Algebra: $ax^2+bx+c=0 \qquad x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Geometry

Triangle: the law of cosines

$$c^2=a^2+b^2-2ab\cos\phi \qquad \text{Area}=\tfrac{1}{2}(\text{base})(\text{height})$$

Parallelogram: Area=(base)(height)

Circle: center (h,k), radius r

$$(x-h)^2+(y-k)^2=r^2$$

$$\text{Area}=\pi r^2 \qquad \text{Circumference } s=2\pi r$$

Ellipse: center (h,k), axes a,b

$$\left(\frac{x-h}{a}\right)^2+\left(\frac{y-k}{b}\right)^2=1$$

Hyperbola: center (h,k), axes a,b

$$\left(\frac{x-h}{a}\right)^2-\left(\frac{y-k}{b}\right)^2=\pm1$$

Parabola: tip $(h,k) \qquad y=a(x-h)^2+k$

Cylinder Volume=(base area)(height)
Cone Volume=$\frac{1}{3}$(base area)(height)
Sphere Volume=$\frac{4}{3}\pi r^3 \qquad \text{SurfaceArea}=4\pi r^2$

Vectors, Matrices, and Linear Algebra

Multiplication: row $\times$ column

$$\begin{pmatrix} \underline{a} & \underline{b} \\ c & d\end{pmatrix}\begin{pmatrix} \underline{x} & z \\ \underline{y} & w\end{pmatrix}=\begin{pmatrix} \underline{ax}+\underline{by} & \underline{a}z+\underline{b}w \\ c\underline{x}+d\underline{y} & cz+dw\end{pmatrix}$$

Determinants $\begin{vmatrix} a & b\\ c & d\end{vmatrix}=ad-bc$

Cramer's rule[1] to solve linear system $\mathbf{Ax}=\mathbf{f}$

$$\begin{pmatrix} a & b\\ c & d\end{pmatrix}\begin{pmatrix} x\\ y\end{pmatrix}=\begin{pmatrix} \underline{e}\\ \underline{f}\end{pmatrix}$$

$$x=\begin{vmatrix} \underline{e} & b\\ \underline{f} & d\end{vmatrix}\Big/\begin{vmatrix} a & b\\ c & d\end{vmatrix} \qquad y=\begin{vmatrix} a & \underline{e}\\ c & \underline{f}\end{vmatrix}\Big/\begin{vmatrix} a & b\\ c & d\end{vmatrix}$$

Inverse matrix: $\mathbf{Ax}=\mathbf{f} \quad\Rightarrow\quad \mathbf{x}=\mathbf{A}^{-1}\mathbf{f}$

$$\mathbf{A}^{-1}=\begin{pmatrix} a & b\\ c & d\end{pmatrix}^{-1}=\frac{1}{ad-bc}\begin{pmatrix} d & -b\\ -c & a\end{pmatrix}$$

Complex Algebra: $\mathrm{i}=\sqrt{-1}$

Complex number $z=x+\mathrm{i}y$ (x,y real numbers)

Usual algebra for z_1+z_2, use $\mathrm{i}^2=-1$ for z_1z_2

Conjugate: $\bar z=x-\mathrm{i}y$ with $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}, \overline{z_1}\,\overline{z_2}=\overline{z_1z_2}$

Real, imaginary parts of z:

$$\mathrm{Re}(z)=\tfrac{1}{2}(z+\bar z)=x \qquad \mathrm{Im}(z)=\tfrac{1}{2\mathrm{i}}(z-\bar z)$$

Modulus (magnitude or length) of z:

$$|z|^2\equiv z\overline{z}=(x+\mathrm{i}y)(x-\mathrm{i}y)=x^2+y^2 \qquad |z|=\sqrt{x^2+y^2}$$

$$\frac{1}{z}=\frac{1}{z}\frac{\overline{z}}{\overline{z}}=\frac{\overline{z}}{|z|^2}=\frac{x-\mathrm{i}y}{x^2+y^2}=\frac{x}{x^2+y^2}-\mathrm{i}\frac{y}{x^2+y^2}$$

