[1]. [2] [3]

1

- $n r_1, r_2, \cdots, r_n f(r_1, \cdots, r_n) \forall i, j \in \{1, 2, \cdots, n\} r_i, r_j \forall i, j \in \{1, 2, \cdots, n\} r_i r_j' r_j r_i r_j' r_j f(r_1, \cdots, r_n) f(r_1, \cdots, r_n)$

- $n f(r_1, \cdots, r_n)$

$$f(r_1, \cdots, r_n) = \sum_{i=1}^{m} a_i r_1^{k_{i,1}} r_2^{k_{i,2}} \cdots r_n^{k_{i,n}},$$

$\forall i \in \{1, 2, \cdots, m\} k_{i,1}, \cdots, k_{i,n} a_i m$

$$\sum_{j=1}^{n} k_{1,j}, \sum_{j=1}^{n} k_{2,j}, \cdots, \sum_{i=1}^{n} k_{m,j}$$

$n f(r_1, \cdots, r_n)$

-

$$\begin{cases} \sigma_1 = r_1 + r_2 + \cdots + r_n, \\ \sigma_2 = r_1 r_2 + r_1 r_3 + \cdots + r_{n-1} r_n, \\ \sigma_3 = r_1 r_2 r_3 + r_1 r_2 r_4 + \cdots + r_{n-2} r_{n-1} r_n, \\ \\ \sigma_n = r_1 r_2 \cdots r_n. \end{cases}$$

$p_1, \cdots, p_i$

$$\sigma_i = \sum_{1 \leq p_1 < p_2 < \cdots < p_i \leq n} r_{p_1} r_{p_2} \cdots r_{p_i}.$$

2

2.1 ([4]). E $\mathbf{R}^n$, $f : E \to \mathbf{R}^n$ E . $\mathbf{x_0} \in E$ $f'(\mathbf{x_0}) : \mathbf{R}^n \to \mathbf{R}^n$, $\mathbf{x_0}$ $U \subset E$ $f(\mathbf{x_0})$ $V \subset \mathbf{R}^n$, f U V , $f^{-1} : V \to U$ $f(\mathbf{x_0})$,

$$(f^{-1})'(f(\mathbf{x_0})) = (f'(\mathbf{x_0}))^{-1}.$$

. [4]. □

2.2. $r_1, \cdots, r_n$, $f : M \to \mathbf{R}^n$, $f((r_1, \cdots, r_n)) = (\sigma_1, \cdots, \sigma_n)$, $M \subset \mathbf{R}^n$ $\mathbf{R}^n$, $\forall \mathbf{x} \in M$, $\mathbf{x}$, $\mathbf{x} = (x_1, \cdots, x_n)$, $x_1, \cdots, x_n$. $\mathbf{r} \in M$,$f'(\mathbf{r})$. $r_1, \cdots, r_n$, $\sigma_1, \cdots, \sigma_n$, $r_1, \cdots, r_n$, $r_1, \cdots, r_n$ $\sigma_1, \cdots, \sigma_n$.

. $\mathbf{r} = (r_1, \cdots, r_n)$,

$$\begin{vmatrix} \frac{\partial \sigma_1}{\partial r_1} & \frac{\partial \sigma_1}{\partial r_2} & \cdots & \frac{\partial \sigma_1}{\partial r_n} \\ \frac{\partial \sigma_2}{\partial r_1} & \frac{\partial \sigma_2}{\partial r_2} & \cdots & \frac{\partial \sigma_2}{\partial r_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial \sigma_n}{\partial r_1} & \frac{\partial \sigma_n}{\partial r_2} & \cdots & \frac{\partial \sigma_n}{\partial r_n} \end{vmatrix} \neq 0.$$

$$\begin{vmatrix} \frac{\partial \sigma_1}{\partial r_1} & \frac{\partial \sigma_1}{\partial r_2} & \cdots & \frac{\partial \sigma_1}{\partial r_n} \\ \frac{\partial \sigma_2}{\partial r_1} & \frac{\partial \sigma_2}{\partial r_2} & \cdots & \frac{\partial \sigma_2}{\partial r_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial \sigma_n}{\partial r_1} & \frac{\partial \sigma_n}{\partial r_2} & \cdots & \frac{\partial \sigma_n}{\partial r_n} \end{vmatrix} = \prod_{1 \leq i < j \leq n} (r_i - r_j) \neq 0. \tag{1}$$

