CHAPTER 1

Tension, Compression, and Shear

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This telecommunications tower is an assemblage of many members that act primarily in tension or compression.

Chapter Objectives

- Define *mechanics of materials*, which examines the stresses, strains, and displacements in structures made of various materials acted on by a variety of different loads.
- Study normal stress (σ) and normal strain (ε) in materials used for structural applications.
- Identify key properties of various materials, such as the modulus of elasticity (E) and yield (σ_y) and ultimate (σ_u) stresses, from plots of stress (σ) versus strain (ε).
- Plot shear stress (τ) versus shear strain (γ) and identify the shearing modulus of elasticity (G).
- Study Hooke's Law for normal stress and strain ($\sigma = E\varepsilon$) and also for shear stress and strain ($\tau = G\gamma$).
- Investigate changes in lateral dimensions and volume of a bar, which depend upon Poisson's ratio (ν) for the material of the bar.
- Study normal, shear, and bearing stresses in simple bolted connections between members.
- Use factors of safety to establish allowable values of stresses.
- Introduce basic concepts of design: the iterative process by which the appropriate size of structural members is determined to meet a variety of both strength and stiffness requirements.

Chapter Outline

1.1 Introduction to Mechanics of Materials

Mechanics of materials is a branch of applied mechanics that deals with the behavior of solid bodies subjected to various types of loading. Other names for this field of study are *strength of materials* and *mechanics of deformable bodies*. The solid bodies considered in this book include bars with axial loads, shafts in torsion, beams in bending, and columns in compression.

The principal objective of mechanics of materials is to determine the stresses, strains, and displacements in structures and their components due to the loads acting on them. An understanding of mechanical behavior is essential for the safe design of all types of structures, whether airplanes and antennas, buildings and bridges, machines and motors, or ships and spacecraft. That is why mechanics of materials is a basic subject in so many engineering fields. Most problems in mechanics of materials begin with an examination of the external and internal forces acting on a stable deformable body. First the loads acting on the body are defined, along with its support conditions, then reaction forces at supports and internal forces in its members or elements are determined using the basic laws of static equilibrium (provided that the body is statically determinate).

In mechanics of materials you study the stresses and strains inside real bodies, that is, bodies of finite dimensions that deform under loads. To determine the stresses and strains, use the physical properties of the materials as well as numerous theoretical laws and concepts. Mechanics of materials provides additional essential information, based on the deformations of the body, to solve statically indeterminate problems (not possible using the laws of static equilibrium alone).

Theoretical analyses and experimental results have equally important roles in mechanics of materials. Theories are used to derive formulas and equations for predicting mechanical behavior but these expressions cannot be used in practical design unless the physical properties of the materials are known. Such properties are available only after careful experiments have been carried out in the laboratory. Furthermore, not all practical problems are amenable to theoretical analysis alone, and in such cases physical testing is a necessity.

The historical development of mechanics of materials is a fascinating blend of both theory and experiment—theory has pointed the way to useful results in some instances, and experiment has done so in others. Such famous persons as Leonardo da Vinci (1452–1519) and Galileo Galilei (1564–1642) performed experiments to determine the strength of wires, bars, and beams, although they did not develop adequate theories (by today's standards) to explain their test results. By contrast, the famous mathematician Leonhard Euler (1707–1783) developed the mathematical theory of columns and calculated the critical load of a column in 1744, long before any experimental evidence existed to show the significance of his results. Without appropriate tests to back up his theories, Euler's results remained unused for over a hundred years, although today they are the basis for the design and analysis of most columns (see Refs. 1-1, 1-2, and 1-3).

1.2 Problem-Solving Approach*

The study of mechanics divides naturally into two parts: first, *understanding* the general concepts and principles, and second, *applying* those concepts and principles to physical situations. You can gain an understanding of the general

*The four step problem-solving approach presented here is patterned after that presented by R. Serway and J. Jewett in *Principles of Physics*, *5e*, *Cengage Learning*, *2013*.

concepts by studying the discussions and derivations presented in this book. You can gain skill only by solving problems on your own. Of course, these two aspects of mechanics are closely related, and many experts in mechanics will argue that *you do not really understand the concepts if you cannot apply them*. It is easy to recite the principles, but applying them to real situations requires an in-depth understanding. Problem solving gives meaning to the concepts and also provides an opportunity to gain experience and develop judgment.

A major objective of this text is to assist you in developing a *structured solution process* for problems in statics and mechanics of materials. This process is referred to as a *problem-solving approach* (PSA) and is used in all example problems in the text. The PSA involves the following four steps:

1. ***Conceptualize*** [*hypothesize, sketch*]: List all relevant data and draw a sketch showing all applied forces, support/boundary conditions, and interactions between adjacent bodies. Development and refinement of the *free-body diagram* is an essential part of this step.
2. ***Categorize*** [*simplify, classify*]: Identify the unknowns in the problem and make any necessary assumptions to simplify the problem and streamline the solution process.
3. ***Analyze*** [*evaluate; select relevant equations, carry out mathematical solution*]: Apply appropriate theories, set up the necessary equations for the chosen mathematical model, and then solve for the unknowns.
4. ***Finalize*** [*conclude; examine answer—Does it make sense? Are units correct? How does it compare to similar problem solutions?*]: Study the answers, compare them to those for similar problems you have solved in the past, and test the robustness of the solution by varying key parameters to see how the results change (perhaps even plot the main result as a function of that parameter to investigate the sensitivity of the answer).

You are encouraged to study the *problem-solving approach* presented in the example problems and then apply it to homework and in-class laboratory problems. This structured systematic approach also will be useful during examinations. See Appendix B.2 for further discussion of the Problem Solving Approach summarized above.

All problems appear at the ends of the chapters, with the problem numbers and subheadings identifying the sections to which they belong. In the case of problems requiring numerical solutions, odd-numbered problems are in U.S. Customary System (USCS) units and even-numbered problems are in International System of Units (SI).

In this book, final numerical results are usually presented with three significant digits when a number begins with the digits 2 through 9, and with four significant digits when a number begins with the digit 1. Intermediate values are often recorded with additional digits to avoid losing numerical accuracy due to rounding of numbers.

1.3 Statics Review

In your prerequisite course on statics, you studied the *equilibrium* of rigid bodies acted upon by a variety of different forces and supported or restrained in such a way that the body was stable and at rest. As a result, a properly restrained body could not undergo rigid-body motion due to the application of static forces. You drew *free-body diagrams* of the entire body, or of key parts of the body, and then applied the *equations of equilibrium* to find external reaction forces and moments or internal forces and moments at critical points. In this section, the basic static equilibrium equations are reviewed and then applied to the solution of example structures (both two and three-dimensional) using both scalar and vector operations (both acceleration and velocity of the body are assumed to be zero). Most problems in mechanics of materials require a static analysis as the first step, so all forces acting on the system and causing its deformation are known. Once all external and internal forces of interest have been found, you can proceed with the evaluation of stresses, strains, and deformations of bars, shafts, beams, and columns as described in subsequent chapters.

Equilibrium Equations

The resultant force R and resultant moment M of *all* forces and moments acting on either a rigid or deformable body in equilibrium are both zero. The sum of the moments may be taken about any arbitrary point. The resulting equilibrium equations can be expressed in *vector form* as:

$$R = \Sigma F = 0 \tag{1-1}$$

$$M = \Sigma M = \Sigma(r \times F) = 0 \tag{1-2}$$

where F is one of a number of vectors of forces acting on the body and r is a position vector from the point at which moments are taken to a point along the line of application of any force F. It is often convenient to write the equilibrium equations in *scalar form* using a rectangular Cartesian coordinate system, either in two dimensions (x, y) or three dimensions (x, y, z) as

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M_z = 0 \tag{1-3}$$

Equation (1-3) can be used for two-dimensional or planar problems, but in three dimensions, three force and three moment equations are required:

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0 \tag{1-4}$$

$$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0 \tag{1-5}$$

If the number of unknown forces is equal to the number of independent equilibrium equations, these equations are sufficient to solve for all unknown reaction or internal forces in the body, and the problem is referred to as *statically determinate* (provided that the body is stable). If the body or structure is constrained by additional (or redundant) supports, it is *statically indeterminate*, and a solution is not possible using the laws of static equilibrium alone.

Applied Forces

External loads applied to a body or structure may be either concentrated or distributed forces or moments. For example, force F_B (with units of pounds, lb, or newtons, N) in Fig. 1-1 is a point or concentrated load and is assumed to act at point B on the body, while moment M_A is a concentrated moment or couple (with units of lb-ft or N · m) acting at point A. Distributed forces may act along or normal to a member and may have constant intensity, such as line load q_1 normal to member BC (Fig. 1-1) or line load q_2 acting in the $-y$ direction on inclined member DF; both q_1 and q_2 have units of force intensity (lb/ft or N/m). Distributed loads also

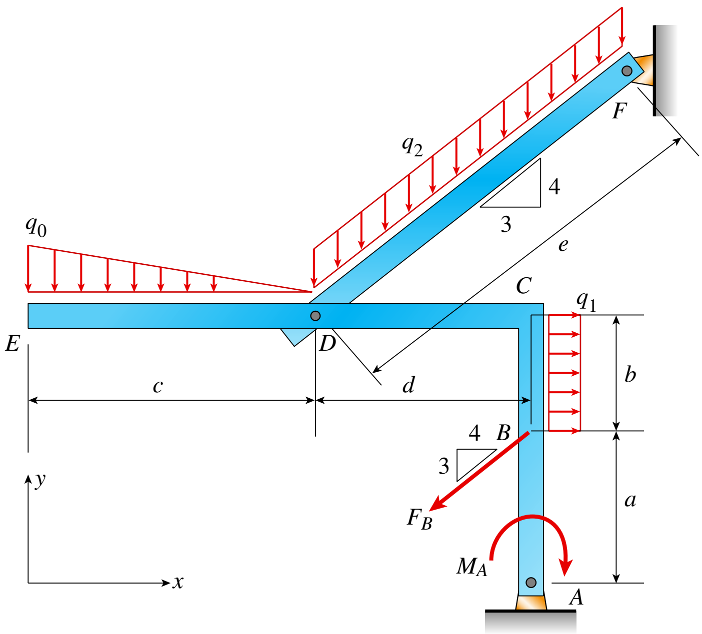


FIGURE 1-1

Plane frame structure

may have a linear (or other) variation with some peak intensity q_0 (as on member ED in Fig. 1-1). Surface pressures p (with units of lb/ft^2 or Pa), such as wind acting on a sign (Fig. 1-2), act over a designated region of a body. Finally, a body force w (with units of force per unit volume, lb/ft^3 or N/m^3), such as the distributed self-weight of the sign or post in Fig. 1-2, acts throughout the volume of the body and can be replaced by the component weight W acting at the center of gravity (c.g.) of the sign (W_s) or post (W_p). In fact, any distributed loading (line, surface, or body force) can be replaced by a statically equivalent force at the center of gravity (or center of pressure for wind) of the distributed loading when overall static equilibrium of the structure is evaluated using Eqs. (1-1) to (1-5).

Free-Body Diagrams

A free-body diagram (FBD) is an essential part of a static analysis of a rigid or deformable body. All forces acting on the body, or component part of the body, must be displayed on the FBD if a correct equilibrium solution is to be obtained. This includes applied forces and moments, reaction forces and moments, and any connection forces between individual components. For example, an *overall* FBD of the plane frame in Fig. 1-1 is shown in Fig. 1-3a; all applied and reaction forces are shown on this FBD and statically equivalent concentrated loads are displayed for all distributed loads. Statically equivalent forces F_{q0}, F_{q1}, and F_{q2}, each acting at the c.g. of the corresponding distributed loading, are used in the equilibrium equation solution to represent distributed loads q_0, q_1, and q_2, respectively.

Next, the plane frame has been disassembled in Fig. 1-3b, so that *separate* FBDs can be drawn for each part of the frame, thereby exposing pin-connection forces at D (D_x, D_y). Both FBDs must show all applied forces as well as reaction forces A_x and A_y at pin-support joint A and F_x and F_y at

FIGURE 1-2

Wind on sign

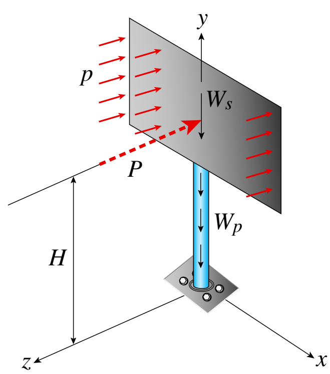
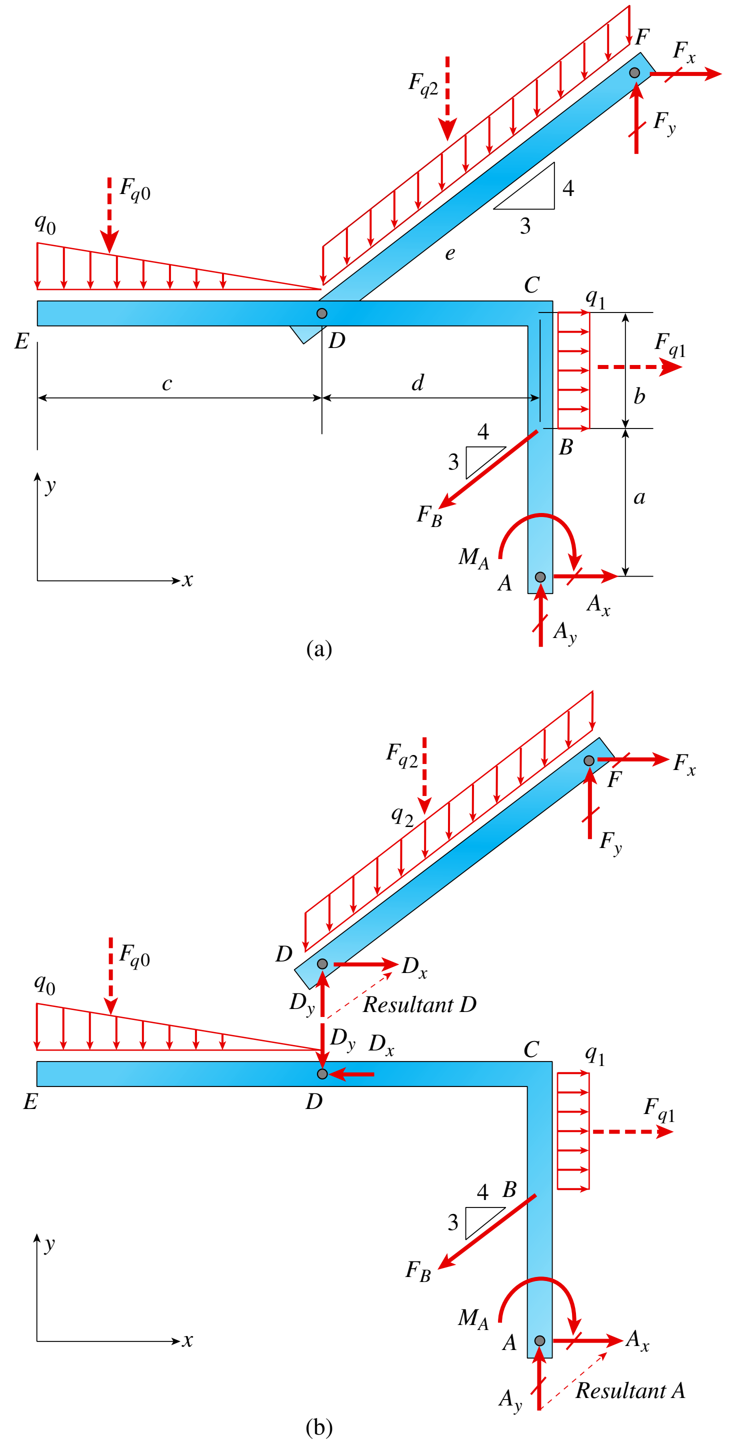


FIGURE 1-3

(a) Overall FBD of plane frame structure from Fig. 1-1, and (b) Separate free-body diagrams of part *ABCDE* and part *DF* of the plane frame structure in Fig. 1-1



pin-support joint *F*. The forces transmitted between frame elements *EDC* and *DF* at pin connection *D* must be determined if the proper interaction of these two elements is to be accounted for in the static analysis.

The FBDs presented in Figs. 1-3a and 1-3b are essential parts of this solution process. A *statics sign convention* is usually employed in the solution for support reactions; forces acting in the positive directions of the coordinate axes are assumed positive, and the right-hand rule is used for moment vectors.

Reactive Forces and Support Conditions

Proper restraint of the body or structure is essential if the equilibrium equations are to be satisfied. A sufficient number and arrangement of supports must be present to prevent rigid-body motion under the action of static forces. A reaction force at a support is represented by a single arrow with a slash drawn through it (see Fig. 1-3) while a moment restraint at a support is shown as a double-headed or curved arrow with a slash. Reaction forces and moments usually result from the action of applied forces of the types described above (i.e., concentrated, distributed, surface, and body forces).

A variety of different support conditions may be assumed depending on whether the problem is 2D or 3D. Supports A and F in the 2D plane frame structure shown in Fig. 1-1 and Fig. 1-3 are pin supports, while the base of the 3D sign structure in Fig. 1-2 may be considered to be a fixed or clamped support. Some of the most commonly used idealizations for 2D and 3D supports, as well as interconnections between members or elements of a structure, are illustrated in Table 1-1. The restraining or transmitted forces and moments associated with each type of support or connection are displayed in the third column of the table (these are not FBDs, however). The reactions forces and moments for the 3D sign structure in Fig. 1-2 are shown on the FBD in Fig. 1-4a; only reactions R_y, R_z, and M_x are nonzero because the sign structure and wind loading are symmetric with respect to the y-z plane. If the sign is eccentric to the post (Fig. 1-4b), only reaction R_x is zero for the case of wind loading in the $-z$ direction. (See Problems 1.8-19 and 1.9-17 at the end of Chapter 1 for a more detailed examination of the reaction forces due to wind pressure acting on several sign structures similar to that shown in Fig. 1-2; forces and stresses in the base plate bolts are also computed).

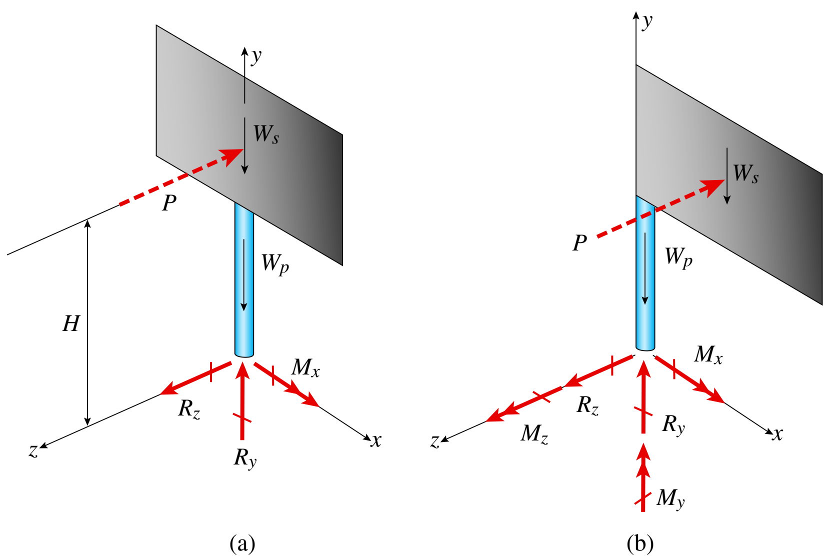
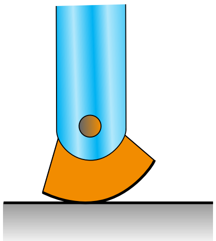
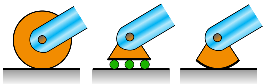
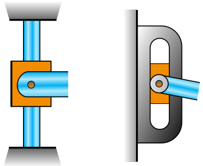
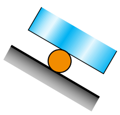
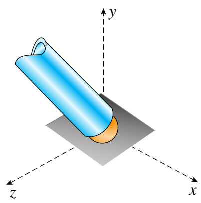
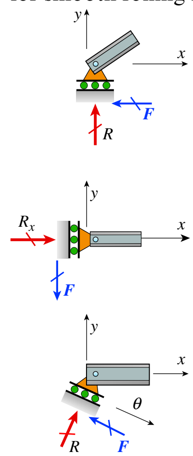
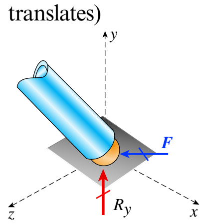
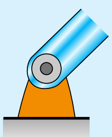
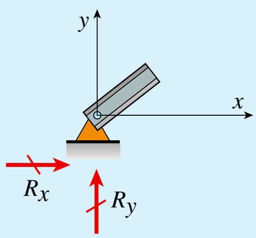


FIGURE 1-4

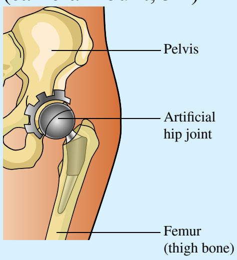
(a) FBD of symmetric sign structure, and (b) FBD of eccentric sign structure

Table 1-1

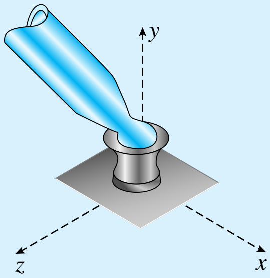
Modeling reaction forces and support conditions in 2D or 3D static analysis

Type of support or connection	Simplified sketch of support or connection	Display of restraint forces and moments, or connection forces
1. Roller Support: A single reaction force R is developed and is normal to the rolling surface; force R opposes motion into or away from the rolling surface. The rolling surface may be horizontal, vertical, or inclined at some angle θ. If friction is present, then include a force F opposing the movement of the support and tangential to the rolling surface. In 3D, the roller moves in the x-z plane and reaction R_y is normal to that plane.		
The Earthquake Engineering Online Archive  Bridge with roller support (see 1.1, 1.2)  Bridge with *rocker* support (see 1.3)	 (1.1) (1.2) (1.3) Horizontal ***roller*** support [(1.1), (1.2)]; or alternate representation as ***rocker*** support [(1.3)] Both downward and uplift motions are restrained.  Vertical roller restraints  Rotated or inclined roller support  3D roller support	(a) **Two-dimensional roller support** (friction force $\boldsymbol{F} = 0$ for smooth rolling surface)  (b) **Three-dimensional roller support** (friction force $\boldsymbol{F} = 0$ for smooth rolling surface; reaction R_y acts normal to plane x-z on which roller translates) 
2. Pin Support: A single resultant force, usually shown using two rectangular components R_x and R_y in 2D but three components in 3D, resists motion in any direction normal to the pin. The pin support cannot resist moment, and the pin is free to rotate about the z axis. In 3D, the pin becomes a ball-and-socket joint or support.		
The Earthquake Engineering Online Archive 	 Two-dimensional pin	(a) **Two-dimensional pin support** 

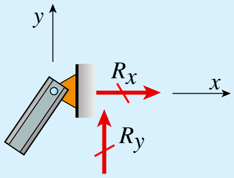
Ball-and-socket joint model (camera mount, 3D)



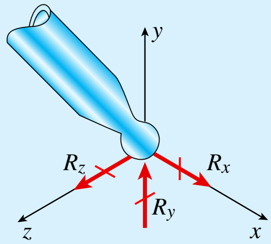
Hip prosthesis for hip replacement



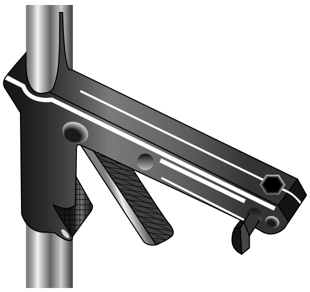
Ball-and-socket joint model



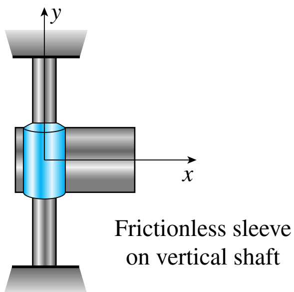
(b) **Three-dimensional pin support**



3. Sliding Support: A support that translates without rotation is a sliding support. Examples are a collar sliding along a sleeve or a flange moving within a slot. Reactions in 2D are a force R_x normal to the sleeve and a moment M_z representing resistance to rotation relative to the sleeve. In 3D, the sliding support translates on frictionless plane *y-z* and reaction moment components M_y and M_z prevent rotation relative to that plane.



Sliding support for column light stand

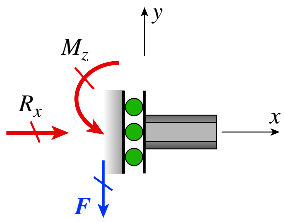


Two-dimensional sliding support

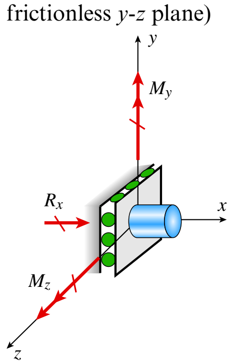
Friction ***F*** opposes motion in $+y$ direction in 2D along sliding surface; ***F*** is zero if smooth surface is assumed.

In 3D, add restraint moment M_x to prevent rotation about *x* axis.

(a) **Two-dimensional sliding support**
(support translates on frictionless path along $+y$ or $-y$ direction)



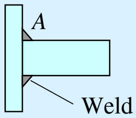
(b) **Three-dimensional sliding support**
(support translates on frictionless *y-z* plane)

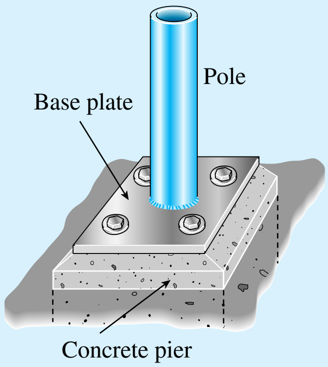


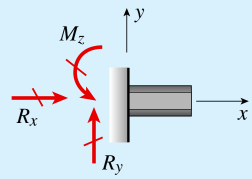
(Continued)

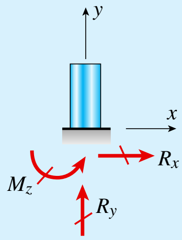
<table>
<tr><th>Type of support or connection</th><th>Simplified sketch of support or connection</th><th>Display of restraint forces and moments, or connection forces</th></tr>
<tr><td colspan="3">4. Fixed Support: No translation or rotation occurs between member and support in a fixed support. This requires three reaction components in 2D: force components R_x and R_y and moment M_z. In 3D, three force reaction components and three moment reaction components are required.</td></tr>
<tr><td>Franco Nadalin/Shutterstock.com

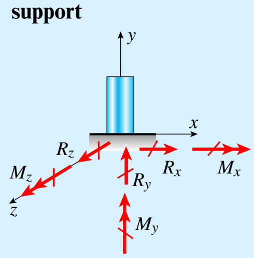
Steel bollard anchored in concrete
Radovan1/Shutterstock.com

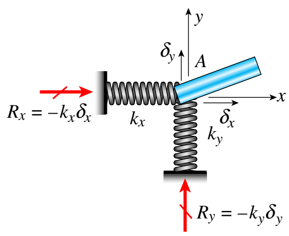
Column bolted to footing</td><td>



Fixed support at base of sign post</td><td>(a) Two-dimensional fixed support


Horizontal member


Vertical member
(b) Three-dimensional fixed support

</td></tr>
<tr><td colspan="3">5. Elastic or Spring Support: In 2D, there may be a longitudinal or normal translational spring or a combination of both. For linear springs, the support reaction at the base of the spring is the product of the spring constant k times the displacement δ in the direction of the spring axis. If joint A translates in $+x$ (δ_x) and $+y$ (δ_y) directions, reaction forces R_x and R_y are created in $-x$ and $-y$ directions, respectively, at the supports of linear translational springs. Alternatively, the support may be pinned for translation but have moment spring k_r for rotation. If joint A rotates about the $+z$ axis (θ_z), reaction moment M_z is created in the $-z$ direction at the base of the rotational spring. In 3D, a fully elastic support consists of three translational springs (k_x, k_y, k_z) and three rotational springs (k_{rx}, k_{ry}, k_{rz}), and an arbitrary joint displacement results in three reaction forces and three reaction moments. In the limit, as each spring constant value approaches infinity, the elastic support becomes a fully fixed support like that shown in Section 4b above.</td></tr>
<tr><td>Nipon Laicharoenchokchai/ Shutterstock.com

Translational spring support for heavy equipment</td><td>In 3D, add spring in $+z$ direction k_z with reaction force $R_z = -k_z\delta_z$.
In 3D, add rotational flexural spring about $+y$ direction k_{ry} with reaction moment $M_y = -k_{ry}\theta_y$ and add rotational torsional spring about $+x$ direction with reaction moment $M_x = -k_{rx}\theta_x$.</td><td>(a) Translational spring (k) in 2D

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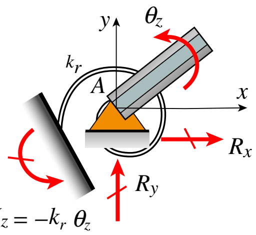
montego/ Shutterstock.com

Rotational spring in a clothespin

Torsion springs are found in window shades and as part of the lift mechanism in power garage door-opening systems.

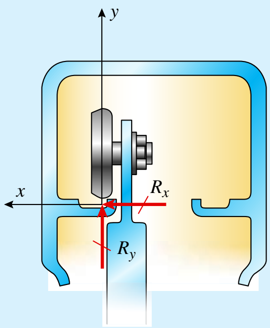
(b) **Rotational spring (k_r) in 2D**



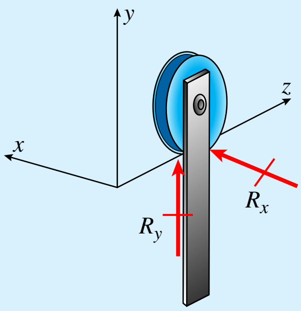
6. Wheel on Rail Support: This support is a particular form of the 3D roller support (see Section 1b above). Now general movement in the *x-z* plane is constrained by normal force R_y and lateral force R_x, both acting normal to the rail or slot on which the wheel travels. If friction is considered, friction force F is added along the rail in the direction opposing the wheel translation.

© Paul Rollins/Alamy Stock Photo

Wheel rolls on rail or in slot along z axis; friction force opposing motion is neglected; R_x is lateral constraint force, R_y is normal force.



Cross section through guide rail



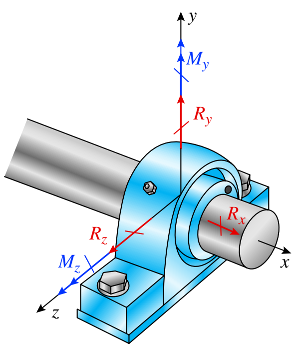
7. Thrust-Bearing Support: A thrust bearing constrains translational motion along the shaft axis while allowing rotary motion to occur about that axis. Support reaction forces and moment components act in all directions except for reaction moment $M_x = 0$ about the thrust axis (in the absence of friction). A special case is the journal bearing for which axial thrust restraint component $R_x = 0$.

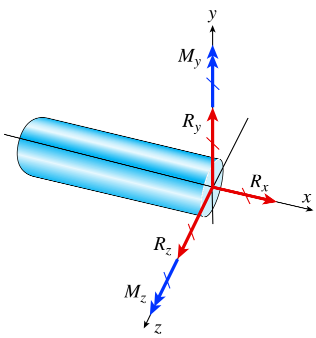
Thrust bearing has support reaction force (R_x, R_y, R_z) and reaction moment components (M_y, M_z)—no moment M_x about the thrust or rotation (x) axis.

dcwcreations/Shutterstock.com

Pillow block bearing

Journal bearing has no axial thrust reaction force ($R_x = 0$) in addition to ($M_x = 0$).





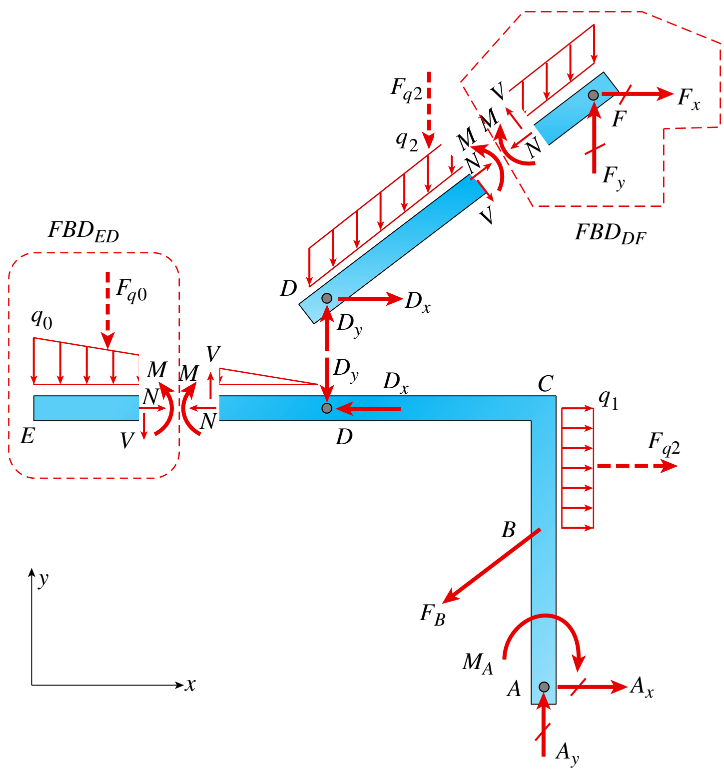
Internal Forces (Stress Resultants)

Mechanics of materials is concerned with study of the deformations of the members or elements that make up the overall deformable body. In order to compute the member deformations, first find the internal forces and moments (i.e., the internal stress resultants) at key points along the members of the overall structure. It is useful to create graphical displays of the internal axial force, torsional moment, transverse shear, and bending moment along the axis of each member of the body so that critical points or regions within the structure are readily identified. The first step is to make a section cut normal to the axis of each member so that a FBD can be drawn that displays the internal forces of interest. For example, Fig. 1-5 shows two cuts made through members *ED* and *DF* in the plane frame; the resulting FBDs now can be used to find N, V, and M in members *ED* and *DF* of the plane frame. Stress resultants N, V, and M are usually taken along and normal to the member under consideration (i.e., local or member axes are used), and a *deformation sign convention* (e.g., tension is positive, compression is negative) is employed in their solution.

The following examples review the application of the equations of static equilibrium to solve for external reactions and internal forces in truss, beam, circular shaft, and frame structures. First reaction forces are computed for a **truss structure** then member forces are found using the *method of joints*. Properly drawn FBDs are essential to the overall solution process. The second example involves static analysis of a **beam structure** to find reactions and internal forces at a particular section along the beam. In the third example, reactive and internal torsional moments in a **stepped shaft** are computed. And, finally, the fourth example presents the solution of a **plane frame structure**.

FIGURE 1-5

FBDs for internal stress resultants in *ED* and *DF*



Example 1-1

FIGURE 1-6

Plane truss model

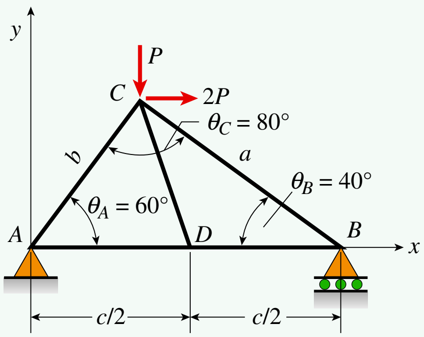
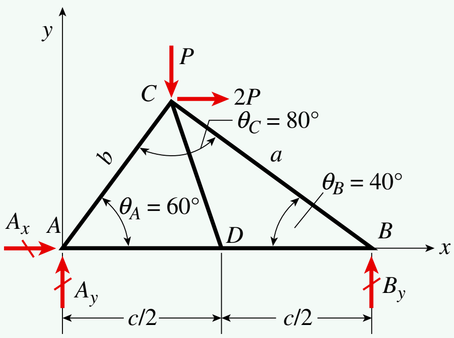


FIGURE 1-7

Free-body diagram of truss model



The plane truss shown in Fig. 1-6 has four joints and five members. Find support reactions at A and B and then use the methods of joints and sections to find all member forces. Let $P = 35$ kips and $c = 10$ ft.

Solution:

Use the following four-step problem-solving approach.

1. ***Conceptualize*** [*hypothesize, sketch*]: First sketch a free-body diagram of the entire truss model (Fig. 1-7). Only known applied forces at C and unknown reaction forces at A and B are shown and then used in an equilibrium analysis to find the reactions.

2. ***Categorize*** [*simplify, classify*]: Overall equilibrium requires that the force components in x and y directions and the moment about the z axis must sum to zero; this leads to reaction force components A_x, A_y, and B_y. The truss is statically determinate (unknowns: $m + r = 5 + 3 = 8$, knowns: $2j = 8$), so all member forces can be obtained using the method of joints. If only a few selected member forces are of interest, the method of sections can be used. Use a statics sign convention when computing external reactions and a deformation sign convention when solving for member forces.

3. ***Analyze*** [*evaluate; select relevant equations, carry out mathematical solution*]: First find the lengths of members AC and BC needed to compute distances to lines of action of forces.

   Law of sines to find member lengths *a* and *b*: Use known angles θ_A, θ_B, and θ_C and $c = 10$ ft to find lengths a and b:

$$b = c\frac{\sin(\theta_B)}{\sin(\theta_C)} = (10\text{ ft})\frac{\sin(40°)}{\sin(80°)} = 6.527\text{ ft},\quad a = c\frac{\sin(\theta_A)}{\sin(\theta_C)} = (10\text{ ft})\frac{\sin(60°)}{\sin(80°)} = 8.794\text{ ft}$$

   Check that computed lengths a and b give length c by using the law of cosines:

$$c = \sqrt{(6.527\text{ ft})^2 + (8.794\text{ ft})^2 - 2(6.527\text{ ft})(8.794\text{ ft})\cos(80°)} = 10\text{ ft}$$

   Support reactions: Using the truss model free-body diagram in Fig. 1-7, sum forces in x and y directions and moments about joint A:

$$\Sigma M_A = 0 \quad B_y = \frac{1}{c}\left[P(b\cos(\theta_A)) + 2P(b\sin(\theta_A))\right] = 51\text{ kips}$$

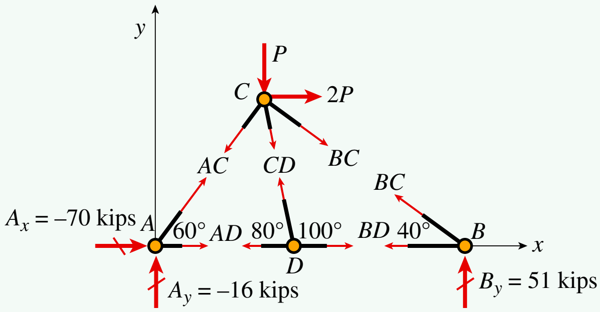
$$\Sigma F_x = 0 \quad A_x = -2P = -70\text{ kips}$$

$$\Sigma F_y = 0 \quad A_y = P - B_y = -16\text{ kips}$$

   Reaction force components A_x and A_y are both negative, so they act in the negative x and y directions, respectively, based on a statics sign convention.

FIGURE 1-8

Free-body diagram of pin at each truss joint



Member forces using method of joints: Begin by drawing free-body diagrams of the pin at each joint (Fig. 1-8). Use a deformation sign convention in which each member is assumed to be in tension (so the member force arrows act away from the two joints to which each member is connected). The forces are concurrent at each joint, so use force equilibrium at each location to find the unknown member forces.

First sum forces in the y direction at joint A to find member force AC, and then sum forces in the x direction to get member force AD:

$$\Sigma F_y = 0 \quad AC = \frac{-1}{\sin(60°)} A_y = 18.46 \text{ kips}$$

$$\Sigma F_x = 0 \quad AD = -A_x - AC\cos(60°) = 60.8 \text{ kips}$$

Summing forces at joint B gives member forces BC and BD as

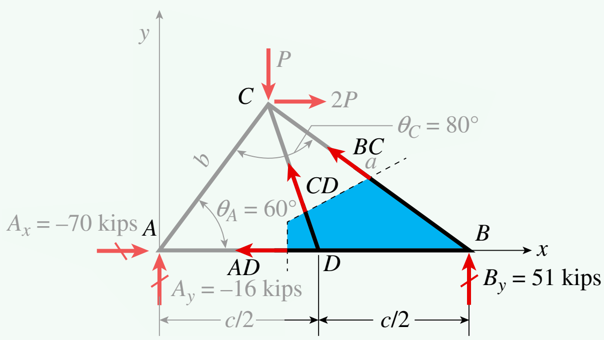
$$\Sigma F_y = 0 \quad BC = \frac{-1}{\sin(40°)} B_y = -79.3 \text{ kips}$$

$$\Sigma F_x = 0 \quad BD = -BC\cos(40°) = 60.8 \text{ kips}$$

The minus sign means that member BC is in compression, not in tension as assumed. Finally, observe that CD is a zero-force member because forces in the y direction must sum to zero at joint D.

FIGURE 1-9

Section cut leading to right-hand free-body diagram



Selected member forces using method of sections: An alternative approach is to make a section cut all the way through the structure to expose member forces of interest, such as AD, CD, and BC in Fig. 1-9. Summing moments about joint B confirms that the force in member CD is zero.

Summing moments about joint C (which is not on the free-body diagram but is a convenient point about which to sum moments because forces CD and BC act through joint C) confirms the solution for force AD as

$$\Sigma M_C = 0 \quad AD = \frac{1}{b\sin(60°)}\left[B_y(a)\cos(40°)\right] = 60.8 \text{ kips}$$

Finally, summing moments about A in Fig. 1-9 confirms member force BC:

$$\Sigma M_A = 0 \quad BC = \frac{1}{c\sin(40°)}[-B_y c] = -79.3\,\text{kips}$$

4. ***Finalize*** [*conclude; examine answer—Does it make sense? Are units correct? How does it compare to similar problem solutions?*]: There are $2j = 8$ equilibrium equations for the simple plane truss considered, and using the method of joints, these are obtained by applying $\Sigma F_x = 0$ and $\Sigma F_y = 0$ at each joint in succession. A computer solution of these simultaneous equations leads to the three reaction forces and five member forces. The method of sections is an efficient way to find selected member forces. A key step is the choice of an appropriate section cut, which isolates the member of interest and eliminates as many unknowns as possible. This is followed by construction of a free-body diagram for use in the static equilibrium analysis to compute the member force of interest. The methods of sections and joints were used, a common solution approach in plane and space truss analysis.

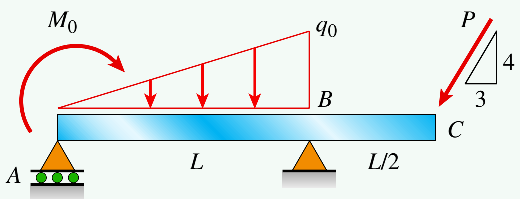
Example 1-2

A simple beam with an overhang is supported at points A and B (Fig. 1-10). A linearly varying distributed load of peak intensity $q_0 = 160$ N/m acts on span AB. Concentrated moment $M_0 = 380\ \text{N}\cdot\text{m}$ is applied at A, and an inclined concentrated load $P = 200$ N acts at C. The length of segment AB is $L = 4$ m, and the length of the overhang BC is 2 m.

Find support reactions at A and B and then calculate the axial force N, shear force V, and bending moment M at midspan of AB.

FIGURE 1-10

Beam with an overhang and uniform and concentrated loads



Solution:

Use the following four-step problem-solving approach.

1. ***Conceptualize***: Find the reaction forces A_y, B_x, and B_y using the FBD of the overall structure shown in Fig. 1-11. Internal axial force N, shear force V, and bending moment M at midspan of AB (Fig. 1-12) are obtained by cutting the beam at that location. Either the left-hand or right-hand free-body diagram in Fig. 1-12 may be used to find N, V, and M.

FIGURE 1-11

Free-body diagram of beam

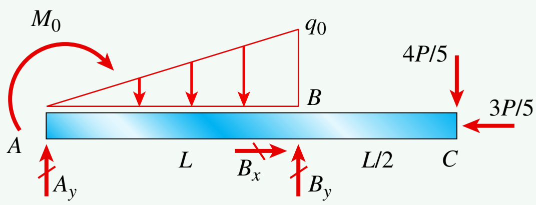
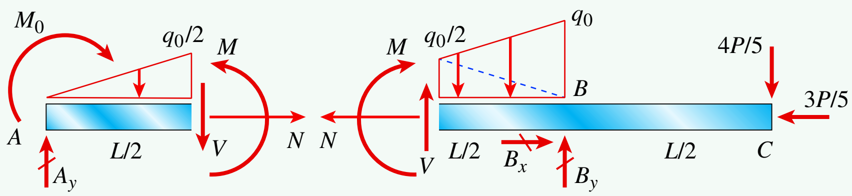


FIGURE 1-12

Left- and right-hand free-body diagrams from section cut at midspan of AB



2. ***Categorize*:** The free-body diagrams in Fig. 1-12 show internal axial force N, shear force V, and bending moment M in their assumed positive directions based on a deformation sign convention. Start by finding reaction forces A_y, B_x, and B_y then use either the left-hand or right-hand free-body diagram in Fig. 1-12 to find N, V, and M.

3. ***Analyze*:**

Solution for external reactions: Sum forces in the x direction to find reaction force component B_x. Next sum moments about A to find reaction component B_y. Finally, sum forces in the y direction to find reaction A_y. Use a statics sign convention in the solution as

$$\Sigma F_x = 0 \qquad B_x = \frac{3}{5}P = 120 \text{ N}$$

$$\Sigma M_A = 0 \qquad B_y = \frac{1}{L}\left[M_0 + \frac{1}{2}q_0 L\left(\frac{2L}{3}\right) + \frac{4}{5}P\left(L + \frac{L}{2}\right)\right] = 548 \text{ N}$$

$$\Sigma F_y = 0 \qquad A_y = -B_y + \frac{1}{2}q_0 L + \frac{4}{5}P = -68 \text{ N}(\downarrow)$$

Reaction A_y is negative, so in accordance with a statics sign convention, it acts downward. Reaction components B_x and B_y are positive, so they act in the directions shown in Figs. 1-11 and 1-12. The resultant reaction force at B is $B_{\text{res}} = \sqrt{B_x^2 + B_y^2} = 561$ N.

Solution for internal axial force N, shear force V, and moment M at midspan of AB: Using the left-hand free-body diagram in Fig. 1-12,

$$\Sigma F_x = 0 \quad N = 0 \quad \Sigma F_y = 0 \quad V = A_y - \frac{1}{2}\frac{q_0}{2}\frac{L}{2} = -148 \text{ N}(\uparrow)$$

$$\Sigma M = 0 \quad M = M_0 + A_y\frac{L}{2} - \frac{1}{2}\frac{q_0}{2}\frac{L}{2}\left[\frac{1}{3}\frac{L}{2}\right] = 190 \text{ N}\cdot\text{m}$$

Alternatively N, V, and M can be obtained if the right-hand free-body diagram is used (Fig. 1-12). Note that the trapezoidal distributed load segment is treated as two triangular loads when moments are summed to find internal moment M:

$$\Sigma F_x = 0 \quad N = B_x - \frac{3}{5}P = 0 \quad \Sigma F_y = 0 \quad V = -B_y + \frac{4}{5}P + \frac{1}{2}\left[\frac{q_0}{2} + q_0\right]\frac{L}{2} = -148\ \mathrm{N}(\downarrow)$$

$$\Sigma M = 0 \quad M = B_y\frac{L}{2} - \frac{4}{5}P\left[\frac{L}{2} + \frac{L}{2}\right] - \frac{1}{2}\frac{q_0}{2}\frac{L}{2}\left[\frac{1}{3}\frac{L}{2}\right] - \frac{1}{2}q_0\frac{L}{2}\left[\frac{2}{3}\frac{L}{2}\right] = 190\ \mathrm{N \cdot m}$$

The minus sign on internal shear force V shows that it acts opposite to that assumed in Fig. 1-12, as indicated by the arrows in the previous equations.

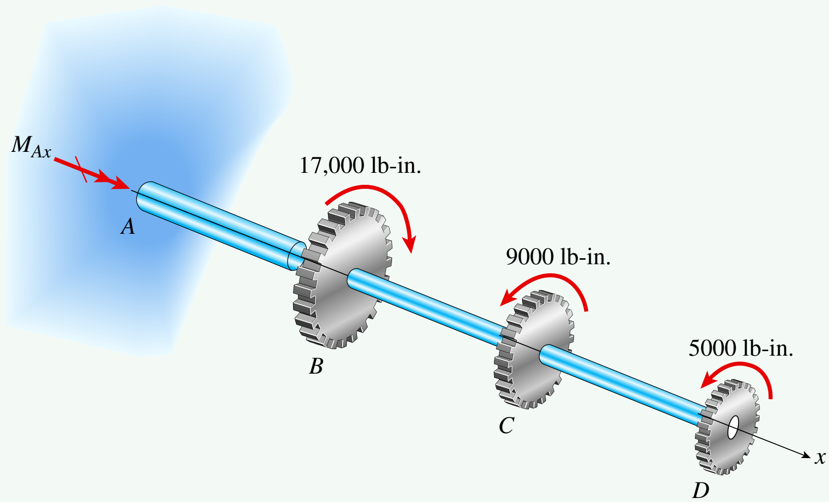
4. ***Finalize***: The results show that computed internal forces (N and V) and internal moment (M) can be determined using either the left- or right-hand free-body diagram. This applies for any section taken through the beam at any point along its length. Plots or diagrams that show the variation of N, V, and M over the length of the beam are very useful in the design of shafts and beams, because they readily show the critical regions of the beam where N, V, and M have maximum values.

Example 1-3

A stepped circular shaft is fixed at A and has three gears that transmit the torques shown in Fig. 1-13. Find the reaction torque M_{Ax} at A and then find the internal torsional moments in segments AB, BC, and CD. Use properly drawn free-body diagrams in your solution.

FIGURE 1-13

Stepped circular shaft subjected to concentrated torques



Solution:

Use the following four-step problem-solving approach to find internal torsional moments $T(x)$.

1. ***Conceptualize***: The cantilever shaft structure is stable and statically determinate. The solution for the reaction moment at $A(M_{Ax})$ must begin with a proper drawing of the FBD of the overall structure (Fig. 1-14). The FBD shows all applied and reactive torques. Separate FBDs showing internal torques T in each segment are obtained by cutting the shaft in regions AB, BC, and CD in succession and are given in Fig. 1-15(a–c). Each cut produces a left-hand and a right-hand free-body diagram.

