NLSE_Split_Step

August 17, 2021

0.1 Imports

[1]:
```
import numpy as np
import pylab as plt
from matplotlib import cm
import seaborn as sns
from tqdm import tqdm
```

0.2 System

NLSE differential equation

$$\partial_z A + \frac{\alpha}{2} A + \frac{i}{2}\beta_2 \partial_T^2 A = i\gamma \|A\|^2 A \tag{1}$$

For now, let's suppose $\alpha = 0$

$$\partial_z A = (\hat{D} + \hat{N})A \tag{2}$$

with $\hat{D} = -\frac{i}{2}\beta_2\partial_T^2$ and $\hat{N} = i\gamma\|A\|^2$

The operator $\hat{D}$ will be computed in the fourier domain such that $\hat{D}(\omega) = \frac{i}{2}\omega^2\beta_2$

Finally we have at first glance :

$$A(z+h, T) = e^{h\hat{N}} F^{-1} \cdot e^{h\hat{D}(\omega)} \cdot F \cdot A(z,t) \tag{3}$$

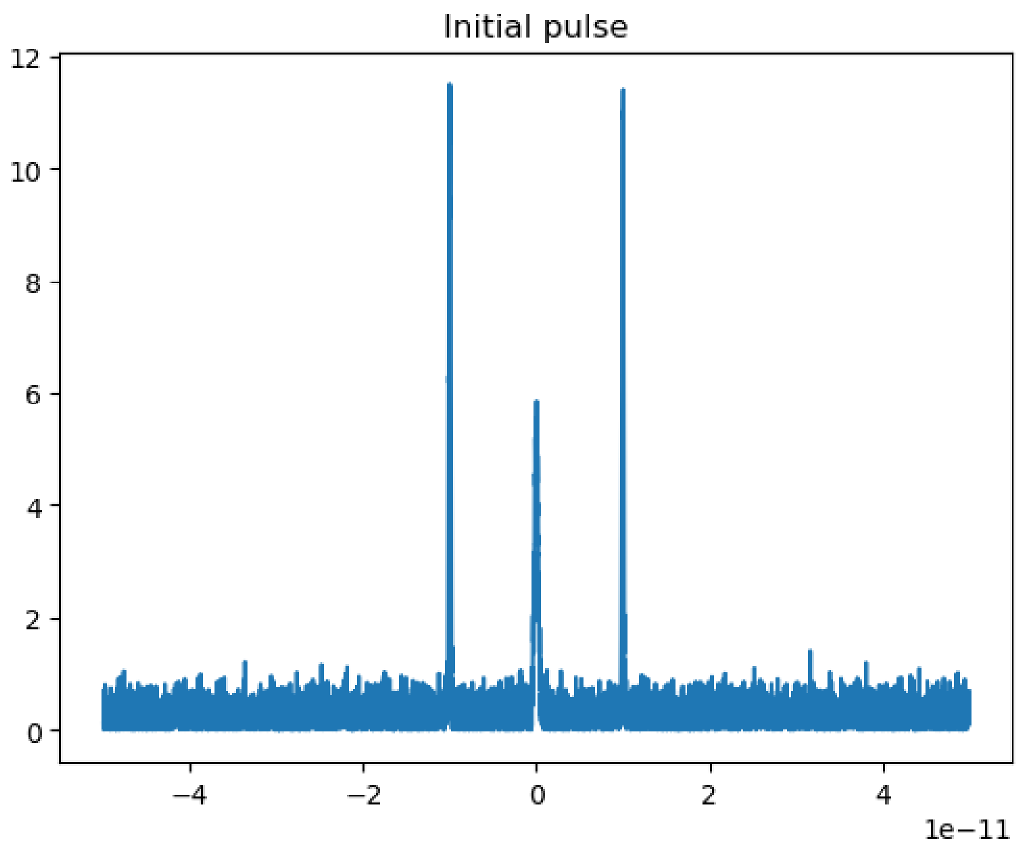
Where F is the Fourier operator, h is the propagation step (of a few meters) and $\beta_2 = -D\lambda^2/2\pi c$ wich is related to the optical fiber chromatic dispersion.

