Georgia Tech
College of Engineering

# COE 3001

# MECHANICS OF DEFORMABLE BODIES

***Lecture 3*** – Mechanical Properties of Materials

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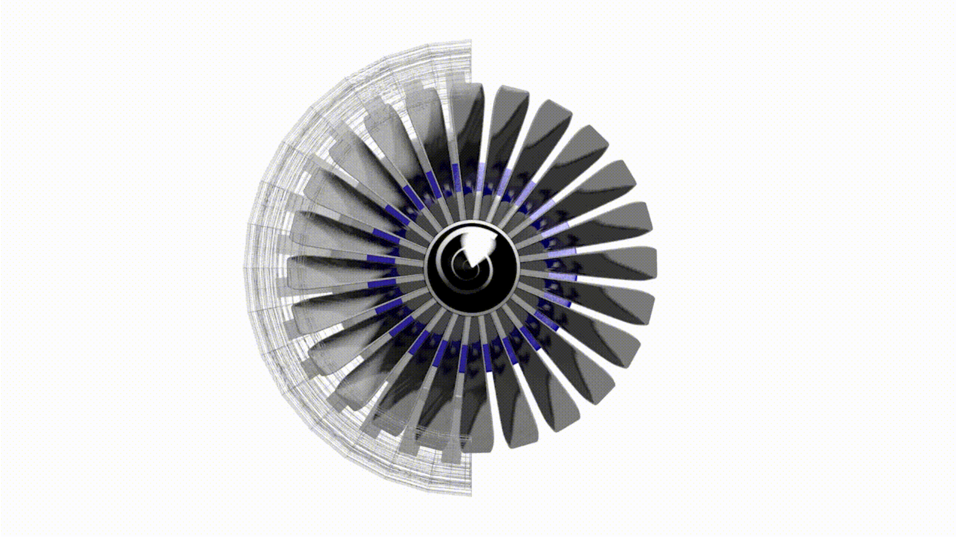
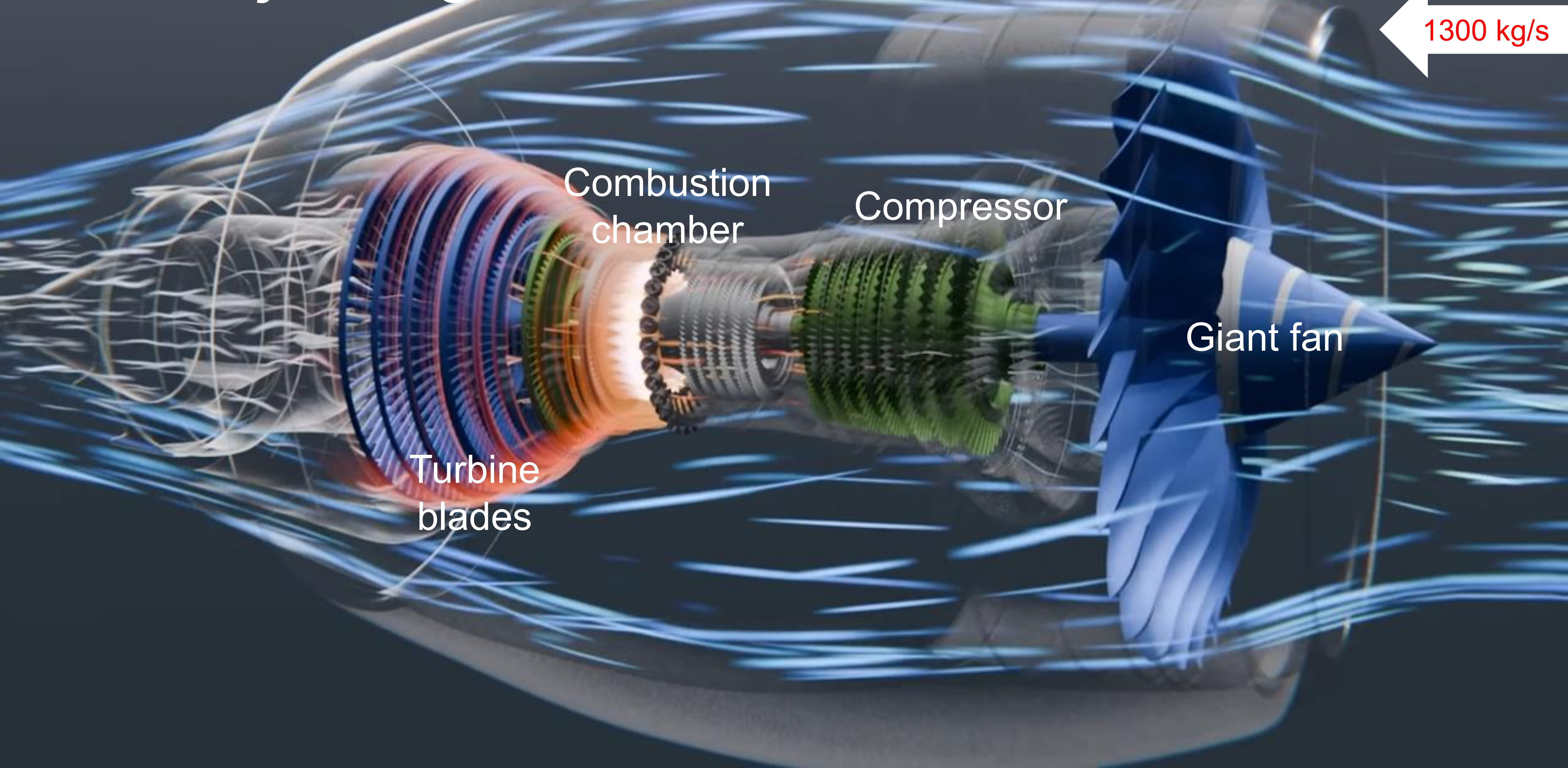
We study Mechanics NOT to solve equations, but to empower the dreams of flight with the certainty of science

