in direction**.
- Shear stresses on adjacent (and perpendicular) faces of an element are equal in magnitude and pointing **head-to-head** or **tail-to-tail**.
- The stress state with only shear stresses is called **pure shear.**

Stress-strain curve for mild steel



Below the yield stress:

$$\sigma = E\varepsilon$$

- E = Young's modulus

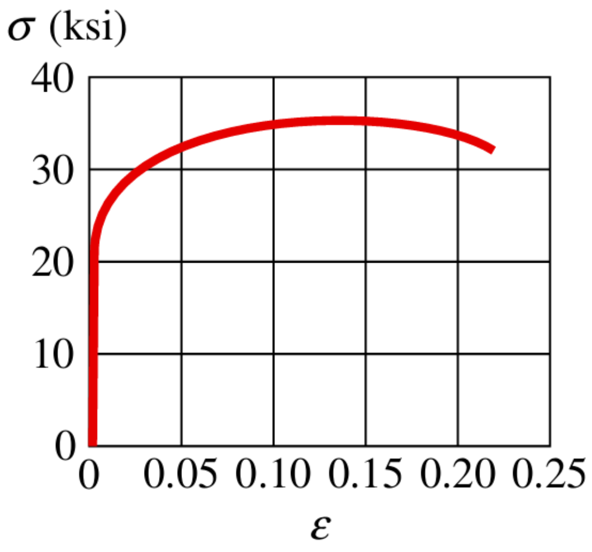
E is a measure of inherent stiffness of a material.

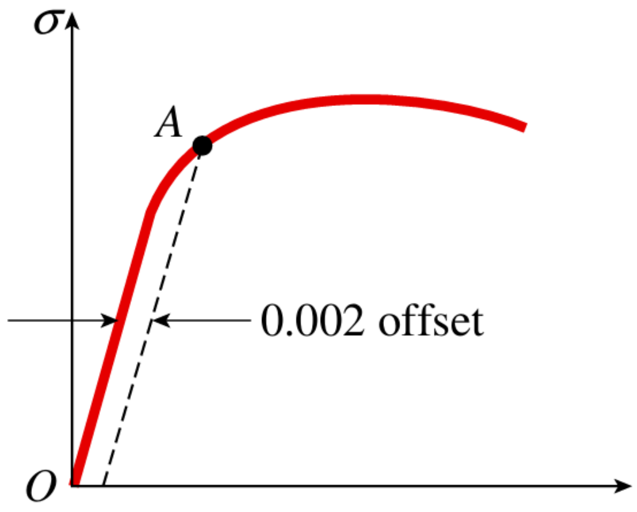
Cold working: If you unload the specimen during the strain hardening stage, the material will unload **elastically** along a line parallel to the original slope, leaving a permanent plastic deformation.

Offset method

For materials with no constant yielding or no well-defined yield point, **Offset yield strength**: a stress corresponding to the intersection of the stress-strain curve and a line parallel to the elastic part of the curve offset by a specified strain. (usually 0.2% offset)

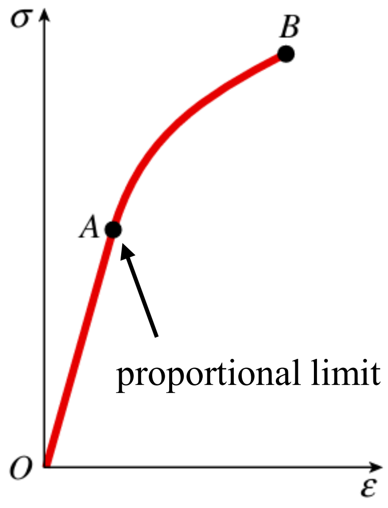
Aluminum alloy



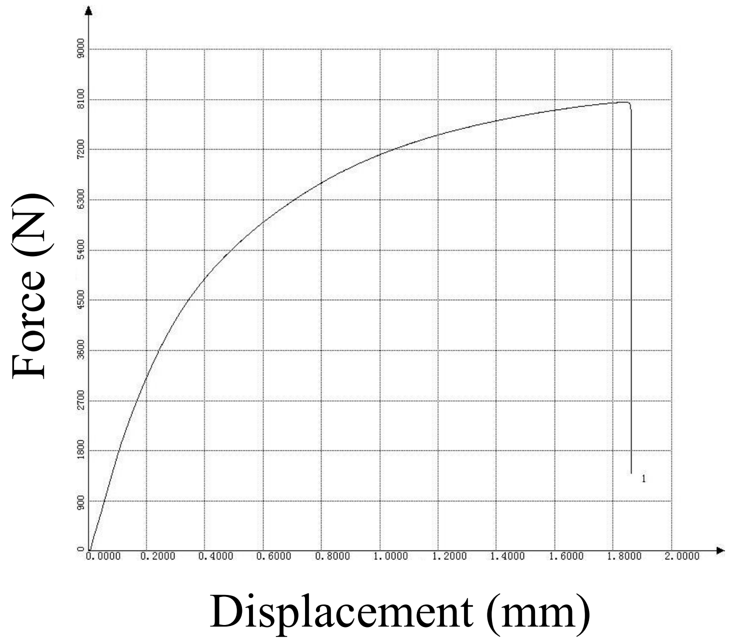


Stress-strain curve: brittle materials

Materials that fail in tension at relatively low values of strain are classified as **brittle**. Examples are concrete, stone, cast iron, glass, ceramics, and a variety of metallic alloys.