Source : doi:10.1109/ICEE.2007.4287333

[312]:
```
# Constants

c=3e8 # light celerity
l0 = 1.55e-6 # wavelength
nm = 1e-9 # nanometer
ns = 1e-9
km = 5e3 #kilometer
ps = 1e-12 #picosecond
```

```
D = 17*ps/nm/km # Dispersion
b2 = -D*l0**2/(2*np.pi*c) # group velocity dispersion (2nd order)
w0 = 2*np.pi*1e10
# Parameters

alpha=0#0.00005 # Losses
gamma=0.78e-3 # [1]/m/W Nonlinear factor https://ieeexplore.ieee.org/document/
↪7764544

L = 5e3 # Fiber length

Nt= 10000 # Time sampling
Nl = 5000 # Length sampling

# Calculated factors and vectors

h = L/Nl # Lengthstep
dT = 0.1*ps # FWHM
T = np.linspace(-50, 50, Nt)*ps # Pulse local time vector
z = np.arange(0, Nl, 1)*h # Propagation distance vector
P0=120 # W
n = 0.03

dt = T[1]-T[0] # timestep
w =  np.fft.fftshift(np.fft.fftfreq(Nt, d=dt)) # Pulsation vector # ?? 2*np.
↪pi*c/l0 +
Dw = 0.5*1j*w**2*b2 # Calculated dispersion operator

Noise = np.random.randn(1,Nt) # Amplitude noise vector

A =np.asarray(np.zeros((Nl, Nt), dtype=complex)) # System matrix
B =np.asarray(np.zeros((Nl, Nt), dtype=complex))
A[0,:]= n*np.sqrt(P0)*Noise + 0.5*np.sqrt(P0)*(np.exp(-T**2/(10*(dT)**2))) #␣
↪Initial state
A[0, :] +=  np.sqrt(P0)*(np.exp(-(T-10*ps)**2/(2*(dT)**2)))
A[0, :] +=  np.sqrt(P0)*(np.exp(-(T+10*ps)**2/(2*(dT)**2)))
#A[0,:]= n*np.sqrt(P0)*Noise + np.sqrt(P0)*np.sin(w0*T) # Initial state
B[0,:]=np.fft.fftshift(np.fft.fft(A[0,:]))
```

[313]:
```
plt.plot(T,np.abs(A[0,:]))
plt.title("Initial pulse")
plt.show()
```



[314]:

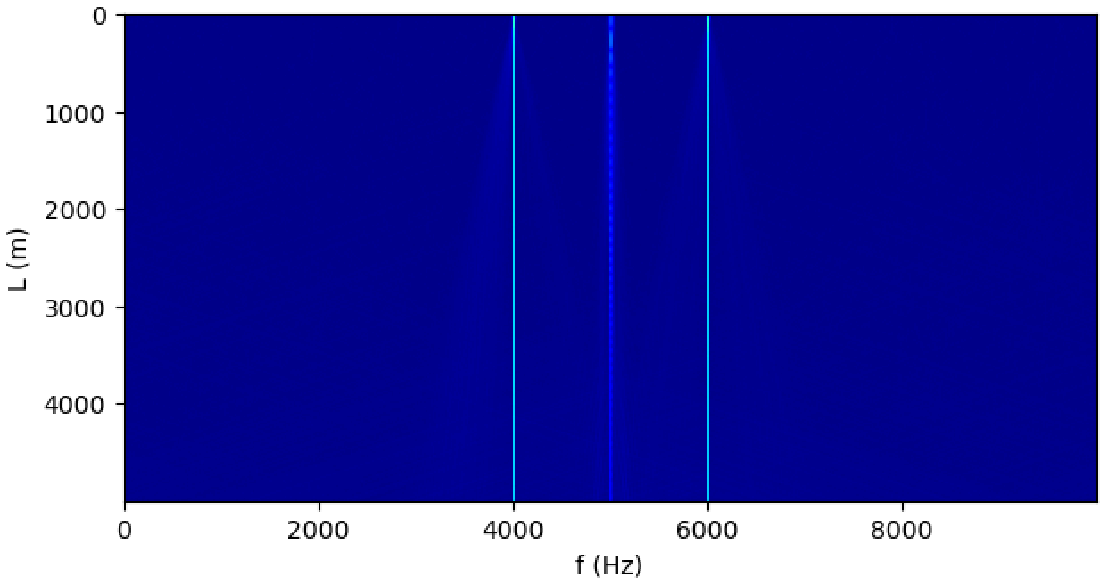
```
for i in tqdm(range(1,Nl)):

    N = 1j*gamma*np.abs(A[i-1,:])**2-0.5*alpha
    Ai = np.exp(0.5*h*N)*A[i-1,:] # half Nonlinearity 1
    Ai = np.fft.fftshift(np.fft.fft(Ai)) # Fourier domain
    Ai = np.exp(h*Dw)*Ai # Dispersion in Fourier domain
    Ai = np.fft.ifft(np.fft.ifftshift(Ai)) # Temporal domain
    Ai = np.exp(0.5*h*N)*Ai # half Nonlinearity 2
    A[i,:] = Ai
    B[i,:] = np.fft.fftshift(np.fft.fft(Ai))
```

```
100%|
 | 4999/4999 [00:30<00:00, 164.10it/s]
```

