Theorem7.9:Weierstrass's M-Test

叶卢庆
杭州师范大学理学院,学 号:1002011005
Email:h5411167@gmail.com
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Theorem (Weierstrass's M-Test). *Let $\{y_m(x)\}$ be a sequence of functions with $|y_m(x)| \leq M_m$ for all x in $[\alpha, \beta]$ with $\sum_{m=0}^{\infty} M_m < \infty$.Then $\sum_{m=0}^{\infty} y_m(x)$ converges uniformly in $[\alpha, \beta]$ to a unique function $y(x)$.*

Proof. Very simple indeed.Still,I'd like to write my proof down.In order to prove that $\sum_{m=0}^{\infty} y_m(x)$ converges uniformly to $y(x)$ in $[\alpha, \beta]$,we only need to prove that for any given positive real number ε,there exists a positive integer N such that for all $m > n > N$,and **for all** $x \in [\alpha, \beta]$,we have

$$|\sum_{i=n}^{m} y_i(x)| < \varepsilon.$$

In order to prove it,we just need to prove that

$$\sum_{i=n}^{m} |y_i(x)| < \varepsilon.$$

We just need to prove that

$$\sum_{i=n}^{m} M_i < \varepsilon.$$

From $\sum_{m=0}^{\infty} M_m$,we know this is true. □