Grid Fundamentals

Ian A. Hiskens

Vennema Professor of Engineering

Professor, Electrical Engineering and Computer Science

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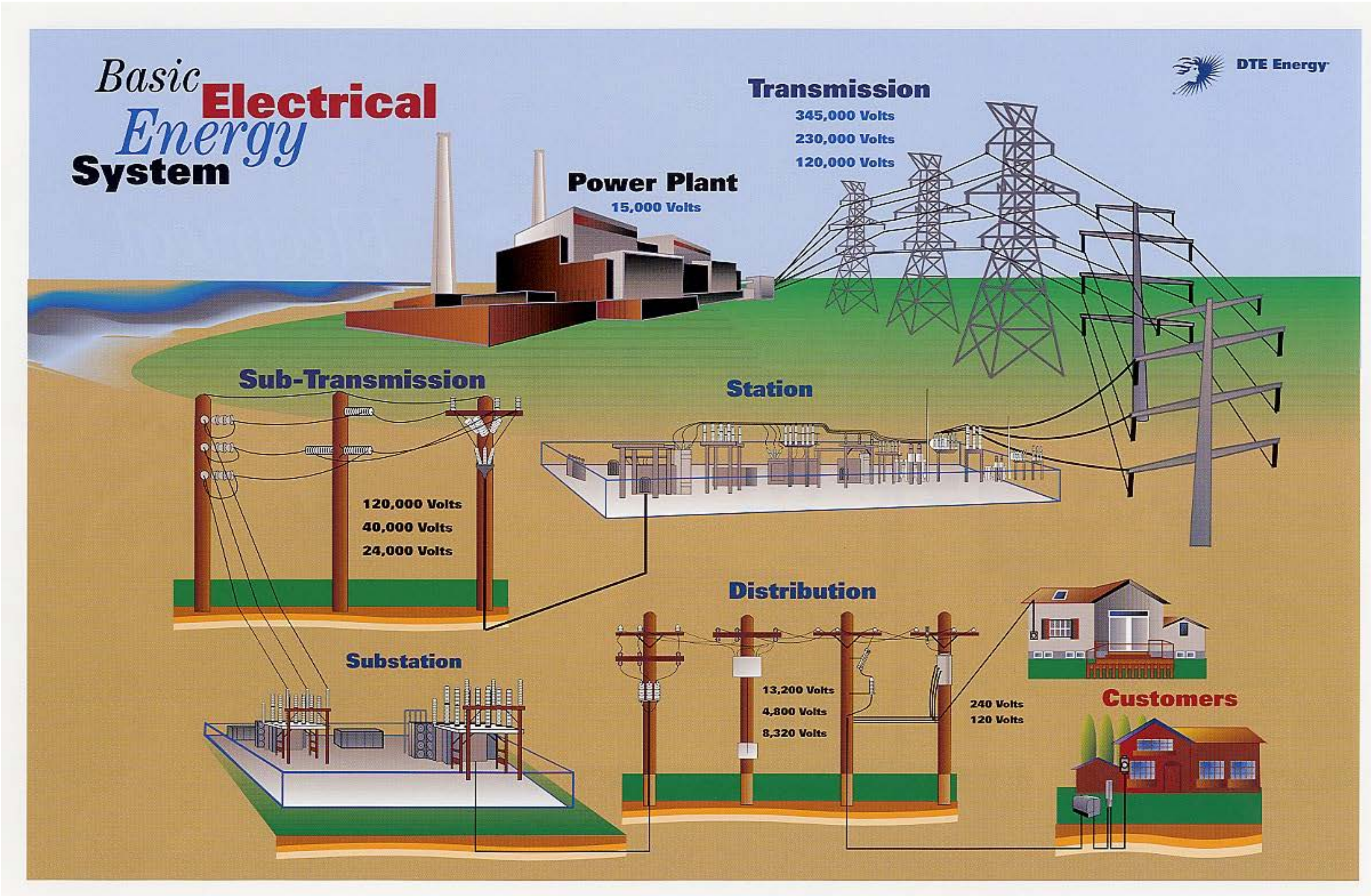
Grid Science Winter School
Santa Fe, NM
January 12, 2015

Power system overview

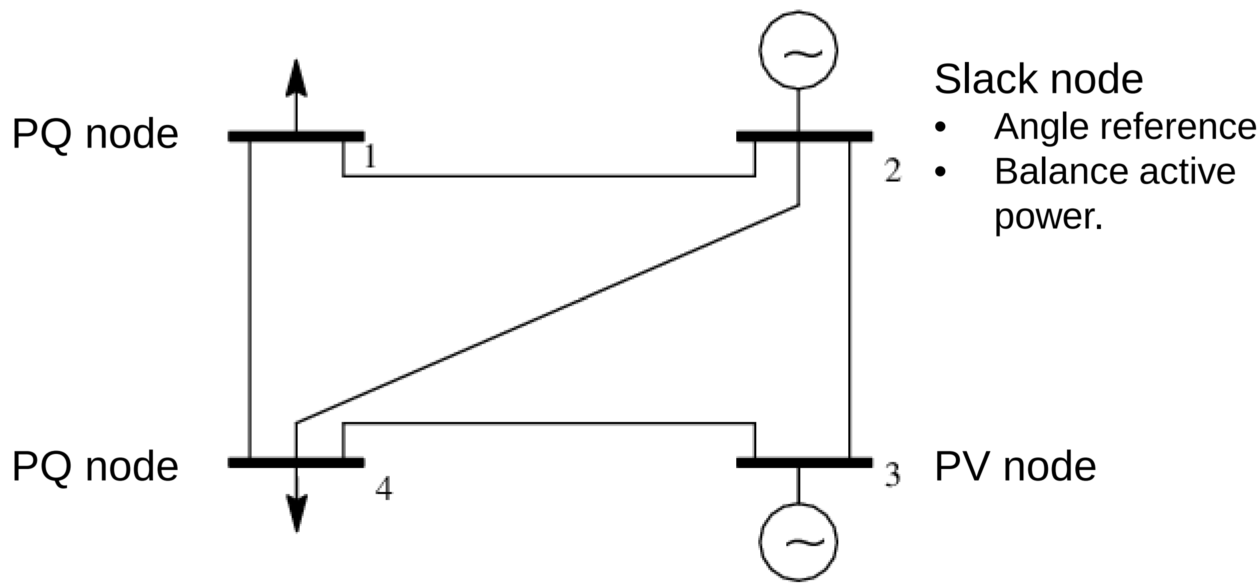


Structure

1. **Power Flow**
2. Generator modelling
3. Generator control
4. Load modelling
5. Substations
6. Protection

Power flow formulation

- The power flow determines the (complex) voltage at every node in a network, given:
  - Generator power injections and voltage set-points.
  - Load active and reactive power demands.
  - Network impedances.
- Linear network model given by Kirchhoff's laws.
- Nonlinear boundary conditions at nodes due to constant power injections.



Power flow equations

Let the phasor voltage at the i-th bus be given by $V_i\angle\theta_i = V_{di}+jV_{qi}$. The power flow equations can be written in (respectively) polar and rectangular form as:

$$
\begin{aligned}
P_i^{sp} &= P_i(\theta, V) = V_i \sum_{k=1}^{n} V_k (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) \\
&= P_i(V_d, V_q) = V_{di}\sum_{k=1}^{n}(G_{ik}V_{dk} - B_{ik}V_{qk}) + V_{qi}\sum_{k=1}^{n}(B_{ik}V_{dk} + G_{ik}V_{qk}) \\
Q_i^{sp} &= Q_i(\theta, V) = V_i \sum_{k=1}^{n} V_k (G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) \\
&= Q_i(V_d, V_q) = V_{di}\sum_{k=1}^{n}(-B_{ik}V_{dk} - G_{ik}V_{qk}) + V_{qi}\sum_{k=1}^{n}(G_{ik}V_{dk} - B_{ik}V_{qk})
\end{aligned}
$$

where angle differences are given by $\theta_{ik} = \theta_i - \theta_k$, and $Y_{ik} = G_{ik} + jB_{ik}$ is the (ik)-th element of the *network admittance matrix* defined by

$$
\begin{aligned}
Y_{ii} &= \sum_{\substack{k=0\\k\neq i}}^{n} y_{ik} = \text{ self admittance of node } i \\
Y_{ik} &= -y_{ik} = \text{ mutual admittance between nodes } i \text{ and } k.
\end{aligned}
$$

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Jacobian

- Power flow sensitivities are given by the Jacobian:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}$$

For $i \neq k$

$$\frac{\partial P_i}{\partial \theta_k} = V_i V_k (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik})$$

$$\frac{\partial P_i}{\partial V_k} = V_i (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik})$$

$$\frac{\partial Q_i}{\partial \theta_k} = -V_i V_k (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik})$$

$$\frac{\partial Q_i}{\partial V_k} = V_i (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik})$$

$$\frac{\partial P_i}{\partial \theta_i} = -B_{ii} V_i^2 - Q_i$$

$$\frac{\partial P_i}{\partial V_i} = G_{ii} V_i + P_i / V_i$$

$$\frac{\partial Q_i}{\partial \theta_i} = -G_{ii} V_i^2 + P_i$$

$$\frac{\partial Q_i}{\partial V_i} = -B_{ii} V_i + Q_i / V_i.$$

Voltage – reactive power coupling

- Under "normal" power system conditions, $G_{ik} \approx 0$ and $\theta_{ik} \approx 0$.
- It follows that $\frac{\partial P_i}{\partial V_k}$ and $\frac{\partial Q_i}{\partial \theta_k}$ are small, and power flow interactions are dominated by the $P - \theta$ and $Q - V$ couplings.
  - When the network is heavily loaded, θ_{ik} may not be small.
  - For distribution and subtransmission networks, R/X ratios are higher, so ignoring G_{ik} may not provide a good approximation.

Radial networks

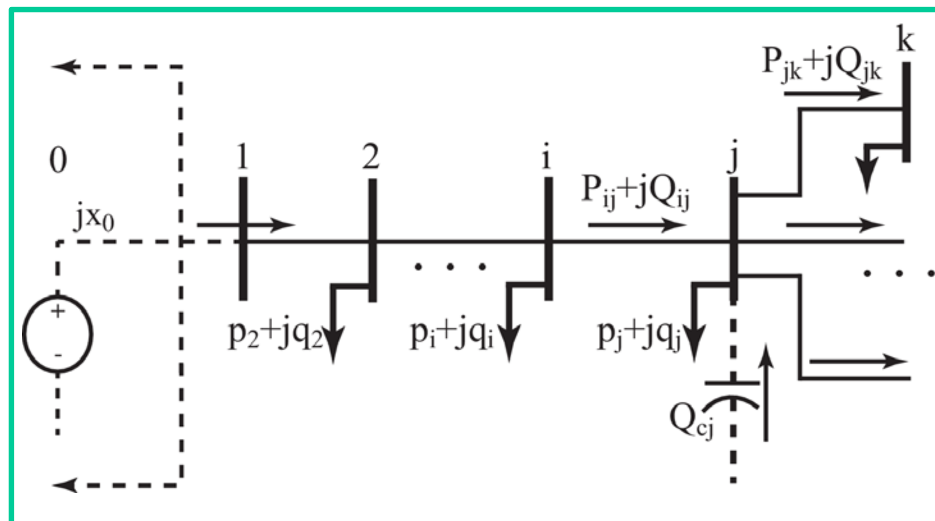
- For radial networks, the power flow equations can be written in the recursive form:

$$P_{ij} = \sum_{k \in \mathcal{C}_j} P_{jk} + r_{ij} \left(P_{ij}^2 + Q_{ij}^2\right) / V_i^2 + p_j$$

$$Q_{ij} = \sum_{k \in \mathcal{C}_j} Q_{jk} + x_{ij} \left(P_{ij}^2 + Q_{ij}^2\right) / V_i^2 + q_j - b_j V_j^2$$

$$V_j^2 = V_i^2 - 2\left(r_{ij} P_{ij} + x_{ij} Q_{ij}\right) + \frac{\left(r_{ij}^2 + x_{ij}^2\right)\left(P_{ij}^2 + Q_{ij}^2\right)}{V_i^2}$$

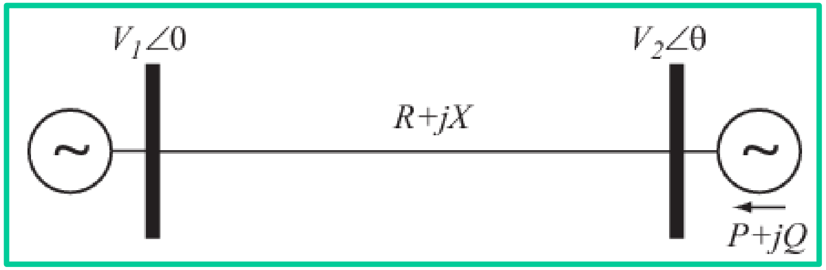
where $\mathcal{C}_j$ is the set of nodes connected "downstream" of node j, and the branch impedance is $z_{ij} = r_{ij} + j x_{ij}$.



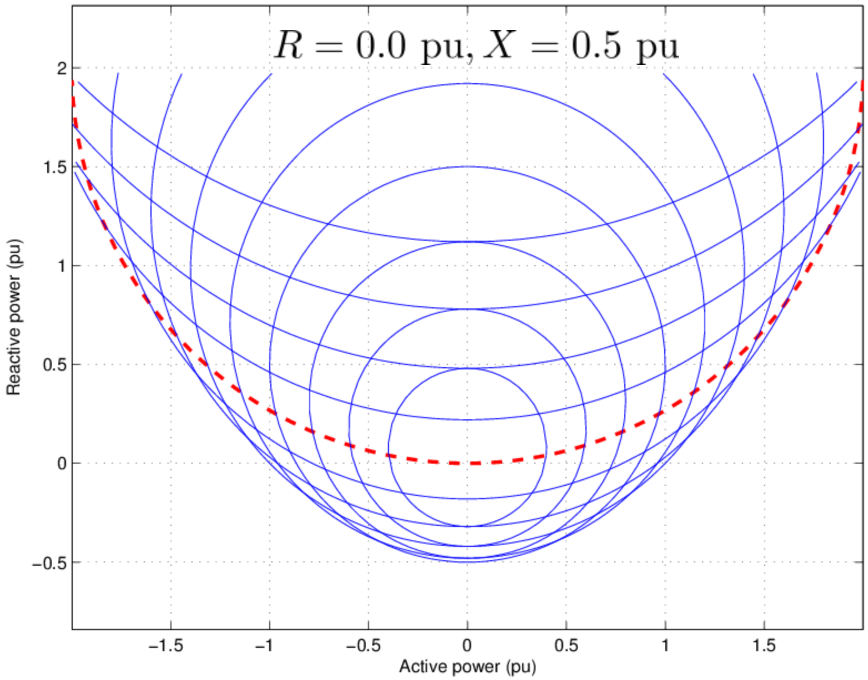
- This set of equations can be solved using a simple forward/backward iterative algorithm.

