作业 3

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11-1

由

$$I = e\nu = e\frac{v}{2\pi r}$$
$$\frac{e^2}{4\pi\varepsilon_0 r} = m\frac{v^2}{2}$$

解得

$$I = \frac{e^2}{2\pi\sqrt{4\pi\varepsilon_0 m r^3}} = 1.05\times 10^{-3}A$$

11-4

(1)

$$j_{Al} = \frac{4I}{\pi d_1^2} = 2.6\times 10^5 \mathrm{A/m^2}$$
$$j_{Cu} = \frac{4I}{\pi d_2^2} = 5.1\times 10^5 \mathrm{A/m^2}$$

(2)

$$v_d = \frac{j}{ne} = \frac{j_{Cu}M}{eN_A\rho} = 3.65\times 10^{-5}\mathrm{m/s}$$

12-3

(1)

$$B_P = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2) = \frac{\mu_0 I}{4\pi a}\left(\frac{\frac{L}{2}}{\sqrt{a^2+\frac{L^2}{4}}} + \frac{\frac{L}{2}}{\sqrt{a^2+\frac{L^2}{4}}}\right) = \frac{\mu_0 IL}{2\pi a\sqrt{4a^2+L^2}}$$

(2)

$$B_Q = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2) = \frac{\mu_0 I}{4\pi a}\left(\frac{L}{\sqrt{a^2+L^2}} - 0\right) = \frac{\mu_0 IL}{4\pi a\sqrt{a^2+L^2}}$$

12-8

$$\mathrm{d}I = \frac{I}{\pi R}\cdot R\mathrm{d}\theta = \frac{I}{\pi}\mathrm{d}\theta$$
$$\mathrm{d}B = \frac{\mu_0 \mathrm{d}I}{2\pi R} = \frac{\mu_0 I}{2\pi^2 R}\mathrm{d}\theta$$
$$B = \int_0^\pi \mathrm{d}B_x = \int_0^\pi \frac{\mu_0 I}{2\pi^2 R}\sin\theta\mathrm{d}\theta = \frac{\mu_0 I}{\pi^2 R} = 6.37\times 10^{-5}T$$

12-9

取半径为ρ, 径向宽度为 $\mathrm{d}\rho$的同心圆电流元

$$\mathrm{d}I = \frac{NI}{R-r} \cdot \mathrm{d}\rho$$

其在圆心处产生的磁场为

$$\mathrm{d}B = \frac{\mu_0 \mathrm{d}I}{2\rho} = \frac{\mu_0 NI}{2(R-r)} \cdot \frac{\mathrm{d}\rho}{\rho}$$

则

$$B = \int_r^R \mathrm{d}B = \frac{\mu_0 NI}{2(R-r)} \int_r^R \frac{\mathrm{d}\rho}{\rho} = \frac{\mu_0 NI}{2(R-r)} \ln \frac{R}{r}$$

12-11

取半径为r, 径向宽度为 $\mathrm{d}r$的同心圆

$$\mathrm{d}q = \sigma \cdot 2\pi r \mathrm{d}r$$
$$\mathrm{d}I = \frac{\omega r}{2\pi r} \cdot \mathrm{d}q$$

其在点 P 处产生的磁场为

$$\mathrm{d}B = \frac{\mu_0 \mathrm{d}I r^2}{2(r^2+x^2)^{\frac{3}{2}}}$$

则

$$B = \int_0^a \mathrm{d}B_x = \frac{\mu_0 \omega \sigma}{2} \int_0^a \frac{r^3}{(r^2+x^2)^{\frac{3}{2}}} \mathrm{d}r = \frac{\mu_0 \omega \sigma}{2} \left(\frac{2x^2+a^2}{\sqrt{a^2+x^2}} - 2x \right)$$

12-13

取无限长直电流元

$$\mathrm{d}I = \frac{I}{a \sin\theta} \mathrm{d}r$$
$$\mathrm{d}B = \frac{\mu_0 \mathrm{d}I}{2\pi r}$$
$$B = \int_{a\sin\theta}^{2a\sin\theta} \mathrm{d}B = \int_{a\sin\theta}^{2a\sin\theta} \frac{\mu_0 I}{2\pi a} \frac{\mathrm{d}r}{r} = \frac{\mu_0 I}{2\pi a} \ln 2$$

12-16

$$\oint_L B \cdot \mathrm{d}l = \mu_0 \sum I$$
$$B \cdot 2\pi r = \mu_0 \frac{I}{\pi R^2} \pi r^2 \Rightarrow B = \frac{\mu_0 I r}{2\pi R^2}$$

在 S 上取宽度为$\mathrm{d}r$，长为l的平行轴线元

$$\mathrm{d}S = l \cdot \mathrm{d}r, \qquad d\Phi = B\mathrm{d}S$$

$$S = \int_0^R B\mathrm{d}S = \int_0^R \frac{\mu_0 Il}{2\pi R^2} r\mathrm{d}r = \frac{\mu_0 Il}{4\pi}$$

