Linear algebra,Exercise 5

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Exercise. [a] *Let $T : \mathbf{R}^4 \to \mathbf{R}^4$ be the transformation*

$$T(x_1, x_2, x_3, x_4) = (0, x_1, x_2, x_3).$$

- *What is the rank and nullity of T?*
- *Let $\beta = ((1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1))$ be the standard ordered basis for T. Compute $[T]_\beta^\beta$,$[T^2]_\beta^\beta$,$[T^3]_\beta^\beta$,and $[T^4]_\beta^\beta$.*

[a]From http://www.math.ucla.edu/ tao/resource/general/115a.3.02f/midterm.pdf

Solve.

- Let $v_1 = (1,0,0,0), v_2 = (0,1,0,0), v_3 = (0,0,1,0), v_4 = (0,0,0,1)$.Then

  $$T(v_1) = (0,1,0,0) = v_2, T(v_2) = (0,0,1,0) = v_3, T(v_3) = (0,0,0,1) = v_4, T(v_4) = (0,0,0,0) = \mathbf{0}.$$

  So the rank of T is 3,the nullity of T is 1.

- $$[T]_\beta^\beta = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

  $$T^2(v_1) = T(v_2) = v_3, T^2(v_2) = T(v_3) = v_4, T^2(v_3) = T(v_4) = \mathbf{0}, T^2(v_4) = \mathbf{0}.$$

  So

  $$[T^2]_\beta^\beta = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

  $$T^3(v_1) = T(v_3) = v_4, T^3(v_2) = T(v_4) = \mathbf{0}, T^3(v_3) = T^3(v_4) = \mathbf{0}.$$

  So

  $$[T^3]_\beta^\beta = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

  And

  $$[T^4]_\beta^\beta = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

□

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