cast iron



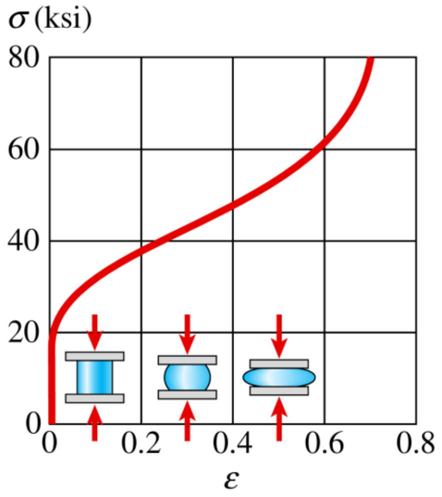
Compression

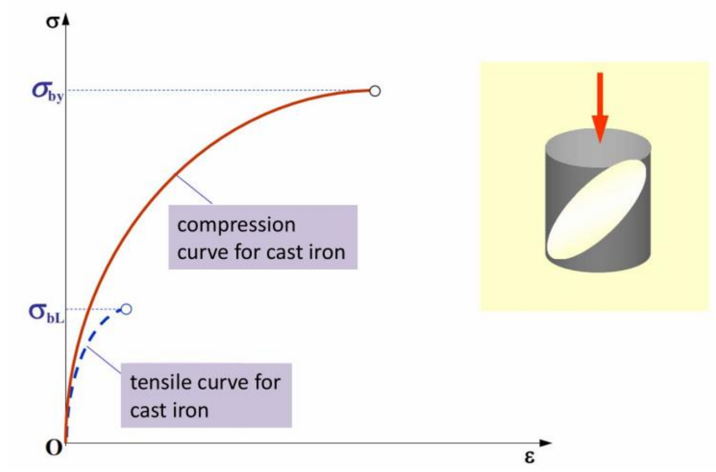
Ductile material:

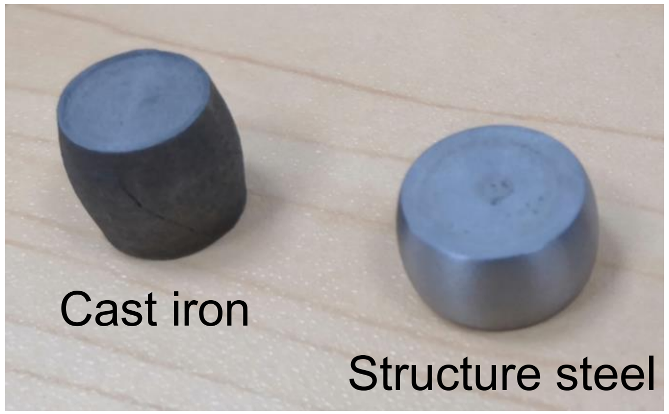
- initial stage:
  - compressive/tensile diagrams — about the same
- after yielding:
  - Tensile: necking, fracture.
  - Compression: bulge out, even stronger.

Brittle material:

- Compression/tension diagrams — similar shapes
- Ultimate stresses in compression are much higher than those in tension.
- break at the maximum load