# Jet engine

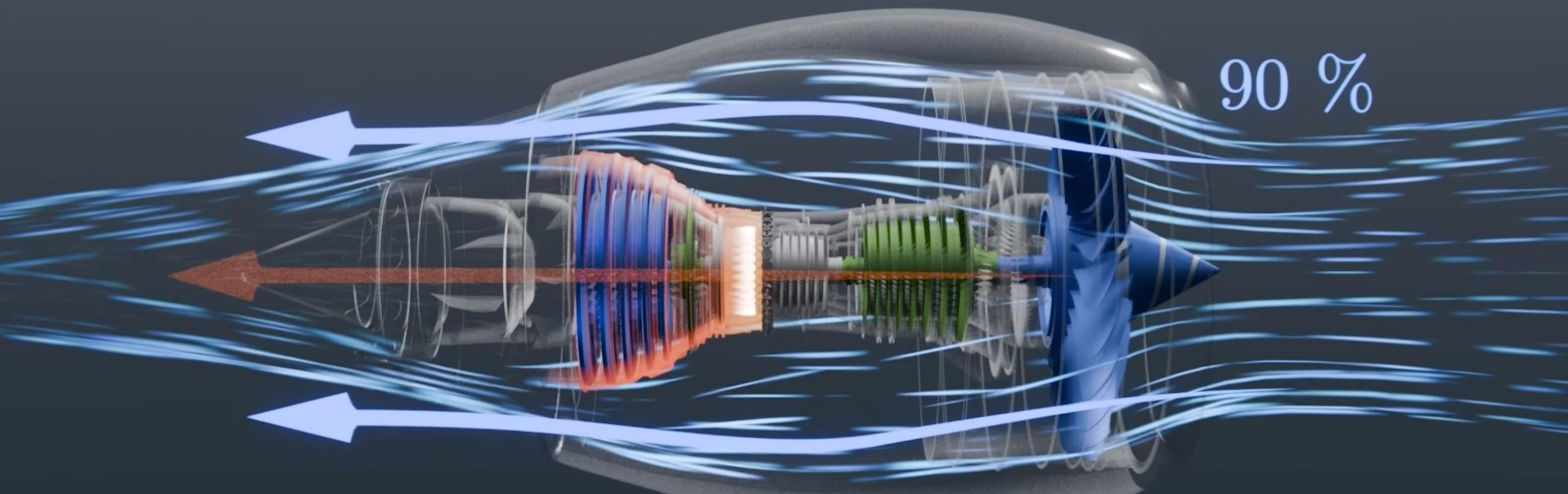
The crown jewel of human industry

# How a jet engine works?



# Where comes the thrust?




Impulse = Δ momentum = mass × Δ velocity

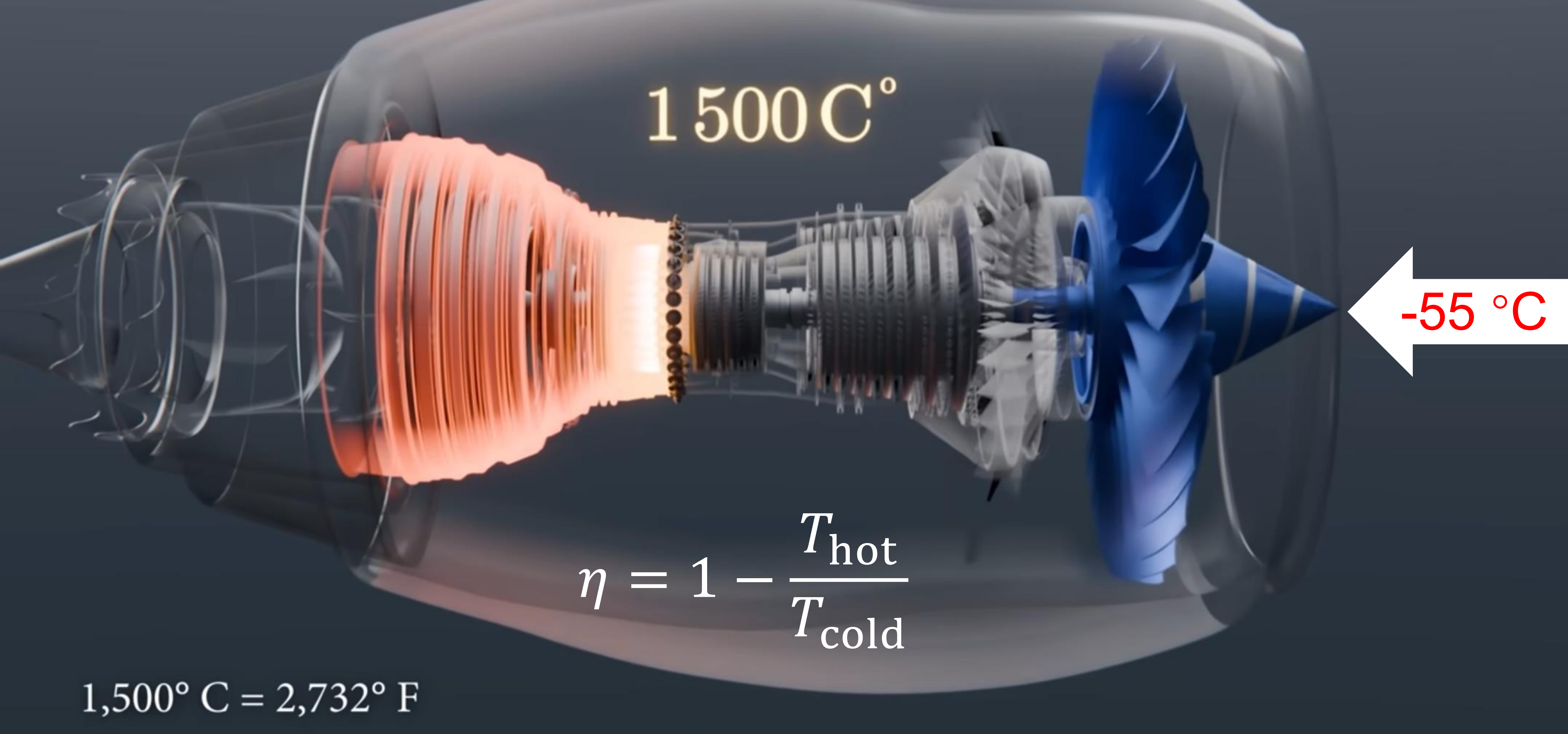
Impulse = 2m × ½ Δv = m Δv

Kinetic energy = $\frac{1}{4}\mathrm{m}(\Delta v)^2$

Impulse = ½ m × 2Δv = m Δv

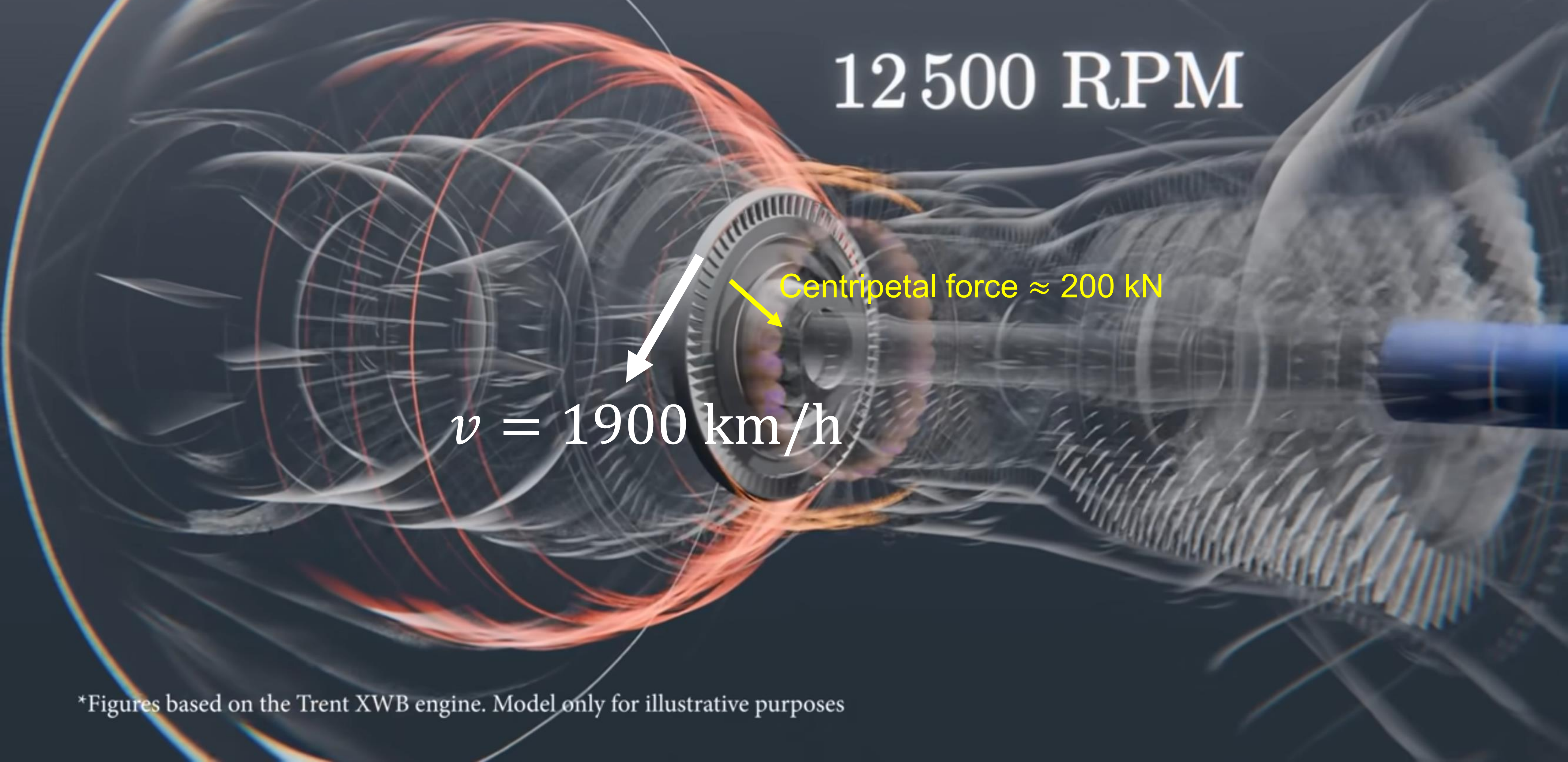
Kinetic energy = $\mathrm{m}(\Delta v)^2$

# How to make the engine more efficient?




$$\eta = 1 - \frac{T_{\text{hot}}}{T_{\text{cold}}}$$

# How strong the material should be?




*Figures based on the Trent XWB engine. Model only for illustrative purposes

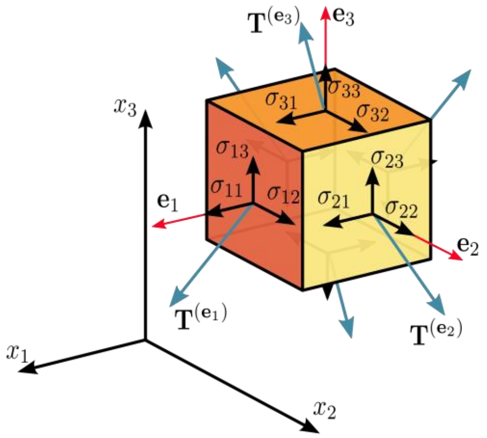
# Stress

- Microscopically, stress is a physical quantity that expresses the **internal forces** that neighboring particles of a continuous material exert on each other.

