例17.3

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例 (17.3). 求 $I = \int_0^1 \frac{x^b - x^a}{\ln x} (a > 0, b > 0).$

解.令

$$f(x,y) = \frac{x^y}{\ln x}$$

则

$$\frac{\partial f}{\partial y} = x^y.$$

可见,

$$\int x^y dy = \frac{x^y}{\ln x}.$$

因此,

$$I = \int_0^1 \int_a^b x^y dy dx.$$

积分换序,可得

$$I = \int_{a}^{b} \int_{0}^{1} x^{y} dx dy = \int_{a}^{b} \frac{1}{1+y} dy = \ln \frac{b+1}{a+1}.$$