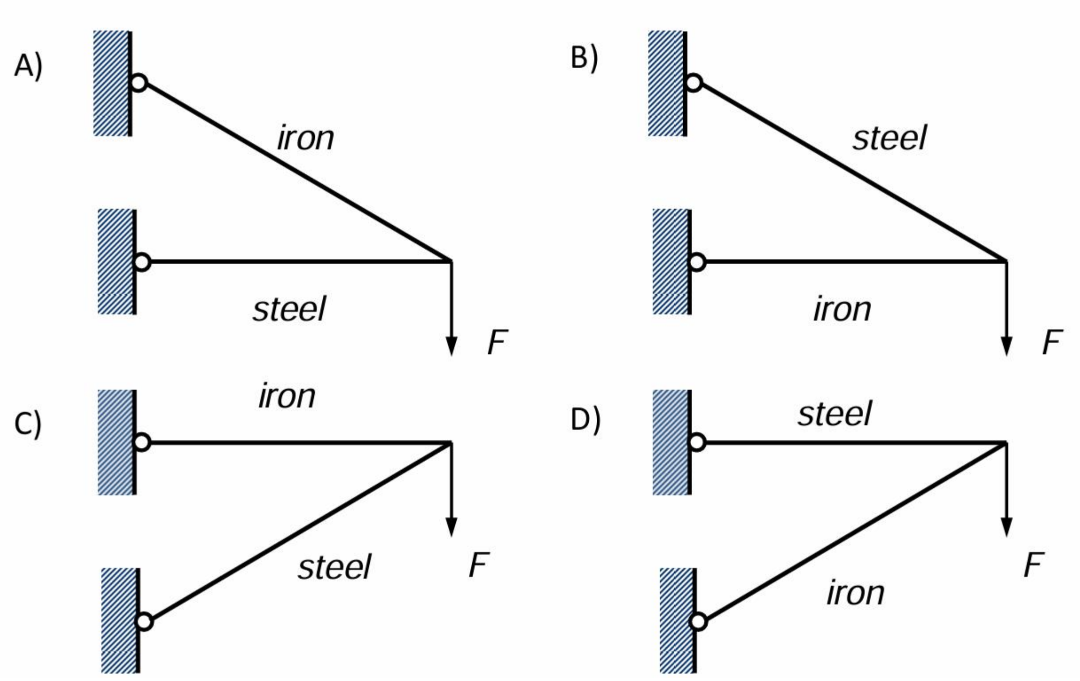






Exercise

In the following four structures, the cross-sectional area of beam, length of horizontal beam, the included angle are the same. Which structure is reasonable?



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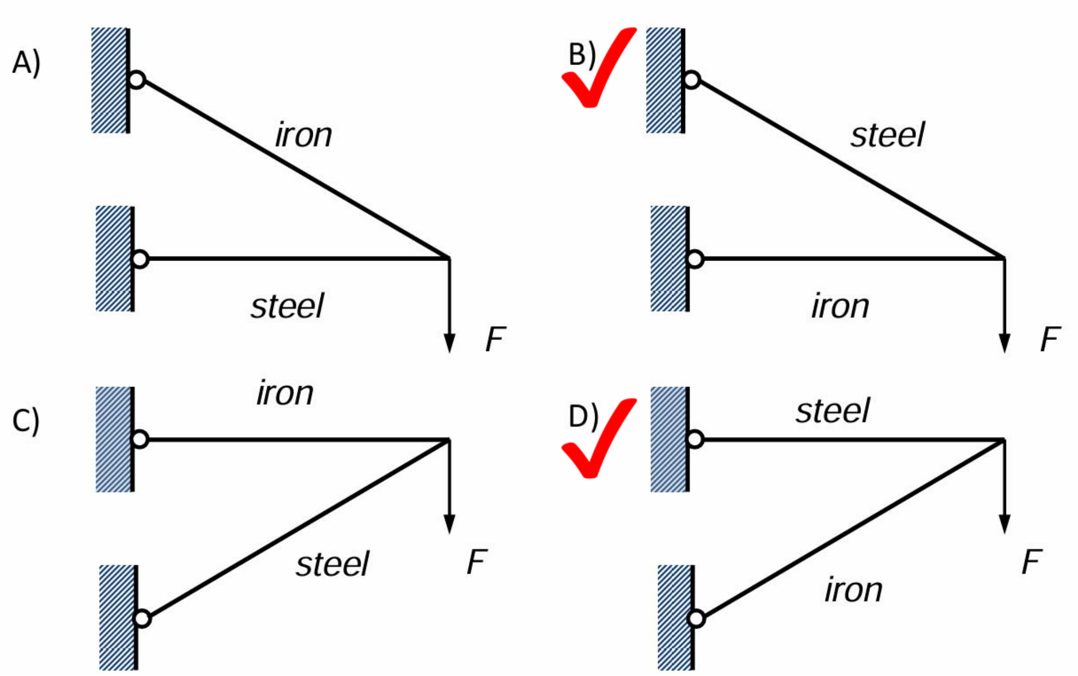


- Iron: brittle – good for compression
- Steel: ductile – good for both tension and compression

✓ B, D are OK for now.

In future lessons, you will learn that

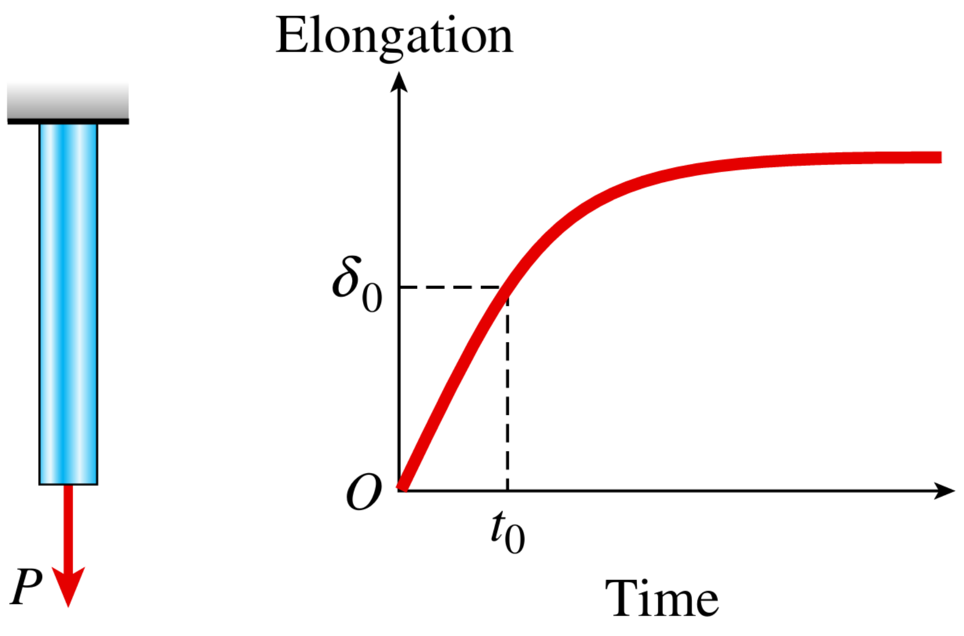
- compression members should be as short as possible

✓ B is more reasonable.