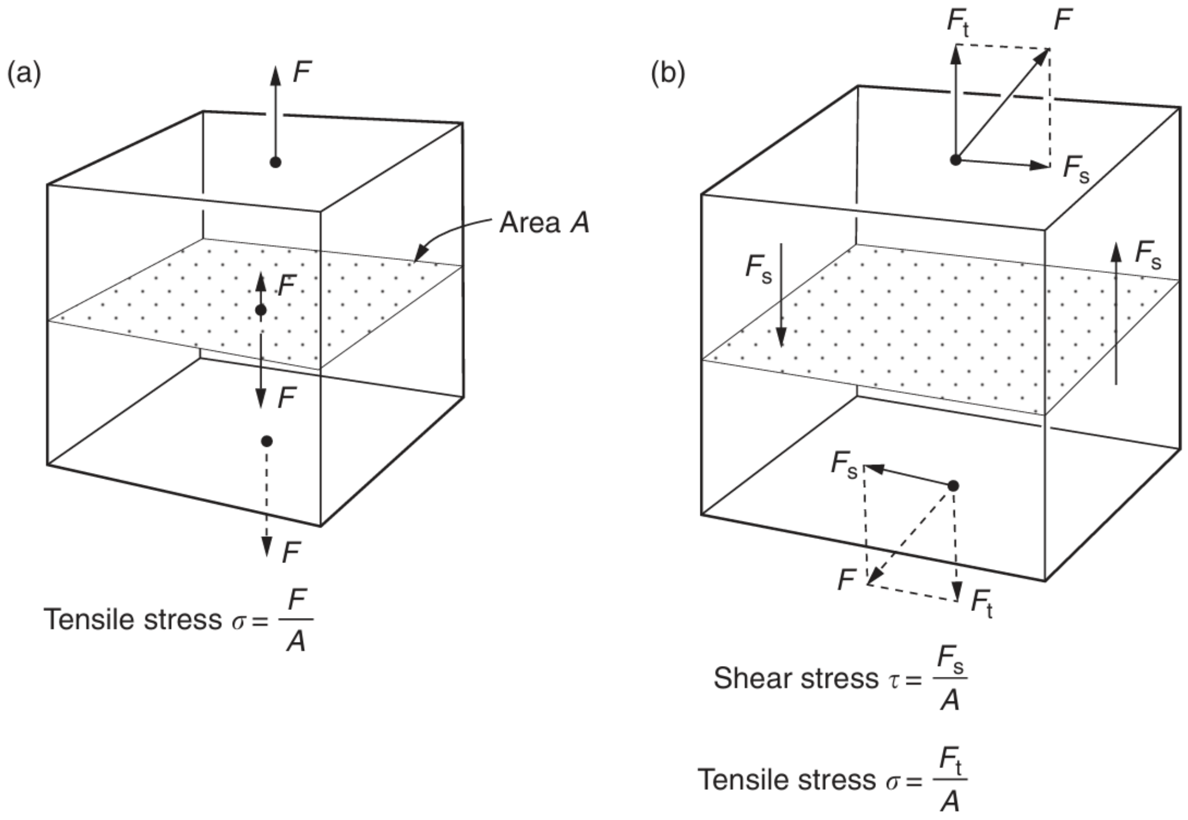
- Stress is defined as the force across a "small" boundary per unit area of that boundary, i.e.,

$$\sigma = \frac{F_t}{A}, \qquad \tau = \frac{F_s}{A}$$

- For more complex stress states, stress tensor




$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$



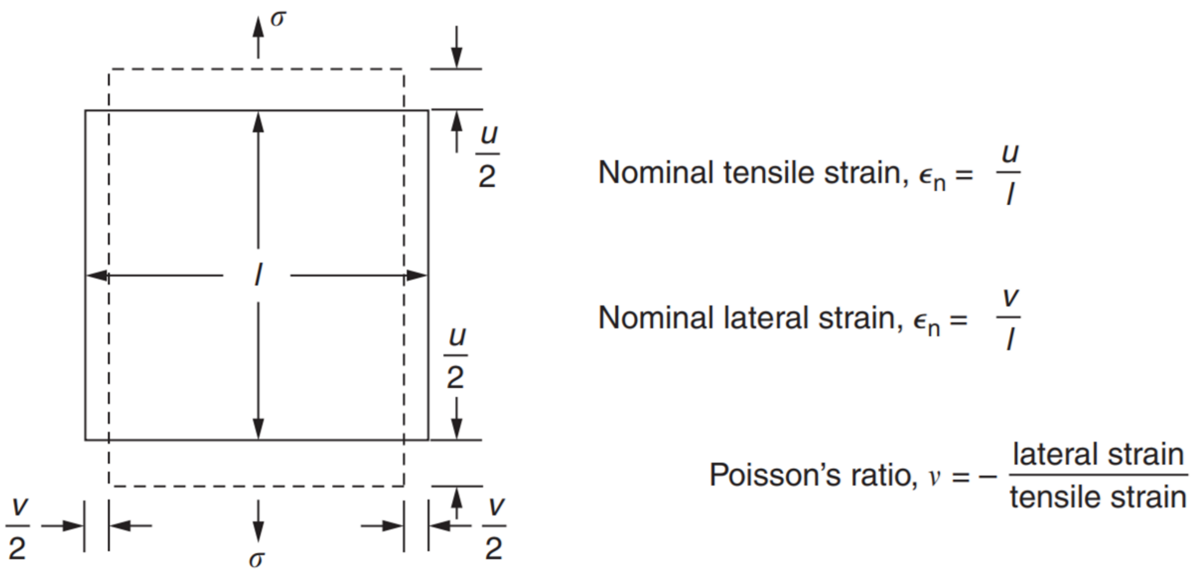
# Strain

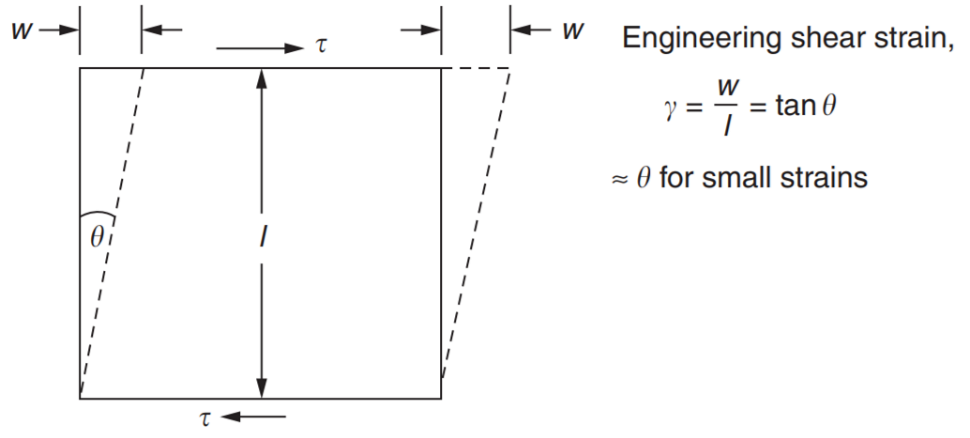
- Engineering strain, also known as Cauchy strain or nominal strain, is expressed as the ratio of total deformation to the initial dimension of the material body on which forces are applied.

$$\varepsilon_n = \frac{u}{l} = \frac{\Delta l}{l}$$

- Shear strain

$$\gamma = \frac{w}{l} = \tan\theta \approx \theta$$



# Why stress?

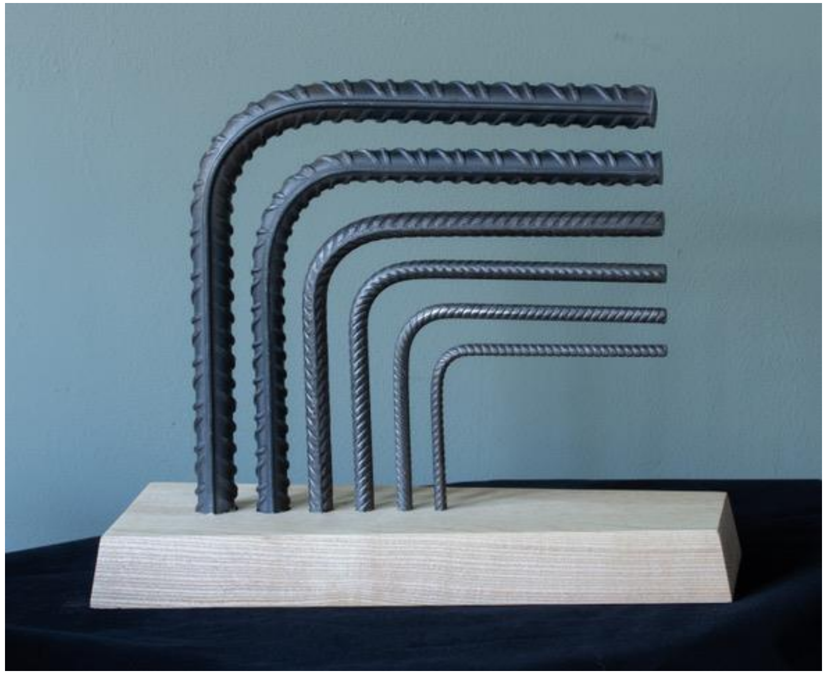
Think about bending the steel bars:

- all bars are made of **the same steel**,
- but have different **cross-sectional area**.