(1), . $n=2$

$$\begin{vmatrix} \frac{\partial\sigma_1}{\partial r_1} & \frac{\partial\sigma_1}{\partial r_2} \\ \frac{\partial\sigma_2}{\partial r_1} & \frac{\partial\sigma_2}{\partial r_2} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ r_2 & r_1 \end{vmatrix} = r_1 - r_2.$$

$n=3$,

$$\begin{vmatrix} \frac{\partial\sigma_1}{\partial r_1} & \frac{\partial\sigma_1}{\partial r_2} & \frac{\partial\sigma_1}{\partial r_3} \\ \frac{\partial\sigma_2}{\partial r_1} & \frac{\partial\sigma_2}{\partial r_2} & \frac{\partial\sigma_2}{\partial r_3} \\ \frac{\partial\sigma_3}{\partial r_1} & \frac{\partial\sigma_3}{\partial r_2} & \frac{\partial\sigma_3}{\partial r_3} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ r_2+r_3 & r_1+r_3 & r_1+r_2 \\ r_2r_3 & r_1r_3 & r_1r_2 \end{vmatrix} = (r_1-r_2)(r_2-r_3)(r_1-r_3) \quad (2)$$

(2) ? $r_1 = r_2$ $r_1 = r_3$ $r_2 = r_3$,

$$\begin{vmatrix} 1 & 1 & 1 \\ r_2+r_3 & r_1+r_3 & r_1+r_2 \\ r_2r_3 & r_1r_3 & r_1r_2 \end{vmatrix} = 0, \quad (3)$$

(2) r_1, r_2, r_3 $\nu(r_1,r_2,r_3)$, $\nu(r_1,r_2,r_3)$ $(r_1-r_2),(r_1-r_3),(r_2-r_3)$, $\nu(r_1,r_2,r_3)$ $(r_1-r_2)(r_1-r_3)(r_2-r_3)$. (2) $r_1^2r_2$, $\nu(r_1,r_2,r_3)=(r_1-r_2)(r_1-r_3)(r_2-r_3)$. (2) .

n, $n=3$, (1). □

2.1. $r_1,\cdots,r_n$, $f: M\to \mathbf{R}^n$, $f((r_1,\cdots,r_n))=(\sigma_1,\cdots,\sigma_n)$, $M\subset\mathbf{R}^n$ $\mathbf{R}^n$. $\forall \mathbf{x}\in\mathbf{R}^n$, $\mathbf{x}$, $\mathbf{x}=(x_1,\cdots,x_n)$, $x_1,\cdots,x_n$. $\mathbf{r}=(r_1,\cdots,r_n)\in M$, $\mathbf{r}$ $U\subset M$, $f(\mathbf{r})$ $V\subset\mathbf{R}^n$, f U V , $f^{-1}:V\to U$ $f(\mathbf{r})$,

$$(f^{-1})'(f(\mathbf{r})) = (f'(\mathbf{r}))^{-1},$$

, $f^{-1}:V\to U$, $f^{-1}((\sigma_1,\cdots,\sigma_n))=(r_1,\cdots,r_n)$,

$$\begin{pmatrix} \frac{\partial r_1}{\partial\sigma_1} & \frac{\partial r_1}{\partial\sigma_2} & \cdots & \frac{\partial r_1}{\partial\sigma_n} \\ \frac{\partial r_2}{\partial\sigma_1} & \frac{\partial r_2}{\partial\sigma_2} & \cdots & \frac{\partial r_2}{\partial\sigma_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial r_n}{\partial\sigma_1} & \frac{\partial r_n}{\partial\sigma_2} & \cdots & \frac{\partial r_n}{\partial\sigma_n} \end{pmatrix} = \begin{pmatrix} \frac{\partial\sigma_1}{\partial r_1} & \frac{\partial\sigma_1}{\partial r_2} & \cdots & \frac{\partial\sigma_1}{\partial r_n} \\ \frac{\partial\sigma_2}{\partial r_1} & \frac{\partial\sigma_2}{\partial r_2} & \cdots & \frac{\partial\sigma_2}{\partial r_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial\sigma_n}{\partial r_1} & \frac{\partial\sigma_n}{\partial r_2} & \cdots & \frac{\partial\sigma_n}{\partial r_n} \end{pmatrix}^{-1}.$$

. 2.1

2.2. $r_1,\cdots,r_n$, $r_1,\cdots,r_n$ $f(r_1,\cdots,r_n)$ M $\mathbf{R}$, M $\mathbf{R}^n$, $\forall\mathbf{x}\in M$, $\mathbf{x}$ $(x_1,\cdots,x_n)$, $x_1,\cdots,x_n$. n

$$\frac{\partial f}{\partial\sigma_1},\cdots,\frac{\partial f}{\partial\sigma_n}$$

.