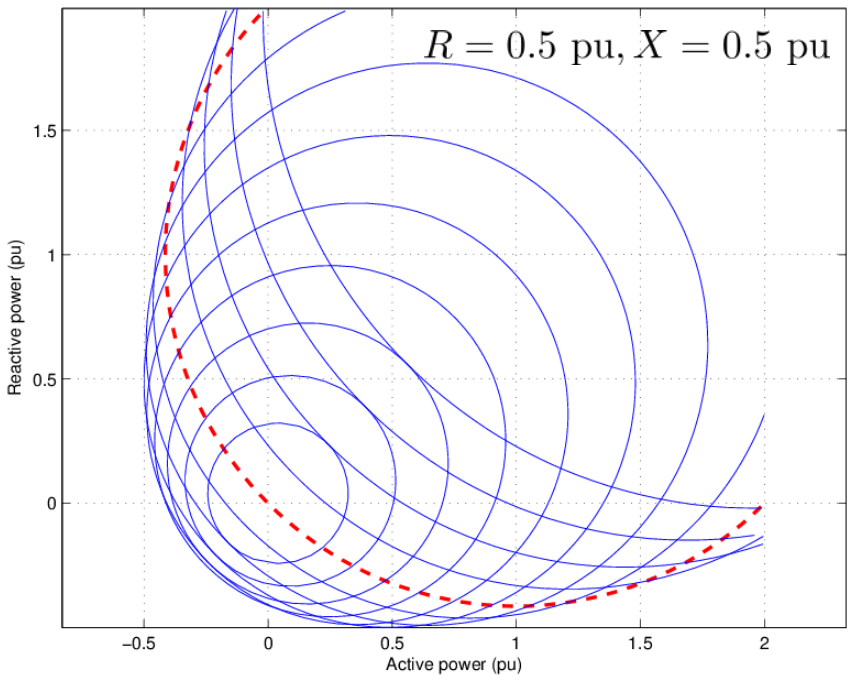
Resistance in the network

- The relationship between injected power $P + jQ$ and the bus voltage magnitude V_2 is given by,



$$\left(\frac{V_2^2 R}{R^2+X^2} - P\right)^2 + \left(\frac{V_2^2 X}{R^2+X^2} - Q\right)^2 = \frac{V_1^2 V_2^2}{R^2+X^2}$$





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Power flow solution

- The power flow problem (neglecting limits) can be expressed as:

$$f(x, \lambda) = 0$$

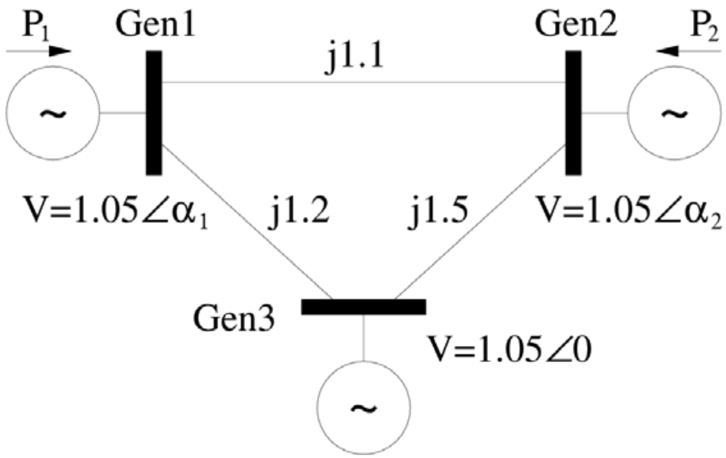
where x consists of voltage magnitudes and angles, λ are parameters such as:

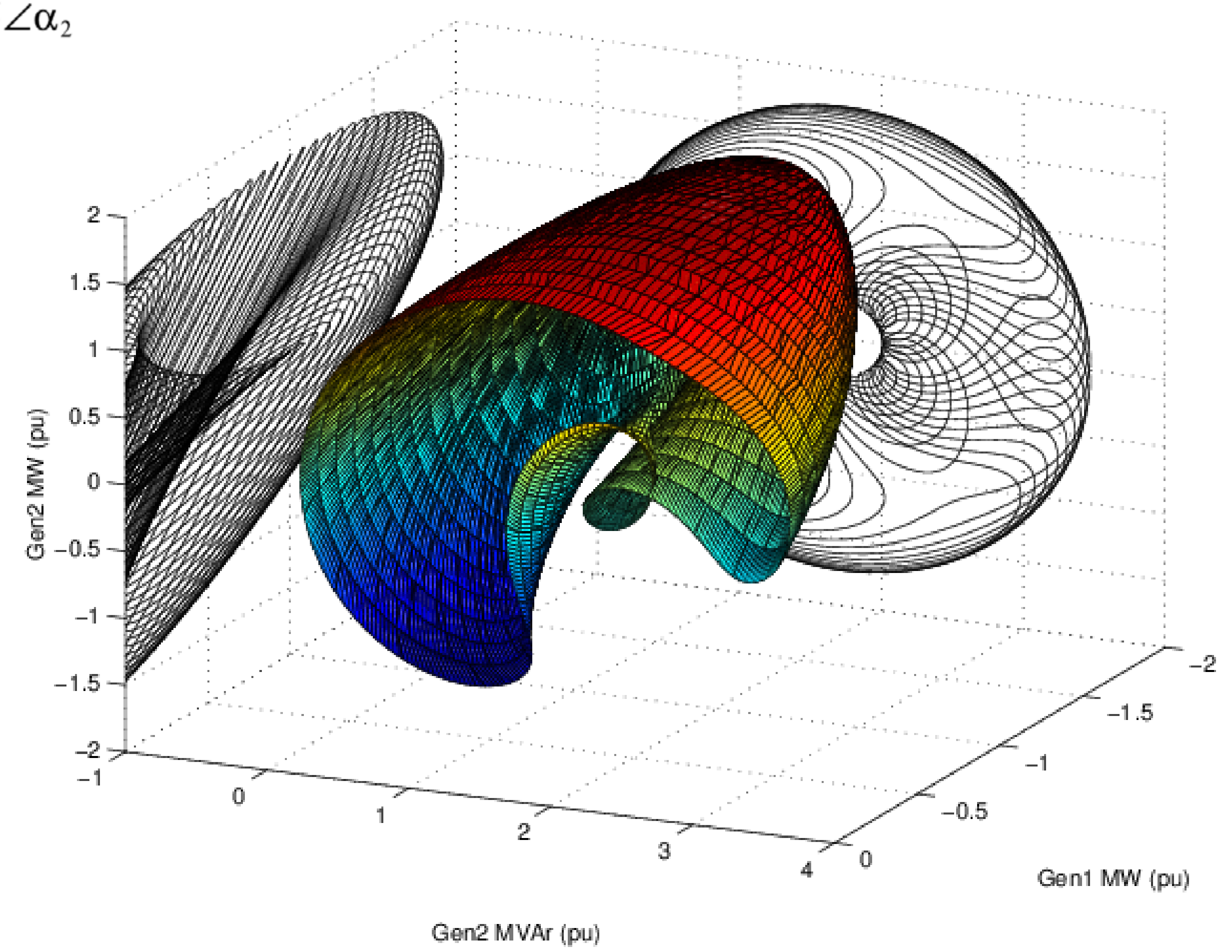
  - Active and reactive power at *PQ* buses.
  - Active power and voltage magnitude at *PV* buses.
  - Distance γ in a particular loading direction:

$$P_d = P_d^o(1 + k_p\gamma), \qquad Q_d = Q_d^o(1 + k_q\gamma)$$

- For a system of n buses, x has dimension $2n$ (if the mismatch at the slack is included as a state), and f has dimension $2n$.
- With all parameters specified (fixed), $f(x) = 0$ consists of $2n$ equations in $2n$ variables.
  - Solutions are isolated points, but there are generally multiple solutions.

Multiple solutions





Newton solution

- Solutions can be obtained using a Newton iterative process:

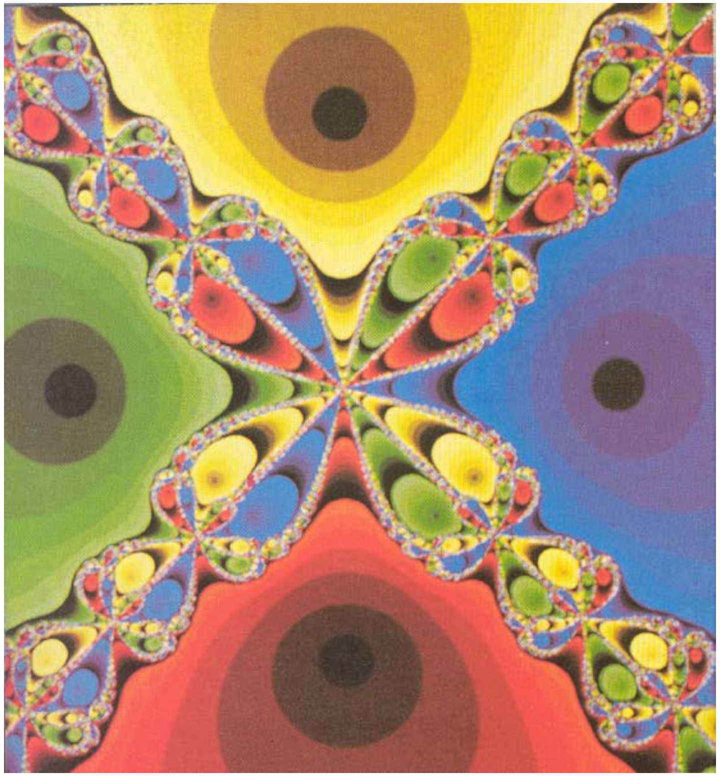
$$x_{k+1} = x_k - J(x_k)^{-1} f(x_k)$$

where k indicates the iteration number, and J is the Jacobian matrix

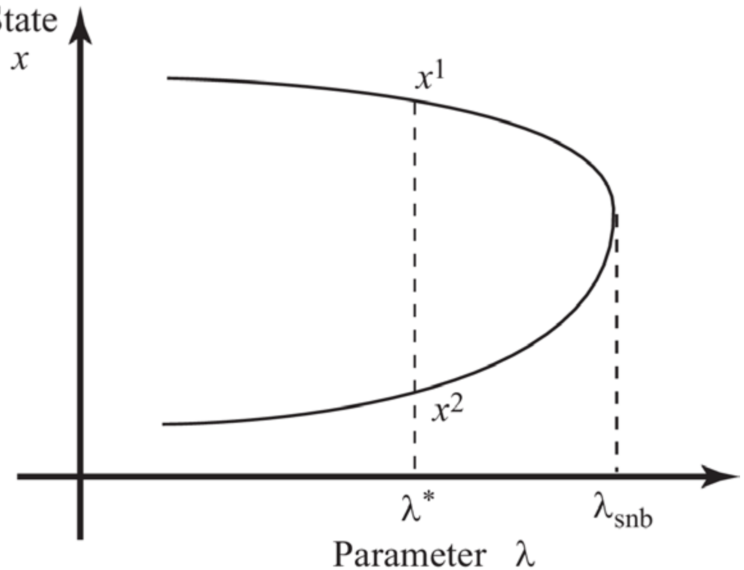
$$J = \frac{\partial f}{\partial x}$$

which has dimension $2n \times 2n$, and is very sparse.

- Iterative techniques such as Newton's method are generally not globally convergent.
- They require an initial guess x_0 that lies in the "region of convergence".



Maximum loadability



- Notice from the "bifurcation diagram" that as λ^* increases, the solutions x^1 and x^2 move closer together, eventually coalescing at $\lambda = \lambda_{snb}$.
- As x^1 and x^2 move closer together, their associated regions of convergence shrink.
- It is more difficult to find an initial guess x_0 that gives convergence.
- No solutions exist for $\lambda > \lambda_{snb}$.
- The critical point at λ_{snb}, the maximum value of λ for which a solution exists, is called a *saddle node bifurcation*.
- The Jacobian J is singular at that point.
- Voltages (given by x) become infinitely sensitive to load changes (given by λ.)

Point of collapse

- If the load increases beyond λ_{snb}, no steady-state (equilibrium) solutions exist, so transient response has nowhere to settle.
- Typically voltages across a region of the power system decline uncontrollably to low values, initiating protection operation.
- The maximum loadability point λ_{snb} is also called the *point of collapse*.
- The point of collapse describes the absolute limiting case for system loading.
- Many security margins are defined in terms of a norm (distance) from the operating point to that limiting case.

Power flow divergence

- The power flow solution process becomes unreliable near maximum loadability.
- Newton solution relies on J^{-1}, but J approaches singularity (becomes ill-conditioned.)
- It is important to note that maximum loadability implies power flow non-convergence, but power flow non-convergence does not necessarily imply maximum loadability.
  - Many power flow programs may exhibit poor convergence a long way from maximum loadability.

Direct solution for maximum loadability

- In order to find the point of maximum loadability, one of the parameters λ must be allowed to vary.
- The desired point is described by the set of equations:

$$\begin{aligned} f(x,\lambda) &= 0 \\ J(x,\lambda)v &= 0 \\ v^\top v &= 1 \end{aligned}$$

where v is the right eigenvector of J corresponding to a zero eigenvalue.

  – The first group of equations ensures a power flow solution.
  – The second and third groups of equations together enforce singularity of J.

Direct solution

- Number of equations is $2n+2n+1$.
- Number of variables:

  x contributes $2n$.

  v contributes $2n$.

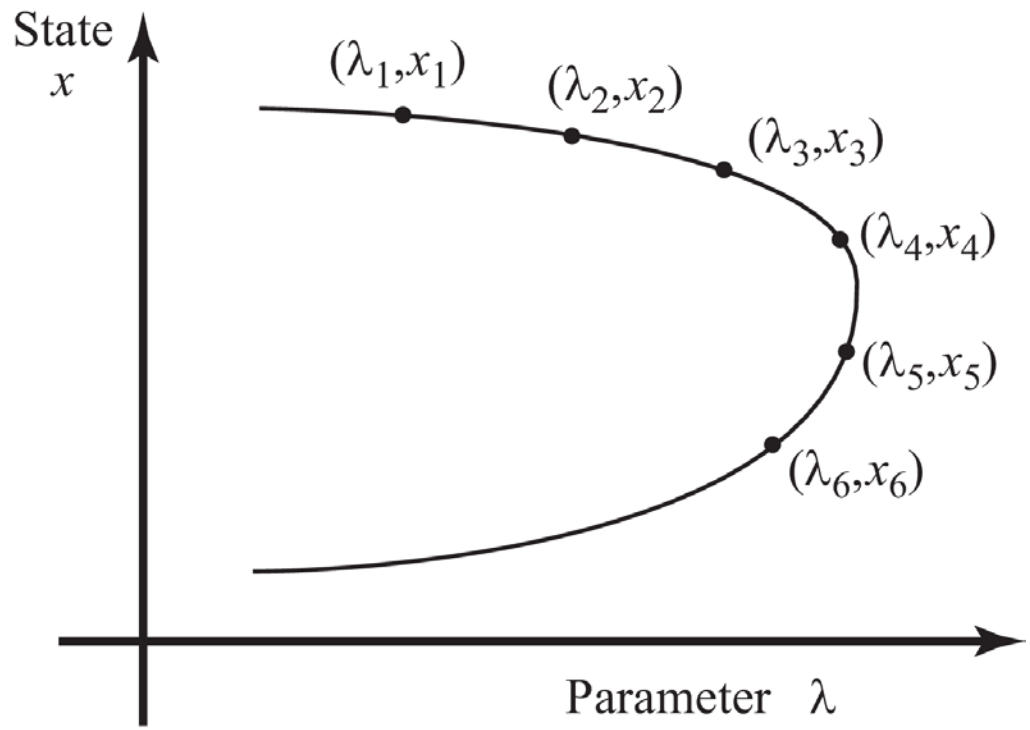
  λ contributes 1.
- Number of equations and variables are equal, giving point solutions.
- Obtaining a solution may be difficult though, as choosing appropriate initial values for x, v, and λ (required by the iterative solution process) can be challenging.
- Direct solution gives the point of maximum loadability, but tells nothing about the variation in system conditions as the load increases towards the maximum.
- This extra information can be obtained using a continuation process.