$z=x+\mathrm{i}y$ is the rectangular coordinate form

Euler's formula: $e^{\mathrm{i}z}=\cos(z)+\mathrm{i}\sin(z)$

$z=re^{\mathrm{i}\theta}=r\cos\theta+\mathrm{i}r\sin\theta$ is the polar form

Rect to polar coords ("modulus" (r) and "argument" (θ))

$$\left.\begin{aligned} x&=r\cos\theta\\ y&=r\sin\theta\end{aligned}\right\} \quad\leftrightarrow\quad \left\{\begin{aligned} r&=|z|=\sqrt{x^2+y^2}\ge 0\\ \theta&=\arg(z)=\tan^{-1}(y/x)\end{aligned}\right.$$

Multiplication and division in polar coords

$$z_1z_2=r_1r_2e^{\mathrm{i}(\theta_1+\theta_2)} \qquad \frac{z_1}{z_2}=\frac{r_1}{r_2}e^{\mathrm{i}(\theta_1-\theta_2)}$$

Trigonometry

Identities

$$\sin^2x+\cos^2x=1 \qquad \sec^2x=1+\tan^2x$$

$$\tan x=\sin x/\cos x \qquad \sec x=1/\cos x$$
$$\cot x=\cos x/\sin x \qquad \csc x=1/\sin x$$

Complementary angles

$$\sin x=\cos(\pi/2-x) \qquad \cos x=\sin(\pi/2-x)$$
$$\tan x=\cot(\pi/2-x) \qquad \cot x=\tan(\pi/2-x)$$
$$\sec x=\csc(\pi/2-x) \qquad \csc x=\sec(\pi/2-x)$$

Addition formulas

$$\sin(x+y)=\sin x\cos y+\cos x\sin y$$
$$\cos(x+y)=\cos x\cos y-\sin x\sin y$$

Half-angle formulas (from $\cos(x+x)$)

$$\cos^2x=\tfrac{1}{2}(1+\cos(2x)) \qquad \sin^2x=\tfrac{1}{2}(1-\cos(2x))$$

Product formulas (from $\pm$ Addition formulas)

$$\sin x\cos y=\tfrac{1}{2}(\sin(x-y)+\sin(x+y))$$

$$\sin x\sin y=\tfrac{1}{2}(\cos(x-y)-\cos(x+y))$$

$$\cos x\cos y=\tfrac{1}{2}(\cos(x-y)+\cos(x+y))$$

$$\sin(2x)=2\sin x\cos x \qquad \cos(2x)=2\cos^2x-1$$

Derivatives

$$(d/dx)\sin x=\cos x \qquad (d/dx)\cos x=-\sin x$$

$$(d/dx)\tan x=\sec^2 x \quad (d/dx)\sec x=\sec x\tan x$$

$$(d/dx)\sin^{-1}x=\frac{1}{\sqrt{1-x^2}} \quad (d/dx)\tan^{-1}x=\frac{1}{1+x^2}$$

Integrals $\displaystyle\int \sin^m x\cos^n x\,dx$

Via u-Substitution: if $m=$ odd, then let $u=\cos x$
if $n=$ odd, then let $u=\sin x$

$\sec x,\tan x$ integrals[2]: use SET, TOS, $\sec^2x=1+\tan^2x$

Polar coordinates: $x=r\cos\theta,\ y=r\sin\theta$

$$r=\sqrt{x^2+y^2} \qquad \theta=\arctan(y/x)$$

[1]The general formula for the solution is: n^{th} variable = (det with n^{th} column replaced by RHS) divided by (det of LHS matrix).
[2]u =Sec if Even power of Tan and u =Tan if Odd power of Sec.

