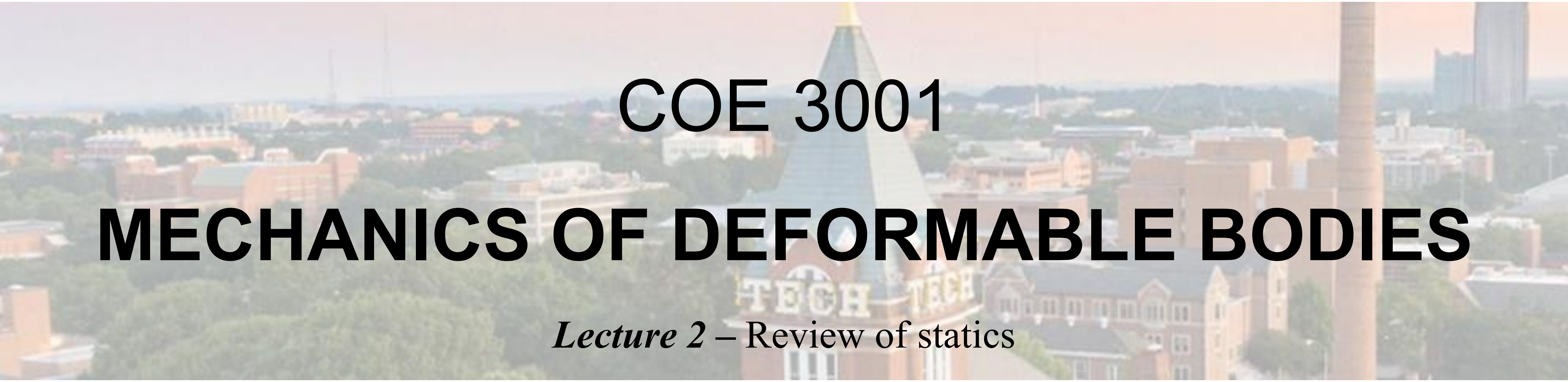
Georgia Tech College of Engineering



COE 3001

MECHANICS OF DEFORMABLE BODIES

Lecture 2 – Review of statics

Yazhuo Liu

Georgia Institute of Technology

Jan. 14, 2025

General Information

Course Materials

- All lecture notes, assignments, and solutions will be available at **Canvas**

Office Hours

- In-person: MW 12:30 – 1:30 pm, MRDC 4114A (starts from **Week 2**)
- Online: Zoom meeting based on appointment

Prerequisite

- COE 2001 or CEE 2020 or ME 2211 or AE 2120 (**Statics**)
- MATH 2403 or MATH 2552 or MATH 2413 (**Differential equations**)

General Information

Grading

- (Bonus) Attendance: 6 pts.
- Homework: 5×6 pts. + 4 pts. bonus
- Midterm Exam: 30 pts.
- Final Exam: 40 pts.

Points	Letter grade
90 ~ 100	A
80 ~ 89	B
70 ~ 79	C
60 ~ 69	D
0 ~ 59	F

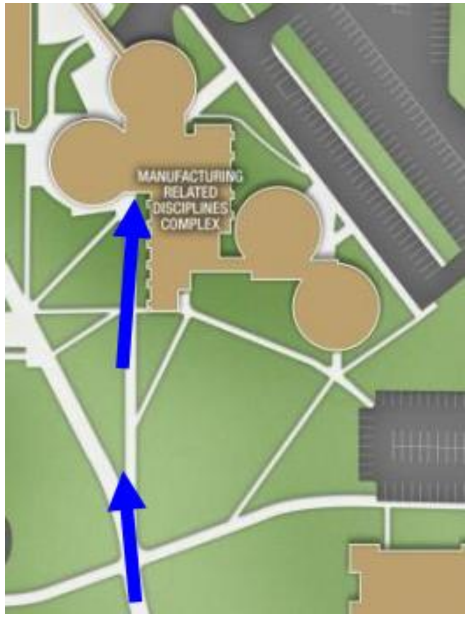
- ➢ 3 attendance check (bonus points)
  - ✓ 1 present – 1 pt.
  - ✓ 2 presents – 3 pts.
  - ✓ 3 presents – 6 pts.
- ➢ 6 assignments: 6 pts each
  - ✓ Lowest grade assignment will be dropped.
  - ✓ Typed assignments have 1 bonus pt each time (maximum total: 4 pts)
- ➢ Exam
  - ✓ In class and close book
  - ✓ 1 page (2 sides) equation sheet allowed

Shell Tutoring

The George W. Woodruff School of Mechanical Engineering will offer **Shell Tutoring** for COE 3001 this semester. Tutoring will run from **January 19 to April 24**

Schedule: Monday–Thursday, 6:00 PM – 8:00 PM

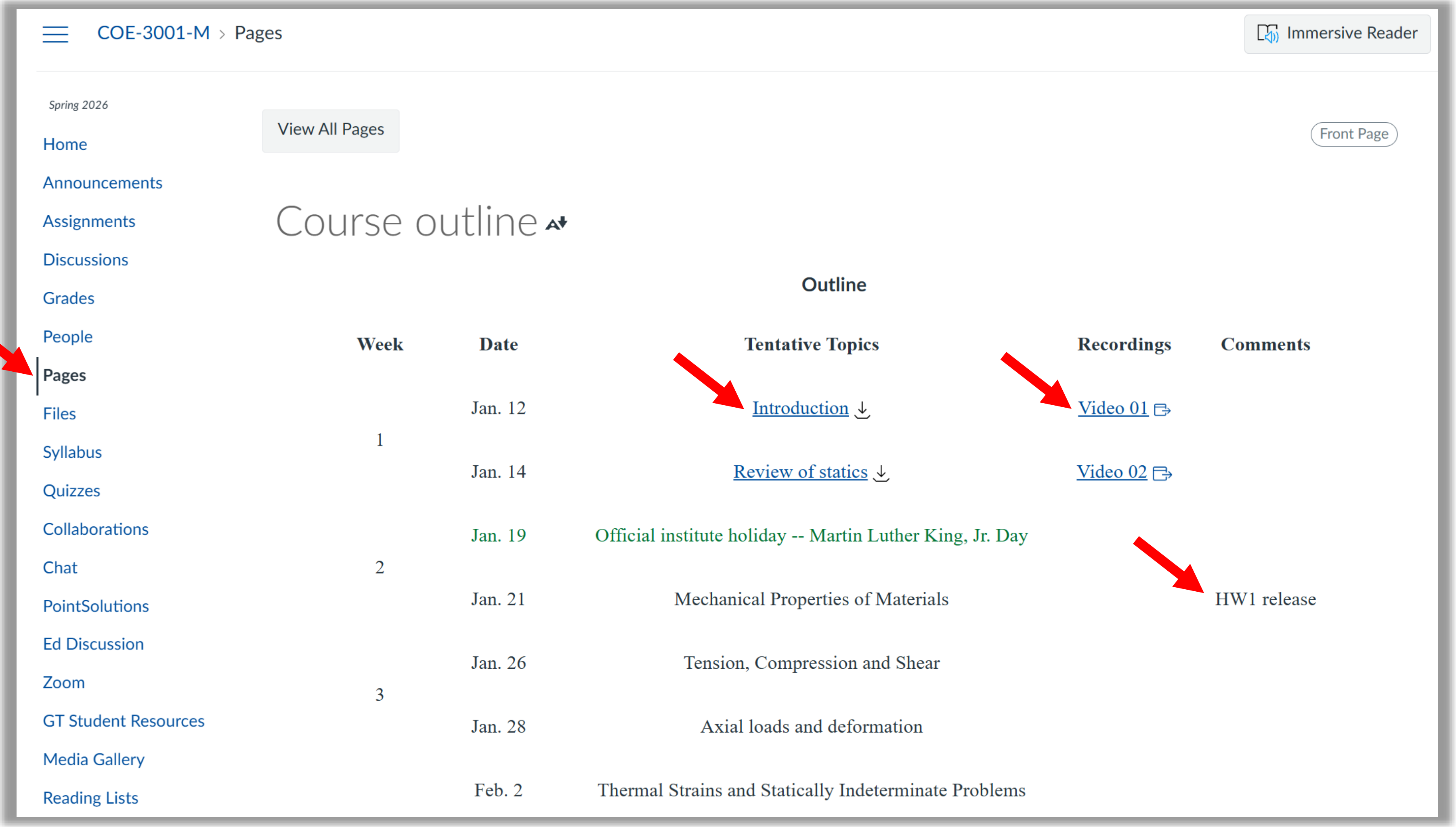
Location: MRDC 4th Floor (Tables near zone 1 and zone 2)



Tutors:

- Chu, Elizabeth E (echu45@gatech.edu)
- Islam, Marzanul (mislam314@gatech.edu)
- Please note that they are not TAs for this class.

