第五周

叶畅飞　3240103132

A13

$$f_X(x)=\begin{cases}\dfrac{1}{4} & -1<x<3\\ \\ 0 & x\in(-\infty,-1]\vee[3,\infty)\end{cases}$$

$$P\{Y=k\}=C_n^k\left(\frac{3}{4}\right)^k\left(\frac{1}{4}\right)^{n-k},k=0,1,2,...,n$$

B17

(1) 由

$$1=\int_{-\infty}^{+\infty}f_X(x)\mathrm{d}x=\int_0^2c(4-x^2)\mathrm{d}x=\frac{16}{3}c$$

可得

$$c=\frac{3}{16}$$

(2)

$$F(x)=\begin{cases}0 & x\le0\\ \\ \dfrac{3}{4}x-\dfrac{1}{16}x^3 & 0<x\le2\\ \\ 1 & x>2\end{cases}$$

(3)

$$P\{-1<X<1\}=F(1)-F(-1)=\frac{11}{16}$$

(4) 设对 X 独立观察 5 次，{-1 < X < 1}出现的次数为 Y

$$P\{Y=2\}=C_5^2\left(\frac{11}{16}\right)^2\left(\frac{5}{16}\right)^3=\frac{151250}{1048576}=0.144$$

B23

(1)

$$\begin{aligned}P_1&=0.1\times\Phi(1)+0.15\times(\Phi(2)-\Phi(1))+0.3\times(1-\Phi(2)\\&=0.111\end{aligned}$$

(2)

$$P_2=\frac{0.15\times(0.9772-0.8413)+0.3\times(1-\Phi(2))}{P_1}=0.245$$

(3)

$$P_3 = 1 - C_3^0(0.111)^0(0.889)^3 = 0.297$$

B24

(1)

$$\alpha = 0.1 \times \Phi(-0.8) + 0.001 \times (\Phi(0.8) - \Phi(-0.8)) + 0.2 \times (1 - \Phi(0.8)) = 0.0641$$

(2)

$$\beta = \frac{0.2 \times (1 - \Phi(0.8))}{\alpha} = 0.661$$

(3)

$$\theta = C_3^2(0.0641)^1(0.9359)^2 + C_3^3(0.9359)^3 = 0.988$$

B27

(1) 由

$$1 = \int_{-\infty}^{+\infty} ae^{-x^2}\mathrm{d}x = a\sqrt{\pi}$$

可得

$$a = \frac{1}{\sqrt{\pi}}$$

(2)

$$P\left\{X > \frac{1}{2}\right\} = 1 - \Phi\left(\frac{\sqrt{2}}{2}\right) = 0.2398$$

B29

(1)

$$P_1 = 0.4 \times (1 - F_1(6)) + 0.6 \times (1 - F_2(6)) = 0.4e^{-2} + 0.6e^{-1}$$

(2)

$$P_2 = 1 - F_1\left(\frac{2}{3}\right) = 1 - e^{-\frac{2}{9}}$$

(3)

$$\begin{aligned} P_3 &= \frac{0.4 \times P\{X_1 > 1\} + 0.6 \times P\{X_2 > 1\}}{0.4 \times P\left\{X_1 > \frac{1}{3}\right\} + 0.6 \times P\left\{X_2 > \frac{1}{3}\right\}} \\ &= \frac{0.4e^{-\frac{1}{3}} + 0.6e^{-\frac{1}{6}}}{0.4e^{-\frac{1}{9}} + 0.6e^{-\frac{1}{18}}} \end{aligned}$$

B32

$$
\begin{aligned}
P\{Y=10\} &= p^2 = 0.49 \\
P\{Y=8\} &= C_2^1 p^2 (1-p)^1 = 0.294 \\
P\{Y=2\} &= 1 - P\{Y=10\} - P\{Y=8\} = 0.216
\end{aligned}
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