EXERCISE 2.6.1

LUQING YE

Exercise 1 (2.6.1)**.** *At which points are the following function f differentiable?*

- $f(z) = x$.

  Proof. For a complex variable $z = x+yi$,$f(x+yi) = x+0i$.Define $g(x,y) = (x,0)$.Then
  $$\begin{pmatrix} \frac{\partial g_1}{\partial x} & \frac{\partial g_1}{\partial y} \\ \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$
  So f is nowhere differentiable. □

- $f(z) = \overline{z}$.

  Proof. For a complex number $z = x+iy$,$f(x+iy) = x-iy$.Define $g(x,y) = (x,-y)$,then
  $$\begin{pmatrix} \frac{\partial g_1}{\partial x} & \frac{\partial g_1}{\partial y} \\ \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$
  So f is nowhere differentiable. □

- $f(z) = \overline{z}^2$.

  Proof. For a complex number $z = x+yi$,$f(z) = (x-yi)^2 = (x^2-y^2) + (-2xy)i$.Define $g(x,y) = (x^2-y^2,-2xy)$.Then
  $$\begin{pmatrix} \frac{\partial g_1}{\partial x} & \frac{\partial g_1}{\partial y} \\ \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} \end{pmatrix} = \begin{pmatrix} 2x & -2y \\ -2y & -2x \end{pmatrix}.$$
  Let f is differentiable at $(0,0)$. □

College of Science, Hangzhou Normal University,Hangzhou City,Zhejiang Province,China

E-mail address: yeluqingmathematics@gmail.com