12-17

(1)

$$B_1 \cdot 2\pi r = \mu_0 \frac{I}{\pi a^2} \cdot \pi r^2 \Rightarrow B_1 = \frac{\mu_0 Ir}{2\pi a^2}$$

(2)

$$B_2 \cdot 2\pi r = \mu_0 I \Rightarrow B_2 = \frac{\mu_0 I}{2\pi r}$$

(3)

$$B_3 \cdot 2\pi r = \mu_0 \left[I - \frac{I}{\pi(c^2 - b^2) \cdot \pi(r^2 - b^2)} \right] \Rightarrow B_3 = \frac{\mu_0 I}{2\pi} \left(\frac{1}{r} - \frac{r^2 - b^2}{c^2 - b^2} \right)$$

(4)

$$B_4 \cdot 2\pi r = 0 \Rightarrow B_4 = 0$$

12-20

$$\oint B \cdot \mathrm{d}l = \mu_0 \sum I$$

$$B \cdot \overline{ab} + B \cdot \overline{bc} \cos 90^\circ + B \cdot \overline{cd} + B \cdot \overline{da} \cos 90^\circ = \mu_0 j \overline{ab}$$

$$\Rightarrow B = \frac{\mu_0 j}{2}$$

于是

$$B_P = B_1 - B_2 = \frac{\mu_0 j_1}{2} - \frac{\mu_0 j_2}{2}$$

$$B_Q = B_1 + B_2 = \frac{\mu_0 j_1}{2} + \frac{\mu_0 j_2}{2}$$

12-26

$$\mathrm{d}I = \mathrm{d}x\boldsymbol{i} + \mathrm{d}y\boldsymbol{j}, \qquad B = \frac{\mu_0 I}{2\pi x}\boldsymbol{k}$$

$$\mathrm{d}F = I_2 \mathrm{d}l \times B = -\frac{\mu_0 I_1 I_2}{2\pi x}\mathrm{d}y\boldsymbol{i} + \frac{\mu_0 I_1 I_2}{2\pi x}\mathrm{d}x\boldsymbol{j}$$

$$F_{AB} = \int \mathrm{d}F_x = \int_0^a -\frac{\mu_0 I_1 I_2}{2\pi \left(b - \frac{\sqrt{3}}{6}a \right) \mathrm{d}y} = -\frac{\mu_0 I_1 I_2 a}{2\pi \left(b - \frac{\sqrt{3}}{6}a \right)}$$

AC 与 BC 在 y 方向的分力相互抵消，在 x 方向的分力相加，得到

$$F_x = 2\int \mathrm{d}F_x = \int_{\frac{a}{2}}^0 -\frac{\mu_0 I_1 I_2}{2\pi(b + \frac{\sqrt{3}}{3}a - \sqrt{3}y})\mathrm{d}y = \frac{\mu_0 I_1 I_2}{\pi\sqrt{3}} \ln \left(\frac{b + \frac{\sqrt{3}}{3}a}{b - \frac{\sqrt{3}}{6}a} \right)$$

$$F = F_{AB} + F_x = \frac{\mu_0 I_1 I_2}{\pi\sqrt{3}} \ln\left(\frac{b + \frac{\sqrt{3}}{3}a}{b - \frac{\sqrt{3}}{6}a}\right) - \frac{\mu_0 I_1 I_2 a}{2\pi\left(b - \frac{\sqrt{3}}{6}a\right)}$$

12-28

$$\begin{aligned} M_{磁} &= P_m B \sin\theta = NIl2RB\sin\theta \\ M_{重} &= mgR\sin\theta \\ M_{磁} &= M_{重} \\ I &= 2.45A \end{aligned}$$

12-30

(1)

$$B_p = \frac{1}{2}\frac{\mu_0 I}{2a} + \frac{1}{2}\frac{\mu_0 I}{2b} = \frac{\mu_0 I}{4}\frac{a+b}{ab}$$

(2)

$$P_m = \frac{1}{2}\pi a^2 I + \frac{1}{2}\pi b^2 I = \frac{1}{2}\pi I(a^2 + b^2)$$

12-31

$$\begin{aligned} \sigma &= \frac{q}{\pi(R_2^2 - R_1^2)} \\ \mathrm{d}I &= \mathrm{d}q \cdot n\omega\sigma r \mathrm{d}r \\ \mathrm{d}P_m &= S\mathrm{d}I = \pi\omega\sigma r^3 \mathrm{d}r \\ P_m &= \int_{R_1}^{R_2} \mathrm{d}P_m = \frac{\omega q}{4}(R_1^2 + R_2^2) \\ M &= P_m B \sin a = \frac{\omega q B}{4}(R_1^2 + R_2^2)\sin a \end{aligned}$$