作业 2

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10-6

(1)

$$E = \begin{cases} \frac{Q+q}{4\pi\varepsilon_0 r^2} & r > R_3 \\ \frac{q}{4\pi\varepsilon_0 r^2} & R_1 < r < R_2 \\ 0 \end{cases}$$

$$U_1 = \frac{Q+q}{4\pi\varepsilon_0 R_3} + \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$U_2 = \frac{Q+q}{4\pi\varepsilon_0 R_3}$$

(2) 若用导线将两个导体球连接，则电荷将全部分布在球壳外表面：

$$U_1 = U_2 = \frac{Q+q}{4\pi\varepsilon_0 R_3}$$

(3) 内球接地则 $U_1 = 0$，此时设内球电荷为 q'，则有：

$$U_1 = \frac{Q+q'}{4\pi\varepsilon_0 R_3} + \frac{q'}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0$$
$$\Rightarrow q' = \frac{QR_1R_2}{R_1R_2 + R_2R_3 - R_1R_3}$$

$$U_2 = -\int_{R_1}^{R_2} \frac{q'}{4\pi\varepsilon_0 r^2} dr = \frac{q'}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$= \frac{Q(R_2 - R_1)}{4\pi\varepsilon_0(R_1R_2 + R_2R_3 - R_1R_3)}$$

10-11

(1)

$U_1 = \frac{q_1}{4\pi\varepsilon_0 a}$，$U_2 = \frac{q_2}{4\pi\varepsilon_0 b}$，两球由细导线相连可知 $U_1 = U_2$ 且 $q_1 + q_2 = Q$，于是有：

$$q_1 = \frac{aQ}{a+b} \qquad q_2 = \frac{bQ}{a+b}$$

(2)

$$C = \frac{Q}{U_1} = 4\pi\varepsilon_0(a+b)$$

10-13

设 A 的电荷线密度为 λ， B 的电荷线密度为 $-\lambda$，则 AB 间距 AB 间距为 x 处的电场强度为：

$$E=\frac{\lambda}{2\pi\varepsilon_0 x}+\frac{\lambda}{2\pi\varepsilon_0(d-x)}=\frac{\lambda d}{2\pi\varepsilon_0 x(d-x)}$$

$$U_{AB}=\int_r^{d-r}E\mathrm{d}x=\frac{\lambda}{2\pi\varepsilon_0}\int_r^{d-r}\left(\frac{1}{x}+\frac{1}{d-x}\right)\mathrm{d}x$$

$$=\frac{\lambda}{\pi\varepsilon_0}\ln\left(\frac{d-r}{r}\right)\approx\frac{\lambda}{\pi\varepsilon_0}\ln\left(\frac{d}{r}\right)$$

$$C=\frac{\lambda}{U_{AB}}=\frac{\pi\varepsilon_0}{\ln(\frac{d}{r})}$$

10-14

设内球电荷为 q，则有：

$$U=\int_b^a\left(\frac{q}{4\pi\varepsilon_0 r^2}\right)\mathrm{d}r=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

内球表面附近的电场强度为：

$$E=\frac{q}{4\pi\varepsilon_0 a^2}=\frac{U}{a(1-\frac{a}{b})}$$

显然，$a=\dfrac{b}{2}$ 时，E 取最小值 $E=\dfrac{4U}{b}$。

10-15

$\varepsilon_0=8.85\times10^{-12}C^2/(Nm^2)$

(1)

$$E=\frac{U}{d}=5\times10^3V/m$$

$$D_1=\varepsilon_0\varepsilon_{r1}E=1.77\times10^{-7}C/m^2$$

$$D_2=\varepsilon_0\varepsilon_{r2}E=2.655\times10^{-7}C/m^2$$

(2) 由(1)可知，$E=5\times10^3V/m$

(3)

$$P_1=\varepsilon_0(\varepsilon_{r1}-1)E=1.3275\times10^{-7}C/m^2$$

$$P_2=\varepsilon_0(\varepsilon_{r2}-1)E=2.2125\times10^{-7}C/m^2$$

10-18

设两圆柱导体沿轴线方向上的单位长度的电荷量为 λ，则有：

$$D = \frac{\lambda}{2\pi r} \qquad E = \frac{D}{\varepsilon} = \frac{\lambda}{2\pi\varepsilon r}$$

