Linear algebra,Exercise 7

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Exercise (Tao). *Let* A *and* B *be similar* $n \times n$ *matrices.Show that* A *and* B *have the same set of eigenvalues.*

Proof. A and B are similar matrices,so there exists an invertible $n \times n$ matrix Q such that

$$A = Q^{-1}BQ.$$

Suppose that v_1 is an eigenvector of A with corresponding eigenvalue λ_1,which means that

$$\det(A - \lambda_1 I) = 0,$$

So

$$\det(Q^{-1}BQ - \lambda_1 Q^{-1}Q) = \det Q^{-1} \det(B - \lambda_1 I) \det Q = 0,$$

So

$$\det(B - \lambda_1 I) = 0,$$

So λ_1 is an eigenvalue of B.And vise versa. □

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