

A clean cut

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ARTICLE INFO

Article history:

Received 19 March 2021

Received in revised form 20 April 2021

Accepted 25 April 2021

Available online 27 April 2021

Keywords:

Cut

Slicing

Soft solids

Friction

ABSTRACT

The effectiveness of slicing in cutting soft solids (i.e., transverse motion along the blade) is natural to anyone with even the slightest kitchen experience, but the underlying mechanism remains intriguing. This study seeks to unveil the mystic role of slicing by looking into the effect of friction in cutting soft materials. With the increase of indentation depth, the nonlinearity of the large deformation in the superficial layer diverts the lateral stress from the compression of the classic linear elastic solution and introduces a tension that accounts for the ultimate fracture. However, when friction is present between the blade and material, there is always a finite no-slip region on the contact surface, under which stress remains compressive in all directions. The slicing motion of the blade, on the other hand, directs the friction toward the horizontal direction, thus minimizing its contribution in resisting the cutting process in the vertical plane, and enabling a clean cut. Through a numerical model of frictional contact between a cutting wire and a hyperelastic solid, we show that the slicing action diverts the friction force, enables sliding, and facilitates the development of local tension. Without a trustworthy stress-based fracture criterion for soft solids, we then study the energetics of the cutting process. By introducing a small pre-existing crack underneath the blade, we compute the energy landscape of the system and show that the friction reduction in the vertical plane greatly decreases the energy barrier of crack opening and thus promotes cutting.

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1. Introduction

Although cutting soft solids is being practiced daily and has various applications varying from the food industry [1] to precision surgery [2], it has drawn little attention from the mechanics or physics community. An apparent paradox stands between linear elasticity and the cutting phenomenon — the classic Hertzian contact theory [3] suggests a hydrostatic compressive state of stress directly below the cutting blade, where separation is to take place. On the other hand, common experience tells us that cutting soft solids is much easier with a combination of indentation and lateral slicing; yet why so is not well understood. This phenomenon has been studied by A. G. Atkins et al. [4] and more recently by Reyssat et al. [5], Spagnoli et al. [6], and Mora et al. [7]. To circumvent the complications of crack nucleation, Atkins et al. considered a steady-state cutting process in which the stress and strain fields invariantly translate with a moving frame attached to the cutting blade. This enables the use of energy balance à la Griffith [8] to determine the relation between the cutting

force and slicing–pressing displacement ratio. It is shown that the vertical cutting force diminishes rapidly (with a simultaneous increase in the horizontal force) after the introduction of lateral slicing. In their work, friction is treated as a small correction to the energy balance and not studied in detail.

The apparent paradox of the hydrostatic compression beneath the cutting blade may be resolved by considering the geometric nonlinearity due to large deformation. With sufficiently deep indentation, tensile stress is developed along the transverse direction perpendicular to the cutting blade, as revealed by numerical simulations [5]. The role of friction, however, is not discussed in detail. The importance of friction to cutting is qualitatively discussed in various possible aspects by Chaudhury [9].

In fact, the presence of friction at the contact area always induces a no-slip zone beneath the cutting blade, thus inhibiting the tensile deformation locally and keeping the material in the state of hydrostatic compression — the paradox persists. Fortunately, another well-known property of friction — the direction of friction is always opposite of the relative sliding — suggests a possible solution to the paradox. When a sizeable lateral slicing motion is applied, the friction is primarily directed sideways, allowing slippage beneath the cutting blade, and recovering the local tension. Such a picture will be studied quantitatively in this letter. Just as in Reyssat et al. [5], we consider cutting with a thin