What Do You Expect?

- The **larger bars** require **more force** to bend.
- The **thinner bars** bend more easily.

Does it mean they are weaker?



Not necessarily — they are just bigger **or** smaller.

# Why strain?

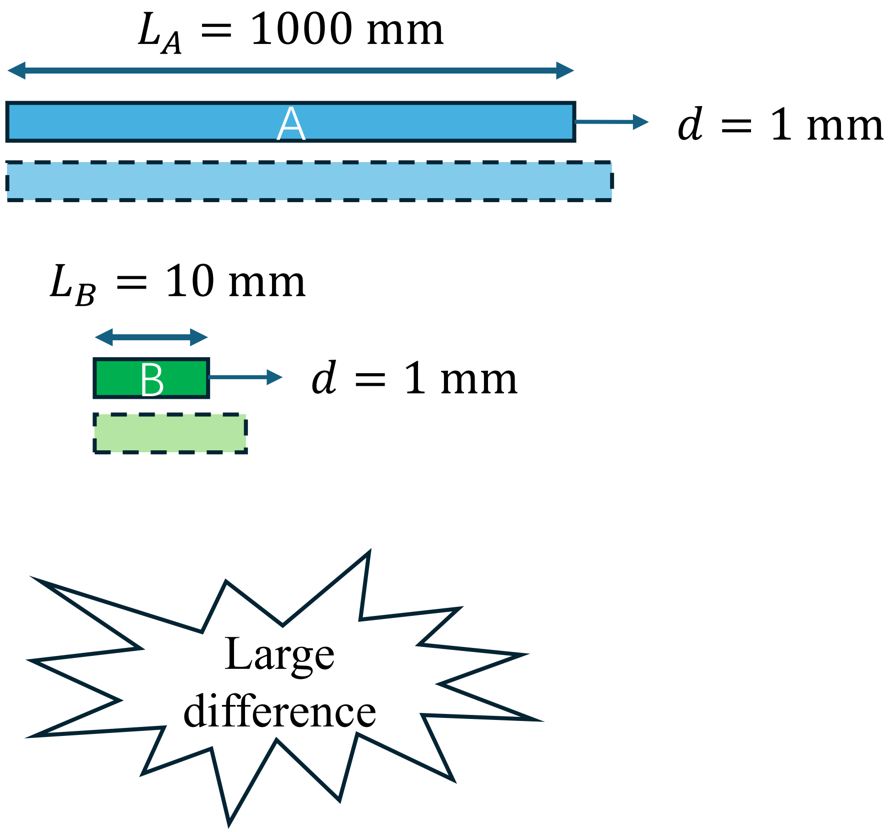
Think about stretch two bars by 1 mm

- all bars are made of **the same steel**,
- but have different **initial length**.

Even if the displacement is the same, the material experience is different.

- Bar A: 0.1%  vs.  Bar B: 10%

Does they deform the same?




Not necessarily — one is just shorter than the other.

# Stress & strain

- Force and displacement are **extrinsic** — they change with **size**.
- To compare material behavior, we need **intrinsic quantities.** That's why engineers and material scientists use:
  - **Stress** instead of force

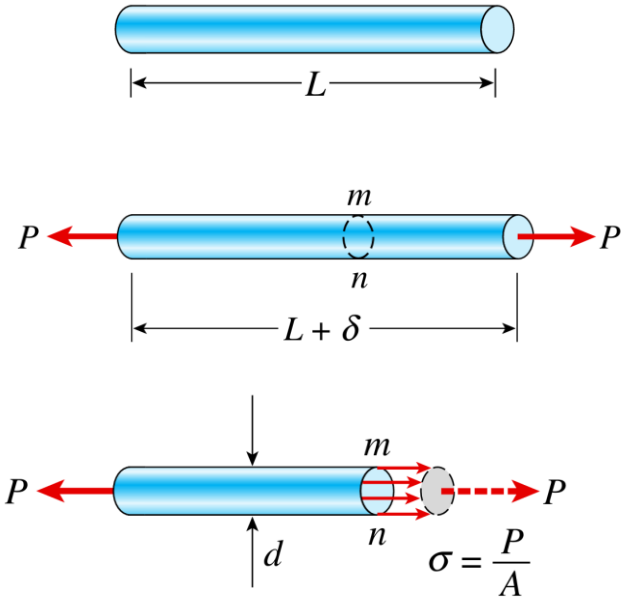
$$\sigma = \frac{\text{Force}}{\text{Area}} = \frac{P}{A}$$

Usually in MPa, GPa

  - **Strain** instead of displacement

$$\varepsilon = \frac{\text{elongation}}{\text{length}} = \frac{\delta}{L}$$

Dimensionless



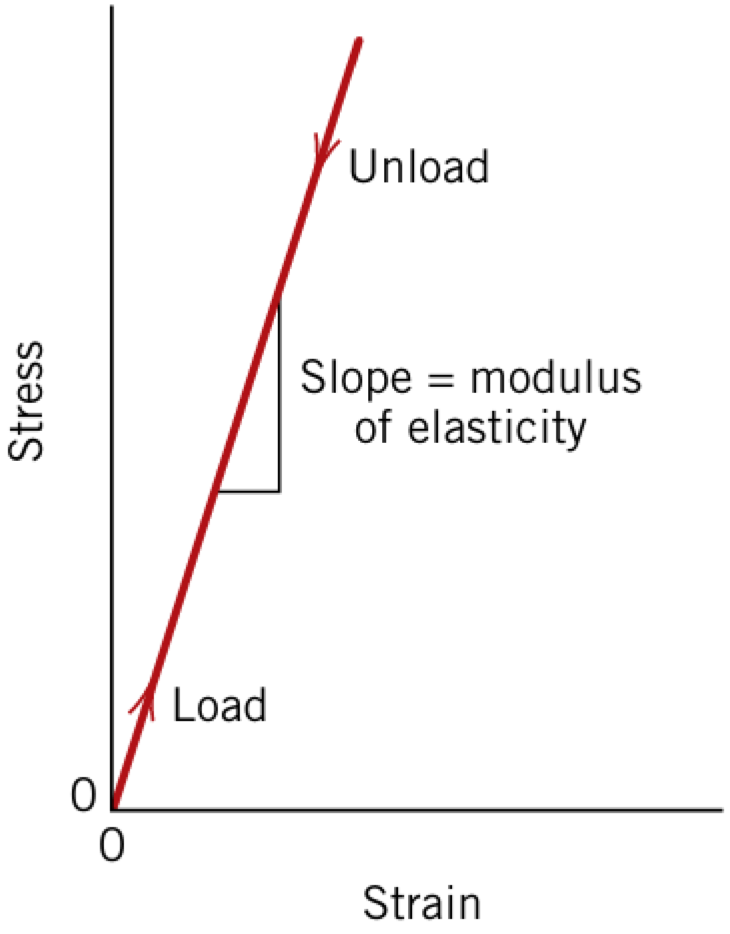

Sign convention: tensile stress, elongation strain – **Positive**

# Hooke's law

For most metals that are stressed in tension and at relatively low levels, stress and strain are proportional to each other through the relationship

$$\sigma = E\varepsilon$$

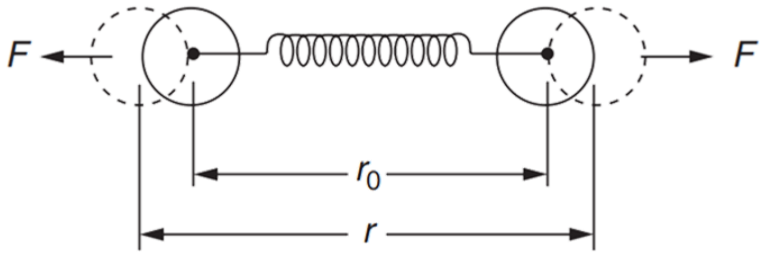
- This is known as Hooke's law, and the constant of proportionality $E$ is the modulus of elasticity, or **Young's modulus**.
- For most typical metals, the magnitude of Young's modulus ranges between 45 GPa for magnesium and 407 GPa for tungsten.