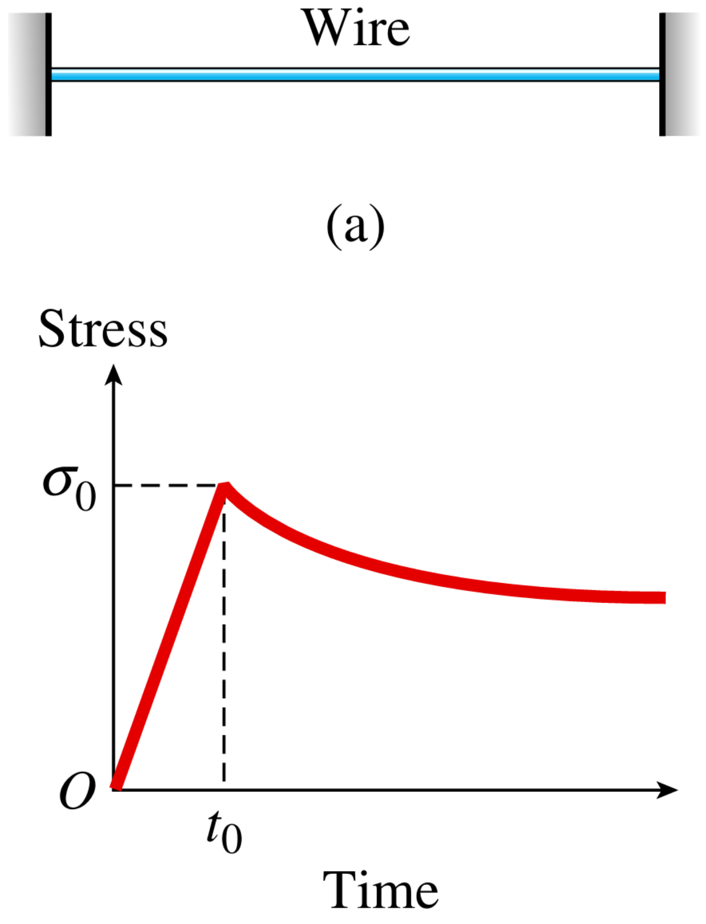
Why do wires have a service life?