FIGURE 1-14

Free-body diagram of shaft

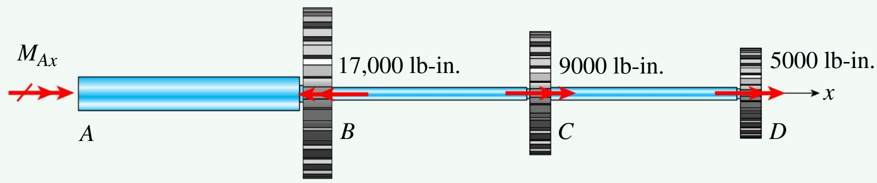
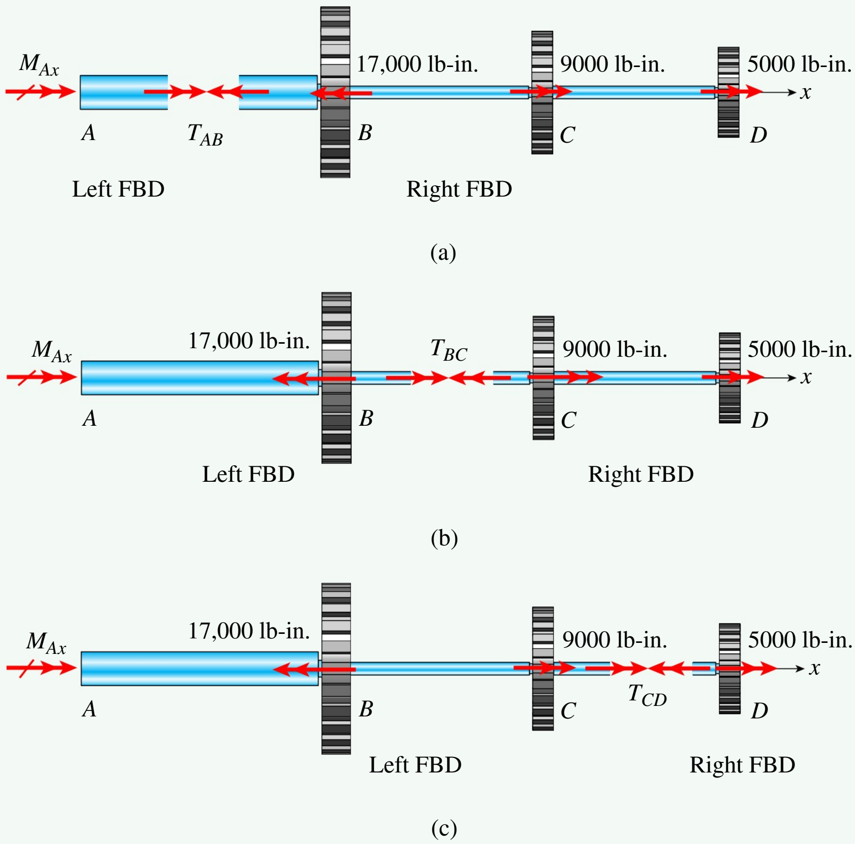


FIGURE 1-15

(a, b, c) Left and right free-body diagrams for each shaft segment



2. ***Categorize***: The shaft is subjected to applied torques that act along the centroidal axis of the shaft, so only internal torsional moment $T(x)$ is present at any section cut along the shaft. There is no distributed torque acting on this shaft, so the internal torsional moment T is constant within each segment.
3. ***Analyze***:

Solution for external reaction moment M_{Ax}:

Sum the moments about the x-axis to find the reaction moment M_{Ax}. This structure is statically determinate because there is one available equation from statics ($\Sigma M_x = 0$) and one reaction unknown (M_{Ax}). A statics sign convention is used [i.e., right-hand rule or counterclockwise (CCW) is positive].

$$M_{Ax} - 17{,}000 \text{ lb-in.} + 9000 \text{ lb-in.} + 5000 \text{ lb-in.} = 0$$
$$M_{Ax} = -(-17000 \text{ lb-in.} + 9000 \text{ lb-in.} + 5000 \text{ lb-in.})$$
$$= 3000 \text{ lb-in.}$$

The computed result for M_{Ax} is positive, so the reaction moment vector is in the positive x direction as assumed.

Solution for internal torsional moments T in each shaft segment:

Start with segment AB and use either FBD in Fig. 1-15a to find:

Left FBD:
$$T_{AB} = -M_{Ax} = -3000 \text{ lb-in.}$$

Right FBD:
$$T_{AB} = -17{,}000 \text{ lb-in.} + 9000 \text{ lb-in.} + 5000 \text{ lb-in.} = -3000 \text{ lb-in.}$$

Next consider segment BC. Summing moments about the x axis in Fig. 1-15b gives

Left FBD:
$$T_{BC} = -M_{Ax} + 17{,}000 \text{ lb-in.} = 14{,}000 \text{ lb-in.}$$

Right FBD:
$$T_{BC} = 9000 \text{ lb-in.} + 5000 \text{ lb-in.} = 14{,}000 \text{ lb-in.}$$

Finally, moment equilibrium about the x axis leads to a solution for the internal torsional moment in segment CD:

Left FBD:
$$T_{CD} = -M_{Ax} + 17{,}000 \text{ lb-in.} - 9000 \text{ lb-in.} = 5000 \text{ lb-in.}$$

Right FBD:
$$T_{CD} = 5000 \text{ lb-in.}$$

In each segment, the internal torsional moments computed using either the left or right FBDs are the same.

4. ***Finalize***: Segment BC has the maximum positive internal torsional moment, and segment AB has the maximum negative torsional moment. This is important information for the designer of the shaft. Properly drawn free-body diagrams are essential to a correct solution. Either the left or right free-body diagram can be used to find the internal torque at any section.

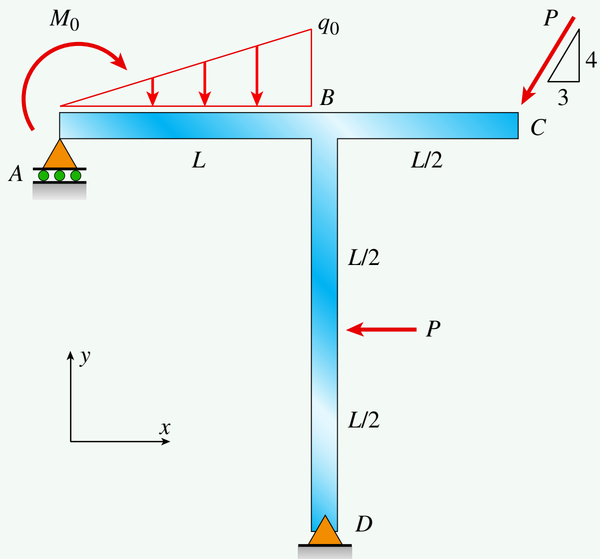
Example 1-4

A plane frame with an overhang is supported at points A and D (Fig. 1-16). (This is the beam of Example 1-2 to which column BD has been added.) A linearly varying distributed load of peak intensity $q_0 = 160$ N/m acts on span AB. Concentrated moment $M_0 = 380$ N · m is applied at A, and an inclined concentrated load $P = 200$ N acts at C. Force P also acts at mid-height of column BD. The lengths of segments AB and BD are $L = 4$ m, and the length of the overhang BC is 2 m.

Find support reactions at A and D and then calculate the axial force N, shear force V, and bending moment M at the top of column BD.

FIGURE 1-16

Plane frame with an overhang and uniform and concentrated loads



Solution:

The four-step problem-solving approach for this plane frame follows the procedures presented for the beam in Example 1-2.

1. ***Conceptualize***: Find the reaction forces A_y, D_x, and D_y using the FBD of the overall structure shown in Fig. 1-17. Internal axial force N, shear force V, and bending moment M at the top of column BD (Fig. 1-18) are obtained by cutting the column at that location. View column BD with joint D on your left and B on your right to establish the assumed positive directions of N, V, and M on either side of the cut section, as shown in Fig. 1-18.
2. ***Categorize***: First find reaction forces A_y, D_x, and D_y. Then use either the upper or lower free-body diagram in Fig. 1-18 to find N, V, and M. The free-body diagrams in Fig. 1-18 show internal axial force N, shear force V, and bending moment M in their assumed positive directions based on a deformation sign convention.

FIGURE 1-17

Free-body diagram of plane frame

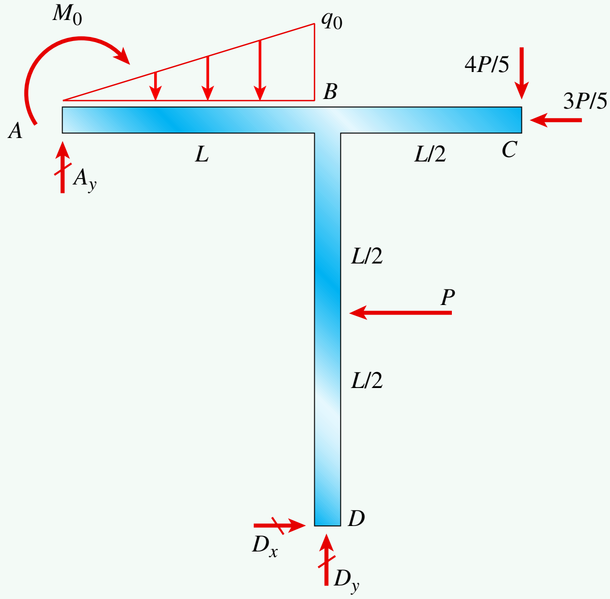
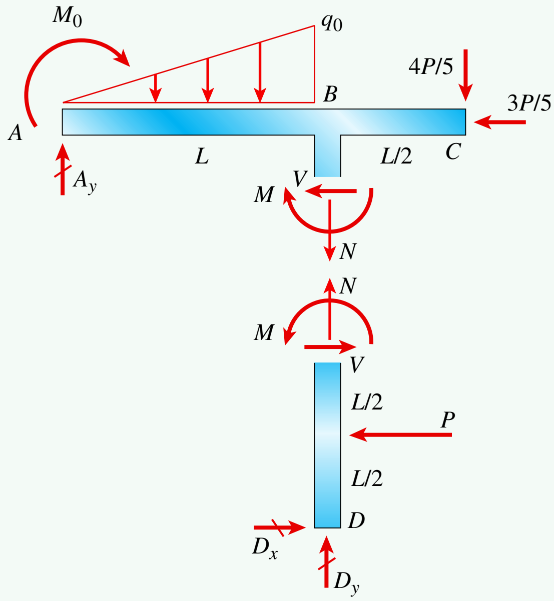


FIGURE 1-18

Upper and lower free-body diagrams from section cut at top of *BD*



3. ***Analyze*:**

Solution for external reactions: Sum forces in the x direction to find reaction force component D_x. Next, sum moments about D to find reaction component A_y.

Finally, sum forces in the y direction to find reaction D_y. Use a statics sign convention in the solution as follows:

$$\Sigma F_x = 0 \quad D_x = \frac{3}{5}P + P = 320 \text{ N}$$

$$\Sigma M_D = 0 \quad A_y = \frac{1}{L}\left[-M_0 + \frac{1}{2}q_0 L\left[\frac{L}{3}\right] + P\frac{1}{2} - \frac{4}{5}P\left[\frac{L}{2}\right] + \frac{3}{5}PL\right] = 152 \text{ N}$$

$$\Sigma F_y = 0 \quad D_y = -A_y + \frac{1}{2}q_0 L + \frac{4}{5}P = 328 \text{ N}$$

All reaction force components are positive, so they act in the directions shown in Figs. 1-17 and 1-18. The resultant reaction force at D is $D_{\text{res}} = \sqrt{D_x^2 + D_y^2} = 458$ N.

Solution for internal axial force *N*, shear force *V*, and moment *M* at top of column *BD*: Using the lower free-body diagram in Fig. 1-18,

$$\Sigma F_y = 0 \quad N = -D_y = -328 \text{ N} \quad \Sigma F_x = 0 \quad V = -D_x + P = -120 \text{ N}$$

$$\Sigma M = 0 \quad M = -D_x L + P\frac{L}{2} = -880 \text{ N} \cdot \text{m}$$

Alternatively, the upper free-body diagram can be used to compute N, V, and M (Fig. 1-18):

$$\Sigma F_y = 0 \quad N = A_y - \frac{1}{2}q_0 L - \frac{4}{5}P = -328 \text{ N} \quad \Sigma F_x = 0 \quad V = -\frac{3}{5}P = -120 \text{ N}$$

$$\Sigma M = 0 \quad M = -M_0 - A_y L - \frac{4}{5}P\frac{L}{2} + \frac{1}{2}q_0 L\left[\frac{L}{3}\right] = -880 \text{ N}\cdot\text{m}$$

The minus signs on internal axial force N, shear force V, and moment M indicate that all three quantities act opposite to directions assumed in Fig. 1-18.

4. ***Finalize***: Either the lower or upper free-body diagram can be used to find internal forces (N and V) and internal moment (M) at the top of column BD. Section forces and moments at any other location on the frame are found using the same approach. A properly drawn free-body diagram is an important first step in the solution.

1.4 Normal Stress and Strain

Now that reactive and internal forces are known from statics, it is time to examine internal actions more closely for a deformable body of interest. The most fundamental concepts in mechanics of materials are **stress** and **strain**. These concepts can be illustrated in their most elementary form by considering a prismatic bar subjected to axial forces. A **prismatic bar** is a straight structural member having the same cross section throughout its length, and an **axial force** is a load directed along the axis of the member, resulting in either tension or compression

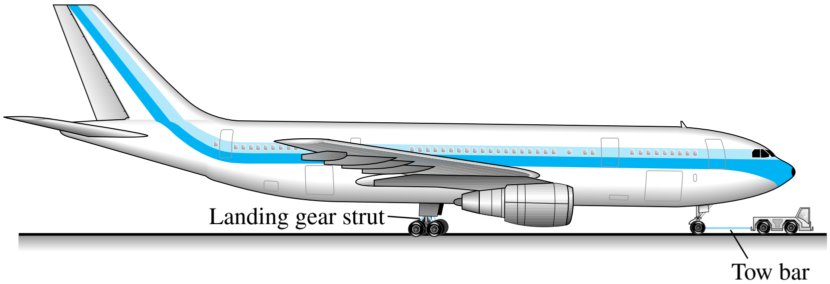


FIGURE 1-19

Structural members subjected to axial loads (the tow bar is in tension and the landing gear strut is in compression)

in the bar. Examples are shown in Fig. 1-19, where the tow bar is a prismatic member in tension and the landing gear strut is a member in compression. Other examples include the members of a bridge truss, connecting rods in automobile engines, spokes of bicycle wheels, columns in buildings, and wing struts in small airplanes.

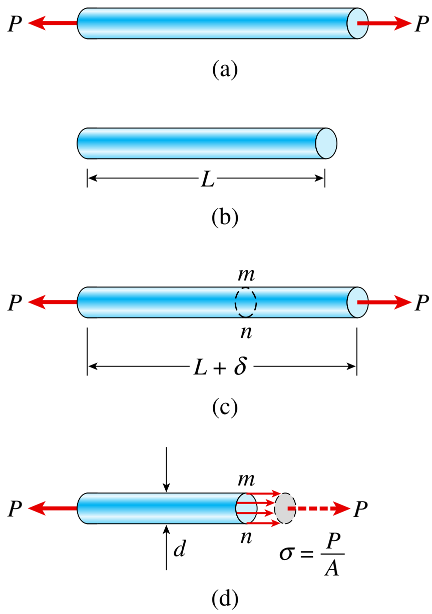
For discussion purposes, consider the tow bar of Fig. 1-19 and isolate a segment of it as a free body (Fig. 1-20a). When drawing this free-body diagram, disregard the weight of the bar itself and assume that the only active forces are the axial forces P at the ends. Next, consider two views of the bar: the first showing the same bar *before* the loads are applied (Fig. 1-20b) and the second showing it *after* the loads are applied (Fig. 1-20c). The original length of the bar is denoted by the letter L, and the increase in length due to the loads is denoted by the Greek letter δ (delta).

The internal actions in the bar are exposed by making an imaginary cut through the bar at section mn (Fig. 1-20c). Because this section is taken perpendicular to the longitudinal axis of the bar, it is called a **cross section**.

Now isolate the part of the bar to the left of cross section mn as a free body (Fig. 1-20d). At the right-hand end of this free body (section mn) you can see the action of the removed part of the bar (that is, the part to the right of section mn) upon the part that remains. This action consists of continuously distributed *stresses* acting over the entire cross section, and the axial force P acting at the cross section is the *resultant* of those stresses. (The resultant force is shown with a dashed line in Fig. 1-20d.)

FIGURE 1-20

Prismatic bar in tension: (a) free-body diagram of a segment of the bar, (b) segment of the bar before loading, (c) segment of the bar after loading, and (d) normal stresses in the bar



Stress has units of force per unit area and is denoted by the Greek letter σ (sigma). In general, the stresses σ acting on a plane surface may be uniform throughout the area or may vary in intensity from one point to another. Assume that the stresses acting on cross section mn (Fig. 1-20d) are *uniformly distributed* over the area. Then the resultant of those stresses must be equal to the magnitude of the stress times the cross-sectional area A of the bar, that is, $P = \sigma A$. Therefore, you can obtain the following expression for the magnitude of the stresses:

$$\sigma = \frac{P}{A} \tag{1-6}$$

This equation gives the intensity of uniform stress in an axially loaded, prismatic bar of arbitrary cross-sectional shape.

When the bar is stretched by the forces P, the stresses are **tensile stresses**; if the forces are reversed in direction, causing the bar to be compressed, they are **compressive stresses**. **Normal stresses** stresses act in a direction perpendicular to the cut surface. Normal stresses may be either tensile or compressive. **Shear stresses** discussed in Section 1.8 act parallel to the surface.

It is customary to define tensile stresses as positive and compressive stresses as negative. Because the normal stress σ is obtained by dividing the axial force by the cross-sectional area, it has units of force per unit of area. Stress is customarily expressed in pounds per square inch (psi) or kips per square inch (ksi).[1] For instance, suppose that the bar of Fig. 1-20 has a diameter d of 2.0 inches, and the load P has a magnitude of 6 kips. Then the stress in the bar is

$$\sigma = \frac{P}{A} = \frac{P}{\pi d^2/4} = \frac{6\text{k}}{\pi(2.0 \text{ in.})^2/4} = 1.91 \text{ ksi (or 1910 psi)}$$

In this example the stress is tensile, or positive.

When SI units are used, force is expressed in newtons (N) and area in square meters (m^2). Consequently, stress has units of newtons per square meter (N/m^2), which is equal to a pascal (Pa). However, the pascal is such a small unit of stress that it is necessary to work with large multiples, usually the megapascal (MPa).

A pascal is so small that it takes almost 7000 pascals to make 1 psi.[2] The stress in the bar described in the preceding example (1.91 ksi) converts to 13.2 MPa, which is 13.2×10^6 pascals. Although it is not recommended in SI, you will sometimes find stress given in newtons per square millimeter (N/mm^2), which is a unit equal to the megapascal (MPa).

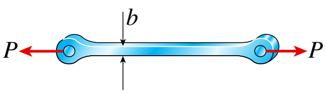
Limitations

The equation $\sigma = P/A$ is valid only if the stress is uniformly distributed over the cross section of the bar. This condition is realized if the axial force P acts through the centroid of the cross-sectional area, as demonstrated later in this section. However, in this book (as in common practice), it is understood that axial forces are applied at the centroids of the cross sections unless specifically stated otherwise.

The uniform stress condition pictured in Fig. 1-20d exists throughout the length of the bar except near the ends. The stress distribution at the end of a bar depends upon how the load P is transmitted to the bar. If the load is distributed uniformly over the end, the stress pattern at the end is the same as everywhere else. However, it is more likely that the load is transmitted through a pin or a bolt, producing high localized stresses called *stress concentrations*.

FIGURE 1-21

Steel eyebar subjected to tensile loads P



One possibility is illustrated by the eyebar shown in Fig. 1-21. In this instance, the loads P are transmitted to the bar by pins that pass through the holes (or eyes) at the ends of the bar. Thus, the forces shown in the figure are actually the resultants of bearing pressures between the pins and the eyebar,

[1]One kip, or kilopound, equals 1000 lb.

[2]Conversion factors between USCS units and SI units are listed in Table A-5, Appendix A.

and the stress distribution around the holes is quite complex. However, as you move away from the ends and toward the middle of the bar, the stress distribution gradually approaches the uniform distribution pictured in Fig. 1-20d.

As a practical rule, the formula $\sigma = P/A$ may be used with good accuracy at any point within a prismatic bar that is at least as far away from the stress concentration as the largest lateral dimension of the bar. In other words, the stress distribution in the steel eyebar of Fig. 1-21 is uniform at distances b or greater from the enlarged ends where b is the width of the bar, and the stress distribution in the prismatic bar of Fig. 1-20 is uniform at distances d or greater from the ends where d is the diameter of the bar (Fig. 1-20d). Of course, even when the stress is *not* uniformly distributed, the equation $\sigma = P/A$ may still be useful because it gives the *average* normal stress on the cross section.

Normal Strain

As already observed, a straight bar changes in length when loaded axially, becoming longer when in tension and shorter when in compression. For instance, consider again the prismatic bar of Fig. 1-20. The elongation δ of this bar (Fig. 1-20c) is the cumulative result of the stretching of all elements of the material throughout the volume of the bar. Assume that the material is the same everywhere in the bar. Then half of the bar (length $L/2$) has an elongation equal to $\delta/2$, and one-fourth of the bar has an elongation equal to $\delta/4$.

In general, the elongation of a segment is equal to its length divided by the total length L and multiplied by the total elongation δ. Therefore, a unit length of the bar has an elongation equal to $1/L \times \delta$. This quantity is called the *elongation per unit length*, or **strain**, and is denoted by the Greek letter ε (epsilon). Strain is given by

$$\varepsilon = \frac{\delta}{L} \tag{1-7}$$

If the bar is in tension, the strain is called a **tensile strain**, representing an elongation or stretching of the material. If the bar is in compression, the strain is a **compressive strain** and the bar shortens. Tensile strain is usually taken as positive and compressive strain as negative. The strain ε is called a **normal strain** because it is associated with normal stresses.

Because normal strain is the ratio of two lengths, it is a **dimensionless quantity**, that is, it has no units. Therefore, strain is expressed simply as a number that is independent of any system of units. Numerical values of strain are usually very small, because bars made of structural materials undergo only small changes in length when loaded.

As an example, consider a steel bar having length L equal to 2.0 m. When heavily loaded in tension, this bar might elongate by 1.4 mm, which means that the strain is

$$\varepsilon = \frac{\delta}{L} = \frac{1.4\text{ mm}}{2.0\text{ m}} = 0.0007 = 700 \times 10^{-6}$$

In practice, the original units of δ and L are sometimes attached to the strain itself, and then the strain is recorded in forms such as mm/m, μm/m, and in./in. For instance, the strain ε in the preceding example could be given as 700 μm/m or 700×10^{-6} in./in. Strain is sometimes expressed as a percent, especially when the strains are large. (In the preceding example, the strain is 0.07%.)

Uniaxial Stress and Strain

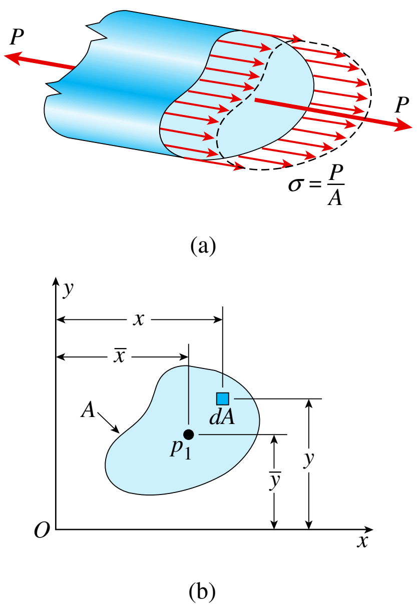
The definitions of normal stress and normal strain are based upon purely static and geometric considerations, which means that Eqs. (1-6) and (1-7) can be used for loads of any magnitude and for any material. The principal requirement is that the deformation of the bar be uniform throughout its volume, which in turn requires that the bar be prismatic, the loads act through the centroids of the cross sections, and the material be **homogeneous** (that is, the same throughout all parts of the bar). The resulting state of stress and strain is called **uniaxial stress and strain** (although lateral strain exists, as discussed later in Section 1.7).

Line of Action of the Axial Forces for a Uniform Stress Distribution

Throughout the preceding discussion of stress and strain in a prismatic bar, the normal stress σ was assumed to be distributed uniformly over the cross section. Note that this condition is met if the line of action of the axial forces is through the centroid of the cross-sectional area.

FIGURE 1-22

Uniform stress distribution in a prismatic bar: (a) axial forces P, and (b) cross section of the bar



Consider a prismatic bar of arbitrary cross-sectional shape subjected to axial forces P that produce uniformly distributed stresses σ (Fig. 1-22a). Let p_1 represent the point in the cross section where the line of action of the forces intersects the cross section (Fig. 1-22b). Construct a set of x-y axes in the plane of the cross section and denote the coordinates of point p_1 by $\bar{x}$ and $\bar{y}$. To determine these coordinates, observe that the moments M_x and M_y of the force P about the x and y axes, respectively, must be equal to the corresponding moments of the uniformly distributed stresses.

The moments of the force P are

$$M_x = P\bar{y} \quad M_y = -P\bar{x} \tag{1-8a,b}$$

in which a moment is considered positive when its vector (using the right-hand rule) acts in the positive direction of the corresponding axis.

The moments of the distributed stresses are obtained by integrating over the cross-sectional area A. The differential force acting on an element of area dA (Fig. 1-22b) is equal to σdA. The moments of this elemental force about the x and y axes are $\sigma y dA$ and $-\sigma x dA$, respectively, in which x and y denote the coordinates of the element dA. The total moments are obtained by integrating over the cross-sectional area:

$$M_x = \int \sigma y dA \quad M_y = -\int \sigma x dA \tag{1-8c,d}$$

These expressions give the moments produced by the stresses σ.

Next, equate the moments M_x and M_y obtained from the force P (Eqs. 1-8a and b) to the moments obtained from the distributed stresses (Eqs. 1-8c and d):

$$P\overline{y} = \int \sigma y dA \quad P\overline{x} = -\int \sigma x dA$$

Because the stresses σ are uniformly distributed, they are constant over the cross-sectional area A and can be placed outside the integral signs. Also, σ is equal to P/A. Therefore, you can obtain the following formulas for the coordinates of point p_1:

$$\overline{y} = \frac{\int y dA}{A} \quad \overline{x} = \frac{\int x dA}{A} \qquad \textbf{(1-9a,b)}$$

These equations are the same as the equations defining the coordinates of the centroid of an area (see Eqs. D-3a, b in Appendix D). Therefore, the important conclusion here is:

In order to have uniform tension or compression in a prismatic bar, the axial force must act through the centroid of the cross-sectional area.

Always assume that these conditions are met unless it is specifically stated otherwise.

The following examples illustrate the calculation of stresses and strains in prismatic bars. In the first example, disregard the weight of the bar and in the second, include it. (It is customary when solving textbook problems to omit the weight of the structure unless specifically instructed to include it.)

Example 1-5

An elastic spring rests on a base plate that is on top of rigid tube B (see Fig. 1-23). The spring is enclosed by rigid tube A but is longer than tube A by an amount s. Force P is then applied to a cap plate to compress the spring. Both tubes have outer diameter d_O, but the inner diameters are d_A and d_B for tubes A and B, respectively. Assume that spring stiffness $k = 24$ kips/in., $d_O = 3$ in., $d_A = 2.5$ in., $d_B = 2.25$ in., and $s = 0.125$ in.

(a) If applied load $P = 2500$ lb, what are the axial normal stresses in tubes A and B?

(b) Repeat part (a) if $P = 5000$ lb.

(c) What is P if the normal stress in tube A is 800 psi? What is the associated stress in tube B?

FIGURE 1-23

Elastic spring inside rigid tubes

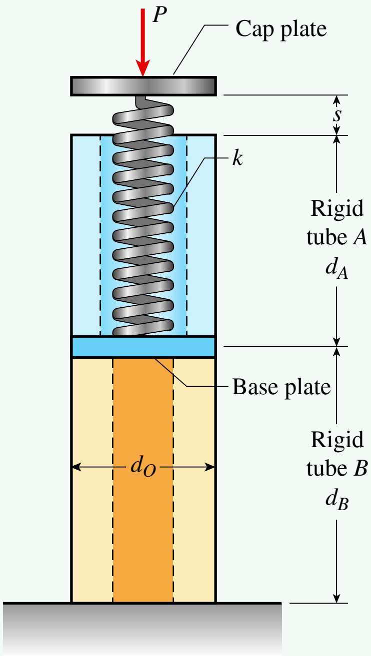
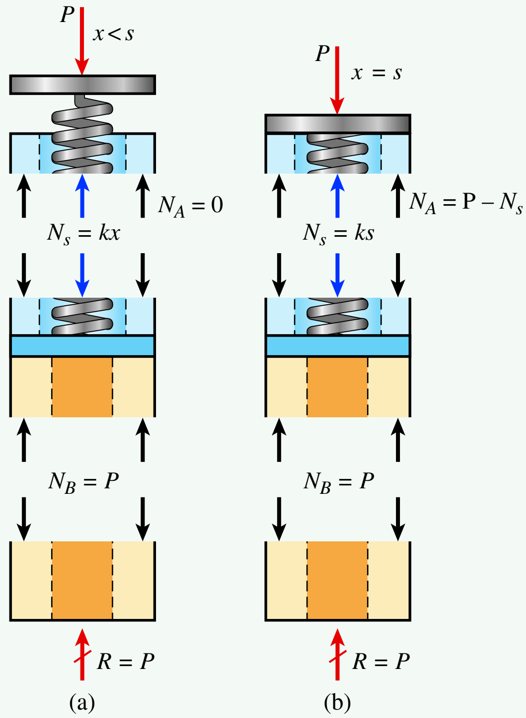


FIGURE 1-24

(a, b) Free-body diagrams ($x < s$, $x = s$)



Solution:

Use the following four-step problem-solving approach.

1. ***Conceptualize***: The two possible states of the assemblage are shown in the free-body diagrams in Fig. 1-24. In Fig. 1-24a, an upper-section cut through both the spring and tube A creates an upper free-body diagram that reveals a spring force $(k)(x)$ for the case of downward cap displacement x that is less than gap width s. In Fig. 1-24b, cap displacement x is equal to gap width s, so the spring force now equals $(k)(s)$. Figure 1-24 also shows lower free-body diagrams for both cases in which a section cut through tube B shows that the internal compressive force in tube B is equal to applied load P. (Internal forces in tubes A and B are shown as two arrows, one at each tube wall, indicating that N_A and N_B are actually uniformly distributed forces acting on the circular cross section of each tube.)
2. ***Categorize***: The force P required to close gap s is $(k)(s)$. This is also the maximum force that can be developed in the spring. If applied force P is too small to close the gap s, force P will be transferred to the base plate and into rigid tube B; tube A will be unaffected by the load. However, if force P is large enough to compress the spring to close the gap s, the spring and tube A will

share the load P applied to the cap plate and together will transfer it to tube B through the base plate. In summary, the free-body diagrams in Fig. 1-24 show that, if the spring is compressed by load P an amount x, the compressive internal forces in the spring and the two tubes are

$$N_s = P = kx,\ N_A = 0,\ N_B = P \qquad \text{for} \quad x < s$$
$$N_s = ks,\ N_A = P - ks,\ N_B = N_s + N_A = P \qquad \text{for} \quad x = s$$

3. ***Analyze***:

Force P required to close gap s: The gap closes when force P is equal to ks

$$P = ks = \left(24\frac{\text{kips}}{\text{in.}}\right)(0.125\text{ in.}) = 3000\text{ lb}$$

Tube stresses for applied load $P = 2500$ lb: The cap will displace downward a distance $x = P/k = 0.104$ in. ($< s$), so tube internal forces are $N_A = 0$ and $N_B = P$. Tube cross-sectional areas are

$$A_A = \frac{\pi}{4}\left(d_O^2 - d_A^2\right) = \frac{\pi}{4}\left(3^2 - 2.5^2\right)\text{in}^2 = 2.16\text{ in}^2$$
$$A_B = \frac{\pi}{4}\left(d_O^2 - d_B^2\right) = \frac{\pi}{4}\left(3^2 - 2.25^2\right)\text{in}^2 = 3.093\text{ in}^2$$

The resulting axial normal compressive stresses in tubes A and B are

$$\sigma_A = \frac{N_A}{A_A} = 0 \quad \sigma_B = \frac{N_B}{A_B} = 808\text{ psi}$$

Tube stresses for applied load $P = 5000$ lb: Cap downward displacement is now $x = P/k = 0.208$ in. ($> s$), so tube internal forces are $N_A = P - (k)(s) = (5000 - 3000)$ lb $= 2000$ lb and $N_B = P$. The normal stresses in tubes A and B are now:

$$\sigma_A = \frac{2000\text{ lb}}{2.16\text{ in}^2} = 926\text{ psi} \quad \sigma_B = \frac{5000\text{ lb}}{3.093\text{ in}^2} = 1617\text{ psi}$$

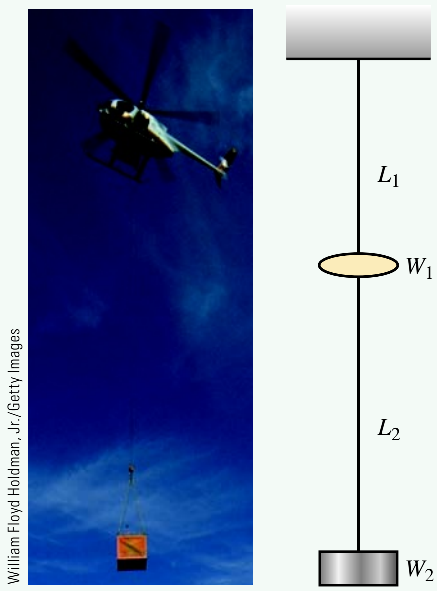
Applied load P if stress in tube A is 800 psi: Force P must exceed $(k)(s) = 3000$ lb for the gap to close, leading to a force in tube A and a normal stress of $\sigma_A = 800$ psi. The normal compressive force in tube A is $N_A = (\sigma_A)(A_A) = 1728$ lb. It follows that applied force P is now $P = N_A + ks = 1728\text{ lb} + 3000\text{ lb} = 4728$ lb. Internal force $N_B = P$, so the normal compressive stress in tube B is now $\sigma_B = \dfrac{4728\text{ lb}}{3.093\text{ in}^2} = 1529$ psi.

4. ***Finalize***: If tube A is elastic instead of rigid as assumed here, tube A can be modeled as another spring that is parallel to the spring it encloses. Now a more advanced analysis procedure will be needed to find tube force N_A for the case of $P > (k)(s)$. Force N_A is no longer equal to $P - (k)(s)$, and downward displacement x can be larger than s.

Example 1-6

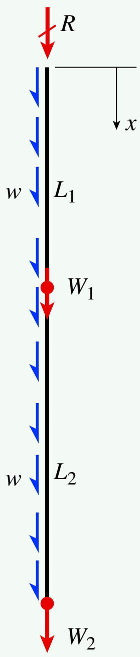
An antenna and receiver are suspended on a steel wire from a helicopter to measure the effects of wind turbines on a local radar installation (see Fig. 1-25). Obtain a formula for the maximum stress in the wire, taking into account the weight of the wire itself. Calculate the maximum stress in the wire in MPa using the following numerical properties: $L_1 = 6$ m, $L_2 = 5$ m, $d = 9.5$ mm; antenna weight is $W_1 = 380$ N; receiver weight is $W_2 = 700$ N. Note that the weight density γ of steel is 77.0 kN/m^3 (from Table I-1 in Appendix I).

FIGURE 1-25
(a, b) Instruments suspended on wire from helicopter



(a) (b)

FIGURE 1-26
Free-body diagram



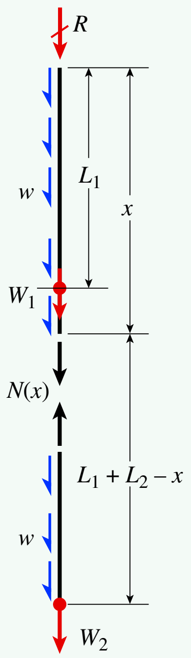
Solution:

Use the following four-step problem-solving approach.

1. ***Conceptualize:*** A free-body diagram of the suspended instrument package is shown in Fig. 1-26. The antenna (W_1) and receiver (W_2) weights are concentrated forces at specified locations along the wire; the weight of the wire is a uniformly distributed axial force expressed as $w(x) = \gamma A$, where A is the cross-sectional area of the wire. Cutting the wire at some point x leads to upper and lower free-body diagrams (Fig. 1-27); either can be used to find the internal axial force $N(x)$ at the location of the cut section. The internal axial force in the wire is a maximum at the point at which it is attached to the helicopter ($x = 0$).
2. ***Categorize:*** Start by solving for the reaction force R at the top of the wire and then cut the wire a short distance below the support to find N_{max}. The wire is prismatic, so simply divide N_{max} by cross-sectional area A to find the maximum axial normal stress σ_{max}.

FIGURE 1-27

Axial force $N(x)$ in wire



3. ***Analyze***:

Reaction force *R*: Use the free-body diagram in Fig. 1-26 to obtain

$$R = -[W_1 + W_2 + w(L_1 + L_2)] = -[W_1 + W_2 + \gamma A(L_1 + L_2)]$$

The minus sign indicates that reaction force R acts in the $(-x)$ direction, or upward in Figs. 1-26 and 1-27.

Internal axial forces $N(x)$ in hanging wire: The internal axial force in the wire varies over the length of the wire. Cutting through the wire in upper and lower segments (the lower segment is cut in Fig. 1-27) gives

$$N(x) = W_1 + W_2 + w(L_1 + L_2 - x) \qquad 0 \le x \le L_1$$
$$N(x) = W_2 + w(L_1 + L_2 - x) \qquad L_1 \le x \le L_1 + L_2$$

Internal force $N(x)$ is shown as a pair of forces acting away from the cut section in accordance with a deformation sign convention in which the wire is initially assumed to be in tension and that tension is positive. The maximum force in the wire is at $x = 0$: $N_{max} = N(0) = W_1 + W_2 + w(L_1 + L_2)$.

Formula for maximum stress in the wire: The cross-sectional area A of the wire is constant, so dividing N_{max} by A leads to a formula for maximum stress in the wire:

$$\sigma_{max} = \frac{N_{max}}{A} = \frac{W_1 + W_2 + w(L_1 + L_2)}{A} = \frac{W_1 + W_2}{A} + \gamma(L_1 + L_2)$$

Numerical calculations: The cross-sectional area of the wire is

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(9.5\text{ mm})^2 = 70.88\text{ mm}^2$$

Solving for maximum normal stress gives

$$\sigma_{max} = \frac{W_1 + W_2}{A} + \gamma(L_1 + L_2) = \frac{380\text{ N} + 700\text{ N}}{70.88\text{ mm}^2} + 77.0\frac{\text{kN}}{\text{m}^3}(6\text{ m} + 5\text{ m}) = 16.08\text{ MPa}$$

4. ***Finalize***: If the weight of the wire is ignored, the maximum normal stress is reduced to 15.24 MPa, which is a decrease of more than 5%. Although the stresses are low here, eliminating the self-weight of the wire from the stress calculation is not recommended.

1.5 Mechanical Properties of Materials

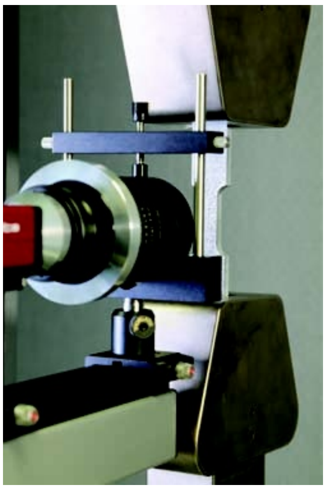
The design of machines and structures so that they function properly requires an understanding of the **mechanical behavior** of the materials being used. Ordinarily, the only way to determine how materials behave when they are subjected to loads is to perform experiments in the laboratory. The usual procedure is to place small specimens of the material in testing machines, apply the loads, and then measure the resulting deformations (such as changes in length and changes in diameter).

FIGURE 1-28

Tensile-test machine with automatic data-processing system (Courtesy of MTS Systems Corporation)

FIGURE 1-29

Typical tensile-test specimen with extensometer attached (Courtesy of MTS Systems Corporation)



A typical **tensile-test machine** is shown in Fig. 1-28. The test specimen is installed between the two large grips of the testing machine and then loaded in tension. Measuring devices record the deformations, and the automatic control and data-processing systems tabulate and graph the results.

A more detailed view of a **tensile-test specimen** is shown in Fig. 1-29. The ends of the circular specimen are enlarged where they fit in the grips so that failure will not occur near the grips themselves. A failure at the ends would not produce the desired information about the material because the stress distribution near the grips is not uniform, as explained in Section 1.4. In a properly designed specimen, failure occurs in the prismatic portion of the specimen where the stress distribution is uniform and the bar is subjected only to pure tension. This situation is shown in Fig. 1-29, where the steel specimen has just fractured under load. The device at the right, which is attached by two arms to the specimen, is an **extensometer** that measures the elongation during loading.

In order that test results stay comparable, the dimensions of test specimens and the methods of applying loads must be standardized. One of the major standards organizations in the United States is the American Society for Testing and Materials (ASTM), which is a technical society that publishes specifications and standards for materials and testing. Other standardizing organizations include the American Standards Association (ASA) and the National Institute of Standards and Technology (NIST). Similar organizations exist in other countries.

The ASTM standard tension specimen has a diameter of 0.505 in. and a **gage length** of 2.0 in. between the gage marks, which are the points where the extensometer arms are attached to the specimen (see Fig. 1-29). As the specimen is pulled, the axial load is measured and recorded, either automatically or by reading from a dial. The elongation over the gage length is measured simultaneously, either by mechanical gages of the kind shown in Fig. 1-29 or by electrical-resistance strain gages.

In a **static test**, the load is applied slowly and the precise *rate* of loading is not of interest because it does not affect the behavior of the specimen. However, in a **dynamic test**, the load is applied rapidly and sometimes in a cyclical manner. Since the nature of a dynamic load affects the properties of the materials, the rate of loading must be measured.

Compression tests of metals are customarily made on small specimens in the shape of cubes or circular cylinders. For instance, cubes may be 2.0 in. on a side, and cylinders may have diameters of 1 in. and lengths from 1 to 12 in. Both the load applied by the machine and the shortening of the specimen may be measured. The shortening should be measured over a gage length that is less than the total length of the specimen in order to eliminate end effects.

Concrete is tested in compression on important construction projects to ensure that the required strength has been obtained. One type of concrete test specimen is 6 in. in diameter, 12 in. in length, and 28 days old (the age of concrete is important because concrete gains strength as it cures). Similar but somewhat smaller specimens are used when performing compression tests of rock (see Fig. 1-30 on the next page).

Stress-Strain Diagrams

Test results generally depend upon the dimensions of the specimen being tested. Since it is unlikely that you will design a structure having parts that are the same size as the test specimens, you need to express the test results in a form that can be applied to members of any size. A simple way to achieve this objective is to convert the test results to stresses and strains.

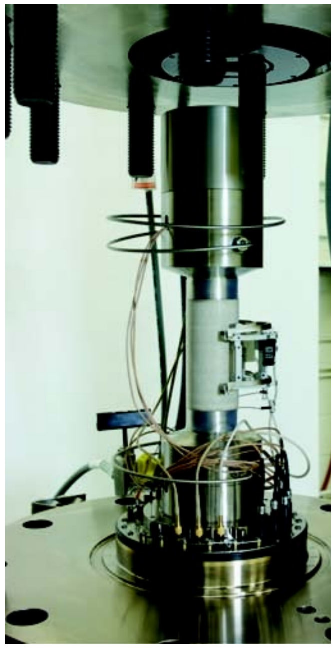
The axial stress σ in a test specimen is calculated by dividing the axial load P by the cross-sectional area A (Eq. 1-6). When the initial area of the specimen is used in the calculation, the stress is called the **nominal stress** (other names are *conventional stress* and *engineering stress*). A more exact value of the axial stress, called the **true stress**, can be calculated by using the actual area of the bar at the cross section where failure occurs. Since the actual area in a tension test is always less than the initial area (see Fig. 1-29), the true stress is larger than the nominal stress.

The average axial strain ε in the test specimen is found by dividing the measured elongation δ between the gage marks by the gage length L (see Fig. 1-29 and Eq. 1-7). If the initial gage length is used in the calculation (for instance, 2.0 in.), the **nominal strain** is obtained. Since the distance between the gage marks increases as the tensile load is applied, you can calculate the **true strain** (or *natural strain*) at any value of the load by using the actual distance between the gage marks. In tension, true strain is always smaller than nominal strain. However, for most engineering purposes, nominal stress and nominal strain are adequate.

After performing a tension or compression test and determining the stress and strain at various magnitudes of the load, you can plot a diagram of stress versus strain. Such a **stress-strain diagram** is a characteristic of the particular material being tested and conveys important information about the mechanical properties and type of behavior.[3]

FIGURE 1-30

Rock sample being tested in compression to obtain compressive strength, elastic modulus and Poisson's ratio (Courtesy of MTS Systems Corporation)



Stresses and Strains for Structural Steel

Structural steel, also known as *mild steel* or *low-carbon steel*, is one of the most widely used metals found in buildings, bridges, cranes, ships, towers, vehicles, and many other types of construction. A stress-strain diagram for a typical structural steel in tension is shown in Fig. 1-31. Strains are plotted on the horizontal axis and stresses on the vertical axis.

The diagram begins with a straight line from the origin O to point A, showing that the relationship between stress and strain in this initial region is not only

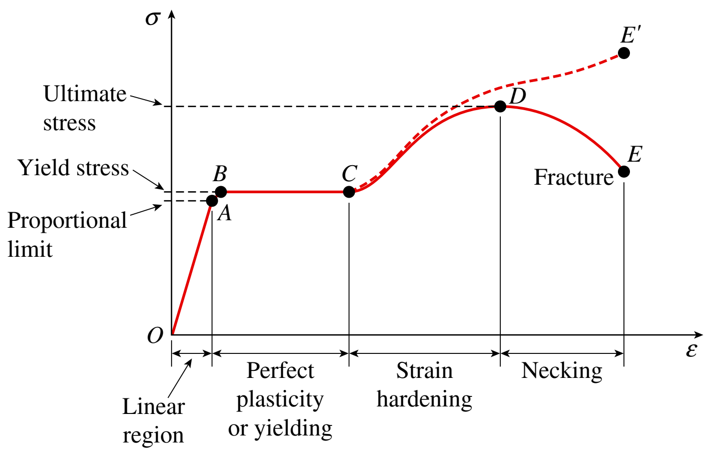


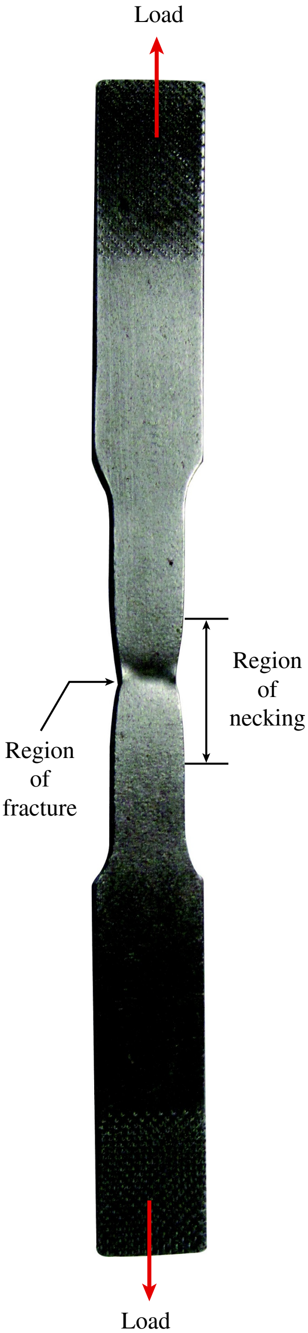
FIGURE 1-31

Stress-strain diagram for a typical structure steel in tension (not to scale)

[3]Stress-strain diagrams were originated by Jacob Bernoulli (1654–1705) and J. V. Poncelet (1788–1867) (Ref. 1-4).

FIGURE 1-32

Necking of a mild-steel bar in tension (© Barry Goodno)



linear but also *proportional*.[4] Beyond point A, the proportionality between stress and strain no longer exists; hence the stress at A is called the **proportional limit**. For low-carbon steels, this limit is in the range 30 to 50 ksi (210 to 350 MPa), but high-strength steels (with higher carbon content plus other alloys) can have proportional limits of more than 80 ksi (550 MPa). The slope of the straight line from O to A is called the **modulus of elasticity**. Because the slope has units of stress divided by strain, modulus of elasticity has the same units as stress. (See Section 1.6.)

With an increase in stress beyond the proportional limit, the strain begins to increase more rapidly for each increment in stress. Consequently, the stress-strain curve has a smaller and smaller slope, until, at point B, the curve becomes horizontal (see Fig. 1-31). Beginning at this point, considerable elongation of the test specimen occurs with no noticeable increase in the tensile force (from B to C). This phenomenon is known as **yielding** of the material, and point B is called the **yield point**. The corresponding stress is known as the **yield stress** of the steel.

In the region from B to C (see Fig. 1-31), the material becomes **perfectly plastic**, which means that it deforms without an increase in the applied load. The elongation of a mild-steel specimen in the perfectly plastic region is typically 10 to 15 times the elongation that occurs in the linear region (between the onset of loading and the proportional limit). The presence of very large strains in the plastic region (and beyond) is the reason for not plotting this diagram to scale.

After undergoing the large strains that occur during yielding in the region BC, the steel begins to **strain harden**. During strain hardening, the material undergoes changes in its crystalline structure, resulting in increased resistance of the material to further deformation. Elongation of the test specimen in this region requires an increase in the tensile load, and therefore the stress-strain diagram has a positive slope from C to D. The load eventually reaches its maximum value, and the corresponding stress (at point D) is called the **ultimate stress**. Further stretching of the bar is actually accompanied by a reduction in the load, and fracture finally occurs at a point such as E in Fig. 1-31.

The yield stress and ultimate stress of a material are also called the **yield strength** and **ultimate strength**, respectively. **Strength** is a general term that refers to the capacity of a structure to resist loads. For instance, the yield strength of a beam is the magnitude of the load required to cause yielding in the beam, and the ultimate strength of a truss is the maximum load it can support, that is, the failure load. However, when conducting a tension test of a particular material, define load-carrying capacity by the stresses in the specimen rather than by the total loads acting on the specimen. As a result, the strength of a material is usually stated as a stress.

When a test specimen is stretched, **lateral contraction** occurs. The resulting decrease in cross-sectional area is too small to have a noticeable effect on the calculated values of the stresses up to about point C in Fig. 1-31, but beyond that point the reduction in area begins to alter the shape of the curve. In the vicinity of the ultimate stress, the reduction in area of the bar becomes clearly visible and a pronounced **necking** of the bar occurs (see Fig. 1-32).

If the actual cross-sectional area at the narrow part of the neck is used to calculate the stress, the **true stress-strain curve** (the dashed line CE' in Fig. 1-31)

[4]Two variables are said to be *proportional* if their ratio remains constant. Therefore, a proportional relationship may be represented by a straight line through the origin. Although a proportional relationship is linear, the converse is not necessarily true, because a relationship represented by a straight line that does *not* pass through the origin is linear but not proportional. "Directly proportional" is synonymous with "proportional" (Ref. 1-5).

is obtained. The total load the bar can carry does indeed diminish after the ultimate stress is reached (as shown by curve *DE*), but this reduction is due to the decrease in area of the bar and not to a loss in strength of the material itself. In reality, the material withstands an increase in true stress up to failure (point *E'*). Because most structures are expected to function at stresses below the proportional limit, the **conventional stress-strain curve** *OABCDE*, which is based upon the original cross-sectional area of the specimen and is easy to determine, provides satisfactory information for use in engineering design.

The diagram of Fig. 1-31 shows the general characteristics of the stress-strain curve for mild steel, but its proportions are not realistic because the strain that occurs from *B* to *C* may be more than ten times the strain occurring from *O* to *A*. Furthermore, the strains from *C* to *E* are many times greater than those from *B* to *C*. The correct relationships are portrayed in Fig. 1-33, which shows a stress-strain diagram for mild steel drawn to scale. In this figure, the strains from the zero point to point *A* are so small in comparison to the strains from point *A* to point *E* that they cannot be seen, and the initial part of the diagram appears to be a vertical line.

FIGURE 1-33

Stress-strain diagram for a typical structural steel in tension (drawn to scale)

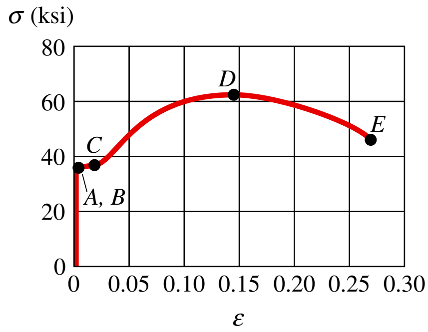


FIGURE 1-34

Typical stress-strain diagram for an aluminum alloy

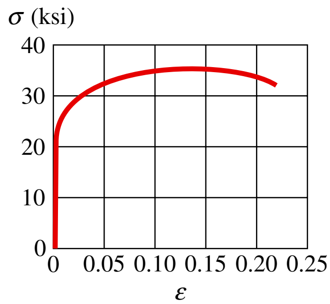
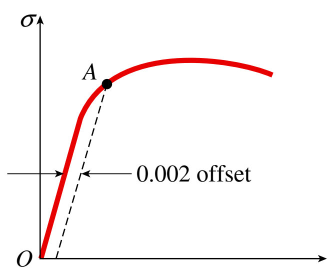


FIGURE 1-35

Arbitrary yields stress determined by the offset method



Ductility

The presence of a clearly defined yield point followed by large plastic strains is an important characteristic of structural steel that is sometimes utilized in practical design (see discussions of elastoplastic behavior in Sections 2.12 and 6.10). Metals such as structural steel that undergo large *permanent* strains before failure are classified as **ductile**. Ductility is the property that enables a bar of steel to be bent into a circular arc or drawn into a wire without breaking. A desirable feature of ductile materials is that visible distortions occur if the loads become too large, thus providing an opportunity to take remedial action before an actual fracture occurs. Also, materials exhibiting ductile behavior are capable of absorbing large amounts of strain energy prior to fracture.