# Mechanical properties

- Mechanical properties determine a material's behavior when subjected to mechanical stresses.
  - Properties include *elastic modules*, *ductility*, *hardnes*s, and various measures of *strength*
  - Dilemma:
    mechanical properties desirable to the designer, such as high strength, usually make the  manufacturing more difficult.

# Bonding forces and energies



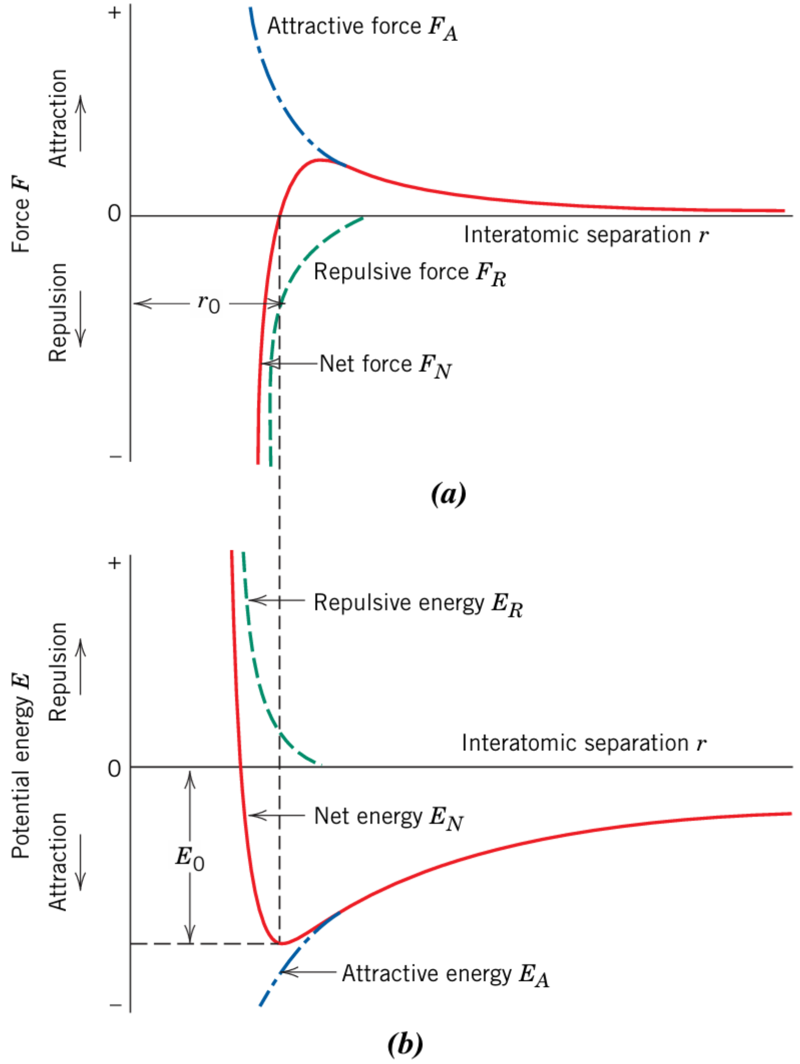

$$F = F_R(r) + F_A(r)$$

Repulsive force

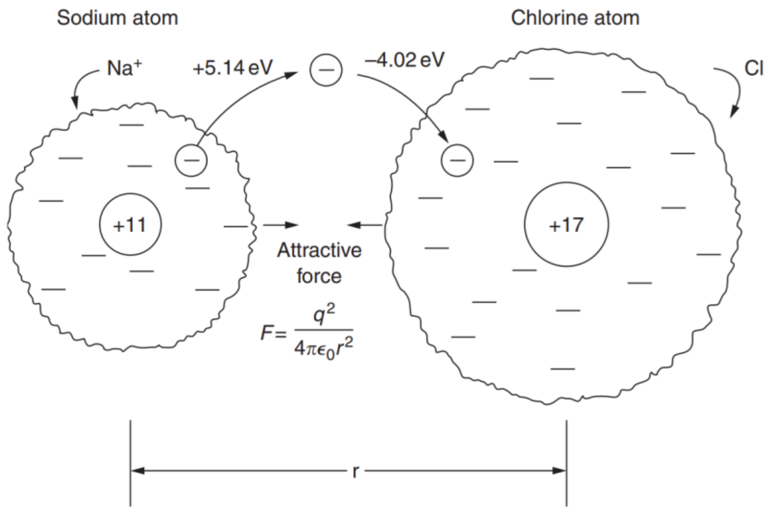
Attractive force

$$U = \int F_R\,dr + \int F_A\,dr$$

- The overlapping of electron clouds causes electrons that originally belonged to different atoms to compete for the same space, thus repelling each other.
- the formation of bonds leads to a more rational electron arrangement, thereby lowering the total energy of the system.



# Example of elastic modulus




NaCl

$$U(r) = U_0 - \frac{q^2}{4\pi\epsilon_0 r} + \frac{B}{r^n}$$

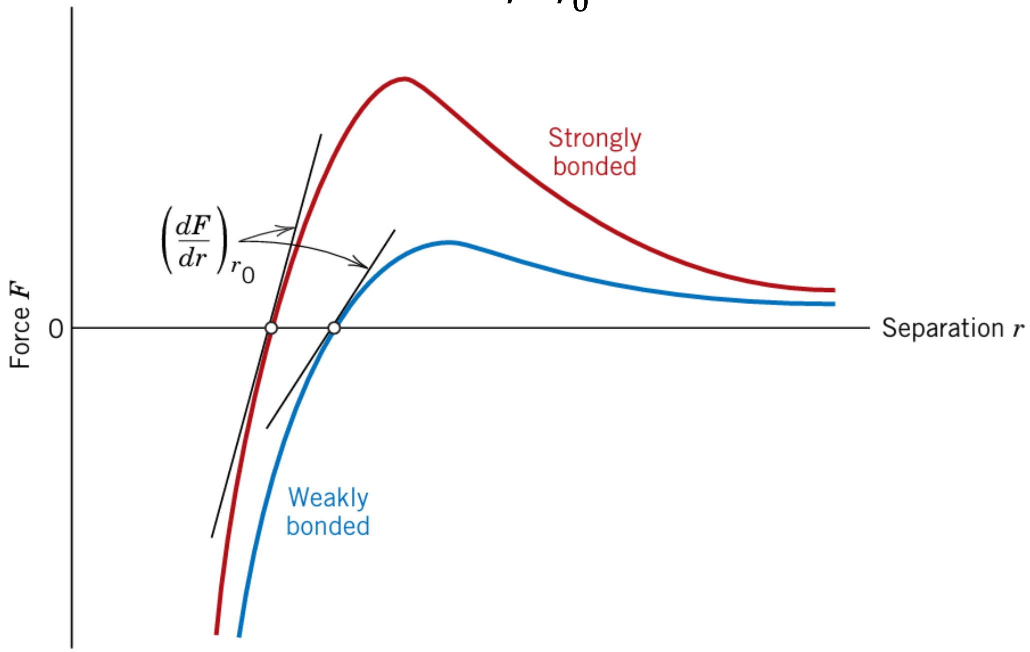
$$n \approx 12, \qquad B = \frac{q^2 r_0^{n-1}}{4\pi n\epsilon_0}$$

The bond force:

$$F = \frac{dU}{dr} = \frac{q^2}{4\pi\epsilon_0 r^2} - \frac{nB}{r^{n+1}}$$

The slope of the interatomic force–separation curve

$$E \propto \left(\frac{dF}{dr}\right)_{r=r_0} = \frac{(n-1)q^2}{4\pi\epsilon_0 r_0^3}$$