Creep

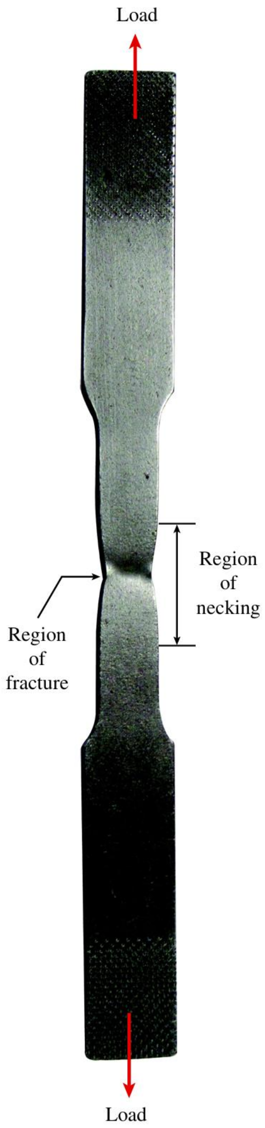
Creep in a bar under constant load

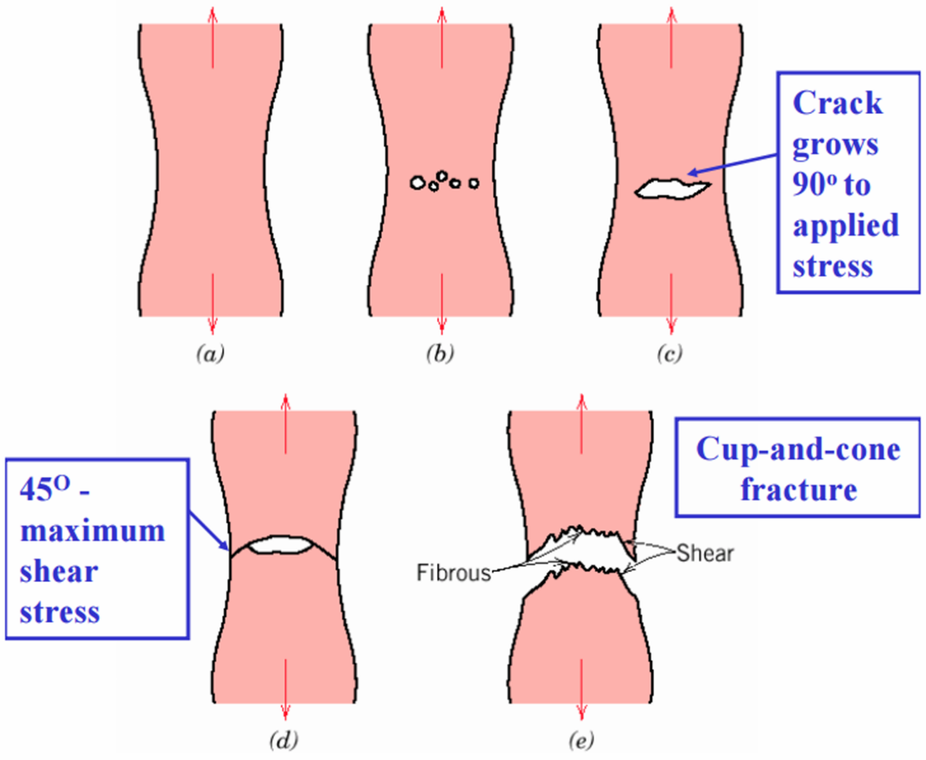


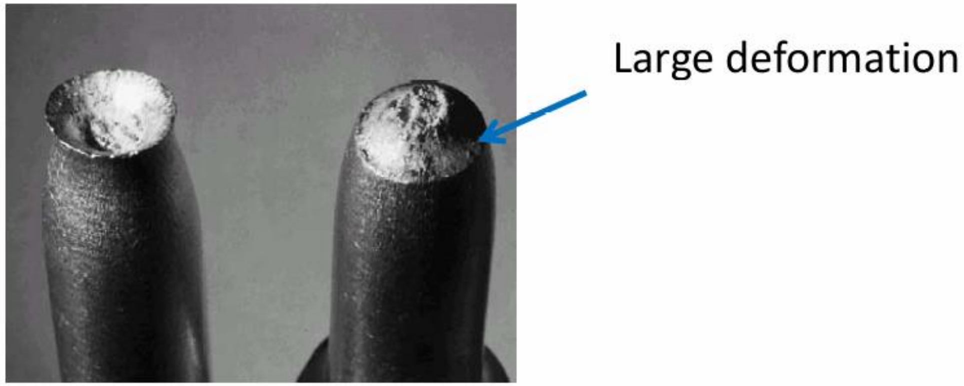
Relaxation of stress in a wire under constant strain