.

$$\begin{pmatrix} \frac{\partial f}{\partial\sigma_1} & \cdots & \frac{\partial f}{\partial\sigma_n} \end{pmatrix}$$

. $g:M\to\mathbf{R}^n$ M $\mathbf{R}^n$, $g((r_1,\cdots,r_n))=(\sigma_1,\cdots,\sigma_n)$. 2.1,$\forall\mathbf{r}=(r_1,\cdots,r_n)\in M$, $\mathbf{r}$ $U\subset M$, $g:U\to V$ U V , V $\mathbf{R}^n$,

$$(g^{-1})'(g(\mathbf{r})) = (g'(\mathbf{r}))^{-1},$$

,

$$\begin{pmatrix} \frac{\partial r_1}{\partial\sigma_1} & \frac{\partial r_1}{\partial\sigma_2} & \cdots & \frac{\partial r_1}{\partial\sigma_n} \\ \frac{\partial r_2}{\partial\sigma_1} & \frac{\partial r_2}{\partial\sigma_2} & \cdots & \frac{\partial r_2}{\partial\sigma_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial r_n}{\partial\sigma_1} & \frac{\partial r_n}{\partial\sigma_2} & \cdots & \frac{\partial r_n}{\partial\sigma_n} \end{pmatrix} = \begin{pmatrix} \frac{\partial\sigma_1}{\partial r_1} & \frac{\partial\sigma_1}{\partial r_2} & \cdots & \frac{\partial\sigma_1}{\partial r_n} \\ \frac{\partial\sigma_2}{\partial r_1} & \frac{\partial\sigma_2}{\partial r_2} & \cdots & \frac{\partial\sigma_2}{\partial r_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial\sigma_n}{\partial r_1} & \frac{\partial\sigma_n}{\partial r_2} & \cdots & \frac{\partial\sigma_n}{\partial r_n} \end{pmatrix}^{-1}. \quad (4)$$

$$\begin{pmatrix} \frac{\partial f}{\partial r_1} & \frac{\partial f}{\partial r_2} & \cdots & \frac{\partial f}{\partial r_n} \end{pmatrix}$$

, (4) ,

$$\begin{pmatrix} \frac{\partial f}{\partial\sigma_1} & \frac{\partial f}{\partial\sigma_2} & \cdots & \frac{\partial f}{\partial\sigma_n} \end{pmatrix}$$

,

$$\begin{pmatrix}\frac{\partial f}{\partial \sigma_1} & \frac{\partial f}{\partial \sigma_2} & \cdots & \frac{\partial f}{\partial \sigma_n}\end{pmatrix} = \begin{pmatrix}\frac{\partial f}{\partial r_1} & \frac{\partial f}{\partial r_2} & \cdots & \frac{\partial f}{\partial r_n}\end{pmatrix} \begin{pmatrix} \frac{\partial r_1}{\partial \sigma_1} & \frac{\partial r_1}{\partial \sigma_2} & \cdots & \frac{\partial r_1}{\partial \sigma_n} \\ \frac{\partial r_2}{\partial \sigma_1} & \frac{\partial r_2}{\partial \sigma_2} & \cdots & \frac{\partial r_2}{\partial \sigma_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial r_n}{\partial \sigma_1} & \frac{\partial r_n}{\partial \sigma_2} & \cdots & \frac{\partial r_n}{\partial \sigma_n} \end{pmatrix}$$

$$= \begin{pmatrix}\frac{\partial f}{\partial r_1} & \frac{\partial f}{\partial r_2} & \cdots & \frac{\partial f}{\partial r_n}\end{pmatrix} \begin{pmatrix} \frac{\partial \sigma_1}{\partial r_1} & \frac{\partial \sigma_1}{\partial r_2} & \cdots & \frac{\partial \sigma_1}{\partial r_n} \\ \frac{\partial \sigma_2}{\partial r_1} & \frac{\partial \sigma_2}{\partial r_2} & \cdots & \frac{\partial \sigma_2}{\partial r_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial \sigma_n}{\partial r_1} & \frac{\partial \sigma_n}{\partial r_2} & \cdots & \frac{\partial \sigma_n}{\partial r_n} \end{pmatrix}^{-1}.$$