Continuation power flow

- Consider the power flow problem

$$f(x, \lambda) = 0$$

  where λ is a single free parameter.
- This problem has $2n + 1$ variables but only $2n$ equations, so is under-determined.
  - The solution is a curve.

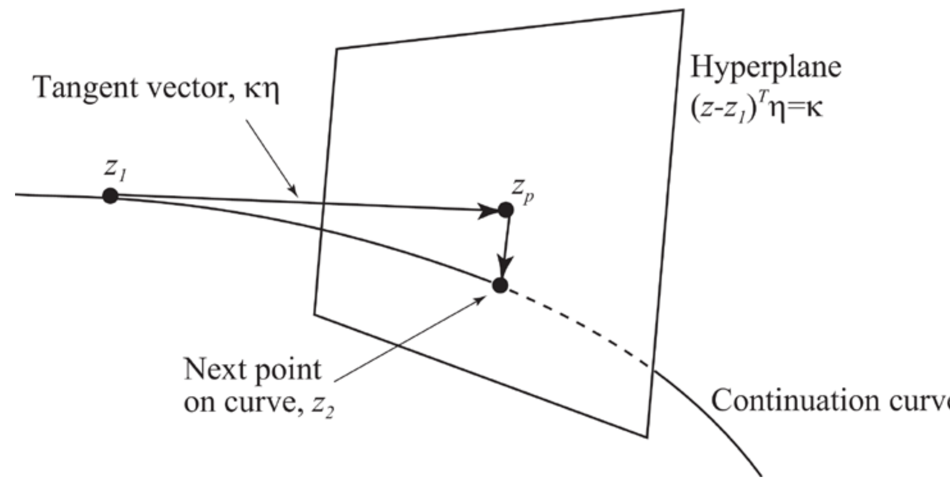


Euler homotopy

- Rewrite the power flow problem:

$$f(z) = 0$$

where z incorporates both x and λ.



- Find an initial point z_1 on the curve, by solving a regular power flow problem.
- Determine the unit vector η that is tangent to the curve at the point z_1.
- This is given by:

$$\frac{\partial f}{\partial z}(z_1)\eta = 0$$

$$\eta^\top \eta = 1$$

- Note that $\frac{\partial f}{\partial z}$ is a $(2n) \times (2n+1)$ Jacobian.
- A numerically robust method of obtaining η follows from the factorisation $(\frac{\partial f}{\partial z})^\top = QR$ where Q is orthonormal and R is upper triangular. The last column of Q is exactly the desired η.

Euler homotopy (2)

- The prediction of the next point on the curve is given by:

$$z_p = z_1 + \kappa\eta$$

  where κ is the size of the step taken in the direction of the tangent vector η.
- The correction to a point z_2 on the curve is then made. This is achieved by solving for the point of intersection of:
  - The solution curve, and
  - A hyperplane that passes through z_p and that is orthogonal to η. Points z on this hyperplane are given by

$$(z - z_p)^\top \eta = 0$$

    which is equivalent to $(z - z_1)^\top \eta = \kappa$.
- The desired point z_2 is given by solving:

$$f(z) = 0$$

$$(z - z_1)^\top \eta - \kappa = 0$$

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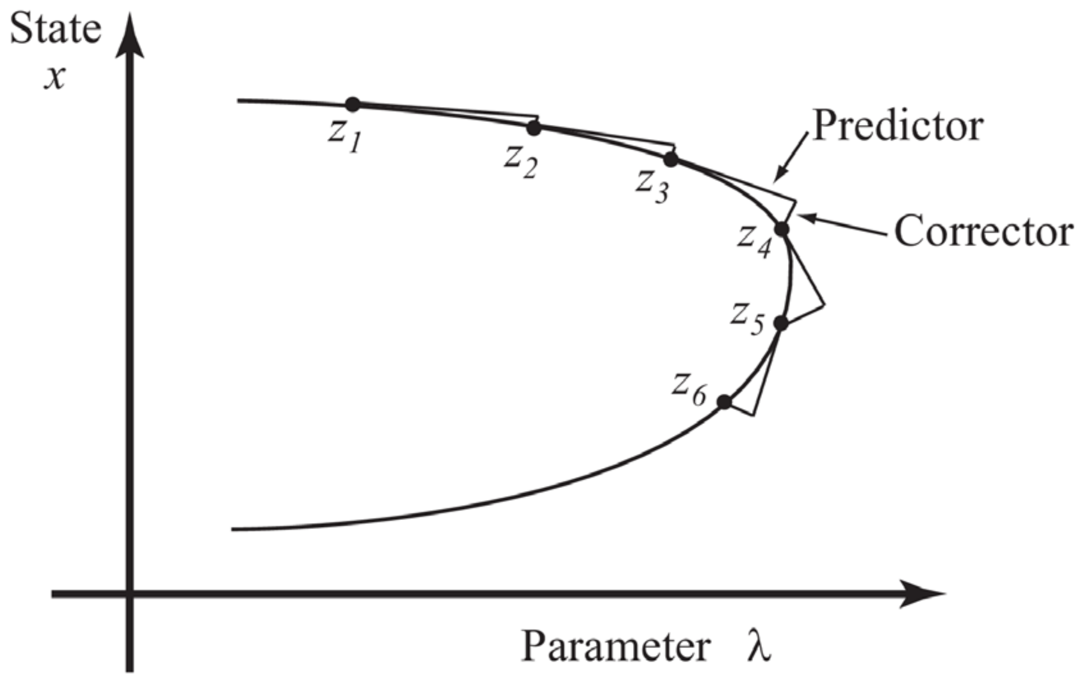
Euler homotopy (3)



- Once two points have been calculated, the tangent vector η can be formed by the secant approximation:

$$\eta_i \approx \frac{z_i - z_{i_1}}{\|z_i - z_{i_1}\|_2}$$

- The approximation may not be adequate when the curve has high curvature.

Structure

1. Power Flow
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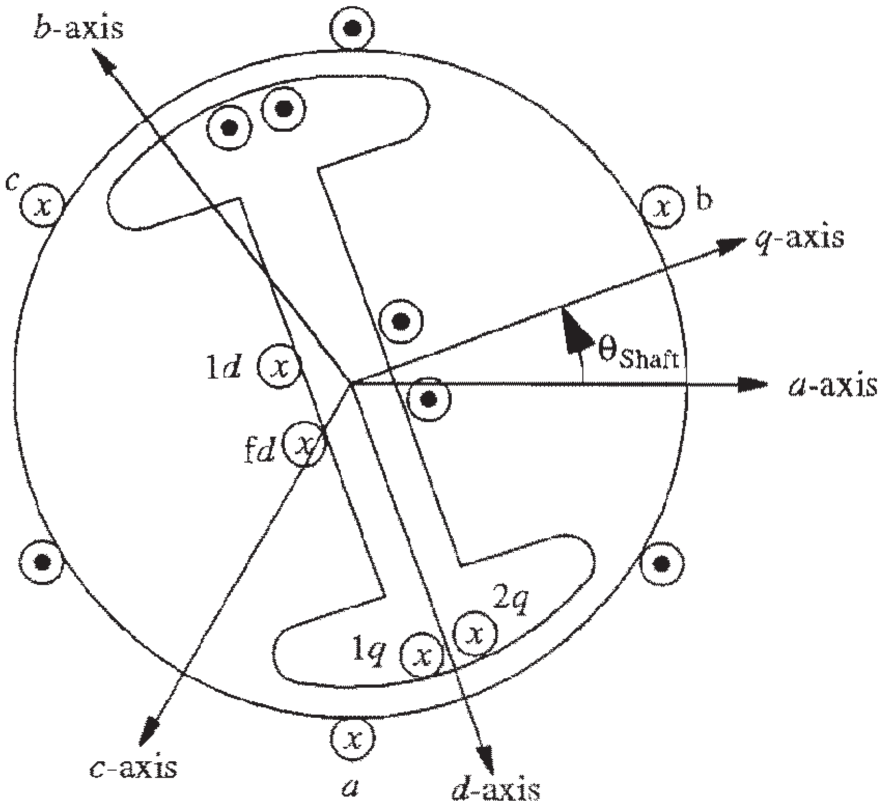
Basic synchronous machine equations



$$v_a = i_a r_s + \frac{d\lambda_a}{dt}$$

$$v_b = i_b r_s + \frac{d\lambda_b}{dt}$$

$$v_c = i_c r_s + \frac{d\lambda_c}{dt}$$

$$v_{fd} = i_{fd} r_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = i_{1d} r_{1d} + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = i_{1q} r_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = i_{2q} r_{2q} + \frac{d\lambda_{2q}}{dt}$$

$$\frac{d\theta_{shaft}}{dt} = \frac{2}{P}\omega$$

$$J \frac{2}{P} \frac{d\omega}{dt} = T_m - T_e - T_{fw}$$

where λ is flux linkage, r is winding resistance, J is the inertia constant, P is the number of magnetic poles per phase, T_m is the mechanical torque applied to the shaft, T_e is the electrical torque and T_{fw} is a friction-windage torque.

From Sauer and Pai

Park's transformation

- The sinusoidal steady-state of balanced symmetrical machines can be transformed to produce constant states. Define:

$$v_{dqo} \triangleq T_{dqo} v_{abc}, \quad i_{dqo} \triangleq T_{dqo} i_{abc}, \quad \lambda_{dqo} \triangleq T_{dqo} \lambda_{abc}$$

where

$$T_{dqo} \triangleq \frac{2}{3} \begin{bmatrix} \sin \frac{P}{2}\theta_{shaft} & \sin(\frac{P}{2}\theta_{shaft} - \frac{2\pi}{3}) & \sin(\frac{P}{2}\theta_{shaft} + \frac{2\pi}{3}) \\ \cos \frac{P}{2}\theta_{shaft} & \cos(\frac{P}{2}\theta_{shaft} - \frac{2\pi}{3}) & \cos(\frac{P}{2}\theta_{shaft} + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Further steps

- Energy balance gives:

$$T_e = -\left(\frac{3}{2}\right)\left(\frac{P}{2}\right)(\lambda_d i_q - \lambda_q i_d)$$

- Define an angle that is constant for constant shaft speed:

$$\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$$

- Scale using per unit – many degrees of freedom.
- Establish the relationship between flux linkages and currents:

$$\lambda_{abc} = L_{ss}(\theta_{shaft})i_{abc} + L_{sr}(\theta_{shaft})i_{rotor}$$
$$\lambda_{rotor} = L_{rs}(\theta_{shaft})i_{abc} + L_{rr}(\theta_{shaft})i_{rotor}$$

Resulting 9th order model

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = R_s I_d + \frac{\omega}{\omega_s}\psi_q + V_d$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = R_s I_q - \frac{\omega}{\omega_s}\psi_d + V_q$$

$$\frac{1}{\omega_s}\frac{d\psi_o}{dt} = R_s I_o + V_o$$

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)\left[I_d - \frac{X'_d - X''_d}{(X'_d - X_{\ell s})^2}\left(\psi_{1d} + (X'_d - X_{\ell s})I_d - E'_q\right)\right] + E_{fd}$$

$$T''_{do}\frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - (X'_d - X_{\ell s})I_d$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + (X_q - X'_q)\left[I_q - \frac{X'_q - X''_q}{(X'_q - X_{\ell s})^2}\left(\psi_{2q} + (X'_q - X_{\ell s})I_q + E'_d\right)\right]$$

$$T''_{qo}\frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - (X'_q - X_{\ell s})I_q$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_m - (\psi_d I_q - \psi_q I_d) - T_{fw}$$

$$\psi_d = -X''_d I_d + \frac{(X''_d - X_{\ell s})}{(X'_d - X_{\ell s})}E'_q + \frac{(X'_d - X''_d)}{(X'_d - X_{\ell s})}\psi_{1d}$$

$$\psi_q = -X''_q I_q - \frac{(X''_q - X_{\ell s})}{(X'_q - X_{\ell s})}E'_d + \frac{(X'_q - X''_q)}{(X'_q - X_{\ell s})}\psi_{2q}$$

$$\psi_o = -X_{\ell s} I_o$$

Terminal conditions

- Consider a balanced set of sinusoidal voltages and currents:

$$
\begin{aligned}
v_a &= \sqrt{2}V_s \cos\left(\omega_s t + \theta_s\right) \\
v_b &= \sqrt{2}V_s \cos\left(\omega_s t + \theta_s - \frac{2\pi}{3}\right) \\
v_c &= \sqrt{2}V_s \cos\left(\omega_s t + \theta_s + \frac{2\pi}{3}\right) \\
i_a &= \sqrt{2}I_s \cos\left(\omega_s t + \phi_s\right) \\
i_b &= \sqrt{2}I_s \cos\left(\omega_s t + \phi_s - \frac{2\pi}{3}\right) \\
i_c &= \sqrt{2}I_s \cos\left(\omega_s t + \phi_s + \frac{2\pi}{3}\right)
\end{aligned}
$$

- Park's transformation together with appropriate per unit scaling give:

$$
\begin{aligned}
(V_d + jV_q)e^{j(\delta - \pi/2)} &= V_s e^{j\theta_s} \\
(I_d + jI_q)e^{j(\delta - \pi/2)} &= I_s e^{j\phi_s} \\
V_o = I_o &= 0
\end{aligned}
$$

Elimination of stator transients

- The time constants associated with ψ_d and ψ_q are small relative to the other machine dynamics.
- Under balanced conditions, ψ_o is zero.
- The resulting 6th order model becomes:

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)\left[I_d - \frac{X'_d - X''_d}{(X'_d - X_{\ell s})^2}\left(\psi_{1d} + (X'_d - X_{\ell s})I_d - E'_q\right)\right] + E_{fd}$$

$$T''_{do}\frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - (X'_d - X_{\ell s})I_d$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + (X_q - X'_q)\left[I_q - \frac{X'_q - X''_q}{(X'_q - X_{\ell s})^2}\left(\psi_{2q} + (X'_q - X_{\ell s})I_q + E'_d\right)\right]$$

$$T''_{qo}\frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - (X'_q - X_{\ell s})I_q$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_m - (\psi_d I_q - \psi_q I_d) - T_{fw}$$

$$\psi_d = -X''_d I_d + \frac{(X''_d - X_{\ell s})}{(X'_d - X_{\ell s})}E'_q + \frac{(X'_d - X''_d)}{(X'_d - X_{\ell s})}\psi_{1d}$$

$$\psi_q = -X''_q I_q - \frac{(X''_q - X_{\ell s})}{(X'_q - X_{\ell s})}E'_d + \frac{(X'_q - X''_q)}{(X'_q - X_{\ell s})}\psi_{2q}$$

The original differential equations are replaced by algebraic equations:

$$R_s I_d + \psi_q + V_d = 0$$

$$R_s I_q - \psi_d + V_q = 0$$

Two-axis model

- If T''_{do} and T''_{qo} are sufficiently small, the damper winding states ψ_{1d} and ψ_{2q} can be eliminated by setting $T''_{do} = T''_{qo} = 0$, giving:

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)I_d + E_{fd}$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + (X_q - X'_q)I_q$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_m - (\psi_d I_q - \psi_q I_d) - T_{fw}$$

$$\psi_d = -X'_d I_d + E'_q$$

$$\psi_q = -X'_q I_q - E'_d$$

$$0 = R_s I_d + \psi_q + V_d$$

$$0 = R_s I_q - \psi_d + V_q$$

One-axis model

- If T'_{qo} is sufficiently small, the remaining damper winding state E'_d can be eliminated by setting $T'_{qo} = 0$, giving:

$$
\begin{aligned}
T'_{do}\frac{dE'_q}{dt} &= -E'_q - (X_d - X'_d)I_d + E_{fd} \\
\frac{d\delta}{dt} &= \omega - \omega_s \\
\frac{2H}{\omega_s}\frac{d\omega}{dt} &= T_m - (\psi_d I_q - \psi_q I_d) - T_{fw} \\
\psi_d &= -X'_d I_d + E'_q \\
\psi_q &= -X_q I_q \\
0 &= R_s I_d + \psi_q + V_d \\
0 &= R_s I_q - \psi_d + V_q
\end{aligned}
$$

Classical model

- Assume T'_{do} is large, so that E'_q is effectively constant.
- Assume $X'_d = X_q$ and $R_s = 0$.
- The resulting model has no electrical dynamics, only mechanical dynamics:

$$\frac{d\delta}{dt} = \omega - \omega_s$$
$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_m - (V_d I_d + V_q I_q) - T_{fw}$$

with

$$\frac{E'_q e^{j\pi/2} - (V_d + jV_q)}{jX'_d} = I_d + jI_q$$

or equivalently, multiplying throughout by $e^{j(\delta-\pi/2)}$,

$$\frac{E'_q e^{j\delta} - V_s e^{j\theta_s}}{jX'_d} = I_s e^{j\phi_s}$$

  – Voltage source $E'_q e^{j\delta}$ (with constant magnitude) behind transient reactance jX'_d.