Canvas Pages



COE-3001-M › Pages

Immersive Reader

Spring 2026

- Home
- Announcements
- Assignments
- Discussions
- Grades
- People
- Pages
- Files
- Syllabus
- Quizzes
- Collaborations
- Chat
- PointSolutions
- Ed Discussion
- Zoom
- GT Student Resources
- Media Gallery
- Reading Lists

View All Pages

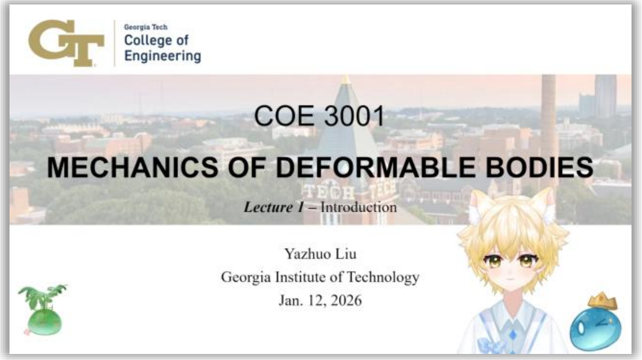
Front Page

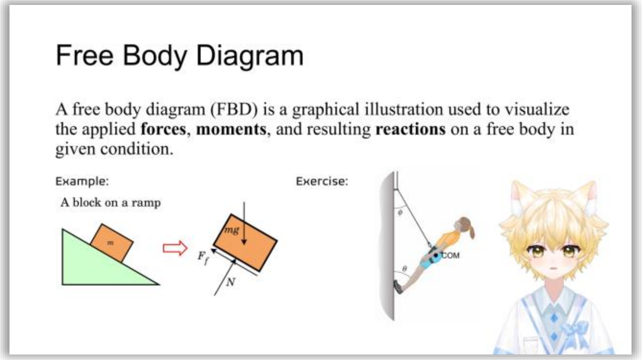
Course outline

Outline

Week	Date	Tentative Topics	Recordings	Comments
1	Jan. 12	Introduction	Video 01	
	Jan. 14	Review of statics	Video 02	
2	Jan. 19	Official institute holiday -- Martin Luther King, Jr. Day		
	Jan. 21	Mechanical Properties of Materials		HW1 release
3	Jan. 26	Tension, Compression and Shear		
	Jan. 28	Axial loads and deformation		
	Feb. 2	Thermal Strains and Statically Indeterminate Problems		

Condensed version of lectures



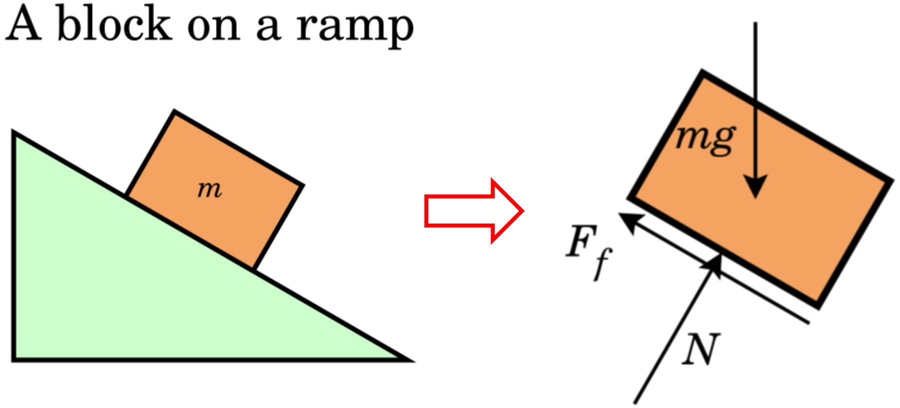


Free Body Diagram

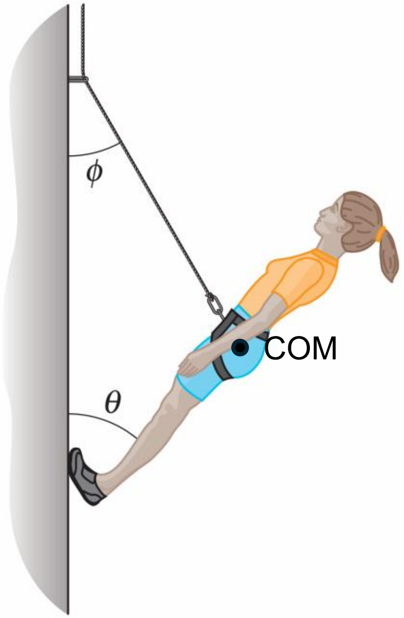
A free body diagram (FBD) is a graphical illustration used to visualize the applied **forces**, **moments**, and resulting **reactions** on a free body in given condition.

Example:

A block on a ramp



Exercise:



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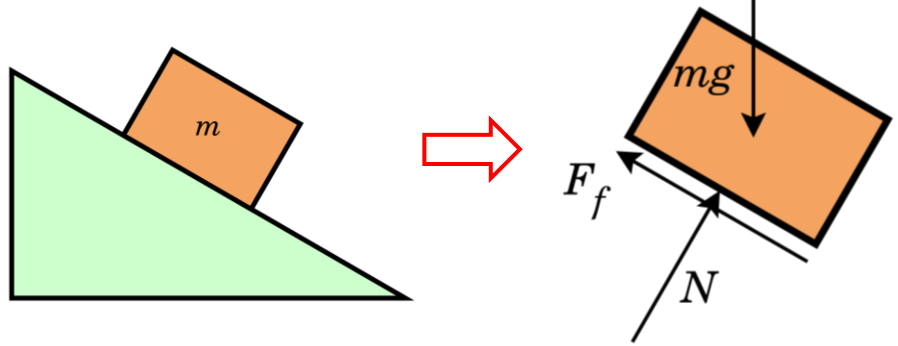

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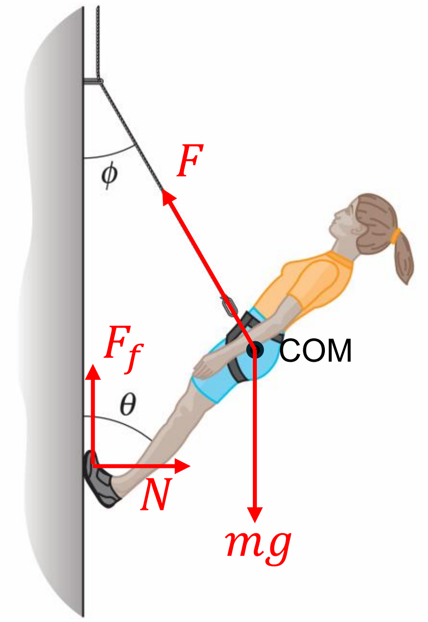
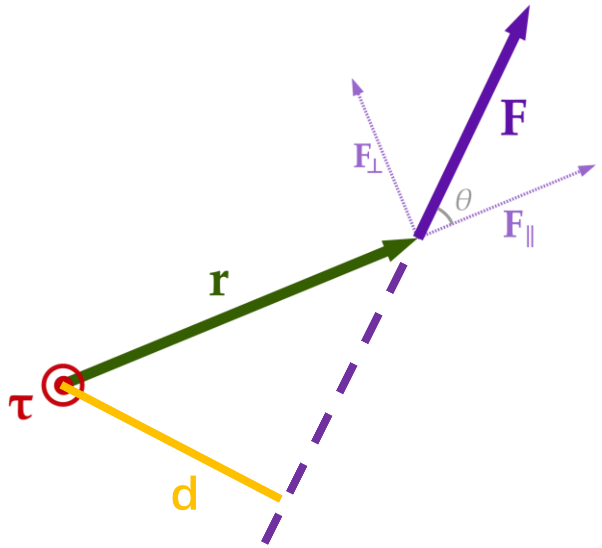


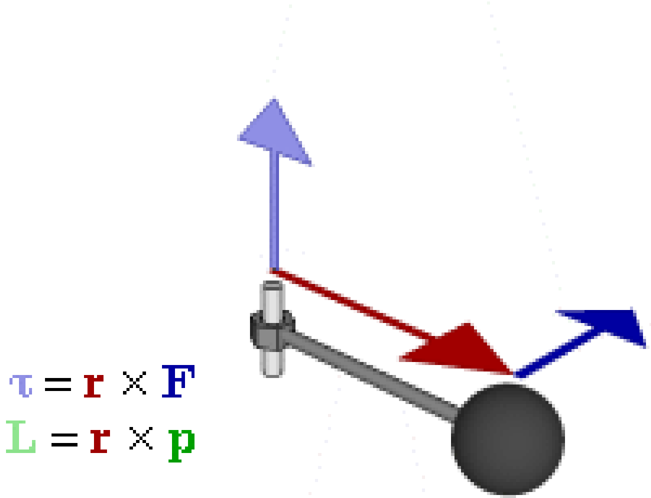
Moment of a Force

The **moment (torque)** of a force about a point is a measure of the **tendency** of the force to cause **rotation** about that point.