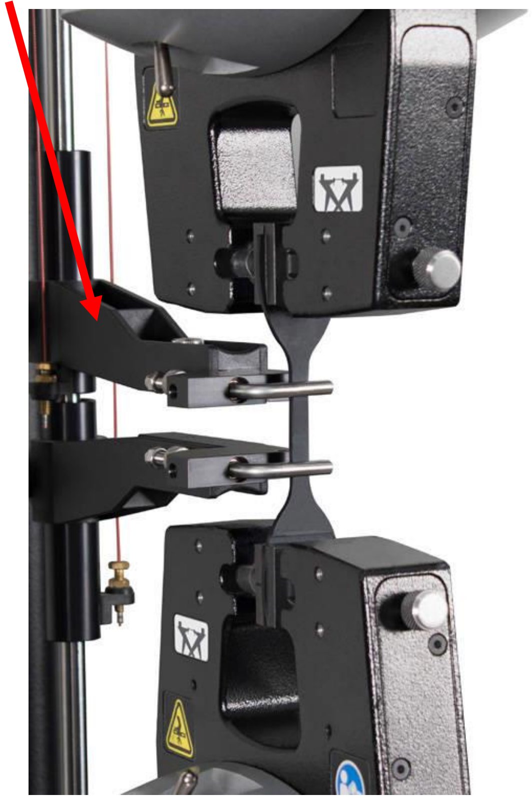


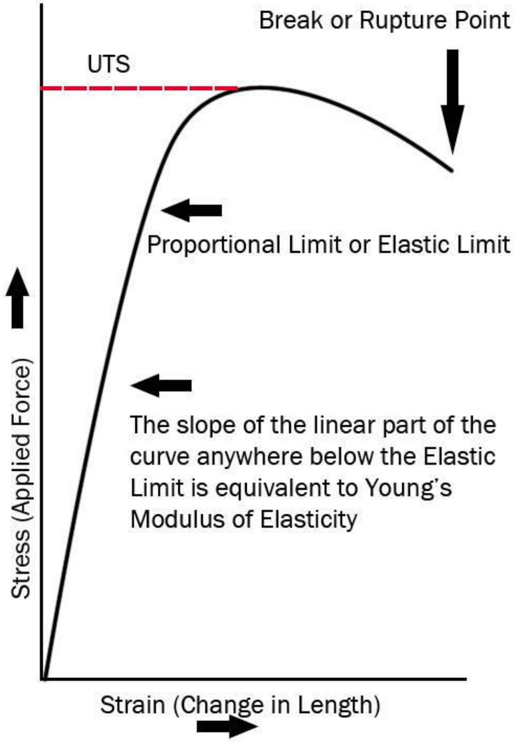
# Mechanical test

- Purposes for tension/compression test
  - to obtain basic data on the strength of materials
  - to test the reliability
  - failure analysis
- The standard procedure
  - **specimen preparation**: ASTM, ASA, NIST, ASME, etc. standard
  - **loading:** application of a continually increasing uniaxial tensile/compressive force, until necking or rupture
  - **data recording**: elongation/contraction of the specimen (extensometer)
  - **post processing**: stress-strain diagram

# Mechanical test


Extensometer

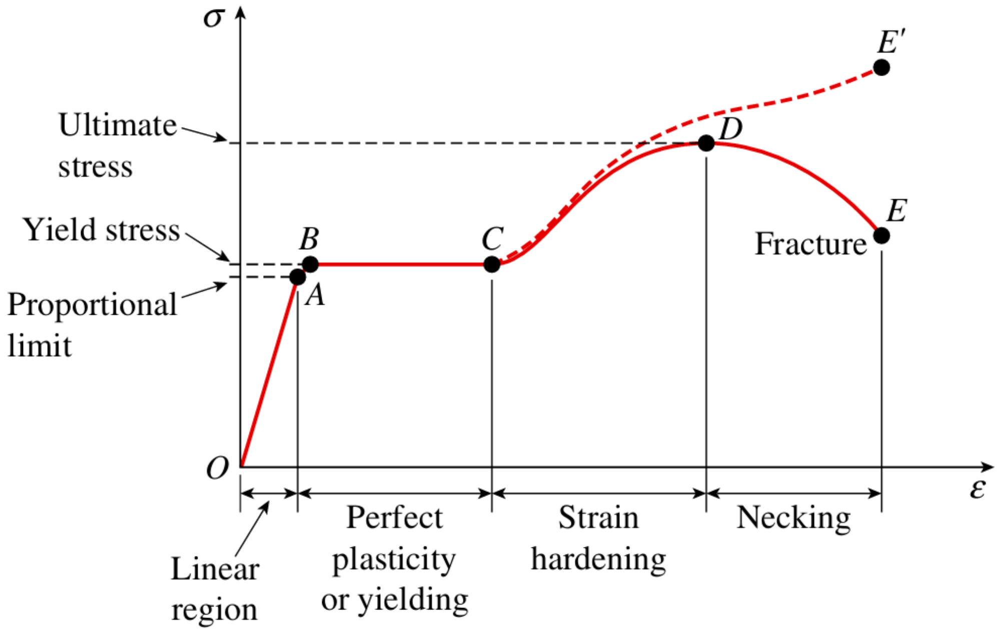


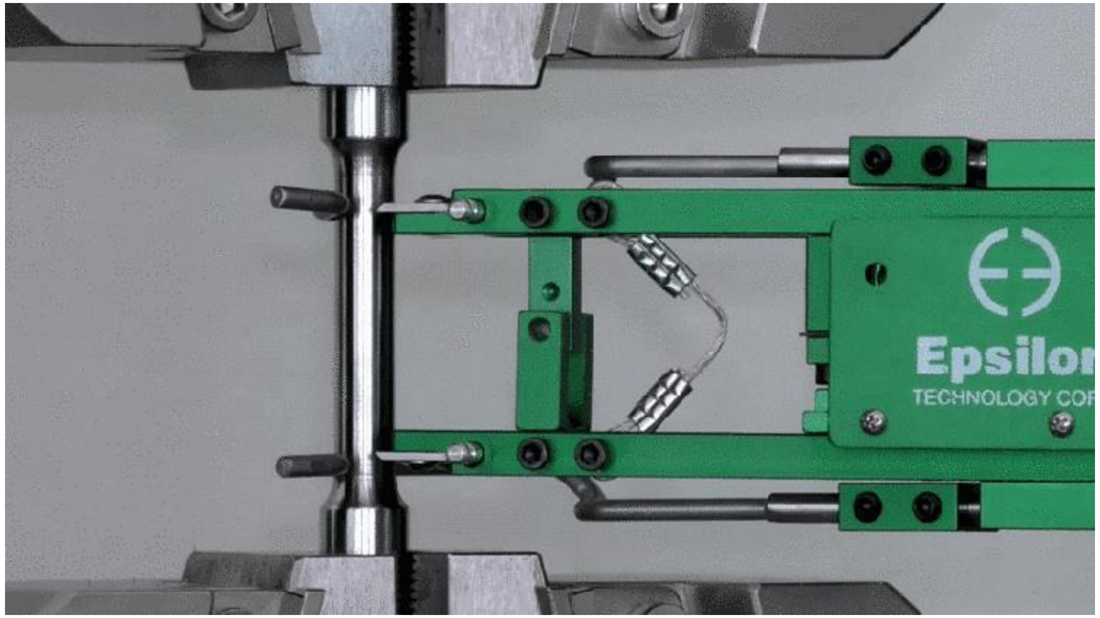




From Instron website: https://www.instron.com/en/resources/test-types/tensile-test/