Ductile fracture

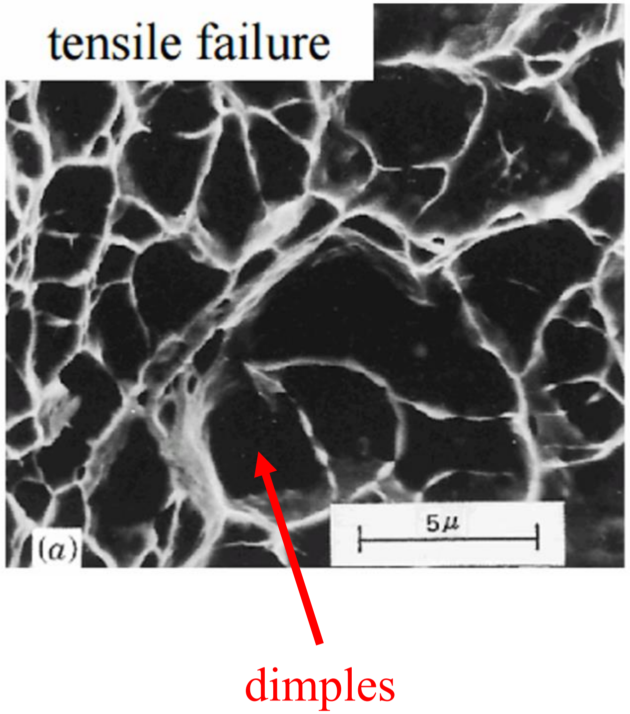




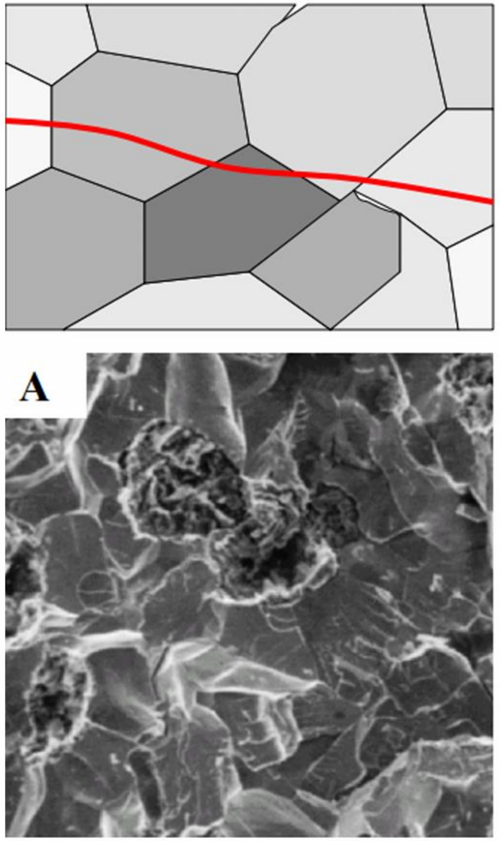


Cup-and-cone fracture in Aluminum

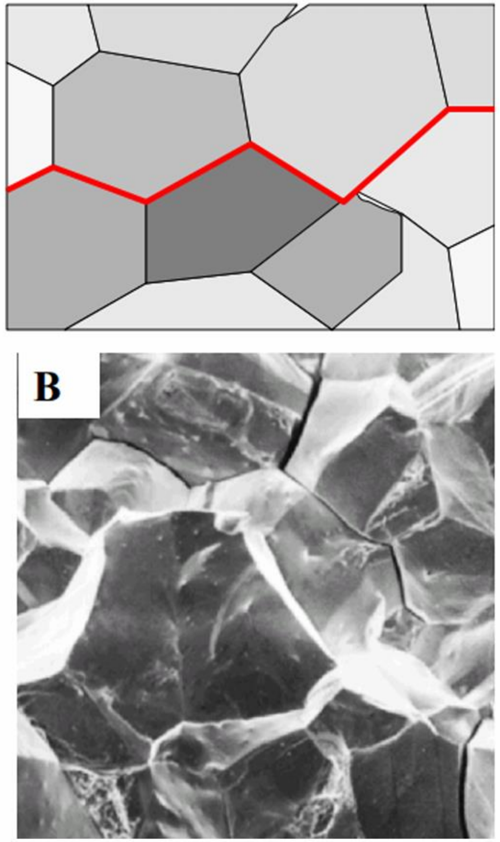
(a) Necking
(b) Formation of microvoids
(c) Coalescence of microvoids to form a crack
(d) Crack propagation by shear deformation
(e) Fracture



