第 8 章 作业

叶畅飞　3240103132

8-3

基点 A 的运动方程

$$\varphi = \frac{1}{2}\alpha t^2$$

$$x_A = (R+r)\cos\varphi = (R+r)\cos\left(\frac{1}{2}\alpha t^2\right)$$

$$y_A = (R+r)\sin\varphi = (R+r)\sin\left(\frac{1}{2}\alpha t^2\right)$$

齿轮绕点 A 的转动方程，设齿轮上一点 K

$$v_K = v_A + v_{KA} = 0$$
$$\Rightarrow \omega_1 r = \alpha t \cdot (R+r)$$
$$\omega_1 = \alpha t \frac{R+r}{r}$$

$$\theta = \int \omega_1 \mathrm{d}t = \int \alpha t \frac{R+r}{r}\mathrm{d}t = \frac{R+r}{2r}\alpha t^2$$

8-10

由速度投影定理，杆 AB 上点 A 和点 B 的速度在杆 AB 上投影相等，即

$$v_A \cos\left(\frac{\pi}{2} - \gamma\right) = v_B$$
$$\Rightarrow v_B = \omega_{O_1} \cdot O_1 A \cdot \sin\gamma = 2.25\sqrt{3}m/s$$

所以

$$\omega_{OB} = \frac{v_B}{r_1 + r_2} = \frac{2.25\sqrt{3}}{0.6\sqrt{3}} = 3.75\mathrm{rad}/s$$

又 A B 与 齿轮 II 固结，故

$$v_B = v_A + \omega_{\mathbb{I}} \cdot AB$$
$$v_B - \omega_{\mathbb{I}} r_1 = \omega_I r_2$$
$$\Rightarrow \omega_I = \frac{v_B - \omega_{\mathbb{I}} r_1}{r_2} = \frac{2.25\sqrt{3} - \frac{2.25}{1.5}\cdot 0.3\sqrt{3}}{0.3\sqrt{3}} = 6\mathrm{rad}/s$$

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显然三角板 A B C 的瞬心为 O A 和 O B 的连线交点 P

$$\omega_{ABC} = \frac{v_A}{AP} = \frac{\omega OA}{2OA \cos 30°} = \frac{2\sqrt{3}}{3} \text{rad}/s$$

$$v_C = \omega_{ABC} \cdot CP = \frac{2\sqrt{3}}{3} m/s$$

又

$$v_C = v_c + v_r$$

$$\frac{v_c}{O_2 C} = \omega_{O_2 D}$$

所以

$$\omega_{O_2 D} = \frac{\sqrt{3}}{3} \text{rad}/s$$

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$$v_1 \cos\theta = v_2 \cos\theta + v_{DA} = v_2 \cos\theta + \omega_{AB} \cdot AD$$

$$\omega_{AB} = (v_1 - v_2)\frac{\cos\theta}{AD} = \frac{(v_1 - v_2)\cos^2\theta}{2R}$$

$$a_D = a_e + a_r + a_C = \frac{v_1^2}{R}$$

$$a_C = 2\omega_e \times v_r = 2\frac{(v_1 - v_2)\cos^2\theta}{2R} \times v_1 \sin\theta = \frac{(v_1 - v_2)^2 \cos^2\theta \sin\theta}{R}$$

$$a_e^t = a_C - a_D \sin\theta = \frac{(v_1 - v_2)^2 \cos^2\theta \sin\theta}{R} - \frac{v_1^2}{R}\sin\theta$$

$$\alpha_{AB} = \frac{a_e^t}{AD} = \frac{\frac{(v_1-v_2)^2 \cos^2\theta \sin\theta}{R} - \frac{v_1^2}{R}\sin\theta}{\frac{2R}{\cos\theta}} = \frac{1}{2R^2}\left[(v_1 - v_2)^2 \cos^2\theta - v_1^2\right]\sin\theta\cos\theta$$

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设两齿轮啮合点为 P,

$$\omega_{AB} = \frac{v_A}{r} = 3\omega_0, \qquad v_B = \omega_{AB} \cdot PB = 3\sqrt{6}r\omega_0$$

不会做