A note to page45,lecture 7.

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2013. 11. 22

In page 45,lecture 7,the book says:

For example,the initial value problem

$$y' = \frac{2}{x}(y-1), y(0) = 0$$

has no solution,while the problem

$$y' = \frac{2}{x}(y-1), y(0) = 1$$

has an infinite number of solutions $y(x) = 1 + cx^2$,where c is an arbitrary constant.

Now I verify these claims.

Verify. We try solving the ODE

$$\frac{dy}{dx} = \frac{2}{x}(y-1), i.e,$$

$$xdy - 2(y-1)dx = 0,$$

where $x \neq 0$.We just need to solve

$$\frac{1}{y-1}dy - \frac{2}{x}dx = 0,$$

where $y \neq 1, x \neq 0$.It is easy to see that the solution is

$$\ln|y-1| = \ln x^2 - \ln(|C|) = \ln\frac{x^2}{|C|}.$$

So,when $y > 1$,we have

$$y - 1 = \frac{x^2}{C},$$

when $y < 1$,we have

$$1 - y = \frac{x^2}{C}.$$

$$\ln y - 1 = \ln x^2 - \ln |C|$$

When $y = 1$,we see that the solution is $y = 1, x \neq 0$. $\square$

Remark. *Note that at point $x = 0$ the derivatives of these functions **are not defined**, but that **does not necessarily** imply that these functions are not defined at $x = 0$,neither does it imply that the derivatives of these functions do not exist at $x = 0$.*