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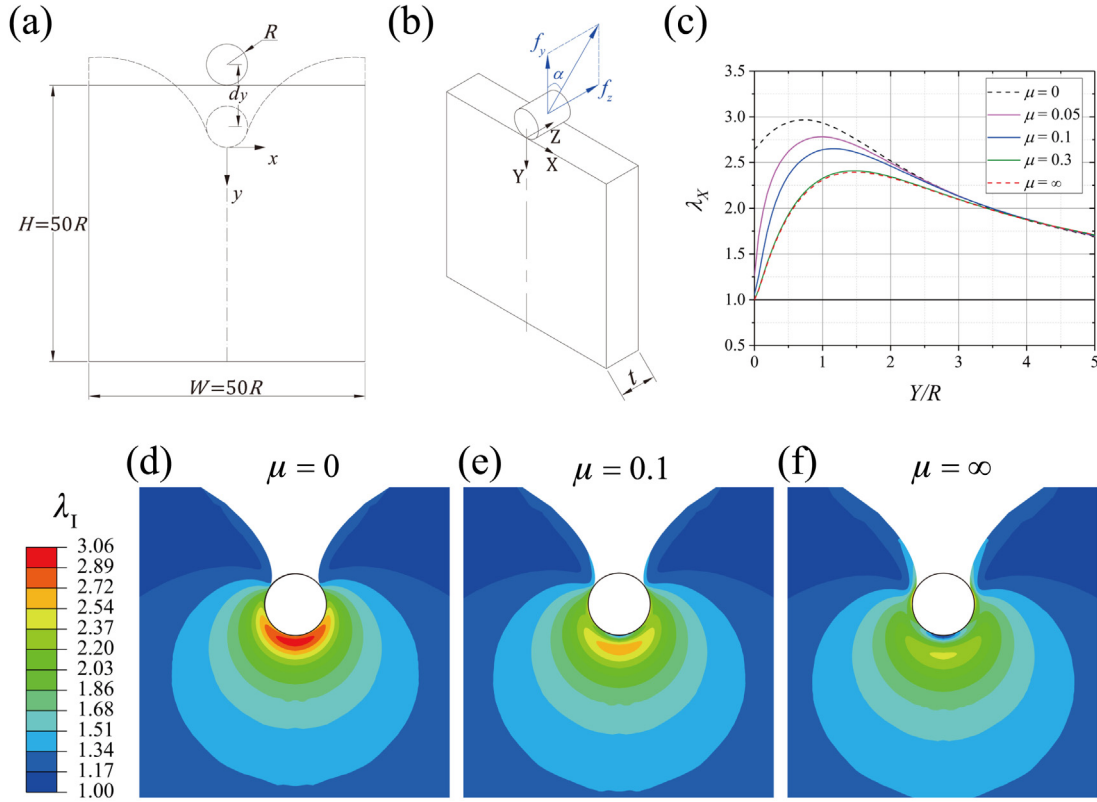


Fig. 1. Schematics of (a) 2D and (b) 3D computational domains and the representative results. The deformed shape in the 2D case is represented by the dash curve. The indentation displacement d_y and the slicing angle α are defined as shown. (c) Maximum principal stretches as functions of the normalized Y coordinate when $d_y = 7.5R$, when different values of the friction coefficient μ are taken. Distributions of the maximum principal stretch at the vicinity of the cutting wire when $d_y = 7.5R$ in the (d) frictionless ($\mu = 0$), (e) frictional ($\mu = 0.1$), and (f) no-slip ($\mu = \infty$) cases.

circular wire. By comparing the numerical results of stress and deformation fields between the frictionless and frictional cases, we first show that the existence of friction induces a finite no-slip zone beneath the cutting wire, where the material locally remains in a state of hydrostatic compression, even under deep indentation. By imposing a slicing motion on the cutting wire to induce relative sliding, simulations show that the friction in the plane perpendicular to the wire is dramatically reduced, and the stress distribution is close to that of the frictionless case. Slicing thus promotes the development of lateral tensile stress and lowers the energy barrier for crack initiation and propagation.