□

2.3. $r_1, \cdots, r_n$,

$$\begin{pmatrix} \frac{\partial r_1}{\partial \sigma_1} & \frac{\partial r_1}{\partial \sigma_2} & \cdots & \frac{\partial r_1}{\partial \sigma_n} \\ \frac{\partial r_2}{\partial \sigma_1} & \frac{\partial r_2}{\partial \sigma_2} & \cdots & \frac{\partial r_2}{\partial \sigma_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial r_n}{\partial \sigma_1} & \frac{\partial r_n}{\partial \sigma_2} & \cdots & \frac{\partial r_n}{\partial \sigma_n} \end{pmatrix} = \begin{pmatrix} \frac{\partial \sigma_1}{\partial r_1} & \frac{\partial \sigma_1}{\partial r_2} & \cdots & \frac{\partial \sigma_1}{\partial r_n} \\ \frac{\partial \sigma_2}{\partial r_1} & \frac{\partial \sigma_2}{\partial r_2} & \cdots & \frac{\partial \sigma_2}{\partial r_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial \sigma_n}{\partial r_1} & \frac{\partial \sigma_n}{\partial r_2} & \cdots & \frac{\partial \sigma_n}{\partial r_n} \end{pmatrix}^{-1} \tag{5}$$

$$= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}, \tag{6}$$

$$a_{ij} = \frac{(-1)^{j+1} r_i^{n-j}}{\prod_{k \neq i; 1 \leq k \leq n} (r_i - r_k)}.$$

$, n = 2$,

$$\begin{pmatrix} \frac{\partial r_1}{\partial \sigma_1} & \frac{\partial r_1}{\partial \sigma_2} \\ \frac{\partial r_2}{\partial \sigma_1} & \frac{\partial r_2}{\partial \sigma_2} \end{pmatrix} = \begin{pmatrix} \frac{r_1}{r_1 - r_2} & \frac{-1}{r_1 - r_2} \\ \frac{r_2}{r_2 - r_1} & \frac{-1}{r_2 - r_1} \end{pmatrix}.$$

$n = 3$,

$$\begin{pmatrix} \frac{\partial r_1}{\partial \sigma_1} & \frac{\partial r_1}{\partial \sigma_2} & \frac{\partial r_1}{\partial \sigma_3} \\ \frac{\partial r_2}{\partial \sigma_1} & \frac{\partial r_2}{\partial \sigma_2} & \frac{\partial r_2}{\partial \sigma_3} \\ \frac{\partial r_3}{\partial \sigma_1} & \frac{\partial r_3}{\partial \sigma_2} & \frac{\partial r_3}{\partial \sigma_3} \end{pmatrix} = \begin{pmatrix} \frac{r_1^2}{(r_1 - r_2)(r_1 - r_3)} & \frac{-r_1}{(r_1 - r_2)(r_1 - r_3)} & \frac{1}{(r_1 - r_2)(r_1 - r_3)} \\ \frac{r_2^2}{(r_2 - r_1)(r_2 - r_3)} & \frac{-r_2}{(r_2 - r_1)(r_2 - r_3)} & \frac{1}{(r_2 - r_1)(r_2 - r_3)} \\ \frac{r_3^2}{(r_3 - r_2)(r_3 - r_1)} & \frac{-r_3}{(r_3 - r_2)(r_3 - r_1)} & \frac{1}{(r_3 - r_2)(r_3 - r_1)} \end{pmatrix}.$$

.

