Theorem7.2

叶卢庆

杭州师范大学理学院,学号:1002011005

Email:h5411167@gmail.com

2013. 11. 23

Theorem (7.2). *Let the domain D be convex and the function $f(x, y)$ be differentiable with respect to y in D.Then for the Lipschitz condition*

$$|f(x, y_1) - f(x, y_2)| \leq L|y_1 - y_2|$$

to be satisfied,it is necessary and sufficient that

$$\sup_D \left| \frac{\partial f(x, y)}{\partial y} \right| \leq L.$$

Proof. $\Rightarrow$: $\forall (x, y) \in D$,

$$\left| \frac{\partial f(x, y)}{\partial y} \right| = \lim_{\Delta y \to 0; \Delta y \neq 0} \left| \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y} \right| \leq L.$$

So we have

$$\sup_D \left| \frac{\partial f(x, y)}{\partial y} \right| \leq L.$$

$\Leftarrow$:This can be easily proved by the differential mean value theorem. $\square$