2. Numerical modeling

A rigid cutting wire with a circular cross-section of radius R , is brought into contact with the flat surface of an elastic solid by normal displacement d_y . The slicing angle α is defined by $\alpha = \tan^{-1}(f_z/f_y)$, where f_z and f_y are the out-of-plane and in-plane components of friction force acting on the wire, respectively, as shown in Fig. 1(b). In this letter, upper and lower-case subscripts are used to differentiate between the undeformed and deformed configurations. For orthogonal cutting ($\alpha = 0$), plane strain is assumed so that all field quantities are independent of Z , the out-of-plane coordinate. In our finite element model, the cross section of the substrate is modeled as a square domain of width $50R$ (Fig. 1(a)). The top surface ($Y = 0$) is traction free except the region of contact, and symmetry conditions are prescribed on all other surfaces. For cutting with slicing angle $\alpha \neq 0$, the solution depends on Z and the problem is 3D. For this case, the square domain is replaced by a cuboid (Fig. 1(b)) with periodic boundary conditions on the front and back surfaces.

The substrate is modeled as an incompressible neo-Hookean solid with shear modulus μ_s . This model is reasonable since the stretch in our simulations rarely exceeds 2, and the effect of strain hardening is negligible. Coulomb friction is imposed on the contact region, where slip occurs when shear stress reaches the product of friction coefficient μ and normal compressive stress. Further details of numerical modeling are given in Supporting Information (SI).

3. Orthogonal cutting

Let us first consider the case of orthogonal cutting ($\alpha = 0$) in the absence of slicing. Fig. 1(c) plots the maximum principal stretch λ_I (i.e., that in the X -direction, λ_x) along the Y -axis. In the frictional cases, λ_I below the indenter tip is dramatically reduced, even for very small friction coefficients, e.g., $\mu = 0.05$. A finite no-slip zone is always present due to large normal compressive stress at the indenter tip. The contour plots of the maximum principal stretch in the vicinity of the indenter tip (Fig. 1(d)–(f)) confirm the suppression, even at a rather large indentation displacement, e.g., $d_y = 7.5R$.

Friction not only affects deformation but also has a significant impact on the stress state along the symmetry plane, where a crack is expected to nucleate. Fig. 2(a) plots the normal Cauchy stress in the x -direction, σ_x , which is also the in-plane maximum principal stress, along the symmetry plane for different friction coefficients. It is striking that even the slightest friction causes σ_x to rapidly change from tensile to highly compressive within the superficial layer beneath the blade. As shown in Fig. 2(b), a state of hydrostatic compression, which naturally hinders crack nucleation, is always present, irrespective of μ . The effect of friction on σ_x along the symmetry plane is shown in Fig. 2(c)