. (5) 2.1

$$\begin{vmatrix} \frac{\partial \sigma_1}{\partial r_1} & \frac{\partial \sigma_1}{\partial r_2} & \cdots & \frac{\partial \sigma_1}{\partial r_n} \\ \frac{\partial \sigma_2}{\partial r_1} & \frac{\partial \sigma_2}{\partial r_2} & \cdots & \frac{\partial \sigma_2}{\partial r_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial \sigma_n}{\partial r_1} & \frac{\partial \sigma_n}{\partial r_2} & \cdots & \frac{\partial \sigma_n}{\partial r_n} \end{vmatrix}$$

$\frac{\partial \sigma_j}{\partial r_i}$. , $\frac{\partial \sigma_j}{\partial r_i}$

$$(-1)^{i+j} r^{n-j} \prod_{1 \leq p < q \leq n; p, q \neq i} (r_p - r_q), \tag{7}$$

, (1) (7),

2.4. $r_1, \cdots, r_n$, $f(r_1, \cdots, r_n)$ $r_1, \cdots, r_n$,

$$\sum_{i=1}^{n} \frac{\frac{\partial f}{\partial r_i}}{\prod_{1 \leq p \leq n; p \neq i} (r_i - r_p)} \tag{8}$$

$r_1, \cdots, r_n$.

. (8) $r_1, \cdots, r_n$, $r_1, \cdots, r_n$. (8)

$$\frac{1}{\prod_{1\le c<d\le n}(r_c - r_d)} \sum_{i=1}^{n} \left((-1)^{i+1} \frac{\partial f}{\partial r_i} \prod_{1\le p<q\le n; p,q\neq i} (r_p - r_q) \right). \tag{9}$$

$$\sum_{i=1}^{n} \left((-1)^{i+1} \frac{\partial f}{\partial r_i} \prod_{1\le p<q\le n; p,q\neq i} (r_p - r_q) \right) \tag{10}$$

$\prod_{1\le c<d\le n}(r_c - r_d)$. 10 $r_i = r_j$, i, j $\{1, \cdots, n\}$, (10) 0. (10) $\prod_{1\le c<d\le n}(r_c - r_d)$. □

2.5. $r_1, \cdots, r_n$. $r_1, \cdots, r_n$ m $\sigma_1, \cdots, \sigma_n$, $r_1, \cdots, r_n$ $m+1$ $\sigma_1, \cdots, \sigma_n$.

. $f(r_1, \cdots, r_n)$ $r_1, \cdots, r_n$ $m+1$. $\forall 1 \le j \le n \frac{\partial f}{\partial \sigma_j}$

$$\frac{\partial f}{\partial \sigma_j} = \sum_{i=1}^{n} \frac{\partial f}{\partial r_i} \frac{\partial r_i}{\partial \sigma_j}. \tag{11}$$

2.3, (11)

$$\begin{aligned} \frac{\partial f}{\partial \sigma_j} &= \sum_{i=1}^{n} \left((-1)^{j+1} \frac{\partial f}{\partial r_i} \frac{r_i^{n-j}}{\prod_{1\le p\le n; p\neq i}(r_i - r_p)} \right) \\ &= (-1)^{j+1} \sum_{i=1}^{n} \left(\frac{\partial f}{\partial r_i} r_i^{n-j} \frac{1}{\prod_{1\le p\le n; p\neq i}(r_i - r_p)} \right). \end{aligned} \tag{12}$$

$\frac{\partial f}{\partial r_i} r_i^{n-j}$ r_i , 2.4, (12) $r_1, \cdots, r_n$. $f(r_1, \cdots, r_n)$ 1, (12) $f(r_1, \cdots, r_n)$. (12) m . ,(12) $\sigma_1, \cdots, \sigma_n$, (12) σ_j ,

$$\frac{\partial f}{\partial \sigma_j} = A_1(\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n)\sigma_j^{l_1} + \cdots + A_s(\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n)\sigma_j^{l_s}, \tag{13}$$

$l_1 \ge \cdots \ge l_s$, $\forall 1 \le i \le n$, $A_i(\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n)$ $\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n$. $\sigma_1, \cdots, \sigma_n$ ((13)

$$f(r_1, \cdots, r_n) = \sum_{i=1}^{s} \frac{1}{1+l_i} A_1(\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n)\sigma_j^{l_i+1} + C(\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n), \tag{14}$$

$C(\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n)$ $\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n$.