Structural steel is an alloy of iron containing about 0.2% carbon, and therefore it is classified as a low-carbon steel. With increasing carbon content, steel becomes less ductile but stronger (higher yield stress and higher ultimate stress). The physical properties of steel are also affected by heat treatment, the presence of other metals, and manufacturing processes such as rolling. Other materials that behave in a ductile manner (under certain conditions) include aluminum, copper, magnesium, lead, molybdenum, nickel, brass, bronze, monel metal, nylon, and teflon.

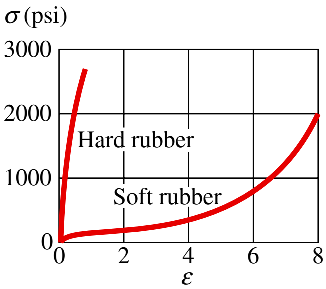
Aluminum Alloys

Although they have considerable ductility, **aluminum alloys** typically do not have a clearly definable yield point, as shown by the stress-strain diagram of Fig. 1-34. However, they do have an initial linear region with a recognizable proportional limit. Alloys produced for structural purposes have proportional limits in the range 10 to 60 ksi (70 to 410 MPa) and ultimate stresses in the range 20 to 80 ksi (140 to 550 MPa).

When a material such as aluminum does not have an obvious yield point yet undergoes large strains after the proportional limit is exceeded, an *arbitrary* yield stress may be determined by the **offset method**. A straight line is drawn on the stress-strain diagram parallel to the initial linear part of the curve (Fig. 1-35) but offset by some standard strain, such as 0.002 (or 0.2%). The intersection of the offset line and the stress-strain curve (point *A* in the figure) defines the yield stress. Because this stress is determined by an arbitrary rule and is not an inherent physical property of the material, it should be distinguished from a true yield stress by referring to it

FIGURE 1-36

Stress-strain curves for two kinds of rubber in tension



as the **offset yield stress**. For a material such as aluminum, the offset yield stress is slightly above the proportional limit. In the case of structural steel, with its abrupt transition from the linear region to the region of plastic stretching, the offset stress is essentially the same as both the yield stress and the proportional limit.

Rubber Rubber maintains a linear relationship between stress and strain up to relatively large strains (as compared to metals). The strain at the proportional limit may be as high as 0.1 or 0.2 (10 or 20%). Beyond the proportional limit, the behavior depends upon the type of rubber (Fig. 1-36). Some kinds of soft rubber stretch enormously without failure, reaching lengths several times their original lengths. The material eventually offers increasing resistance to the load, and the stress-strain curve turns markedly upward. You can easily sense this characteristic behavior by stretching a rubber band with your hands. (Note that although rubber exhibits very large strains, it is not a ductile material because the strains are not permanent. It is, of course, an elastic material; see Section 1.6.)

Ductility and Elongation The ductility of a material in tension can be characterized by its elongation and by the reduction in area at the cross section where fracture occurs. The **percent elongation** is defined as

$$\text{Percent elongation} = \frac{L_1 - L_0}{L_0}(100) \qquad \textbf{(1-10)}$$

in which L_0 is the original gage length and L_1 is the distance between the gage marks at fracture. Because the elongation is not uniform over the length of the specimen but is concentrated in the region of necking, the percent elongation depends upon the gage length. Therefore, when stating the percent elongation, the gage length should always be given. For a 2-in. gage length, steel may have an elongation in the range from 3 to 40%, depending upon composition; in the case of structural steel, values of 20 or 30% are common. The elongation of aluminum alloys varies from 1 to 45%, depending upon composition and treatment.

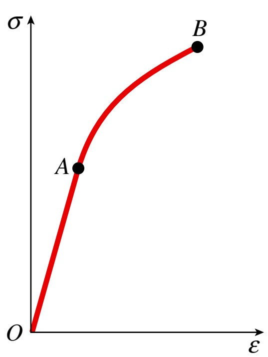
The **percent reduction in area** measures the amount of necking that occurs and is defined as

$$\text{Percent reduction} = \frac{A_0 - A_1}{A_0}(100) \qquad \textbf{(1-11)}$$

in which A_0 is the original cross-sectional area and A_1 is the final area at the fracture section. For ductile steels, the reduction is about 50%.

FIGURE 1-37

Typical stress-strain diagram for a brittle material showing the proportional limit (point A) and fracture stress (point B)



Brittle Materials Materials that fail in tension at relatively low values of strain are classified as **brittle**. Examples are concrete, stone, cast iron, glass, ceramics, and a variety of metallic alloys. Brittle materials fail with only little elongation after the proportional limit (the stress at point A in Fig. 1-37) is exceeded. Furthermore, the reduction in area is insignificant, and so the nominal fracture stress (point B) is the same as the true ultimate stress. High-carbon steels have very high yield stresses—over 100 ksi (700 MPa) in some cases—but they behave in a brittle manner and fracture occurs at an elongation of only a few percent.

Ordinary glass is a nearly ideal brittle material because it exhibits almost no ductility. The stress-strain curve for glass in tension is essentially a straight line, with failure occurring before any yielding takes place. The ultimate stress is about 10,000 psi (70 MPa) for certain kinds of plate glass, but great variations exist, depending upon the type of glass, the size of the specimen, and the presence of microscopic defects. **Glass fibers** can develop enormous strengths, and ultimate stresses over 1,000,000 psi (7 GPa) have been attained.

Plastics Many types of **plastics** are used for structural purposes because of their light weight, resistance to corrosion, and good electrical insulation properties. Their mechanical properties vary tremendously, with some plastics being brittle and others ductile. When designing with plastics, it is important to realize that their properties are greatly affected by both temperature changes and the passage of time. For instance, the ultimate tensile stress of some plastics is cut in half merely by raising the temperature from 50° F to 120° F. Also, a loaded plastic may stretch gradually over time until it is no longer serviceable. For example, a bar of polyvinyl chloride subjected to a tensile load that initially produces a strain of 0.005 may have that strain doubled after one week, even though the load remains constant. (This phenomenon, known as *creep*, is discussed in the next section.)

Ultimate tensile stresses for plastics are generally in the range 2 to 50 ksi (14 to 350 MPa) and weight densities vary from 50 to 90 lb/ft^3 (8 to 14 kN/m^3). One type of nylon has an ultimate stress of 12 ksi (80 MPa) and weighs only 70 lb/ft^3 (11 kN/m^3), which is only 12% heavier than water. Because of its light weight, the strength-to-weight ratio for nylon is about the same as for structural steel (see Prob. 1.4-4).

Composites A **filament-reinforced material** consists of a base material (or *matrix*) in which high-strength filaments, fibers, or whiskers are embedded. The resulting composite material has much greater strength than the base material. As an example, the use of glass fibers can more than double the strength of a plastic matrix. Composites are widely used in aircraft, boats, rockets, and space vehicles where high strength and light weight are needed.

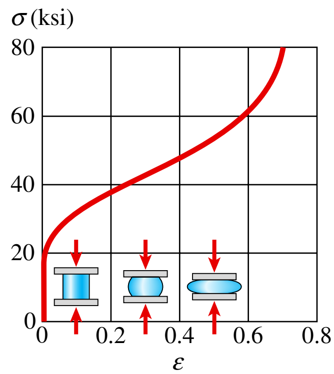
Compression

Stress-strain curves for materials in compression differ from those in tension. Ductile metals such as steel, aluminum, and copper have proportional limits in compression very close to those in tension, and the initial regions of their compressive and tensile stress-strain diagrams are about the same. However, after yielding begins, the behavior is quite different. In a tension test, the specimen is stretched, necking may occur, and fracture ultimately takes place. When the material is compressed, it bulges outward on the sides and becomes barrel shaped, because friction between the specimen and the end plates prevents lateral expansion. With increasing load, the specimen is flattened out and offers greatly increased resistance to further shortening (which means that the stress-strain curve becomes very steep). These characteristics are illustrated in Fig. 1-38, which shows a compressive stress-strain diagram for copper. Since the actual cross-sectional area of a specimen tested in compression is larger than the initial area, the true stress in a compression test is smaller than the nominal stress.

Brittle materials loaded in compression typically have an initial linear region followed by a region in which the shortening increases at a slightly higher rate than does the load. The stress-strain curves for compression and tension often have similar shapes, but the ultimate stresses in compression are much higher than those in tension. Also, unlike ductile materials, which flatten out when compressed, brittle materials actually break at the maximum load.

FIGURE 1-38

Stress-strain diagrams for copper in compression



Tables of Mechanical Properties

Properties of materials are listed in the tables of Appendix I at the back of the book. The data in the tables are typical of the materials and are suitable for solving problems in this book. However, properties of materials and stress-strain curves vary greatly—even for the same material—because of different manufacturing processes, chemical composition, internal defects, temperature, and many other factors.

For these reasons, data obtained from Appendix I (or other tables of a similar nature) should not be used for specific engineering or design purposes. Instead, the manufacturers or materials suppliers should be consulted for information about a particular product.

1.6 Elasticity, Plasticity, and Creep

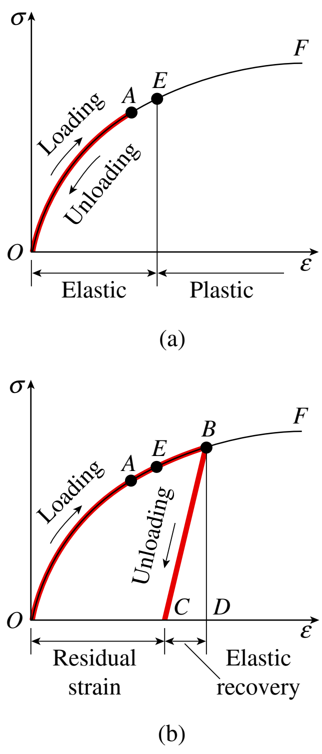
Stress-strain diagrams portray the behavior of engineering materials when the materials are loaded in tension or compression, as described in the preceding section. Now consider what happens when the load is removed and the material is *unloaded*.

Assume, for instance, that you apply a load to a tensile specimen so that the stress and strain go from the origin *O* to point *A* on the stress-strain curve of Fig. 1-39a. Suppose that when the load is removed, the material follows exactly the same curve back to the origin *O*. This property of a material, by which it returns to its original dimensions during unloading, is called **elasticity**, and the material itself is said to be *elastic*. *Note that the stress-strain curve from O to A need not be linear in order for the material to be elastic.*

Now suppose that you load this same material to a higher level, so that point *B* is reached on the stress-strain curve (Fig. 1-39b). When unloading occurs from point *B*, the material follows line *BC* on the diagram. This unloading line is parallel to the initial portion of the loading curve; that is, line *BC* is parallel to a tangent to the stress-strain curve at the origin. When point *C* is reached, the load has been entirely removed, but a **residual strain**, or *permanent strain*, which is represented by line *OC*, remains in the material. As a consequence, the bar being tested is longer than it was before loading. This residual elongation of the bar is called the **permanent set**. Of the total strain *OD* developed during loading from *O* to *B*, the strain *CD* has been recovered elastically and the strain *OC* remains as a permanent strain. Thus, during unloading the bar returns partially to its original shape, and the material is said to be **partially elastic**.

FIGURE 1-39

Stress-strain diagrams illustrating (a) elastic behavior, and (b) partially elastic behavior



Between points *A* and *B* on the stress-strain curve (Fig. 1-39b), there must be a point before which the material is elastic and beyond which the material is partially elastic. To find this point, load the material to some selected value of stress and then remove the load. If there is no permanent set (that is, if the elongation of the bar returns to zero), the material is fully elastic up to the selected value of the stress.

The process of loading and unloading can be repeated for successively higher values of stress. Eventually, a stress will be reached such that not all the strain is recovered during unloading. By this procedure, it is possible to determine the stress at the upper limit of the elastic region, for instance, the stress at point *E* in Figs. 1-39a and b. The stress at this point is known as the **elastic limit** of the material.

Many materials, including most metals, have linear regions at the beginning of their stress-strain curves (for example, see Figs. 1-31 and 1-34). The stress at the upper limit of this linear region is the proportional limit. The elastic limit is usually the same as, or slightly above, the proportional limit. Hence, for many materials the two limits are assigned the same numerical value. In the case of mild steel, the yield stress is also very close to the proportional limit, so for practical purposes, the yield stress, the elastic limit, and the proportional limit are assumed to be equal. Of course, this situation does not hold for all materials. Rubber is an outstanding example of a material that is elastic far beyond the proportional limit.

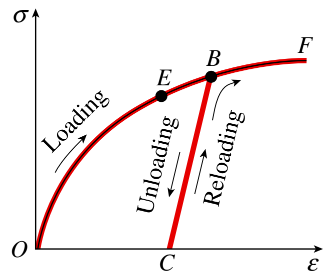
The characteristic of a material that undergoes inelastic strains beyond the strain at the elastic limit is known as **plasticity**. Thus, on the stress-strain curve of Fig. 1-39a, an elastic region is followed by a plastic region. When large deformations occur in a ductile material loaded into the plastic region, the material is said to undergo **plastic flow**.

Reloading of a Material

If the material remains within the elastic range, it can be loaded, unloaded, and loaded again without significantly changing its behavior. However, when loaded into the plastic range, the internal structure of the material is altered and its properties change. For instance, you have already observed that a permanent strain exists in the specimen after unloading from the plastic region (Fig. 1-39b). Now suppose that the material is reloaded after such an unloading (Fig. 1-40). The new loading begins at point *C* on the diagram and continues upward to point *B*, which is the point where unloading began during the first loading cycle. The material then follows the original stress-strain curve toward point *F*. Thus, for the second loading, imagine that there is a new stress-strain diagram with its origin at point *C*.

FIGURE 1-40

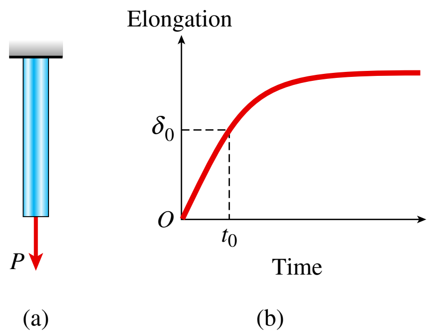
Reloading of a material and raising of the elastic and proportional limits



During the second loading, the material behaves in a linearly elastic manner from *C* to *B*, with the slope of line *CB* being the same as the slope of the tangent to the original loading curve at the origin *O*. The proportional limit is now at point *B*, which is at a higher stress than the original elastic limit (point *E*). Thus, by stretching a material such as steel or aluminum into the inelastic or plastic range, the *properties of the material are changed*—the linearly elastic region is increased, the proportional limit is raised, and the elastic limit is raised. However, the ductility is reduced because in the "new material" the amount of yielding beyond the elastic limit (from *B* to *F*) is less than in the original material (from *E* to *F*).[5]

FIGURE 1-41

Creep in a bar under constant load



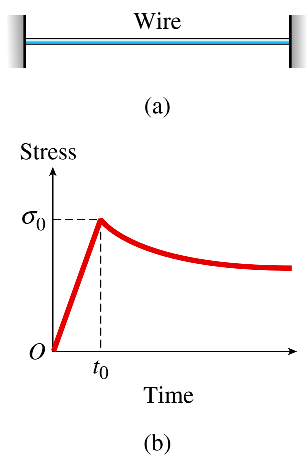
Creep

The stress-strain diagrams described previously were obtained from tension tests involving static loading and unloading of the specimens, and the passage of time did not enter the discussion. However, when loaded for long periods of time, some materials develop additional strains and are said to **creep**.

Creep can manifest itself in a variety of ways. For instance, suppose that a vertical bar (Fig. 1-41a) is loaded slowly by a force *P*, producing an elongation equal to δ_0. Assume that the loading and corresponding elongation take place during a time interval of duration t_0 (Fig. 1-41b). Subsequent to time t_0, the load remains constant. However, due to creep, the bar may gradually lengthen, as shown in Fig. 1-41b, even though the load does not change. This behavior occurs with many materials, although sometimes the change is too small to be of concern.

FIGURE 1-42

Relaxation of stress in a wire under constant strain



As another manifestation of creep, consider a wire that is stretched between two immovable supports so that it has an initial tensile stress σ_0 (Fig. 1-42). Again, denote the time during which the wire is initially stretched as t_0. With the elapse of time, the stress in the wire gradually diminishes, eventually reaching a constant value, even though the supports at the ends of the wire do not move. This process is called **relaxation** of the material.

Creep is usually more important at high temperatures than at ordinary temperatures, therefore it should always be considered in the design of engines,

[5]The study of material behavior under various environmental and loading conditions is an important branch of applied mechanics. For more detailed engineering information about materials, consult a textbook devoted solely to this subject.

furnaces, and other structures that operate at elevated temperatures for long periods of time. However, materials such as steel, concrete, and wood will creep slightly even at atmospheric temperatures. For example, creep of concrete over long periods of time can create undulations in bridge decks because of sagging between the supports. (One remedy is to construct the deck with an upward **camber**, which is an initial displacement above the horizontal, so that when creep occurs, the spans lower to the level position.)

Example 1-7

A machine component slides along a horizontal bar at A and moves in a vertical slot B. The component is represented as a rigid bar AB (length $L = 5$ ft, weight $W = 985$ lb) with roller supports at A and B (neglect friction). When not in use, the machine component is supported by a single wire (diameter $d = 1/8$ in.) with one end attached at A and the other end supported at C (see Fig. 1-43). The wire is made of a copper alloy; the stress-strain relationship for the wire is

$$\sigma(\varepsilon) = \frac{17{,}500\varepsilon}{1 + 240\varepsilon} \qquad 0 \le \varepsilon \le 0.03 \quad (\sigma \text{ in ksi})$$

(a) Plot a stress-strain diagram for the material; What is the modulus of elasticity E (ksi)? What is the 0.2% offset yield stress (ksi)?

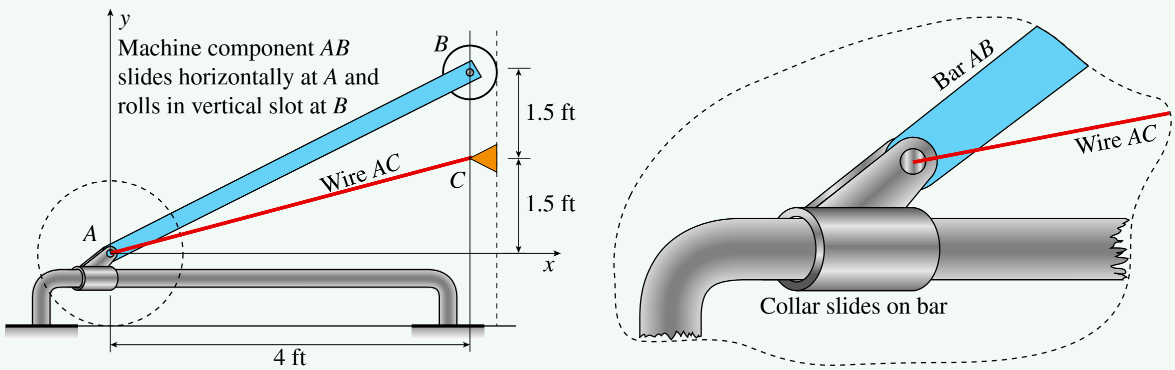
(b) Find the tensile force T (lb) in the wire.

(c) Find the normal axial strain ε and elongation δ (in.) of the wire.

(d) Find the permanent set of the wire if all forces are removed.

FIGURE 1-43

Example 1-7: Rigid bar supported by copper alloy wire



Solution:

Use a four-step problem-solving approach to find the modulus of elasticity, yield stress, tensile force, normal strain and elongation, and the permanent set of copper alloy wire AC.

1. ***Conceptualize***: The copper alloy has considerable ductility but will have a stress-strain curve without a well-defined yield point. Define the yield point

using an *offset method* as illustrated in Fig. 1-35. Find the residual strain and then the *permanent set* of the wire, as shown in Fig. 1-39.

2. ***Categorize***: The given analytical expression for the stress-strain curve $\sigma(\varepsilon)$ is based on measured laboratory data for the copper alloy used to manufacture this wire. Hence, the analytical expression is an approximation of the actual behavior of this material and was formulated based on test data. Analytical representations of actual stress-strain curves are often used in computer programs to model and analyze structures of different materials under applied loads of various kinds.

3. ***Analyze***:

Part (a): Plot a stress-strain diagram for the material; What is the modulus of elasticity E (ksi)? What is the 0.2% offset yield stress (ksi)?

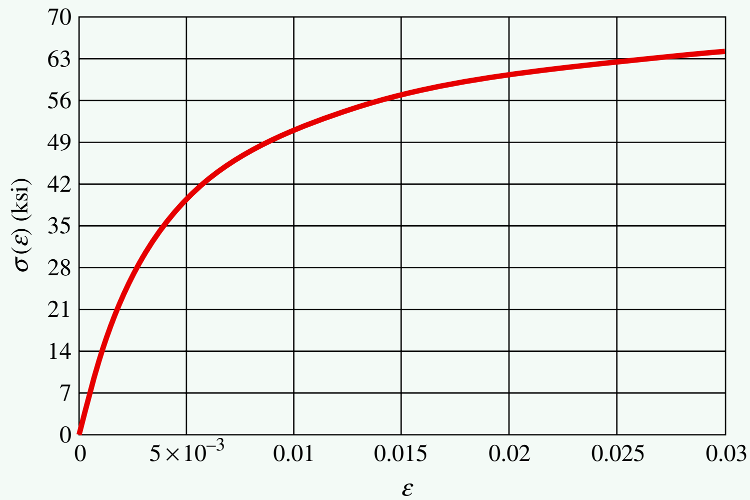
Plot the function $\sigma(\varepsilon)$ for strain values between 0 and 0.03 (Fig. 1-44). The stress at strain $\varepsilon = 0.03$ is 64 ksi.

$$\sigma(\varepsilon) = \frac{17{,}500\varepsilon}{1 + 240\varepsilon} \qquad \varepsilon = 0, 0.001, \ldots, 0.03$$

$$\sigma(0) = 0 \qquad \sigma(0.03) = 64 \text{ ksi}$$

FIGURE 1-44

Stress-strain curve for copper alloy wire in Example 1-7



The slope of the tangent to the stress-strain curve at strain $\varepsilon = 0$ is the modulus of elasticity E (see Fig. 1-45). Take the derivative of $\sigma(\varepsilon)$ to get the slope of the tangent to the $\sigma(\varepsilon)$ curve, and evaluate the derivative at strain $\varepsilon = 0$ to find E:

$$E(\varepsilon) = \frac{d}{d\varepsilon}\sigma(\varepsilon) = \frac{17{,}500}{(240\varepsilon + 1)^2}$$

$$E = E(0) \qquad E = 17{,}500 \text{ ksi}$$

Next, find an expression for the yield strain ε_y, the point at which the 0.2% offset line crosses the stress-strain curve (see Fig. 1-45). Substitute the expression ε_y into the $\sigma(\varepsilon)$ expression and then solve for yield $\sigma(\varepsilon_y) = \sigma_y$:

$$\varepsilon_y = 0.002 + \frac{\sigma_y}{E} \text{ and } \sigma(\varepsilon_y) = \sigma_y \quad \text{or} \quad \sigma_y = \frac{17{,}500\varepsilon_y}{1+240\varepsilon_y}$$

Rearranging the equation in terms of σ_y gives

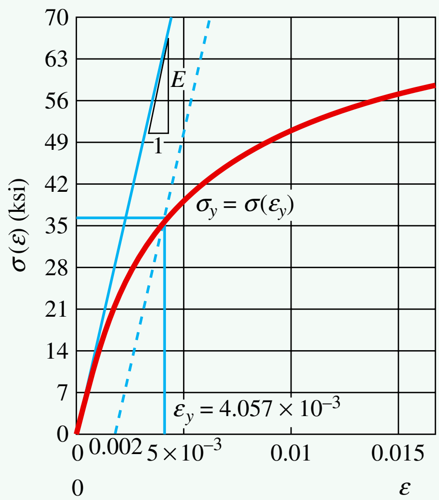
$$\sigma_y^2 + \left(\frac{37E}{6000} - \frac{875}{12}\right)\sigma_y - \frac{7E}{48} = 0$$

Solving this quadratic equation for the 0.2% offset yield, stress σ_y gives $\sigma_y = 36$ ksi.

The yield strain is computed as

$$\varepsilon_y = 0.002 + \frac{\sigma_y}{E(\text{ksi})} = 4.057 \times 10^{-3}$$

FIGURE 1-45
Modulus of elasticity E, 0.2% offset line, and yield stress σ_y and strain ε_y for copper alloy wire in Example 1-7



Part (b): Find the tensile force T (lb) in the wire. Recall that bar weight $W = 985$ lb.

Find the angle between the x-axis and cable attachment position at C:

$$\alpha_C = \arctan\left(\frac{1.5}{4}\right) = 20.556°$$

Sum the moments about A to obtain one equation and one unknown. The reaction B_x acts to the left:

$$B_x = \frac{-W(2\text{ ft})}{3\text{ ft}} = -656.667\text{ lb}$$

Next, sum the forces in the x direction to find the cable force T_C:

$$T_C = \frac{-B_x}{\cos(\alpha_C)} \qquad T_C = 701\text{ lb}$$

Part (c): Find the normal axial strain ε and elongation δ (in.) of the wire.

Compute the normal stress then find the associated strain from stress-strain plot (or from the $\sigma(\varepsilon)$ equation). The wire elongation is strain times wire length.

The wire diameter, cross-sectional area, and length are

$$d = \frac{1}{8}\,\text{in.} \quad A = \frac{\pi}{4}d^2 = 0.0123\ \text{in}^2$$

$$L_C = \sqrt{(4\,\text{ft})^2 + (1.5\,\text{ft})^2} = 4.272\ \text{ft}$$

Now compute the stress and strain in the wire and the elongation of the wire as

$$\sigma_C = \frac{T_C}{A} = 57.1\ \text{ksi}$$

Note that the stress in the wire exceeds the 0.2% offset yield stress of 36 ksi. The corresponding normal strain is found from the $\sigma(\varepsilon)$ plot or by rearranging the $\sigma(\varepsilon)$ equation to give

$$\varepsilon(\sigma) = \frac{\sigma}{17{,}500 - 240\sigma}$$

Then,

$$\varepsilon(\sigma_C) = \varepsilon_C, \quad \text{or} \quad \varepsilon_C = \frac{\sigma_C}{17{,}500\ \text{ksi} - 240\sigma_C} = 0.015$$

Finally, the wire elongation is

$$\delta_C = \varepsilon_C L_C = 0.774\ \text{in.}$$

Part (d): Find the permanent set of the wire if all forces are removed.

If the load is removed from the wire, the stress in the wire will return to zero following the unloading line in Fig. 1-46 (see also Fig. 1-39b). The elastic recovery strain is

$$\varepsilon_{er} = \frac{\sigma_C}{E} = 3.266 \times 10^{-3}$$

The residual strain is the difference between the total strain (ε_C) and the elastic recovery strain (ε_{er})

$$\varepsilon_{res} = \varepsilon_C - \varepsilon_{er} = 0.012$$

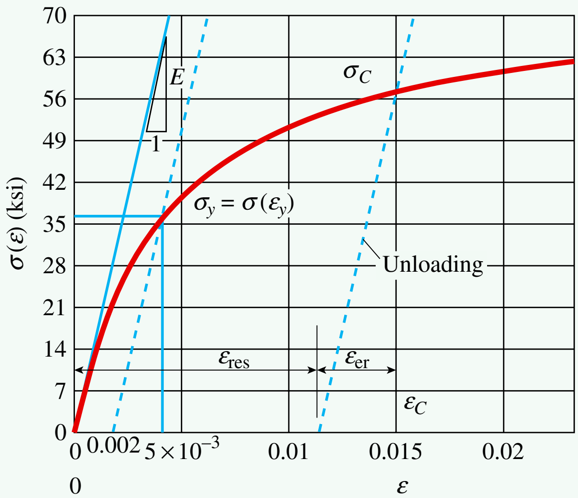
Finally, the permanent set of the wire is the product of the residual strain and the length of the wire:

$$\varepsilon_{res} L_C = 0.607\ \text{in.}$$

4. ***Finalize***: This example presents an analytical model of the stress-strain relationship for a copper alloy. The computed values of modulus of elasticity E and yield stress σ_y are consistent with values listed in

FIGURE 1-46

Residual strain (ε_{res}) and elastic recovery strain (ε_{er}) for copper alloy wire in Example 1-7



Appendix I. The tensile force, normal strain and elongation, and permanent set are computed for the wire when stressed beyond the apparent yield point of the material.

1.7 Linear Elasticity, Hooke's Law, and Poisson's Ratio

Many structural materials, including most metals, wood, plastics, and ceramics, behave both elastically and linearly when first loaded. Consequently, their stress-strain curves begin with a straight line passing through the origin. An example is the stress-strain curve for structural steel (Fig. 1-31), where the region from the origin O to the proportional limit (point A) is both linear and elastic. Other examples are the regions below *both* the proportional limits and the elastic limits on the diagrams for aluminum (Fig. 1-34), brittle materials (Fig. 1-37), and copper (Fig. 1-38).

When a material behaves elastically and also exhibits a linear relationship between stress and strain, it is said to be **linearly elastic**. By designing structures and machines to function in this region, engineers avoid permanent deformations due to yielding.

Hooke's Law

The linear relationship between stress and strain for a bar in simple tension or compression is expressed by the equation

$$\sigma = E\varepsilon \tag{1-12}$$

in which σ is the axial stress, ε is the axial strain, and E is a constant of proportionality known as the **modulus of elasticity** for the material. The modulus of elasticity is the slope of the stress-strain diagram in the linearly elastic region, as mentioned previously in Section 1.5. Since strain is dimensionless, the units of E are the same as the units of stress. Typical units of E are psi or ksi in USCS units and pascals (or multiples thereof) in SI units.

The equation $\sigma = E\varepsilon$ is commonly known as **Hooke's law**, named for the famous English scientist Robert Hooke (1635–1703). Hooke was the first person to investigate scientifically the elastic properties of materials, and he tested such diverse materials as metal, wood, stone, bone, and sinew. He measured the stretching of long wires supporting weights and observed that the elongations "always bear the same proportions one to the other that the weights do that made them." (Ref. 1-6). Thus, Hooke established the linear relationship between the applied loads and the resulting elongations.

Equation (1-12) is actually a very limited version of Hooke's law because it relates only to the longitudinal stresses and strains developed in simple tension or compression of a bar (*uniaxial stress*). To deal with more complicated states of stress, such as those found in most structures and machines, more extensive equations of Hooke's law are needed (see Sections 7.5 and 7.6).

The modulus of elasticity has relatively large values for materials that are very stiff, such as structural metals. Steel has a modulus of approximately 30,000 ksi (210 GPa); for aluminum, values around 10,600 ksi (73 GPa) are typical. More flexible materials have a lower modulus—values for plastics range from 100 to 2000 ksi (0.7 to 14 GPa). Some representative values of E are listed in Table I-2, Appendix I. For most materials, the value of E in compression is nearly the same as in tension.

Modulus of elasticity is often called **Young's modulus**, after another English scientist, Thomas Young (1773–1829). In connection with an investigation of tension and compression of prismatic bars, Young introduced the idea of a "modulus of the elasticity." However, his modulus was not the same as the one in use today because it involved properties of the bar as well as of the material (Ref. 1-7).

Poisson's Ratio

When a prismatic bar is loaded in tension, the axial elongation is accompanied by **lateral contraction** (that is, contraction normal to the direction of the applied load). This change in shape is pictured in Fig. 1-47, where part (a) shows the bar before loading and part (b) shows it after loading. In part (b), the dashed lines represent the shape of the bar prior to loading.

Lateral contraction is easily seen by stretching a rubber band, but in metals, the changes in lateral dimensions (in the linearly elastic region) are usually too small to be visible. However, they can be detected with sensitive measuring devices.

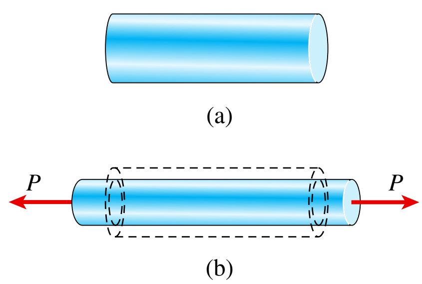
The **lateral strain** ε' at any point in a bar is proportional to the axial strain ε at that same point if the material is linearly elastic. The ratio of these strains is a property of the material known as **Poisson's ratio**. This dimensionless ratio, usually denoted by the Greek letter ν (nu), can be expressed by

$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon'}{\varepsilon} \tag{1-13}$$

The minus sign is inserted in the equation to compensate for the fact that the lateral and axial strains normally have opposite signs. For instance, the axial strain in a bar in tension is positive, and the lateral strain is negative

FIGURE 1-47

Axial elongation and lateral contraction of a prismatic bar in tension: (a) bar before loading and (b) bar after loading (The deformations of the bar are highly exaggerated)



(because the width of the bar decreases). The opposite is true for compression, with the bar becoming shorter (negative axial strain) and wider (positive lateral strain). Therefore, for ordinary materials, Poisson's ratio has a positive value. Some materials such as low-density open-cell polymer foams, however, can have a negative Poisson's ratio so, in the linear elastic range, Poisson's ratio lies between -1 and $+0.5$.

When Poisson's ratio for a material is known, you can obtain the lateral strain from the axial strain as

$$\varepsilon' = -\nu\varepsilon \tag{1-14}$$

When using Eqs. (1-13) and (1-14), always keep in mind that they apply only to a bar in uniaxial stress, that is, a bar for which the only stress is the normal stress σ in the axial direction.

Poisson's ratio is named for the famous French mathematician Siméon Denis Poisson (1781–1840), who attempted to calculate this ratio by a molecular theory of materials (Ref. 1-8). For isotropic materials, Poisson found $v = 1/4$. More recent calculations based upon better models of atomic structure give $v = 1/3$. Both of these values are close to actual measured values, which are in the range 0.25 to 0.35 for most metals and many other materials. Materials with an extremely low value of Poisson's ratio include cork, for which v is practically zero, and concrete, for which ν is about 0.1 or 0.2. A theoretical upper limit for Poisson's ratio is 0.5, as explained later in Section 7.5. Rubber comes close to this limiting value. Note however, that Poisson's ratio may be as low as –1.0 for materials such as low-density open cell polymer foams. Hence, in the elastic range, Poisson's ratio varies between -1.0 and $+0.5$.

A table of Poisson's ratios for various materials in the linearly elastic range is given in Appendix I (see Table I-2). For most purposes, Poisson's ratio is assumed to be the same in both tension and compression.

When the strains in a material become large, Poisson's ratio changes. For instance, in the case of structural steel, the ratio becomes almost 0.5 when plastic yielding occurs. Thus, Poisson's ratio remains constant only in the linearly elastic range. When the material behavior is nonlinear, the ratio of lateral strain to axial strain is often called the *contraction ratio*. Of course, in the special case of linearly elastic behavior, the contraction ratio is the same as Poisson's ratio.

Limitations

For a particular material, Poisson's ratio remains constant throughout the linearly elastic range, as explained previously. Therefore, at any given point in the prismatic bar of Fig. 1-47, the lateral strain remains proportional to the axial strain as the load increases or decreases. However, for a given value of the load (which means that the axial strain is constant throughout the bar), additional conditions must be met if the lateral strains are to be the same throughout the entire bar.

First, the material must be **homogeneous**, that is, it must have the same composition (and hence the same elastic properties) at every point. However, having a homogeneous material does not mean that the elastic properties at a particular point are the same in all *directions*. For instance, the modulus of elasticity could be different in the axial and lateral directions, as in the case of a wood pole. Therefore, a second condition for uniformity in the lateral strains is that the elastic properties must be the same in all directions *perpendicular* to the longitudinal axis. When the preceding conditions are met, as is often the case with metals, the lateral strains in a prismatic bar subjected to uniform tension will be the same at every point in the bar and the same in all lateral directions.

Materials having the same properties in all directions (whether axial, lateral, or any other direction) are said to be **isotropic**. If the properties differ in various directions, the material is **anisotropic**.

In this book, all examples and problems are solved with the assumption that the material is linearly elastic, homogeneous, and isotropic—unless a specific statement is made to the contrary.

Example 1-8

A hollow plastic circular pipe (length L_p, inner and outer diameters d_1 and d_2, respectively; see Fig. 1-48) is inserted as a liner inside a cast iron pipe (length L_c, inner and outer diameters d_3 and d_4, respectively).

(a) Derive a formula for the required initial length L_p of the plastic pipe so that, when it is compressed by some force P, the final length of both pipes is the same and at the same time the final outer diameter of the plastic pipe is equal to the inner diameter of the cast iron pipe.

(b) Using the numerical data given, find the initial length L_p (m) and final thickness t_p (mm) for the plastic pipe.

(c) What is the required compressive force P (N)? What are the final normal stresses (MPa) in both pipes?

(d) Compare the initial and final volumes (mm^3) for the plastic pipe.

Numerical data and pipe cross-section properties are

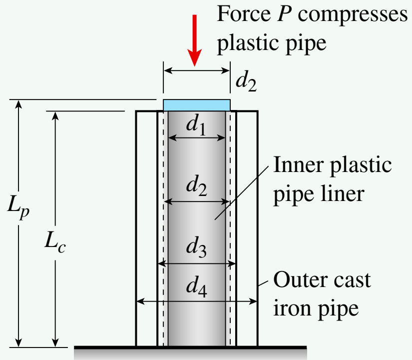
$$L_c = 0.25\text{ m} \quad E_c = 170\text{ GPa} \quad E_p = 2.1\text{ GPa} \quad \nu_c = 0.3 \quad \nu_p = 0.4$$

$$d_1 = 109.8\text{ mm} \qquad d_2 = 110\text{ mm} \qquad d_3 = 110.2\text{ mm}$$

$$d_4 = 115\text{ mm} \qquad t_p = \frac{d_2 - d_1}{2} = 0.1\text{ mm}$$

FIGURE 1-48

Example 1-8: Plastic pipe compressed inside cast iron pipe



Solution:

Use the four-step problem-solving approach to find the dimensions and force required to fit a plastic liner into a cast iron pipe.

1. ***Conceptualize***: Application of a compressive force P results in compressive normal strains and extensional lateral strains in the plastic pipe, while the cast iron pipe is stress-free. The initial length of the plastic pipe (L_p) is greater than that of the cast iron pipe (L_c). With full application of force P, the lengths are made equal.

   The initial cross-sectional areas of the plastic and cast iron pipes are

$$A_p = \frac{\pi}{4}(d_2^2 - d_1^2) = 34.526 \text{ mm}^2 \qquad A_c = \frac{\pi}{4}(d_4^2 - d_3^2) = 848.984 \text{ mm}^2$$

2. ***Categorize***: The two requirements are (a) compression of the plastic pipe must close the gap ($d_3 - d_2$) between the plastic pipe and the inner surface of the cast iron pipe and (b) the final lengths of the two pipes are the same. The first requirement depends on lateral strain and the second on normal strain. Each requirement leads to an expression for shortening of the plastic pipe. Equating the two expressions (i.e., enforcing *compatibility of displacements*) leads to a solution for the required length of the plastic pipe.

3. ***Analyze***:

   Part (a): Derive a formula for the required initial length L_p of the plastic pipe.

   The lateral strain resulting from compression of the plastic pipe must close the gap ($d_3 - d_2$) between the plastic pipe and the inner surface of the cast iron pipe. The required *extensional* lateral strain is positive (here, $\varepsilon_{\text{lat}} = \varepsilon'$):

$$\varepsilon_{\text{lat}} = \frac{d_3 - d_2}{d_2} = 1.818 \times 10^{-3}$$

   The accompanying *compressive* normal strain in the plastic pipe is obtained using Eq. 1-14, which requires Poisson's ratio for the plastic pipe and also the required lateral strain:

$$\varepsilon_p = \frac{-\varepsilon_{\text{lat}}}{\nu_p} \quad \text{or} \quad \varepsilon_p = \frac{-1}{\nu_p}\left(\frac{d_3 - d_2}{d_2}\right) = -4.545 \times 10^{-3}$$

   Use the compressive normal strain ε_p to compute the *shortening* δ_{p1} of the plastic pipe as

$$\delta_{p1} = \varepsilon_p L_p$$

   The required *shortening* of the plastic pipe (so that it has the same final length as that of the cast iron pipe) is

$$\delta_{p2} = -(L_p - L_c)$$

Equating δ_{p1} and δ_{p2} leads to a formula for the required initial length L_p of the plastic pipe:

$$L_p = \frac{L_c}{1 + \varepsilon_p} \quad \text{or} \quad L_p = \frac{L_c}{1 - \dfrac{d_3 - d_2}{\nu_p d_2}}$$

Part (b): Now substitute the numerical data to find the initial length L_p, change in thickness Δt_p, and final thickness t_{pf} for the plastic pipe.

As expected, L_p is greater than the length of the cast iron pipe, $L_c = 0.25$ m, and the thickness of the compressed plastic pipe increases by Δt_p:

$$L_p = \frac{L_c}{1 - \left(\dfrac{d_3 - d_2}{\nu_p d_2}\right)} = 0.25114 \text{ m}$$

$$\Delta t_p = \varepsilon_{\text{lat}} t_p = 1.818 \times 10^{-4} \text{ mm} \quad \text{so} \quad t_{pf} = t_p + \Delta t_p = 0.10018 \text{ mm}$$

Part (c): Next find the required compressive force P and the final normal stresses in both pipes.

A check on the normal compressive stress in the plastic pipe, computed using Hooke's law (Eq. 1-12), shows that it is well below the ultimate stress for selected plastics (see Table I-3, Appendix I); this is also the final normal stress in the plastic pipe:

$$\sigma_p = E_p \varepsilon_p = -9.55 \text{ MPa}$$

The required downward force to compress the plastic pipe is

$$P_{\text{reqd}} = \sigma_p A_p = -330 \text{ N}$$

Both the initial and final stresses in the cast iron pipe are zero because no force is applied to the cast iron pipe.

Part (d): Lastly, compare the initial and final volumes of the plastic pipe.

The initial cross-sectional area of the plastic pipe is

$$A_p = 34.526 \text{ mm}^2$$

The final cross-sectional area of the plastic pipe is

$$A_{pf} = \frac{\pi}{4}[d_3^2 - (d_3 - 2t_{pf})^2] = 34.652 \text{ mm}^2$$

The initial volume of the plastic pipe is

$$V_{\text{pinit}} = L_p A_p = 8671 \text{ mm}^3$$

and the final volume of the plastic pipe is

$$V_{p\text{final}} = L_c A_{pf} \text{ or } V_{p\text{final}} = 8663 \text{ mm}^3$$

4. ***Finalize***: The ratio of final to initial volume reveals little change in the volume of the plastic pipe:

$$\frac{V_{p\text{final}}}{V_{p\text{init}}} = 0.99908$$

The numerical results obtained in this example illustrate that the dimensional changes in structural materials under normal loading conditions are extremely small. In spite of their smallness, changes in dimensions can be important in certain kinds of analysis (such as the analysis of statically indeterminate structures) and in the experimental determination of stresses and strains.

1.8 Shear Stress and Strain

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Diagonal bracing for an elevated walkway showing a clevis and a pin in double shear

The preceding sections discussed the effects of normal stresses produced by axial loads acting on straight bars. These stresses are called "normal stresses" because they act in directions *perpendicular* to the surface of the material. Now consider another kind of stress, called a **shear stress**, that acts *tangential* to the surface of the material.

As an illustration of the action of shear stresses, consider the bolted connection shown in Fig. 1-49a. This connection consists of a flat bar A, a clevis C, and a bolt B that pass through holes in the bar and clevis. Under the action of the tensile loads P, the bar and clevis press against the bolt in **bearing**, and contact stresses, called **bearing stresses**, are developed. In addition, the bar and clevis tend to shear the bolt, that is, cut through it, and this tendency is resisted by shear stresses in the bolt. As an example, see the bracing for an elevated pedestrian walkway shown in the photograph.

To show more clearly the actions of the bearing and shear stresses, look at this type of connection in a schematic side view (Fig. 1-49b). With this view in mind, draw a free-body diagram of the bolt (Fig. 1-49c). The bearing stresses exerted by the clevis against the bolt appear on the left-hand side of the free-body diagram and are labeled 1 and 3. The stresses from the bar appear on the right-hand side and are labeled 2. The actual distribution of the bearing stresses is difficult to determine, so it is customary to assume that the stresses are uniformly distributed. Based upon the assumption of uniform distribution, calculate an **average bearing** stress σ_b by dividing the total bearing force F_b by the bearing area A_b:

$$\sigma_b = \frac{F_b}{A_b} \qquad \textbf{(1-15)}$$

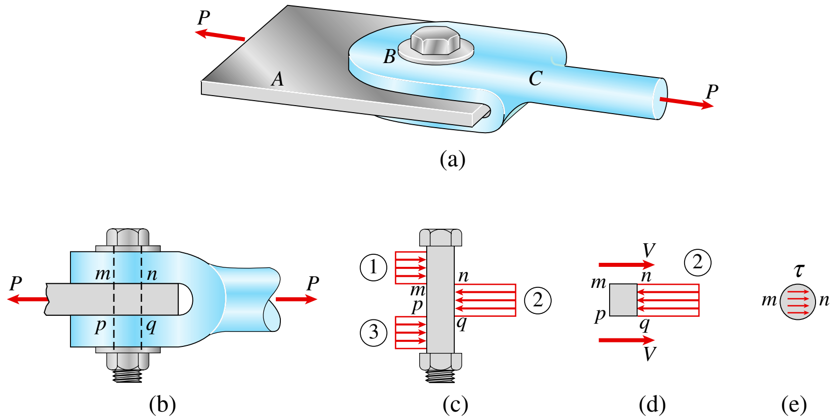


FIGURE 1-49

Bolted connection in which the bolt is loaded in double shear

The **bearing area** is defined as the projected area of the curved bearing surface. For instance, consider the bearing stresses labeled 1. The projected area A_b on which they act is a rectangle having a height equal to the thickness of the clevis and a width equal to the diameter of the bolt. Also, the bearing force F_b represented by the stresses labeled 1 is equal to $P/2$. The same area and the same force apply to the stresses labeled 3.

Now look at the bearing stresses between the flat bar and the bolt (the stresses labeled 2). For these stresses, the bearing area A_b is a rectangle with height equal to the thickness of the flat bar and width equal to the bolt diameter. The corresponding bearing force F_b is equal to the load P.

The free-body diagram of Fig. 1-49c shows that there is a tendency to shear the bolt along cross sections *mn* and *pq*. From a free-body diagram of the portion *mnpq* of the bolt (see Fig. 1-49d), note that shear forces V act over the cut surfaces of the bolt. There are two planes of shear (*mn* and *pq*), and the bolt is said to be in **double shear**. In double shear, each of the shear forces is equal to one-half of the total load transmitted by the bolt, that is, $V = P/2$.

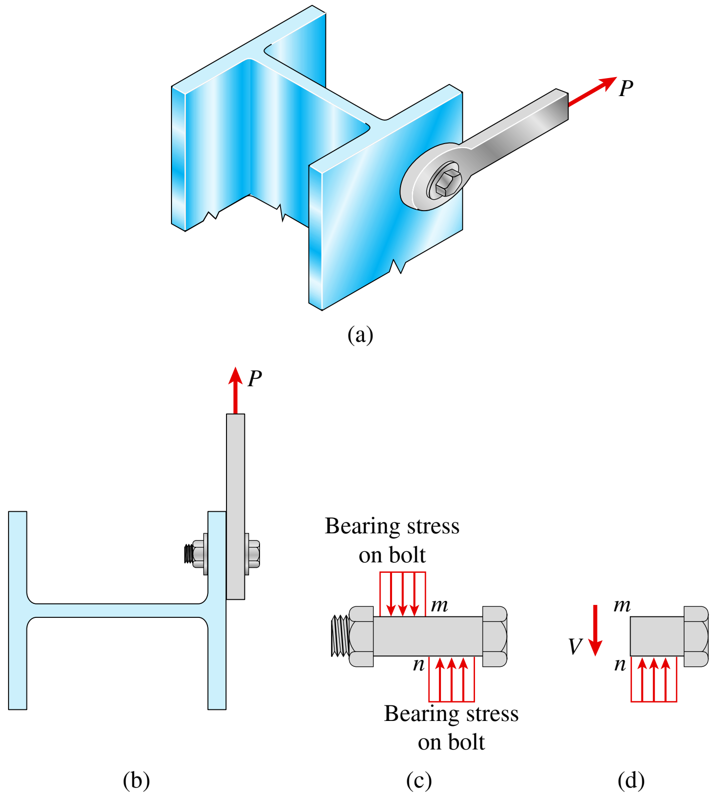
The shear forces V are the resultants of the shear stresses distributed over the cross-sectional area of the bolt. For instance, the shear stresses acting on cross section *mn* are shown in Fig. 1-49e. These stresses act parallel to the cut surface. The exact distribution of the stresses is not known, but they are highest near the center and become zero at certain locations on the edges. As indicated in Fig. 1-49e, shear stresses are customarily denoted by the Greek letter τ (tau).

A bolted connection in single shear is shown in Fig. 1-50a, where the axial force P in the metal bar is transmitted to the flange of the steel column through a bolt. A cross-sectional view of the column (Fig. 1-50b) shows the connection in more detail. Also, a sketch of the bolt (Fig. 1-50c) shows the assumed distribution of the bearing stresses acting on the bolt. The actual distribution of these bearing stresses is much more complex than shown in the figure. Furthermore, bearing stresses are also developed against the inside surfaces of the bolt head and nut. Thus, Fig. 1-50c is *not* a free-body diagram—only the idealized bearing stresses acting on the shank of the bolt are shown in the figure.

Cutting through the bolt at section *mn* reveals the diagram shown in Fig. 1-50d. This diagram includes the shear force V (equal to the load P) acting

FIGURE 1-50

Bolted connection in which the bolt is loaded in single shear



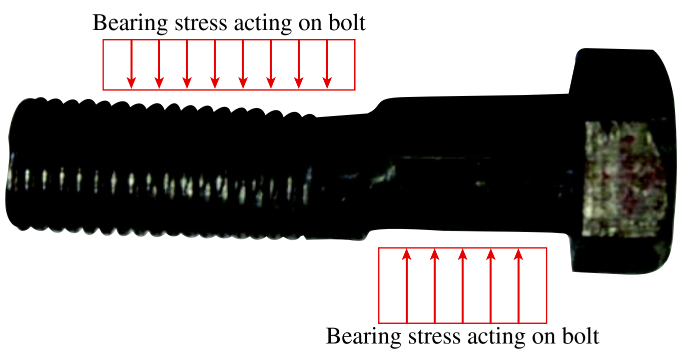
on the cross section of the bolt. This shear force is the resultant of the shear stresses that act over the cross-sectional area of the bolt.

The deformation of a bolt loaded almost to fracture in single shear is shown in Fig. 1-51 (compare with Fig. 1-50c).

The preceding discussion of bolted connections disregarded **friction** (produced by tightening of the bolts) between the connecting elements. The presence of friction means that part of the load is carried by friction forces, thereby reducing the loads on the bolts. Since friction forces are unreliable and difficult to estimate, it is common practice to err on the conservative side and omit them from the calculations.

FIGURE 1-51

Failure of a bolt in single shear (© Barry Goodno)



The **average shear stress** on the cross section of a bolt is obtained by dividing the total shear force V by the area A of the cross section on which it acts, as

$$\tau_{\text{aver}} = \frac{V}{A} \tag{1-16}$$

In the example of Fig. 1-50, which shows a bolt in *single shear*, the shear force V is equal to the load P, and the area A is the cross-sectional area of the bolt. However, in the example of Fig. 1-49, where the bolt is in *double shear*, the shear force V equals $P/2$.

Equation (1-19) shows that shear stresses, like normal stresses, represent intensity of force, or force per unit of area. Thus, the **units of** shear stress are the same as those for normal stress, namely, psi or ksi in USCS units and pascals or multiples thereof in SI units.

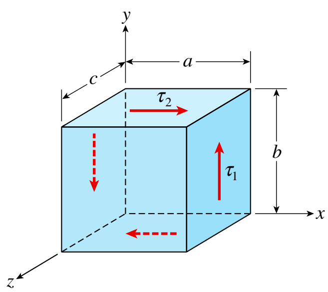
The loading arrangements shown in Figs. 1-49 and 1-50 are examples of **direct shear** (or *simple shear*) in which the shear stresses are created by the direct action of the forces trying to cut through the material. Direct shear arises in the design of bolts, pins, rivets, keys, welds, and glued joints.

