EXERCISE 2.6.3

LUQING YE

Problem 1 (2.6.3)**.** *Prove that the Cauchy-Riemann equations in polar coordinates are*

$$r\frac{\partial u}{\partial r} = \frac{\partial v}{\partial \theta}, \frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}.$$

Proof. We know that

$$y = r\sin\theta, x = r\cos\theta,$$

So

$$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta}. \iff \frac{\partial u}{\partial \theta} = -r\sin\theta\frac{\partial u}{\partial x} + r\cos\theta\frac{\partial u}{\partial y}.$$

$$\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial \theta}. \iff \frac{\partial v}{\partial \theta} = r\sin\theta\frac{\partial u}{\partial y} + r\cos\theta\frac{\partial u}{\partial x}.$$

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r} \iff \frac{\partial u}{\partial r} = \cos\theta\frac{\partial u}{\partial x} + \sin\theta\frac{\partial u}{\partial y}.$$

$$\frac{\partial v}{\partial r} = -\frac{\partial u}{\partial y}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial x}\frac{\partial y}{\partial r} \iff \frac{\partial v}{\partial r} = -\cos\theta\frac{\partial u}{\partial y} + \sin\theta\frac{\partial u}{\partial x}.$$

Done. □

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