Structure

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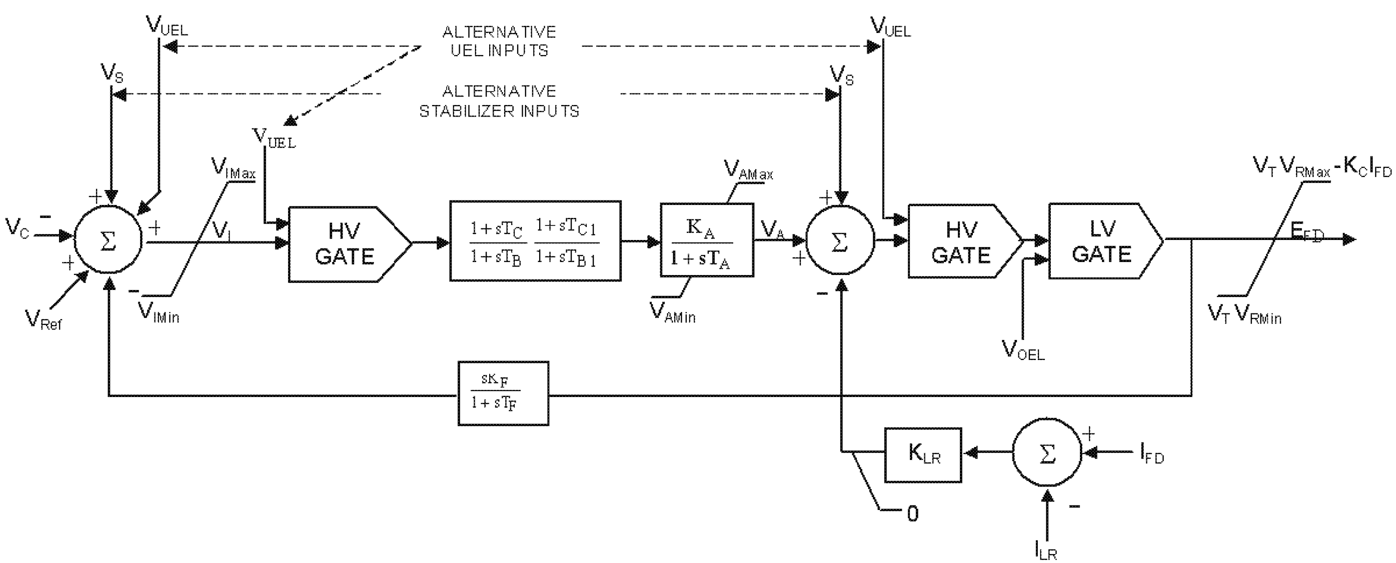
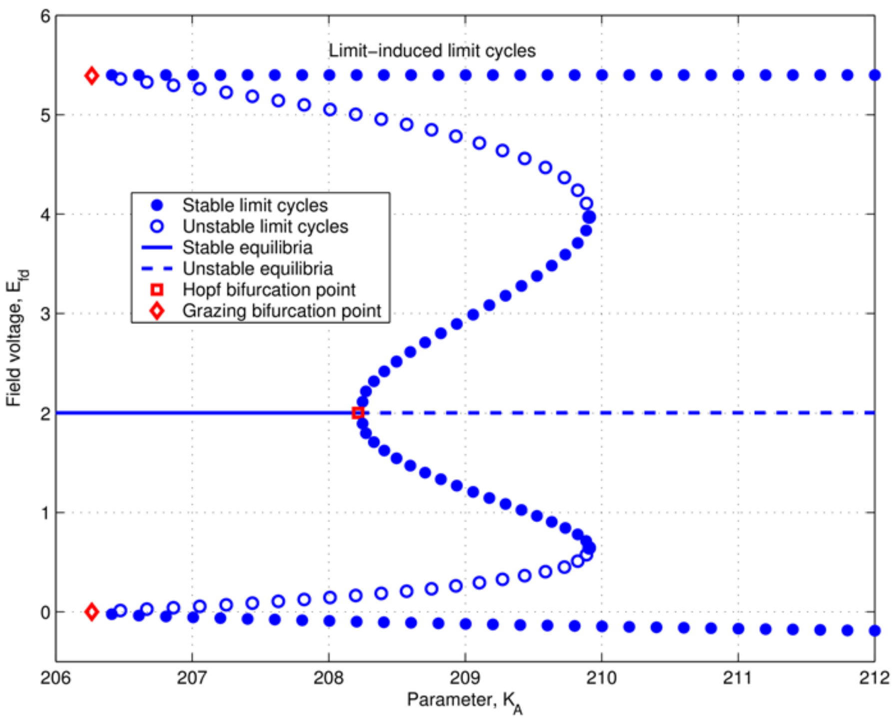
Generator voltage control

- Voltage control is achieved by the automatic voltage regulator (AVR).
  - Terminal voltage is measured and compared with a set-point.
  - The voltage error is driven to zero by adjusting the field voltage E_{fd} .
- An increase in the field voltage will result in an increase in the terminal voltage and in the reactive power produced by the generator.
- If field voltage becomes excessive, an over-excitation limiter will operate to reduce the field current.
  - The terminal voltage will subsequently fall.

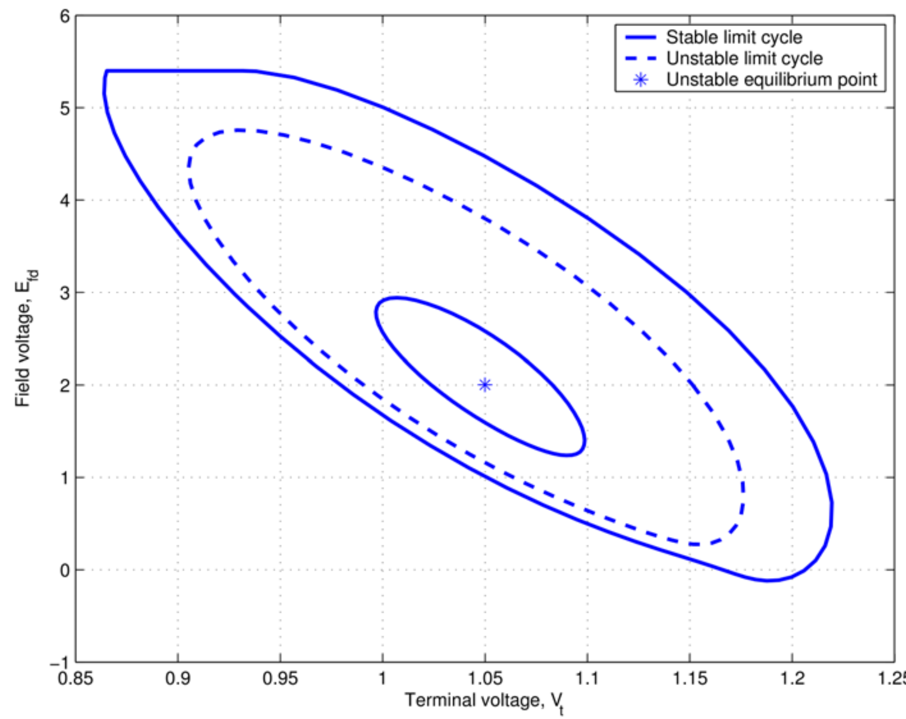


High gain instability

- As the AVR gain is increased, a Hopf bifurcation may lead to oscillatory instability.

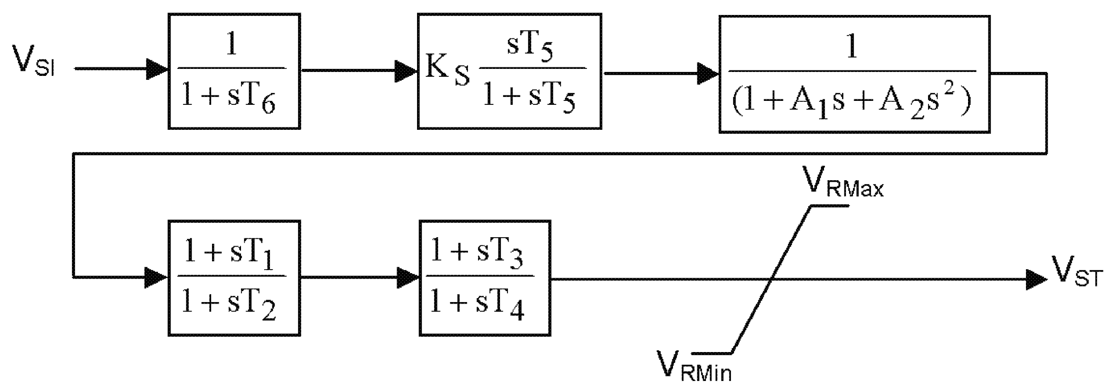


AVR gain $K_A = 209$



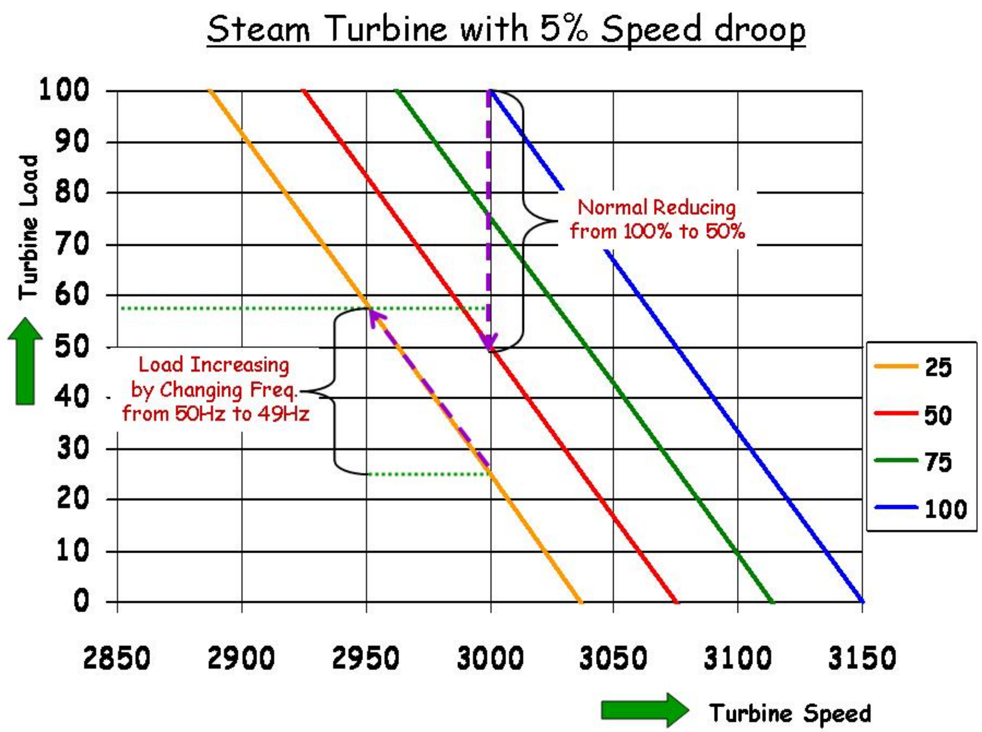
Power system stabilizers

- High-gain voltage control can destabilize angle dynamics.
- To compensate, many generators have a power system stabilizer (PSS) to improve damping.



Governor

- Active power regulation is achieved by a governor.
  - If frequency is less than desired, increase mechanical torque.
  - Decrease mechanical torque if frequency is high.
- For a steam plant, torque is controlled by adjusting the steam value, for a hydro unit control vanes regulate the flow of water delivered by the penstock.
- Frequency is a common signal seen by all generators.
  - If all generators tried to regulate frequency to its nominal setpoint, hunting would result.
  - This is overcome through the use of a droop characteristic.



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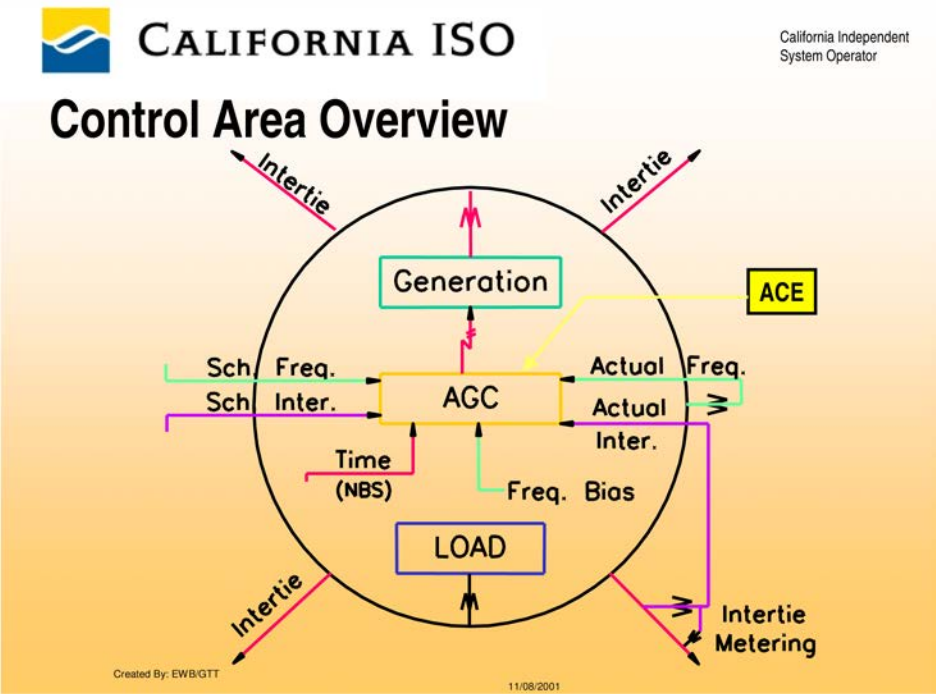
Automatic generation control (AGC)

- Based on a control area concept (now called a balancing authority.)
- Each balancing authority generates an "area control error" (ACE) signal,

$$ACE = -\Delta P_{net\ int} - B\Delta f$$

where B is the frequency bias factor.