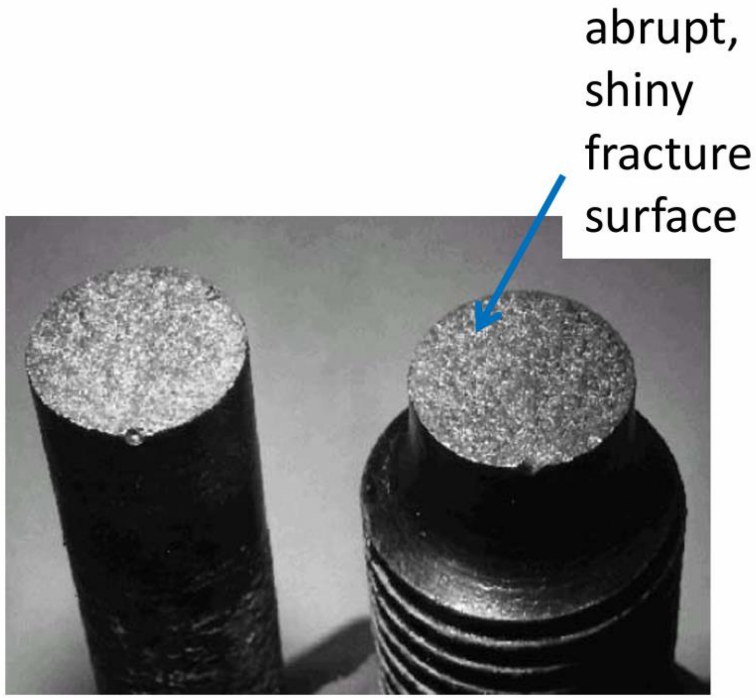
Brittle fracture



Transgranular fracture

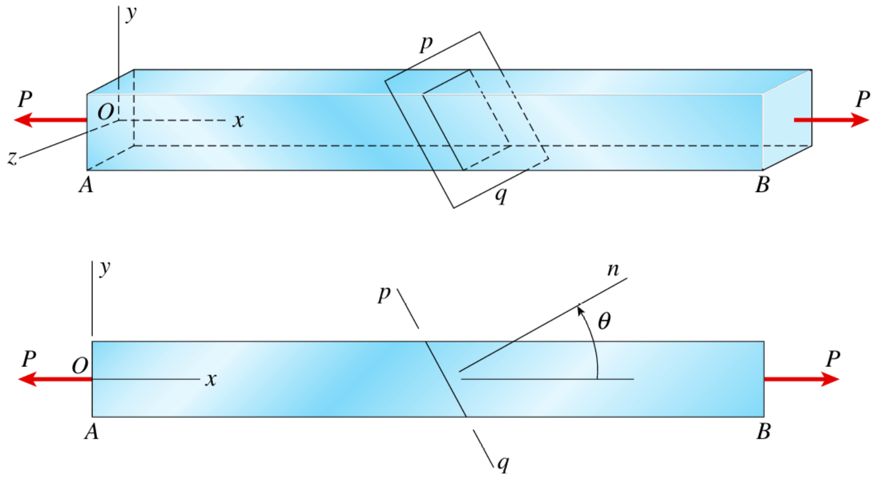


Intergranular fracture



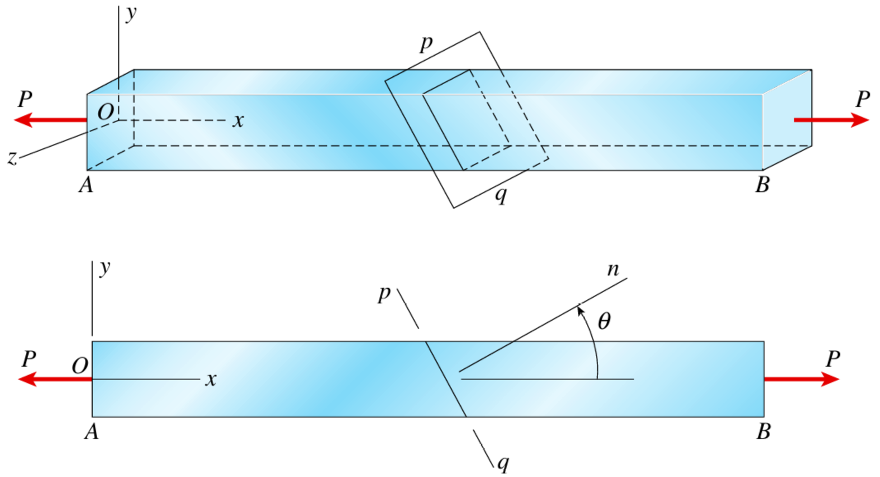
Brittle fracture in a mild steel

Exercise: Stresses on inclined sections

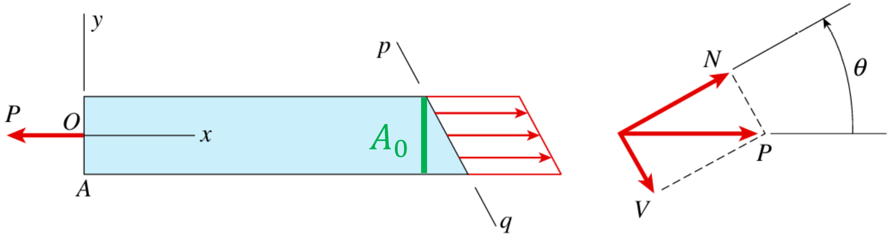


- A Prismatic bar in tension with loading force P.
- The cross-sectional area of the bar is A.
- pq is the section inclined at angle θ.
- Find the normal and shear stress acting on the inclined section pq.

Exercise: Stresses on inclined sections



- A Prismatic bar in tension with loading force P.
- The cross-sectional area of the bar is A_0.
- pq is the section inclined at angle θ.
- Find the normal and shear stress acting on the inclined section pq.

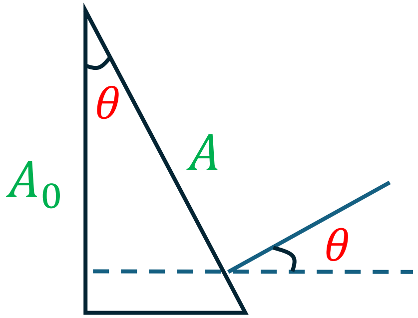


Geometric relation:

$$A = \frac{A_0}{\cos\theta}$$

$$N = P\cos\theta$$

$$V = P\sin\theta$$



Normal stress:

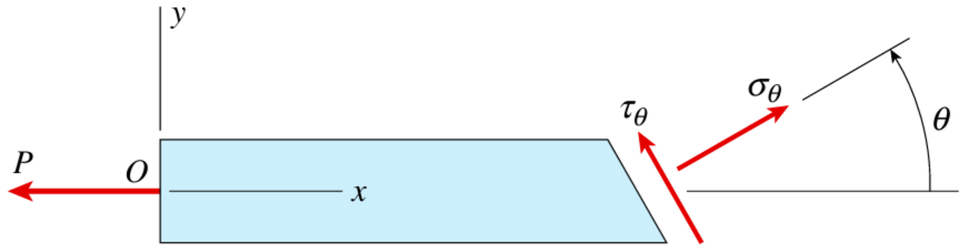
$$\sigma_\theta = \frac{N}{A} = \frac{P}{A_0}\cos^2\theta = \frac{\sigma_x}{2}(1+\cos 2\theta)$$

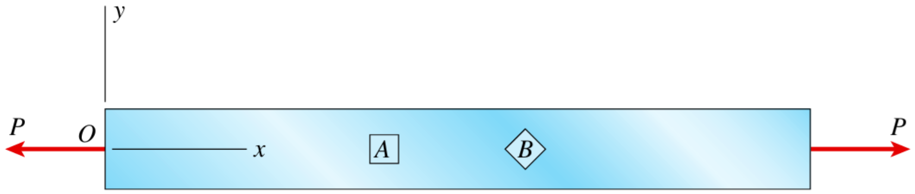
Shear stress:

$$\tau_\theta = -\frac{V}{A} = -\frac{P}{A}\sin\theta\cos\theta = -\frac{\sigma_x}{2}\sin 2\theta$$

(sign convention)