则最大电场强度出现在 $r = R_1$ 处，于是$\lambda_M = 2\pi\varepsilon R_1 E_M$

于是：

$$\begin{aligned} U_M &= \int_{R_1}^{R_2} E\mathrm{d}r = \frac{\lambda_M}{2\pi\varepsilon}\ln\left(\frac{R_2}{R_1}\right) = R_1 E_M \ln\left(\frac{R_2}{R_1}\right) \\ &= 2cm \times 2 \times 10^5 V/cm \times \ln 2 = 2.77 \times 10^5 V \end{aligned}$$

10-19

(1)

$r < R, D = 0, E = 0$

$R < r < a, D = \dfrac{\lambda}{4\pi r^2}, E = \dfrac{D}{\varepsilon} = \dfrac{\lambda}{4\pi\varepsilon_0 r^2}$

$a < r < b, D = \dfrac{\lambda}{4\pi r^2}, E = \dfrac{D}{\varepsilon} = \dfrac{\lambda}{4\pi\varepsilon_0\varepsilon_r r^2}$

$r > a, D = \dfrac{\lambda}{4\pi r^2}, E = \dfrac{D}{\varepsilon} = \dfrac{\lambda}{4\pi\varepsilon_0 r^2}$

(2)

$$P = D - \varepsilon_0 E = \frac{\lambda}{4\pi r^2}\left(1 - \frac{1}{\varepsilon_r}\right)$$

$$\text{介电质内表面}\sigma_a' = \frac{\lambda}{4\pi a^2}\left(1 - \frac{1}{\varepsilon_r}\right), \text{介电质外表面}\sigma_b' = \frac{\lambda}{4\pi b^2}\left(1 - \frac{1}{\varepsilon_r}\right)$$

10-23

(1) 设平行板电容器的电荷面密度为 σ，则有 $D = \sigma$，$E = \dfrac{D}{\varepsilon}$，因此：

$$\begin{aligned} U &= E_1 d_1 + E_2 d_2 = \frac{\sigma}{\varepsilon_0\varepsilon_{r1}} d_1 + \frac{\sigma}{\varepsilon_0\varepsilon_{r2}} d_2 \\ &\Rightarrow \sigma = 8.85 \times 10^{-7} C/m^2 = \mathbb{D} \end{aligned}$$

于是

$$\omega_1 = \frac{1}{2} D E_1 = \frac{\sigma^2}{2\varepsilon_0\varepsilon_{r1}} = 1.10625 \times 10^{-2} J/m^2$$

$$\omega_2 = \frac{1}{2} D E_2 = \frac{\sigma^2}{2\varepsilon_0\varepsilon_{r2}} = 2.2125 \times 10^{-2} J/m^2$$

(2)

$$W_1 = \omega_1 S d_1 = 8.85 \times 10^{-8} J$$
$$W_2 = \omega_2 S d_2 = 2.655 \times 10^{-7} J$$

10-26

$$D=\frac{\lambda}{2\pi r}\qquad E=\frac{D}{\varepsilon}=\frac{\lambda}{2\pi\varepsilon r}$$

$$\omega=\frac{1}{2}DE=\frac{\lambda^2}{8\varepsilon_0\varepsilon_r\pi^2r^2}$$

$$W=\int_a^b\omega\cdot2\pi r\mathrm{d}r=\frac{\lambda^2}{4\pi\varepsilon_0\varepsilon_r}\ln\left(\frac{b}{a}\right)$$

10-28

(1)

金属板抽出前$C_1=\frac{\varepsilon_0S}{d-t}$，抽出后$C_2=\frac{\varepsilon_0S}{d}$，外力做功:

$$\begin{aligned}A=W_2-W_1&=\frac{1}{2}\frac{Q^2}{C_2}-\frac{1}{2}\frac{Q^2}{C_1}=\frac{\varepsilon_0StU_1^2}{2(d-t)^2}\\&=\frac{8.85\times10^{-12}C^2/(Nm^2)\times3\times10^{-2}m\times10^{-3}m\times(600V)^2}{2\times(3\times10^{-3}m-10^{-3}m)^2}\\&=1.19475\times10^{-5}J\end{aligned}$$

(2)

$$E=\frac{\sigma}{2\varepsilon_0}\qquad Q=\frac{\varepsilon_0S}{d-t}U_1$$

$$F=EQ=\frac{\varepsilon_0SU_1^2}{2(d-t)^2}$$