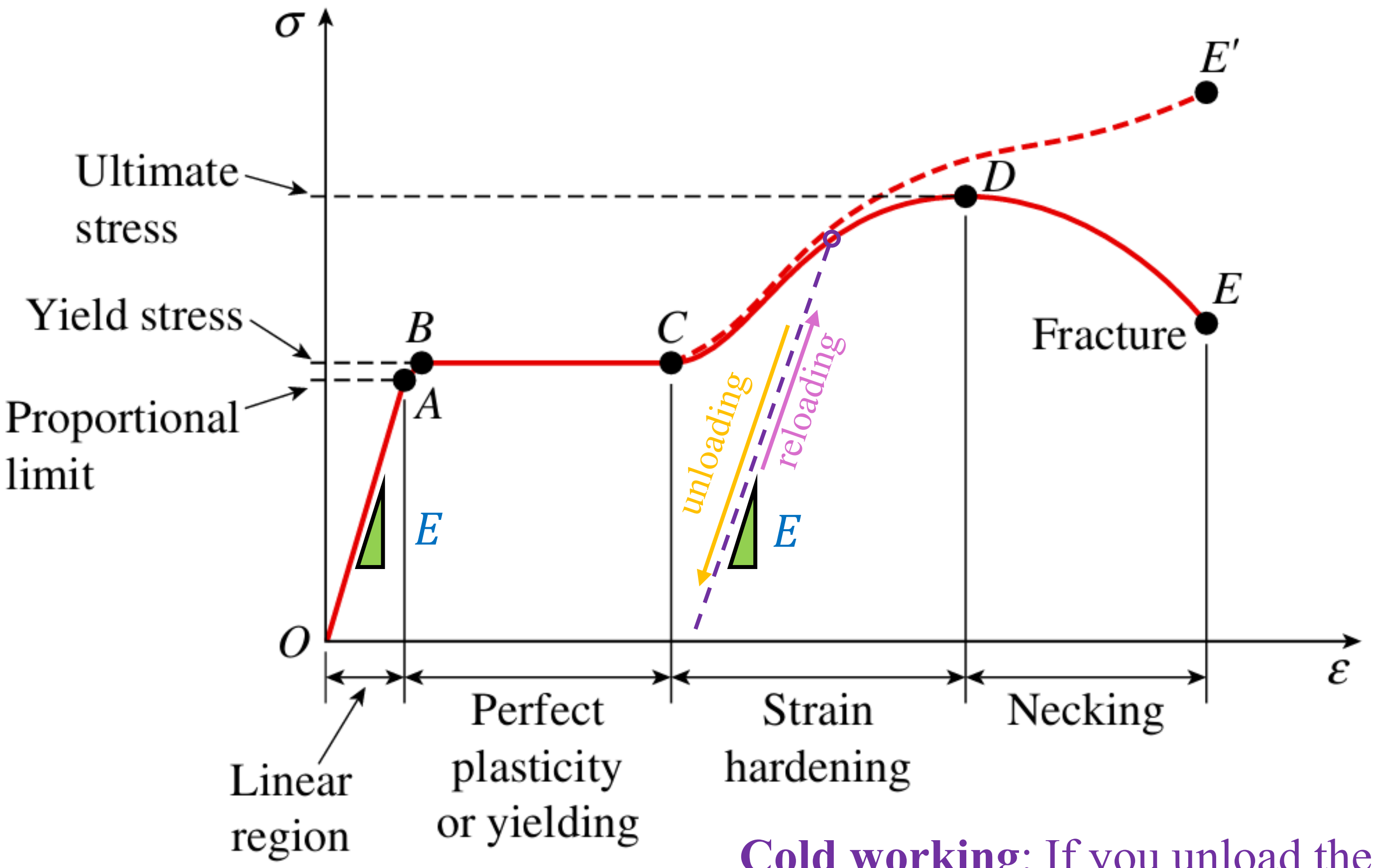
# Stresses and Strains for Structural Steel

*Structural steel*, also known as *mild steel* or *low-carbon steel*, is one of the most widely used **ductile** metals found in buildings, bridges, cranes, ships, towers, vehicles, and many other types of construction.



# 4 stages of the stress-strain curve




Below the yield stress:

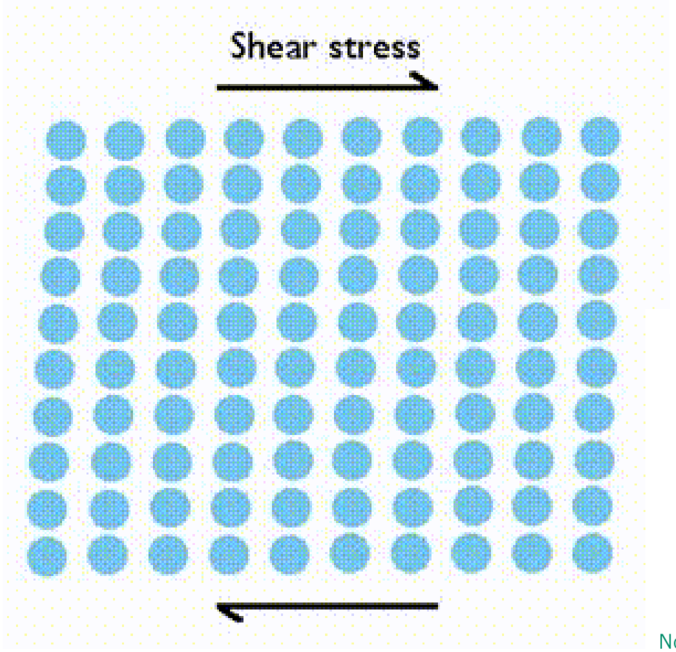
$$\sigma = E\varepsilon$$

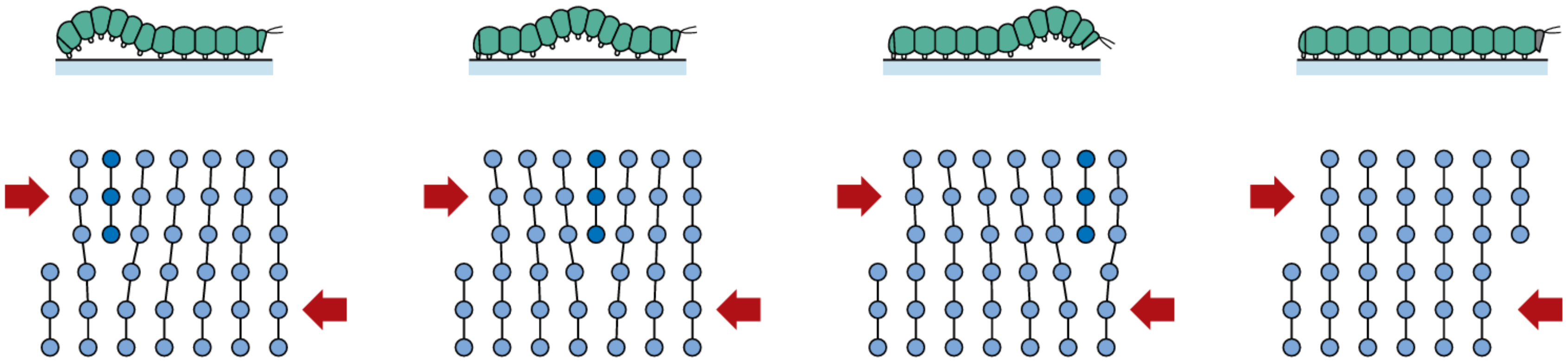
- $E$ = Young's modulus

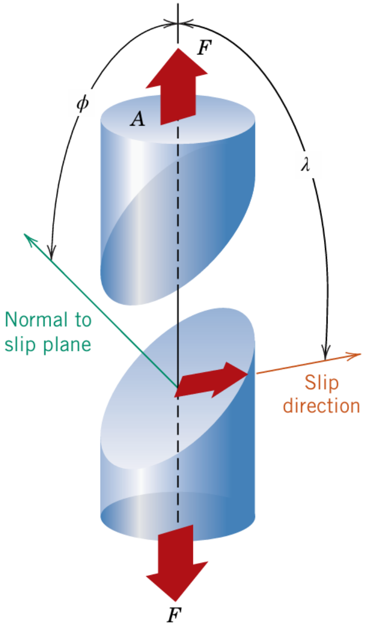
$E$ is a measure of inherent stiffness of a material.

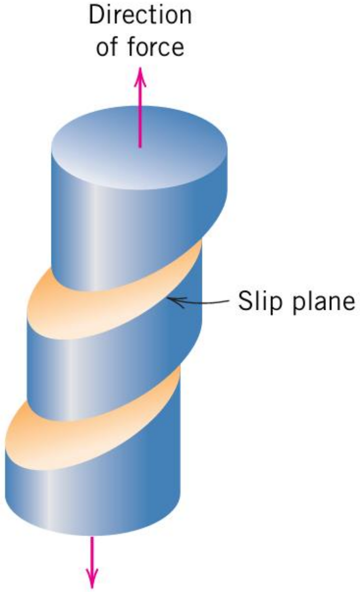
**Cold working**: If you unload the specimen during the strain hardening stage, the material will unload **elastically** along a line parallel to the original slope, leaving a permanent plastic deformation.