$C(\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n)$ $\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n$, (14),$f(r_1, \cdots, r_n)$ $\sigma_h (1 \le h \le n)$, $C(\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n)$ $\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n$, $h \in \{1, \cdots, j-1, j+1, \cdots, n\}$, $f(r_1, \cdots, r_n)$ σ_h , $C(\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n)$ $\sigma_1, \cdots, \sigma_{j-1}, \sigma_{j+1}, \cdots, \sigma_n$, $f(r_1, \cdots, r_n)$ $\sigma_1, \cdots, \sigma_n$. □

3

3.1 (. $r_1, r_2, \cdots, r_n$ $f(r_1, \cdots, r_n)$ $\sigma_1, \sigma_2, \cdots, \sigma_n$.

. , $r_1, \cdots, r_n$. $f(r_1, \cdots, r_n)$ 1 ,

$$f(r_1, \cdots, r_n) = a(r_1 + \cdots + r_n) = a\sigma_1,$$

a . $f(r_1, \cdots, r_n)$, $\sigma_1, \cdots, \sigma_n$.

4

4.1. $n=2$, $f(r_1,r_2)=r_1^2+r_2^2$ σ_1,σ_2 . $f(r_1,r_2)=\sigma_1^2-2\sigma_2$. , r_1,r_2 ,

$$
\begin{aligned}
\begin{pmatrix}\frac{\partial f}{\partial \sigma_1} & \frac{\partial f}{\partial \sigma_2}\end{pmatrix} &= \begin{pmatrix}\frac{\partial f}{\partial r_1} & \frac{\partial f}{\partial r_2}\end{pmatrix}\begin{pmatrix}\frac{\partial r_1}{\partial \sigma_1} & \frac{\partial r_1}{\partial \sigma_2}\\ \frac{\partial r_2}{\partial \sigma_1} & \frac{\partial r_2}{\partial \sigma_2}\end{pmatrix}\\
&= \begin{pmatrix}\frac{\partial f}{\partial r_1} & \frac{\partial f}{\partial r_2}\end{pmatrix}\begin{pmatrix}\frac{\partial \sigma_1}{\partial r_1} & \frac{\partial \sigma_1}{\partial r_2}\\ \frac{\partial \sigma_2}{\partial r_1} & \frac{\partial \sigma_2}{\partial r_2}\end{pmatrix}^{-1}\\
&= \begin{pmatrix}2r_1 & 2r_2\end{pmatrix}\begin{pmatrix}\frac{r_1}{r_1-r_2} & \frac{-1}{r_1-r_2}\\ \frac{r_2}{r_2-r_1} & \frac{-1}{r_2-r_1}\end{pmatrix}\\
&= \begin{pmatrix}2(r_1+r_2) & -2\end{pmatrix}\\
&= \begin{pmatrix}2\sigma_1 & -2\end{pmatrix}.
\end{aligned}
$$

, $f(r_1,r_2)=\sigma_1^2-2\sigma_2+C$, $C=0$.

4.2. $n=3$, $f(r_1,r_2,r_3)=r_1^2+r_2^2+r_3^2$ $\sigma_1,\sigma_2,\sigma_3$. r_1,r_2,r_3 ,

$$
\begin{aligned}
\begin{pmatrix}\frac{\partial f}{\partial \sigma_1} & \frac{\partial f}{\partial \sigma_2} & \frac{\partial f}{\partial \sigma_3}\end{pmatrix} &= \begin{pmatrix}\frac{\partial f}{\partial r_1} & \frac{\partial f}{\partial r_2} & \frac{\partial f}{\partial r_3}\end{pmatrix}\begin{pmatrix}\frac{\partial r_1}{\partial \sigma_1} & \frac{\partial r_1}{\partial \sigma_2} & \frac{\partial r_1}{\partial \sigma_3}\\ \frac{\partial r_2}{\partial \sigma_1} & \frac{\partial r_2}{\partial \sigma_2} & \frac{\partial r_2}{\partial \sigma_3}\\ \frac{\partial r_3}{\partial \sigma_1} & \frac{\partial r_3}{\partial \sigma_2} & \frac{\partial r_3}{\partial \sigma_3}\end{pmatrix}\\
&= \begin{pmatrix}\frac{\partial f}{\partial r_1} & \frac{\partial f}{\partial r_2} & \frac{\partial f}{\partial r_3}\end{pmatrix}\begin{pmatrix}\frac{\partial \sigma_1}{\partial r_1} & \frac{\partial \sigma_1}{\partial r_2} & \frac{\partial \sigma_1}{\partial r_3}\\ \frac{\partial \sigma_2}{\partial r_1} & \frac{\partial \sigma_2}{\partial r_2} & \frac{\partial \sigma_2}{\partial r_3}\\ \frac{\partial \sigma_3}{\partial r_1} & \frac{\partial \sigma_3}{\partial r_2} & \frac{\partial \sigma_3}{\partial r_3}\end{pmatrix}^{-1}\\
&= \begin{pmatrix}2r_1 & 2r_2 & 2r_3\end{pmatrix}\begin{pmatrix}\frac{r_1^2}{(r_1-r_2)(r_1-r_3)} & \frac{-r_1}{(r_1-r_2)(r_1-r_3)} & \frac{1}{(r_1-r_2)(r_1-r_3)}\\ \frac{r_2^2}{(r_2-r_1)(r_2-r_3)} & \frac{-r_2}{(r_2-r_1)(r_2-r_3)} & \frac{1}{(r_2-r_1)(r_2-r_3)}\\ \frac{r_3^2}{(r_3-r_2)(r_3-r_1)} & \frac{-r_3}{(r_3-r_2)(r_3-r_1)} & \frac{1}{(r_3-r_2)(r_3-r_1)}\end{pmatrix}\\
&= \begin{pmatrix}2(x_1+x_2+x_3) & -2 & 0\end{pmatrix}\\
&= \begin{pmatrix}2\sigma_1 & -2 & 0\end{pmatrix}.
\end{aligned}
$$