Equality of Shear Stresses on Perpendicular Planes

To obtain a more complete picture of the action of shear stresses, consider a small element of material in the form of a rectangular parallelepiped having sides of lengths a, b, and c in the x, y, and z directions, respectively (Fig. 1-52).[6] The front and rear faces of this element are free of stress.

FIGURE 1-52

Small element of material subjected to shear stresses



Now assume that a shear stress τ_1 is distributed uniformly over the right-hand face, which has area bc. In order for the element to be in equilibrium in the y direction, the total shear force $\tau_1 bc$ acting on the right-hand face must be balanced by an equal but oppositely directed shear force on the left-hand face. Since the areas of these two faces are equal, it follows that the shear stresses on the two faces must be equal.

The forces $\tau_1 bc$ acting on the left- and right-hand side faces (Fig. 1-52) form a couple having a moment about the z axis of magnitude $\tau_1 bc$, acting counterclockwise in the figure.[7] Equilibrium of the element requires that this moment be balanced by an equal and opposite moment resulting from shear stresses acting on the top and bottom faces of the element. If the stresses on the top and bottom faces are labeled as τ_2, the corresponding horizontal shear forces equal $\tau_2 ac$. These forces form a clockwise couple of moment $\tau_2 abc$. From moment equilibrium of the element about the z axis, $\tau_1 abc$ equals $\tau_2 abc$, or

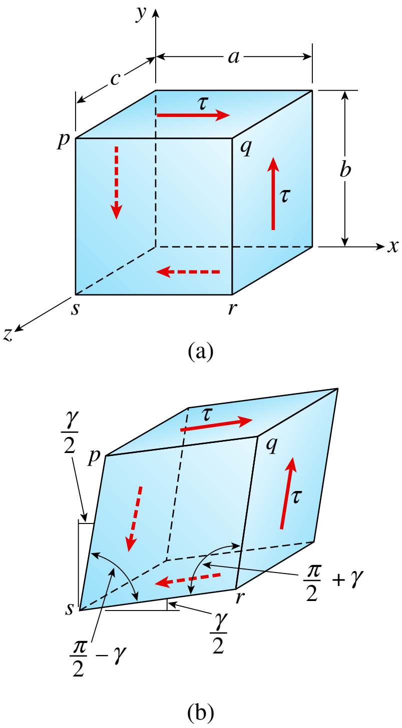
$$\tau_1 = \tau_2 \tag{1-17}$$

[6]A **parallelepiped** is a prism whose bases are parallelograms; thus, a parallelepiped has six faces, each of which is a parallelogram. Opposite faces are parallel and identical parallelograms. A **rectangular parallelepiped** has all faces in the form of rectangles.

[7]A **couple** consists of two parallel forces that are equal in magnitude and opposite in direction.

FIGURE 1-53

Element of material subjected to shear stresses and strains



Therefore, the magnitudes of the four shear stresses acting on the element are equal, as shown in Fig. 1-53a.

The following are observations regarding shear stresses acting on a rectangular element:

1. Shear stresses on opposite (and parallel) faces of an element are equal in magnitude and opposite in direction.
2. Shear stresses on adjacent (and perpendicular) faces of an element are equal in magnitude and have directions such that both stresses point toward, or both point away from, the line of intersection of the faces.

These observations apply to an element subjected only to shear stresses (no normal stresses), as pictured in Figs. 1-52 and 1-53. This state of stress is called **pure shear**.

For most purposes, the preceding conclusions remain valid even when normal stresses act on the faces of the element. The reason is that the normal stresses on opposite faces of a small element usually are equal in magnitude and opposite in direction.

Shear Strain

Shear stresses acting on an element of material (Fig. 1-53a) are accompanied by *shear strains*. As an aid in visualizing these strains, note that the shear stresses have no tendency to elongate or shorten the element in the x, y, and z directions—in other words, the lengths of the sides of the element do not change. Instead, the shear stresses produce a change in the *shape* of the element (Fig. 1-53b). The original element, which is a rectangular parallelepiped, is deformed into an oblique parallelepiped, and the front and rear faces become rhomboids.[8]

Because of this deformation, the angles between the side faces change. For instance, the angles at points q and s, which were $\pi/2$ before deformation, are reduced by a small angle γ to $\pi/2 - \gamma$ (Fig. 1-53b). At the same time, the angles at points p and r are increased to $\pi/2 + \gamma$. The angle γ is a measure of the **distortion**, or change in shape, of the element and is called the shear strain. Because **shear strain** is an angle, it is usually measured in degrees or radians.

Sign Conventions for Shear Stresses and Strains

As an aid in establishing sign conventions for shear stresses and strains, you need a scheme for identifying the various faces of a stress element (Fig. 1-53a). Henceforth, the faces oriented toward the positive directions of the axes are referred to as the positive faces of the element. In other words, a positive face has its outward normal directed in the positive direction of a coordinate axis. The opposite faces are negative faces. Thus, in Fig. 1-53a, the right-hand, top, and front faces are the positive x, y, and z faces, respectively, and the opposite faces are the negative x, y, and z faces.

[8]An **oblique angle** can be either acute or obtuse, but it is *not* a right angle. A **rhomboid** is a parallelogram with oblique angles and adjacent sides *not* equal. (A *rhombus* is a parallelogram with oblique angles and all four sides equal, sometimes called a *diamond-shaped figure*.)

Using the terminology described in the preceding paragraph, the sign convention for shear stresses is as follows:

> A shear stress acting on a positive face of an element is positive if it acts in the positive direction of one of the coordinate axes and negative if it acts in the negative direction of an axis. A shear stress acting on a negative face of an element is positive if it acts in the negative direction of an axis and negative if it acts in a positive direction.

Thus, all shear stresses shown in Fig. 1-53a are positive.

The sign convention for shear strains is as follows:

> Shear strain in an element is positive when the angle between two positive faces (or two negative faces) is reduced. The strain is negative when the angle between two positive (or two negative) faces is increased.

Thus, the strains shown in Fig. 1-53b are positive, and the positive shear stresses are accompanied by positive shear strains.

Hooke's Law in Shear

The properties of a material in shear can be determined experimentally from direct-shear tests or from torsion tests. The latter tests are performed by twisting hollow, circular tubes, thereby producing a state of pure shear. The results of these tests are used to plot **shear stress-strain diagrams** (that is, diagrams of shear stress τ versus shear strain γ). These diagrams are similar in shape to tension-test diagrams (σ versus ε) for the same materials, although they differ in magnitudes.

From shear stress-strain diagrams, you can obtain material properties such as the proportional limit, modulus of elasticity, yield stress, and ultimate stress. Numerical values of these properties in shear are usually about half as large as those in tension. For instance, the yield stress for structural steel in shear is 0.5 to 0.6 times the yield stress in tension.

For many materials, the initial part of the shear stress-strain diagram is a straight line through the origin, just as it is in tension. For this linearly elastic region, the shear stress and shear strain are proportional, resulting in the following equation for **Hooke's law in shear**:

$$\tau = G\gamma \qquad \textbf{(1-18)}$$

in which G is the **shear modulus of elasticity** (also called the *modulus of rigidity*).

The shear modulus G has the same units as the tension modulus E, namely, psi or ksi in USCS units and pascals (or multiples thereof) in SI units. For mild steel, typical values of G are 11,000 ksi or 75 GPa; for aluminum alloys, typical values are 4000 ksi or 28 GPa. Additional values are listed in Table I-2, Appendix I.

The moduli of elasticity in tension and shear are related by

$$G = \frac{E}{2(1 + \nu)} \qquad \textbf{(1-19)}$$

in which ν is Poisson's ratio. This relationship shows that E, G, and ν are not independent elastic properties of the material. Because the value of Poisson's ratio for ordinary materials is between zero and one-half, from Eq. (1-19) G must be from one-third to one-half of E.

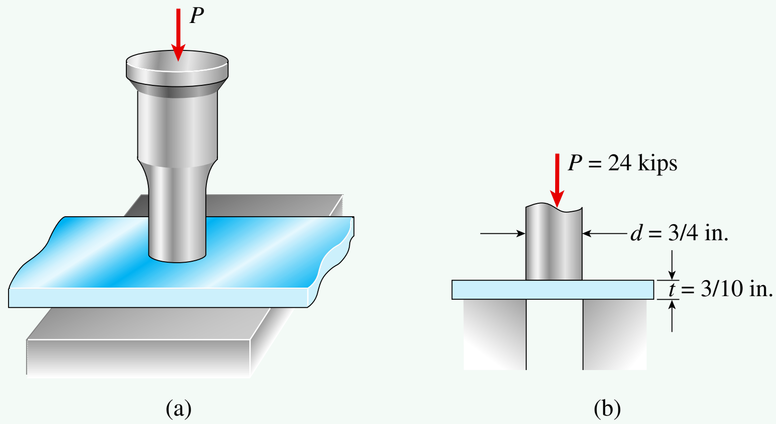
The following examples illustrate some typical analyses involving the effects of shear. Example 1-9 is concerned with shear stresses in a plate, Example 1-10 involves finding shear stresses and shear strains in an elastomeric bearing pad subjected to a horizontal shear force, and Example 1-11 deals with normal and shear stresses in a bolted bracket.

Example 1-9

A punch for making holes in steel plates is shown in Fig. 1-54a. Assume that a punch having diameter $d = 3/4$ in. is used to punch a hole in an 3/10 in. plate, as shown in the cross-sectional view (Fig. 1-54b).

If a force $P = 24$ kips is required to create the hole, what is the average shear stress in the plate and the average compressive stress in the punch?

FIGURE 1-54
Example 1-9: Punching a hole in a steel plate



Solution:

Use the four-step problem-solving approach to find average shear stress in the plate and average compressive stress in the punch.

1. ***Conceptualize***: Assume that the shaft of the punch is in compression over its entire length due to applied load P. Force P acts downward on the plate and is applied as a uniformly distributed force along a circle of diameter d as the punch passes through the plate.
2. ***Categorize***: The average shear stress in the plate is obtained by dividing the force P by the shear area of the plate. The shear area is the cylindrical area of the plate that is exposed when the punch passes through the plate. The compressive stress of interest is the one acting on a circular cross section through the lower segment of the punch (Fig. 1-54).
3. ***Analyze***: The shear area A_s is equal to the circumference of the hole times the thickness of the plate, or

$$A_s = \pi dt = \pi(3/4 \text{ in.})(3/10 \text{ in.}) = 0.707 \text{ in}^2$$

in which d is the diameter of the punch and t is the thickness of the plate. Therefore, the average shear stress in the plate is

$$\tau_{\text{aver}} = \frac{P}{A_s} = 24 \text{ kips}/0.707 \text{ in}^2 = 34 \text{ ksi}$$

The average compressive stress in the punch is

$$\sigma_c = \frac{P}{A_{\text{punch}}} = \frac{P}{\pi d^2/4} = 24 \text{ kips}/\pi(0.75 \text{ in.})^2/4 = 54.3 \text{ ksi}$$

in which A_{punch} is the cross-sectional area of the lower segment of the punch.

4. ***Finalize***: The normal and shear stress distributions are not uniform due to stress concentration effects; hence, the calculations result in "average" stresses. In addition, this analysis is highly idealized because impact effects that occur when a punch is rammed through a plate are not part of this analysis.

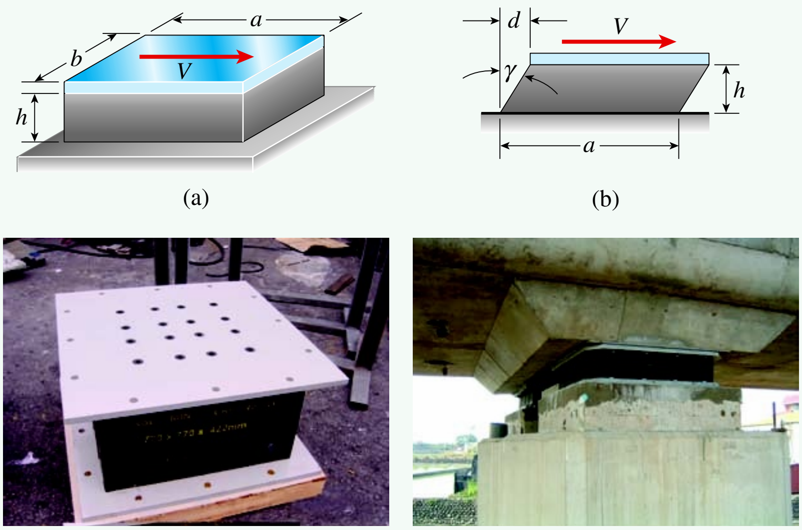
Example 1-10

A bearing pad of the kind used to support machines and bridge girders consists (see photos) of a linearly elastic material (usually an elastomer, such as rubber) capped by a steel plate (Fig. 1-55a). Assume that the thickness of the elastomer is h, the dimensions of the plate are $a \times b$, and the pad is subjected to a horizontal shear force V.

Obtain formulas for the average shear stress τ_{aver} in the elastomer and the horizontal displacement d of the plate (Fig. 1-55b).

FIGURE 1-55

Example 1-10: Bearing pad in shear (Courtesy of Mageba)



Solution:

Use the following four-step problem-solving approach to find average shear stress in the elastomer and horizontal displacement d of the plate.

1. ***Conceptualize***: The distortion of the bearing pad under shear force V is assumed to be linear through the thickness h, as shown in Fig. 1-55b.
2. ***Categorize***: Assume that the shear stresses in the elastomer are uniformly distributed throughout its entire volume and that the shear strain γ is small.
3. ***Analyze***: The shear stress on any horizontal plane through the elastomer equals the shear force V divided by the area ab of the plane (Fig. 1-55a):

$$\tau_{\text{aver}} = \frac{V}{ab} \tag{1-20}$$

The corresponding shear strain [from Hooke's law in shear; Eq. (1-18)] is

$$\gamma = \frac{\tau_{\text{aver}}}{G_e} = \frac{V}{abG_e} \tag{1-21}$$

in which G_e is the shear modulus of the elastomeric material. Finally, the horizontal displacement d is equal to h tan γ (from Fig. 1-55b):

$$d = h\tan\gamma = h\tan\left(\frac{V}{abG_e}\right) \tag{1-22}$$

In most practical situations, the shear strain γ is a small angle, and in such cases, replace $\tan\gamma$ with γ and obtain

$$d = h\gamma = \frac{hV}{abG_e} \tag{1-23}$$

For example, if $V = 0.8$ kN, $a = 75$ mm, $b = 60$ mm, $h = 20$ mm, and $G_e = 1.25$ MPa, Eq. (1-22) results in $d = 2.86$ mm, while Eq. (1-23) gives $d = 2.84$ mm.

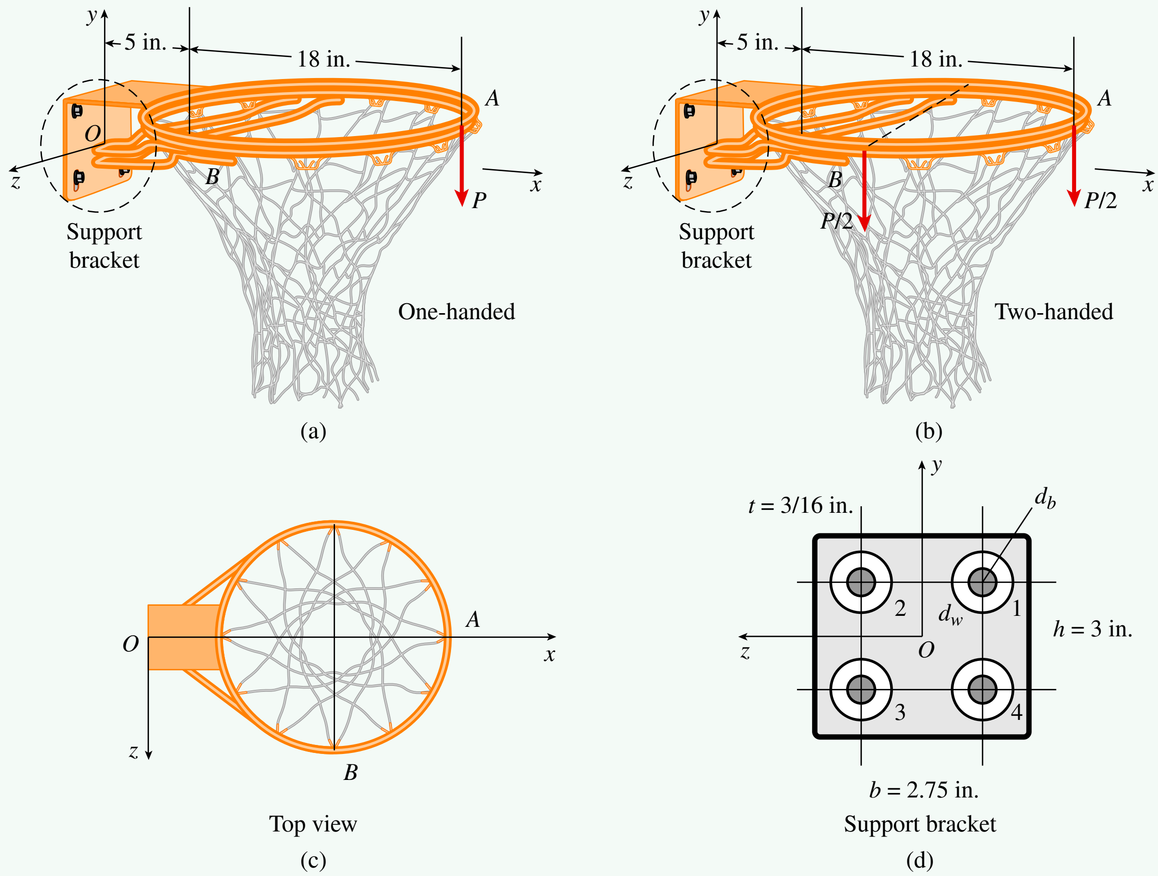
4. ***Finalize***: Equations (1-22) and (1-23) give approximate results for the horizontal displacement of the plate because they are based upon the assumption that the shear stress and strain are constant throughout the volume of the elastomeric material. In reality, the shear stress is zero at the edges of the material (because there are no shear stresses on the free vertical faces), and therefore, the deformation of the material is more complex than pictured in Fig. 1-55b. However, if the length a of the plate is large compared with the thickness h of the elastomer, the preceding results are satisfactory for design purposes.

Example 1-11

A basketball player hangs on the rim after dunking the ball. The player applies a downward force at point A with an estimated magnitude of $P = 400$ lb (Fig. 1-56a). Later, the player dunks again and hangs on the rim with two hands: one at A and one at B (see Fig. 1-56b, c). The rim and support bracket are bolted to the backboard using four bolts with washers (Fig. 1-56d). Find and compare the stresses in the bolted connection at bolt location 2 for the one-handed and two-handed load cases. Assume that the backboard is a fixed support and that bolt and washer diameters are $d_b = 3/8$ in. and $d_w = 5/8$ in., respectively. The support bracket has thickness $t = 3/16$ in.

FIGURE 1-56

(a) Load case 1—downward force on rim at A; (b) load case 2—forces applied at A and B; (c) top view of rim; (d) support bracket and bolt detail



Solution:

Use the following four-step problem-solving approach.

1. ***Conceptualize***: Find reaction forces and moments at support point O and then distribute forces and moments to each bolt location. The rim and bracket act

as a cantilever beam. From the free-body diagrams in Fig. 1-57, the reactions for each load case are as follows.

Reactions—Load case 1: Load P is applied in the $(-y)$ direction at A. Sum forces and moments to find:

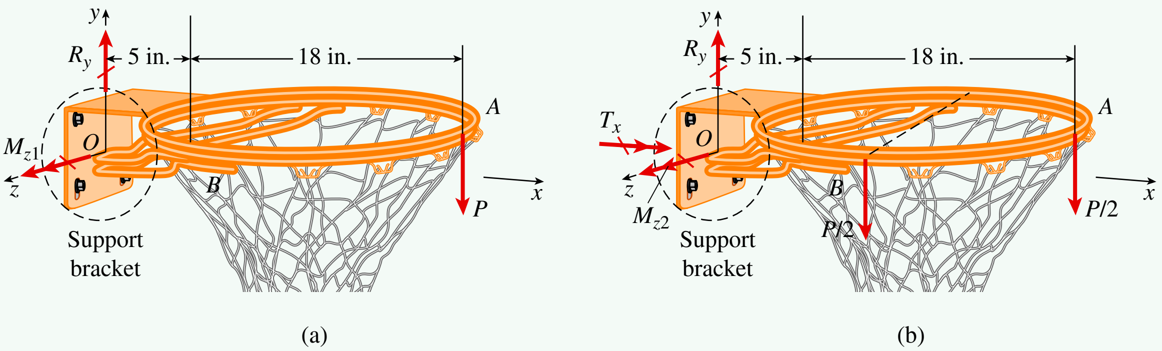
$$\Sigma F_y = 0 \quad R_y = P = 400 \text{ lb}$$
$$\Sigma M_z = 0 \quad M_{z1} = P(5 + 18) \text{ in.} = (400)(23) \text{ lb-in.} = 9200 \text{ lb-in.} \tag{a}$$

Reactions—Load case 2: Loads $P/2$ are applied at both A and B. Reactions at O are

$$\Sigma F_y = 0 \quad R_y = P = 400 \text{ lb}$$
$$\Sigma M_z = 0 \quad M_{z2} = \frac{P}{2}(5 + 9) \text{ in.} + \frac{P}{2}(23 \text{ in.}) = [200(14) + 200(23)] \text{ lb-in.} = 7400 \text{ lb-in.} \tag{b}$$
$$\Sigma M_x = 0 \quad T_x = -\frac{P}{2}(9 \text{ in.}) = -200(9) \text{ lb-in.} = -1800 \text{ lb-in.}$$

FIGURE 1-57

(a) Support reactions at O for load case 1; (b) support reactions at O for load case 2



Forces at bolt 2—Load case 1: Use the negatives of the reactions at O to find normal and shear forces acting on the bolts. From reaction R_y, downward shear force $P/4 = 100$ lb acts at each of the four bolt locations (Fig. 1-58a). Replace moment M_{z1} [Eq. (a)] with two force couples, each equal to $(N_1)(h)$ (see Fig. 1-58a) so normal force N_1 acts in the $(+x)$ direction at bolt 2 and is computed as

$$N_1 = \frac{M_{z1}}{2h} = \frac{23P}{2(3 \text{ in.})} = \frac{9200 \text{ lb-in.}}{6 \text{ in.}} = 1533 \text{ lb} \tag{c}$$

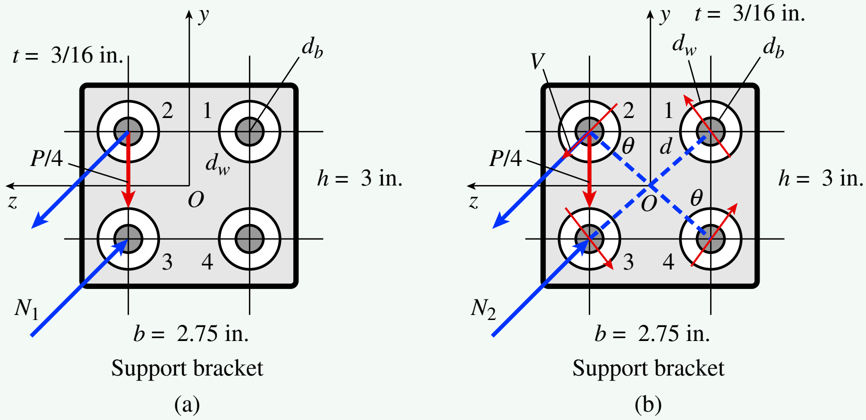
Forces at bolt 2—Load case 2: Reaction R_y is the same in load cases 1 and 2, so downward shear force $P/4 = 100$ lb acts at bolt 2 (Fig. 1-58b). Replace moment M_{z2} [Eq. (b)] with two force couples (Fig. 1-58b), so the tension force on bolt 2 is

$$N_2 = \frac{M_{z2}}{2h} = \frac{7400 \text{ lb-in.}}{6 \text{ in.}} = 1233 \text{ lb} \tag{d}$$

Load case 2 also creates a torsional reaction moment T_x [see Eq. (b)] that can be replaced by two counterclockwise force couples each equal to

FIGURE 1-58

(a) Bolt 2 forces for load case 1; (b) bolt 2 forces for load case 2



$(V)(d)$ (see Fig. 1-58b) where $d = \sqrt{b^2 + h^2} = 4.0697$ in. Compute the additional in-plane shear force on bolt 2 as

$$V = \frac{T_x}{2d} = \frac{1800 \text{ lb-in.}}{2(4.0697 \text{ in.})} = 221.15 \text{ lb} \quad \textbf{(e)}$$

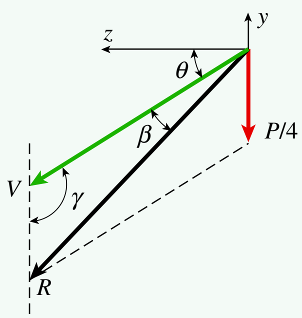
The line of action of force V is shown in Fig. 1-58(b) at angle $\theta = \tan^{-1}\left(\frac{b}{h}\right) = 42.51°$. The total in-plane shear force on bolt 2 is the resultant R of forces V and $P/4$ computed as

$$R = \sqrt{(V\cos\theta)^2 + \left(\frac{P}{4} + V\sin\theta\right)^2} = \sqrt{(163.02)^2 + (100 + 149.43)^2} \text{ lb} = 298 \text{ lb} \quad \textbf{(f)}$$

Resultant R also can be found using the parallelogram law, as shown in Fig. 1-59 with $\beta = 14.32°$ and $\gamma = 132.51°$.

FIGURE 1-59

Resultant R using parallelogram law



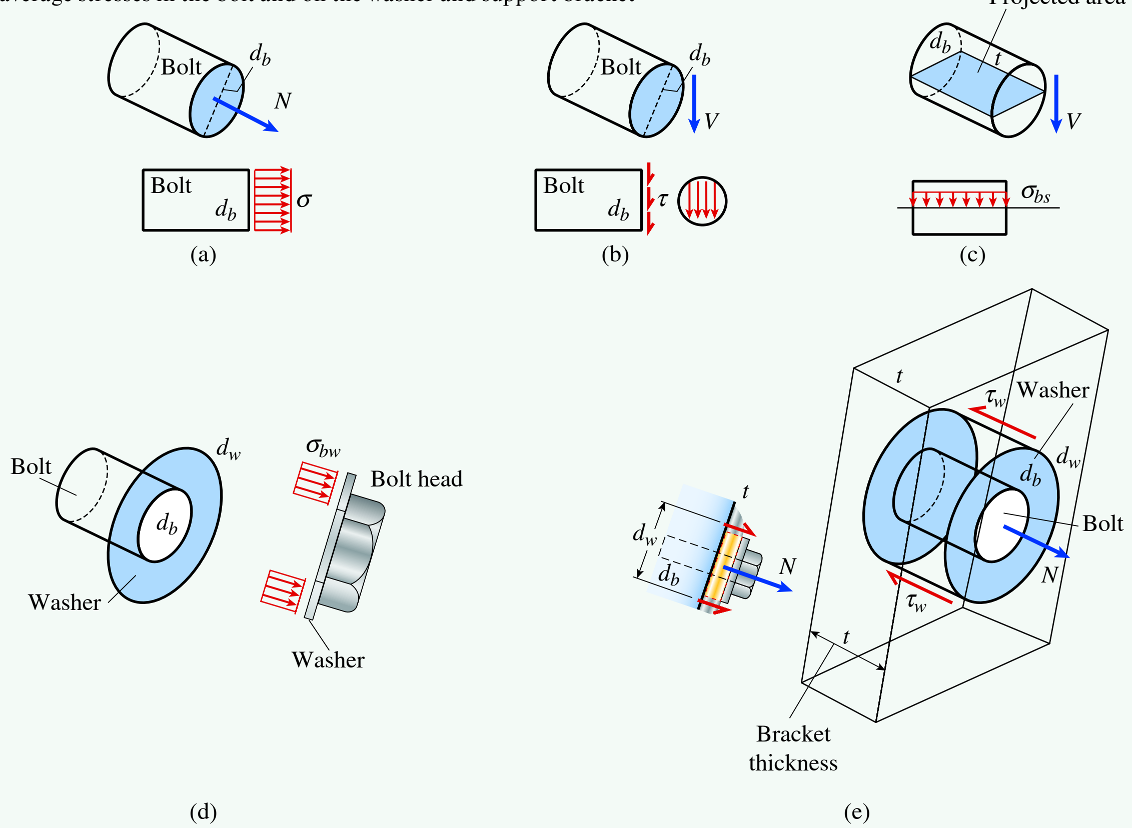
2. ***Categorize***: Use the forces acting on bolt 2 in simple formulas to compute average stresses in the bolt and on the washer and support bracket at bolt location 2. The five connection stresses of interest are (a) normal stress in bolt (Fig. 1-60a); (b) shear stress in bolt (Fig. 60b); (c) bearing stress on shank of bolt (Fig. 1-60c); (d) bearing stress on washer (Fig. 1-60d); and (e) shear stress through bracket on periphery of washer (Fig. 1-60e).
3. ***Analyze***: The five connection stresses at bolt location 2 are listed in Table 1-2 for load cases 1 and 2. Numerical dimensions for the bolt, washer, and bracket are $d_b = 3/8$ in., $d_w = 5/8$ in., and $t = 3/16$ in. Areas needed in stress calculations are

Cross-sectional area of bolt:

$$A_b = \frac{\pi}{4} d_b^2 = 0.1104 \text{ in}^2$$

FIGURE 1-60

Five average stresses in the bolt and on the washer and support bracket



Surface area of washer:

$$A_w = \frac{\pi}{4}\left(d_w^2 - d_b^2\right) = 0.1963\ \text{in}^2$$

Cylindrical area through bracket on periphery of washer:

$$A_p = \pi d_w t = 0.3682\ \text{in}^2$$

4. ***Finalize***: Shear and bearing stresses on bolt 2 (items b and c in Table 1-2) are increased three-fold for load case 2 when torsional moment T_x [see Eq. (b)] is applied to the bracket. The other three bolt stresses differ by about 25% for the two load cases. The average stresses illustrated in Fig. 1-60 and listed in Table 1-2 are only approximations to the true state of stress at one of the four bolt locations on the support bracket. The true maximum stresses are likely to be higher for a variety of reasons, such as localized stress concentrations, pretensioning of bolts, and impact aspects of the loading. The stress levels computed here are low. If stress values are higher, a more detailed and sophisticated analysis using computer models employing the finite element method may be required.

Table 1-2

Five connection stresses at bolt location 2

Connection Stress	Load Case 1	Load Case 2
a. Normal stress in bolt	$\dfrac{N_1}{A_b} = 13{,}880$ psi	$\dfrac{N_2}{A_b} = 11{,}164$ psi
b. Shear stress in bolt	$\dfrac{P}{4A_b} = 905$ psi	$\dfrac{R}{A_b} = 2698$ psi
c. Bearing stress on shank of bolt	$\dfrac{P}{4d_b t} = 1422$ psi	$\dfrac{R}{d_b t} = 4238$ psi
d. Bearing stress on washer	$\dfrac{N_1}{A_w} = 7808$ psi	$\dfrac{N_2}{A_w} = 6280$ psi
e. Shear stress through bracket on periphery of washer	$\dfrac{N_1}{A_p} = 4164$ psi	$\dfrac{N_2}{A_p} = 3349$ psi

1.9 Allowable Stresses and Allowable Loads

Engineers design a seemingly endless variety of objects to serve the basic needs of society. These needs include housing, agriculture, transportation, communication, and many other aspects of modern life. Factors to be considered in design include functionality, strength, appearance, economics, and environmental effects. However, when studying mechanics of materials, our principal design interest is **strength**, that is, *the capacity of the object to support or transmit loads*. Objects that must sustain loads include buildings, machines, containers, trucks, aircraft, ships, and the like. For simplicity, all such objects are referred to as **structures**; thus, a *structure is any object that must support or transmit loads*.

Factors of Safety

Strength is the ability of a structure to resist loads. *The actual strength of a structure must exceed the required strength*. The ratio of the actual strength to the required strength is called the **factor of safety** n:

$$\text{Factor of safety } n = \frac{\text{Actual strength}}{\text{Required strength}} \tag{1-24}$$

Of course, the factor of safety must be greater than 1.0 if failure is to be avoided. Depending upon the circumstances, factors of safety from slightly above 1.0 to as much as 10 are used.

The incorporation of factors of safety into design is not a simple matter, because both strength and failure have many different meanings. Strength may be measured by the load-carrying capacity of a structure, or it may be measured by the stress in the material. Failure may mean the fracture and complete collapse of a structure, or it may mean that the deformations have become so large that the structure can no longer perform its intended functions. The latter kind of failure may occur at loads much smaller than those that cause actual collapse.

The determination of a factor of safety must also take into account such matters as the following: probability of accidental overloading of the structure by loads that exceed the design loads; types of loads (static or dynamic); whether the loads are applied once or are repeated; how accurately the loads are known; possibilities for fatigue failure; inaccuracies in construction; variability in the quality of workmanship; variations in properties of materials; deterioration due to corrosion or other environmental effects; accuracy of the methods of analysis; whether failure is gradual (ample warning) or sudden (no warning); consequences of failure (minor damage or major catastrophe); and other such considerations. If the factor of safety is too low, the likelihood of failure will be high and the structure will be unacceptable; if the factor is too large, the structure will be wasteful of materials and perhaps unsuitable for its function (for instance, it may be too heavy).

Because of these complexities and uncertainties, factors of safety must be determined on a probabilistic basis. They usually are established by groups of experienced engineers who write the codes and specifications used by other designers, and sometimes they are even enacted into law. The provisions of codes and specifications are intended to provide reasonable levels of safety without unreasonable costs.

In aircraft design, it is customary to speak of the **margin of safety** rather than the factor of safety. The margin of safety is defined as the factor of safety minus one:

$$\text{Margin of safety} = n - 1 \qquad \textbf{(1-25)}$$

Margin of safety is often expressed as a percent, in which case the value given above is multiplied by 100. Thus, a structure having an actual strength that is 1.75 times the required strength has a factor of safety of 1.75 and a margin of safety of 0.75 (or 75%). When the margin of safety is reduced to zero or less, the structure (presumably) will fail.

Allowable Stresses

Factors of safety are defined and implemented in various ways. For many structures, it is important that the material remain within the linearly elastic range in order to avoid permanent deformations when the loads are removed. Under these conditions, the factor of safety is established with respect to yielding of the structure. Yielding begins when the yield stress is reached at *any* point within the structure. Therefore, by applying a factor of safety with respect to the yield stress (or yield strength), you obtain an **allowable stress** (or *working stress*) that must not be exceeded anywhere in the structure. Thus,

$$\text{Allowable stress} = \frac{\text{Yield strength}}{\text{Factor of safety}} \qquad \textbf{(1-26)}$$

or for tension and shear, respectively,

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{n_1} \quad \text{and} \quad \tau_{\text{allow}} = \frac{\tau_Y}{n_2} \qquad \textbf{(1-27a,b)}$$

in which σ_Y and τ_Y are the yield stresses and n_1 and n_2 are the corresponding factors of safety. In building design, a typical factor of safety with respect to yielding in tension is 1.67; thus, a mild steel having a yield stress of 36 ksi has an allowable stress of 21.6 ksi.

Sometimes the factor of safety is applied to the **ultimate stress** instead of the yield stress. This method is suitable for brittle materials, such as concrete and some plastics, and for materials without a clearly defined yield stress, such as wood and high-strength steels. In these cases, the allowable stresses in tension and shear are

$$\sigma_{allow} = \frac{\sigma_U}{n_3} \quad \text{and} \quad \tau_{allow} = \frac{\tau_U}{n_4} \tag{1-28}$$

in which σ_U and τ_U are the ultimate stresses (or ultimate strengths). Factors of safety with respect to the ultimate strength of a material are usually larger than those based upon yield strength. In the case of mild steel, a factor of safety of 1.67 with respect to yielding corresponds to a factor of approximately 2.8 with respect to the ultimate strength.

Allowable Loads

After the allowable stress has been established for a particular material and structure, the **allowable load** on that structure can be determined. The relationship between the allowable load and the allowable stress depends upon the type of structure. This chapter is concerned only with the most elementary kinds of structures, namely, bars in tension or compression and pins (or bolts) in direct shear and bearing.

In these kinds of structures, the stresses are uniformly distributed (or at least *assumed* to be uniformly distributed) over an area. For instance, in the case of a bar in tension, the stress is uniformly distributed over the cross-sectional area provided the resultant axial force acts through the centroid of the cross section. The same is true of a bar in compression provided the bar is not subject to buckling. In the case of a pin subjected to shear, consider only the average shear stress on the cross section, which is equivalent to assuming that the shear stress is uniformly distributed. Similarly, consider only an average value of the bearing stress acting on the projected area of the pin.

Therefore, in all four of the preceding cases, the **allowable load** (also called the *permissible load* or the *safe load*) is equal to the allowable stress times the area over which it acts:

$$\text{Allowable load} = (\text{Allowable stress})(\text{Area}) \tag{1-29}$$

For bars in direct *tension* and *compression* (no buckling), this equation becomes

$$P_{allow} = \sigma_{allow} A \tag{1-30}$$

in which σ_{allow} is the permissible normal stress and A is the cross-sectional area of the bar. If the bar has a hole through it, the *net area* is normally used when the bar is in tension. The **net area** is the gross cross-sectional area minus the area removed by the hole. For compression, the gross area may be used if the hole is filled by a bolt or pin that can transmit the compressive stresses.

For pins in *direct shear*, Eq. (1-29) becomes

$$P_{allow} = \tau_{allow} A \tag{1-31}$$

in which τ_{allow} is the permissible shear stress and A is the area over which the shear stresses act. If the pin is in single shear, the area is the cross-sectional area of the pin; in double shear, it is twice the cross-sectional area.

Finally, the permissible load based upon *bearing* is

$$P_{\text{allow}} = \sigma_b A_b \tag{1-32}$$

in which σ_b is the allowable bearing stress and A_b is the projected area of the pin or other surface over which the bearing stresses act.

The following example illustrates how allowable loads are determined when the allowable stresses for the material are known.

Example 1-12

A steel bar serving as a vertical hanger to support heavy machinery in a factory is attached to a support by the bolted connection shown in Fig. 1-61. Two clip angles (thickness $t_c = 9.5$ mm) are fastened to an upper support by bolts 1 and 2 each with a diameter of 12 mm; each bolt has a washer with a diameter of $d_w = 28$ mm. The main part of the hanger is attached to the clip angles by a single bolt (bolt 3 in Fig. 1-61a) with a diameter of $d = 25$ mm. The hanger has a rectangular cross section with a width of $b_1 = 38$ mm and thickness of $t = 13$ mm, but at the bolted connection, the hanger is enlarged to a width of $b_2 = 75$ mm. Determine the allowable value of the tensile load P in the hanger based upon the following considerations.

(a) The allowable tensile stress in the main part of the hanger is 110 MPa.

(b) The allowable tensile stress in the hanger at its cross section through the bolt 3 hole is 75 MPa. (The permissible stress at this section is lower because of the stress concentrations around the hole.)

(c) The allowable bearing stress between the hanger and the shank of bolt 3 is 180 MPa.

(d) The allowable shear stress in bolt 3 is 45 MPa.

(e) The allowable normal stress in bolts 1 and 2 is 160 MPa.

(f) The allowable bearing stress between the washer and the clip angle at either bolt 1 or 2 is 65 MPa.

(g) The allowable shear stress through the clip angle at bolts 1 and 2 is 35 MPa.

Solution:

Use a four-step problem-solving approach to find the allowable value of the tensile load P in the hanger based upon a variety of different allowable stresses in the different connection components.

1. ***Conceptualize***: Start by sketching a series of free-body diagrams to find the forces acting on each connection component. Express the force on each component in terms of an allowable stress times the associated area upon which it acts. This force is the allowable value of applied load P for that stress condition. Each of the seven stress states [(a)–(g) in the problem statement] and the associated applied load are illustrated in this example's figures; each is adjacent to the corresponding calculations in Step (3).

2. ***Categorize*:** Compute seven different values of the allowable load P, each based on an allowable stress and a corresponding area. The minimum value of load P will control.

Numerical data for the hanger connection design shown in Fig. 1-61 are as follows.

(a) Connection component dimensions:

$$t_c = 9.5 \text{ mm} \quad t = 13 \text{ mm} \quad b_1 = 38 \text{ mm} \quad b_2 = 75 \text{ mm}$$
$$d_1 = 12 \text{ mm} \quad d = 25 \text{ mm} \quad d_w = 28 \text{ mm}$$

(b) Allowable stresses:

$$\sigma_a = 110 \text{ MPa} \quad \sigma_{a3} = 75 \text{ MPa} \quad \sigma_{ba3} = 180 \text{ MPa} \quad \tau_{a3} = 45 \text{ MPa}$$
$$\tau_{a1} = 35 \text{ MPa} \quad \sigma_{a1} = 160 \text{ MPa} \quad \sigma_{ba1} = 65 \text{ MPa}$$

3. ***Analyze*:**

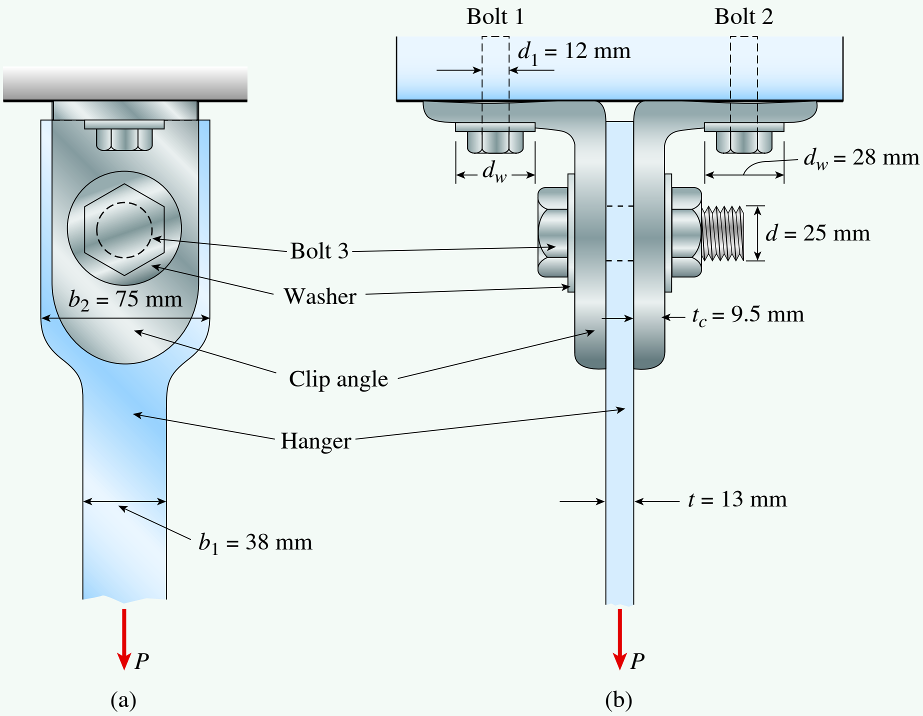
Part (a): Find the allowable load based upon the *stress in the main part of the hanger* (Fig. 1-61c). This is equal to the allowable stress in tension (110 MPa) times the cross-sectional area of the hanger (Eq. 1-30):

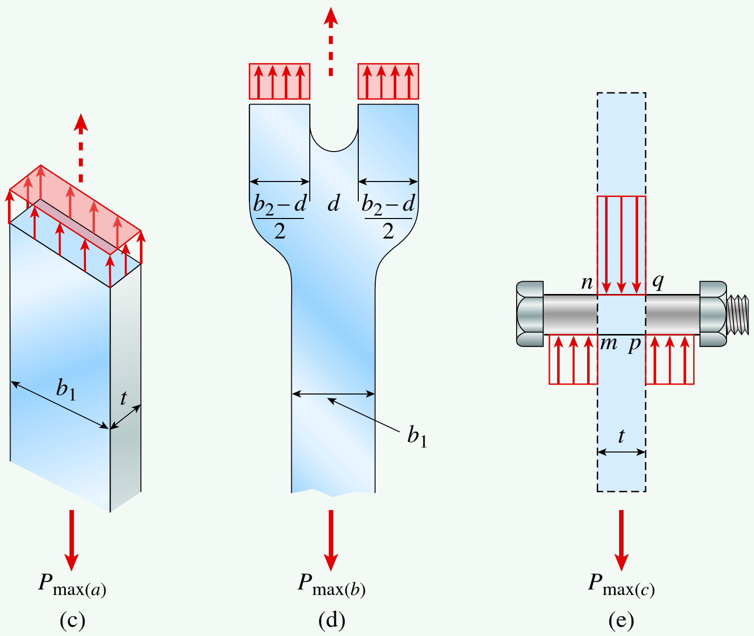
$$P_a = \sigma_a b_1 t = (110 \text{ MPa})(38 \text{ mm} \times 13 \text{ mm}) = 54.3 \text{ kN}$$

A load greater than this value will overstress the main part of the hanger (that is, the actual stress will exceed the allowable stress), thereby reducing the factor of safety.

FIGURE 1-61

Example 1-12: Vertical hanger subjected to a tensile load P: (a) front view of bolted connection and (b) side view of connection





Part (b): Find the allowable load based upon the allowable tensile stress (75 MPa) in the hanger at its cross section through the bolt 3 hole.

At the cross section of the hanger through the bolt hole (Fig. 1-61d), make a similar calculation but with a different allowable stress and a different area. The net cross-sectional area (that is, the area that remains after the hole is drilled through the bar) is equal to the net width times the thickness. The net width is equal to the gross width b_2 minus the diameter d of the hole. Thus, the equation for the allowable load P_b at this section is

$$P_b = \sigma_{a3}(b_2 - d)t = (75 \text{ MPa})(75 \text{ mm} - 25 \text{ mm})(13 \text{ mm}) = 48.8 \text{ kN}$$

Part (c): Now find the allowable load based upon the allowable bearing stress (180 MPa) between the hanger and the shank of bolt 3.

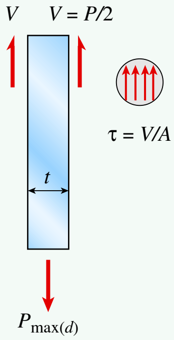
The allowable load based upon bearing between the hanger and the bolt (Fig. 1-61e) is equal to the allowable bearing stress times the bearing area. The bearing area is the projection of the actual contact area, which is equal to the bolt diameter times the thickness of the hanger. Therefore, the allowable load (Eq. 1-32) is

$$P_c = \sigma_{ba3}dt = 58.5 \text{ kN} = (180 \text{ MPa})(25 \text{ mm})(13 \text{ mm}) = 58.5 \text{ kN}$$

Part (d): Determine the allowable load based upon the allowable shear stress (45 MPa) in bolt 3.

The allowable load P_d based upon shear in the bolt (Fig. 1-61f) is equal to the allowable shear stress times the shear area (Eq. 1-31). The shear area is twice the area of the bolt because the bolt is in double shear; thus,

$$P_d = 2\tau_{a3}\left(\frac{\pi}{4}d^2\right) = 2(45 \text{ MPa})\left[\frac{\pi}{4}(25 \text{ mm})^2\right] = 44.2 \text{ kN}$$

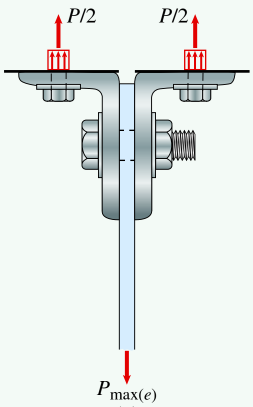


(f)

Part (e): Find the allowable load based upon the allowable normal stress (160 MPa) in bolts 1 and 2.

The allowable normal stress in bolts 1 and 2 is 160 MPa. Each bolt carries one half of the applied load P (see Fig. 1-61g). The allowable total load P_e is the product of the allowable normal stress in the bolt and the sum of the cross-sectional areas of bolts 1 and 2:

$$P_e = \sigma_{a1}(2)\left(\frac{\pi}{4}d_1^2\right) = (160\text{ MPa})(2)\left[\frac{\pi}{4}(12\text{ mm})^2\right] = 36.2\text{ kN}$$

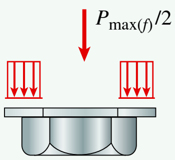


(g)

Part (f): Now find the allowable load based upon the allowable bearing stress (65 MPa) between the washer and the clip angle at either bolt 1 or 2.

The allowable bearing stress between the washer and the clip angle at either bolt 1 or 2 is 65 MPa. Each bolt (1 or 2) carries one half of the applied load P (see Fig. 1-61h). The bearing area here is the ring-shaped circular area of the washer (the washer is assumed to fit snugly against the bolt). The allowable total load P_f is the allowable bearing stress on the washer times twice the area of the washer:

$$P_f = \sigma_{ba1}(2)\left[\frac{\pi}{4}(d_w^2 - d_1^2)\right] = (65\text{ MPa})(2)\left\{\frac{\pi}{4}[(28\text{ mm})^2 - (12\text{ mm})^2]\right\}$$
$$= 65.3\text{ kN}$$



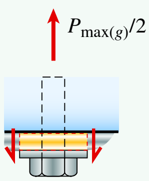
(h)

Part (g): Finally, determine the allowable load based upon the allowable shear stress (35 MPa) through the clip angle at bolts 1 and 2.

The allowable shear stress through the clip angle at bolts 1 and 2 is 35 MPa. Each bolt (1 or 2) carries one half of the applied load P (see Fig. 1-61i). The shear area at each bolt is equal to the circumference of the hole $(\pi \times d_w)$ times the thickness of the clip angle (t_c).

The allowable total load P_g is the the allowable shear stress times twice the shear area:

$$P_g = \tau_{a1}(2)(\pi d_w t_c) = (35\text{ MPa})(2)(\pi \times 28\text{ mm} \times 9.5\text{ mm}) = 58.5\text{ kN}$$



(i)

4. ***Finalize***: All seven conditions were used to find the allowable tensile loads in the hanger. Comparing the seven preceding results shows that the smallest value of the load is $P_{\text{allow}} = 36.2\text{ kN}$. This load is based upon normal stress in bolts 1 and 2 [see part (e)] and is the allowable tensile load for the hanger.

A more refined analysis that includes the weight of the entire hanger assembly can be carried out as shown in Example 1-6. As in Example 1-11, note that these computed stresses are average values only and do not include localized effects such as stress concentrations around bolt holes.

1.10 Design For Axial Loads and Direct Shear

The preceding section discussed the determination of allowable loads for simple structures, and earlier sections showed how to find the stresses, strains, and deformations of bars. The determination of such quantities is known as **analysis**. In the context of mechanics of materials, analysis consists of determining the *response* of a structure to loads, temperature changes, and other physical actions. The response of a structure means the stresses, strains, and deformations produced by the loads.

Response also refers to the load-carrying capacity of a structure; for instance, the allowable load on a structure is a form of response.

A structure is said to be *known* (or *given*) when there is a complete physical description of the structure, that is, all of its *properties* are known. The properties of a structure include the types of members and how they are arranged, the dimensions of all members, the types of supports and where they are located, the materials used, and the properties of the materials. Thus, when analyzing a structure, *the properties are given, and the response is to be determined.*

The inverse process is called **design**. When designing a structure, *you must determine the properties of the structure in order that the structure will support the loads and perform its intended functions.* For instance, a common design problem in engineering is to determine the size of a member to support given loads. Designing a structure is usually a much lengthier and more difficult process than analyzing it—indeed, analyzing a structure, often more than once, is typically part of the design process.

This section covers design in its most elementary form by calculating the required sizes of simple tension and compression members as well as pins and bolts loaded in shear. In these cases, the design process is quite straightforward. Knowing the loads to be transmitted and the allowable stresses in the materials, you can calculate the required areas of members from the following general relationship [compare with Eq. (1-29)]:

$$\text{Required area} = \frac{\text{Load to be transmitted}}{\text{Allowable stress}} \tag{1-33}$$

Apply this equation to any structure in which the stresses are uniformly distributed over the area. (The use of this equation for finding the size of a cable in tension and the size of a pin in shear is illustrated in Example 1-13.)

In addition to **strength** considerations, as shown by Eq. (1-33), the design of a structure is likely to involve **stiffness** and **stability**. Stiffness refers to the ability of the structure to resist changes in shape (for instance, to resist stretching, bending, or twisting), and stability refers to the ability of the structure to resist buckling under compressive stresses. Limitations on stiffness are sometimes necessary to prevent excessive deformations, such as large deflections of a beam that might interfere with its performance. Buckling is the principal consideration in the design of columns, which are slender compression members (Chapter 11).

