Linear algebra,Exercise 1

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Exercise. [a] *Let* $\beta = ((1,0),(0,1))$ *be the standard ordered basis for* $\mathbf{R}^2$*.Let* $T:\mathbf{R}^2 \to \mathbf{R}^2$ *be a linear transformation such that*

$$T(1,2) = (3,2), T(1,1) = (1,1).$$

Compute $[T]_\beta^\beta$.

[a]This exercise is from http://www.math.ucla.edu/ tao/resource/general/115a.3.02f/practice.pdf

Solve. Let $\mathbf{v}_1 = (1,0), \mathbf{v}_2 = (0,1)$.Then

$$T(\mathbf{v}_1 + 2\mathbf{v}_2) = 3\mathbf{v}_1 + 2\mathbf{v}_2,$$

$$T(\mathbf{v}_1 + \mathbf{v}_2) = \mathbf{v}_1 + \mathbf{v}_2.$$

So

$$\begin{cases} T(\mathbf{v}_1) + 2T(\mathbf{v}_2) = 3\mathbf{v}_1 + 2\mathbf{v}_2, \\ T(\mathbf{v}_1) + T(\mathbf{v}_2) = \mathbf{v}_1 + \mathbf{v}_2. \end{cases}$$

So

$$T(\mathbf{v}_1) = -\mathbf{v}_1, T(\mathbf{v}_2) = 2\mathbf{v}_1 + \mathbf{v}_2.$$

So

$$[T]_\beta^\beta = \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}.$$

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