$f(r_1,r_2,r_3)=\sigma_1^2-2\sigma_2+C$, $C=0$.

4.3. $n=3$, $f(r_1,r_2,r_3)=r_1^3+r_2^3+r_3^3$ $\sigma_1,\sigma_2,\sigma_3$. r_1,r_2,r_3 ,

$$
\begin{aligned}
\begin{pmatrix}\frac{\partial f}{\partial \sigma_1} & \frac{\partial f}{\partial \sigma_2} & \frac{\partial f}{\partial \sigma_3}\end{pmatrix} &= \begin{pmatrix}\frac{\partial f}{\partial r_1} & \frac{\partial f}{\partial r_2} & \frac{\partial f}{\partial r_3}\end{pmatrix}\begin{pmatrix}\frac{\partial r_1}{\partial \sigma_1} & \frac{\partial r_1}{\partial \sigma_2} & \frac{\partial r_1}{\partial \sigma_3}\\ \frac{\partial r_2}{\partial \sigma_1} & \frac{\partial r_2}{\partial \sigma_2} & \frac{\partial r_2}{\partial \sigma_3}\\ \frac{\partial r_3}{\partial \sigma_1} & \frac{\partial r_3}{\partial \sigma_2} & \frac{\partial r_3}{\partial \sigma_3}\end{pmatrix}\\
&= \begin{pmatrix}\frac{\partial f}{\partial r_1} & \frac{\partial f}{\partial r_2} & \frac{\partial f}{\partial r_3}\end{pmatrix}\begin{pmatrix}\frac{\partial \sigma_1}{\partial r_1} & \frac{\partial \sigma_1}{\partial r_2} & \frac{\partial \sigma_1}{\partial r_3}\\ \frac{\partial \sigma_2}{\partial r_1} & \frac{\partial \sigma_2}{\partial r_2} & \frac{\partial \sigma_2}{\partial r_3}\\ \frac{\partial \sigma_3}{\partial r_1} & \frac{\partial \sigma_3}{\partial r_2} & \frac{\partial \sigma_3}{\partial r_3}\end{pmatrix}^{-1}\\
&= \begin{pmatrix}3r_1^2 & 3r_2^2 & 3r_3^2\end{pmatrix}\begin{pmatrix}\frac{r_1^2}{(r_1-r_2)(r_1-r_3)} & \frac{-r_1}{(r_1-r_2)(r_1-r_3)} & \frac{1}{(r_1-r_2)(r_1-r_3)}\\ \frac{r_2^2}{(r_2-r_1)(r_2-r_3)} & \frac{-r_2}{(r_2-r_1)(r_2-r_3)} & \frac{1}{(r_2-r_1)(r_2-r_3)}\\ \frac{r_3^2}{(r_3-r_2)(r_3-r_1)} & \frac{-r_3}{(r_3-r_2)(r_3-r_1)} & \frac{1}{(r_3-r_2)(r_3-r_1)}\end{pmatrix}\\
&= \begin{pmatrix}3(x_1^2+x_2^2+x_3^2+x_1x_2+x_2x_3+x_3x_1) & -3(x_1+x_2+x_3) & 3\end{pmatrix}.
\end{aligned}
$$