Another part of the design process is **optimization**, which is the task of designing the best structure to meet a particular goal, such as minimum weight. For instance, there are many structures that can support a given load, but in some circumstances the best structure is the lightest one. Of course, a goal such as minimum weight usually must be balanced against more general considerations, including the aesthetic, economic, environmental, political, and technical aspects of the particular design project.

When analyzing or designing a structure, the forces that act on it are **loads** and **reactions**. Loads are *active forces* that are applied to the structure by some external cause, such as gravity, water pressure, wind, and earthquake ground motion. Reactions are *passive forces* that are induced at the supports of the structure—their magnitudes and directions are determined by the nature of the structure itself. Thus, reactions must be calculated as part of the analysis, whereas loads are known in advance.

Example 1-13

Continuous cable ADB runs over a small frictionless pulley at D to support beam $OABC$, which is part of an entrance canopy for a building (see Fig. 1-62). Load $P = 1000$ lb is applied at the end of the canopy at C. Assume that the canopy segment has weight $W = 1700$ lb.

(a) Find cable force T and pin support reactions at O and D.

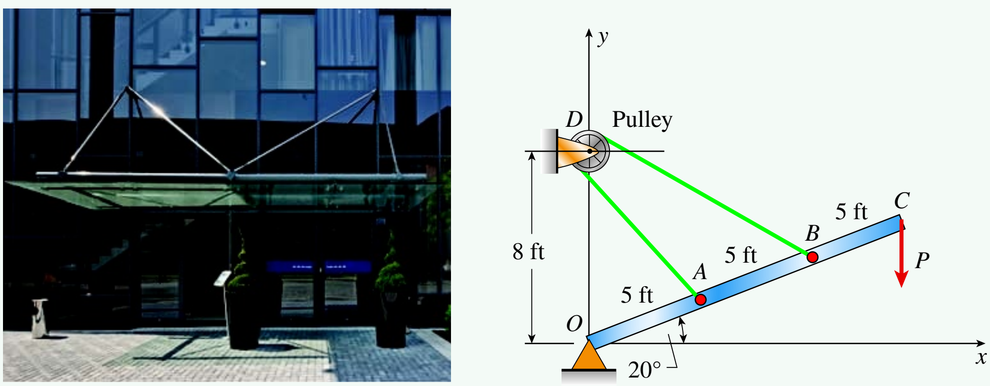
(b) Find the required cross-sectional area of cable ADB if the allowable normal stress is 18 ksi.

(c) Determine the required diameter of the pins at O, A, B, and D if the allowable stress in shear is 12 ksi.

(*Note*: The pins at O, A, B, and D are in double shear. Also, consider only load P and the weight W of the canopy; disregard the weight of cable ADB.)

FIGURE 1-62

(a) Inclined canopy at entrance to building; (b) two-dimensional model of one beam and supporting cable (© Archimage / Alamy Stock Photo)



(a) (b)

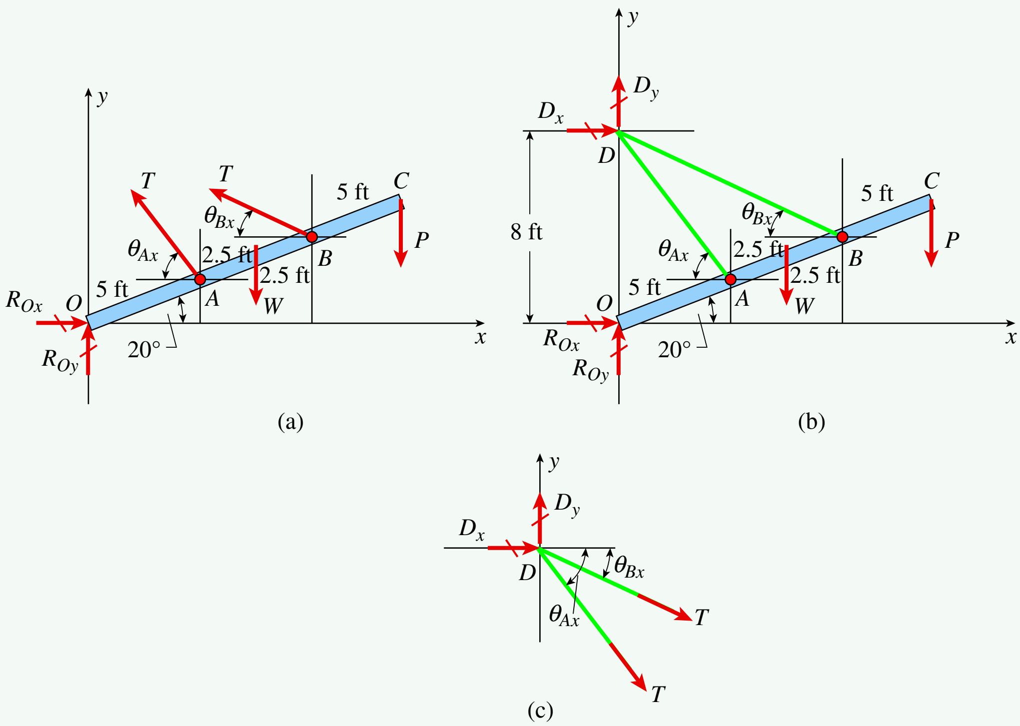
Solution:

Use the following four-step problem-solving approach.

1. ***Conceptualize***: Begin with a free-body diagram of beam $OABC$ (Fig. 1-63a). Also sketch free-body diagrams of the entire structure (Fig. 1-63b) and of joint D alone (Fig. 1-63c). Show cable force T and all applied and reaction force components.

FIGURE 1-63

Free-body diagrams of: (a) beam *OABC*; (b) beam and cable structure; (c) joint *D*



2. ***Categorize*:** First, use the free-body diagram of beam *OABC* (Fig. 1-63a) to find cable force *T* and reaction force components at *O*. Then use Fig. 1-63b or Fig. 1-63c to find reaction forces at *D*. Use cable force *T* and the allowable normal stress to find the required cross-sectional area of the cable. Also use force *T* and the allowable shear stress to find the required diameter of the pins at *A* and *B*. Use the resultant reaction forces at *O* and *D* to find pin diameters at these locations.
3. ***Analyze*:**

Cable force *T*: First, find required distances and angles in Fig. 1-63a:

$$AD = \sqrt{5^2 + 8^2 - 2(5)(8)\cos 70°}\text{ ft} = 7.851\text{ ft} \qquad BD = \sqrt{10^2 + 8^2 - 2(10)(8)\cos 70°}\text{ ft} = 10.454\text{ ft}$$

$$\theta_{Ax} = \sin^{-1}\left(\frac{8\text{ ft}}{AD}\sin 70°\right) - 20° = 53.241° \qquad \theta_{Bx} = \sin^{-1}\left(\frac{8\text{ ft}}{BD}\sin 70°\right) - 20° = 25.983°$$

Now sum moments about *O* in Fig. 1-63a to find tension *T* in continuous cable *ADB*:

$$T = \frac{W(7.5\text{ ft})(\cos 20°) + P(15\text{ ft})(\cos 20°)}{d_1 + d_2} = 2177\text{ lb} \qquad \text{(a)}$$

where

$$d_1 = \cos\theta_{Ax}(8\text{ ft} - AD\sin\theta_{Ax}) + \sin\theta_{Ax}(5\text{ ft}\cos 20°) = 4.788\text{ ft}$$
$$d_2 = \cos\theta_{Bx}(8\text{ ft} - BD\sin\theta_{Bx}) + \sin\theta_{Bx}(10\text{ ft}\cos 20°) = 7.191\text{ ft}$$

Reaction force at *O*: Sum forces in Fig. 1-63a to find reaction force components at *O*:

$$\Sigma F_x = 0 \quad R_{Ox} = T(\cos\theta_{Ax} + \cos\theta_{Bx}) = 3260 \text{ lb}$$
$$\Sigma F_y = 0 \quad R_{Oy} = -T(\sin\theta_{Ax} + \sin\theta_{Bx}) + W + P = 2 \text{ lb}$$

The resultant reaction force at O is $R_{O\text{res}} = \sqrt{R_{Ox}^2 + R_{Oy}^2} = 3260 \text{ lb}$ **(b)**

Reaction force at *D*: Sum forces in Fig. 1-63c to find reaction force components at *D*:

$$\Sigma F_x = 0 \quad D_x = -T(\cos\theta_{Ax} + \cos\theta_{Bx}) = -3260 \text{ lb}$$
$$\Sigma F_y = 0 \quad D_y = T(\sin\theta_{Ax} + \sin\theta_{Bx}) = 2698 \text{ lb}$$

The resultant reaction force at D is

$$D_{\text{res}} = \sqrt{D_x^2 + D_y^2} = 4231 \text{ lb} \qquad \textbf{(c)}$$

Cross-sectional area of cable *ADB*: Use the allowable normal stress of 18 ksi and cable force $T = 2177$ lb [Eq. (a)] to find the required cross-sectional area of the cable:

$$A_{\text{cable}} = \frac{2177 \text{ lb}}{18 \text{ ksi}} = 0.121 \text{ in}^2$$

Required diameter of the pins at *O*, *A*, *B*, and *D*: All pins are in double shear. The allowable shear stress is $\tau_{\text{allow}} = 12$ ksi. Required diameters of each pin are computed as

$$\text{Pins } A, B: \quad A_{\text{reqd}} = \frac{T}{2\tau_{\text{allow}}} = \frac{2177 \text{ lb}}{2(12 \text{ ksi})} = 0.091 \text{ in}^2 \quad \text{so } d = \sqrt{\frac{4}{\pi}(0.091 \text{ in}^2)} = 0.340 \text{ in.}$$

$$\text{Pin } O: \quad A_{\text{reqd}} = \frac{R_{O\text{res}}}{2\tau_{\text{allow}}} = \frac{3260 \text{ lb}}{2(12 \text{ ksi})} = 0.136 \text{ in}^2 \quad \text{so } d = \sqrt{\frac{4}{\pi}(0.136 \text{ in}^2)} = 0.416 \text{ in.}$$

$$\text{Pin } D: \quad A_{\text{reqd}} = \frac{D_{\text{res}}}{2\tau_{\text{allow}}} = \frac{4231 \text{ lb}}{2(12 \text{ ksi})} = 0.176 \text{ in}^2 \quad \text{so } d = \sqrt{\frac{4}{\pi}(0.176 \text{ in}^2)} = 0.474 \text{ in.}$$

4. ***Finalize***: In practice, other loads besides the weight of the canopy would have to be considered before making a final decision about the sizes of the cables and pins. Loads that could be important include wind loads, earthquake loads, and the weights of objects that might have to be supported temporarily by the structure. In addition, if cables *AD* and *BD* are *separate cables* (instead of one continuous cable *ADB*), the forces in the two cables are not equal in magnitude. The structure is now *statically indeterminate*, and the cable forces and the reactions at *O* and *D* cannot be determined using the equations of static equilibrium alone. Problems of this type are discussed in Chapter 2, Section 2.4 (see Example 2-7).

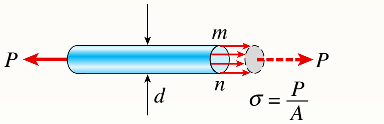
CHAPTER SUMMARY AND REVIEW

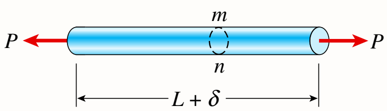
Chapter 1 covered mechanical properties of construction materials. Normal stresses and strains in bars loaded by centroidal axial loads were computed. Shear stresses and strains (as well as bearing stresses) in pin connections used to assemble simple structures such as trusses, were evaluated. Allowable levels of stress were calculated from appropriate factors of safety and used to set allowable loads that could be applied to the structure.

Some of the major concepts presented in this chapter are:

1. The principal objective of mechanics of materials is to determine the **stresses**, **strains**, **and displacements** in structures and their components due to the loads acting on them. These components include bars with axial loads, shafts in torsion, beams in bending, and columns in compression.
2. Prismatic bars subjected to tensile or compressive loads acting through the centroid of their cross section (to avoid bending) experience **normal stress** (σ) and **strain** (ε):

$$\sigma = \frac{P}{A} \qquad \varepsilon = \frac{\delta}{L}$$

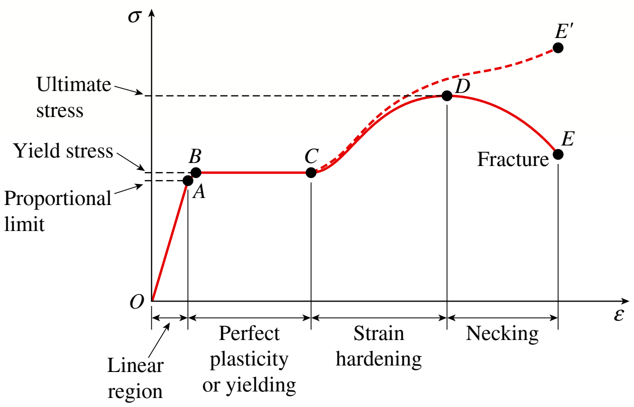




and either extension or contraction proportional to their lengths. These stresses and strains are **uniform** except near points of load application where high localized stresses, or **stress concentrations**, occur.
3. The **mechanical behavior** of various materials was displayed in a stress-strain diagram. **Ductile** materials such as mild steel have an initial linear relationship between normal stress and strain up to the **proportional limit** and are **linearly elastic** with stress and strain related by **Hooke's law**:

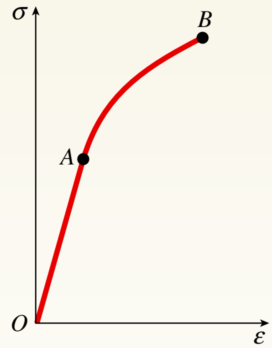
$$\sigma = E\varepsilon$$

They also have a well-defined yield point. Other ductile materials such as aluminum alloys typically do not have a clearly definable yield point, so an arbitrary yield stress is determined using the **offset method**.



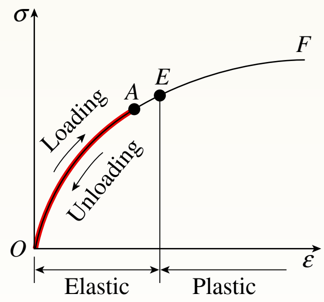
Mild steel

4. Materials that fail in tension at relatively low values of strain (such as concrete, stone, cast iron, glass ceramics, and a variety of metallic alloys) are classified as **brittle**. Brittle materials fail with only little elongation after the proportional limit.



Brittle material

5. If a material remains within the elastic range, it can be loaded, unloaded, and loaded again without significantly changing its behavior. However, when loaded into the plastic range, the internal structure of the material is altered and its properties change. Loading and unloading behavior of materials depends on their **elasticity** and **plasticity** properties such as the **elastic limit** and the possibility of **permanent set** (residual strain). Sustained loading over time may lead to **creep** and **relaxation**.

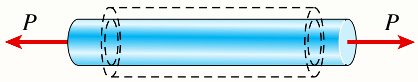


Partially elastic behavior

6. Axial elongation of bars loaded in tension is accompanied by lateral contraction; the ratio of lateral strain to normal strain is known as **Poisson's ratio** (ν):

$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon'}{\varepsilon}$$

Poisson's ratio remains constant throughout the linearly elastic range, provided the material is homogeneous and isotropic. Most of the examples and problems in the text are solved with the assumption that the material is linearly elastic, homogeneous, and **isotropic**.



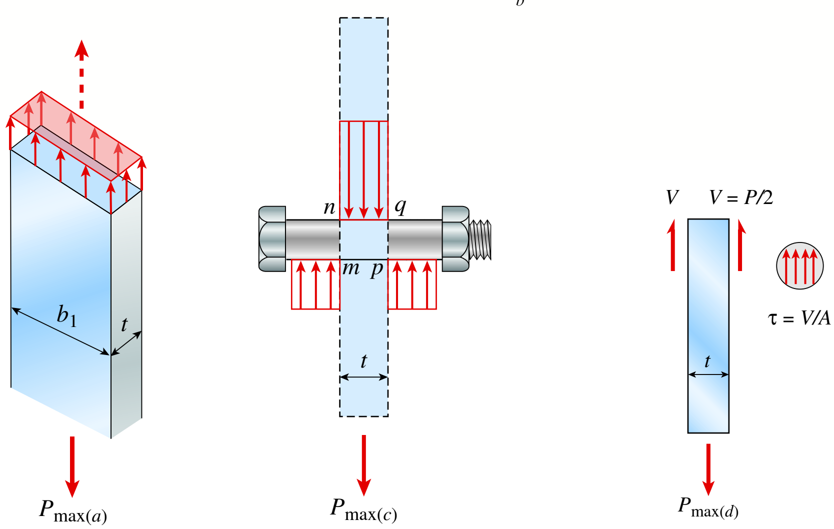
Lateral contraction

7. **Normal** stresses (σ) act perpendicular to the surface of the material, and **shear** stresses (τ) act tangential to the surface. In bolted connections between plates, the bolts are subjected to either single or double shear (τ_{aver}) where the average shear stress is

$$\tau_{\text{aver}} = \frac{V}{A}$$

Average **bearing** stresses (σ_b) act on the rectangular projected area (A_b) of the actual curved contact surface between a bolt and plate:

$$\sigma_b = \frac{F_b}{A_b}$$



Normal stresses

Bearing stresses on a bolt passing through a bar

Shear stresses on a bolt passing through a bar

8. An element of material acted on by only shear stresses and strains is in a state of stress referred to as **pure shear**. Shear strain (γ) is a measure of the distortion or change in shape of the element in pure shear. Hooke's law in shear relates shear stress (τ) to shear strain by the shearing modulus of elasticity G:

$$\tau = G\gamma$$

Moduli E and G are not independent elastic properties of the material. Compute modulus G from E using Poisson's ratio:

$$G = \frac{E}{2(1+\nu)}$$

9. **Strength** is the capacity of a structure or component to support or transmit loads. **Factors of safety** relate actual to required strength of structural members and account for a variety of uncertainties, such as variations in material properties, uncertain magnitudes or distributions of loadings, and probability of accidental overload. Because of these uncertainties, factors of safety (n_1, n_2, n_3, and n_4) must be determined using probabilistic methods.
10. Yield or ultimate-level stresses are divided by factors of safety to produce allowable values for use in design. For **ductile** materials,

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{n_1} \qquad \tau_{\text{allow}} = \frac{\tau_Y}{n_2}$$

while for **brittle** materials,

$$\sigma_{\text{allow}} = \frac{\sigma_U}{n_3} \qquad \tau_{\text{allow}} = \frac{\tau_U}{n_4}$$

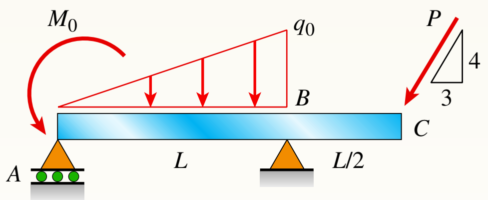
A typical value of n_1 and n_2 is 1.67, while n_3 and n_4 might be 2.8.

For a pin-connected member in axial tension, the **allowable load** depends on the allowable stress times the appropriate area (such as the net cross-sectional area for bars acted on by centroidal tensile loads, cross-sectional area of pin for pins in shear, and projected area for bolts in bearing). If the bar is in compression, the net cross-sectional area need not be used, but buckling may be an important consideration.
11. **Design** is the iterative process by which the appropriate size of structural members is determined to meet a variety of both **strength** and **stiffness** requirements. Incorporation of factors of safety into design is not a simple matter because both strength and failure have many different meanings.

1.3 Statics Review

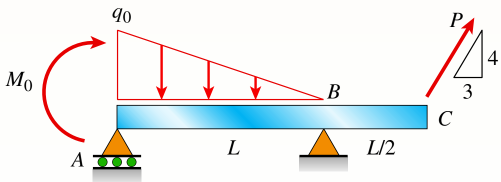
Introductory Problems

1.3-1 Find support reactions at A and B and then calculate the axial force N, shear force V, and bending moment M at mid-span of AB. Let $L = 14$ ft, $q_0 = 12$ lb/ft, $P = 50$ lb, and $M_0 = 300$ lb-ft.



PROBLEM 1.3-1

1.3-2 Find support reactions at A and B and then calculate the axial force N, shear force V, and bending moment M at mid-span of AB. Let $L = 4$ m, $q_0 = 160$ N/m, $P = 200$ N, and $M_0 = 380$ N · m.

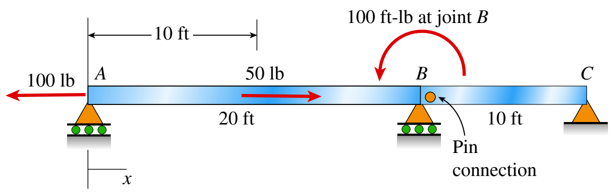


PROBLEM 1.3-2

1.3-3 Segments AB and BC of beam ABC are pin connected a small distance to the right of joint B (see figure). Axial loads act at A and at the mid-span of AB. A concentrated moment is applied at joint B.

(a) Find reactions at supports A, B, and C.

(b) Find internal stress resultants N, V, and M at 15 ft.



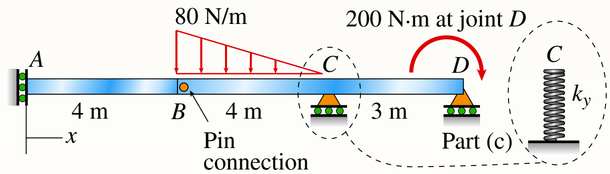
PROBLEM 1.3-3

1.3-4 Segments AB and BCD of beam $ABCD$ are pin connected at $x = 4$ m. The beam is supported by a sliding support at A and roller supports at C and D (see figure). A triangularly distributed load with peak intensity of 80 N/m acts on BC. A concentrated moment is applied at joint D.

(a) Find reactions at supports A, C, and D.

(b) Find internal stress resultants N, V, and M at $x = 5$ m.

(c) Repeat parts (a) and (b) for the case of the roller support at C replaced by a linear spring of stiffness $k_y = 200$ kN/m (see figure).



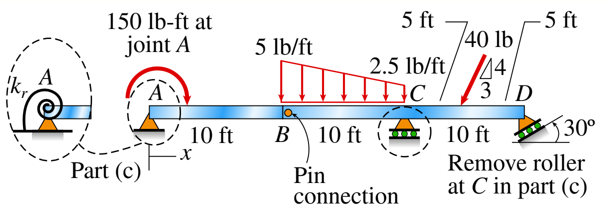
PROBLEM 1.3-4

1.3-5 Segments AB and BCD of beam $ABCD$ are pin connected at $x = 10$ ft. The beam is supported by a pin support at A and roller supports at C and D; the roller at D is rotated by 30° from the x axis (see figure). A trapezoidal distributed load on BC varies in intensity from 5 lb/ft at B to 2.5 lb/ft at C. A concentrated moment is applied at joint A, and a 40-lb inclined load is applied at the mid-span of CD.

(a) Find reactions at supports A, C, and D.

(b) Find the resultant force in the pin connection at B.

(c) Repeat parts (a) and (b) if a rotational spring ($k_r = 50$ ft-lb/radian) is added at A *and* the roller at C is removed.

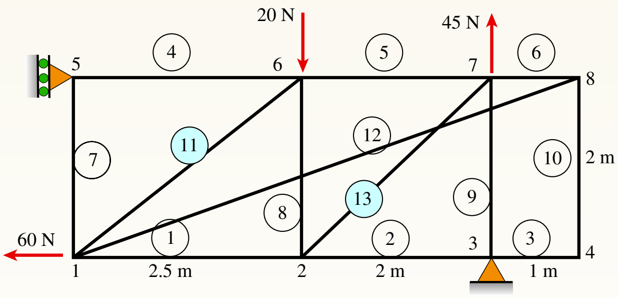


PROBLEM 1.3-5

1.3-6 Consider the plane truss with a pin support at joint 3 and a vertical roller support at joint 5 (see figure).

(a) Find reactions at support joints 3 and 5.

(b) Find axial forces in truss members 11 and 13.

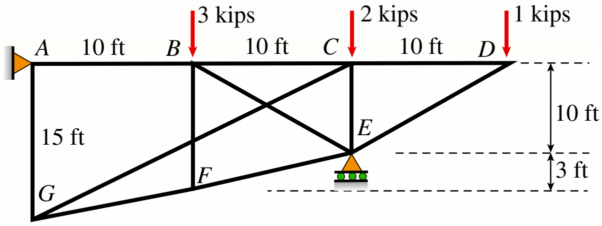


PROBLEM 1.3-6

1.3-7 A plane truss has a pin support at A and a roller support at E (see figure).

(a) Find reactions at all supports.

(b) Find the axial force in truss member FE.

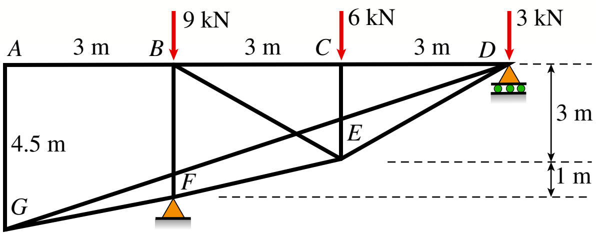


PROBLEM 1.3-7

1.3-8 A plane truss has a pin support at F and a roller support at D (see figure).

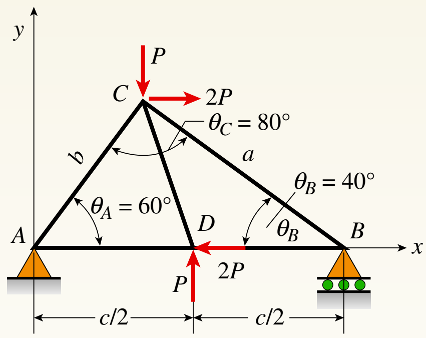
(a) Find reactions at both supports.

(b) Find the axial force in truss member FE.



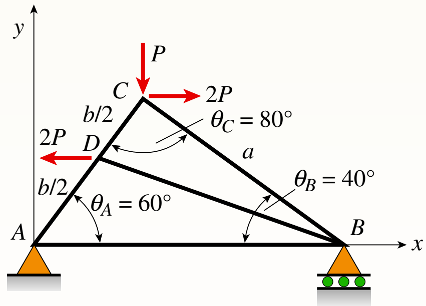
PROBLEM 1.3-8

1.3-9 Find support reactions at A and B and then use the *method of joints* to find all member forces. Let $c = 8$ ft and $P = 20$ kips.



PROBLEM 1.3-9

1.3-10 Find support reactions at A and B and then use the *method of joints* to find all member forces. Let $b = 3$ m and $P = 80$ kN.



PROBLEM 1.3-10

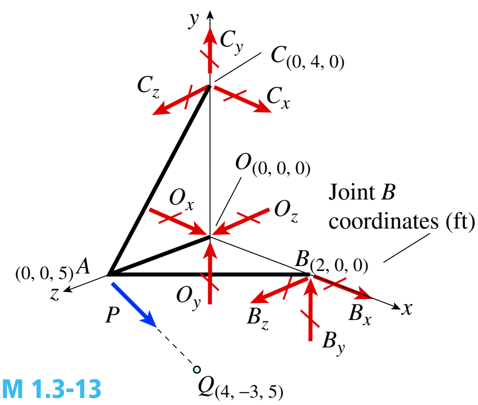
1.3-11 Repeat 1.3-9 but use the *method of sections* to find member forces in AC and BD.

1.3-12 Repeat 1.3-10 but use the *method of sections* to find member forces in AB and DC.

1.3-13 A space truss has three-dimensional pin supports at joints O, B, and C. Load P is applied at joint A and acts toward point Q. Coordinates of all joints are given in feet (see figure).

(a) Find reaction force components B_x, B_z, and O_z.

(b) Find the axial force in truss member AC.

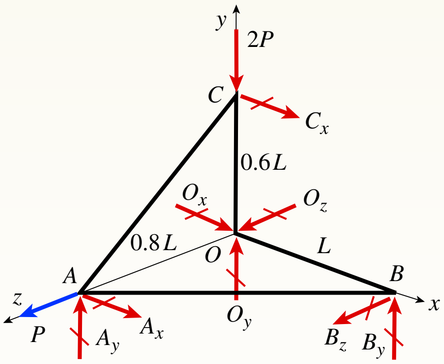


PROBLEM 1.3-13

1.3-14 A space truss is restrained at joints O, A, B, and C, as shown in the figure. Load P is applied at joint A and load $2P$ acts downward at joint C.

(a) Find reaction force components A_x, B_y, and B_z in terms of load variable P.

(b) Find the axial force in truss member AB in terms of load variable P.

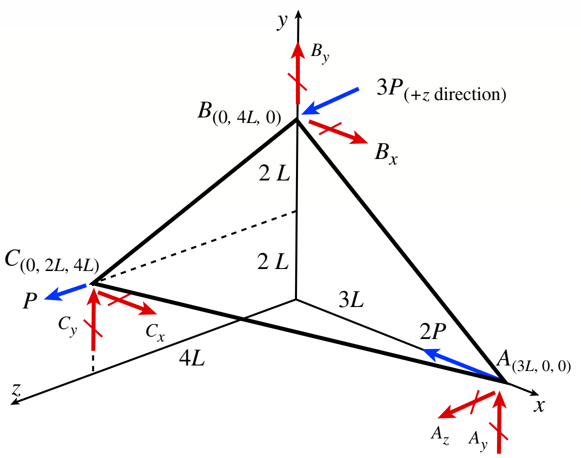


PROBLEM 1.3-14

1.3-15 A space truss is restrained at joints A, B, and C, as shown in the figure. Load $2P$ is applied at in the $-x$ direction at joint A, load $3P$ acts in the $+z$ direction at joint B, and load P is applied in the $+z$ direction at joint C. Coordinates of all joints are given in terms of dimension variable L (see figure).

(a) Find reaction force components A_y and A_z in terms of load variable P.

(b) Find the axial force in truss member AB in terms of load variable P.

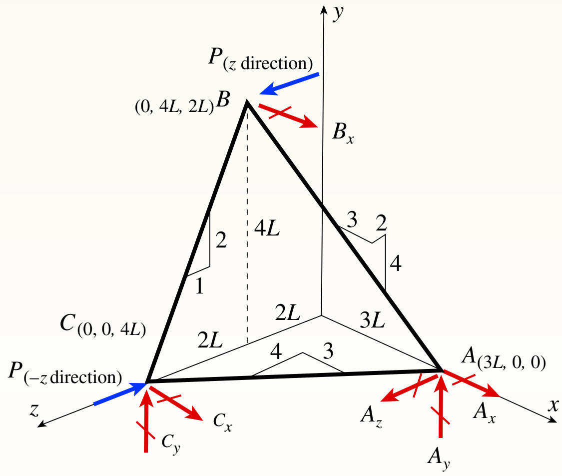


PROBLEM 1.3-15

1.3-16 A space truss is restrained at joints A, B, and C, as shown in the figure. Load P acts in the $+z$ direction at joint B and in the $-z$ directions at joint C. Coordinates of all joints are given in terms of dimension variable L (see figure). Let $P = 5$ kN and $L = 2$ m.

(a) Find the reaction force components A_z and B_x.

(b) Find the axial force in truss member AB.

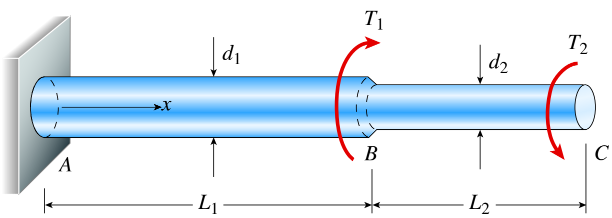


PROBLEM 1.3-16

1.3-17 A stepped shaft ABC consisting of two solid, circular segments is subjected to torques T_1 and T_2 acting in opposite directions, as shown in the figure. The larger segment of the shaft has a diameter of $d_1 = 2.25$ in. and a length $L_1 = 30$ in.; the smaller segment has a diameter $d_2 = 1.75$ in. and a length $L_2 = 20$ in. The torques are $T_1 = 21{,}000$ lb-in. and $T_2 = 10{,}000$ lb-in.

(a) Find reaction torque T_A at support A.

(b) Find the internal torque $T(x)$ at two locations: $x = L_1/2$ and $x = L_1 + L_2/2$. Show these internal torques on properly drawn free-body diagrams (FBDs).

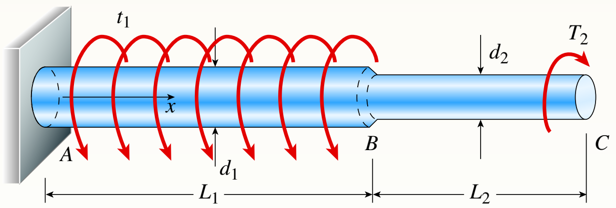


PROBLEM 1.3-17

1.3-18 A stepped shaft ABC consisting of two solid, circular segments is subjected to uniformly distributed torque t_1 acting over segment 1 and concentrated torque T_2 applied at C, as shown in the figure. Segment 1 of the shaft has a diameter of $d_1 = 57$ mm and length of $L_1 = 0.75$ m; segment 2 has a diameter $d_2 = 44$ mm and length $L_2 = 0.5$ m. Torque intensity $t_1 = 3100$ N · m/m and $T_2 = 1100$ N · m.

(a) Find reaction torque T_A at support A.

(b) Find the internal torque $T(x)$ at two locations: $x = L_1/2$ and at $x = L_1 + L_2/2$. Show these internal torques on properly drawn free-body diagrams.

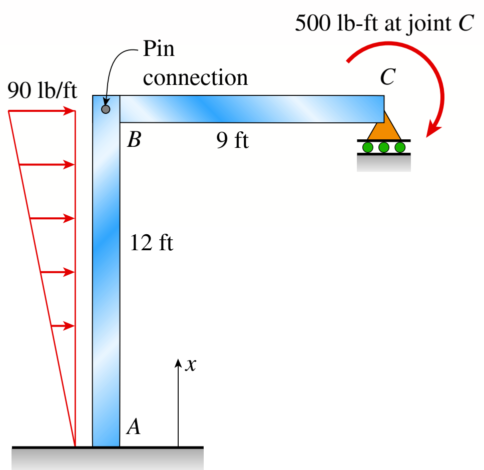


PROBLEM 1.3-18

1.3-19 A plane frame is restrained at joints A and C, as shown in the figure. Members AB and BC are pin connected at B. A triangularly distributed lateral load with a peak intensity of 90 lb/ft acts on AB. A concentrated moment is applied at joint C.

(a) Find reactions at supports A and C.

(b) Find internal stress resultants N, V, and M at $x = 3$ ft on column AB.

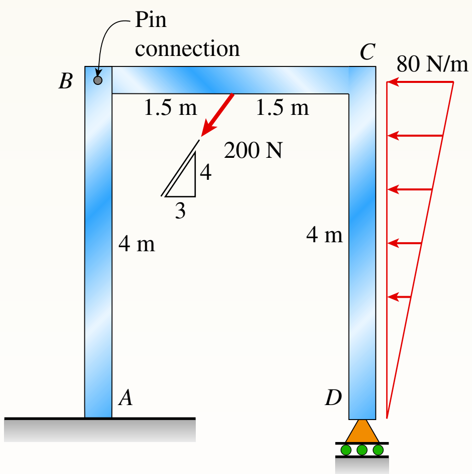


PROBLEM 1.3-19

1.3-20 A plane frame is restrained at joints A and D, as shown in the figure. Members AB and BCD are pin connected at B. A triangularly distributed lateral load with peak intensity of 80 N/m acts on CD. An inclined concentrated force of 200 N acts at the mid-span of BC.

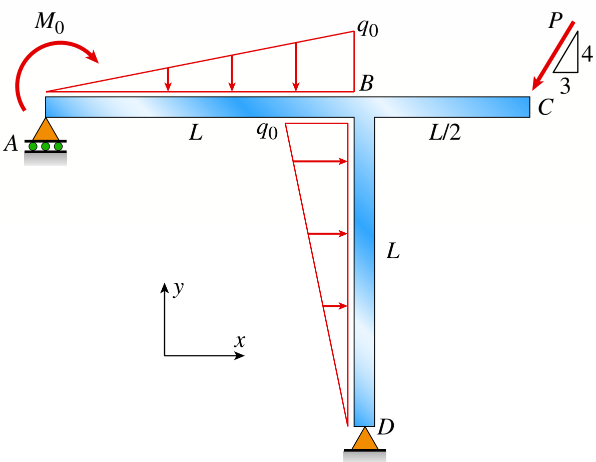
(a) Find reactions at supports A and D.

(b) Find resultant forces in the pins at B and C.



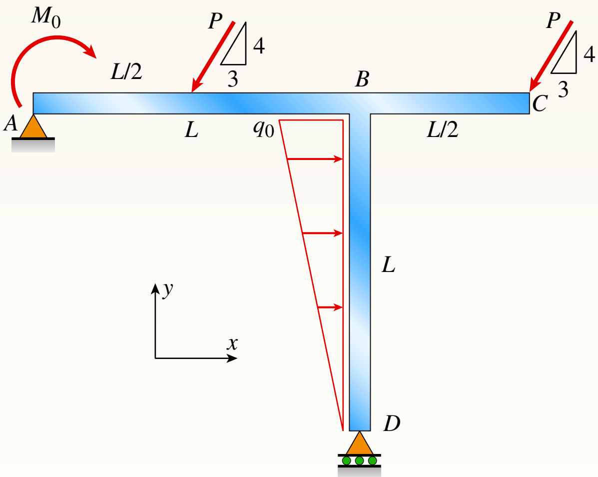
PROBLEM 1.3-20

1.3-21 Find support reactions at A and D and then calculate the axial force N, shear force V, and bending moment M at mid-span of AB. Let $L = 14$ ft, $q_0 = 12$ lb/ft, $P = 50$ lb, and $M_0 = 300$ lb-ft.



PROBLEM 1.3-21

1.3-22 Find support reactions at A and D and then calculate the axial force N, shear force V, and bending moment M at mid-span of column BD. Let $L = 4$ m, $q_0 = 160$ N/m, $P = 200$ N, and $M_0 = 380$ N · m.

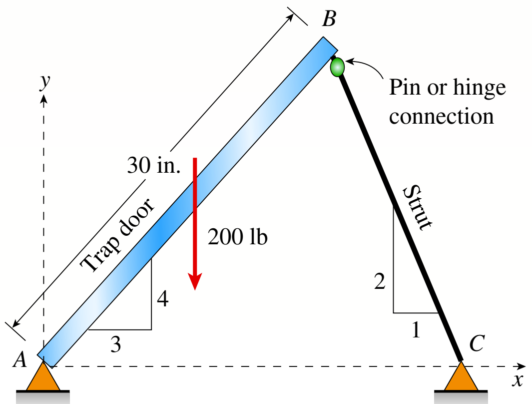


PROBLEM 1.3-22

1.3-23 A 200-lb trap door (AB) is supported by a strut (BC) which is pin connected to the door at B (see figure).

(a) Find reactions at supports A and C.

(b) Find internal stress resultants N, V, and M on the trap door at 20 in. from A.

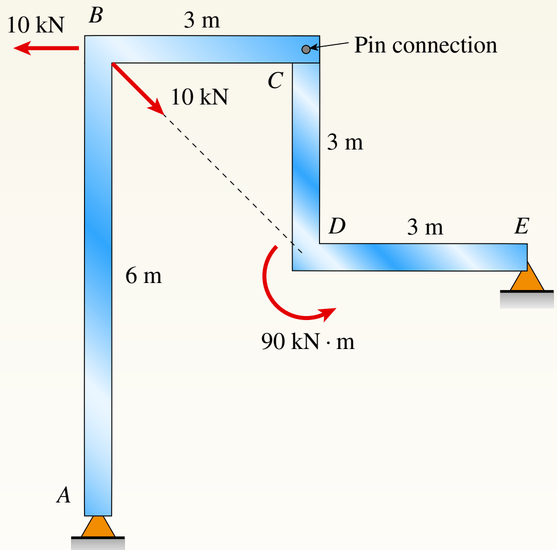


PROBLEM 1.3-23

1.3-24 A plane frame is constructed by using a pin connection between segments ABC and CDE. The frame has pin supports at A and E and joint loads at B and D (see figure).

(a) Find reactions at supports A and E.

(b) Find the resultant force in the pin at C.

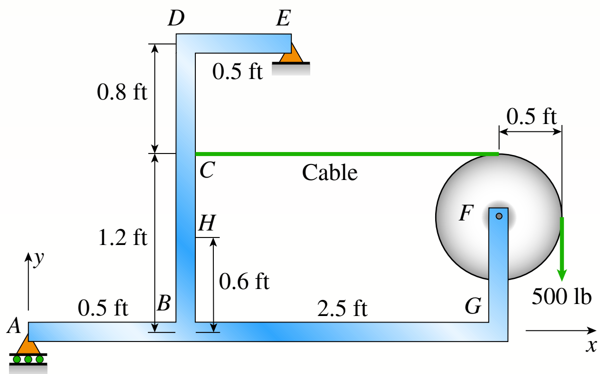


PROBLEM 1.3-24

1.3-25 A plane frame with pin supports at A and E has a cable attached at C, which runs over a frictionless pulley at F (see figure). The cable force is known to be 500 lb.

(a) Find reactions at supports A and E.

(b) Find internal stress resultants N, V, and M at point H.



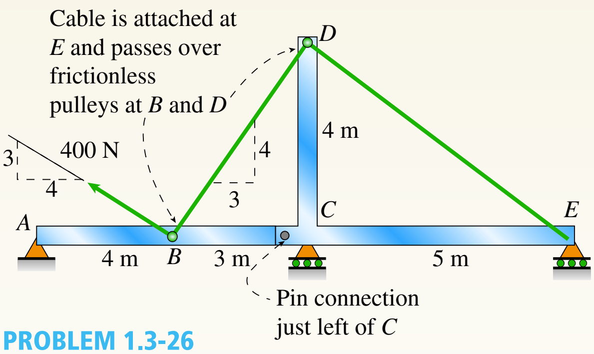
PROBLEM 1.3-25

1.3-26 A plane frame with a pin support at A and roller supports at C and E has a cable attached at E, which runs over frictionless pulleys at D and B (see figure). The cable force is known to be 400 N. There is a pin connection just to the left of joint C.

(a) Find reactions at supports A, C, and E.

(b) Find internal stress resultants N, V, and M just to the right of joint C.

(c) Find resultant force in the pin near C.

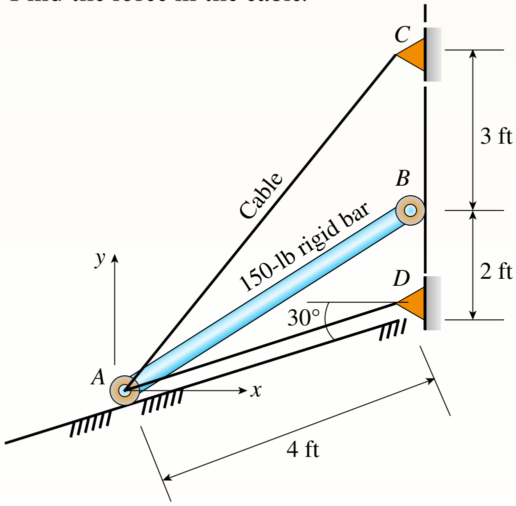


PROBLEM 1.3-26

1.3-27 A 150-lb rigid bar AB, with frictionless rollers at each end, is held in the position shown in the figure by a continuous cable CAD. The cable is pinned at C and D and runs over a pulley at A.

(a) Find reactions at supports A and B.

(b) Find the force in the cable.

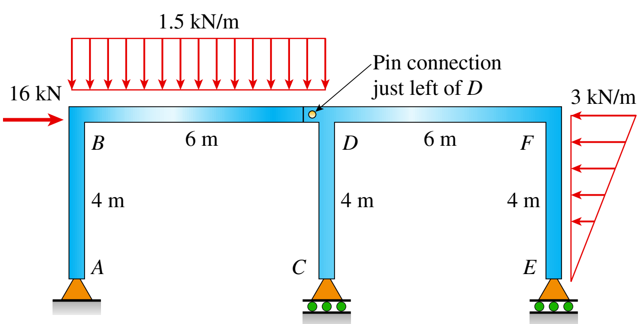


PROBLEM 1.3-27

1.3-28 A plane frame has a pin support at A and roller supports at C and E (see figure). Frame segments ABD and $CDEF$ are joined just left of joint D by a pin connection.

(a) Find reactions at supports A, C, and E.

(b) Find the resultant force in the pin just left of D.

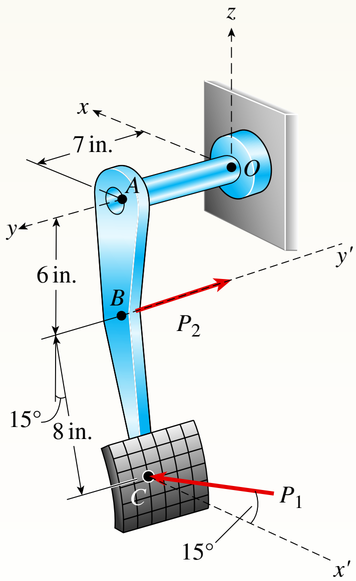


PROBLEM 1.3-28

1.3-29 A special vehicle brake is clamped at O when the brake force P_1 is applied (see figure). Force $P_1 = 50$ lb and lies in a plane that is parallel to the x-z plane and is applied at C normal to line BC. Force $P_2 = 40$ lb and is applied at B in the $-y$ direction.

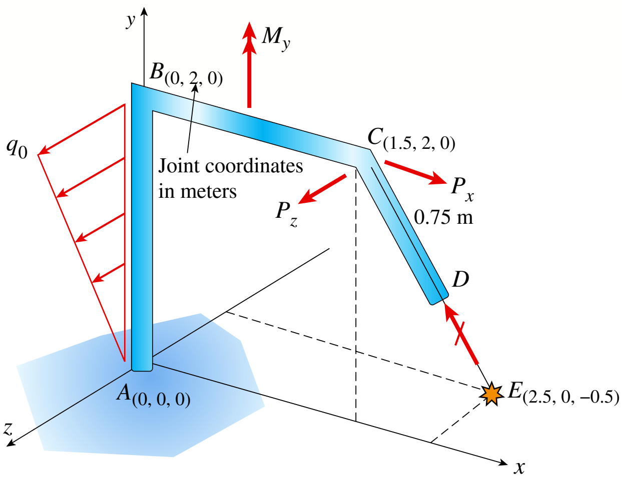
(a) Find reactions at support O.

(b) Find internal stress resultants N, V, T, and M at the mid-point of segment OA.



PROBLEM 1.3-29

1.3-30 Space frame $ABCD$ is clamped at A, except it is free to translate in the x direction. There is also a roller support at D, which is normal to line CDE. A triangularly distributed force with peak intensity $q_0 = 75$ N/m acts along AB in the positive z direction. Forces $P_x = 60$ N and $P_z = -45$ N are applied at joint C, and a concentrated moment $M_y = 120$ N · m acts at the mid-span of member BC.



PROBLEM 1.3-30

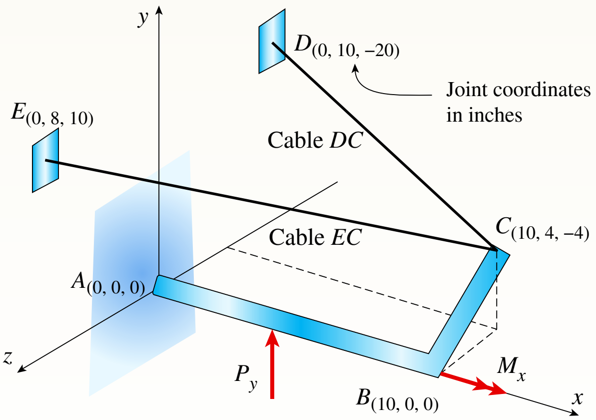
(a) Find reactions at supports A and D.

(b) Find internal stress resultants N, V, T, and M at the mid-height of segment AB.

1.3-31 Space frame ABC is clamped at A, except it is free to rotate at A about the x and y axes. Cables DC and EC support the frame at C. Force $P_y = -50$ lb is applied at the mid-span of AB, and a concentrated moment $M_x = -20$ in.-lb acts at joint B.

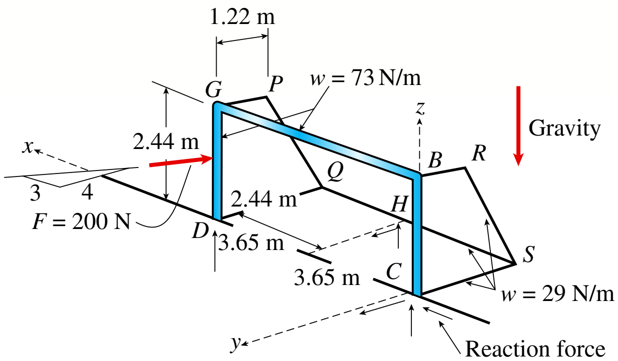
(a) Find reactions at support A.

(b) Find cable tension forces.



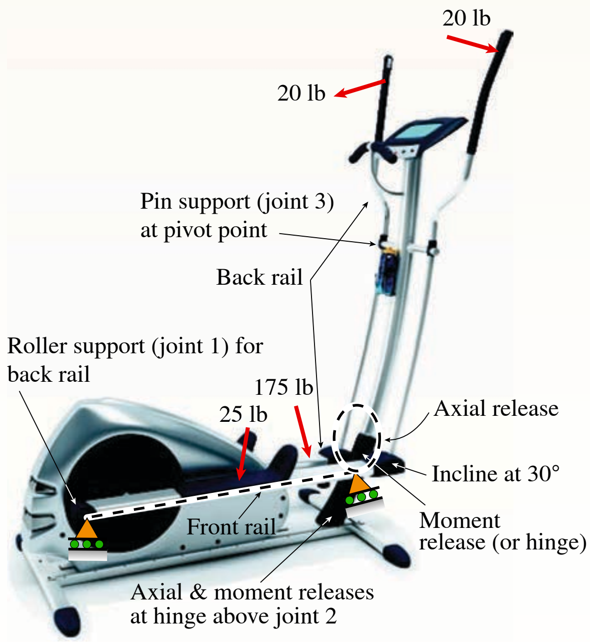
PROBLEM 1.3-31

1.3-32 A soccer goal is subjected to gravity loads (in the $-z$ direction, $w = 73$ N/m for DG, BG, and BC; $w = 29$ N/m for all other members; see figure) and a force $F = 200$ N applied eccentrically at the mid-height of member DG. Find reactions at supports C, D, and H.

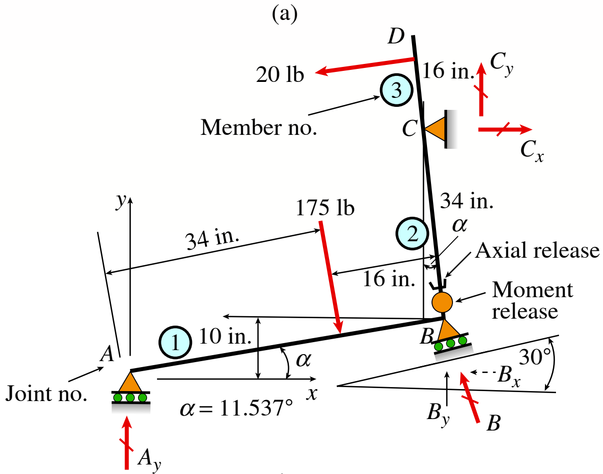


PROBLEM 1.3-32

1.3-33 An elliptical exerciser machine (see figure part a) is composed of front and back rails. A simplified plane-frame model of the back rail is shown in figure part b. Analyze the plane-frame model to find reaction forces at supports A, B, and C for the position and applied loads given in figure part b. Note that there are axial and moment releases at the base of member 2 so that member 2 can lengthen and shorten as the roller support at B moves along the 30° incline. (These releases indicate that the internal axial force N and moment M must be zero at this location.)