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$$

$$|\boldsymbol{\tau}| = rF_{\perp} = Fd = rF\sin\theta$$

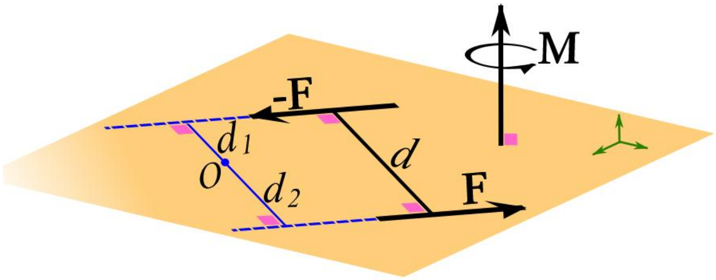


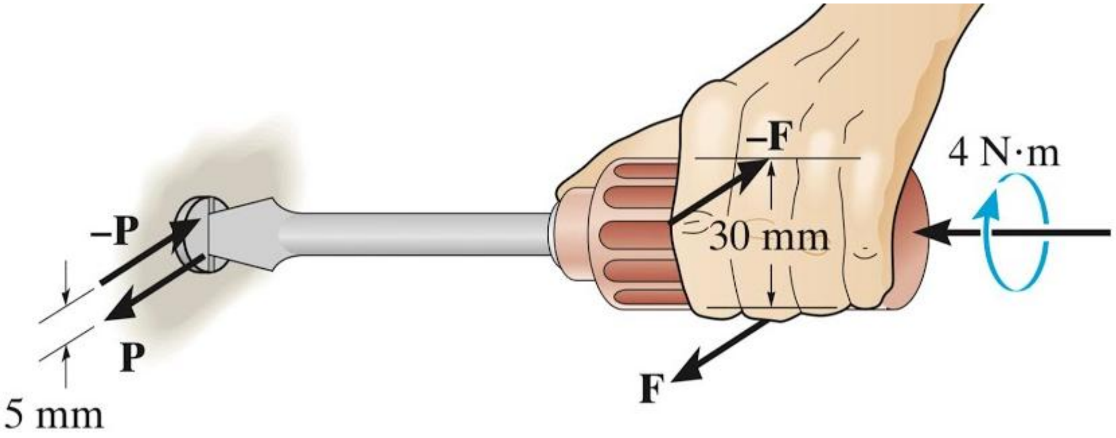


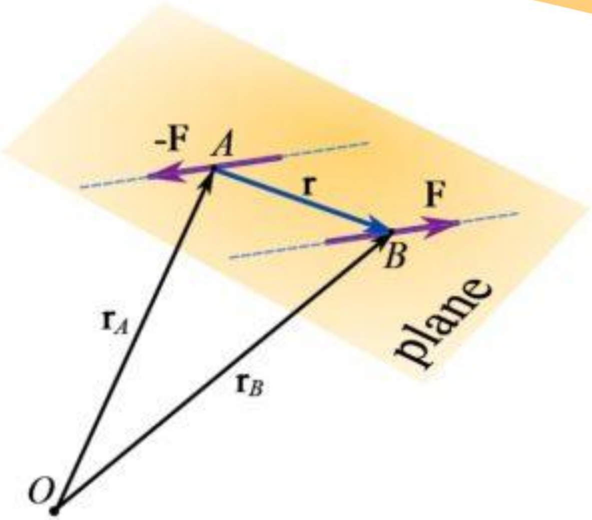
Moment of a Couple

- **A couple** is defined as a pair of **parallel** but **noncollinear** forces that have the **same magnitude but opposite directions**.
- Moment of a couple:

$$\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$$
$$|\boldsymbol{M}| = Fd$$

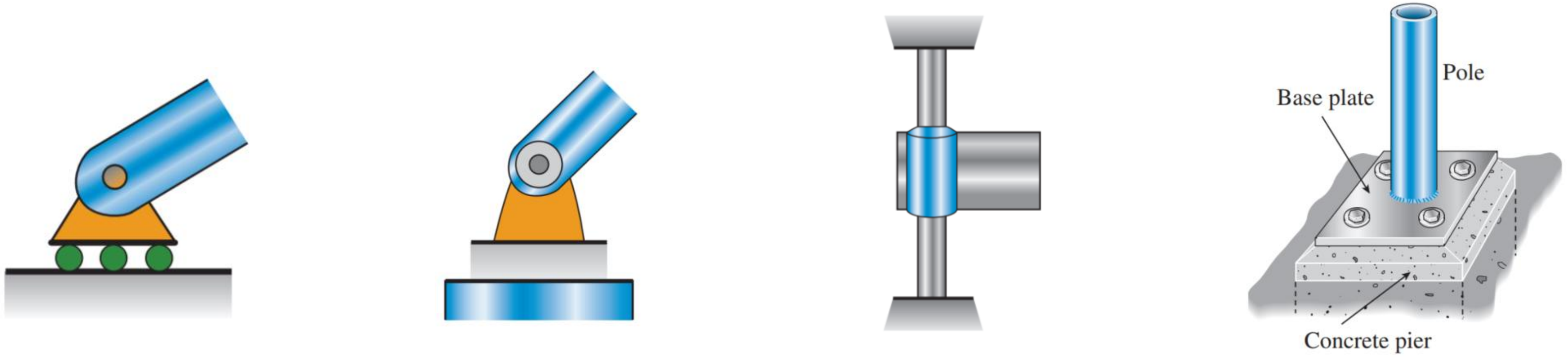




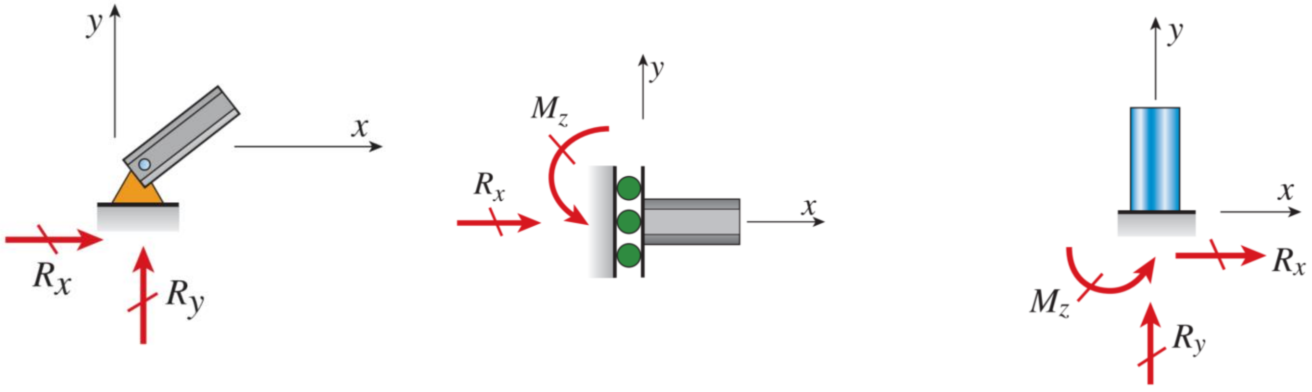


Supporting type and reactions

Supports:



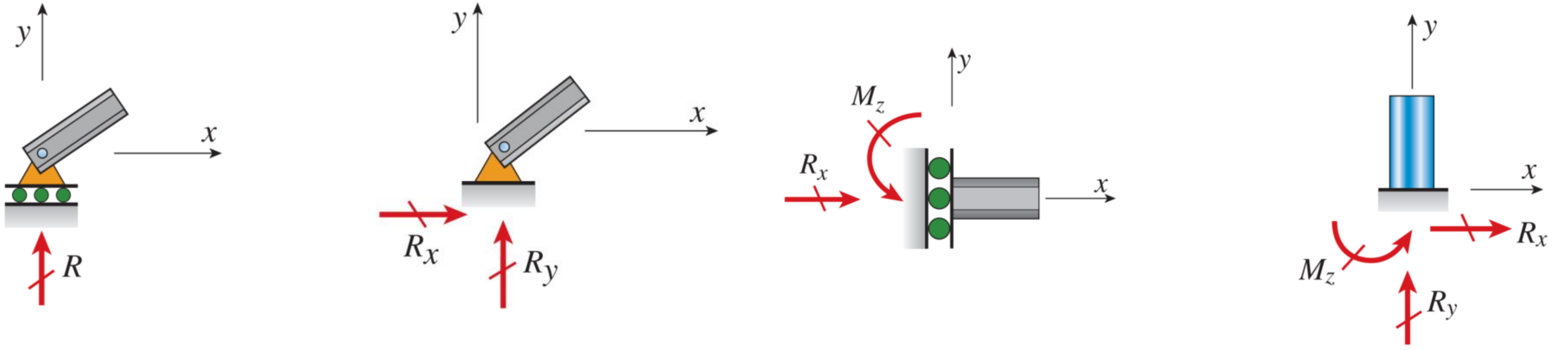
Reactions:



Roller Pin Sliding Fixed

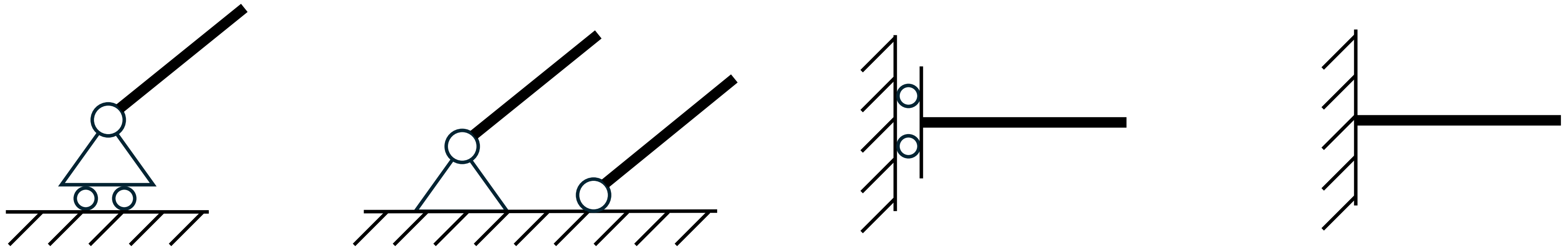
Supporting type and reactions

Reactions:



Roller Pin Sliding Fixed

Symbols:



Static equilibrium

The resultant force $\boldsymbol{R}$ and resultant moment $\boldsymbol{M}$ of all forces and moments acting on either a rigid or deformable body in equilibrium are both **0**.