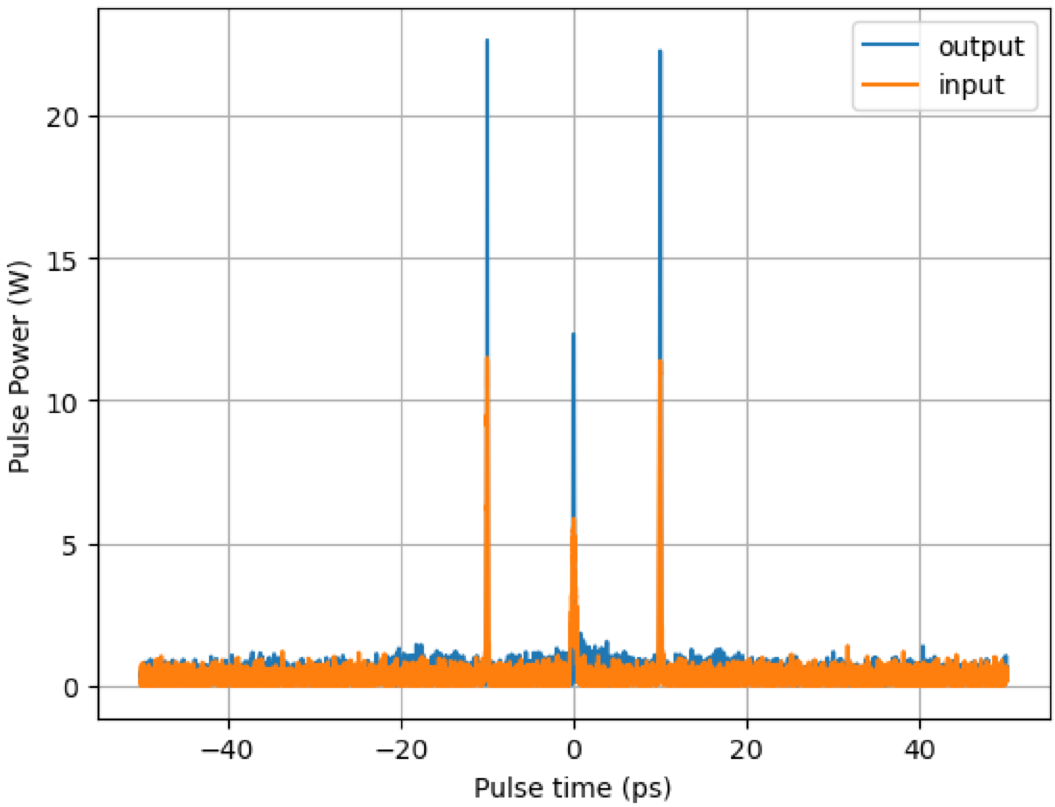
[315]:

```
f = w/(2*np.pi)
extent = [f[0], f[-1], z[-1], z[0]]
plt.imshow(np.abs(A), aspect=1, cmap='jet')
plt.xlabel(r"f (Hz)")
plt.ylabel(r"L (m)")
```