(a)



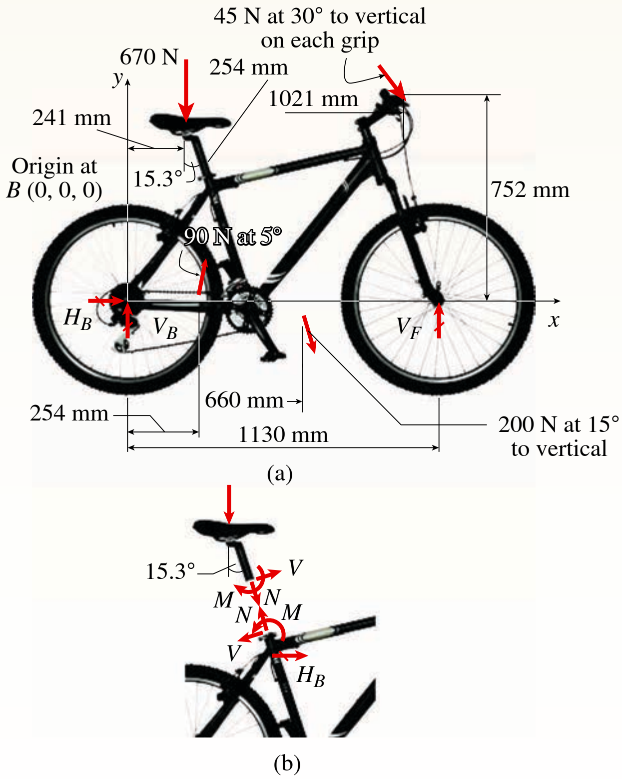
(b)

PROBLEM 1.3-33

1.3-34 A mountain bike is moving along a flat path at constant velocity. At some instant, the rider (weight = 670 N) applies pedal and hand forces, as shown in the figure part a.

(a) Find reaction forces at the front and rear hubs. (Assume that the bike is pin supported at the rear hub and roller supported at the front hub.)

(b) Find internal stress resultants N, V, and M in the inclined seat post (see figure part b).



PROBLEM 1.3-34

1.4 Normal Stress and Strain

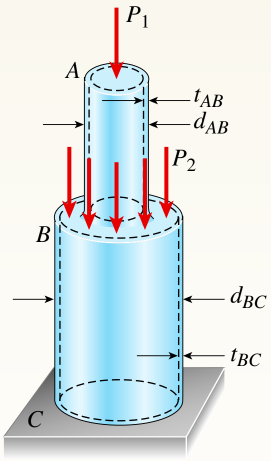
Introductory Problems

1.4-1 A hollow circular post ABC (see figure) supports a load $P_1 = 1700$ lb acting at the top. A second load P_2 is uniformly distributed around the cap plate at B. The diameters and thicknesses of the upper and lower parts of the post are $d_{AB} = 1.25$ in., $t_{AB} = 0.5$ in., $d_{BC} = 2.25$ in., and $t_{BC} = 0.375$ in., respectively.

(a) Calculate the normal stress σ_{AB} in the upper part of the post.

(b) If you want the lower part of the post to have the same compressive stress as the upper part, what should be the required magnitude of load P_2?

(c) If P_1 remains at 1700 lb and P_2 is set at 2260 lb, what new thickness of BC will result in the same compressive stress in both parts?

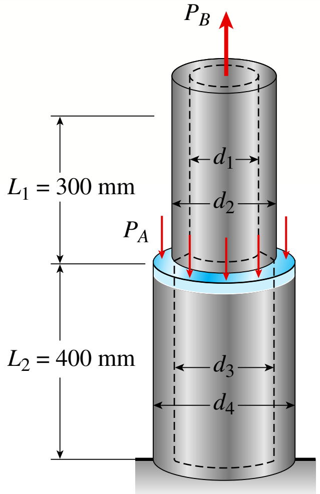


PROBLEM 1.4-1

1.4-2 A circular nylon pipe supports a downward load $P_A = 10$ kN, which is uniformly distributed around a cap plate at the top of the lower pipe. A second load $P_B = 20$ kN is applied upward at the top. The inner and outer diameters of the upper and lower parts of the pipe are $d_1 = 50$ mm, $d_2 = 60$ mm, $d_3 = 55$ mm, and $d_4 = 65$ mm, respectively. The bottom pipe has length 400 mm and the upper pipe has length 300 mm.

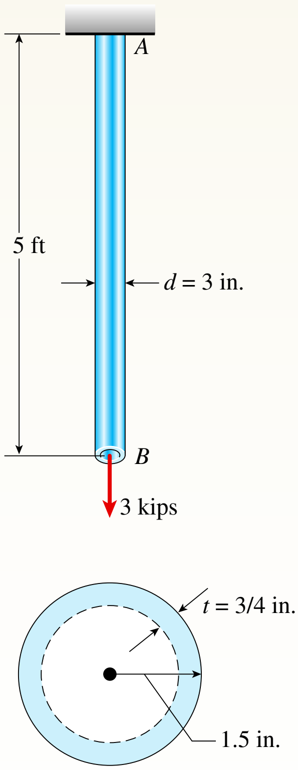
(a) Calculate the axial normal stress in each pipe segment.

(b) Calculate the strain in each pipe segment if the elongation of the upper pipe is 3.29 mm and the elongation of the bottom part is 1.25 mm.



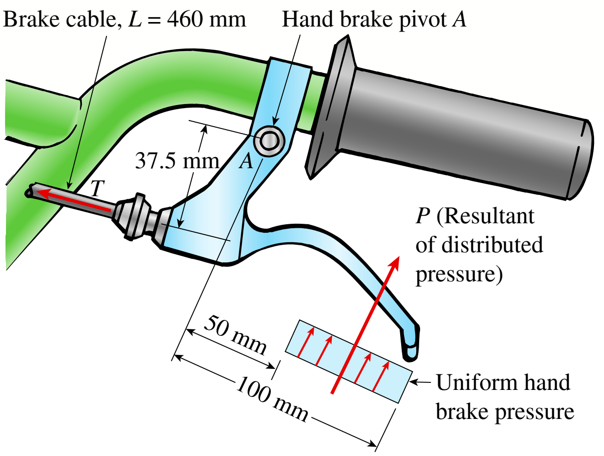
PROBLEM 1.4-2

1.4-3 A circular tube AB is fixed at one end and free at the other end. The tube is subjected to axial force at joint B. If the outer diameter of the tube is 3 in. and the thickness is ¾ in., calculate the maximum normal stress in the tube.



PROBLEM 1.4-3

1.4-4 A force P of 70 N is applied by a rider to the front hand brake of a bicycle (P is the resultant of an evenly distributed pressure). As the hand brake pivots at A, a tension T develops in the 460-mm long brake cable ($A_e = 1.075\ \text{mm}^2$), which elongates by $\delta = 0.214$ mm. Find the normal stress δ and strain ε in the brake cable.

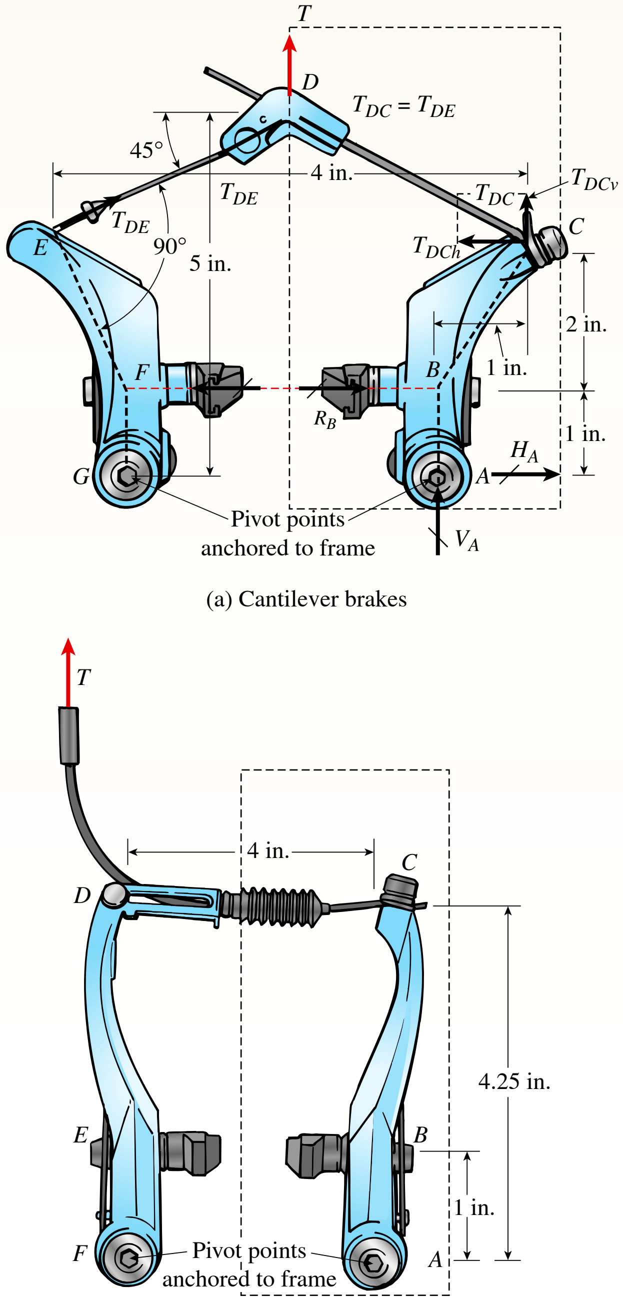


PROBLEM 1.4-4

Representative Problems

1.4-5 A bicycle rider wants to compare the effectiveness of cantilever hand brakes (see figure part a) versus V brakes (figure part b).

(a) Calculate the braking force R_B at the wheel rims for each of the bicycle brake systems shown. Assume that all forces act in the plane of the



(a) Cantilever brakes

(b) V brakes

PROBLEM 1.4-5

figure and that cable tension $T = 45$ lb. Also, what is the average compressive normal stress σ_c on the brake pad ($A = 0.625\ \text{in}^2$)?

(b) For each braking system, what is the stress in the brake cable if the effective cross-sectional area is 0.00167 in^2?

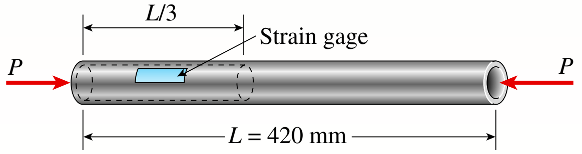
Hint: Because of symmmetry, use only the right half of each figure in your analysis.

1.4-6 A circular aluminum tube with a lenth of $L = 420$ mm is loaded in compression by forces P (see figure). The hollow segment of length $L/3$ has outside and inside diameters of 60 mm and 35 mm, respectively. The solid segment of length $2L/3$ has a diameter of 60 mm. A strain gage is placed on the outside of the hollow segment of the bar to measure normal strains in the longitudinal direction.

(a) If the measured strain in the hollow segment is $\varepsilon_h = 470 \times 10^{-6}$, what is the strain ε_s in the solid part? *Hint:* The strain in the solid segment is equal to that in the hollow segment multiplied by the ratio of the area of the hollow to that of the solid segment.

(b) What is the overall shortening δ of the bar?

(c) If the compressive stress in the bar cannot exceed 48 MPa, what is the maximum permissible value of load P?

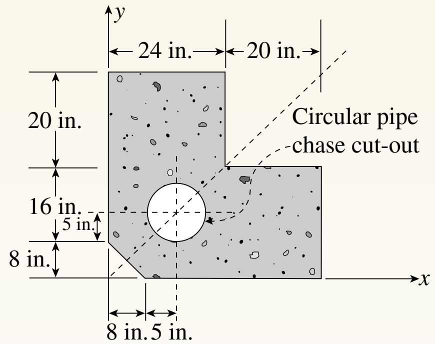


PROBLEM 1.4-6

1.4-7 The cross section of a concrete corner column that is loaded uniformly in compression is shown in the figure. A circular pipe chase cut-out of 10 in. in diameter runs the height of the column (see figure).

(a) Determine the average compression stress σ_c in the concrete if the load is equal to 3500 kips.

(b) Determine the coordinates x_c and y_c of the point where the resultant load must act in order to produce uniform normal stress in the column.

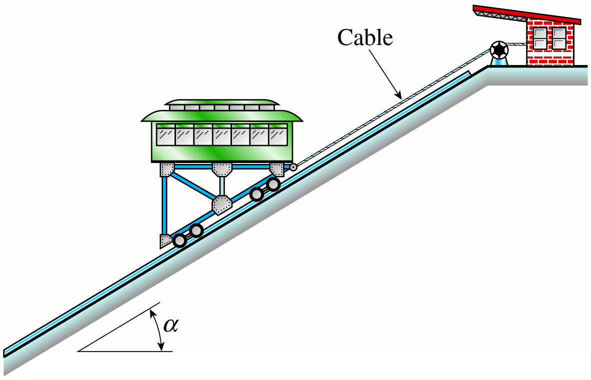


PROBLEM 1.4-7

1.4-8 A car weighing 130 kN when fully loaded is pulled slowly up a steep inclined track by a steel cable (see figure). The cable has an effective cross-sectional area of 490 mm^2, and the angle α of the incline is 30°.

(a) Calculate the tensile stress σ_t in the cable.

(b) If the allowable stress in the cable is 150 MPa, what is the maximum acceptable angle of the incline for a fully loaded car?



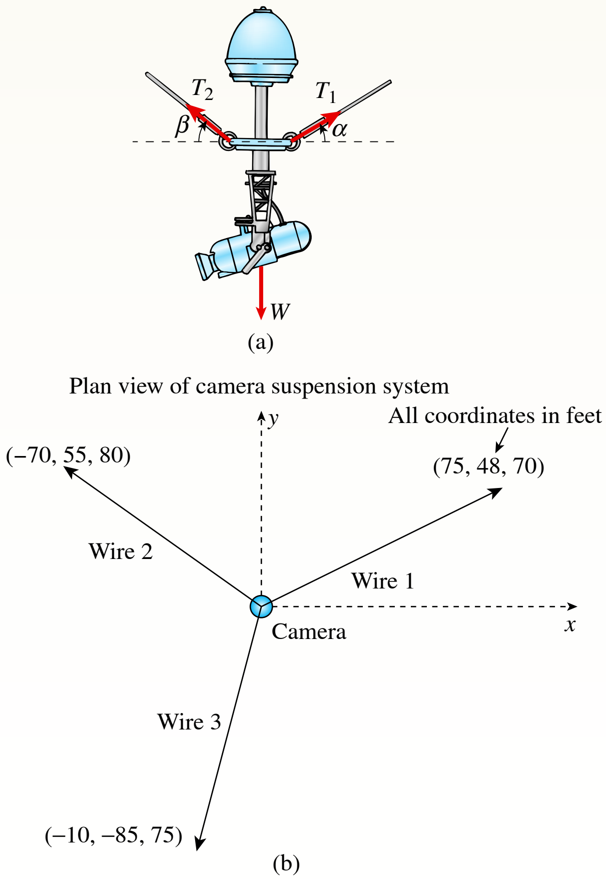
PROBLEM 1.4-8

1.4-9 Two steel wires support a moveable overhead camera weighing $W = 28$ lb (see fgure part a) used for close-up viewing of field action at sporting events. At some instant, wire 1 is at an angle $\alpha = 22°$ to the horizontal and wire 2 is at angle $\beta = 40°$. Wires 1 and 2 have diameters of 30 and 35 mils, respectively. (Wire diameters are often expressed in mils; one mil equals 0.001 in.)

(a) Determine the tensile stresses σ_1 and σ_2 in the two wires.

(b) If the stresses in wires 1 and 2 must be the same, what is the required diameter of wire 1?

(c) To stabilize the camera for windy outdoor conditions, a third wire is added (see figure part b). Assume the three wires meet at a common point coordinates (0, 0, 0) above the camera at the instant shown in figure part b. Wire 1 is attached to a support at coordinates (75 ft, 48 ft, 70 ft). Wire 2 is supported at (−70 ft, 55 ft, 80 ft). Wire 3 is supported at (−10 ft, −85 ft, 75 ft). Assume that all three wires have a diameter of 30 mils. Find the tensile stresses in all three wires.

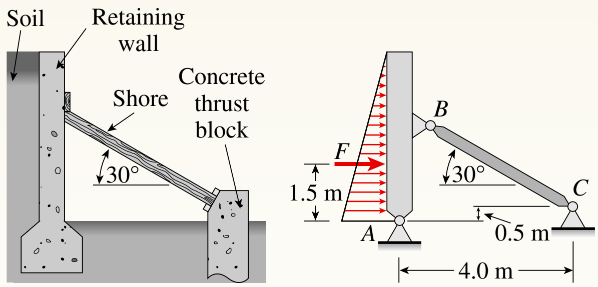


PROBLEM 1.4-9

1.4-10 A long retaining wall is braced by wood shores set at an angle of 30° and supported by concrete thrust blocks, as shown in the first part of the figure. The shores are evenly spaced at 3 m apart.

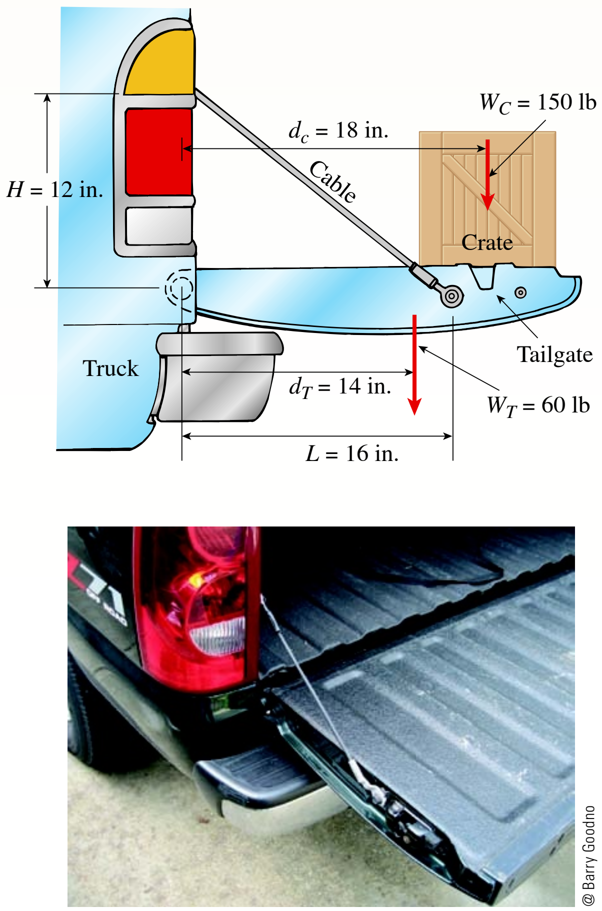
For analysis purposes, the wall and shores are idealized as shown in the second part of the figure. Note that the base of the wall and both ends of the shores are assumed to be pinned. The pressure of the soil against the wall is assumed to be triangularly distributed, and the resultant force acting on a 3-meter length of the walls is $F = 190$ kN.

If each shore has a 150 mm × 150 mm square cross section, what is the compressive stress σ_c in the shores?



PROBLEM 1.4-10

1.4-11 A pickup truck tailgate supports a crate where $W_C = 150$ lb, as shown in the figure. The tailgate weighs $W_T = 60$ lb and is supported by two cables (only one is shown in the figure). Each cable has an effective cross-sectional area $A_e = 0.017 \text{in.}^2$



PROBLEM 1.4-11

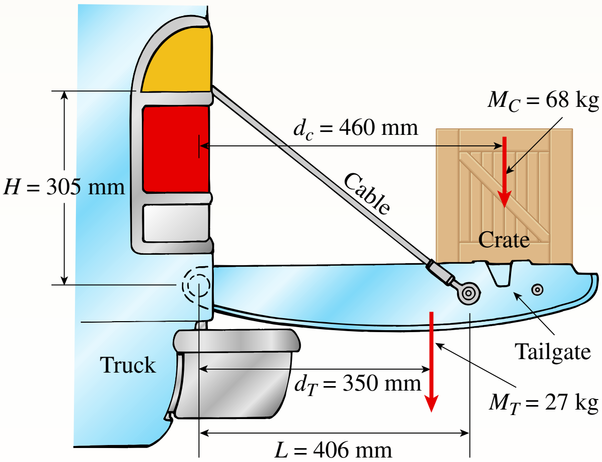
(a) Find the tensile force T and normal stress σ in each cable.

(b) If each cable elongates $\delta = 0.01$ in. due to the weight of both the crate and the tailgate, what is the average strain in the cable?

1.4-12 Solve the preceding problem if the mass of the tailgate is $M_T = 27$ kg and that of the crate is $M_C = 68$ kg. Use dimensions $H = 305$ mm, $L = 406$ mm, $d_C = 460$ mm, and $d_T = 350$ mm. The cable cross-sectional area is $A_e = 11.0\ \text{mm}^2$.

(a) Find the tensile force T and normal stress σ in each cable.

(b) If each cable elongates $\delta = 0.25$ mm due to the weight of both the crate and the tailgate, what is the average strain in the cable?



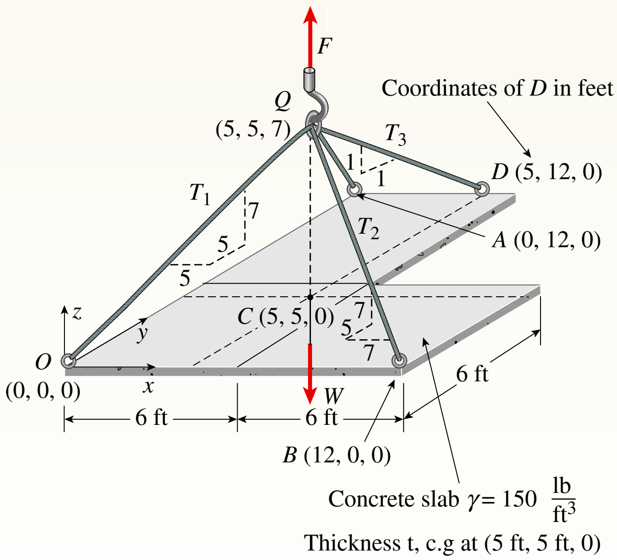
PROBLEM 1.4-12

1.4-13 An L-shaped reinforced concrete slab 12 ft × 12 ft, with a 6 ft × 6 ft cut-out and thickness $t = 9.0$ in, is lifted by three cables attached at O, B, and D, as shown in the figure. The cables are combined at point Q, which is 7.0 ft above the top of the slab and directly above the center of mass at C. Each cable has an effective cross-sectional area of $A_e = 0.12\ \text{in}^2$.

(a) Find the tensile force $T_i (i = 1, 2, 3)$ in each cable due to the weight W of the concrete slab (ignore weight of cables).

(b) Find the average stress σ_i in each cable. (See Table I-1 in Appendix I for the weight density of reinforced concrete.)

(c) Add cable AQ so that OQA is one continuous cable, with each segment having force T_1, which is connected to cables BQ and DQ at point Q. Repeat parts (a) and (b). *Hint*: There are now three forced equilibrium equations and one *constrain equation*, $T_1 = T_4$.

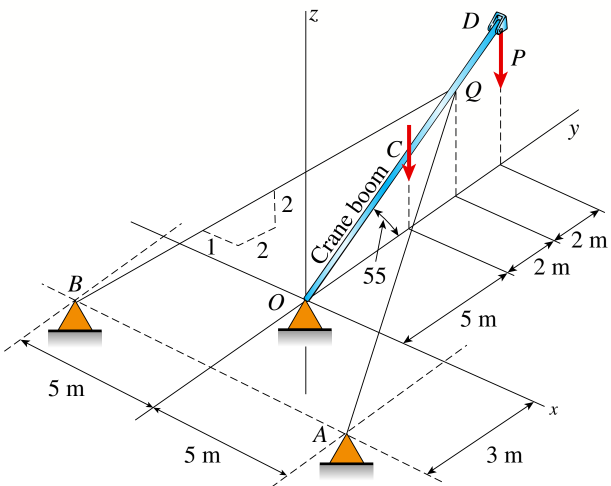


PROBLEM 1.4-13

1.4-14 A crane boom of mass 450 kg with its center of mass at C is stabilized by two cables AQ and BQ ($A_e = 304\ \text{mm}^2$ for each cable) as shown in the figure. A load $P = 20$ kN is supported at point D. The crane boom lies in the y-z plane.

(a) Find the tension forces in each cable: T_{AQ} and T_{BQ} (kN). Neglect the mass of the cables, but include the mass of the boom in addition to load P.

(b) Find the average stress (σ) in each cable.

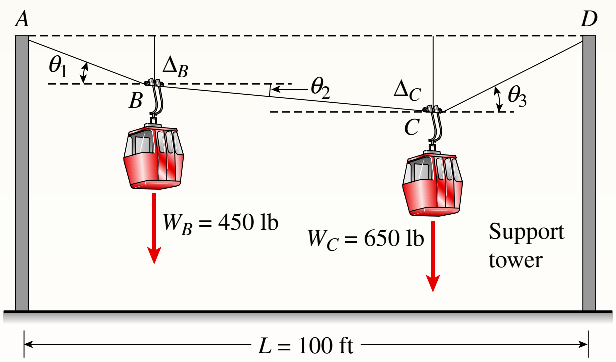


PROBLEM 1.4-14

1.4-15 Two gondolas on a ski lift are locked in the position show in the figure while repairs are being made elsewhere. The distance between support towers is $L = 100$ ft. The length of each cable segment under gondolas weighing $W_B = 450$ lb and $W_C = 650$ lb are $D_{AB} = 12$ ft, $D_{BC} = 70$ ft, and $D_{CD} = 20$ ft. The cable sag at B is $\Delta_B = 3.9$ ft and that at C is $\Delta_C = 7.1$ ft. The effective cross-sectional area of the cables is $A_e = 0.12$ in^2.

(a) Find the tension force in each segment; neglect the mass of the cable.

(b) Find the average stress (σ) in each cable segment.

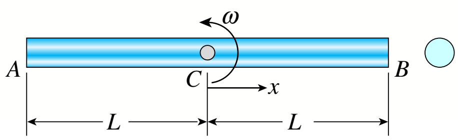


PROBLEM 1.4-15

1.4-16 A round bar ABC of length $2L$ (see figure) rotates about an axis through the midpoint C with constant angular speed ω (radians per second). The material of the bar has weight density γ.

(a) Derive a formula for the tensile stress σ_x in the bar as a function of the distance x from the midpoint C.

(b) What is the maximum tensile stress σ_{max}?

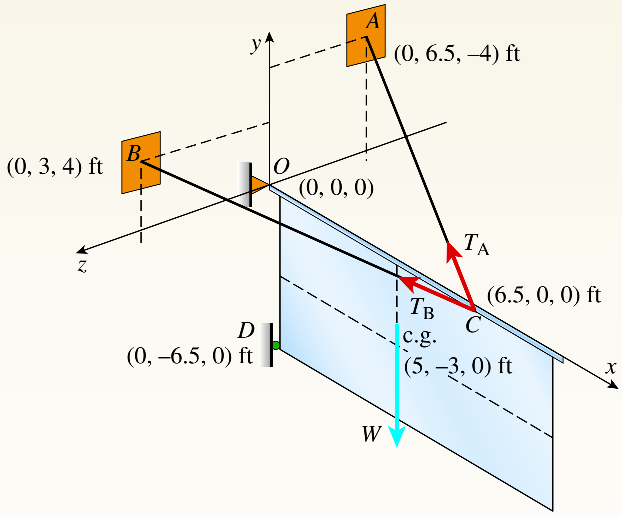


PROBLEM 1.4-16

1.4-17 Two separate cables AC and BC support a sign structure of weight $W = 1575$ lb attached to a building. The sign is also supported by a pin support at O and a lateral restraint in the z-direction at D.

(a) Find the tension in each cable. Neglect the mass of the cables.

(b) Find the average stress in each cable if the area of each cable is $A_e = 0.471$ in^2.



PROBLEM 1.4-17

1.5 Mechanical Properties of Materials

Introductory Problems

1.5-1 Imagine that a long steel wire hangs vertically from a high-altitude balloon.

(a) What is the greatest length (feet) it can have without yielding if the steel yields at 40 ksi?

(b) If the same wire hangs from a ship at sea, what is the greatest length? (Obtain the weight densities of steel and sea water from Table I-1, Appendix I.)

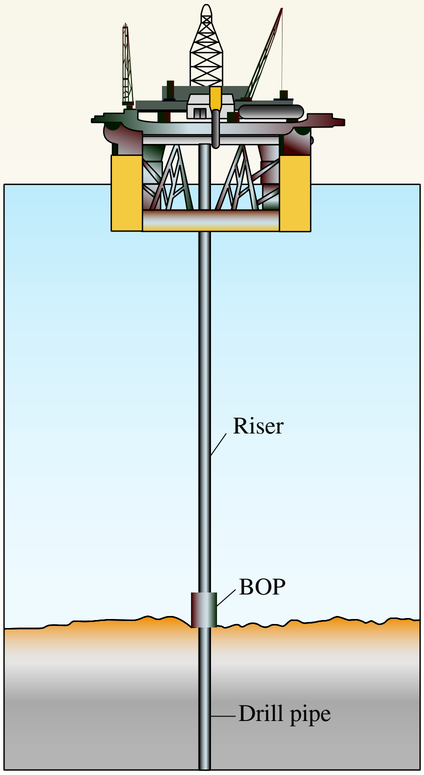
1.5-2 A steel riser pipe hangs from a drill rig located offshore in deep water (see figure).

(a) What is the greatest length (meters) it can have without breaking if the pipe is suspended in the air and the ultimate strength (or breaking strength) is 550 MPa?

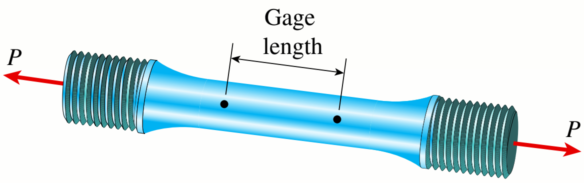
(b) If the same riser pipe hangs from a drill rig at sea, what is the greatest length? (Obtain the weight densities of steel and sea water from Table I-1, Appendix I. Neglect the effect of buoyant foam casings on the pipe.)

1.5-3 Three different materials, designated A, B, and C, are tested in tension using test specimens having diameters of 0.505 in. and gage lengths of 2.0 in. (see figure). At failure, the distances between the gage marks are found to be 2.13, 2.48, and 2.78 in., respectively. Also, at the failure cross sections, the diameters are found to be 0.484, 0.398, and 0.253 in., respectively.

Determine the percent elongation and percent reduction in area of each specimen. Using your own judgment, classify each material as brittle or ductile.



PROBLEM 1.5-2



PROBLEM 1.5-3

Representative Problems

1.5-4 The *strength-to-weight ratio* of a structural material is defined as its load-carrying capacity divided by its weight. For materials in tension, use a characteristic tensile stress obtained from a stress-strain curve as a measure of strength. For instance, either the yield stress or the ultimate stress could be used, depending upon the particular application. Thus, the strength-to-weight ratio $R_{S/W}$ for a material in tension is defined as

$$R_{S/W} = \frac{\sigma}{\gamma}$$

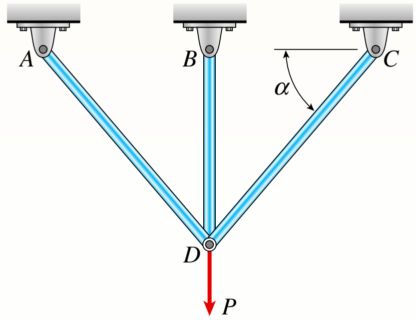
in which σ is the characteristic stress and γ is the weight density. Note that the ratio has units of length.

Using the ultimate stress σ_U as the strength parameter, calculate the strength-to-weight ratio (in units of meters) for each of the following materials: aluminum alloy 6061-T6, Douglas fir (in bending), nylon, structural steel ASTM-A572, and a titanium alloy. Obtain the material properties from Tables I-1 and I-3 of Appendix I. When a range of values is given in a table, use the average value.

1.5-5 A symmetrical framework consisting of three pin-connected bars is loaded by a force P (see figure). The angle between the inclined bars and the horizontal is $\alpha = 52°$. The axial strain in the middle bar is measured as 0.036.

Determine the tensile stress in the outer bars if they are constructed of a copper alloy having the following stress-strain relationship:

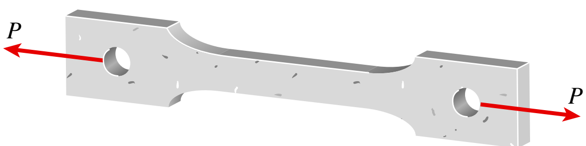
$$\sigma = \frac{18{,}000\varepsilon}{1 + 300\varepsilon} \qquad 0 \le \varepsilon \le 0.03 \qquad (\sigma = \text{ksi})$$



PROBLEM 1.5-5

1.5-6 A specimen of a methacrylate plastic is tested in tension at room temperature (see figure), producing the stress-strain data listed in the accompanying table (see next page).

Plot the stress-strain curve and determine the proportional limit, modulus of elasticity (which is the slope of the initial part of the stress-strain curve), and the yield stress at 0.2% offset. Is the material ductile or brittle?



PROBLEM 1.5-6

STRESS-STRAIN DATA FOR PROB. 1.5-6

Stress (MPa)	Strain
8.0	0.0032
17.5	0.0073
25.6	0.0111
31.1	0.0129
39.8	0.0163
44.0	0.0184
48.2	0.0209
53.9	0.0260
58.1	0.0331
60.2	0.0384
62.0	0.0429
62.1	Fracture

1.5-7 The data shown in the accompanying table are from a tensile test of high-strength steel. The test specimen has a diameter of 0.505 in. and a gage length of 2.00 in. (see figure for Prob. 1.5-3). At fracture, the elongation between the gage marks is 0.12 in. and the minimum diameter is 0.42 in.

Plot the conventional stress-strain curve for the steel and determine the proportional limit, modulus of elasticity (the slope of the initial part of the stress-strain curve), yield stress at 0.1% offset, ultimate stress, percent elongation in 2.00 in., and percent reduction in area.

TENSILE-TEST DATA FOR PROB. 1.5-7

Load (lb)	Elongation (in.)
1000	0.0002
2000	0.0006
6000	0.0019
10,000	0.0033
12,000	0.0039
12,900	0.0043
13,400	0.0047
13,600	0.0054
13,800	0.0063
14,000	0.0090
14,400	0.0102
15,200	0.0130
16,800	0.0230
18,400	0.0336
20,000	0.0507
22,400	0.1108
22,600	Fracture

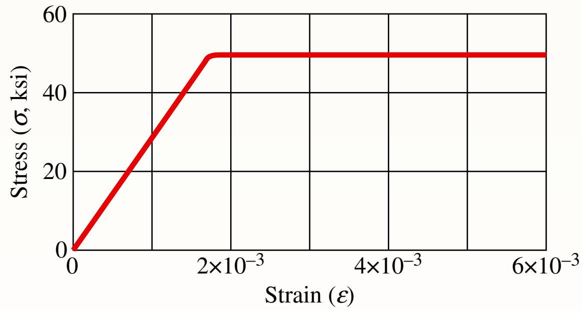
1.6 Elasticity, Plasticity, and Creep

Introductory Problems

1.6-1 A bar made of structural steel having the stress-strain diagram shown in the figure has a length of 60 in. The yield stress of the steel is 50 ksi, and the slope of the initial linear part of the stress-strain curve is 29,000 ksi.

(a) The bar is loaded axially until it elongates 0.2 in. and then the load is removed. How does the final length of the bar compare with its original length?

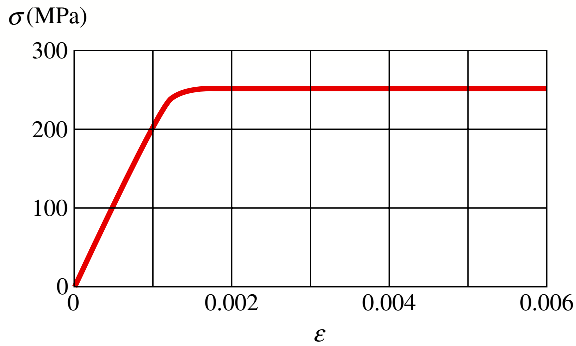
(b) If the bar has a circular cross section with a diameter $d = 1.5$ in. and is loaded by tensile forces $P = 80$ kips, what is the stress in the bar? What is the permanent set of the bar?



PROBLEM 1.6-1

1.6-2 A bar of length 2.0 m is made of a structural steel having the stress-strain diagram shown in the figure. The yield stress of the steel is 250 MPa, and the slope of the initial linear part of the stress-strain curve (modulus of elasticity) is 200 GPa. The bar is loaded axially until it elongates 6.5 mm, and then the load is removed.

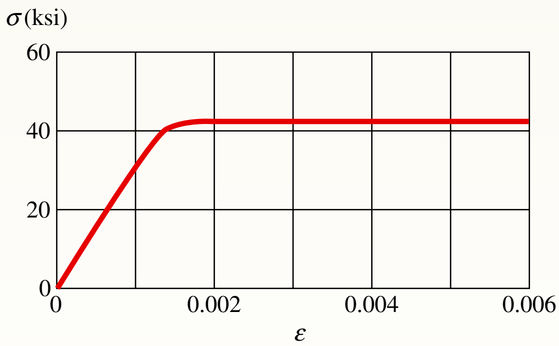
How does the final length of the bar compare with its original length? *Hint:* Use the concepts illustrated in Fig. 1-39b.



PROBLEM 1.6-2

1.6-3 A bar made of structural steel having the stress-strain diagram shown in the figure has a length of 48 in. The yield stress of the steel is 42 ksi, and the slope of the initial linear part of the stress-strain curve (modulus of elasticity) is 30×10^3 ksi. The bar is loaded axially until it elongates 0.20 in., and then the load is removed.

How does the final length of the bar compare with its original length? *Hint:* Use the concepts illustrated in Fig. 1-39b.



PROBLEM 1.6-3

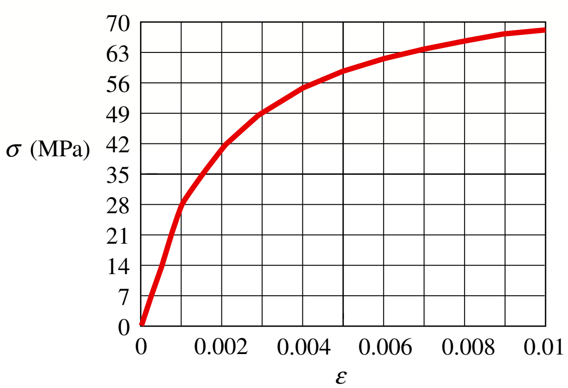
Representative Problems

1.6-4 A circular bar of magnesium alloy is 750 mm long. The stress-strain diagram for the material is shown in the figure. The bar is loaded in tension to an elongation of 6.0 mm, and then the load is removed.

(a) What is the permanent set of the bar?

(b) If the bar is reloaded, what is the proportional limit?

Hint: Use the concepts illustrated in Figs. 1-39b and 1-40.



PROBLEM 1.6-4

1.6-5 An aluminum bar has length $L = 6$ ft and diameter $d = 1.375$ in. The stress-strain curve for the aluminum is shown in Fig. 1-34. The initial straight-line part of the curve has a slope (modulus of elasticity) of 10.6×10^6 psi. The bar is loaded by tensile forces $P = 44.6$ k and then unloaded.

(a) What is the permanent set of the bar?

(b) If the bar is reloaded, what is the proportional limit?

Hint: Use the concepts illustrated in Figs. 1-39b and 1-40.

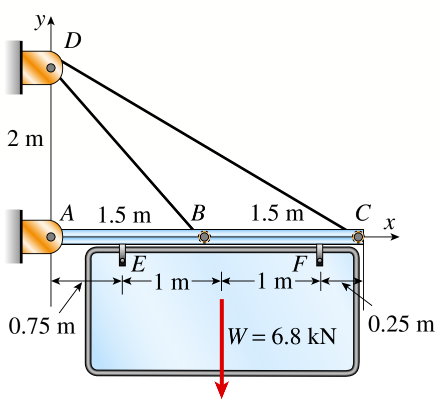
1.6-6 A continuous cable (diameter 6 mm) with tension force T is attached to a horizontal frame member at B and C to support a sign structure. The cable passes over a small frictionless pulley at D. The wire is made of a copper alloy, and the stress-strain relationship for the wire is

$$\sigma(\varepsilon) = \frac{124{,}000\varepsilon}{1 + 300\varepsilon} \quad 0 \leq \varepsilon \leq 0.03 \quad (\sigma \text{ in MPa})$$

(a) Find the axial normal strain in the cable and its elongation due to the load $W = 6.8$ kN.

(b) If the forces are removed, what is the permanent set of the cable?

Hint: Start with constructing the stress-strain diagram and determine the modulus of elasticity, E, and the 0.2% offset yield stress.



PROBLEM 1.6-6

1.6-7 A wire of length $L = 4$ ft and diameter $d = 0.125$ in. is stretched by tensile forces $P = 600$ lb. The wire is made of a copper alloy having a stress-strain relationship that may be described mathematically by

$$\sigma = \frac{18{,}000\varepsilon}{1 + 300\varepsilon} \qquad 0 \le \varepsilon \le 0.03 \qquad (\sigma = \text{ksi})$$

in which ε is nondimensional and σ has units of kips per square inch (ksi).

(a) Construct a stress-strain diagram for the material.
(b) Determine the elongation of the wire due to the forces P.
(c) If the forces are removed, what is the permanent set of the bar?
(d) If the forces are applied again, what is the proportional limit?

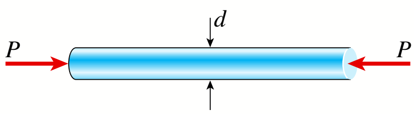
1.7 Linear Elasticity, Hooke's Law, and Poisson's Ratio

When solving the problems for Section 1.7, assume that the material behaves linearly elastically.

Introductory Problems

1.7-1 A high-strength steel bar used in a large crane has a diameter $d = 2.00$ in. (see figure). The steel has a modulus of elasticity $E = 29 \times 10^6$ psi and Poisson's ratio is $\nu = 0.29$. Because of clearance requirements, the diameter of the bar is limited to 2.001 in. when it is compressed by axial forces.

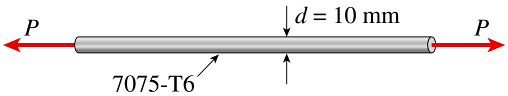
What is the largest compressive load P_{max} that is permitted?



PROBLEM 1.7-1

1.7-2 A round bar of 10 mm diameter is made of aluminum alloy 7075-T6 (see figure). When the bar is stretched by axial forces P, its diameter decreases by 0.016 mm.

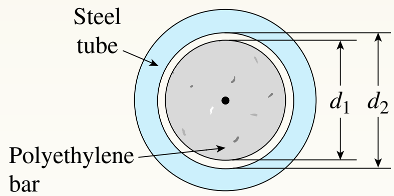
Find the magnitude of the load P. Obtain the material properties from Appendix I.



PROBLEM 1.7-2

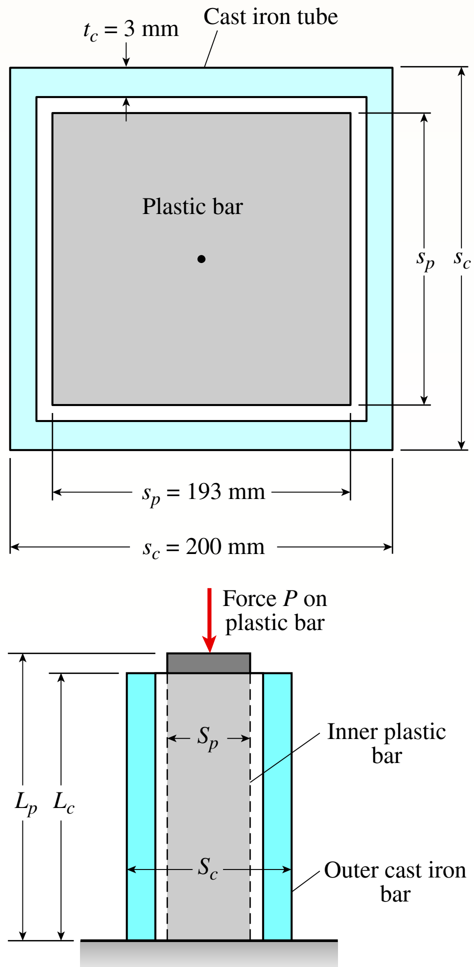
1.7-3 A polyethylene bar with a diameter $d_1 = 4.0$ in. is placed inside a steel tube with an inner diameter $d_2 = 4.01$ in. (see figure). The polyethylene bar is then compressed by an axial force P.

At what value of the force P will the space between the polyethylene bar and the steel tube be closed? For polyethylene, assume $E = 200$ ksi and $\nu = 0.4$.



PROBLEM 1.7-3

1.7-4 A square plastic bar (length L_p, side dimension $s_p = 193$ mm) is inserted inside a hollow, square, cast iron tube (length $L_c = 400$ mm, side $s_c = 200$ mm, and thickness $t_c = 3$ mm).



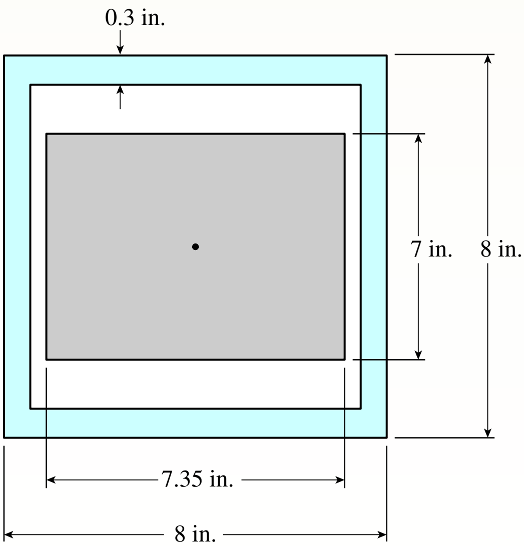
PROBLEM 1.7-4

(a) What is the required initial length L_p of the plastic bar so that, when it is compressed by some force P, the final length of bar and tube are equal to length L_c and, at the same time, the gap between plastic bar and cast iron tube is closed?

(b) Compare initial and final volumes for the plastic bar.

Assume that $E_c = 170$ GPa, $E_p = 2.1$ GPa, $\nu_c = 0.3$, and $\nu_p = 0.4$.

1.7-5 A polyethylene bar having rectangular cross section with a width 7.35 in. and depth 7 in. is placed inside a hollow steel square section with side dimension of 8 in. The polyethylene bar is then compressed by an axial force P. At what value of the force P will the gap between the polyethylene bar and the steel tube be closed for the first time on one side? What is the remaining gap between the polyethylene bar and the steel tube on the other side? For polyethylene, assume $E = 200$ ksi and $\nu = 0.4$.



PROBLEM 1.7-5

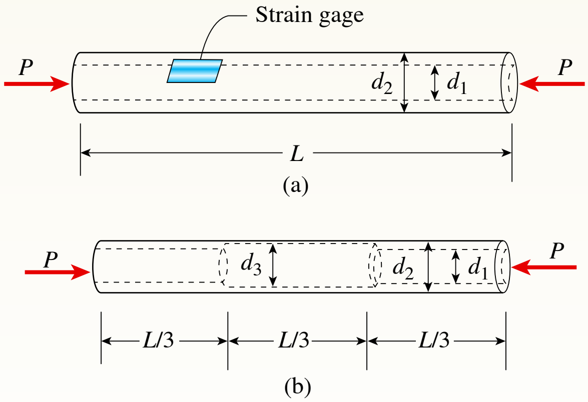
Representative Problems

1.7-6 A circular aluminum tube of length $L = 600$ mm is loaded in compression by forces P (see figure). The outside and inside diameters are $d_2 = 75$ mm and $d_1 = 63$ mm, respectively. A strain gage is placed on the outside of the tube to measure normal strains in the longitudinal direction. Assume that $E = 73$ GPa and Poisson's ratio is $\nu = 0.33$.

(a) If the compressive stress in the tube is 57 MPa, what is the load P?

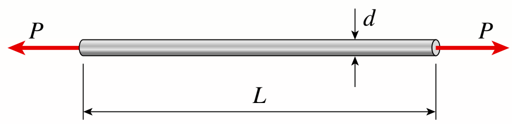
(b) If the measured strain is $\varepsilon = 781 \times 10^{-6}$, what is the shortening δ of the tube? What is the percent change in its cross-sectional area? What is the volume change of the tube?

(c) If the tube has a constant outer diameter of $d_2 = 75$ mm along its entire length L but now has increased *inner* diameter d_3 with a normal stress of 70 MPa over the middle third (see figure, part b) while the rest of the tube remains at normal stress of 57 MPa, what is the diameter d_3?



PROBLEM 1.7-6

1.7-7 A bar of monel metal with a length $L = 9$ in. and a diameter $d = 0.225$ in. is loaded axially by a tensile force P (see figure). If the bar elongates by 0.0195 in., what is the decrease in diameter d? What is the magnitude of the load P? Use the data in Table I-2, Appendix I.

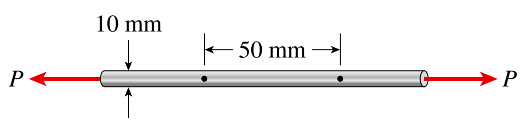


PROBLEM 1.7-7

1.7-8 A tensile test is performed on a brass specimen 10 mm in diameter using a gage length of 50 mm (see figure). When the tensile load P reaches a value of 20 kN, the distance between the gage marks has increased by 0.122 mm.

(a) What is the modulus of elasticity E of the brass?

(b) If the diameter decreases by 0.00830 mm, what is Poisson's ratio?



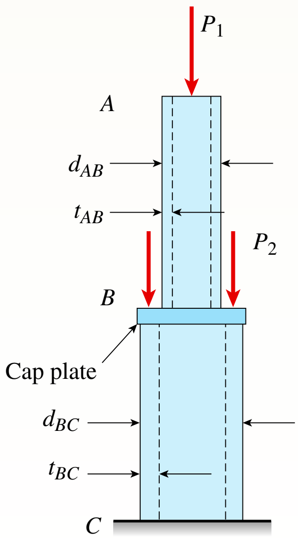
PROBLEM 1.7-8

1.7-9 A hollow, brass circular pipe ABC (see figure) supports a load $P_1 = 26.5$ kips acting at the top. A second load $P_2 = 22.0$ kips is uniformly distributed around the cap plate at B. The diameters and thicknesses of the upper and lower parts of the pipe are $d_{AB} = 1.25$ in., $t_{AB} = 0.5$ in., $d_{BC} = 2.25$ in., and $t_{BC} = 0.375$ in., respectively. The modulus of elasticity is 14,000 ksi. When both loads are fully applied, the wall thickness of pipe segment BC increases by 200×10^{-6} in.

(a) Find the increase in the inner diameter of pipe segment BC.

(b) Find Poisson's ratio for the brass.

(c) Find the increase in the wall thickness of pipe segment AB and the increase in the inner diameter of segment AB.



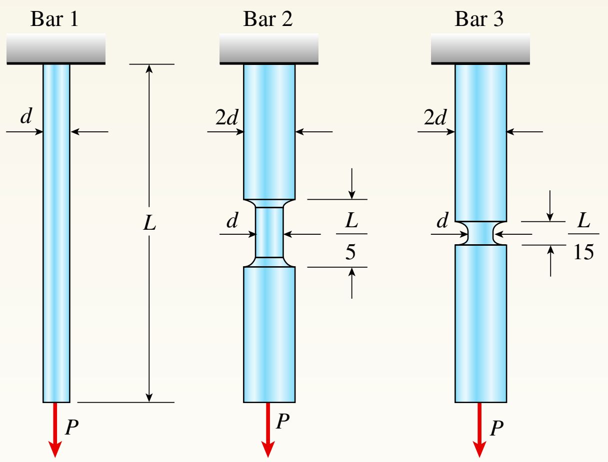
PROBLEM 1.7-9

1.7-10 Three round, copper alloy bars having the same length L but different shapes are shown in the figure. The first bar has a diameter d over its entire length, the second has a diameter d over one-fifth of its length, and the third has a diameter d over one-fifteenth of its length. Elsewhere, the second and third bars have a diameter $2d$. All three bars are subjected to the same axial load P.

Use the following numerical data: $P = 1400$ kN, $L = 5$ m, $d = 80$ mm, $E = 110$ GPa, and $\nu = 0.33$.

(a) Find the change in length of each bar.

(b) Find the change in volume of each bar.