- The sum of the moments may be taken about any arbitrary point.

$$\boldsymbol{R} = \sum \boldsymbol{F} = \boldsymbol{0}$$
$$\boldsymbol{M} = \sum \boldsymbol{M} = \sum (\boldsymbol{r} \times \boldsymbol{F}) = \boldsymbol{0}$$

$$\Longleftrightarrow$$

$$\sum F_x = 0, \qquad \sum F_y = 0, \qquad \sum F_z = 0$$
$$\sum M_x = 0, \qquad \sum M_y = 0, \qquad \sum M_z = 0$$

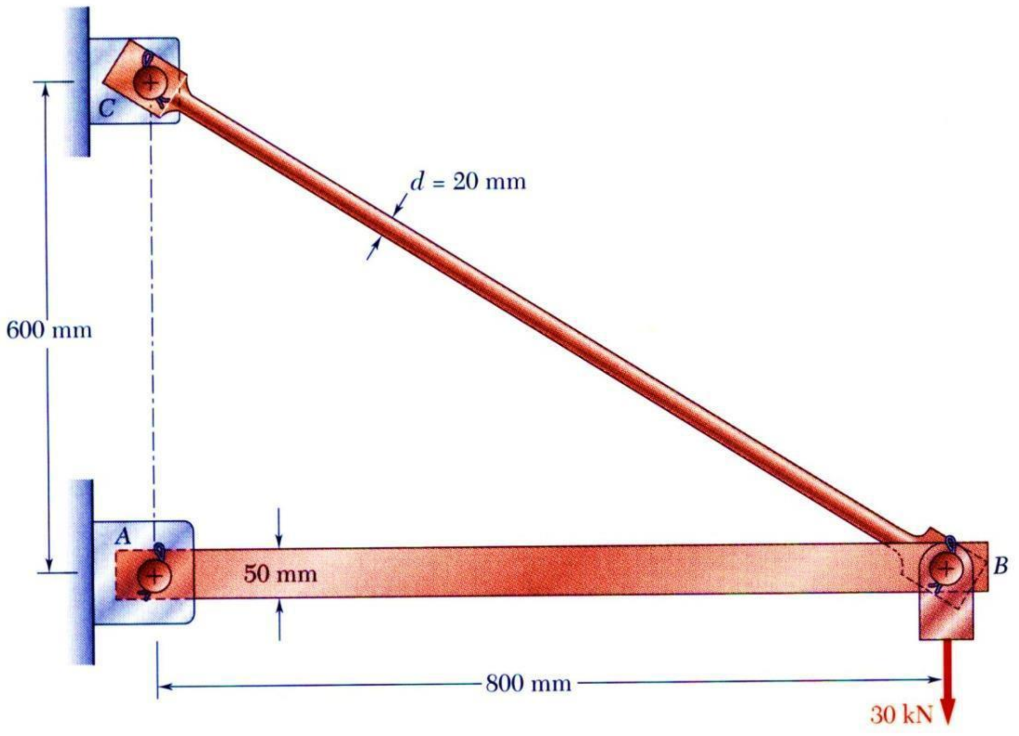
In 2D:

$$\sum F_x = 0, \qquad \sum F_y = 0, \qquad \sum M_z = 0$$

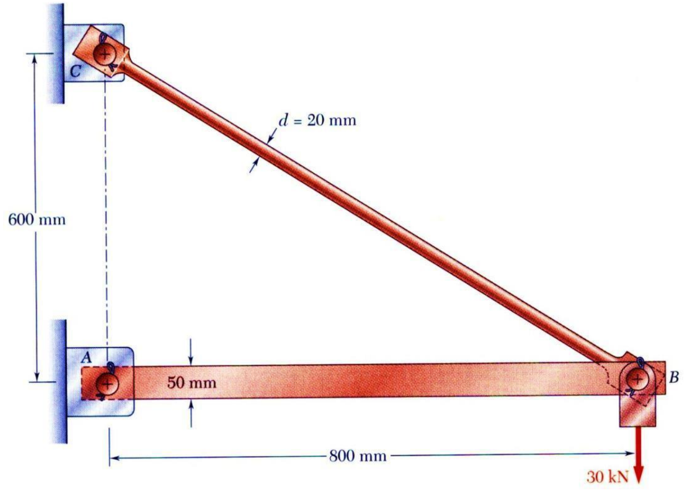
Static equilibrium

Example:

Analyze the reaction force on support A and C

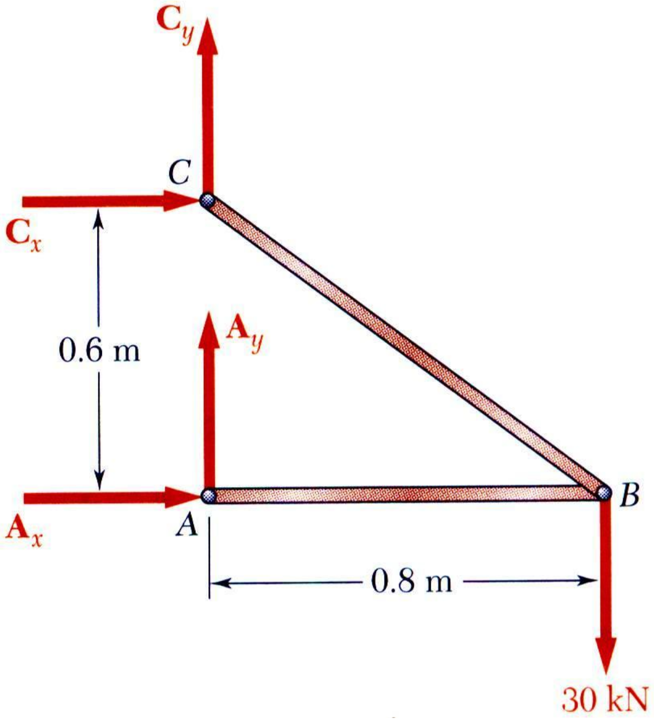


Static equilibrium

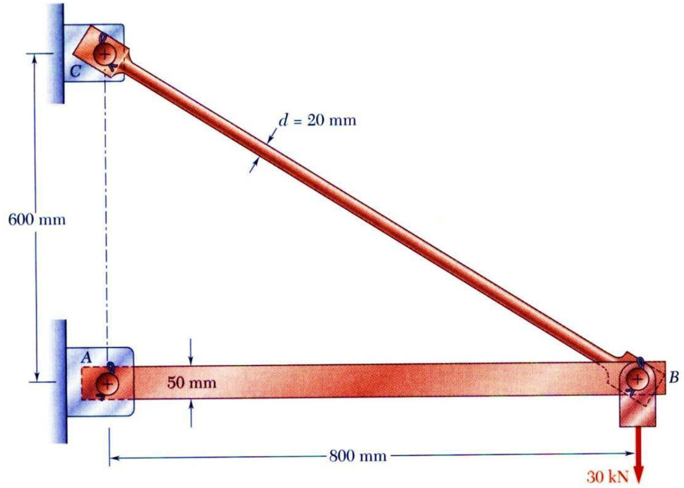


Example:

Analyze the reaction force on support A and C



Static equilibrium



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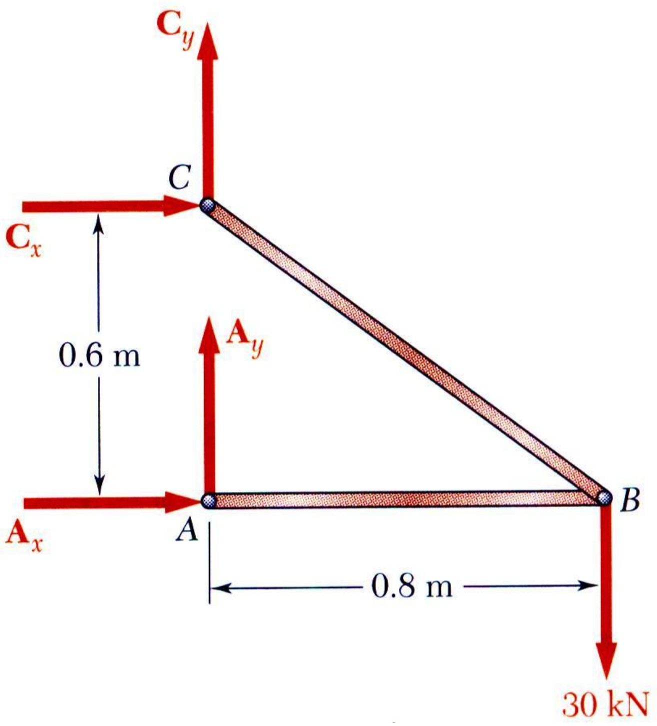
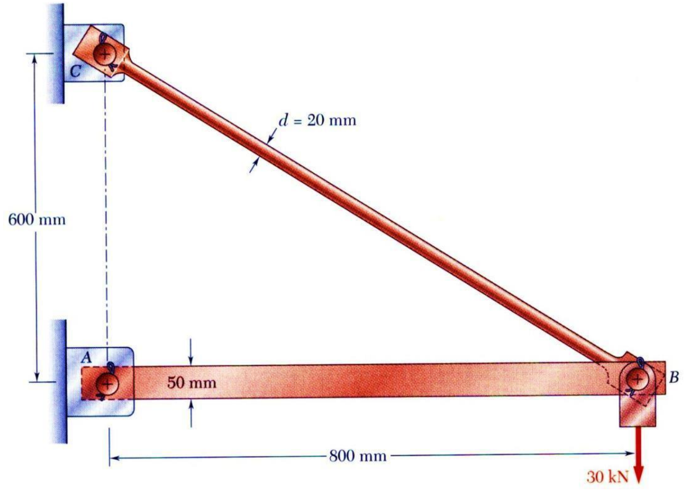


$$+\circlearrowleft \sum M_C = 0 = A_x(0.6\,\mathrm{m}) - (30\,\mathrm{kN})(0.8\,\mathrm{m})$$