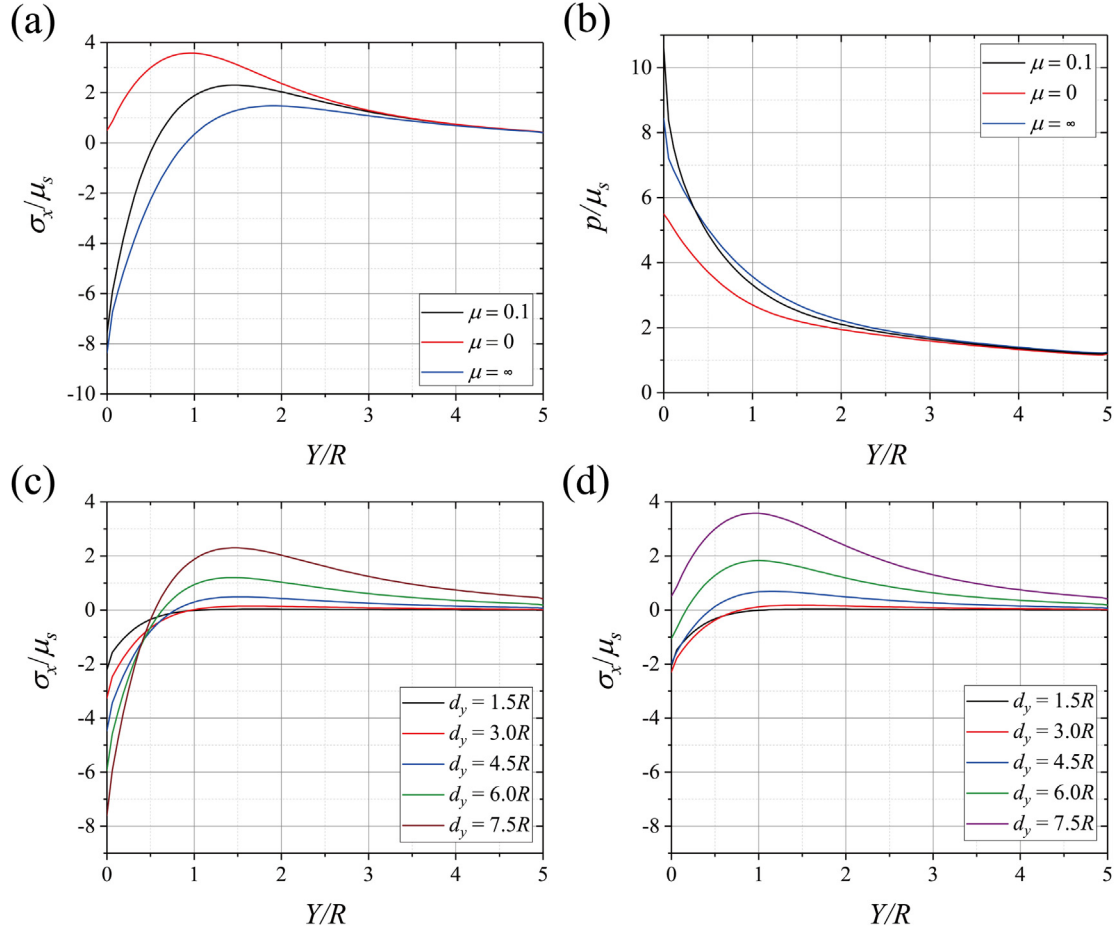


Fig. 2. Stress distributions along the symmetry line. (a) Maximum principal Cauchy stress and (b) hydrostatic pressure when $d_y = 7.5R$. Maximum principal Cauchy stresses at different indentation displacements, for the (c) frictional ($\mu = 0.1$) and (d) frictionless ($\mu = 0$) cases.

and (d) for different d_y . Evidently, even for the case of very small friction, a local compression will arise and hence delay crack nucleation.

A simple argument may be used to rationalize the significant effect of friction on local stresses. Consider *no slip*, the stretch λ_x of the material point right underneath the wire at the instant of making contact is exactly 1. Hertzian contact theory can be used at this moment since the deformation is small. The plane-strain assumption and incompressibility imply that $\lambda_z = \lambda_y = 1$. The stress state at the tip is therefore hydrostatic compression, that is, $\sigma_x = \sigma_y = \sigma_z = -p$. For small contact, the pressure is given by Hertzian theory [3] as

$$p = \frac{2F_y}{\pi a},$$

where a is the half-width of contact. Because of no-slip, $\lambda_x = 1$ irrespective of the indentation displacement, and the stress state at the tip remains hydrostatic. In contrast, consider the opposite situation when the surface is frictionless. The surface is laterally unconstrained, so λ_x can be greater than 1. Incompressibility implies that $\lambda_y = 1/\lambda_x$: the stress state at the tip is no longer hydrostatic. Indeed, for a neo-Hookean solid,

$$\sigma_x - \sigma_y = \mu_s (\lambda_x^2 - \lambda_x^{-2}).$$

4. Cutting with slicing

Our results above indicate that the in-plane friction plays an essential role in the stress and stretch states beneath the

wire. Due to the coupling in the direction of relative motion and friction, slicing motion may significantly change the in-plane friction. To illustrate the effect of slicing on the local deformation and stress fields, we carry out 3D finite element simulations by first indenting the wire to $d_y = 7.5R$, and then imposing a lateral displacement in the z -direction while holding d_y . A large enough lateral displacement is imposed so that sliding occurs over the entire area of contact and $\alpha \approx 90^\circ$. Representative distribution of λ_l near the cutting wire is plotted in Fig. 3(a) ($\mu = 0.1$). It is interesting to compare this result to the case of a frictionless contact (Fig. 1(d)) with the same d_y . Evidently, the deformation field is remarkably close to that of frictionless cutting – a clean cut.

