Theorem7.8

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Theorem. *Let* $\{y_m(x)\}$ *be a sequence converging uniformly to* $y(x)$ *in* $[\alpha,\beta]$,*and let* $f(x,y)$ *be a continuous function in the domain* D *such that for all* m *and* x *in* $[\alpha,\beta]$ *the points* $(x,y_m(x))$ *are in* D.*Then*

$$\lim_{m\to\infty}\int_\alpha^\beta f(t,y_m(t))dt=\int_\alpha^\beta \lim_{m\to\infty} f(t,y_m(t))dt=\int_\alpha^\beta f(t,y(t))dt. \quad (1)$$

Proof. Note that for all m,$f(t,y_m(t)$ is a function from $\mathbf{R}$ to $\mathbf{R}$,and $f(t,y(t))$ is also a function from $\mathbf{R}$ to $\mathbf{R}$.

Because $f(x,y)$ is continuous in the domain D,and $(y_m(t))_{m=1}^\infty$ converges uniformly to $y(t)$,so we have $(f(x,y_m(x)))_{m=1}^\infty$ converges uniformly to $f(t,y(t))$.

So according to Theorem14.6.1 in Terence Tao's *Analysis*, (1) holds. □