$$\boxed{A_x = 40\,\mathrm{kN}}$$

Static equilibrium



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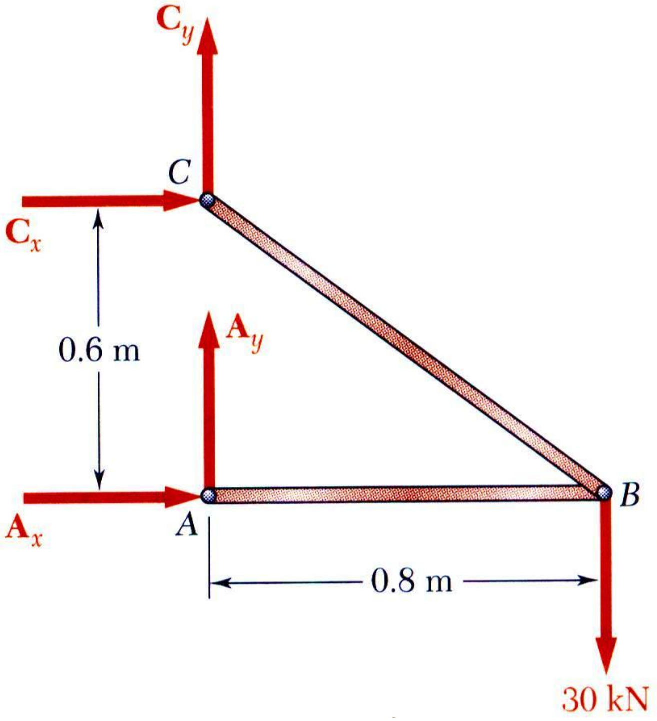


$$+\circlearrowleft \sum M_C = 0 = A_x(0.6\,\mathrm{m}) - (30\,\mathrm{kN})(0.8\,\mathrm{m})$$

$$\boxed{A_x = 40\,\mathrm{kN}}$$

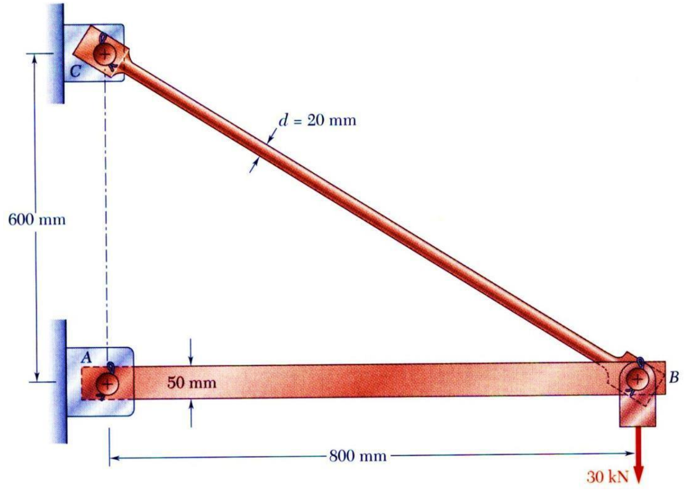
$$\xrightarrow{+} \sum F_x = 0 = A_x + C_x$$

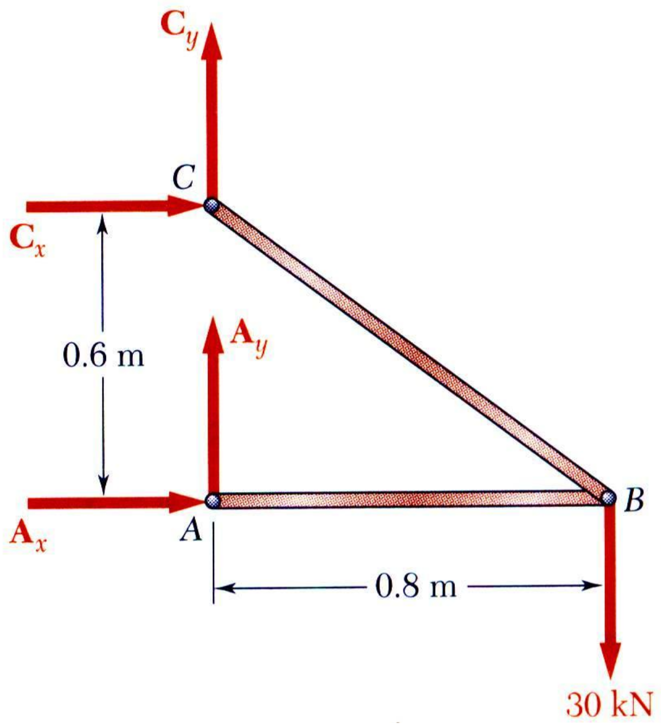
$$\boxed{C_x = -A_x = -40\,\mathrm{kN}}$$

Static equilibrium

Example:

Analyze the reaction force on support A and C





$$+\circlearrowleft \sum M_C = 0 = A_x(0.6\,\mathrm{m}) - (30\,\mathrm{kN})(0.8\,\mathrm{m})$$

$$\boxed{A_x = 40\,\mathrm{kN}}$$

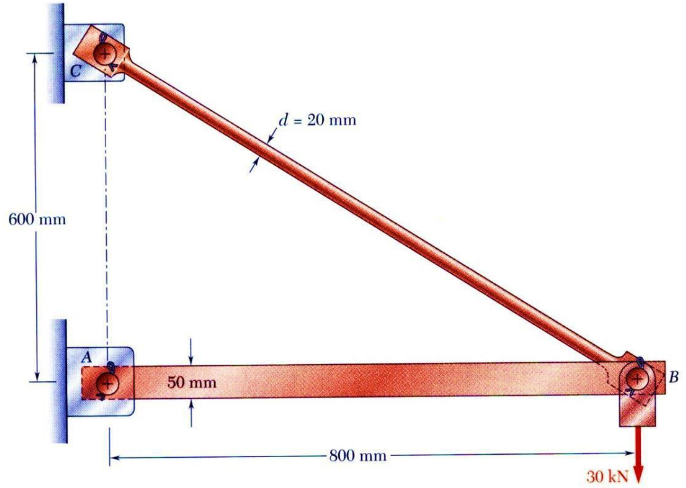
$$\overset{+}{\rightarrow} \sum F_x = 0 = A_x + C_x$$

$$\boxed{C_x = -A_x = -40\,\mathrm{kN}}$$

$$+\uparrow \sum F_y = 0 = A_y + C_y - 30\,\mathrm{kN} = 0$$

$$A_y + C_y = 30\,\mathrm{kN}$$

Static equilibrium



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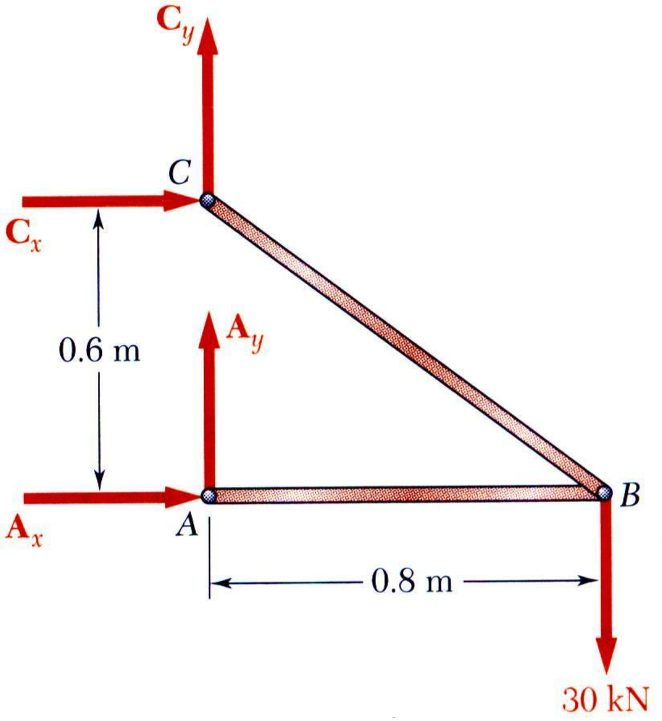

$$+\circlearrowleft \sum M_C = 0 = A_x(0.6\,\mathrm{m}) - (30\,\mathrm{kN})(0.8\,\mathrm{m})$$

$$\boxed{A_x = 40\,\mathrm{kN}}$$

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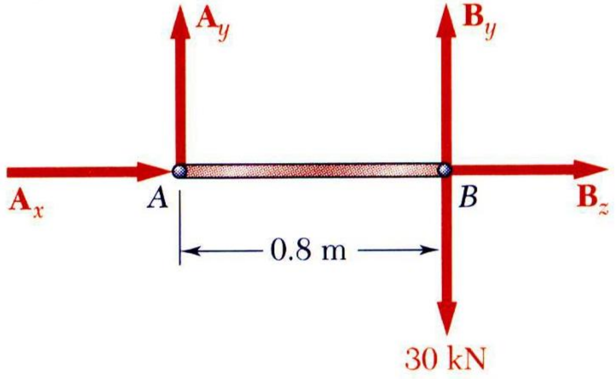
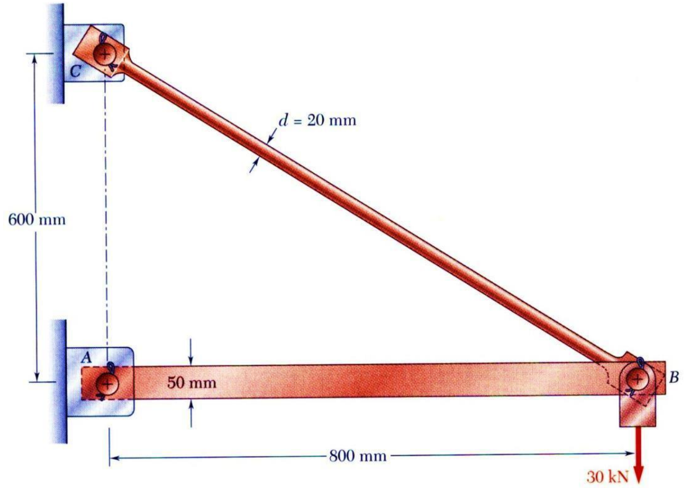
$$A_y + C_y = 30\,\mathrm{kN}$$