The nature of friction dictates that friction is always in the opposite direction of the relative motion. Therefore, when the relative motion is mainly along the out-of-plane direction, the in-plane friction reduces significantly. As a result, the in-plane stress and stretch states are similar to those in a frictionless case, as shown by the maximum principal values of stress and stretch at the wire tip, both of which approach or even exceed the corresponding values in the frictionless case (Fig. 3(b)). The distributions of maximum principal stress and stretch along the symmetry plane show the same tendency, as shown in Fig. 3(c) and (d).

Reyssat et al. [5] suggested that slicing induces additional stress through the out-of-plane deformation or anti-plane shear force, which leads to easier cutting. These stress and deformation fields can be approximated by an anti-plane shear problem where

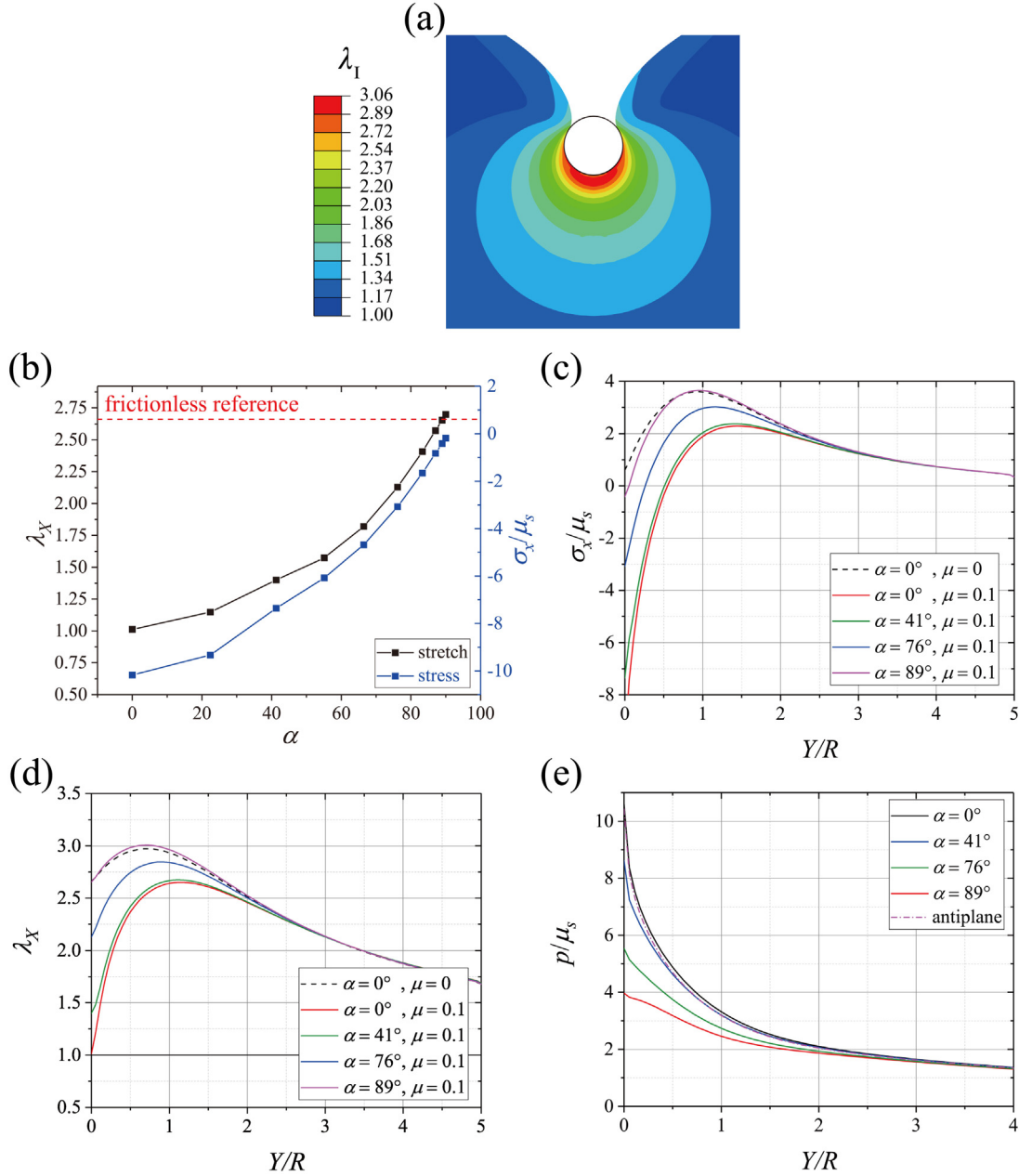


Fig. 3. Numerical results of cutting with Slicing. (a) Distribution of the maximum principal stretch at $d_y = 7.5R$ and $\alpha \approx 90^\circ$. (b) Maximum principal stress and stretch beneath the indenter tip as functions of α (in degrees). The red dash line shows the values of the frictionless case. (c) Distributions of maximum principal stress and (d) stretch along the symmetry line at different α . (e) Distribution of hydrostatic pressure along the symmetry line at different α .