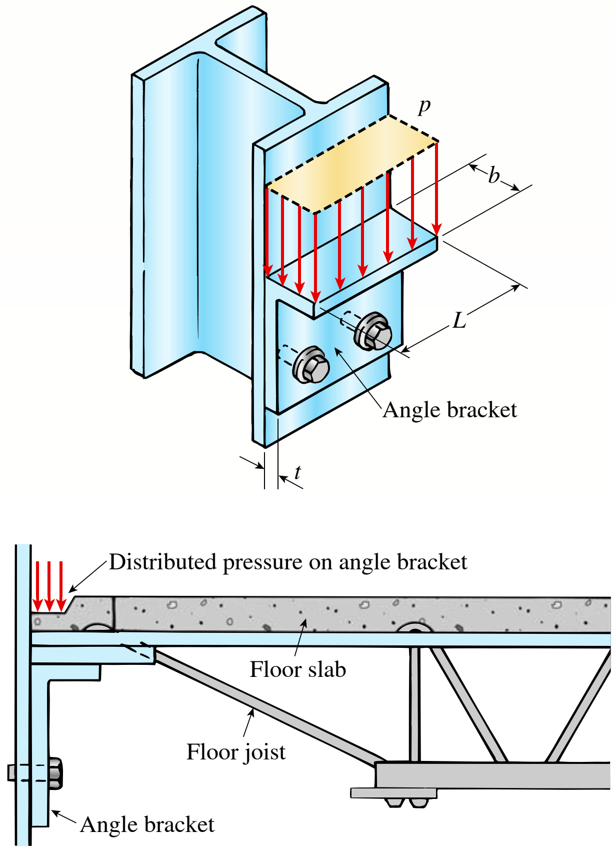


PROBLEM 1.7-10

1.8 Shear Stress and Strain

Introductory Problems

1.8-1 An angle bracket having a thickness $t = 0.75$ in. is attached to the flange of a column by two 5/8-inch diameter bolts (see figure). A uniformly distributed load from a floor joist acts on the top face of the bracket with a pressure $p = 275$ psi. The top face of the bracket has a length $L = 8$ in. and width $b = 3.0$ in.



PROBLEM 1.8-1

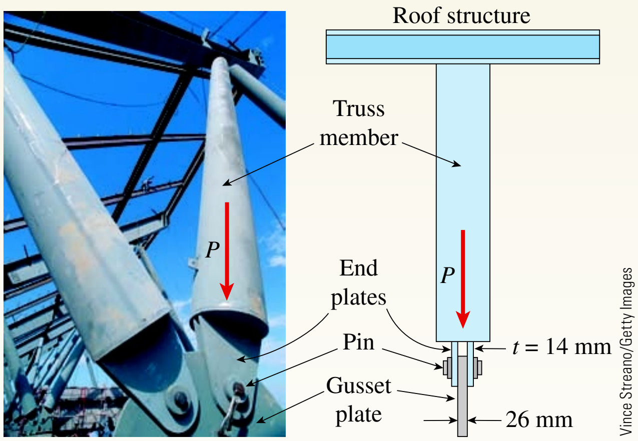
Determine the average bearing pressure σ_b between the angle bracket and the bolts and the average shear stress τ_{aver} in the bolts. Disregard friction between the bracket and the column.

1.8-2 Truss members supporting a roof are connected to a 26-mm-thick gusset plate by a 22-mm diameter pin, as shown in the figure and photo. The two end plates on the truss members are each 14 mm thick.

(a) If the load $P = 80$ kN, what is the largest bearing stress acting on the pin?

(b) If the ultimate shear stress for the pin is 190 MPa, what force P_{ult} is required to cause the pin to fail in shear?

Disregard friction between the plates.

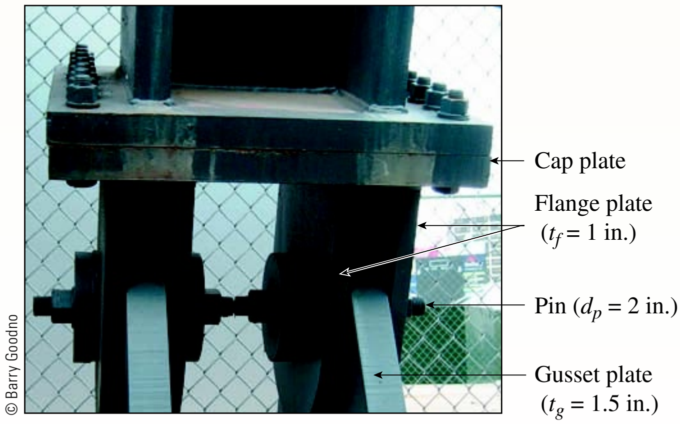


Truss members supporting a roof

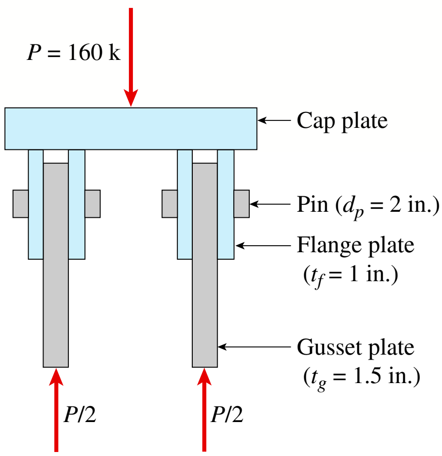
PROBLEM 1.8-2

1.8-3 The upper deck of a football stadium is supported by braces, each of which transfers a load $P = 160$ kips to the base of a column (see figure part a). A cap plate at the bottom of the brace distributes the load P to four flange plates ($t_f = 1$ in.) through a pin ($d_p = 2$ in.) to two gusset plates ($t_g = 1.5$ in.) (see figure parts b and c).

(a) Stadium brace



(b) Detail at bottom of brace



(c) Section through bottom of brace

PROBLEM 1.8-3

Determine the following quantities.

(a) The average shear stress τ_{aver} in the pin.

(b) The average bearing stress between the flange plates and the pin (σ_{bf}), and also between the gusset plates and the pin (σ_{bg}).

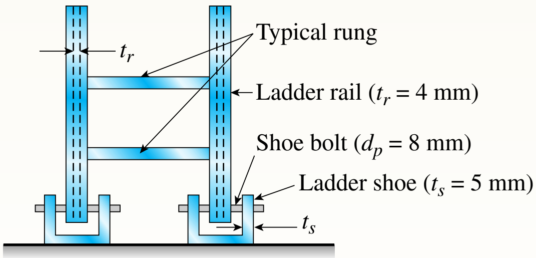
Disregard friction between the plates.

1.8-4 The inclined ladder AB supports a house painter (85 kg) at C and the weight ($q = 40$ N/m) of the ladder itself. Each ladder rail ($t_r = 4$ mm) is supported by a shoe ($t_s = 5$ mm) that is attached to the ladder rail by a bolt of diameter $d_p = 8$ mm.

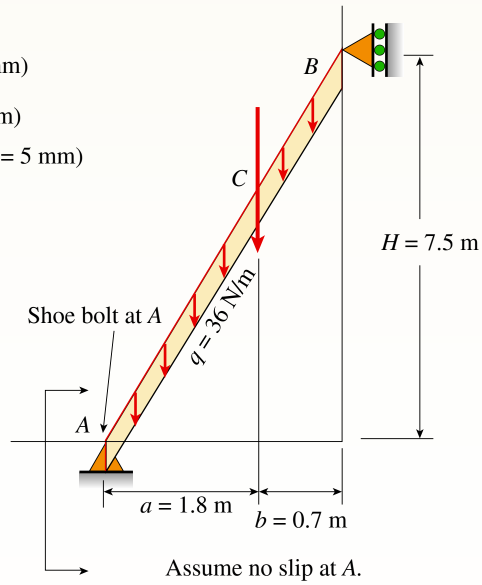
(a) Find support reactions at A and B.

(b) Find the resultant force in the shoe bolt at A.

(c) Find maximum average shear τ and bearing (σ_b) stresses in the shoe bolt at A.



Section at base



Assume no slip at A.

PROBLEM 1.8-4

1.8-5 The force in the brake cable of the V-brake system shown in the figure is $T = 45$ lb. The pivot pin at A has a diameter $d_p = 0.25$ in. and length $L_p = 5/8$ in.

Use the dimensions shown in the figure. Neglect the weight of the brake system.

(a) Find the average shear stress τ_{aver} in the pivot pin where it is anchored to the bicycle frame at B.

(b) Find the average bearing stress $\sigma_{b,aver}$ in the pivot pin over segment AB.

1.8-6 A steel plate of dimensions $2.5 \times 1.5 \times 0.08$ m and weighing 23.1 kN is hoisted by steel cables with lengths $L_1 = 3.2$ m and $L_2 = 3.9$ m that are each attached to the plate by a clevis and pin (see figure). The pins through the clevises are 18 mm in diameter and are located 2.0 m apart. The orientation angles are measured to be $\theta = 94.4°$ and $\alpha = 54.9°$.

For these conditions, first determine the cable forces T_1 and T_2, then find the average shear stress τ_{aver} in both pin 1 and pin 2, and then the average bearing stress σ_b between the steel plate and each pin. Ignore the weight of the cables.

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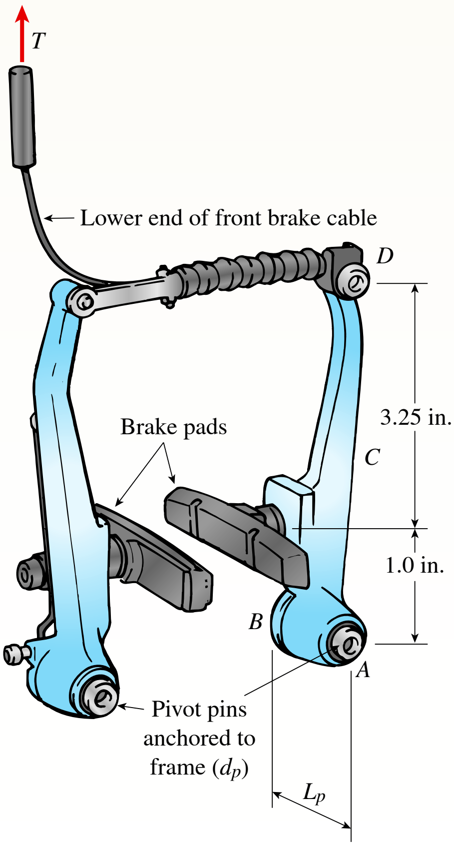
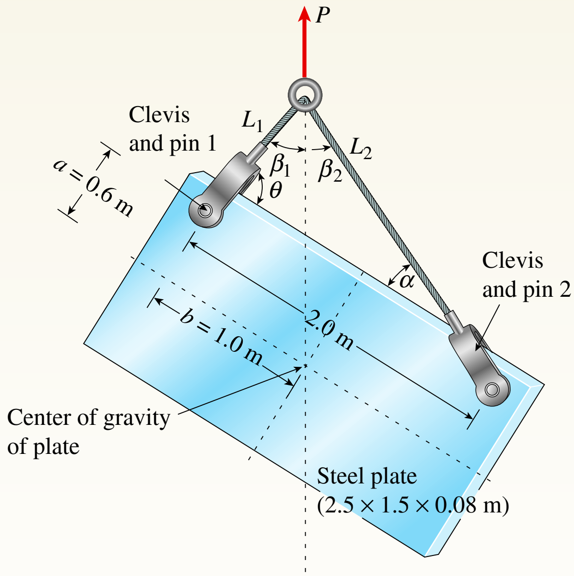


PROBLEM 1.8-5



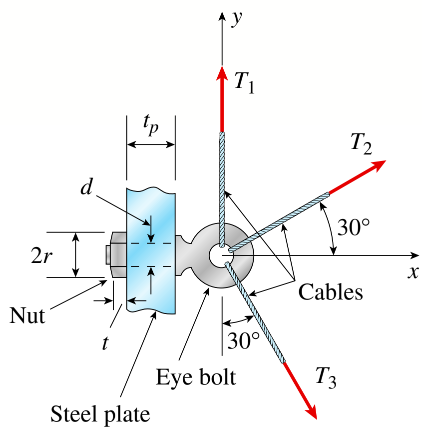
PROBLEM 1.8-6

1.8-7 A special-purpose eye bolt with a shank diameter $d = 0.50$ in. passes through a hole in a steel plate of thickness $t_p = 0.75$ in. (see figure) and is secured by a nut with thickness $t = 0.25$ in. The hexagonal nut bears directly against the steel plate. The radius of the circumscribed circle for the hexagon is $r = 0.40$ in., so each side of the hexagon has a length 0.40 in. The tensile forces in three cables attached to the eye bolt are $T_1 = 800$ lb, $T_2 = 500$ lb, and $T_3 = 1241$ lb.

(a) Find the resultant force acting on the eye bolt.

(b) Determine the average bearing stress σ_b between the hexagonal nut on the eye bolt and the plate.

(c) Determine the average shear stress τ_{aver} in the nut and also in the steel plate.

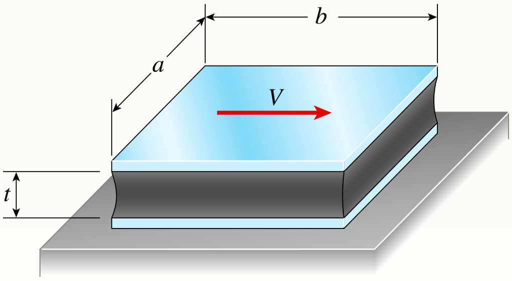


PROBLEM 1.8-7

Representative Problems

1.8-8 An elastomeric bearing pad consisting of two steel plates bonded to a chloroprene elastomer (an artificial rubber) is subjected to a shear force V during a static loading test (see figure). The pad has dimensions $a = 125$ mm and $b = 240$ mm, and the elastomer has a thickness $t = 50$ mm. When the force V equals 12 kN, the top plate is found to have displaced laterally 8.0 mm with respect to the bottom plate.

What is the shear modulus of elasticity G of the chloroprene?

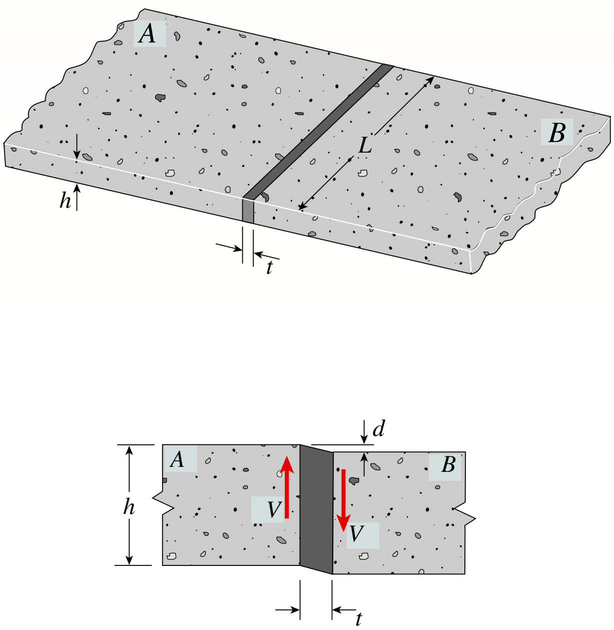


PROBLEM 1.8-8

1.8-9 A joint between two concrete slabs A and B is filled with a flexible epoxy that bonds securely to the concrete (see figure). The height of the joint is $h = 4.0$ in., its length is $L = 40$ in., and its thickness is $t = 0.5$ in. Under the action of shear forces V, the slabs displace vertically through the distance $d = 0.002$ in. relative to each other.

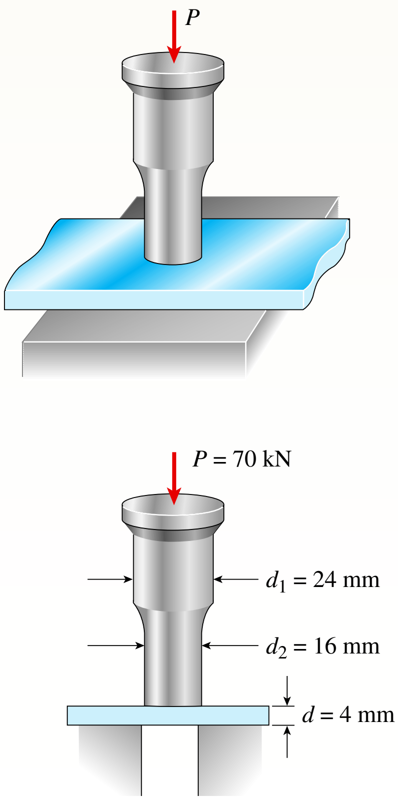
(a) What is the average shear strain γ_{aver} in the epoxy?

(b) What is the magnitude of the forces V if the shear modulus of elasticity G for the epoxy is 140 ksi?



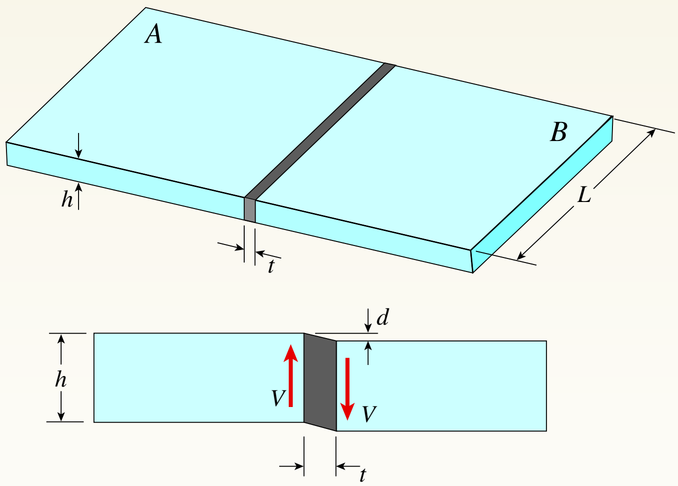
PROBLEM 1.8-9

1.8-10 A steel punch consists of two shafts: upper shaft and lower shaft. Assume that the upper shaft has a diameter $d_1 = 24$ mm and the bottom shaft has a diameter $d_2 = 16$ mm. The punch is used to insert a hole in a 4 mm plate, as shown in the figure. If a force $P = 70$ kN is required to create the hole, what is the average shear stress in the plate and the average compressive stress in the upper and lower shaft of the punch?



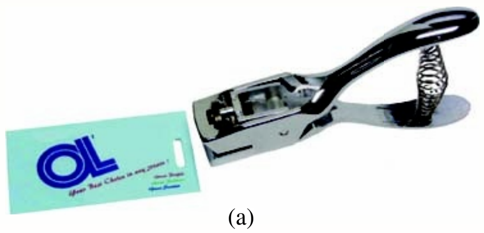
PROBLEM 1.8-10

1.8-11 A joint between two glass plates A and B is filled with a flexible epoxy that bonds securely to the glass. The height of the joint is $h = 0.5$ in., its length is $L = 30$ in., and its thickness is $t = 0.5$ in. Shear force of $V = 25$ kips is applied to the joint. Calculate the displacement of the joint if the shear modulus of elasticity G of the epoxy is 100 ksi. Calculate the average shear strain in the epoxy.

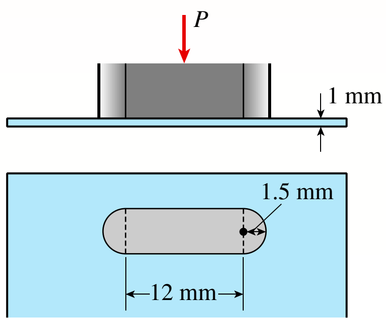


PROBLEM 1.8-11

1.8-12 A punch for making a slotted hole in ID cards is shown in the figure part a. Assume that the hole produced by the punch can be described as a rectangle (12 mm × 3 mm) with two half circles ($r = 1.5$ mm) on the left and the right sides. If $P = 10$ N and the thickness of the ID card is 1 mm, what is the average shear stress in the card?



(a)



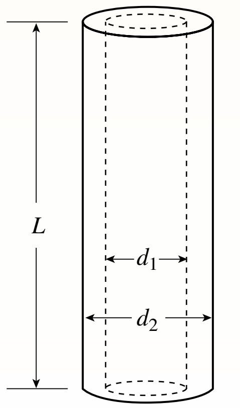
(b)

PROBLEM 1.8-12

1.8-13 A steel riser pipe hangs from a drill rig located offshore in deep water (see figure). Separate segments are joined using bolted flange plages (see figure part b and photo). Assume that there are six bolts at each pipe segment connection. Assume that the total length of the riser pipe is $L = 5000$ ft; outer and inner diameters are $d_2 = 16$ in. and $d_1 = 15$ in.; flange plate thickness $t_f = 1.75$ in.; and bolt and washer diameters are $d_b = 1.125$ in., and $d_w = 1.875$ in., respectively.

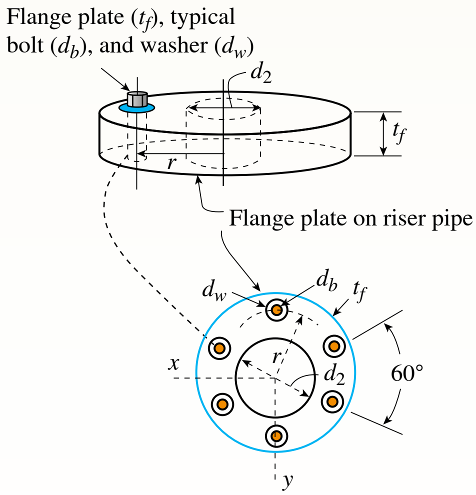
(a) If the entire length of the riser pipe is suspended in air, find the average normal stress σ in each bolt, the average bearing stress σ_b beneath each washer, and the average shear stress τ through the flange plate at each bolt location for the topmost bolted connection.

(b) If the same riser pipe hangs from a drill rig at sea, what are the normal, bearing, and shear stresses in the connection? Obtain the weight densities of steel and sea water from Table I-1, Appendix I. Neglect the effect of buoyant foam casings on the riser pipe.



Riser pipe (d_2, d_1, L)

(a)



Flange plate on riser pipe–
plan view (n = 6 bolts shown)

(b)

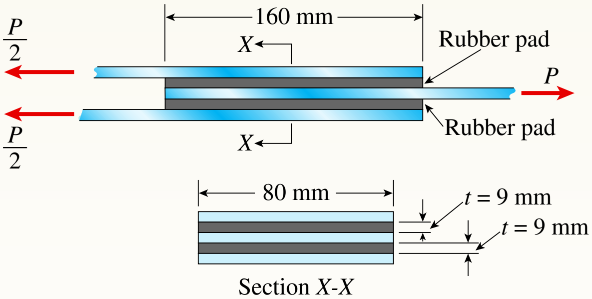
(c)

PROBLEM 1.8-13

1.8-14 A flexible connection consisting of rubber pads (thickness $t = 9$ mm) bonded to steel plates is shown in the figure. The pads are 160 mm long and 80 mm wide.

(a) Find the average shear strain γ_{aver} in the rubber if the force $P = 16$ kN and the shear modulus for the rubber is $G = 1250$ kPa.

(b) Find the relative horizontal displacement δ between the interior plate and the outer plates.

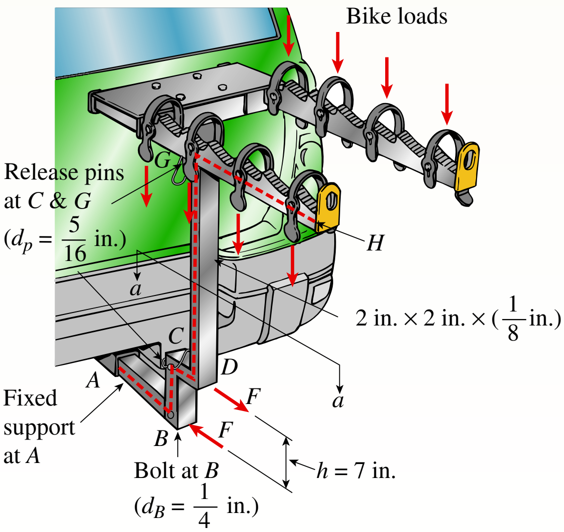


PROBLEM 1.8-14

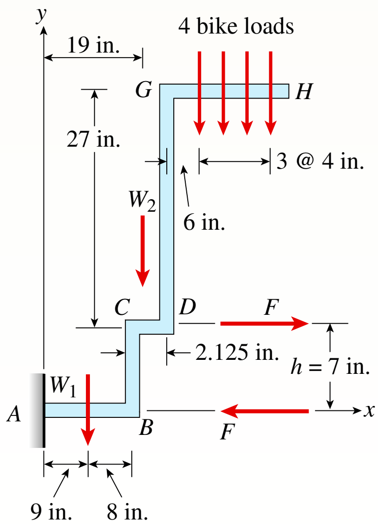
1.8-15 A hitch-mounted bicycle rack is designed to carry up to four 30-lb bikes mounted on and strapped to two arms *GH* (see bike loads in the figure part a). The rack is attached to the vehicle at *A* and is assumed to be like a cantilever beam *ABCDGH* (figure part b). The weight of fixed segment *AB* is $W_1 = 10$ lb, centered 9 in. from *A* (see figure part b) and the rest of the rack weighs $W_2 = 40$ lb, centered 19 in. from *A*. Segment *ABCDG* is a steel tube of 2×2 in. with a thickness $t = 1/8$ in. Segment *BCDGH* pivots about a bolt at *B* with a diameter $d_B = 0.25$ in. to allow access to the rear of the vehicle without removing the hitch rack. When in use, the rack is secured in an upright position by a pin at *C* (diameter of pin $d_p = 5/16$ in.) (see photo and figure part c). The overturning effect of the bikes on the rack is resisted by a force couple $F \cdot h$ at *BC*.

(a) Find the support reactions at *A* for the fully loaded rack.

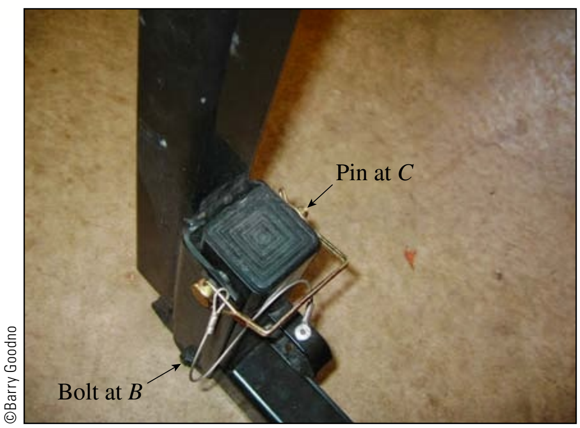
(b) Find forces in the bolt at *B* and the pin at *C*.

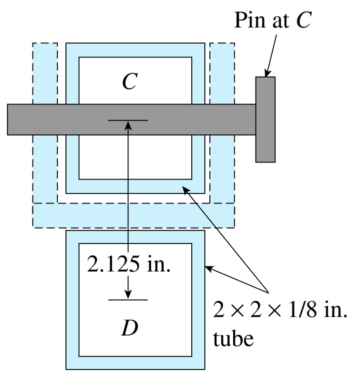


(a)



(b)





(c) Section *a–a*

PROBLEM 1.8-15

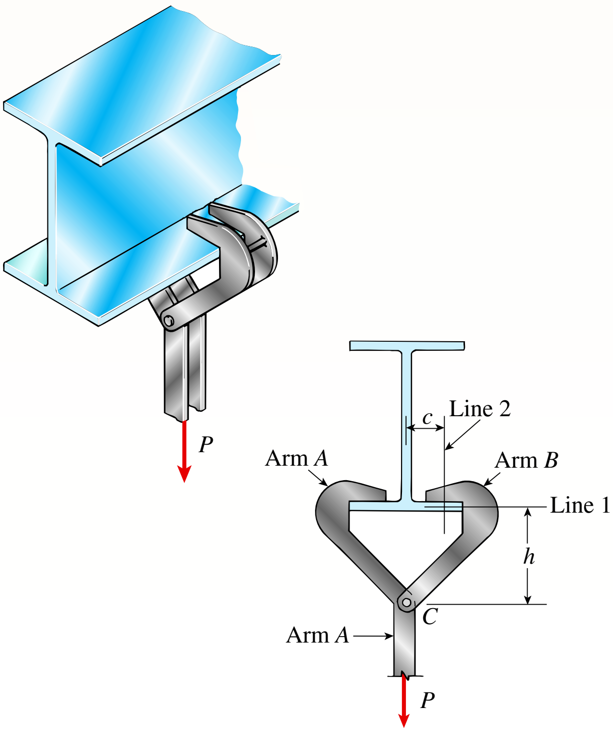
(c) Find average shear stresses τ_{aver} in both the bolt at B and the pin at C.

(d) Find average bearing stresses σ_b in the bolt at B and the pin at C.

1.8-16 The clamp shown in the figure supports a load hanging from the lower flange of a steel beam. The clamp consists of two arms (A and B) joined by a pin at C. The pin has a diameter $d = 12$ mm. Because arm B straddles arm A, the pin is in double shear.

Line 1 in the figure defines the line of action of the resultant horizontal force H acting between the lower flange of the beam and arm B. The vertical distance from this line to the pin is $h = 250$ mm. Line 2 defines the line of action of the resultant vertical force V acting between the flange and arm B. The horizontal distance from this line to the centerline of the beam is $c = 100$ mm. The force conditions between arm A and the lower flange are symmetrical with those given for arm B.

Determine the average shear stress in the pin at C when the load $P = 18$ kN.

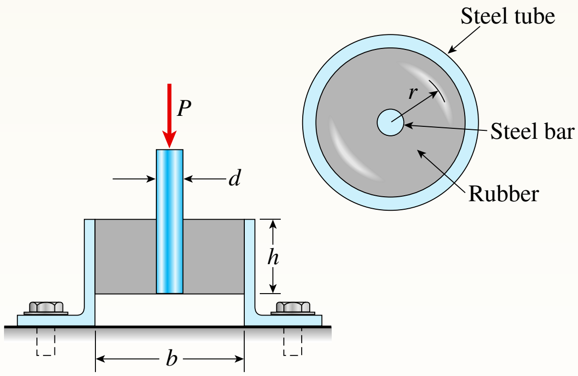


PROBLEM 1.8-16

1.8-17 A shock mount constructed as shown in the figure is used to support a delicate instrument. The mount consists of an outer steel tube with inside diameter b, a central steel bar of diameter d that supports the load P, and a hollow rubber cylinder (height h) bonded to the tube and bar.

(a) Obtain a formula for the shear stress τ in the rubber at a radial distance r from the center of the shock mount.

(b) Obtain a formula for the downward displacement δ of the central bar due to the load P, assuming that G is the shear modulus of elasticity of the rubber and that the steel tube and bar are rigid.



PROBLEM 1.8-17

1.8-18 A removable sign post on a hurricane evacuation route (see figure part a) consists of an upper pole with a slotted base plate bolted to a short post anchored in the ground. The lower post is capped with a separate conventional base plate having four holes of diameter d_b. The upper base plate has slots at locations 1 to 4 and is bolted to the lower base plate at these four points (see figure part b). Each of the four bolts has a diameter of d_b and a washer with a diameter of d_w. The bolts are arranged in a rectangular pattern ($b \times h$). Consider only wind force W_y applied in the y direction at the center of pressure (C.P.) of the sign structure at height $z = L$ above the base. Neglect the weight of the sign and post and the friction between the upper and lower base plates. Assume that the lower base plate and short anchored post are rigid.

(a) Find the average shear stress τ (MPa) at bolts 1 and 4 (see figure part c) due to the wind force W_y.

(b) Find the average bearing stress σ_b (MPa) between the bolt and the upper base plate (thickness t) at bolts 1 and 4.

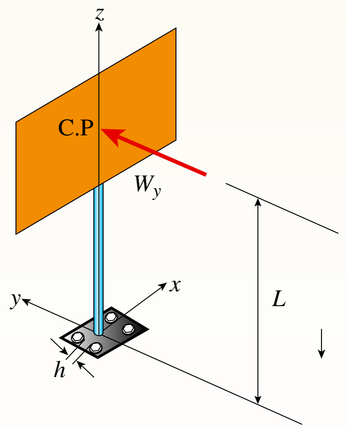
(c) Find the average bearing stress σ_b (MPa) between the upper base plate and washer at bolt 4 due to the wind force W_y (assume the initial bolt pretension is zero).

(d) Find the average shear stress τ (MPa) through the upper base plate at bolt 4 due to the wind force W_y.

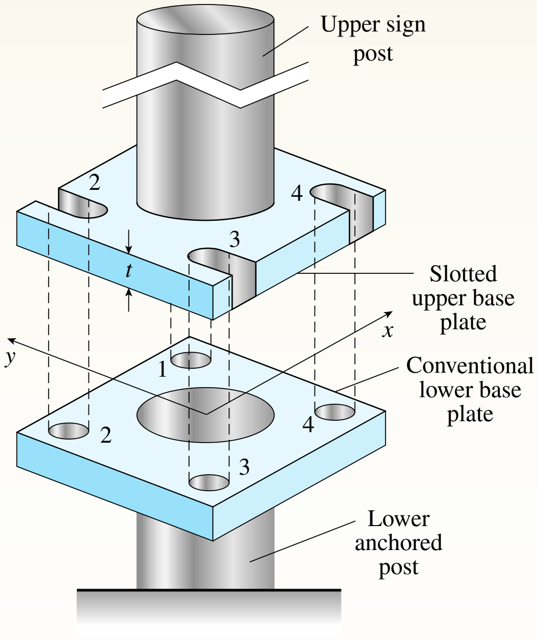
(e) Find an expression for the normal stress σ in bolt 3 due to the wind force W_y.

(See problem 1.9-17 for additional discussion of wind on a sign and the resulting forces acting on a conventional base plate.)

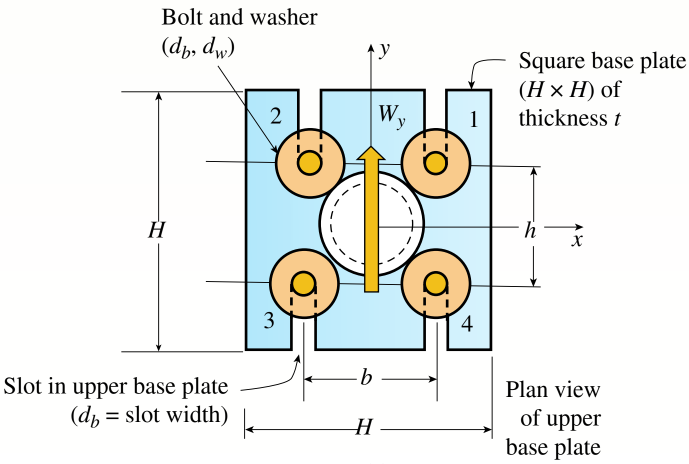
Numerical data	
$H = 150$ mm	$b = 96$ mm
$h = 108$ mm	$t = 14$ mm
$d_b = 12$ mm	$d_w = 22$ mm
$L = 2.75$ m	$W_y = 667$ N



(a)



(b)



(c)

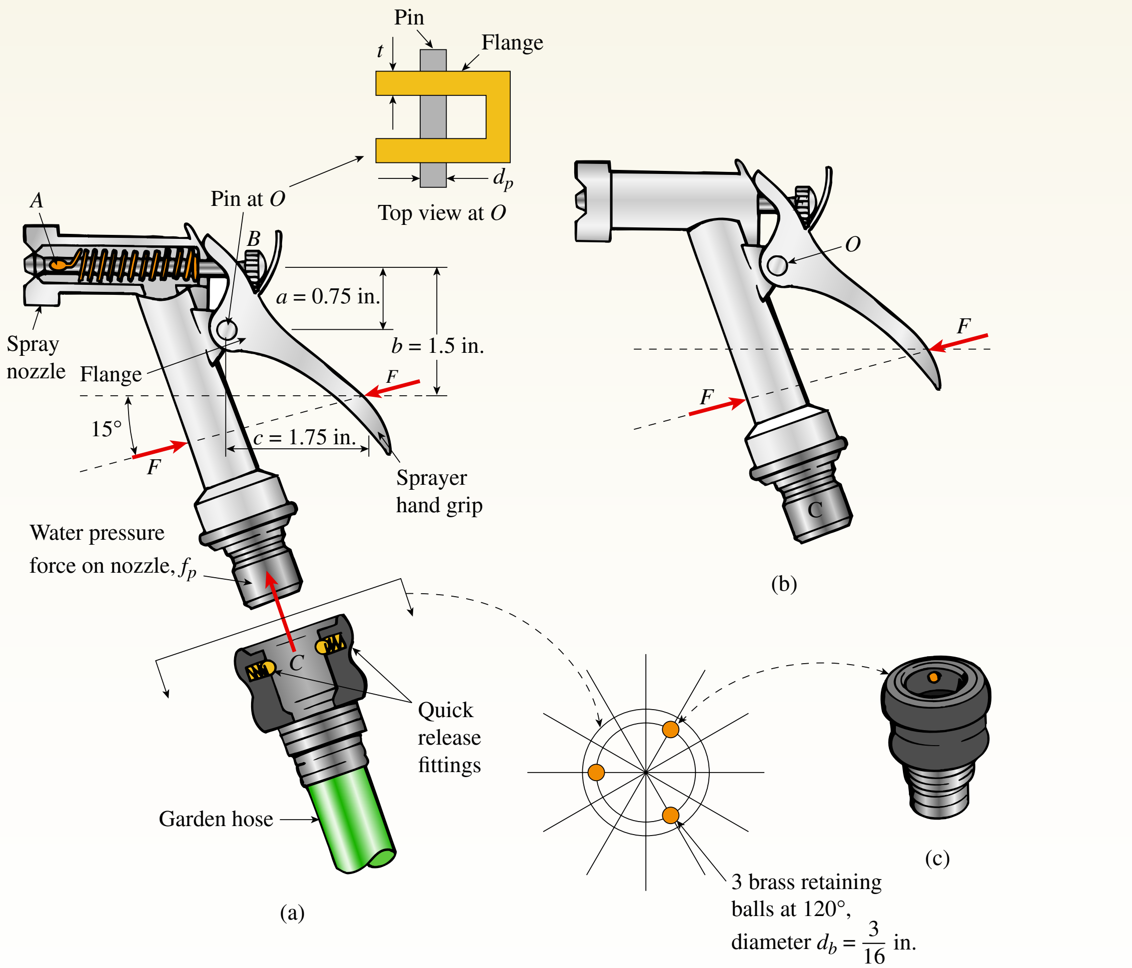
PROBLEM 1.8-18

1.8-19 A spray nozzle for a garden hose requires a force $F = 5$ lb to open the spring-loaded spray chamber AB. The nozzle hand grip pivots about a pin through a flange at O. Each of the two flanges has a thickness $t = 1/16$ in., and the pin has a diameter $d_p = 1/8$ in. (see figure part a). The spray nozzle is attached to the garden hose with a quick release fitting at B (see figure part b). Three brass balls (diameter $d_b = 3/16$ in.) hold the spray head in place under a water pressure force $f_p = 30$ lb at C (see figure part c). Use dimensions given in figure part a.

(a) Find the force in the pin at O due to applied force F.

(b) Find average shear stress τ_{aver} and bearing stress σ_b in the pin at O.

(c) Find the average shear stress τ_{aver} in the brass retaining balls at C due to water pressure force f_p.



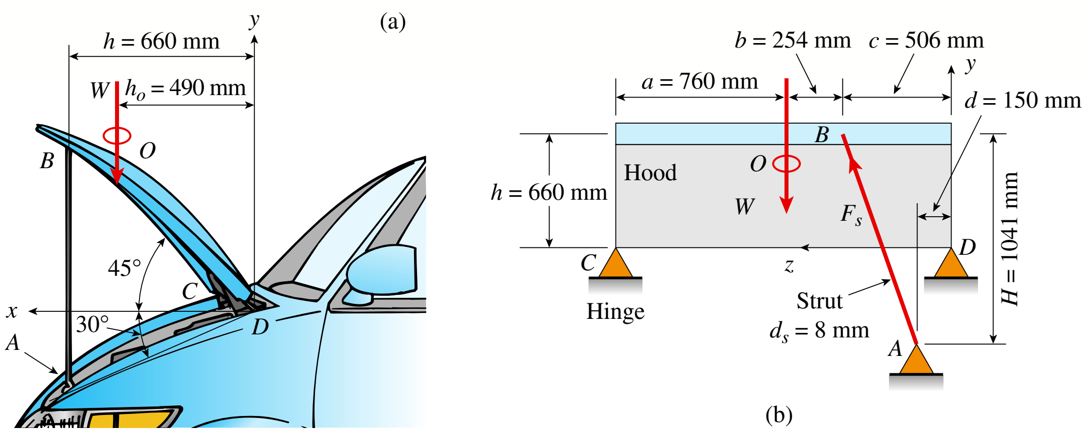
PROBLEM 1.8-19

1.8-20 A single steel strut AB with a diameter $d_s = 8$ mm supports the vehicle engine hood of a mass 20 kg that pivots about hinges at C and D (see figure parts a and b). The strut is bent into a loop at its end and then attached to a bolt at A with a diameter $d_b = 10$ mm. Strut AB lies in a vertical plane.

(a) Find the strut force F_s and average normal stress σ in the strut.

(b) Find the average shear stress τ_{aver} in the bolt at A.

(c) Find the average bearing stress σ_b on the bolt at A.



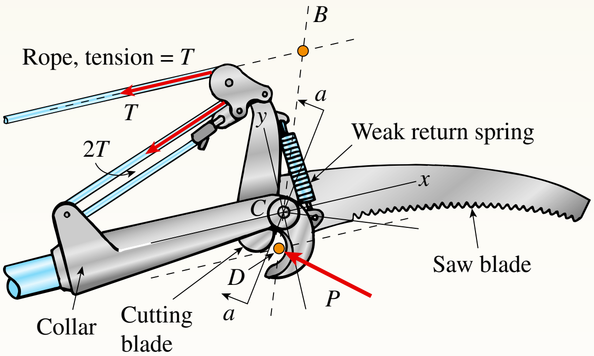
PROBLEM 1.8-20

1.8-21 The top portion of a pole saw used to trim small branches from trees is shown in the figure part a. The cutting blade *BCD* (see figure parts a and c) applies a force *P* at point *D*. Ignore the effect of the weak return spring attached to the cutting blade below *B*. Use properties and dimensions given in the figure.

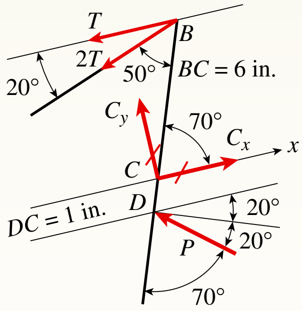
(a) Find the force *P* on the cutting blade at *D* if the tension force in the rope is $T = 25$ lb (see free-body diagram in figure part b).

(b) Find force in the pin at *C*.

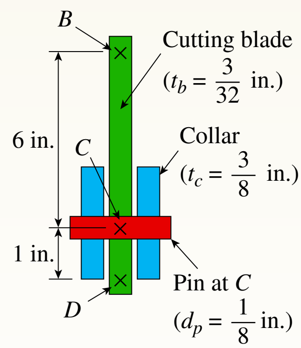
(c) Find average shear stress τ_{aver} and bearing stress σ_b in the support pin at *C* (see section *a–a* through cutting blade in figure part c).



(a) Top part of pole saw



(b) Free-body diagram



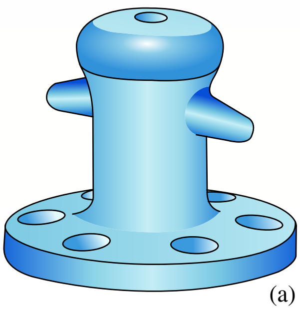
(c) Section *a–a*

PROBLEM 1.8-21

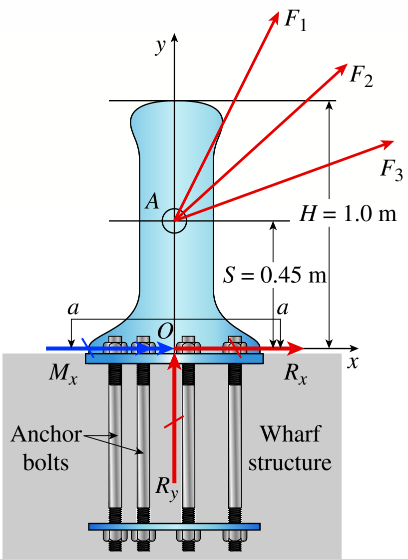
1.8-22 A cargo ship is tied down to marine bollards at a number of points along its length while its cargo is unloaded by a container handling crane. Each bollard is fastened to the wharf using anchor bolts. Three cables having known tension force magnitudes $F_1 = 110$ kN, $F_2 = 85$ kN, and $F_3 = 90$ kN are secured to one bollard at a point *A* with coordinates (0, 0.45 m, 0) in the *x-y-z* coordinate system shown in the figure part b. Each cable force is directed at an attachment point on the ship. Force F_1 is directed from point *A* to a point on the ship having coordinates (3 m, 9 m, 0); force F_2 is directed at a point with coordinates (6.5 m, 8.5 m, 2 m); and force F_3 is directed at a point with coordinates (8 m, 9 m, 5 m). The diameter of each anchor bolts is $d_b = 24$ mm.

(a) Find the reaction forces and reaction moments at the base of the bollard.

(b) Calculate the average shear stress in the anchor bolts (in the *x-z* plane). Assume each bolt carries an equal share of the total force.



(a)



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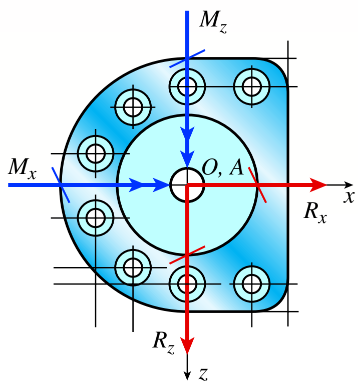


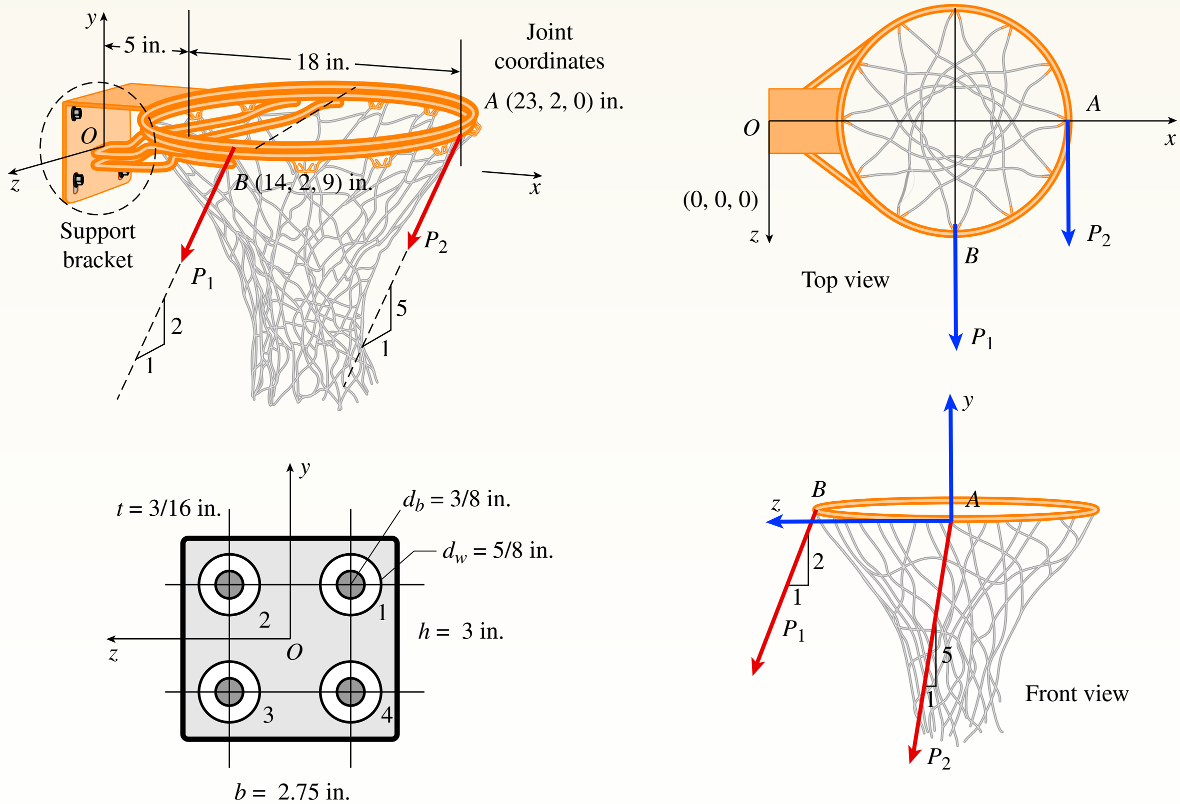
(c)

PROBLEM 1.8-22

1.8-23 A basketball player hangs on the rim after a dunk. He applies equal forces $P_1 = P_2 = 110$ lb at both A and B (see joint coordinates in the figure). Forces P_1 *and* P_2 act parallel to the y-z plane.

(a) Find the *reactions* at the support bracket (assume that the bracket-rim assembly is a cantilever beam).

(b) Find connection shear stresses at bolt 2.



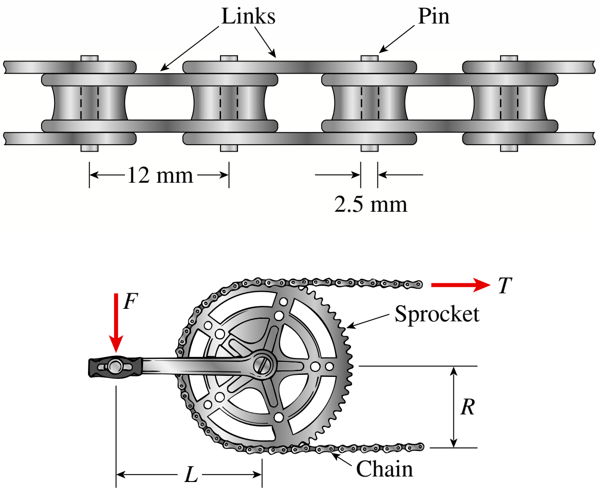
PROBLEM 1.8-23 Support bracket

1.8-24 A bicycle chain consists of a series of small links, where each are 12 mm long between the centers of the pins (see figure). You might wish to examine a bicycle chain and observe its construction. Note particularly the pins, which have a diameter of 2.5 mm.

To solve this problem, make two measurements on a bicycle (see figure): (1) the length L of the crank arm from main axle to pedal axle and (2) the radius R of the sprocket (the toothed wheel, sometimes called the chainring).

(a) Using your measured dimensions, calculate the tensile force T in the chain due to a force $F = 800$ N applied to one of the pedals.

(b) Calculate the average shear stress τ_{aver} in the pins.



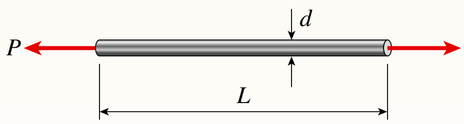
PROBLEM 1.8-24

1.9 Allowable Stresses and Allowable Loads

Introductory Problems

1.9-1 A bar of solid circular cross section is loaded in tension by forces P (see figure). The bar has a length $L = 16.0$ in. and diameter $d = 0.50$ in. The material is a magnesium alloy having a modulus of elasticity $E = 6.4 \times 10^6$ psi. The allowable stress in tension is $\sigma_{allow} = 17{,}000$ psi, and the elongation of the bar must not exceed 0.04 in.

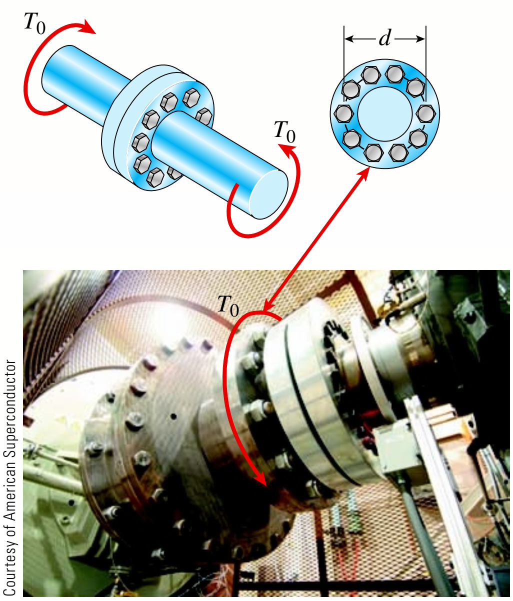
What is the allowable value of the forces P?



PROBLEM 1.9-1

1-9.2 A torque T_0 is transmitted between two flanged shafts by means of ten 20-mm bolts (see figure and photo). The diameter of the bolt circle is $d = 250$ mm.

