# 定理2.6

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2013. 11. 19

**定理 (2.6).** 若 $u = u(x,y)$ 是方程

$$P(x,y)dx + Q(x,y)dy = 0 \tag{1}$$

的一个积分因子,使得

$$uP(x,y)dx + uQ(x,y)dy = d\Phi(x,y), \tag{2}$$

则 $u(x,y)g(\Phi(x,y))$ 也是 (1) 的一个积分因子,其中 $g(\cdot)$ 是任一可微的非零函数.

证明. 我们只用证明

$$u(x,y)g(\Phi(x,y))P(x,y)dx + u(x,y)g(\Phi(x,y))Q(x,y)dy = 0 \tag{3}$$

也是一个恰当微分方程.也即证明,

$$\frac{\partial[u(x,y)g(\Phi(x,y))P(x,y)]}{\partial y} = \frac{\partial[u(x,y)g(\Phi(x,y))Q(x,y)]}{\partial x}. \tag{4}$$

也即证明

$$\begin{aligned}&\frac{\partial[u(x,y)P(x,y)]}{\partial y}g(\Phi(x,y)) + \frac{\partial[g(\Phi(x,y))]}{\partial y}u(x,y)P(x,y)\\ &= \frac{\partial[u(x,y)Q(x,y)]}{\partial x}g(\Phi(x,y)) + \frac{\partial g(\Phi(x,y))}{\partial x}u(x,y)Q(x,y).\end{aligned}$$

我们知道,

$$\frac{\partial[u(x,y)P(x,y)]}{\partial y}g(\Phi(x,y)) = \frac{\partial[u(x,y)Q(x,y)]}{\partial x}g(\Phi(x,y)),$$

因此我们只用证明

$$\frac{\partial g(\Phi(x,y))}{\partial y}P(x,y) = \frac{\partial g(\Phi(x,y))}{\partial x}Q(x,y). \tag{5}$$

根据链法则,可得

$$\frac{\partial g(\Phi(x,y))}{\partial y} = \frac{\partial g}{\partial \Phi}\frac{\partial \Phi(x,y)}{\partial y} = \frac{\partial g}{\partial \Phi}u(x,y)Q(x,y).$$

且

$$\frac{\partial g(\Phi(x,y))}{\partial x} = \frac{\partial g}{\partial \Phi}\frac{\partial \Phi(x,y)}{\partial x} = \frac{\partial g}{\partial \Phi}u(x,y)P(x,y).$$

因此 (5) 是成立的. $\square$