$$+\circlearrowleft \sum M_B = 0 = -A_y(0.8\,\mathrm{m})$$

$$\boxed{A_y = 0}$$

Static equilibrium



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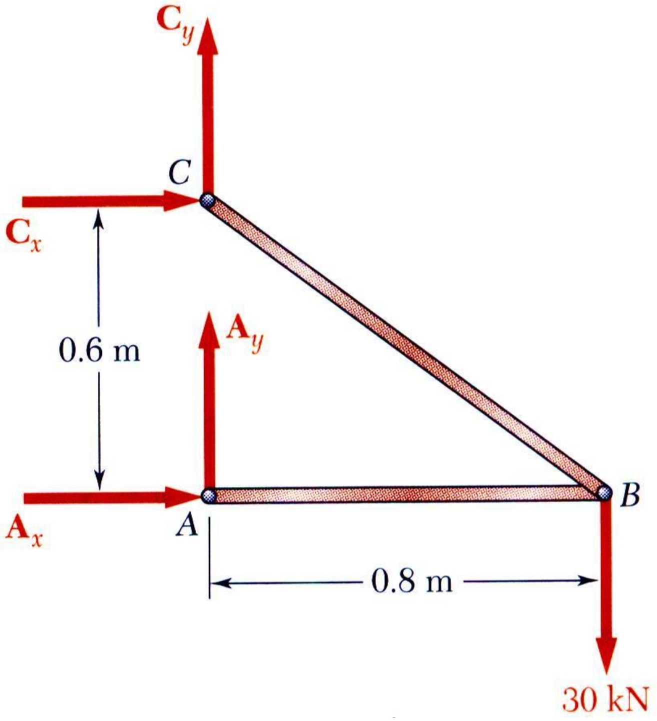


$$+\circlearrowleft \sum M_C = 0 = A_x(0.6\,\text{m}) - (30\,\text{kN})(0.8\,\text{m})$$

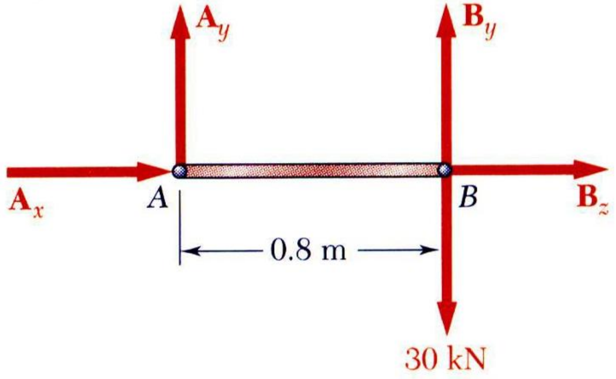
$$\boxed{A_x = 40\,\text{kN}}$$

$$\overset{+}{\rightarrow} \sum F_x = 0 = A_x + C_x$$

$$\boxed{C_x = -A_x = -40\,\text{kN}}$$

$$+\uparrow \sum F_y = 0 = A_y + C_y - 30\,\text{kN} = 0$$

$$A_y + C_y = 30\,\text{kN}$$

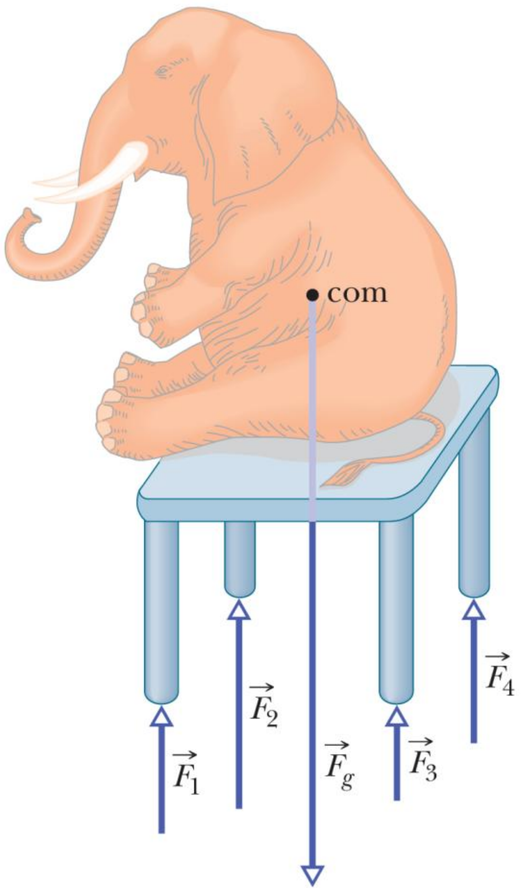


$$+\circlearrowleft \sum M_B = 0 = -A_y(0.8\,\text{m})$$

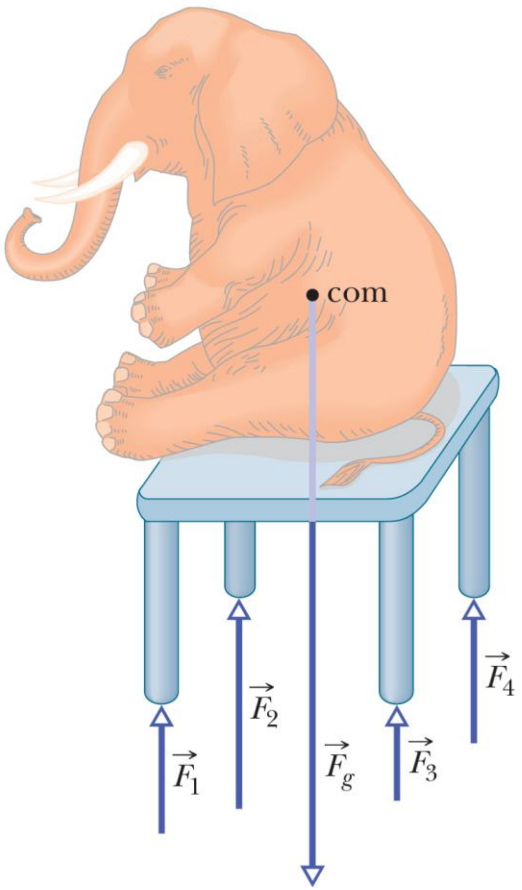
$$\boxed{A_y = 0}$$

$$\boxed{C_y = 30\,\text{kN}}$$

Can statics solve this problem?



Can statics solve this problem?



How many unknowns?

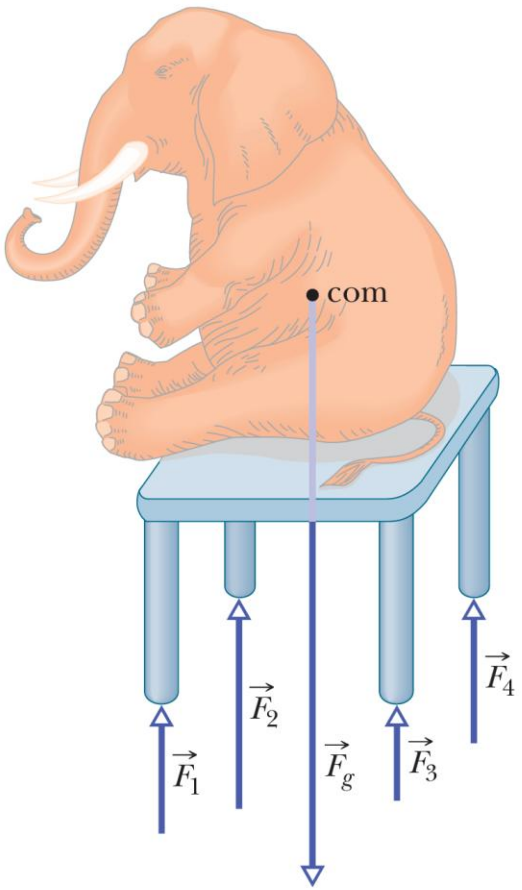
$$F_1, \quad F_2, \quad F_3, \quad F_4$$

How many equations?

$$\sum F_z = 0$$
$$\sum M_x = 0$$
$$\sum M_y = 0$$

# of unknown > # of equations: **indeterminate**

Can statics solve this problem?



What is eluding us in our efforts to find the individual forces by solving equations?

How many unknowns?

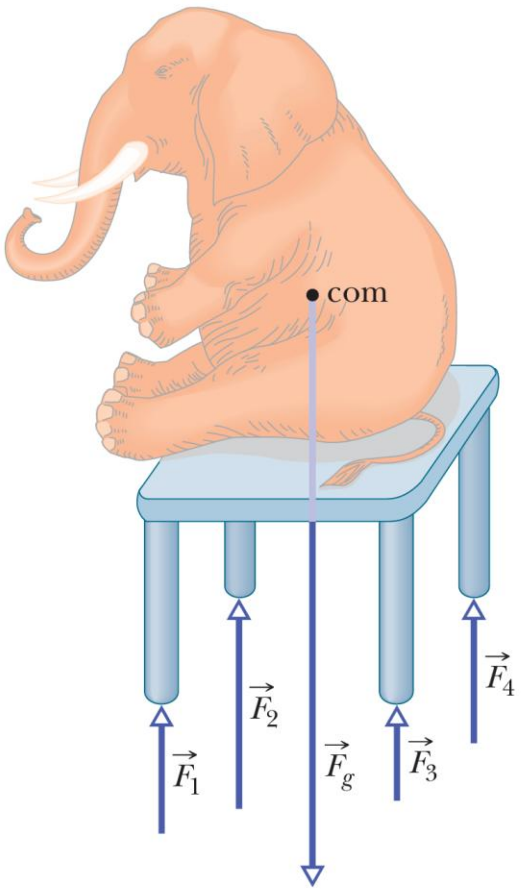
$$F_1, \quad F_2, \quad F_3, \quad F_4$$

How many equations?

$$\sum F_z = 0$$
$$\sum M_x = 0$$
$$\sum M_y = 0$$

# of unknown > # of equations: **indeterminate**

Can statics solve this problem?