in-plane displacements are neglected, and the out-of-plane displacement is independent of Z . Although linear theory suggests that an anti-plane problem only yields non-zero out-of-plane shear stress, in a finite deformation scenario, the normal stress σ_z is non-zero and can be much larger than the out-of-plane shear stresses [10]. In SI, we show that σ_z caused by a line force of magnitude F_z acting in the z -direction is given by

$$\sigma_z = \frac{F_z^2}{\mu_s \pi^2 r^2},$$

where r is the lateral distance from the line of action in the deformed configuration. Using a representative value of the dimensionless sliding force $F_z/\mu_s R \approx 1.41$, we estimate the maximum dimensionless normal stress $\sigma_z/\mu_s \approx 0.20$, which is much smaller than the effect of friction or indentation (Fig. 3(c)). Numerical simulations further confirmed this observation. When

no-slip contact is prescribed, the pressure distribution differs only slightly from the orthogonal cutting case, even when a lateral force equal to that of $\alpha = 89^\circ$ is applied, as shown by the dash-dot curve (anti-plane) in Fig. 3(e).

5. Energetics

The analyses above demonstrate the effect of slicing in cutting soft solids — diverting the friction to the out-of-plane direction and thus reverting the in-plane states of stress and deformation into those of an ideal frictionless case. Although maximum stretch, principal stress, and hydrostatic stress are natural indicators of fracture initiation, there is no single criterion that is widely accepted for the crack nucleation in a soft solid. For an isotropic elastic solid, a reasonable assumption is that crack nucleation depends only on the three principal Cauchy stresses:

