Exercise. *Let D be some region in the xy-plane. Let M denote the portion of the graph of $z = g(x, y)$ that lies above D.*

- *Let $\omega = f(x, y)dx \wedge dy$ be a differential 2-form on $\mathbf{R}^3$.Show that*

$$\int_M \omega = \int_D f(x, y)dxdy.$$

- *Now suppose $\omega = f(x, y, z)dx \wedge dy$.Show that*

$$\int_M \omega = \int_D f(x, y, g(x, y))dxdy.$$

- The surface can be parameterized as

$$p : (x, y) \to (x, y, g(x, y)).$$

$$\frac{\partial p}{\partial x} = \left(1, 0, \frac{\partial g}{\partial x}\right), \frac{\partial p}{\partial y} = \left(0, 1, \frac{\partial g}{\partial y}\right).$$

$$\omega\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}\right) = f(x, y).$$

  So

$$\int_M \omega = \int_D f(x, y)dxdy.$$

- $$\omega\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}\right) = f(x, y, z) = f(x, y, g(x, y)).$$

  So

$$\int_M \omega = \int_D f(x, y, g(x, y))dxdy.$$