

Exercise 3.12

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Exercise. Let ω and ν be 1-forms on $T_p \mathbf{R}^2$. Show that $\omega \wedge \nu(V_1, V_2)$ is the area of the parallelogram spanned by V_1 and V_2 , times the area of the parallelogram spanned by $\langle \omega \rangle$ and $\langle \nu \rangle$.

Proof. Let $V_1 = \langle p_1, p_2 \rangle$, $V_2 = \langle q_1, q_2 \rangle$. Let $\omega \langle dx, dy \rangle = a dx + b dy$, $\nu \langle dx, dy \rangle = c dx + d dy$.

$$\begin{aligned}\omega \wedge \nu(V_1, V_2) &= \begin{vmatrix} \omega(V_1) & \nu(V_1) \\ \omega(V_2) & \nu(V_2) \end{vmatrix} \\ &= \begin{vmatrix} ap_1 + bp_2 & cp_1 + dp_2 \\ aq_1 + bq_2 & cq_1 + dq_2 \end{vmatrix} \\ &= \begin{vmatrix} a & b \\ c & d \end{vmatrix} \cdot \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix}\end{aligned}$$

Done.

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