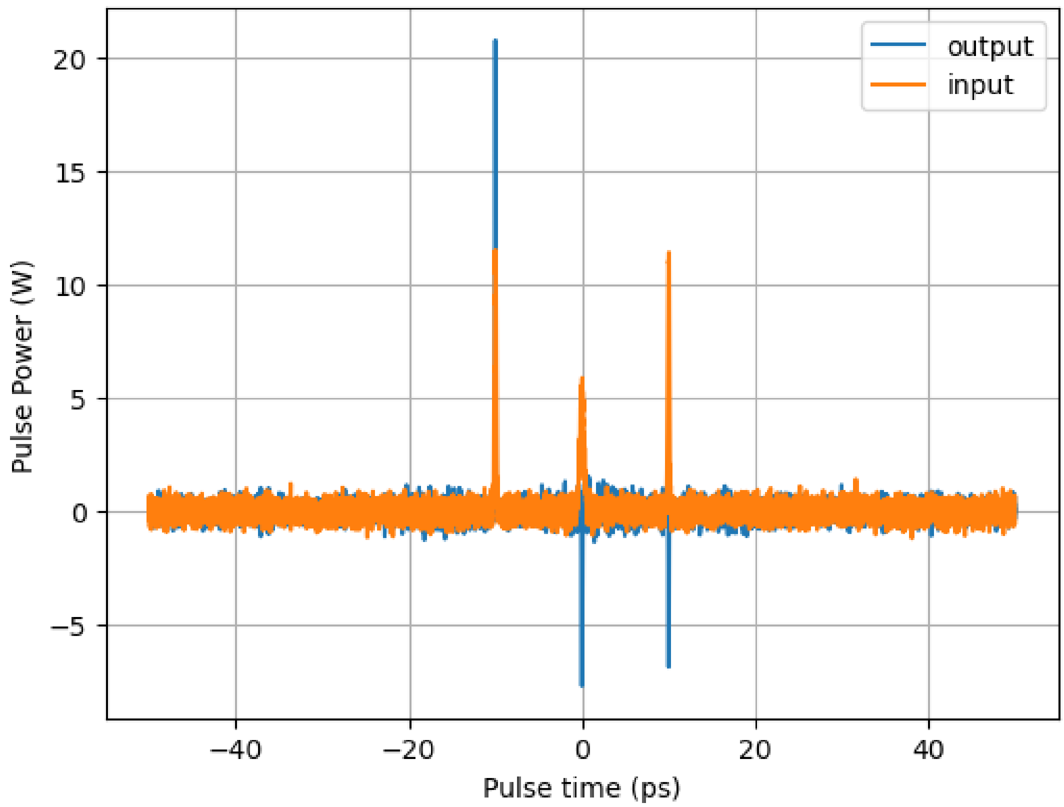
```
plt.tight_layout()
plt.show()
```



[316]:
```
plt.plot(T/ps,np.abs(A[-1,:]), label='output')
plt.plot(T/ps,np.abs(A[0,:]), label='input')


plt.grid()
plt.legend()
plt.xlabel("Pulse time (ps)")
plt.ylabel("Pulse Power (W)")
plt.show()

plt.figure()
plt.plot(T/ps,np.real(A[-1,:]), label='output')
plt.plot(T/ps,np.real(A[0,:]), label='input')


plt.grid()
plt.legend()
plt.xlabel("Pulse time (ps)")
plt.ylabel("Pulse Power (W)")

plt.show()
```



output
input
20
15
10
5
0
−40
−20
0
20
40
Pulse time (ps)
Pulse Power (W)



[279]:
```
w[0]
```

[279]: -4999499999998.172

[302]:
```
Dw
```

[302]:
```
array([-0.-0.05415821j, -0.-0.05413655j, -0.-0.0541149j , …,
       -0.-0.05409324j, -0.-0.0541149j , -0.-0.05413655j])
```

[301]:
```
N
```

[301]:
```
array([0.+4.25759166e-08j, 0.+1.08248965e-08j, 0.+7.47240172e-08j, …,
       0.+1.06654191e-07j, 0.+4.60674140e-09j, 0.+9.80428558e-08j])
```

[25]:
```
h
```

[25]: 50.0

[26]:
```
c/10
```

[26]: 193548387096774.2

[82]:
```python
z[-1]
```

[82]: 4997.5

[ ]:
```python

```