- The ACE signal is used by AGC to adjust governor setpoints at participating generators.
  - This restores frequency and tie-line flows to their scheduled values.
  - Economic dispatch operates on a slower timescale to re-establish the most economic generation schedule.

Structure

1. Power Flow
2. Generator modelling
3. Generator control
4. **Load modelling**
5. Substations
6. Protection

Load modelling

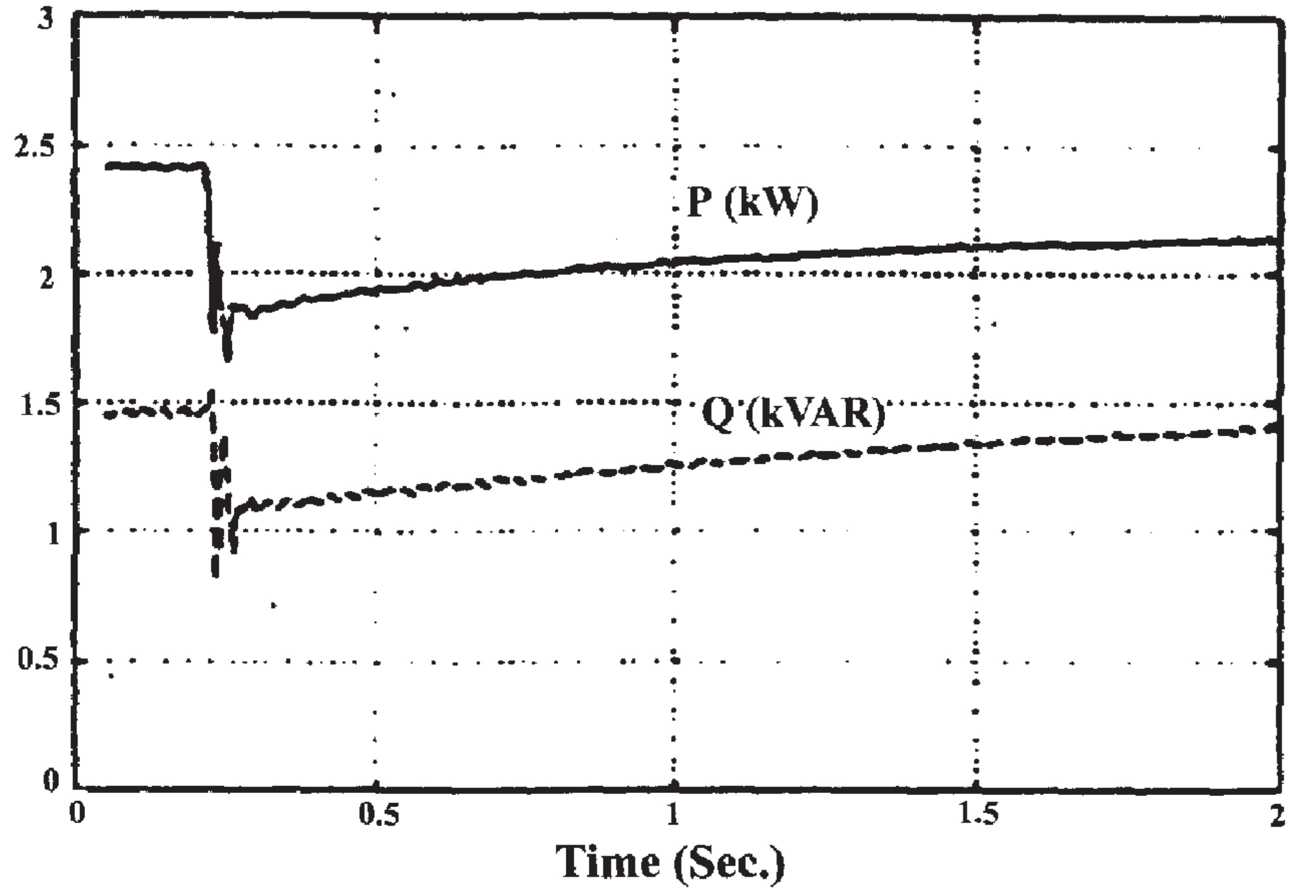
- Work is always on-going…
- Three philosophies:
  1. Very simple generic load models. Static models have the form,

     $$P(V) = P^0\Big(\frac{V}{V^0}\Big)^{\alpha}, \qquad Q(V) = Q^0\Big(\frac{V}{V^0}\Big)^{\beta}.$$

     The so-called ZIP model is composed of three terms: constant impedance, constant current and constant power.
  2. Use disturbance measurements to estimate parameters of generic load models that (supposedly) capture aggregate load behaviour.
  3. Undertake a detailed assessment of the load composition (amounts of various load categories) at load locations (distribution substations) that exert an important influence on system behaviour.
- Distributed generation further complicates load modelling.

Generic load recovery

- Load response to a voltage step typically consists of an initial step followed by a recovery phase.



Load recovery model

- A commonly used generic dynamic load model has the form,

$$\dot{x}_p = \frac{1}{T_p}(P_s(V) - P_d)$$

$$P_d = x_p + P_t(V)$$

where P_d is the active power drawn from the system, $P_t(V)$ describes the transient response of the load, and $P_s(V)$ gives the steady-state load response, with

$$P_s(V) = P_s^0\left(\frac{V}{V_s^0}\right)^{\alpha_s}, \qquad P_t(V) = P_t^0\left(\frac{V}{V_t^0}\right)^{\alpha_t}.$$

- The transient and steady-state changes, and the rate of recovery, are load dependent.
  - For induction motors, the recovery is very fast.
  - For aggregated distribution loads that are dominated by tap-changing transformers, recovery is slow.

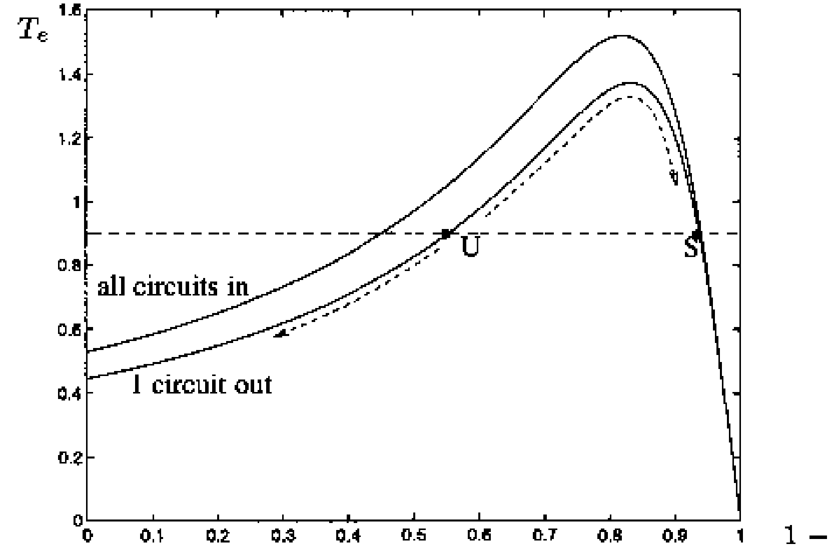
Load recovery model behaviour

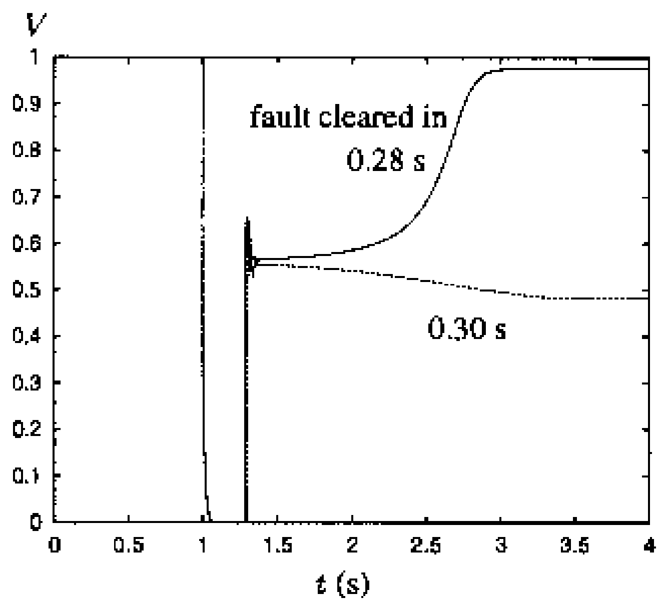
- When a disturbance (step change) in voltage occurs, the load state x_p cannot change instantaneously. Load demand P_d will vary instantaneously in response to the change in voltage V according to $P_d = x_p + P_t(V)$.
- The load state x_p will evolve over time, driven by the mismatch $(P_s(V) - P_d)$, and with a rate of change dictated by the time constant T_p. This process will continue until steady-state is reached, when $P_d = P_s(V)$.
- Reactive power load is handled in different ways.
  - Constant power factor.
  - Reactive power has the same form of response, but $Q_s(V)$ and $Q_t(V)$ differ from their active power counterparts.
  - It is usual to assume the rate of recovery matches active power, $T_p = T_q$.
- Numerous other forms of generic load models have been proposed.

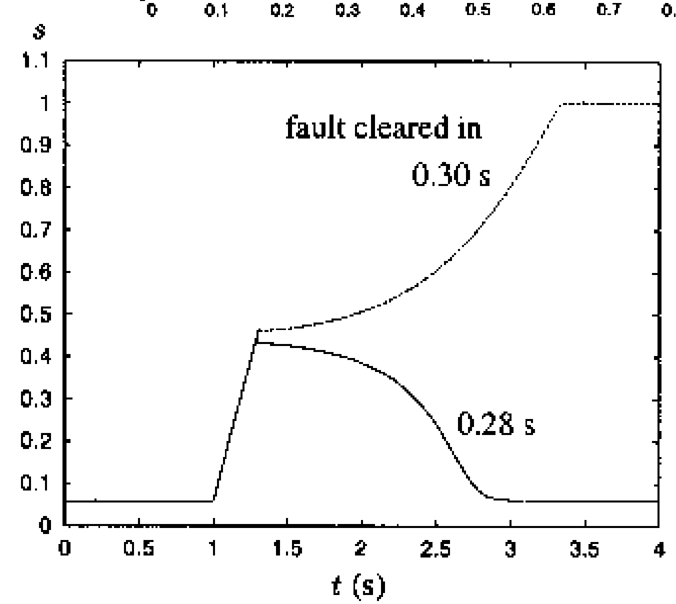
Induction motor loads

- Induction motor slip is driven by:

