Linear algebra,Exercise 4

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Exercise. [a] *Consider the linear transformation $T:\mathbf{R}^3\to\mathbf{R}^3$ defined by*

$$T\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}:=\begin{pmatrix}1&1&2\\2&1&3\\3&1&4\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}.$$

What is the rank and nullity of T?

[a]From http://www.math.ucla.edu/ tao/resource/general/115a.3.02f/practice.pdf

Solve. Let $\mathbf{v}_1=(1,0,0),\mathbf{v}_2=(0,1,0),\mathbf{v}_3=(0,0,1)$.Then $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$ is a basis of $\mathbf{R}^3$.

$$\begin{cases}T(\mathbf{v}_1)=(1,2,3)=\mathbf{v}_1+2\mathbf{v}_2+3\mathbf{v}_3,\\T(\mathbf{v}_2)=(1,1,1)=\mathbf{v}_1+\mathbf{v}_2+\mathbf{v}_3,\\T(\mathbf{v}_3)=(2,3,4)=2\mathbf{v}_1+3\mathbf{v}_2+4\mathbf{v}_3.\end{cases}$$

So

$$\begin{cases}T(\mathbf{v}_1)=\mathbf{v}_1+2\mathbf{v}_2+3\mathbf{v}_3,\\T(\mathbf{v}_1-\mathbf{v}_2)=\mathbf{v}_2+2\mathbf{v}_3,\\T(2\mathbf{v}_1-\mathbf{v}_3)=\mathbf{v}_2+2\mathbf{v}_3.\end{cases}$$

So

$$\begin{cases}T(\mathbf{v}_1)=\mathbf{v}_1+2\mathbf{v}_2+3\mathbf{v}_3,\\T(\mathbf{v}_1-\mathbf{v}_2)=\mathbf{v}_2+2\mathbf{v}_3,\\T(-\mathbf{v}_1-\mathbf{v}_2+\mathbf{v}_3)=\mathbf{0}.\end{cases}$$

So the rank of T is 2,the nullity of T is 1. □

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