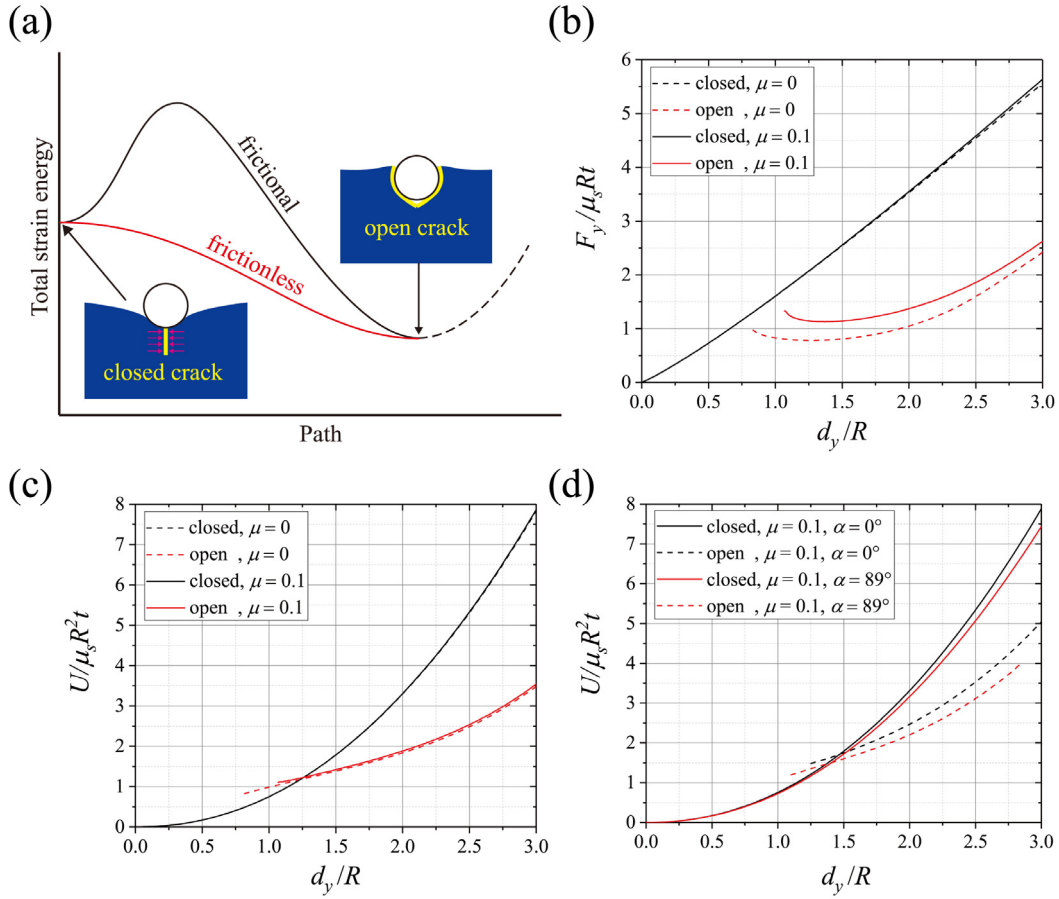


Fig. 4. (a) Schematic energy landscapes along a hypothetical path from a closed crack state to an open crack state (see text for description of the path). The dash line represents the energy states that cannot be achieved spontaneously. Insets illustrate the two crack states. (b) The dimensionless normal force, and (c) the normalized strain energy plotted against the cutting displacement for different crack states and friction coefficients for orthogonal cutting. (d) The normalized total potential energy with respect to the indentation displacement for the two crack states and different slicing angles.

$N(\sigma_I, \sigma_{II}, \sigma_{III}) = 0$, which is similar to the yielding criterion in metals except that the latter is usually pressure-insensitive. In our case, N may depend on the hydrostatic stress. Indeed, it has long been known that cavities nucleate in soft materials at a critical hydrostatic tension [11]. However, there are few experimental observations on local failure processes, and in particular, how these processes interact with local stresses during cutting. In light of these difficulties, we introduce a preexisting crack of length $a = 1.5R$, emerging from the tip of the cutter, and study the stability of this crack from an energy perspective.

During the indentation process, the pre-cracked soft solid may have two equilibrium states as sketched in Fig. 4(a): one with the crack closed and the other with cutting wire wedged into the open crack. These two states have different strain energies, and the energy difference, $\Delta U = U_{\text{closed}} - U_{\text{open}}$ can be regarded as an energetic driving force to open the crack. A necessary condition for opening the crack is that ΔU is greater than the surface energy. By comparing the cutting forces and total strain energies among different friction and crack states, as shown in Fig. 4(b) and (c) for orthogonal cutting, we found that the difference in normal forces between the frictionless and frictional cases is significant only in the open-crack states. In addition, the strain energy of the substrate is only minimally affected by friction as long as the system is in the same crack state. As expected, the open-crack state has lower strain energy when the cutting displacement d_y is above a critical value, e.g., $1.25R$.

Thus, although friction changes the local stress distribution significantly (as shown in Fig. 2), it has little influence on the

resultant reaction force to the cutting wire for the closed-crack state (or in the absence of a pre-crack). This is because the normal force is primarily controlled by the elastic deformation of the substrate. The additional cutting force in the open-crack state, when the cutting wire is wedged between the crack faces, comes from friction on the large area of contact. Combining with the fact from Fig. 4(c) that friction hardly affects strain energy, we conclude that the work done by the additional force is dissipated by friction, which could further increase the critical energetic driving force beyond the surface energy.