Logs, Exponentials

Notations: $\ln x = \log x = \log_e x$

$$\ln(xy) = \ln x + \ln y \qquad e^{x+y} = e^x e^y$$

$$\ln(x^a) = a \ln x \qquad e^{ax} = (e^x)^a \qquad b^a = e^{a \ln b}$$

$$e^{\ln x} = \ln\left(e^x\right) = x$$

$$\int \frac{du}{u} = \ln|u| + C \qquad \int e^{au}\, du = \frac{1}{a} e^{au} + C$$

Hyperbolic Trig

$$\cosh x = \tfrac{1}{2}(e^x + e^{-x}) \qquad \sinh x = \tfrac{1}{2}(e^x - e^{-x})$$

Identity: $\cosh^2 x - \sinh^2 x = 1$

$$(d/dx) \cosh x = \sinh x \qquad (d/dx) \sinh x = \cosh x$$

$$\int \cosh x\, dx = \sinh x + C \qquad \int \sinh x\, dx = \cosh x + C$$

Limits

Function $f(x)$ is continuous at $x = a$ if

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = \lim_{x \to a} f(x) = f(a)$$

If all the limits exist, then

$$\lim_{x \to a} [f(x) + g(x)] = \left[\lim_{x \to a} f(x)\right] + \left[\lim_{x \to a} g(x)\right]$$

$$\lim_{x \to a} f(x) g(x) = \left[\lim_{x \to a} f(x)\right] \left[\lim_{x \to a} g(x)\right]$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \left[\lim_{x \to a} f(x)\right] \Big/ \left[\lim_{x \to a} g(x)\right]$$

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$$

L'Hospital's rule: if $f(a)/g(a)$ "$= 0/0$ or ∞/∞", then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Differential Calculus

Limit definition of derivative for $y = f(x)$

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Power rule: $y = x^r \qquad \dfrac{dy}{dx} = r x^{r-1}$

Product rule: $y = f(x) g(x) \qquad \dfrac{dy}{dx} = f'(x) g(x) + f(x) g'(x)$

Quotient rule: $y = f(x)/g(x) \qquad \dfrac{dy}{dx} = \dfrac{f'(x) g(x) - f(x) g'(x)}{g(x)^2}$

Chain rule: $y = f(g(x)) \qquad \dfrac{dy}{dx} = f'(g(x)) g'(x)$

Taylor series for $f(t)$ at $t = t_0$

$$f(t) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\left. \frac{d^n f}{dt^n} \right|_{t=t_0} \right) (t - t_0)^n$$

Integral Calculus

Fundamental theorem of calculus: if $F'(x) = f(x)$ then

$$F(x) = \int_a^x f(t)\, dt$$

Anti-derivatives/Indefinite integrals

$$\int f(x)\, dx = F(x) + C$$

Definite integrals

$$\int_a^b f(x)\, dx = F(x) \Big|_{x=a}^{x=b} = F(b) - F(a)$$

Riemann sum definition of definite integral

$$\int_a^b f(x)\, dx = \lim_{N \to \infty} \left(\sum_{n=0}^{N} f(x_n)\, \Delta x_n \right)$$

Integration by parts (def integral of $(uv)' = u'v + uv'$)

$$\int_a^b u(x) v'(x)\, dx = u(x) v(x) \Big|_a^b - \int_a^b u'(x) v(x)\, dx$$

Leibniz's rule

$$\frac{d}{dt} \left(\int_{a(t)}^{b(t)} f(x,t)\, dx \right) = f(b,t) \frac{db}{dt} - f(a,t) \frac{da}{dt} + \int_a^b \frac{\partial f}{\partial t}\, dx$$

A table of basic integrals

1) $\displaystyle \int u^n\, du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1)$

2) $\displaystyle \int \frac{du}{u} = \ln|u| + C$

3) $\displaystyle \int e^u\, du = e^u + C$

4) $\displaystyle \int \cos u\, du = \sin u + C$

5) $\displaystyle \int \sin u\, du = -\cos u + C$

6) $\displaystyle \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$

7) $\displaystyle \int \frac{du}{1+u^2} = \tan^{-1} u + C$

8) $\displaystyle \int \sec^2 u\, du = \tan u + C$

9) $\displaystyle \int \sec u \tan u\, du = \sec u + C$

10) $\displaystyle \int \sec u\, du = \ln|\sec u + \tan u| + C$

Brief review of methods of integration: u-substitutions, integration by parts, use of trigonometric identities, trigonometric substitutions, completing the square, partial fractions...
