Corollary7.5

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2013. 11. 23

Corollary. *Let $u(x), p(x), q(x)$ be nonnegative continuous functions in the interval $|x-x_0| \leq a$ and the function $p(x)$ is non-decreasing in $[x_0, x_0+a]$ and non-increasing in $[x_0-a, x_0]$, if*

$$u(x) \leq p(x) + \left| \int_{x_0}^{x} q(t)u(t)dt \right| \ for\ |x-x_0| \leq a. \tag{1}$$

then

$$u(x) \leq p(x) \exp\left(\left| \int_{x_0}^{x} q(t)dt \right| \right) \ for\ |x-x_0| \leq a. \tag{2}$$

Proof. According to symmetry,we assume that $x \geq x_0$.According to Theorem7.3,we know that

$$u(x) \leq p(x) + \int_{x_0}^{x} q(t)p(t)e^{\int_t^x q(s)ds}dt. \tag{3}$$

Because $p(x)$ is non-decreasing in $[x_0, x_0+a]$,from (3),we can deduce that

$$\begin{aligned} u(x) &\leq p(x) + p(x)\int_{x_0}^{x} q(t)e^{\int_t^x q(s)ds}dt \\ &= p(x)\left(1 + \int_{x_0}^{x} q(t)e^{\int_t^x q(s)ds}dt\right). \end{aligned}$$

We only need to prove that

$$1 + \int_{x_0}^{x} q(t)e^{\int_t^x q(s)ds}dt \leq e^{\int_{x_0}^{x} q(t)dt}. \tag{4}$$

When $x = x_0$,(4) obviously holds.When $x > x_0$,differentiate both sides of (4),we get

$$q(x)e^{\int_t^x q(s)ds}$$

and

$$q(x)e^{\int_{x_0}^x q(t)dt}.$$

We know that

$$q(x)e^{\int_t^x q(s)ds} \leq q(x)e^{\int_{x_0}^x q(t)dt},$$

so we have (4) holds. $\square$