If the allowable shear stress in the bolts is 85 MPa, what is the maximum permissible torque? (Disregard friction between the flanges.)



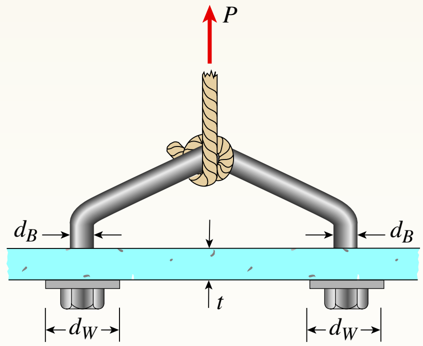
Courtesy of American Superconductor

Drive shaft coupling on a ship propulsion motor

PROBLEM 1.9-2

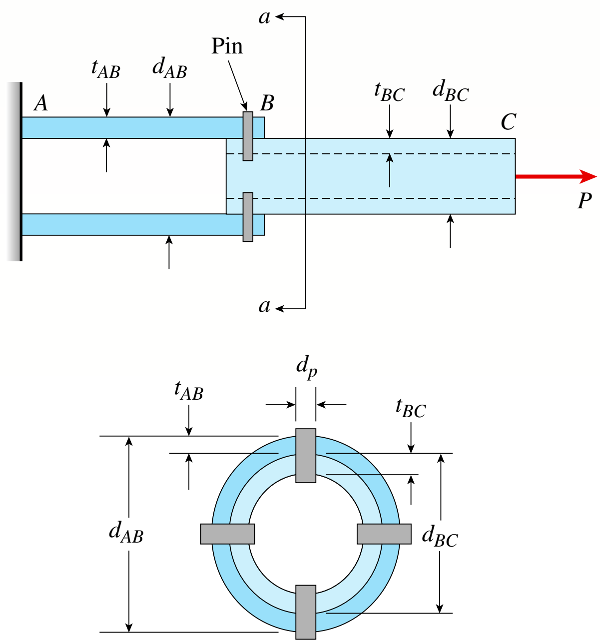
1.9-3 A tie-down on the deck of a sailboat consists of a bent bar bolted at both ends, as shown in the figure. The diameter d_B of the bar is 1/4 in., the diameter d_W of the washers is 7/8 in., and the thickness t of the fiberglass deck is 3/8 in.

If the allowable shear stress in the fiberglass is 300 psi, and the allowable bearing pressure between the washer and the fiberglass is 550 psi, what is the allowable load P_{allow} on the tie-down?



PROBLEM 1.9-3

1.9-4 Two steel tubes are joined at B by four pins ($d_p = 11$ mm), as shown in the cross section a–a in the figure. The outer diameters of the tubes are $d_{AB} = 41$ mm and $d_{BC} = 28$ mm. The wall thickness are $t_{AB} = 6.5$ mm and $t_{BC} = 7.5$ mm. The yield stress in tension for the steel is $\sigma_Y = 200$ MPa and the ultimate stress in *tension* is $\sigma_U = 340$ MPa. The corresponding yield and ultimate values in *shear* for the



Section a–a

PROBLEM 1.9-4

pin are 80 MPa and 140 MPa, respectively. Finally, the yield and ultimate values in *bearing* between the pins and the tubes are 260 MPa and 450 MPa, respectively. Assume that the factors of safety with respect to yield stress and ultimate stress are 3.5 and 4.5, respectively.

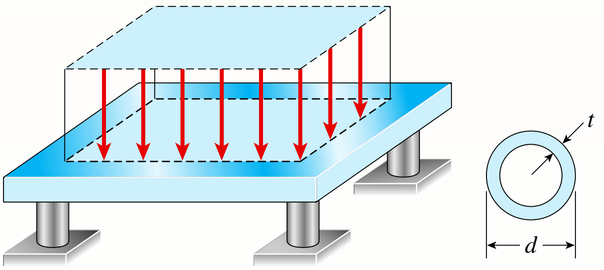
(a) Calculate the allowable tensile force P_{allow} considering tension in the tubes.

(b) Recompute P_{allow} for shear in the pins.

(c) Finally, recompute P_{allow} for bearing between the pin and the tubes. Which is the controlling value of P?

1.9-5 A steel pad supporting heavy machinery rests on four short, hollow, cast iron piers (see figure). The ultimate strength of the cast iron in compression is 50 ksi. The outer diameter of the piers is $d = 4.5$ in., and the wall thickness is $t = 0.40$ in.

Using a factor of safety of 3.5 with respect to the ultimate strength, determine the total load P that can be supported by the pad.



PROBLEMS 1.9-5 and 1.9-6

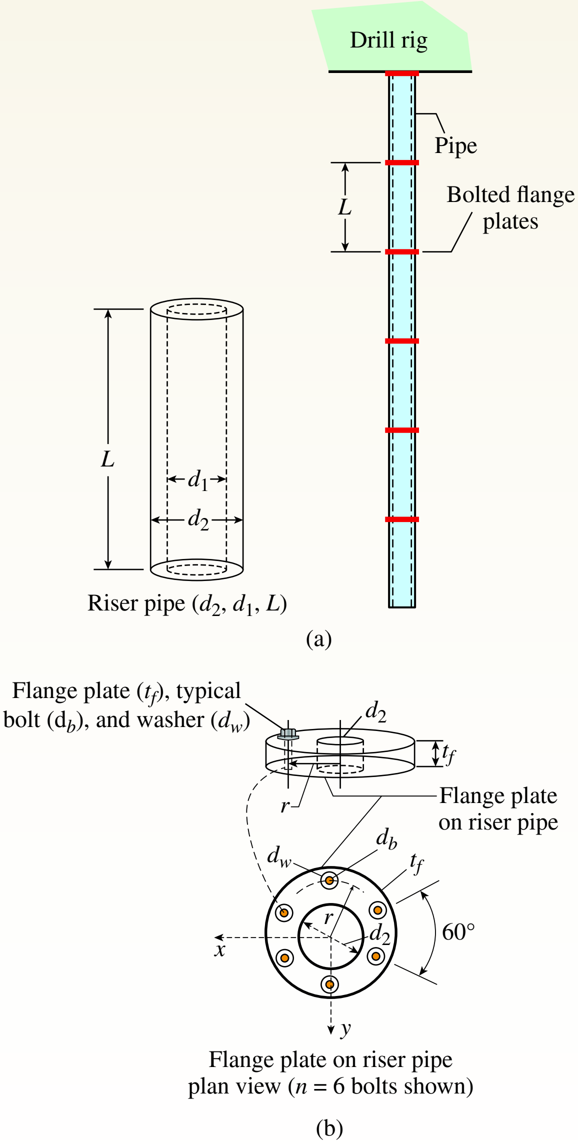
1.9-6 A steel pad supporting heavy machinery rests on four short, hollow, cast iron piers (see figure). The ultimate strength of the cast iron in compression is 400 MPa. The total load P that may be supported by the pad is 900 kN. Using a factor of safety 3.0 with respect to ultimate strength, determine the outer diameter of the pier if the thickness is of the cross section is 12 mm.

1.9-7 A steel riser pipe hangs from a drill rig. Individual segments of equal length $L = 50$ ft are joined together using bolted flange plates (see figure part b). There are six bolts at each pipe segment connection. The outer and inner pipe diameters are $d_2 = 14$ in. and $d_1 = 13$ in.; flange plate thickness $t_f = 1.5$ in.; and bolt and washer diameters are $d_b = 1.125$ in. and $d_w = 1.875$ in. Find the number n of permissible segments of pipe based on following allowable stresses.

(a) The allowable tensile stress in the pipe is 50 ksi.

(b) The allowable tensile stress in a bolt is 120 ksi.

Find number of segments n for two cases: pipe hanging in air and pipe hanging in seawater.



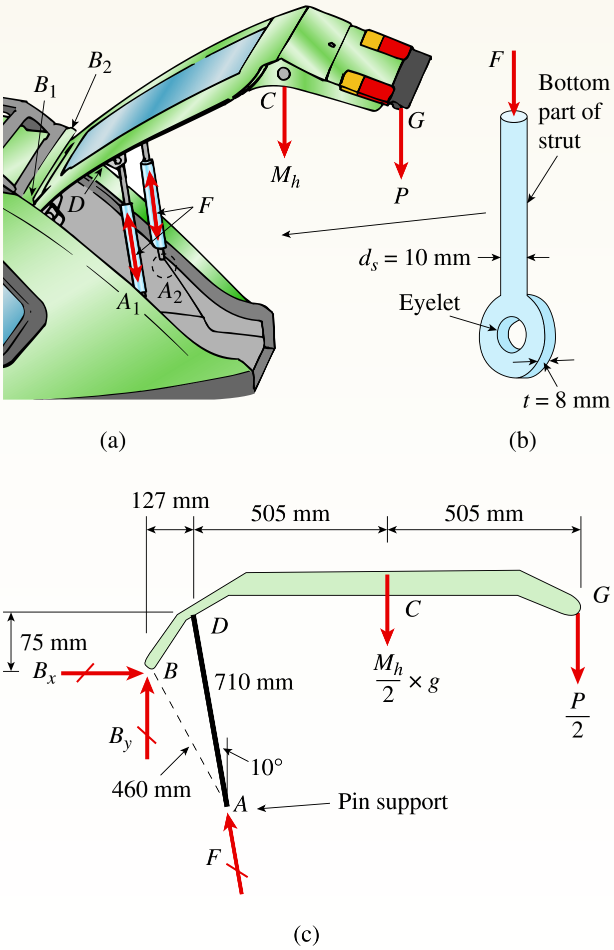
PROBLEM 1.9-7

Representative Problems

1.9-8 The rear hatch of a van ($BDCG$ in figure part a) is supported by two hinges at B_1 and B_2 and by two struts A_1B_1 and A_2B_2 (diameter $d_s = 10$ mm), as shown in figure part b. The struts are supported at A_1 and A_2 by pins, each with a diameter $d_p = 9$ mm and passing through an eyelet of thickness $t = 8$ mm at the end of the strut (figure part b). A closing force $P = 50$ N is applied at G, and the mass of the hatch $M_h = 43$ kg is concentrated at C.

(a) What is the force F in each strut? (Use the free-body diagram of one half of the hatch in the figure part c.)

(b) What is the maximum permissible force in the strut, F_{allow}, if the allowable stresses are compressive stress in the strut, 70 MPa; shear stress in the pin, 45 MPa; and bearing stress between the pin and the end of the strut, 110 MPa.

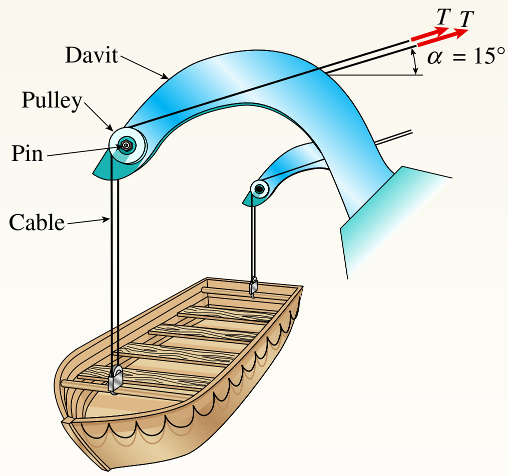


PROBLEM 1.9-8

1.9-9 A lifeboat hangs from two ship's davits, as shown in the figure. A pin of diameter $d = 0.80$ in. passes through each davit and supports two pulleys, one on each side of the davit.

Cables attached to the lifeboat pass over the pulleys and wind around winches that raise and lower the lifeboat. The lower parts of the cables are vertical and the upper parts make an angle $\alpha = 15°$ with the horizontal. The allowable tensile force in each cable is 1800 lb, and the allowable shear stress in the pins is 4000 psi.

If the lifeboat weighs 1500 lb, what is the maximum weight that can be carried in the lifeboat?

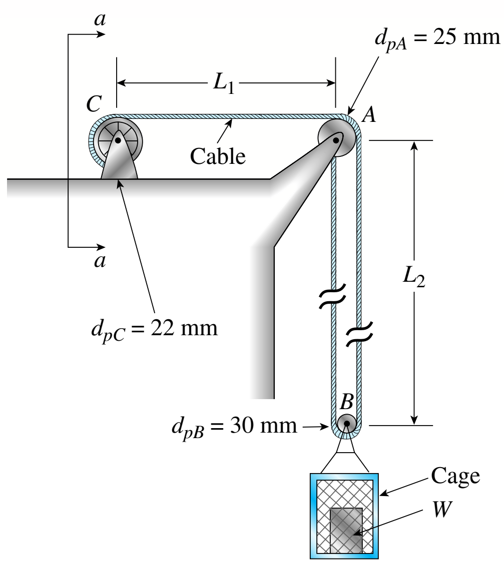


PROBLEM 1.9-9

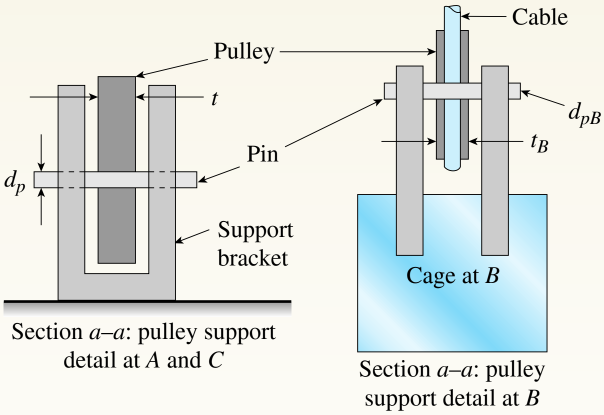
1.9-10 A cable and pulley system in the figure part a supports a cage of a mass 300 kg at B. Assume that this includes the mass of the cables as well. The thickness of each of the three steel pulleys is $t = 40$ mm. The pin diameters are $d_{pA} = 25$ mm, $d_{pB} = 30$ mm, and $d_{pC} = 22$ mm (see figure part a and part b).

(a) Find expressions for the resultant forces acting on the pulleys at A, B, and C in terms of cable tension T.

(b) What is the maximum weight W that can be added to the cage at B based on the following allowable stresses? Shear stress in the pins is 50 MPa; bearing stress between the pin and the pulley is 110 MPa.



(a)

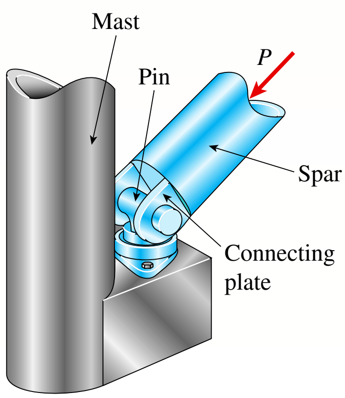


(b)

PROBLEM 1.9-10

1.9-11 A ship's spar is attached at the base of a mast by a pin connection (see figure). The spar is a steel tube of outer diameter $d_2 = 3.5$ in. and inner diameter $d_1 = 2.8$ in. The steel pin has a diameter $d = 1$ in., and the two plates connecting the spar to the pin have a thickness $t = 0.5$ in. The allowable stresses are compressive stress in the spar, 10 ksi; shear stress in the pin, 6.5 ksi; and bearing stress between the pin and the connecting plates, 16 ksi.

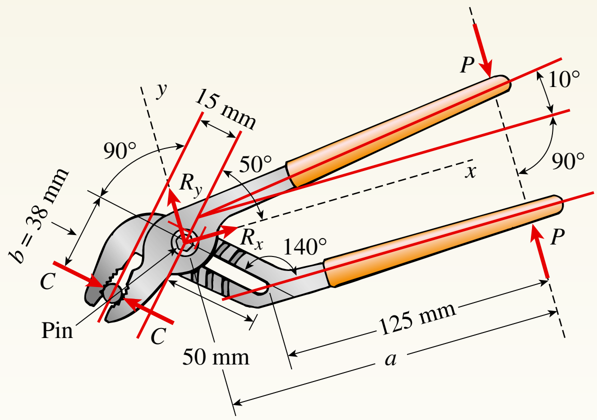
Determine the allowable compressive force P_{allow} in the spar.



PROBLEM 1.9-11

1.9-12 What is the maximum possible value of the clamping force C in the jaws of the pliers shown in the figure if the ultimate shear stress in the 5-mm diameter pin is 340 MPa?

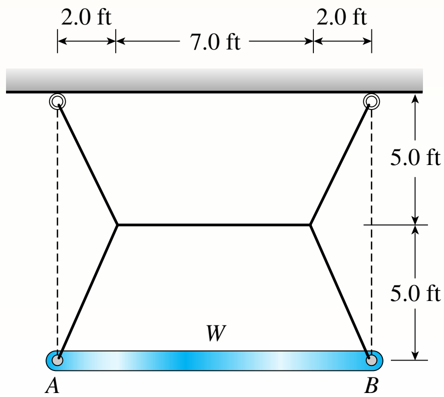
What is the maximum permissible value of the applied load P to maintain a factor of safety of 3.0 with respect to failure of the pin?



PROBLEM 1.9-12

1.9-13 A metal bar AB of a weight W is suspended by a system of steel wires arranged as shown in the figure. The diameter of the wires is 5/64 in., and the yield stress of the steel is 65 ksi.

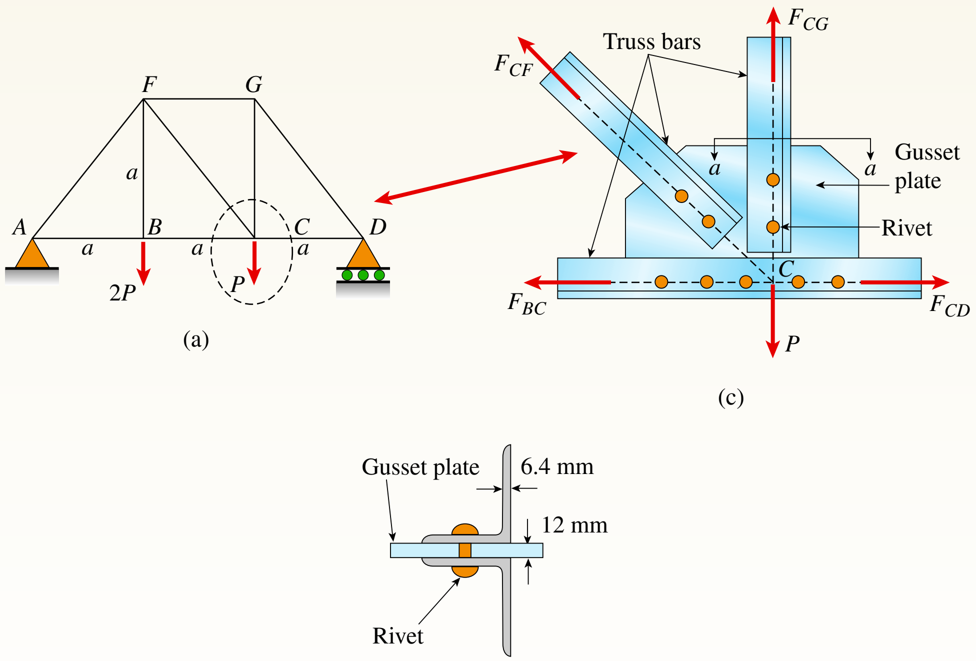
Determine the maximum permissible weight W_{max} for a factor of safety of 1.9 with respect to yielding.



PROBLEM 1.9-13

1.9-14 A plane truss is subjected to loads $2P$ and P at joints B and C, respectively, as shown in the figure part a. The truss bars are made of two L 102 × 76 × 6.4 steel angles (see Table F-5(b): cross-sectional area of the two angles, $A = 2180$ mm^2, and figure part b) having an ultimate stress in tension equal to 390 MPa. The angles are connected to a 12-mm-thick gusset plate at C (figure part c) with 16-mm diameter rivets; assume each rivet transfers an equal share of the member force to the gusset plate. The ultimate stresses in shear and bearing for the rivet steel are 190 MPa and 550 MPa, respectively.

Determine the allowable load P_{allow} if a safety factor of 2.5 is desired with respect to the ultimate load that can be carried. Consider tension in the bars, shear in the rivets, bearing between the rivets and the bars, and also bearing between the rivets and the gusset plate. Disregard friction between the plates and the weight of the truss itself.



(a)

(c)

(b) Section *a*–*a*

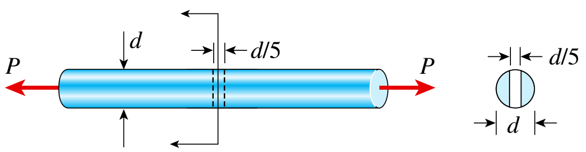
PROBLEM 1.9-14

1.9-15 A solid bar of circular cross section (diameter d) has a hole of diameter $d/5$ drilled laterally through the center of the bar (see figure). The allowable average tensile stress on the net cross section of the bar is σ_{allow}.

(a) Obtain a formula for the allowable load P_{allow} that the bar can carry in tension.

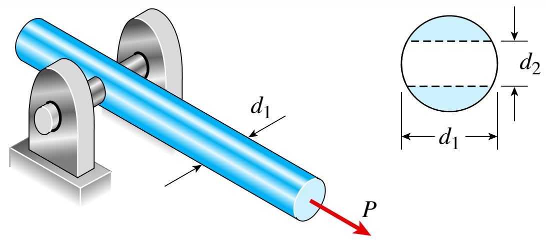
(b) Calculate the value of P_{allow} if the bar is made of brass with a diameter $d = 1.75$ in. and $\sigma_{allow} = 12$ ksi.

Hint: Use the formulas of Case 15, Appendix E.



PROBLEM 1.9-15

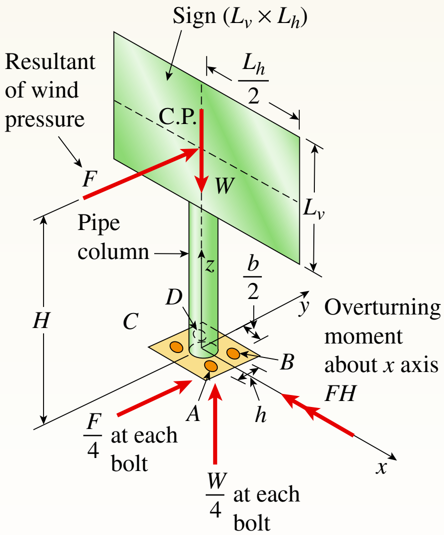
1.9-16 A solid steel bar of a diameter $d_1 = 60$ mm has a hole of a diameter $d_2 = 32$ mm drilled through it (see figure). A steel pin of a diameter d_2 passes through the hole and is attached to supports. Determine the maximum permissible tensile load P_{allow} in the bar if the yield stress for shear in the pin is $\tau_Y = 120$ MPa, the yield stress for tension in the bar is $\sigma_Y = 250$ MPa, and a factor of safety of 2.0 with respect to yielding is required. *Hint*: Use the formulas of Case 15, Appendix E.



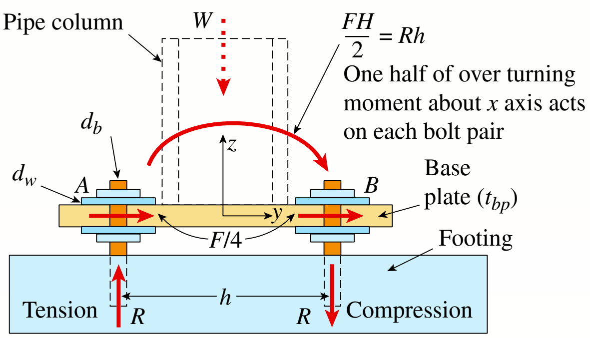
PROBLEM 1.9-16

1.9-17 A sign of weight W is supported at its base by four bolts anchored in a concrete footing. Wind pressure p acts normal to the surface of the sign; the resultant of the uniform wind pressure is force F at the center of pressure (C.P.). The wind force is assumed to create equal shear forces $F/4$ in the y direction at each bolt (see figure parts a and c). The overturning effect of the wind force also causes an uplift force R at bolts A and C

and a downward force ($-R$) at bolts B and D (see figure part b). The resulting effects of the wind and the associated ultimate stresses for each stress condition are normal stress in each bolt ($\sigma_u = 60$ ksi); shear through the base plate ($\tau_u = 17$ ksi); horizontal shear and bearing on each bolt ($\tau_{hu} = 25$ ksi and $\sigma_{bu} = 75$ ksi); and bearing on the bottom washer at B (or D) ($\sigma_{bw} = 50$ ksi).



(a)



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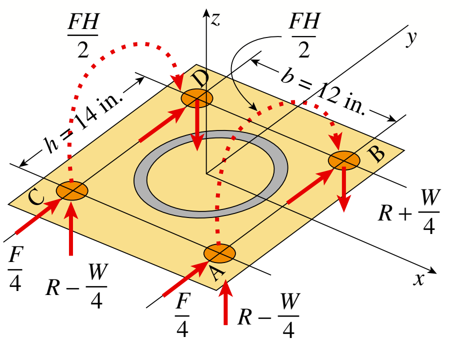


(c)

PROBLEM 1.9-17

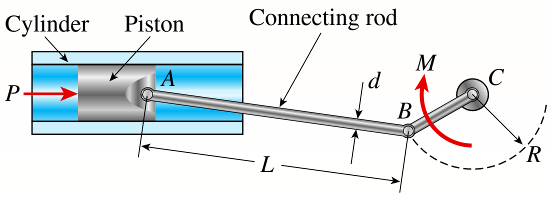
Find the maximum wind pressure p_{max} (psf) that can be carried by the bolted support system for the sign if a safety factor of 2.5 is desired with respect to the ultimate wind load that can be carried.

Use the following numerical data: bolt $d_b = \frac{3}{4}$ in.; washer $d_w = 1.5$ in.; base plate $t_{bp} = 1$ in.; base plate dimensions $h = 14$ in. and $b = 12$ in.; $W = 500$ lb; $H = 17$ ft; sign dimensions $L_v = 10$ ft. $\times$ $L_h = 12$ ft.; pipe column diameter $d = 6$ in.; and pipe column thickness $t = 3/8$ in.

1.9-18 The piston in an engine is attached to a connecting rod AB, which in turn is connected to a crank arm BC (see figure). The piston slides without friction in a cylinder and is subjected to a force P (assumed to be constant) while moving to the right in the figure. The connecting rod, with diameter d and length L, is attached at both ends by pins. The crank arm rotates about the axle at C with the pin at B moving in a circle of radius R. The axle at C, which is supported by bearings, exerts a resisting moment M against the crank arm.

(a) Obtain a formula for the maximum permissible force P_{allow} based upon an allowable compressive stress σ_c in the connecting rod.

(b) Calculate the force P_{allow} for the following data: $\sigma_c = 160$ MPa, $d = 9.00$ mm, and $R = 0.28L$.



PROBLEM 1.9-18

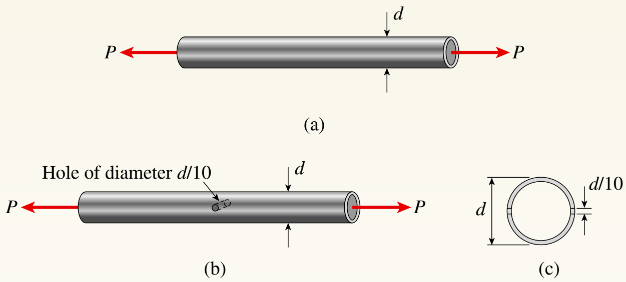
1.10 Design for Axial Loads and Direct Shear

Introductory Problems

1.10-1 An aluminum tube is required to transmit an axial tensile force $P = 33$ k (see figure part a). The thickness of the wall of the tube is 0.25 in.

(a) What is the minimum required outer diameter d_{min} if the allowable tensile stress is 12,000 psi?

(b) Repeat part (a) if the tube has a hole of a diameter $d/10$ at mid-length (see figure parts b and c).

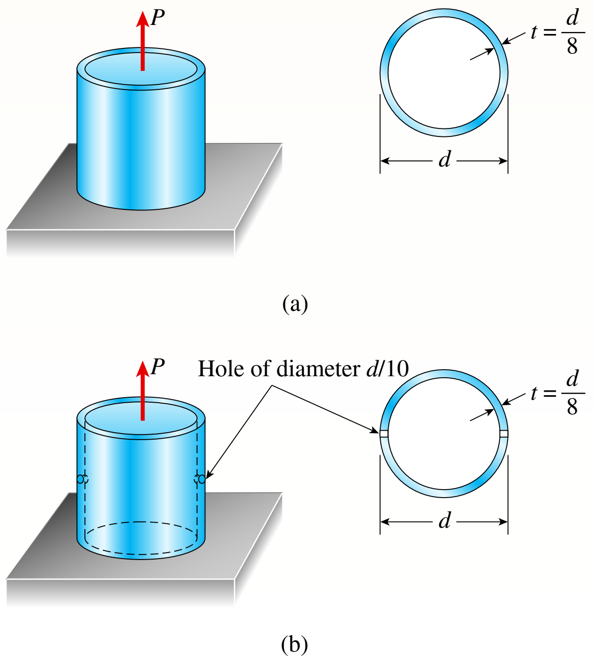


PROBLEM 1.10-1

1.10-2 A copper alloy pipe with a yield stress $\sigma_Y = 290$ MPa is to carry an axial tensile load $P = 1500$ kN (see figure part a). Use a factor of safety of 1.8 against yielding.

(a) If the thickness t of the pipe is one-eighth of its outer diameter, what is the minimum required outer diameter d_{min}?

(b) Repeat part (a) if the tube has a hole of diameter $d/10$ drilled through the entire tube, as shown in the figure part b.

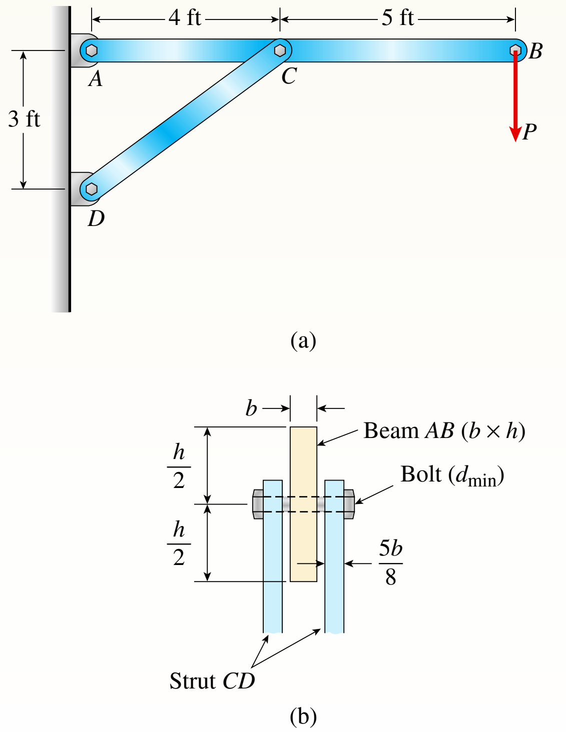


PROBLEM 1.10-2

1.10-3 A horizontal beam AB with cross-sectional dimensions $(b = 0.75 \text{ in.}) \times (h = 8.0 \text{ in.})$ is supported by an inclined strut CD and carries a load $P = 2700$ lb at joint B (see figure part a). The strut, which consists of two bars each of thickness $5b/8$, is connected to the beam by a bolt passing through the three bars meeting at joint C (see figure part b).

(a) If the allowable shear stress in the bolt is 13,000 psi, what is the minimum required diameter d_{min} of the bolt at C?

(b) If the allowable bearing stress in the bolt is 19,000 psi, what is the minimum required diameter d_{min} of the bolt at C?

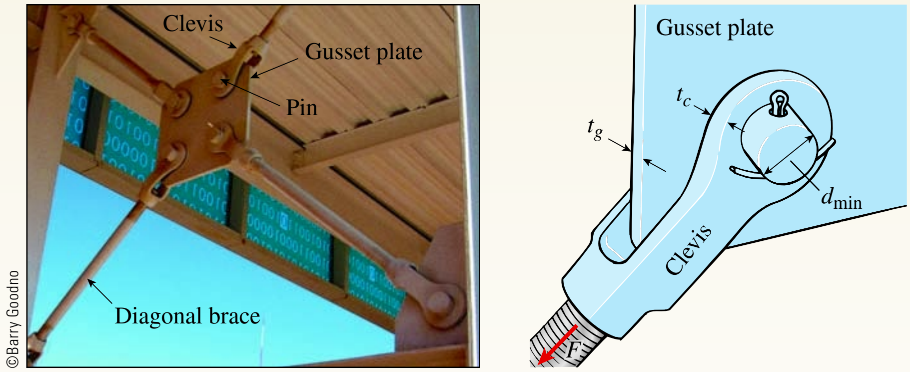


PROBLEM 1.10-3

Representative Problems

1.10-4 Lateral bracing for an elevated pedestrian walkway is shown in the figure part a. The thickness of the clevis plate $t_c = 16$ mm and the thickness of the gusset plate $t_g = 20$ mm (see figure part b). The maximum force in the diagonal bracing is expected to be $F = 190$ kN.

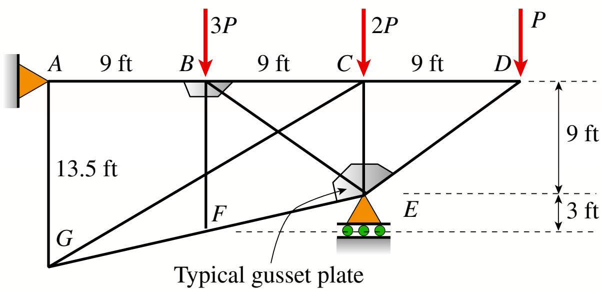
If the allowable shear stress in the pin is 90 MPa and the allowable bearing stress between the pin and both the clevis and gusset plates is 150 MPa, what is the minimum required diameter d_{min} of the pin?



(a) (b)

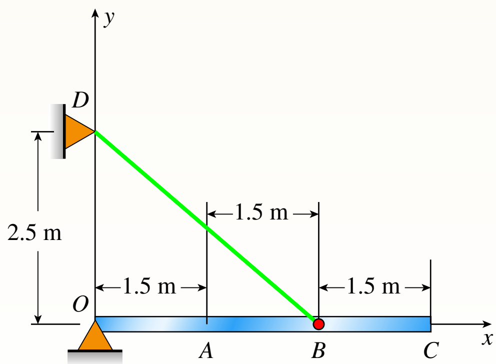
PROBLEM 1.10-4

1.10-5 A plane truss has joint loads P, $2P$, and $3P$ at joints D, C, and B, respectively (see figure) where load variable $P = 5200$ lb. All members have two end plates (see figure for Prob. 1.8-2) that are pin-connected to gusset plates. Each end plate has a thickness $t_p = 0.625$ in., and all gusset plates have a thickness $t_g = 1.125$ in. If the allowable shear stress in each pin is 12,000 psi and the allowable bearing stress in each pin is 18,000 psi, what is the minimum required diameter d_{min} of the pins used at either end of member BE?



PROBLEM 1.10-5

1.10-6 Cable DB supports canopy beam $OABC$ as shown in the figure. Find the required cross-sectional area of cable BD if the allowable normal stress is 125 MPa. Determine the required diameter of the pins at O, B, and D if the allowable stress in shear is 80 MPa. Assume that canopy beam weight is $W = 8$ kN. *Note*: The pins at O, A, B, and D are in double shear. Consider only the weight of the canopy; disregard the weight of cable DB.

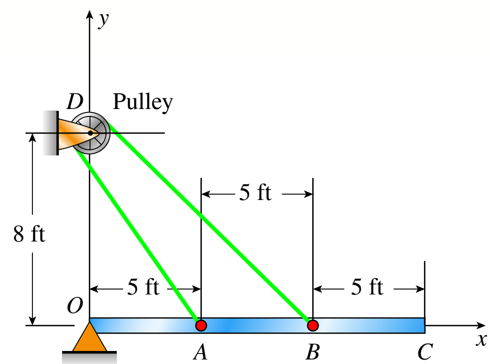


PROBLEM 1.10-6

1.10-7 Continuous cable ADB runs over a small frictionless pulley at D to support beam $OABC$ that is part of an entrance canopy for a building (see figure). Assume that the canopy segment has a weight $W = 1700$ lb.

(a) Find the required cross-sectional area of cable ADB if the allowable stress is 18 ksi.

(b) Determine the required diameter of the pins at O, A, B, and D if the allowable stress in shear is 12 ksi.



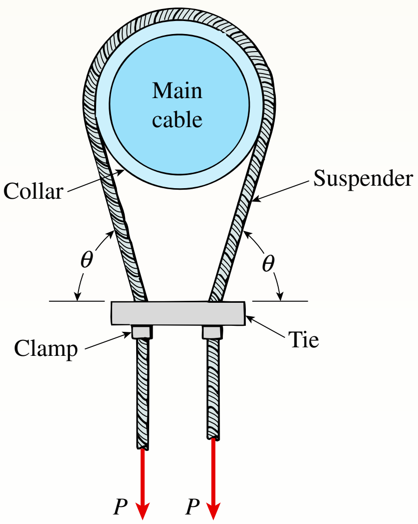
PROBLEM 1.10-7

1.10-8 A suspender on a suspension bridge consists of a cable that passes over the main cable (see figure) and supports the bridge deck, which is far below. The suspender is held in position by a metal tie that is prevented from sliding downward by clamps around the suspender cable.

Let P represent the load in each part of the suspender cable, and let θ represent the angle of the suspender cable just above the tie. Let σ_{allow} represent the allowable tensile stress in the metal tie.

(a) Obtain a formula for the minimum required cross-sectional area of the tie.

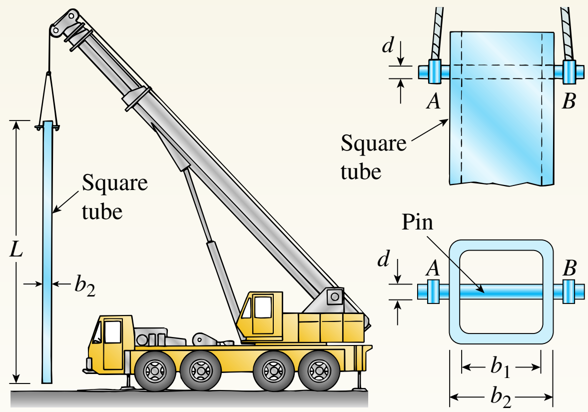
(b) Calculate the minimum area if $P = 130$ kN, $\theta = 75°$, and $\sigma_{allow} = 80$ MPa.



PROBLEM 1.10-8

1.10-9 A square steel tube of a length $L = 20$ ft and width $b_2 = 10.0$ in. is hoisted by a crane (see figure). The tube hangs from a pin of diameter d that is held by the cables at points A and B. The cross section is a hollow square with an inner dimension $b_1 = 8.5$ in. and outer dimension $b_2 = 10.0$ in. The allowable shear stress in the pin is 8,700 psi, and the allowable bearing stress between the pin and the tube is 13,000 psi.

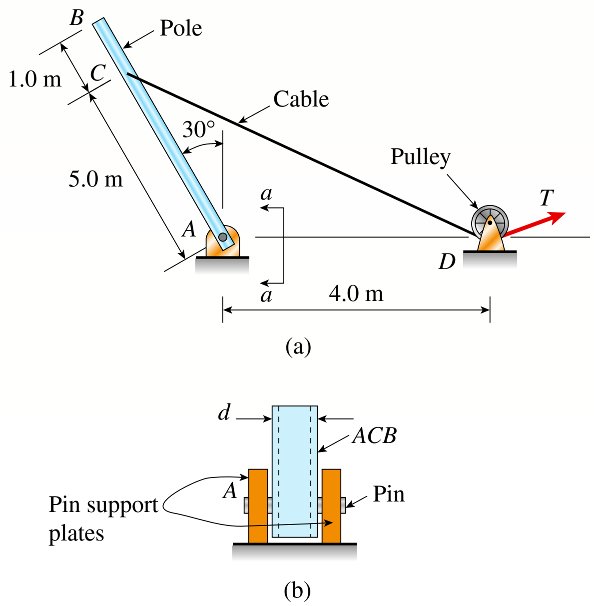
Determine the minimum diameter of the pin in order to support the weight of the tube. *Note:* Disregard the rounded corners of the tube when calculating its weight.



PROBLEM 1.10-9

1.10-10 A cable and pulley system at D is used to bring a 230-kg pole (ACB) to a vertical position, as shown in the figure part a. The cable has tensile force T and is attached at C. The length L of the pole is 6.0 m, the outer diameter is $d = 140$ mm, and the wall thickness is $t = 12$ mm. The pole pivots about a pin at A in figure part b. The allowable shear stress in the pin is 60 MPa and the allowable bearing stress is 90 MPa.

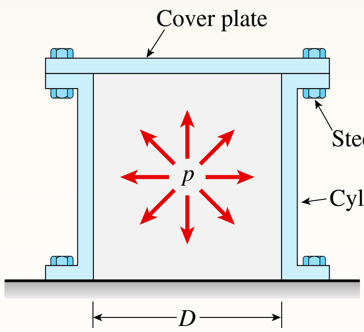
Find the minimum diameter of the pin at A in order to support the weight of the pole in the position shown in the figure part a.



PROBLEM 1.10-10

1.10-11 A pressurized circular cylinder has a sealed cover plate fastened with steel bolts (see figure). The pressure p of the gas in the cylinder is 290 psi, the inside diameter D of the cylinder is 10.0 in., and the diameter d_B of the bolts is 0.50 in.

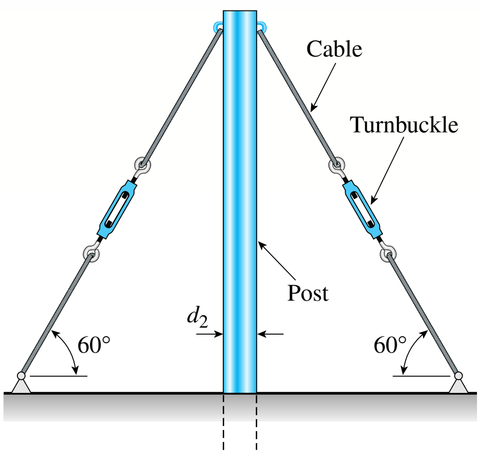
If the allowable tensile stress in the bolts is 10,000 psi, find the number n of bolts needed to fasten the cover.



PROBLEM 1.10-11

1.10-12 A tubular post of outer diameter d_2 is guyed by two cables fitted with turnbuckles (see figure). The cables are tightened by rotating the turnbuckles, producing tension in the cables and compression in the post. Both cables are tightened to a tensile force of 110 kN. The angle between the cables and the ground is 60°, and the allowable compressive stress in the post is $\sigma_c = 35$ MPa.

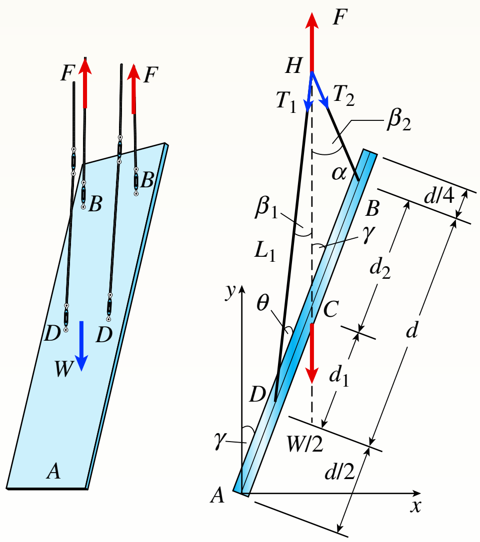
If the wall thickness of the post is 15 mm, what is the minimum permissible value of the outer diameter d_2?



PROBLEM 1.10-12

1.10-13 A large precast concrete panel for a warehouse is raised using two sets of cables at two lift lines, as shown in the figure part a. Cable 1 has a length $L_1 = 22$ ft, cable 2 has a length $L_2 = 10$ ft, and the distance along the panel between lift points B and D is $d = 14$ ft (see figure part b). The total weight of the panel is $W = 85$ kips. Assuming the cable lift forces F at each lift line are about equal, use the simplified model of one half of the panel in figure part b to perform your analysis for the lift position shown.

Find the required cross-sectional area AC of the cable if its breaking stress is 91 ksi and a factor of safety of 4 with respect to failure is desired.



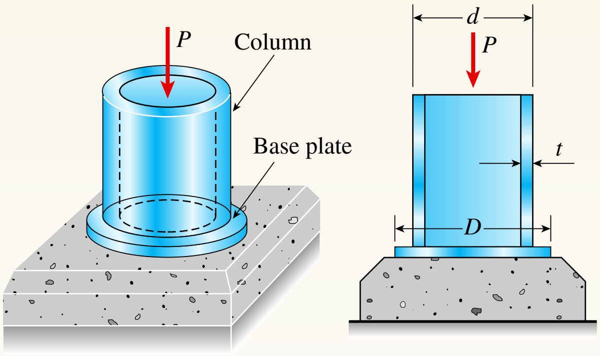
(a) (b)

PROBLEM 1.10-13

1.10-14 A steel column of hollow circular cross section is supported on a circular, steel base plate and a concrete pedestal (see figure). The column has an outside diameter $d = 250$ mm and supports a load $P = 750$ kN.

(a) If the allowable stress in the column is 55 MPa, what is the minimum required thickness t? Based upon your result, select a thickness for the column. (Select a thickness that is an even integer, such as 10, 12, 14, . . ., in units of millimeters.)

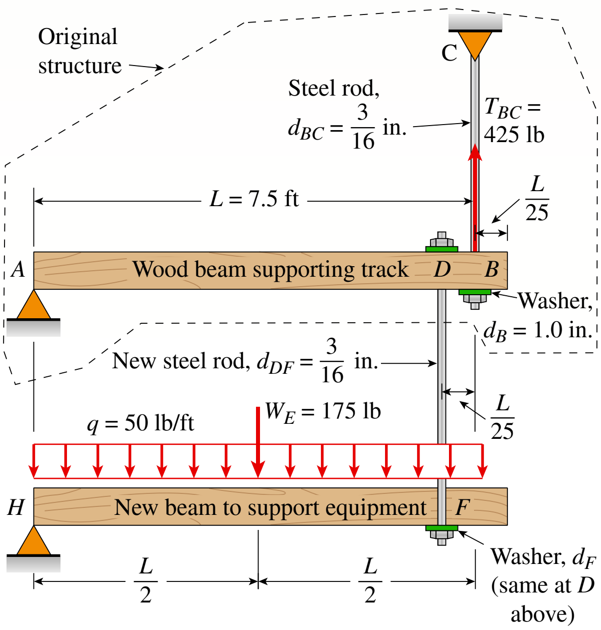
(b) If the allowable bearing stress on the concrete pedestal is 11.5 MPa, what is the minimum required diameter D of the base plate if it is designed for the allowable load P_{allow} that the column with the selected thickness can support?



PROBLEM 1.10-14

1.10-15 An elevated jogging track is supported at intervals by a wood beam AB ($L = 7.5$ ft) that is pinned at A and supported by steel rod BC and a steel washer at B. Both the rod ($d_{BC} = 3/16$ in.) and the washer ($d_B = 1.0$ in.) were designed using a rod tension force of $T_{BC} = 425$ lb. The rod was sized using a factor of safety of 3 against reaching the ultimate stress $\sigma_u = 60$ ksi. An allowable bearing stress $\sigma_{ba} = 565$ psi was used to size the washer at B.

A small platform HF is suspended below a section of the elevated track to support some mechanical and electrical equipment. The equipment load is uniform load $q = 50$ lb/ft and concentrated load $W_E = 175$ lb at mid-span of beam HF. The plan is to drill a hole through beam AB at D and install the same rod (d_{BC}) and washer (d_B) at both D and F to support beam HF.



PROBLEM 1.10-15

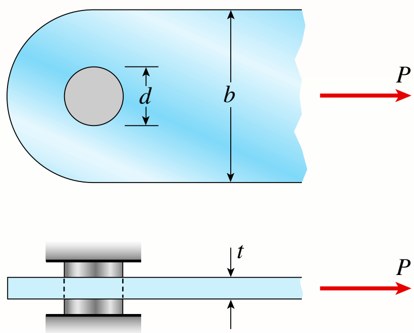
(a) Use σ_u and σ_{ba} to check the proposed design for rod DF and washer d_F; are they acceptable?

(b) Re-check the normal tensile stress in rod BC and bearing stress at B; if either is inadequate under the additional load from platform HF, redesign them to meet the original design criteria.

1.10-16 A flat bar of a width $b = 60$ mm and thickness $t = 10$ mm is loaded in tension by a force P (see figure). The bar is attached to a support by a pin of a diameter d that passes through a hole of the same size in the bar. The allowable tensile stress on the net cross section of the bar is $\sigma_T = 140$ MPa, the allowable shear stress in the pin is $\tau_S = 80$ MPa, and the allowable bearing stress between the pin and the bar is $\sigma_B = 200$ MPa.

(a) Determine the pin diameter d_m for which the load P is a maximum.

(b) Determine the corresponding value P_{max} of the load.

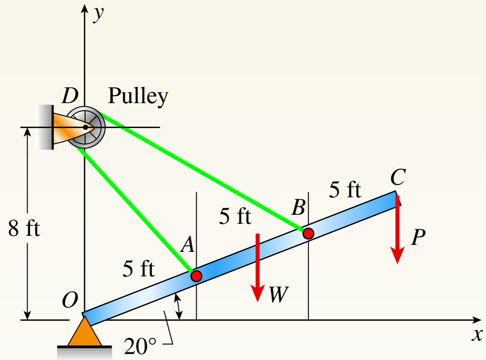


PROBLEM 1.10-16

1.10-17 Continuous cable ADB runs over a small frictionless pulley at D to support beam $OABC$, which is part of an entrance canopy for a building (see figure). The canopy segment has a weight $W = 1700$ lb that acts as a concentrated load in the middle of segment AB.

(a) What is the maximum permissible value of load P at C if the allowable force in the cable is 4200 lb?

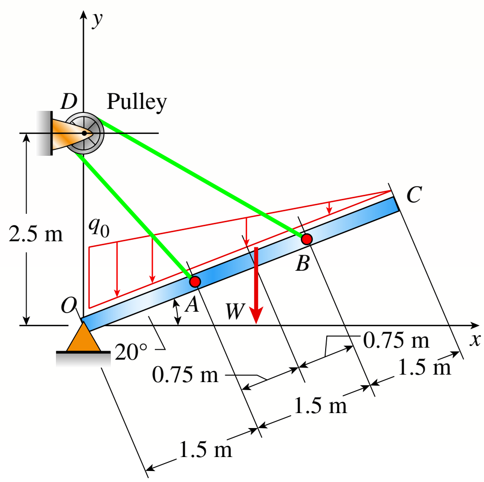
(b) If $P = 2300$ lb, what is the required diameter of pins A, B, and D? Assume that the pins are in *double shear* and the allowable shear stress in the pins is 10 ksi.



PROBLEM 1.10-17

1.10-18 Continuous cable ADB runs over a small frictionless pulley at D to support beam $OABC$, which is part of an entrance canopy for a building (see figure). A downward distributed load with peak intensity $q_0 = 5$ kN/m at O acts on the beam (see figure). Assume that canopy weight $W = 8$ kN and that the cable cross-sectional area is 100 mm^2.

What is the required diameter of pins A, B, and D if the pins are in *double shear* and the allowable shear stress is 80 MPa? Note that dimensions $OA = AB = BC = 1.5$ m.

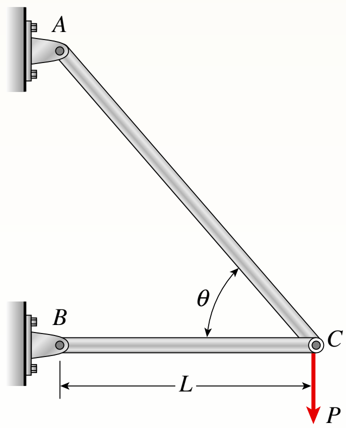


PROBLEM 1.10-18

1.10-19 Two bars AC and BC of the same material support a vertical load P (see figure). The length L of the horizontal bar is fixed, but the angle θ can be varied by moving support A vertically and changing the length of bar AC to correspond with the new position of support A. The allowable stresses in the bars are the same in tension and compression.

When the angle θ is reduced, bar AC becomes shorter, but the cross-sectional areas of both bars increase because the axial forces are larger. The opposite effects occur if the angle θ is increased. Thus, the weight of the structure (which is proportional to the volume) depends upon the angle θ.

Determine the angle θ so that the structure has minimum weight without exceeding the allowable stresses in the bars. *Note*: The weights of the bars are very small compared to the force P and may be disregarded.



PROBLEM 1.10-19