Exercise 3.14

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Exercise. *Show that for any 1-forms* ω *and* ν *on* $\mathsf{T}\mathbf{R}^3$*,there are constants* c_1, c_2 *and* c_3 *such that*

$$\omega \wedge \nu = c_1\,dx \wedge dy + c_2\,dx \wedge dz + c_3\,dy \wedge dz.$$

Proof. Let $\omega = a_1\,dx + b_1\,dy + c_1\,dz$,$\nu = a_2\,dx + b_2\,dy + c_3\,dz$.Then

$$\begin{aligned}\omega \wedge \nu &= (a_1\,dx + b_1\,dy + c_1\,dz) \wedge (a_2\,dx + b_2\,dy + c_2\,dz)\\ &= [(a_1\,dx + b_1\,dy) + c_1\,dz] \wedge [(a_2\,dx + b_2\,dy) + c_2\,dz]\\ &= (a_1b_2 - b_1a_2)dx \wedge dy + (a_1c_2 - c_1a_2)dx \wedge dz + (b_1c_2 - c_1b_2)dy \wedge dz.\end{aligned}$$

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