The compressive region near the tip (Fig. 2(c)) and energy dissipation caused by friction inevitably affect the energetics of crack opening. To illustrate this, we introduce a hypothetical path between the closed and open crack states to study the energetics. Although friction processes are path-dependent and the path chosen may not truly represent the actual process during cutting, the purpose here is to illustrate the qualitative energy landscape. Upon indentation by the wire, material points on the two surfaces of the pre-existing crack move sideways when the crack opens. Without friction, the increasing d_y eventually changes σ_x beneath the wire from compression to tension. If the cutting displacement is large enough, e.g., $d_y = 7.5R$, even the material point right under the cutter tip is in a tensile state (Fig. 2(d)), and thus the pre-existing crack opens spontaneously. If there is no pre-crack, a crack could form and open when the energetic driving force is greater than the energy required for crack nucleation and propagation. The situation is drastically different in the case of frictional contact. A finite region subject to compressive stress persists

below the wire (Fig. 2(c)) and hinders crack opening. Moreover, throughout the hypothetical crack-opening path, the cutting wire must push against lateral compressive stress and thus increases the strain energy. This additional strain energy, together with the frictional dissipation, forms an energy barrier between the closed and open crack states, as illustrated schematically in Fig. 4(a). The lateral compression enhances cutting resistance, as discussed by Atkins et al. [4]. The energy barrier caused by lateral compression, however, can be reduced using a thinner blade or narrowed by deeper indentation, as shown by Fig. 2(c), which is consistent with our daily experience that sharper blades or deeper pressing facilitate cutting.

We also study the effect of slicing on the energetic driving force. Fig. 4(d) plots the total potential energy, i.e., the sum of the strain energy and the potential of lateral slicing force, with constant friction coefficient but different crack states and slicing angles. Although the energetic driving force shows a slight increase when slicing is present, the change is minor. As mentioned above, the effect of slicing is mainly embodied in the reduction of energy barrier.

6. Summary and discussion

When a soft elastic solid is subject to orthogonal cutting, the presence of friction significantly affects the local fields of stress and deformation. Due to friction, a finite no-slip zone always exists beneath the blade, reducing lateral tension and retaining a hydrostatically compressive stress state. Nevertheless, the stress and strain fields change dramatically when a slicing motion is introduced, which diverts the friction to the out-of-plane direction. Through theoretical and numerical analyses, we first confirm the effect of friction on stress and strain distributions – a finite layer of lateral compression that hinders effective cutting. Then, exemplified by a cutting wire of circular cross section and assuming simple Coulomb friction, we show that the slicing motion greatly reduces the in-plane friction and reverts the stress and strain distributions to a state close to the frictionless case so that a clean cut is made possible. Lacking a universally applicable fracture criterion, we examine the energy landscape and find that the superficial compressive layer induced by friction creates an energy barrier against crack-opening. The energy barrier, which may naturally be reduced or narrowed by sharpening the blade or by applying a larger pressing force, is also effectively reduced by slicing.

The mechanics and physics of cutting are areas where experiments far exceed theory. It is a complex process with multiple mechanisms. The absence of a micromechanics-based nucleation criterion for cutting soft solids presents difficulties for quantitative analysis. We study the effect of friction on cutting in this

letter by introducing slicing motion to divert the friction force direction. It is rational to expect that other factors which affect friction may also play a role in cutting. For example, it has been observed that vibration facilitates fracture of soft solids [12], while it is known that vibration can reduce friction [13,14]. Other factors not directly related to friction, such as blade geometry [15], material behavior [7], or anisotropy [16], may also affect cutting.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This research is supported by the National Natural Science Foundation of China through Grant No. 11972015. WH and CYH are thankful to Dr. Jian-Ping Gong and GI-CoRE for hosting their visit to Hokkaido University, which initiated this research.

Appendix A. Supporting information

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.eml.2021.101343>.

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