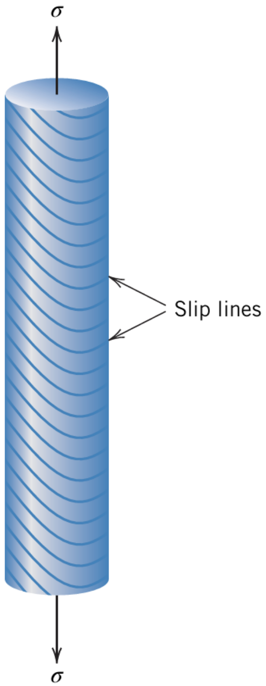
# Dislocation: reason of permanent deformation

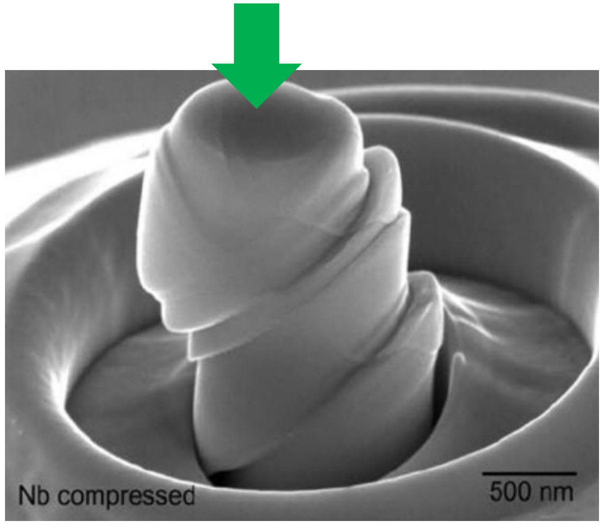


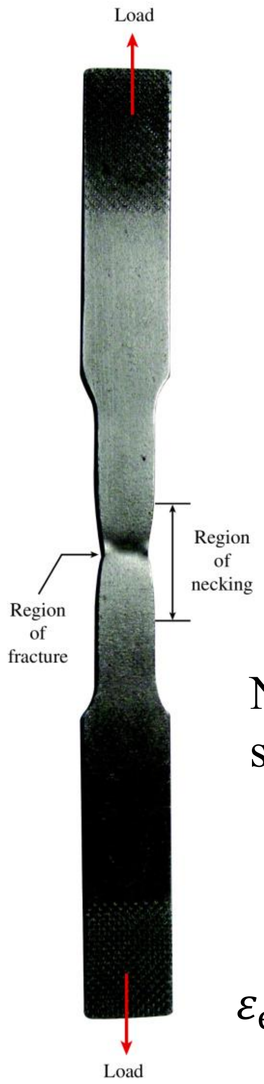


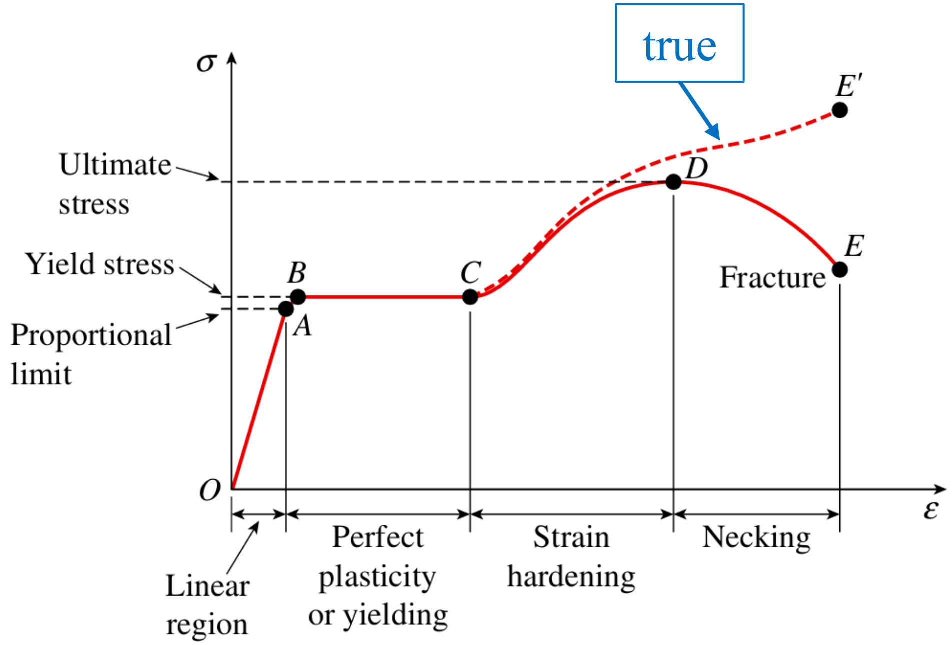



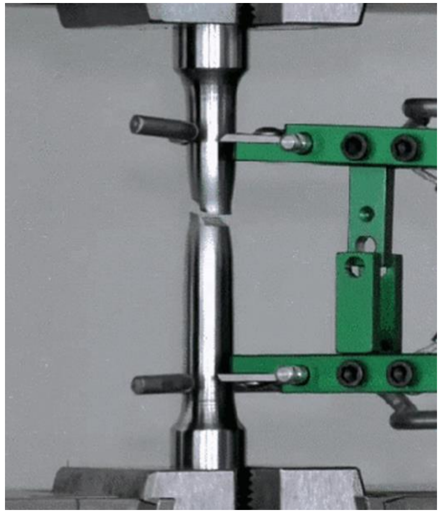



# Engineering vs. true stress/strain



$$\varepsilon_{\text{eng}} = \frac{L - L_0}{L_0} \Rightarrow L = L_0(1 + \varepsilon_{\text{eng}})$$

$$\varepsilon_{\text{true}} = \ln\left(\frac{L}{L_0}\right) = \ln(1 + \varepsilon_{\text{eng}})$$

$$A \cdot L = A_0 \cdot L_0 \Rightarrow A = A_0 \cdot \frac{L_0}{L}$$

$$\sigma_{\text{true}} = \frac{F}{A} = \frac{F}{A_0 \cdot \frac{L_0}{L}} = \left(\frac{F}{A_0}\right) \cdot \frac{L}{L_0}$$

$$\sigma_{\text{true}} = \sigma_{\text{eng}}(1 + \varepsilon_{\text{eng}})$$

$$\varepsilon_{\text{true}} = \ln(1 + \varepsilon_{\text{eng}})$$

No obvious shape change in First 3 stages, shape change starts in Necking stage

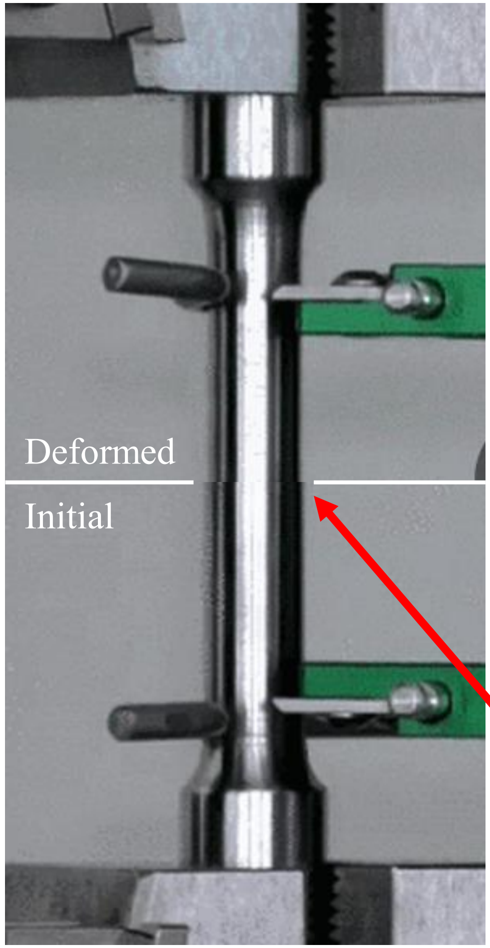
$$\sigma_{\text{eng}} = \frac{P}{A_0}$$ Engineering stress

$$\sigma_{\text{true}} = \frac{P}{A_{\text{instant}}}$$ True stress

$$\varepsilon_{\text{eng}} = \frac{L - L_0}{L_0}$$ Engineering strain

$$\varepsilon_{\text{true}} = \int_{L_0}^{L} \frac{dL}{L} = \ln\frac{L}{L_0}$$ True strain

# Poisson's ratio




Diameter changes

When a prismatic bar is loaded in tension,

$$\varepsilon_x = \frac{\sigma_x}{E}, \qquad \sigma_y = \sigma_z = 0$$

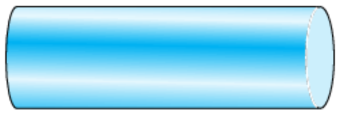
the axial elongation is accompanied by lateral contraction.

Assuming that the material is **homogeneous** and **isotropic**,

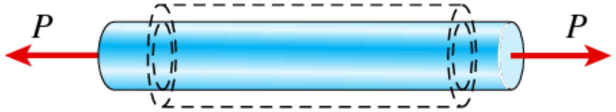
$$\varepsilon_y = \varepsilon_z \neq 0$$

Poisson's ratio is defined as:

$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$



(a)




(b)