$$\dot{s} \approx T_m - T_e$$

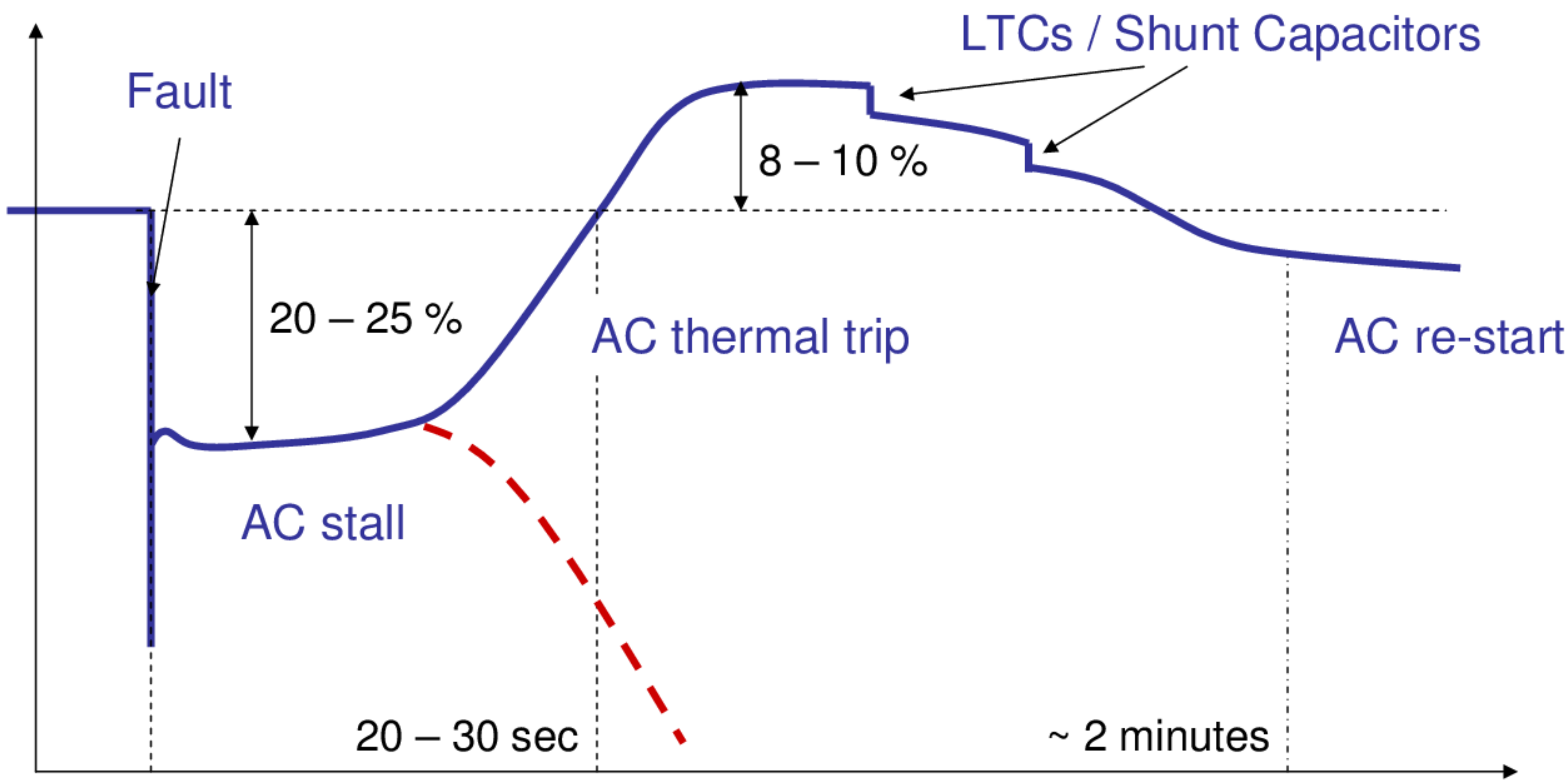






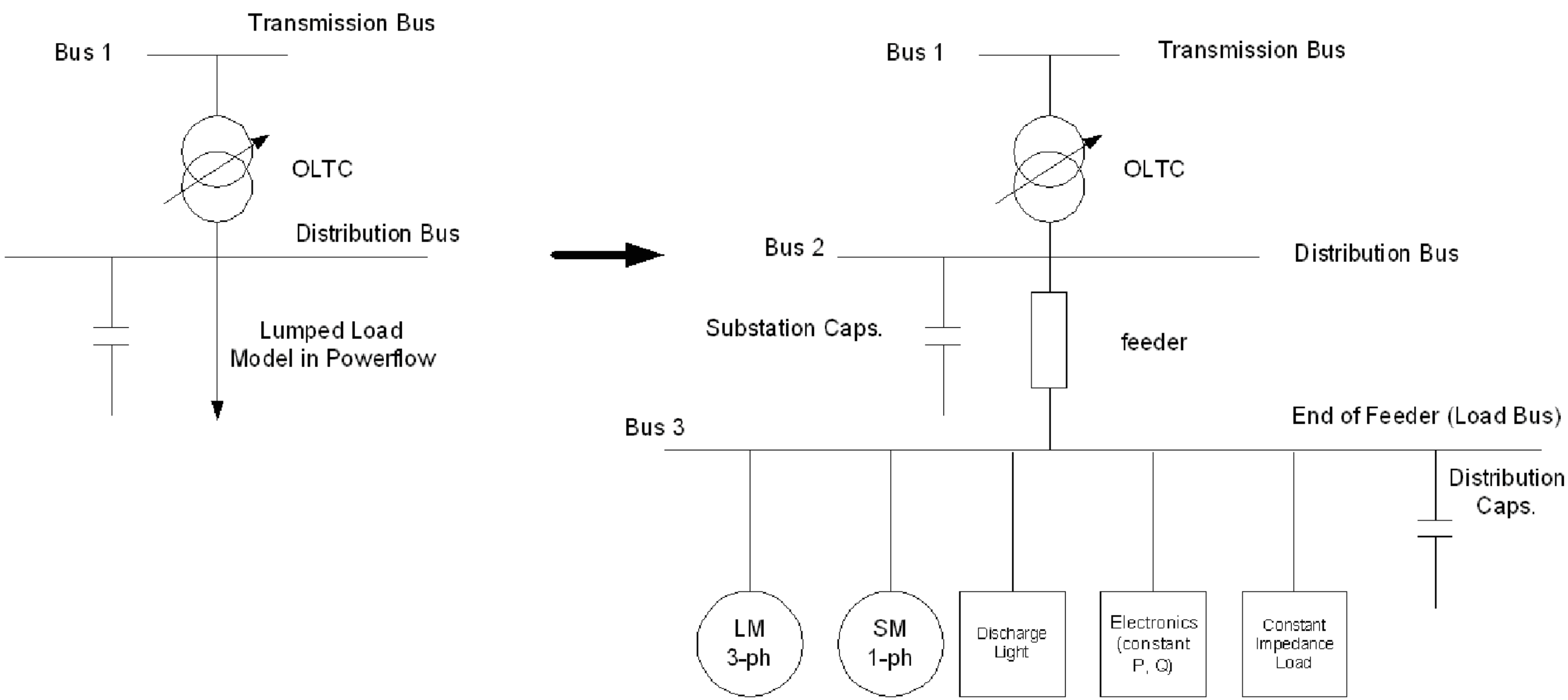
Detailed load modelling

- Member utilities of the Western Electricity Coordinating Council (WECC) face difficulties with delayed voltage recovery.



WECC load model

- The WECC load model task force has proposed the following elaborate model.

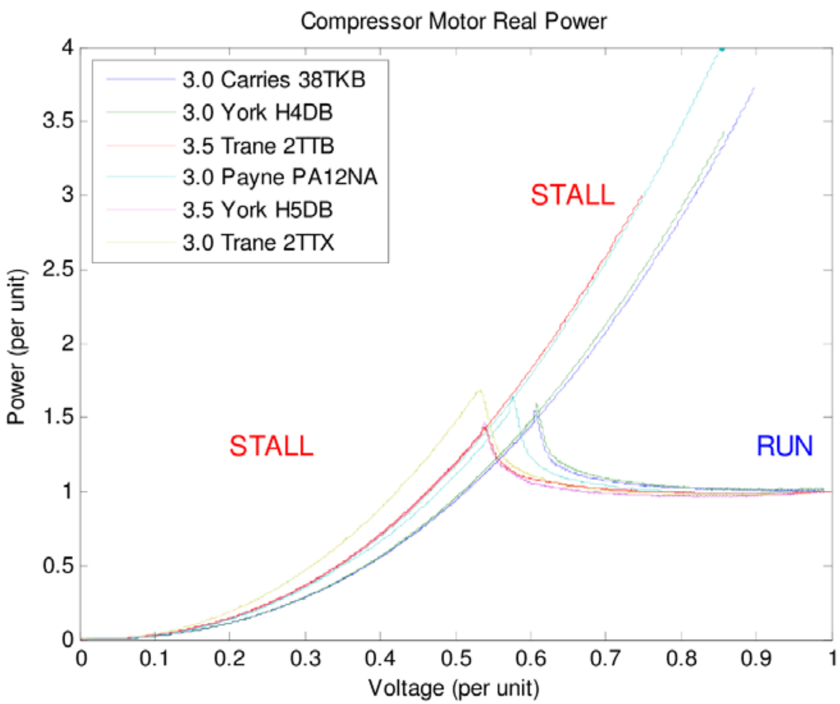


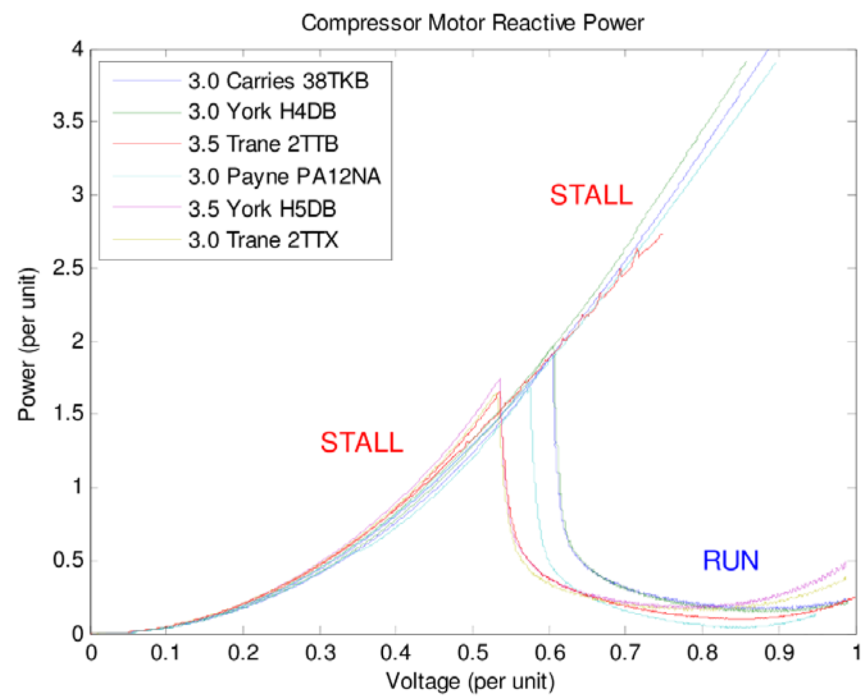
WECC load model components

- Distribution system impedance.
  - As motor loads start to draw high current, the voltage seen by the loads will be lower than the supply point voltage.
- Distribution capacitors.
  - The reactive support drops with the square of the voltage seen down the distribution feeder. This response is different from most loads, so the capacitors should be included separately.
- Distributed generation can also be included on bus~3 of this model.
- Air conditioning motor load.
  - In summer, this can be a significant component of the total load.
  - Most residential air conditioners are single phase induction motors, which behave quite differently to three phase motors.
  - These motors stall in 3-5 cycles, and then draw significant current, eventually tripping on thermal protection.

Residential AC tests

- The following plots were obtained by ramping voltage down to zero, and then ramping back up.





From WECC Load Modeling Task Force.

Load composition

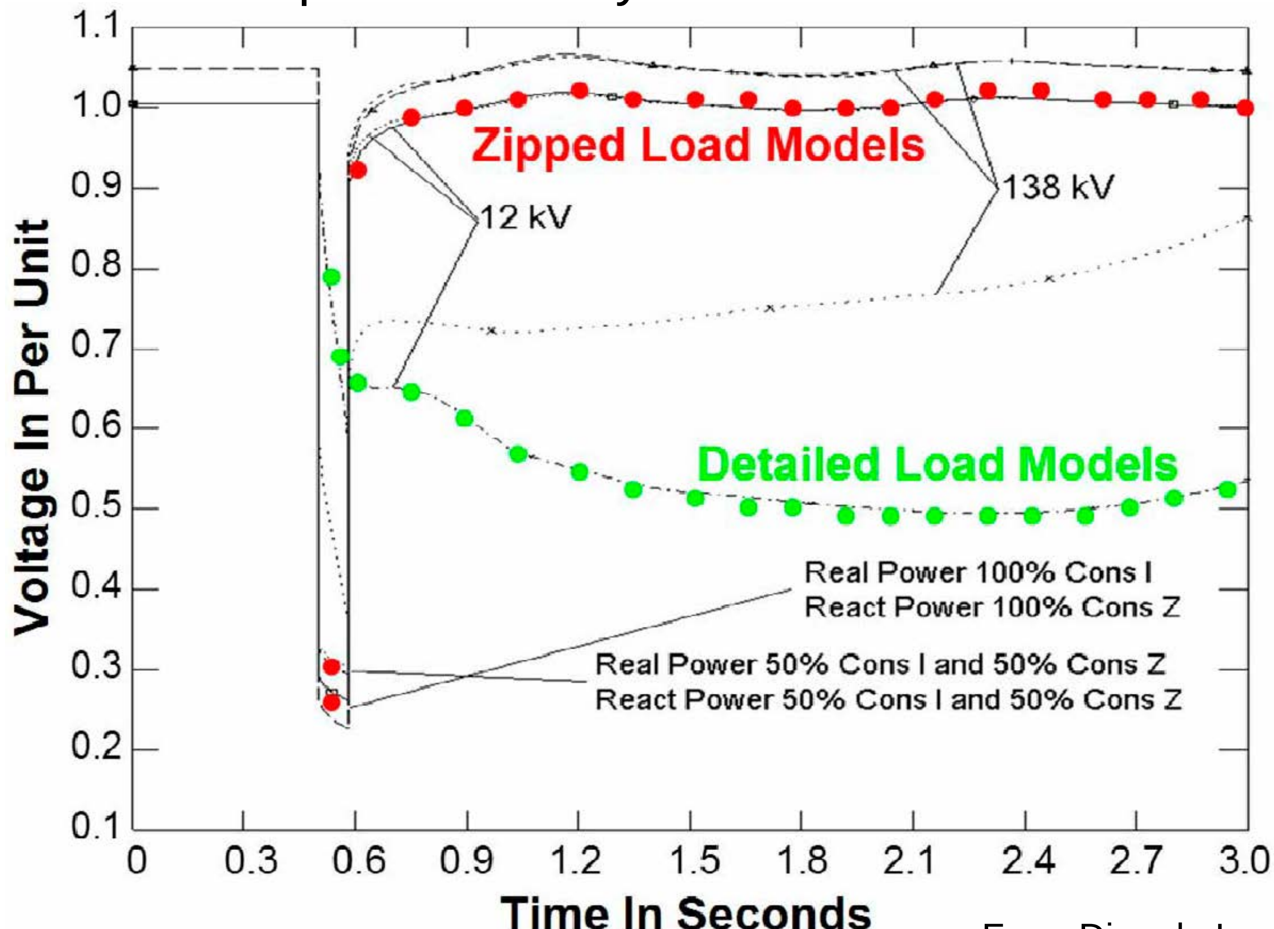
- For a place like Wisconsin, the load contributions during a typical summer peak are given in the following table.

Percentage of: / Customer class:	Large Motor	Small Motor	Discharge Lighting	XFMR Saturation	Constant Power	Remaining
Residential	0	64.4	3.7	1.0	4.1	26.8
Agriculture	10.0	45.0	20.0	1.0	4.5	19.5
Commercial	0	46.7	41.5	1.0	4.5	6.3
Industrial	65.0	15.0	10.0	1.0	5.0	4.0
Power Factor	88.7%	82.0%	92.8%	0%	90.0%	Calculated

LOAD BREAKDOWN BASED UPON LITERATURE REVIEW AND HEURISTICS
From WPS/PTI

Load model comparison (1)

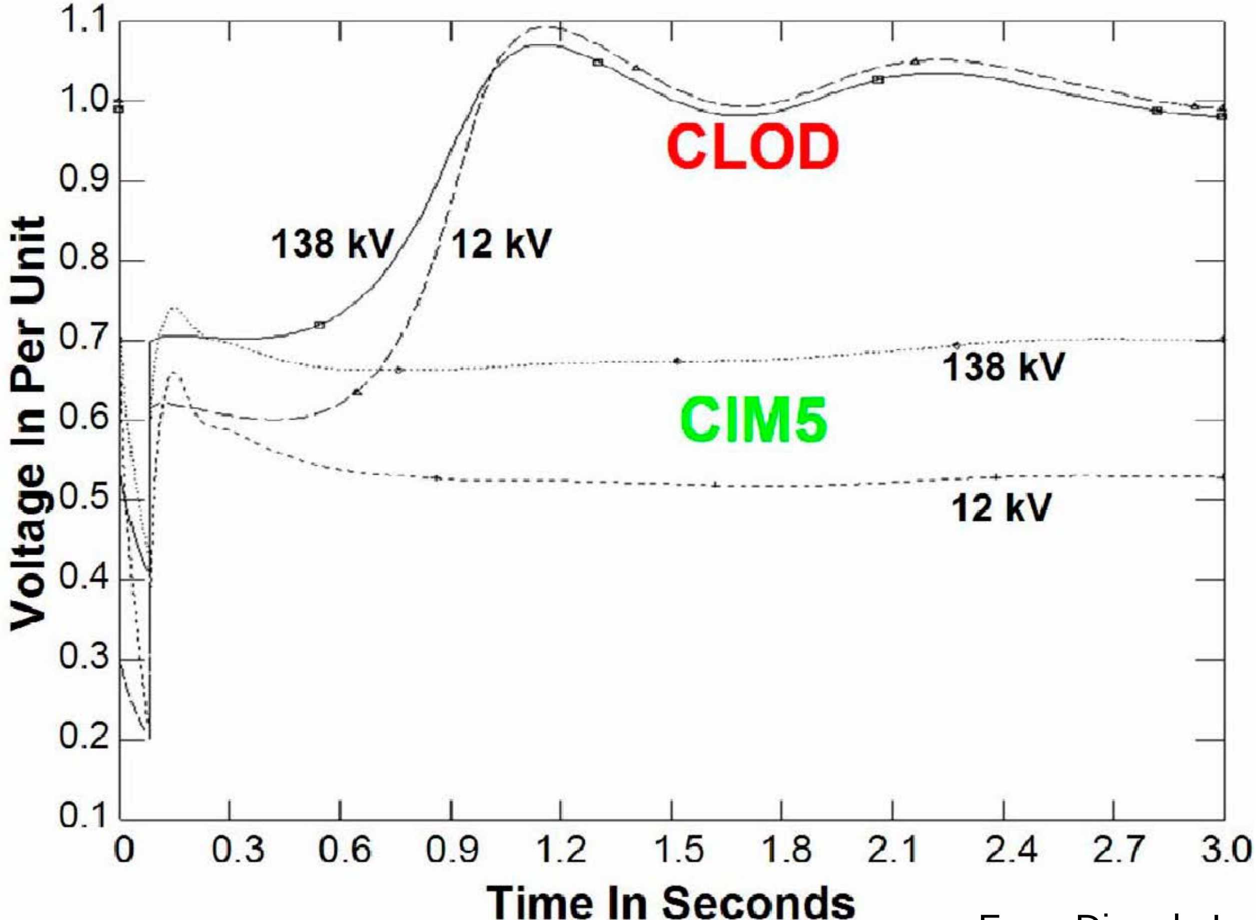
- The ZIP (constant impedance plus constant current plus constant power) voltage-dependent load model and the detailed model behave quite differently.