What is eluding us in our efforts to find the individual forces by solving equations?

How many unknowns?

$$F_1, \quad F_2, \quad F_3, \quad F_4$$

Rigid body assumption

How many equations?

$$\sum F_z = 0$$
$$\sum M_x = 0$$
$$\sum M_y = 0$$

# of unknown > # of equations: **indeterminate**

Welcome to the world with elasticity

All real "rigid" bodies are to some extent **elastic**, which means that we can change their dimensions slightly by pulling, pushing, twisting, or compressing them.

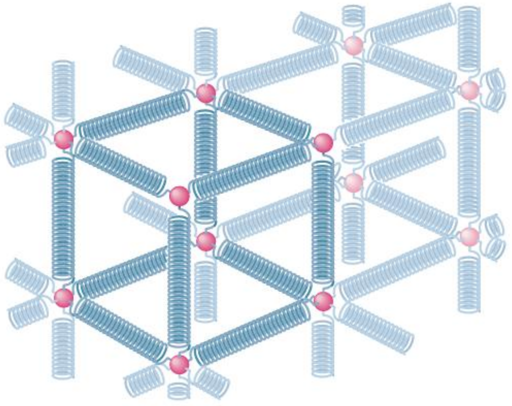
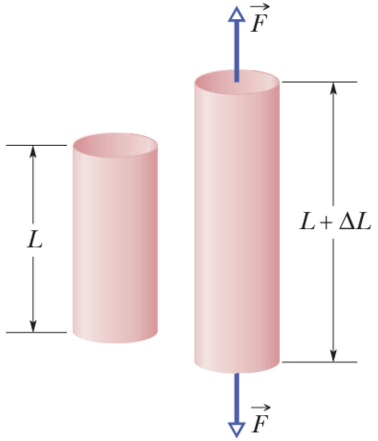
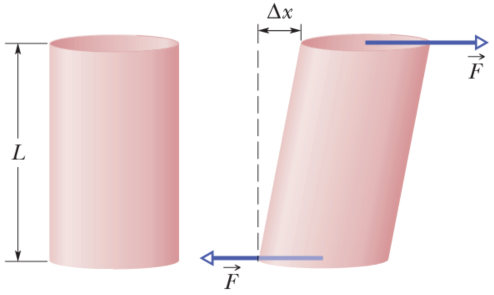
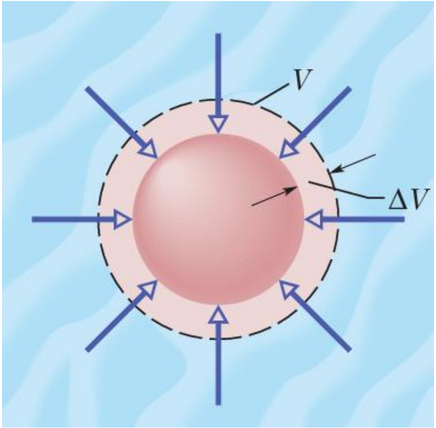


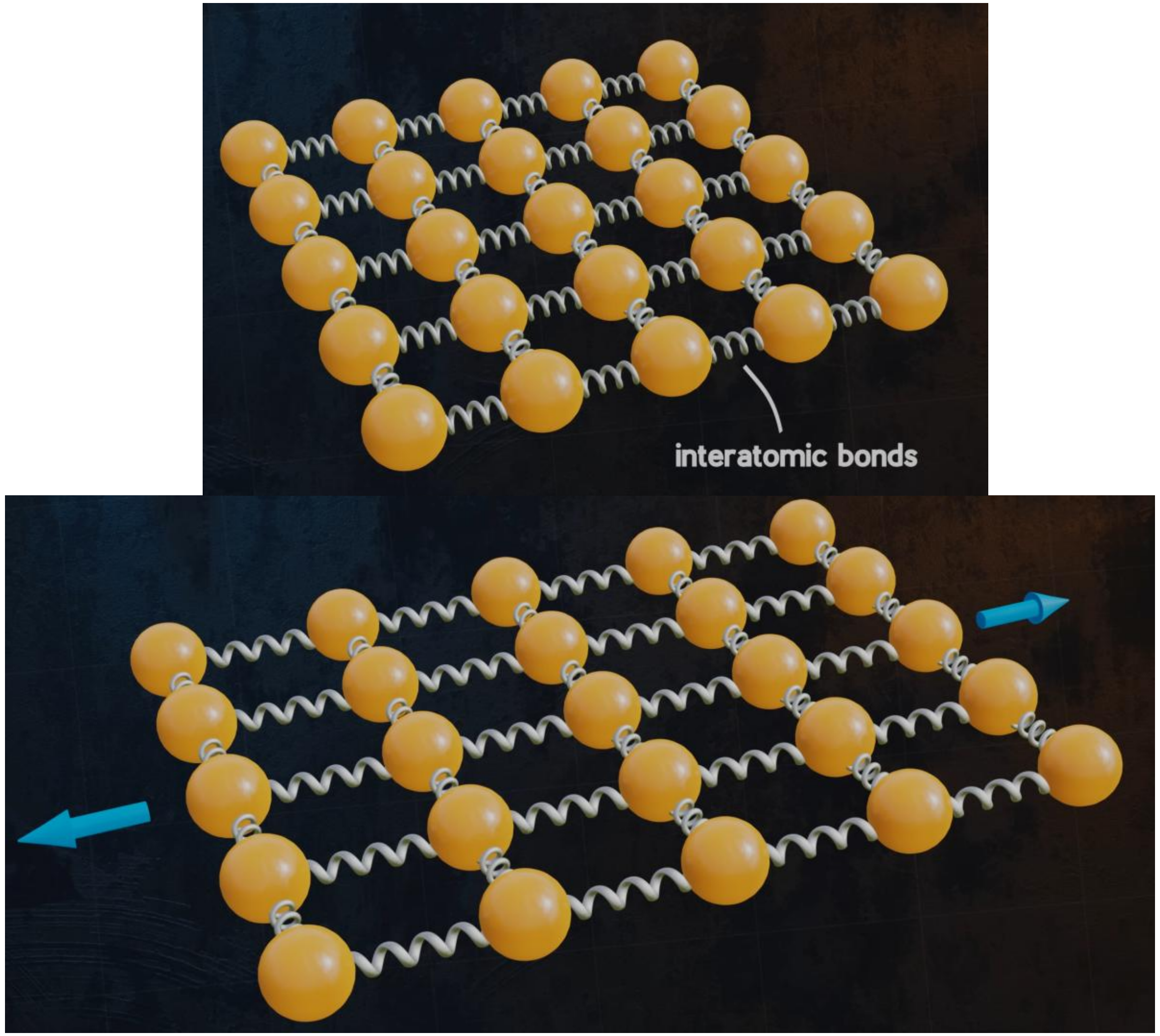
Figure 12-10 The atoms of a metallic solid are distributed on a repetitive three-dimensional lattice. The springs represent interatomic forces.







Origin of elasticity

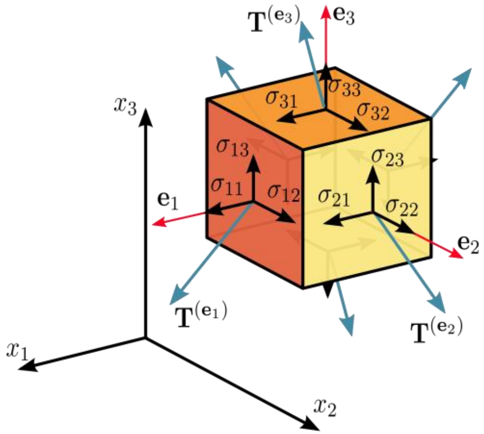


Stress

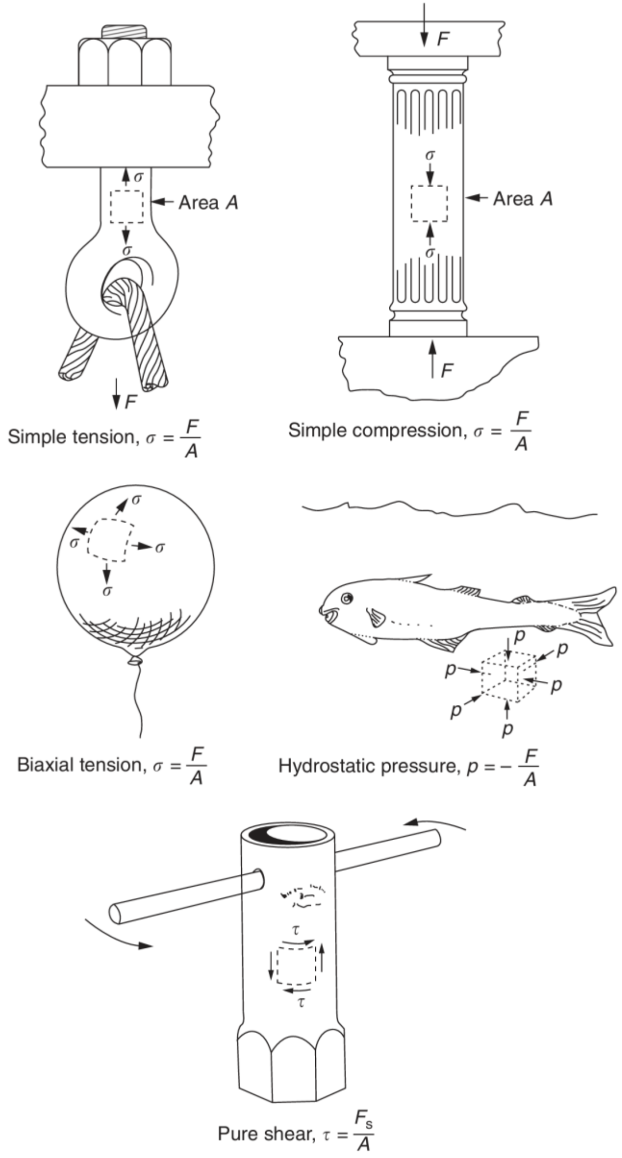
- Microscopically, stress is a physical quantity that expresses the **internal forces** that neighboring particles of a continuous material exert on each other.
- Macroscopically, Stress is defined as the force across a "small" boundary per unit area of that boundary, for all orientations of the boundary, i.e.,

