Exercise3.12

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Exercise. *Let* ω *and* ν *be* 1*-forms on* $T_p\mathbf{R}^2$*.Show that* $\omega \wedge \nu(V_1, V_2)$ *is the area of the parallelogram spanned by* V_1 *and* V_2*,times the area of the parallelogram spanned by* $\langle\omega\rangle$ *and* $\langle\nu\rangle$*.*

Proof. Let $V_1 = \langle p_1, p_2\rangle, V_2 = \langle q_1, q_2\rangle$.Let $\omega\langle dx, dy\rangle = a dx + b dy = \langle\omega\rangle \cdot \langle dx, dy\rangle$,$\nu\langle dx, dy\rangle = c dx + d dy = \langle\nu\rangle \cdot \langle dx, dy\rangle$.

$$\begin{aligned}
\omega \wedge \nu(V_1, V_2) &= \begin{vmatrix} \omega(V_1) & \nu(V_1) \\ \omega(V_2) & \nu(V_2) \end{vmatrix} \\
&= \begin{vmatrix} ap_1 + bp_2 & cp_1 + dp_2 \\ aq_1 + bq_2 & cq_1 + dq_2 \end{vmatrix} \\
&= \begin{vmatrix} a & b \\ c & d \end{vmatrix} \cdot \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix}
\end{aligned}$$

Done. □

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