From Diaz de Leon and Kehrli, 2006.

Load model comparison (2)

- Different motor models may exhibit quite different responses.
  - For example, the two motor models CLOD and CIM5 that are available within PSS/E behave very differently.

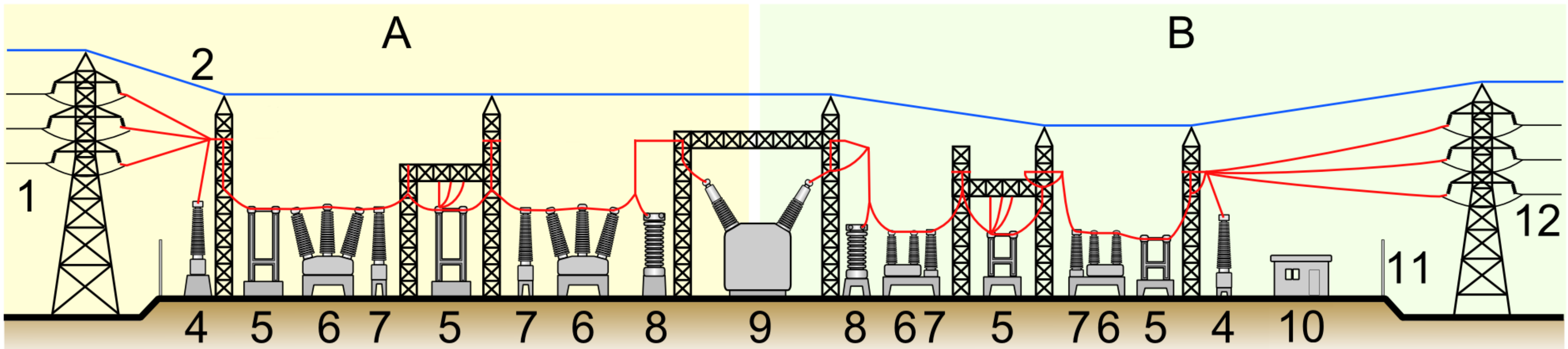


From Diaz de Leon and Kehrli, 2006.

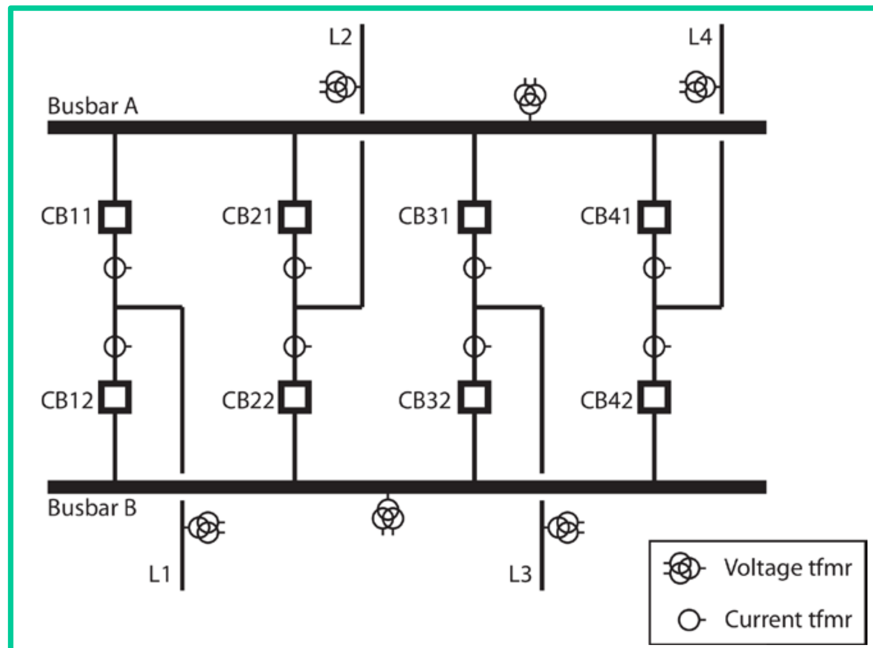
Structure

1. Power Flow
2. Generator modelling
3. Generator control
4. Load modelling
5. **Substations**
6. Protection

Typical substation layout



Equipment:
2 – Overhead earth wire
4 – Voltage transformer
5 – Disconnect switch
6 – Circuit breaker
7 – Current transformer
8 – Lightning arrester
9 – Power transformer



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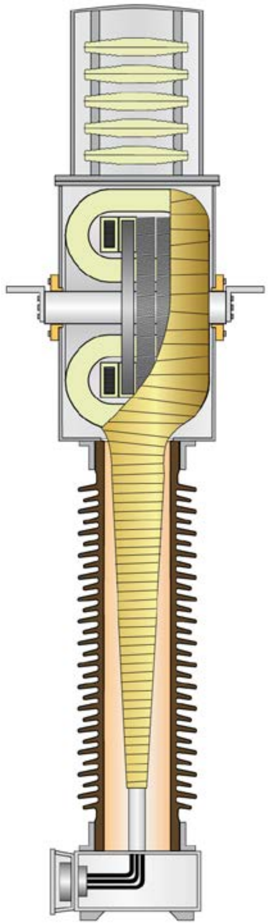
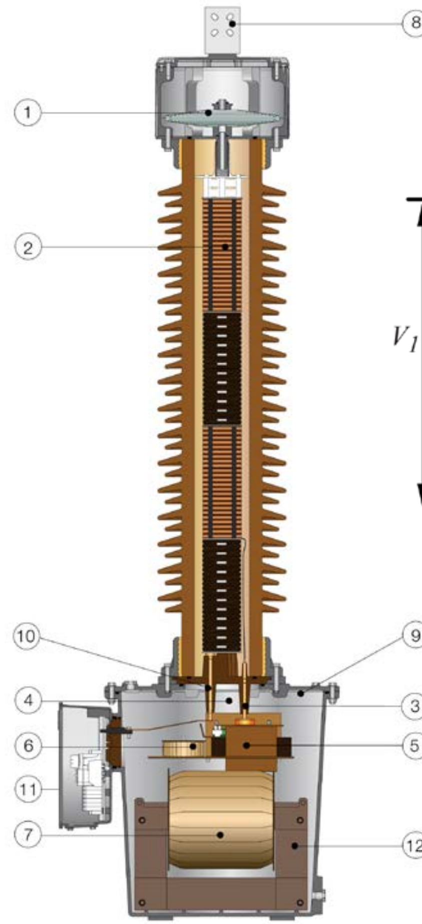
Sensing equipment

Hair-pin/tank CT

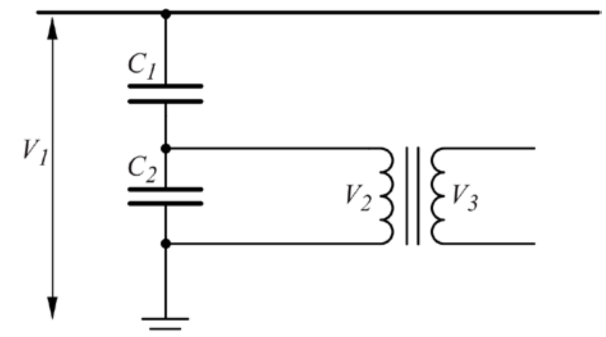


Top-core CT

Current transformers



Capacitor voltage transformers (CVT)



CVT equivalent circuit

From ABB Instrument transformer application guide.

Structure

1. Power Flow
2. Generator modelling
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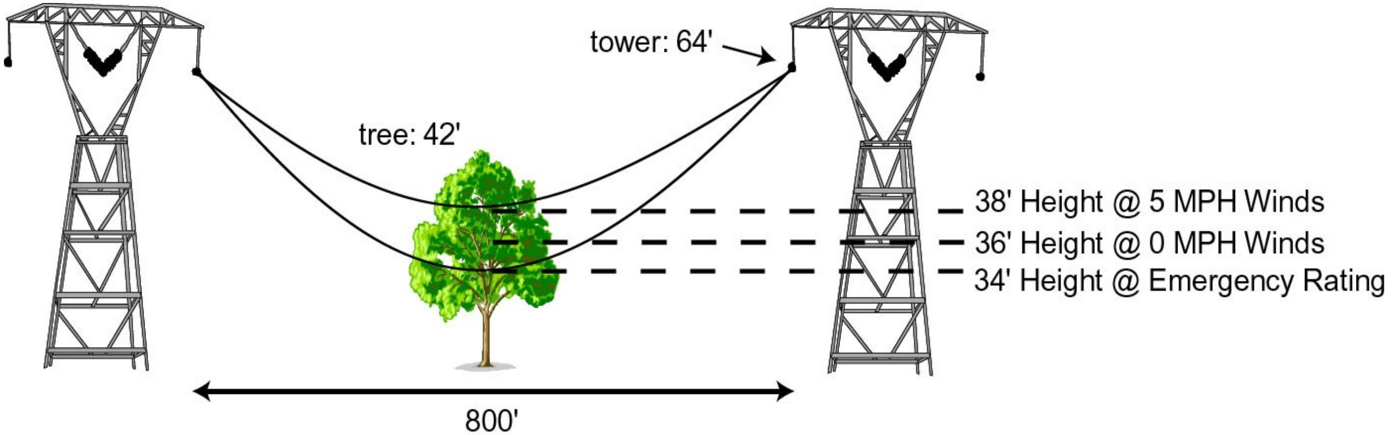
6. **Protection**

Faults on power systems

- Most unplanned transmission outages result from people in substations.
- Lightning strikes are the cause of most transmission system faults.
- Lack of vegetation management also contributes to line outages.
  - Example: August 2003 blackout of north-east America.



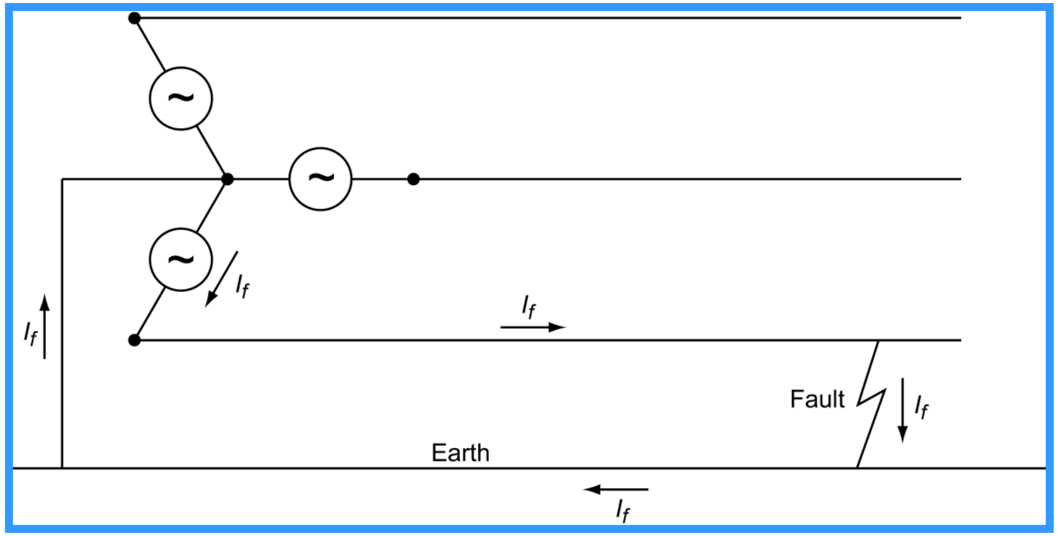
Faults on power systems (cont)

- Other causes of transmission system faults are many and varied:
  - Animals climbing towers.
  - Insulators used for target practice.
  - Ice storms.
  - Dust accumulation on insulators, leading to flashovers.
  - Etc.

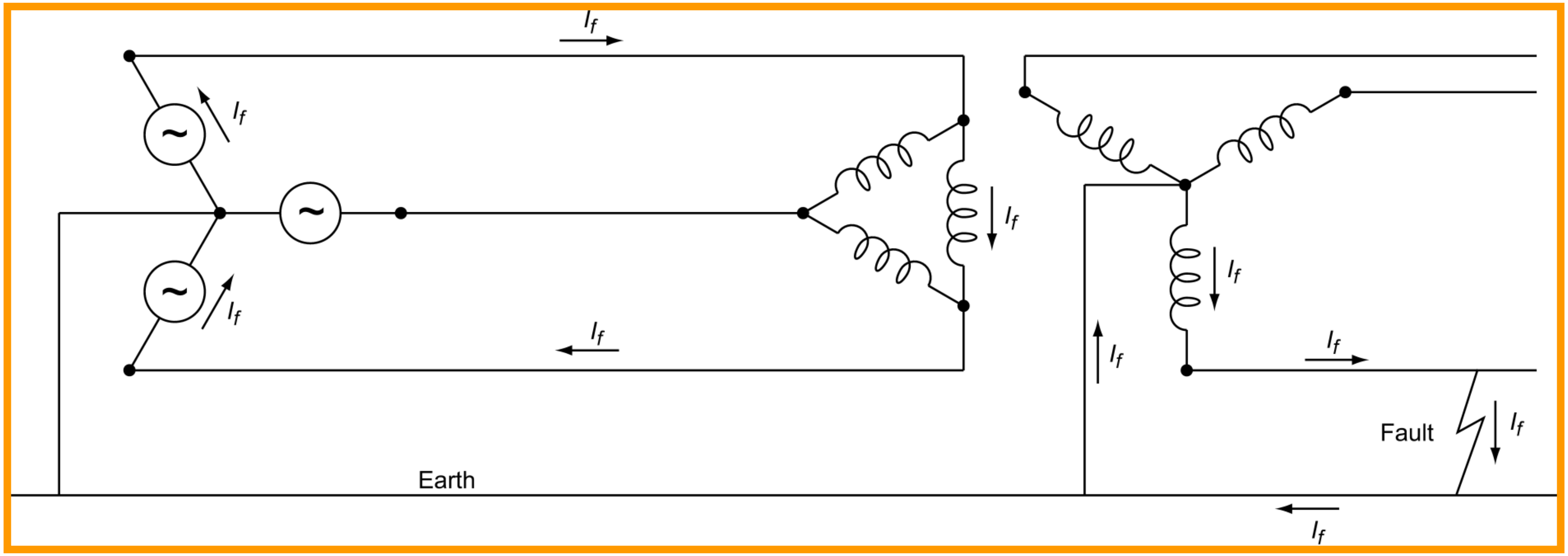
- Most faults are single-line-to-ground.
  - Generally least severe.
- Solid three-phase-to-ground faults are the most severe, but quite uncommon.

Fault current paths

Current flow for a single line to ground fault.



Impact of a Y-Δ transformer on fault current flow.

