Linear algebra,Exercise 3

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Exercise. [a] *Let V_1, V_2, V_3, V_4 be vector spaces such that*

$$\dim(V_1) = 8, \dim(V_2) = 5, \dim(V_3) = 7, \dim(V_4) = 6.$$

Let $T_1 : V_1 \to V_2$,$T_2 : V_2 \to V_3$,and $T_3 : V_3 \to V_4$ be linear transformations.Let $T = T_3T_2T_1$ be their composition.Prove that T is not surjective.

[a]This exercise is from http://www.math.ucla.edu/ tao/resource/general/115a.3.02f/practice.pdf

Proof. Prove by contradiction.If T is surjective,then T_3 must be surjective.Then $N(T_3) = 7 - 6 = 1$,which means that there exists a basis $\{v_1, \cdots, v_6\}$ of V_3 such that there is a bijection T_3' from $Span(\{v_1, \cdots, v_6\})$ to V_4,where for all $x \in Span(\{v_1, \cdots, v_6\})$,$T_3'(x) = T_3(x)$.

So there is a surjection from V_2 to V_3.But $\dim(V_2) < \dim(V_3)$,which leads to contradiction. □

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