4.2,

$$x_1^2+x_2^2+x_3^2=\sigma_1^2-2\sigma_2,$$

$$\begin{pmatrix}\frac{\partial f}{\partial \sigma_1} & \frac{\partial f}{\partial \sigma_2} & \frac{\partial f}{\partial \sigma_3}\end{pmatrix}=3\begin{pmatrix}\sigma_1^2-\sigma_2 & -\sigma_1 & 1\end{pmatrix}.$$

$$f(r_1,r_2,r_3)=\sigma_1^3-3\sigma_1\sigma_2+3\sigma_3+C.$$

C 0.

4.4. $n=3$, $f(r_1,r_2,r_3)=r_1^2r_2+r_2^2r_1+r_2^2r_3+r_3^2r_2+r_3^2r_1+r_1^2r_3$ $\sigma_1,\sigma_2,\sigma_3$. r_1,r_2,r_3 ,

$$\begin{aligned}
\begin{pmatrix} \frac{\partial f}{\partial \sigma_1} & \frac{\partial f}{\partial \sigma_2} & \frac{\partial f}{\partial \sigma_3} \end{pmatrix} &= \begin{pmatrix} \frac{\partial f}{\partial r_1} & \frac{\partial f}{\partial r_2} & \frac{\partial f}{\partial r_3} \end{pmatrix} \begin{pmatrix} \frac{\partial r_1}{\partial \sigma_1} & \frac{\partial r_1}{\partial \sigma_2} & \frac{\partial r_1}{\partial \sigma_3} \\ \frac{\partial r_2}{\partial \sigma_1} & \frac{\partial r_2}{\partial \sigma_2} & \frac{\partial r_2}{\partial \sigma_3} \\ \frac{\partial r_3}{\partial \sigma_1} & \frac{\partial r_3}{\partial \sigma_2} & \frac{\partial r_3}{\partial \sigma_3} \end{pmatrix} \\
&= \begin{pmatrix} \frac{\partial f}{\partial r_1} & \frac{\partial f}{\partial r_2} & \frac{\partial f}{\partial r_3} \end{pmatrix} \begin{pmatrix} \frac{\partial \sigma_1}{\partial r_1} & \frac{\partial \sigma_1}{\partial r_2} & \frac{\partial \sigma_1}{\partial r_3} \\ \frac{\partial \sigma_2}{\partial r_1} & \frac{\partial \sigma_2}{\partial r_2} & \frac{\partial \sigma_2}{\partial r_3} \\ \frac{\partial \sigma_3}{\partial r_1} & \frac{\partial \sigma_3}{\partial r_2} & \frac{\partial \sigma_3}{\partial r_3} \end{pmatrix}^{-1} \\
&= \begin{pmatrix} 2r_1r_2+2r_1r_3+r_2^2+r_3^2 & 2r_2r_3+2r_2r_1+r_1^2+r_3^2 & 2r_3r_1+2r_3r_2+r_2^2+r_1^2 \end{pmatrix} \\
&\quad \begin{pmatrix} \frac{r_1^2}{(r_1-r_2)(r_1-r_3)} & \frac{-r_1}{(r_1-r_2)(r_1-r_3)} & \frac{1}{(r_1-r_2)(r_1-r_3)} \\ \frac{r_2^2}{(r_2-r_1)(r_2-r_3)} & \frac{-r_2}{(r_2-r_1)(r_2-r_3)} & \frac{1}{(r_2-r_1)(r_2-r_3)} \\ \frac{r_3^2}{(r_3-r_2)(r_3-r_1)} & \frac{-r_3}{(r_3-r_2)(r_3-r_1)} & \frac{1}{(r_3-r_2)(r_3-r_1)} \end{pmatrix} \\
&= \begin{pmatrix} r_1r_2+r_2r_3+r_3r_1 & r_1+r_2+r_3 & -3 \end{pmatrix} \\
&= \begin{pmatrix} \sigma_2 & \sigma_1 & -3 \end{pmatrix}.
\end{aligned}$$

$f(r_1,r_2,r_3)=\sigma_1\sigma_2-3\sigma_3+C,\ \ C=0.$