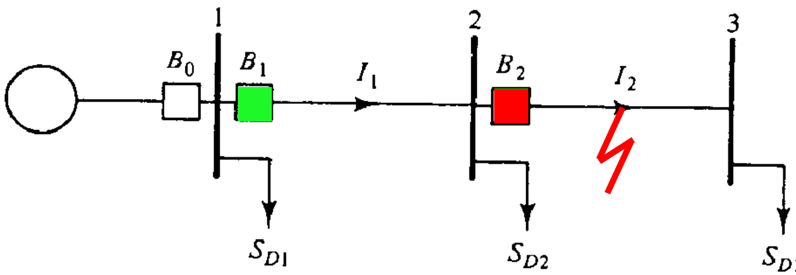


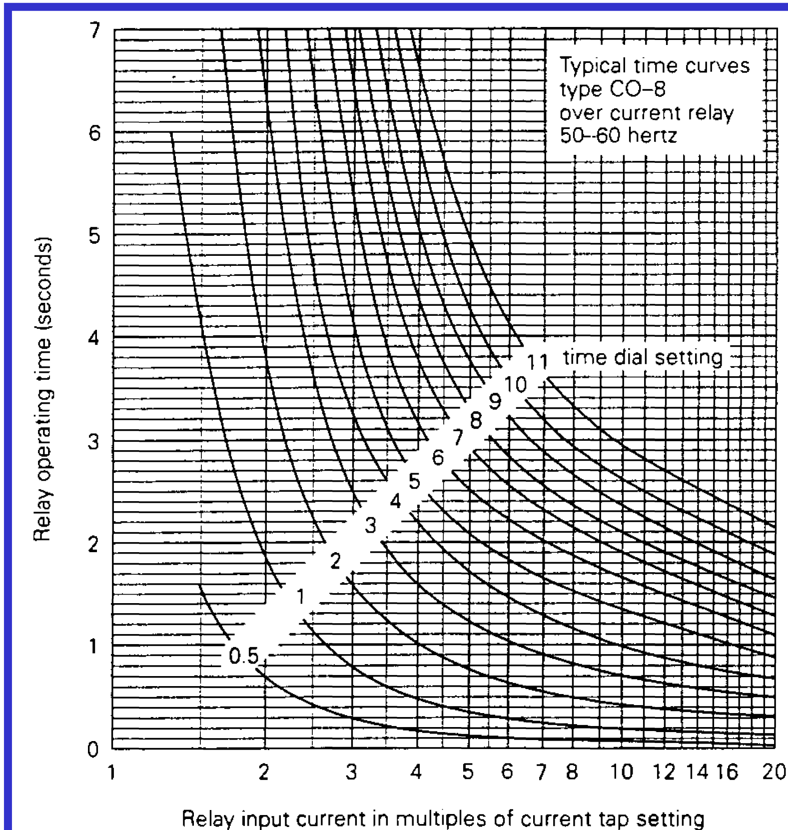
Protective relaying principles

1. Protective systems must be selective.
   - Remove the fault as quickly as possible.
   - Remove no more of the system than is absolutely necessary.
2. Faults can occur in the protection equipment and circuit breakers, so all protection must be backed up.
3. Absolutely every piece of a power system must be covered by protection.
   - If it can't be protected, then it can't be built.

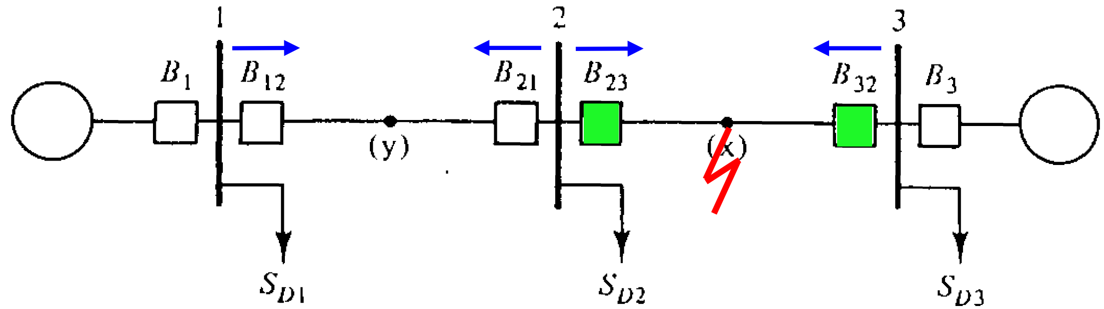
Radial systems

- Simplest system to protect.
- Common to use overcurrent protection.
  - Current flow due to a fault is usually much higher than load current.
- Under fault conditions, the circuit breaker to the left of the fault should operate.
- If a relay/breaker fails to operate, the next breaker up-stream should operate as backup.
  - Backup protection must be slower than primary protection, otherwise they would race to operate.
  - This is called the coordination time.





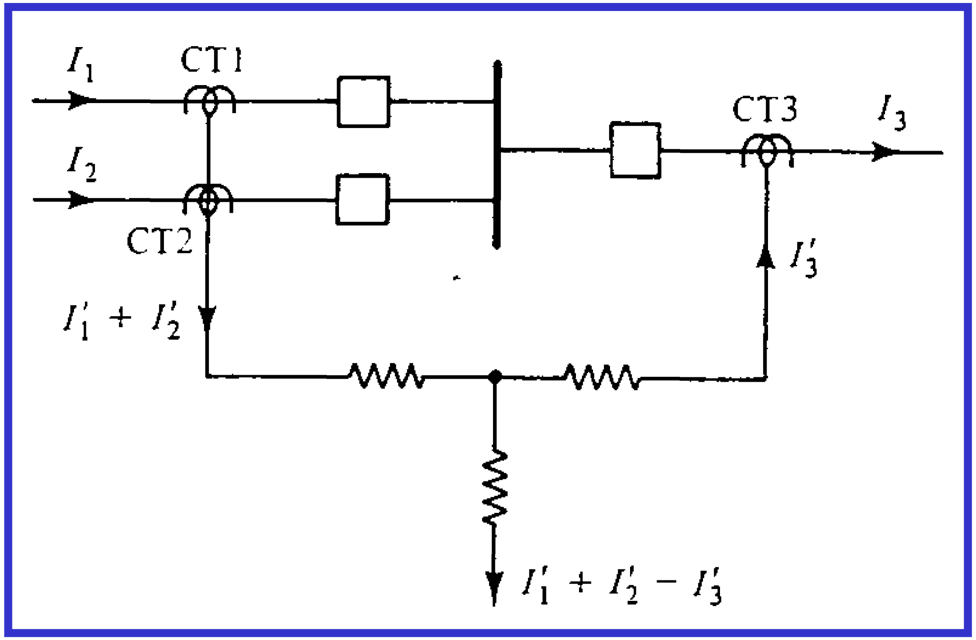
Systems with multiple sources



- This situation arises when distributed generation is added to a radial system.
- Protection must be more sophisticated.
- Consider various fault scenarios:
  - For a fault at point *x*, breakers B_{23} and B_{32} should operate.
  - Assuming radial-system relay coordination, B_{23} would operate faster than B_{21}.
  - But for a fault at point *y*, B_{21} should operate faster than B_{23}.
  - Simple overcurrent relays cannot be coordinated for this case.
- Relays must be directional.
  - They only respond to faults in their "forward" direction.
  - In the example, B_{21} would not see the fault at *x*, and B_{23} would not see the fault at *y*.
  - Backup protection is still provided:
    - B_{12} backs up B_{23}, and B_{32} backs up B_{21}.
- Directionality is achieved by considering the phase angle between bus voltage and fault current.

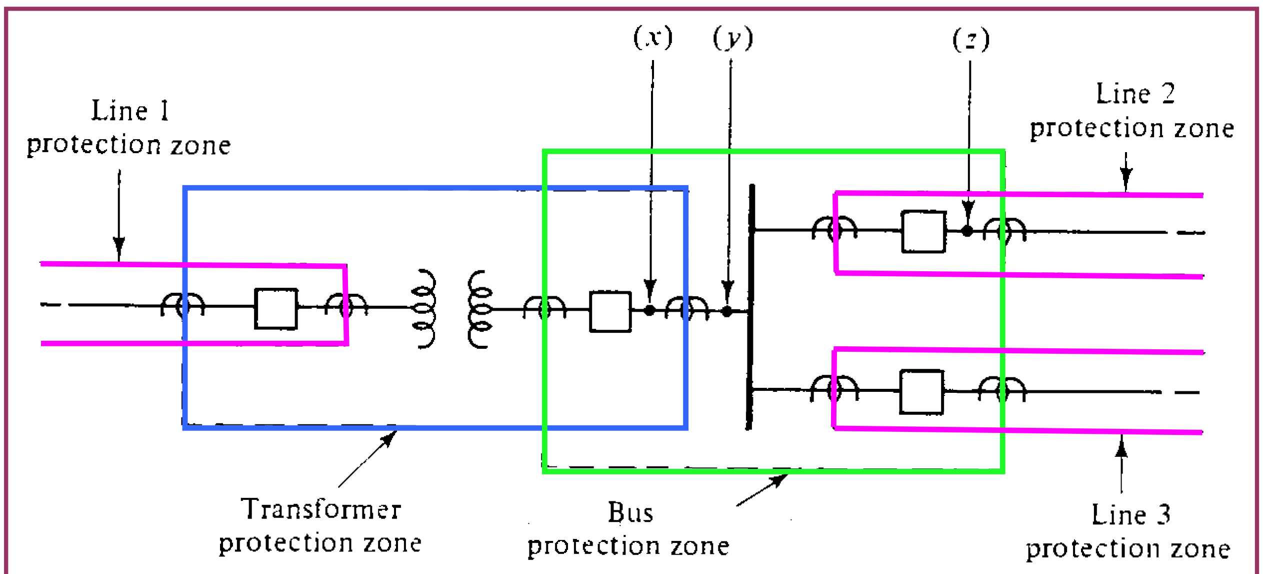
Differential protection

- Fault detection is achieved by summing all the currents into a component, and checking that Kirchhoff's current law holds.
  - If the currents do not add to zero, the mismatch must be due to a fault creating an extraneous path.
- Uses:
  - Generators.
  - Transformers.
    - Must take account of turns ratio, and winding configuration.
  - Buses.
  - Lines and cables.



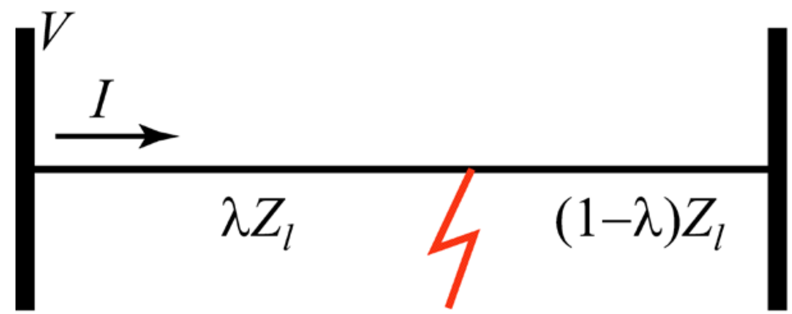
Overlapping protection zones

- Power systems are divided into primary protection zones.
  - Defined by the locations of current transformers (CTs).
  - A fault anywhere in the zone will cause all circuit breakers that bound the zone to open, isolating the fault.
- The arrangement of circuit breakers and CTs ensures that primary protection zones overlap.
  - Every point in the system lies in at least one primary zone.
  - Circuit breakers are also protected.



Distance protection

- During a fault, the voltage drops and the current rises.
- Distance protection monitors the "apparent impedance"



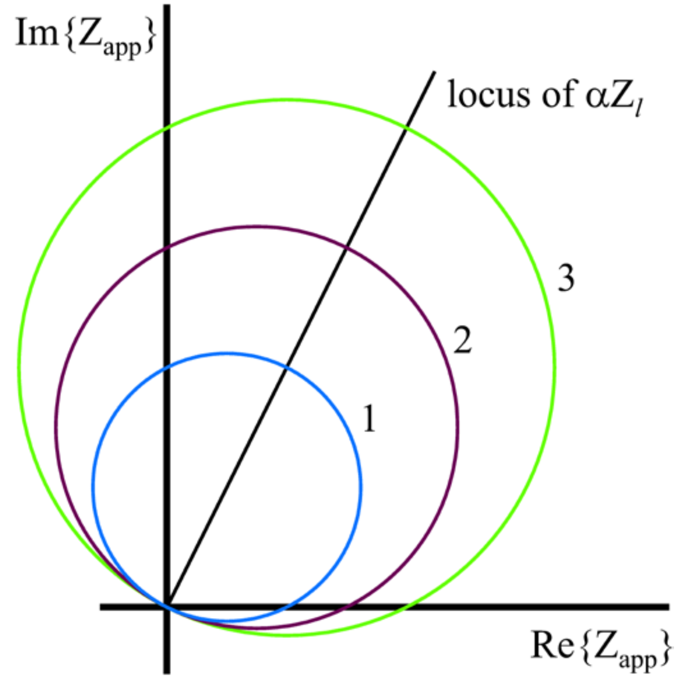
$$Z_{app} = \frac{V}{I} = \lambda Z_l$$

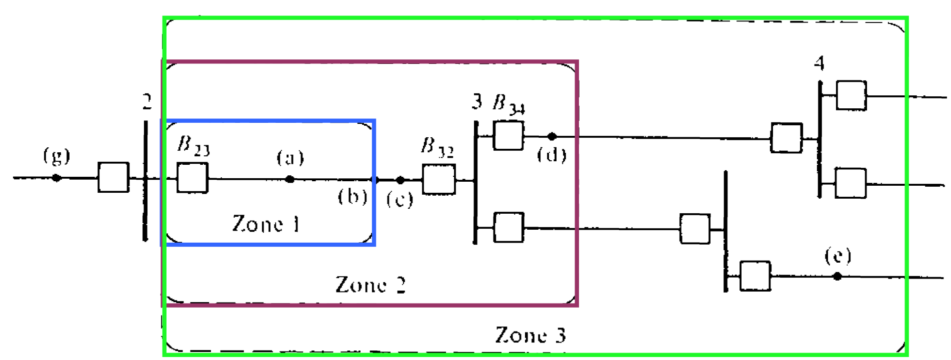
seen looking along a feeder.

  - The "distance" to the fault is given (approximately) by λ.
- Example: If a fault causes the voltage to halve and the current to double.
  - Overcurrent protection sees a 2:1 change in current.
  - Distance protection sees a 4:1 change in apparent impedance.

Distance protection (cont)

- Distance protection operation depends on Z_{app} lying within one of the zone characteristics:
  - Zone 1: Reaches 80% of the line length, $\alpha = 0.8$, instantaneous trip.
  - Zone 2: Reaches 120% of the line length, $\alpha = 1.2$, delayed trip.
  - Zone 3: Reaches 160-200% of the line length, delayed trip.
- Zone 2 and 3 provide backup, in case of primary protection failure.
- The lengthy reach of zone 3 was a major contributing factor in the August 2003 blackout, and has now been disabled by many utilities.





Over/under-voltage protection

- High voltages may cause insulation breakdown, component damage.
- Low voltages are responsible for:
  - Induction motor stalling.
  - Contactors dropping out.
  - Power electronics misfiring.
- Protection operating characteristics often take account of the magnitude and duration of a disturbance.
- Undervoltage load shedding is also used to mitigate system-wide voltage collapse.


Questions?