$$\sigma_n = \frac{F}{A_0}$$

- For more complex stress states, people using stress tensor, i.e.,



$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$



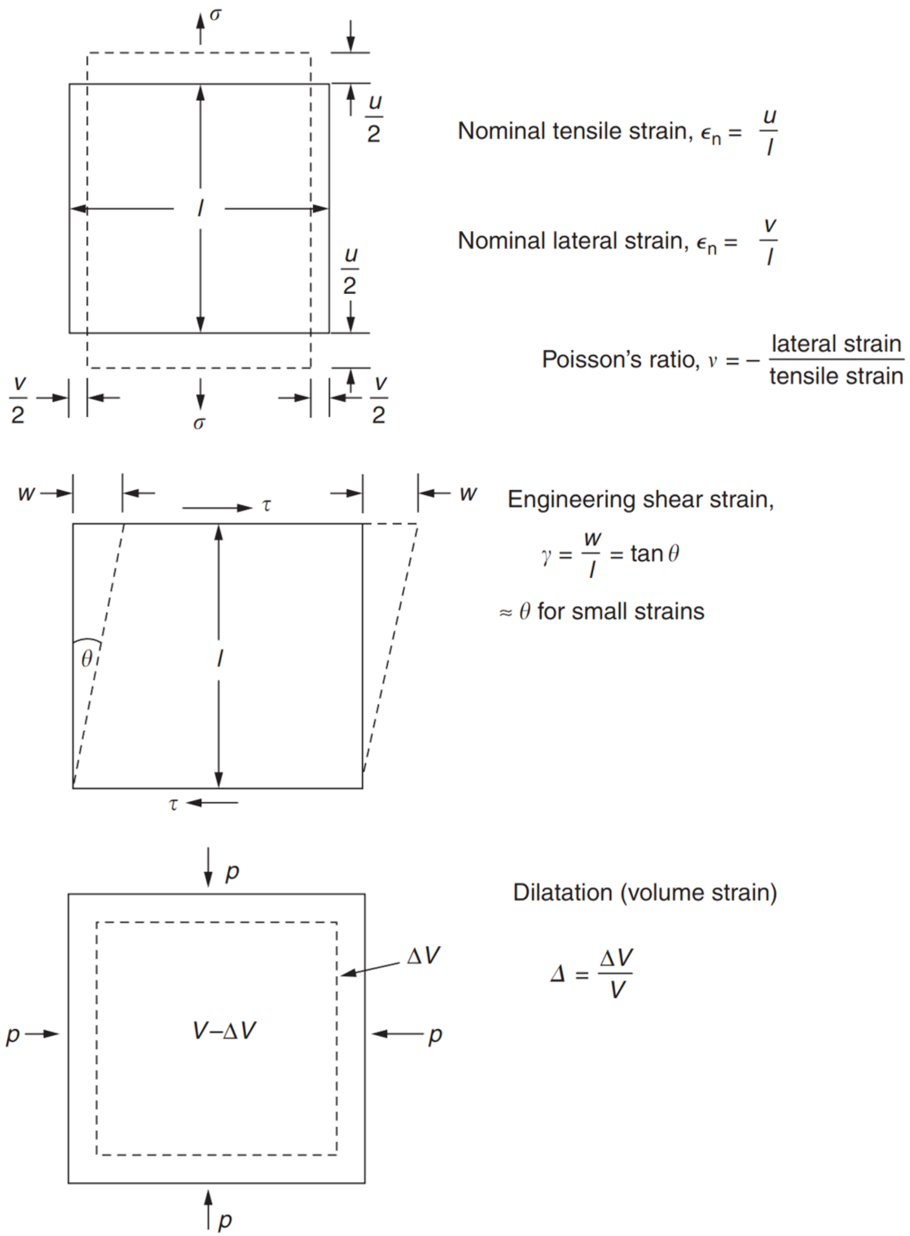
Strain

- Deformation is the continuum mechanics transformation of a body from a reference configuration (capital letter) to a current configuration (lowercase).
- Strain is related to deformation in terms of relative displacement of particles in the body that excludes rigid-body motions.
- Engineering strain, also known as Cauchy strain or nominal strain, is expressed as the ratio of total deformation to the initial dimension of the material body on which forces are applied.

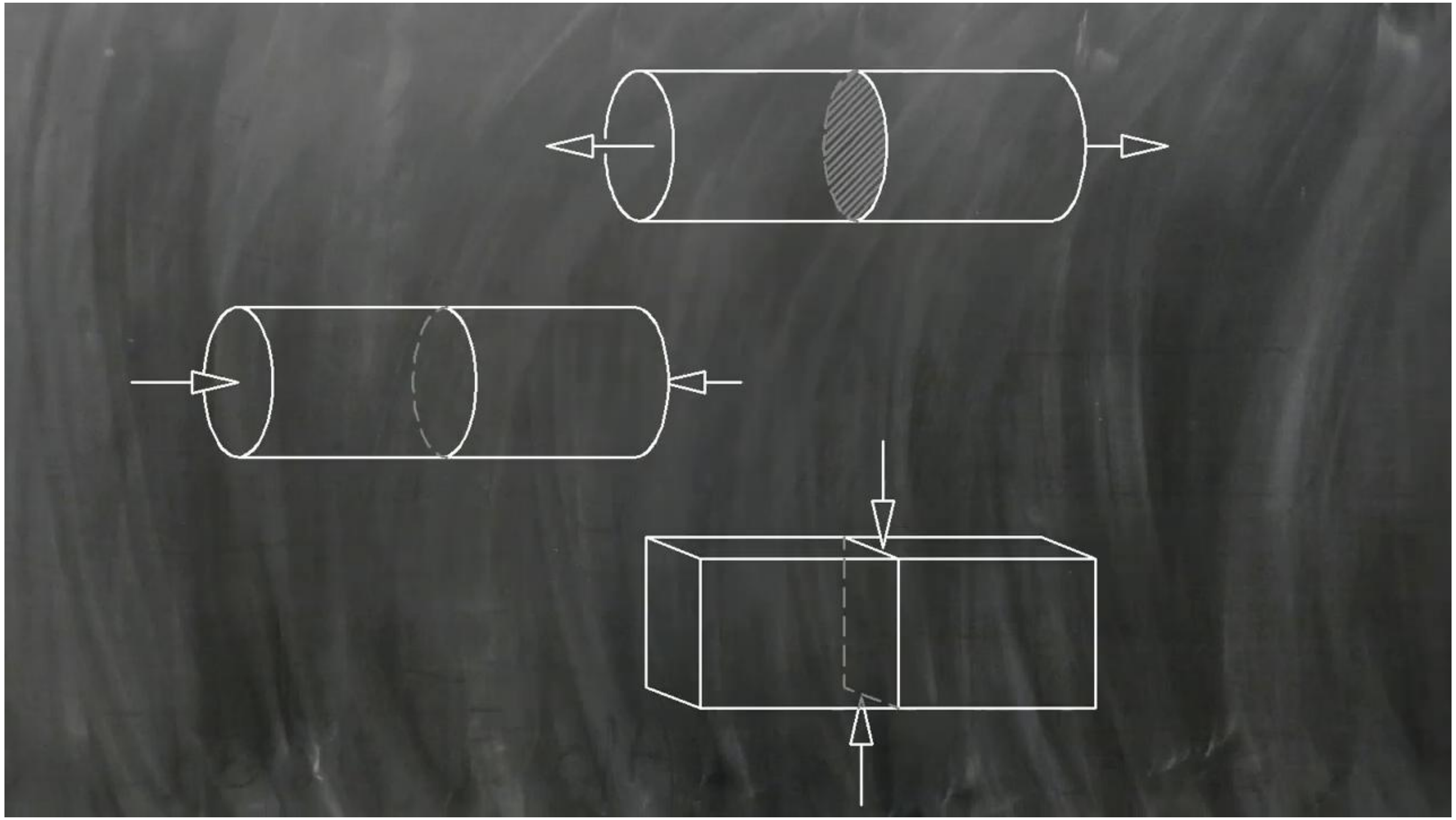
$$\varepsilon_n = \frac{\Delta L}{L} = \frac{l - L}{L}$$

- The logarithmic strain ε, also called, true strain, is defined as

$$\delta\varepsilon = \frac{\delta l}{l} \Longrightarrow \varepsilon = \ln\frac{l}{L} = \ln(1 + \varepsilon_n)$$



Deformation



Art Meets Mechanics

Analyze the mechanical design behind this table.

Art Meets Mechanics

Analyze the mechanical design behind this table.

Which Parts Are in Tension or Compression?

•**Chains**:

- All vertical chains are under **tension**
- They support the top platform and resist gravitational force
- Without them, the upper surface would fall

•**Z-shaped wooden frame**:

- Primarily under **compression**
- Transfers part of the load from top to bottom base
- Also helps maintain the overall geometry

Additional Reference