THE UNIFORM CONVERGENCE OF COMPLEX POWER SERIES

LUQING YE

A complex power series $P(z)$ centering at the origin is an expression of the form

$$P(z) = c_0 + c_1 z + c_2 z^2 + c_3 z^3 + \cdots ,$$

where the c_j's are complex constants,and z is a complex variable.In this note we prove the following theorem,which can be found at [1].

Theorem 1. *If $P(z)$ is a complex power series centered at the origin,and $P(z)$ has disc of convergence $D = \{z : |z| < R\}$,then $P(z)$ is uniformly convergent on the closed disc $C = \{z : |z| \leq r\}$,where $r < R$.*

Proof. For any given $\varepsilon > 0$,and any given positive integer n,let's see the set

$$A_{\varepsilon,n} = \{z : |z| \leq r, |P_n(z) - P(z)| < \varepsilon\}.$$

We first prove that A_n is an open set of C.In order to prove this,we only need to prove that $P(z)$ is a continuous function on C,i.e,

$$\lim_{\Delta z \to 0; z \in C, z+\Delta z \in C} P(z + \Delta z) = P(z), \tag{1}$$

which is equivalent to

$$\lim_{\Delta z \to 0; P(z) \neq 0; z \in C, z+\Delta z \in C} \frac{P(z + \Delta z)}{P(z)} = 1,$$

which is equivalent to

$$\lim_{\Delta z \to 0; P(z) \neq 0; z \in C, z+\Delta z \in C} \frac{c_0 + c_1(z + \Delta z) + c_2(z + \Delta z)^2 + \cdots + c_n(z + \Delta z)^n + \cdots}{c_0 + c_1 z + \cdots + c_n z^n + \cdots} = 1.$$

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References

[1] Needham T. Visual Complex Analysis[M]. New York:Oxford University Press, 1997:70

College of Science, Hangzhou Normal University,Hangzhou City,Zhejiang Province,China

E-mail address: yeluqingmathematics@gmail.com