Exercise. *Integrate the 2-form*

$$\omega = \frac{1}{x} dy \wedge dz - \frac{1}{y} dx \wedge dz$$

over the following surfaces:

- *The top half of the unit sphere using the following parameterizations from cylindrical coordinates:*

$$(r, \theta) \to (r\cos\theta, r\sin\theta, \sqrt{1-r^2}),$$

where $0 \le \theta \le 2\pi$ *and* $0 \le r \le 1$.

- Let the region $R = [0,1] \times [0, 2\pi]$.$f(r,\theta) = (r\cos\theta, r\sin\theta, \sqrt{1-r^2})$.Then

$$\frac{\partial f}{\partial r} = \left(\cos\theta, \sin\theta, \frac{-r}{\sqrt{1-r^2}}\right), \frac{\partial f}{\partial \theta} = (-r\sin\theta, r\cos\theta, 0).$$

$\omega = (\frac{1}{x}dy - \frac{1}{y}dx) \wedge dz$.So

$$\omega\left(\frac{\partial f}{\partial r}, \frac{\partial f}{\partial \theta}\right) = \frac{2r}{\sqrt{1-r^2}}.$$

$$\int_R \omega\left(\frac{\partial f}{\partial r}, \frac{\partial f}{\partial \theta}\right) dr \wedge d\theta = \int_0^{2\pi}\int_0^1 \frac{2r}{\sqrt{1-r^2}} dr d\theta = 4\pi.$$