Exercise: Stresses on inclined sections





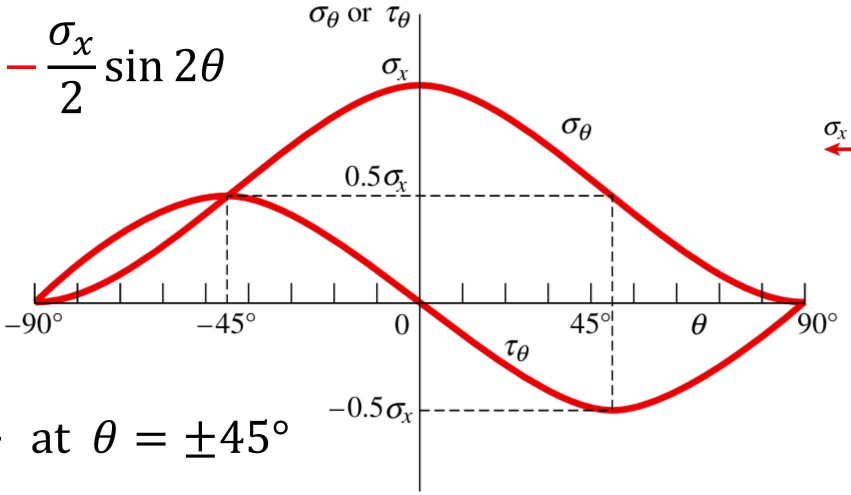
(a)

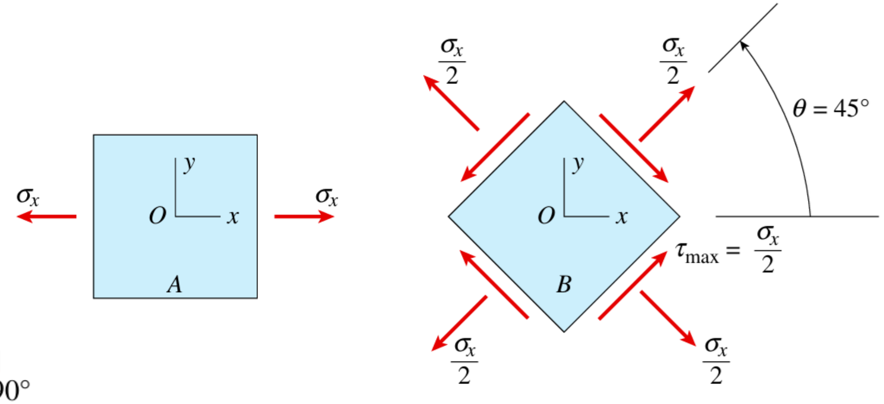
Normal stress:

$$\sigma_\theta = \frac{\sigma_x}{2}(1 + \cos 2\theta)$$

Shear stress:

$$\tau_\theta == -\frac{\sigma_x}{2}\sin 2\theta$$





$$\tau_{max} = \frac{\sigma_x}{2} \;\text{ at }\; \theta = \pm 45°$$

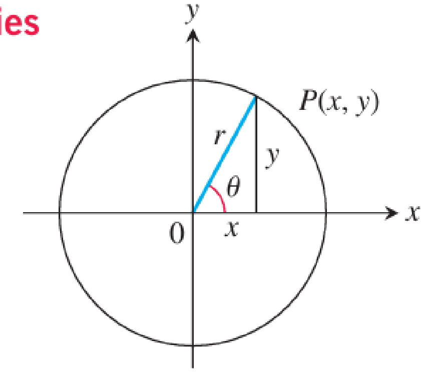
Trigonometry Formulas

Definitions and Fundamental Identities

Sine: $\sin\theta = \dfrac{y}{r} = \dfrac{1}{\csc\theta}$

Cosine: $\cos\theta = \dfrac{x}{r} = \dfrac{1}{\sec\theta}$

Tangent: $\tan\theta = \dfrac{y}{x} = \dfrac{1}{\cot\theta}$



Identities

$$\sin(-\theta) = -\sin\theta, \quad \cos(-\theta) = \cos\theta$$

$$\sin^2\theta + \cos^2\theta = 1, \quad \sec^2\theta = 1 + \tan^2\theta, \quad \csc^2\theta = 1 + \cot^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}, \quad \sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A - B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

$$\sin\left(A - \frac{\pi}{2}\right) = -\cos A, \qquad \cos\left(A - \frac{\pi}{2}\right) = \sin A$$

$$\sin\left(A + \frac{\pi}{2}\right) = \cos A, \qquad \cos\left(A + \frac{\pi}{2}\right) = -\sin A$$

$$\sin A\sin B = \frac{1}{2}\cos(A - B) - \frac{1}{2}\cos(A + B)$$

$$\cos A\cos B = \frac{1}{2}\cos(A - B) + \frac{1}{2}\cos(A + B)$$

$$\sin A\cos B = \frac{1}{2}\sin(A - B) + \frac{1}{2}\sin(A + B)$$

$$\sin A + \sin B = 2\sin\frac{1}{2}(A + B)\cos\frac{1}{2}(A - B)$$

$$\sin A - \sin B = 2\cos\frac{1}{2}(A + B)\sin\frac{1}{2}(A - B)$$

$$\cos A + \cos B = 2\cos\frac{1}{2}(A + B)\cos\frac{1}{2}(A - B)$$

$$\cos A - \cos B = -2\sin\frac{1}{2}(A + B)\sin\frac{1}{2}(A - B)$$