作业 1

叶畅飞 3240103132

9-8

对于半球壳上的任意电荷，其在球心处产生的电场为：

$$d\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}\boldsymbol{e}_r$$

由对称性，电荷产生的场强在垂直于轴向的分量相互抵消，只有沿轴向的分量相加。则有：

$$\begin{aligned}\boldsymbol{E} = \int d\boldsymbol{E} &= \int_0^{\frac{\pi}{2}}\int_0^{2\pi R\sin\theta}\frac{1}{4\pi\varepsilon_0}\frac{\sigma(R\cdot d\theta)dl}{R^2}\cos\theta \\ &= \frac{\sigma}{2\varepsilon_0}\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta d\theta \\ &= \frac{\sigma}{4\varepsilon_0}\end{aligned}$$

9-9

以水平向右为正方向， 对左侧圆环，其在 O 点产生的电场为：

$$\begin{aligned}\boldsymbol{E}_1 = \int d\boldsymbol{E} &= \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}\boldsymbol{e}_r \\ &= -\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{4\pi\varepsilon_0}\frac{\lambda\cdot R\cdot d\theta}{R^2}\cos\theta \\ &= -\frac{\lambda}{2\pi\varepsilon_0 R}\end{aligned}$$

右侧两直导线在 O 点产生的电场为：

$$\begin{aligned}\boldsymbol{E}_2 = \int d\boldsymbol{E} &= \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\boldsymbol{e}_r \\ &= 2\int_0^{\infty}\frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{R^2+l^2}\frac{l}{\sqrt{R^2+l^2}} \\ &= \frac{\lambda}{2\pi\varepsilon_0}\int_0^{\infty}\frac{ldl}{(R^2+l^2)^{\frac{3}{2}}} = \frac{\lambda}{2\pi\varepsilon_0}\int_{\frac{1}{R}}^{0}-d\left(R^2+l^2\right)^{-\frac{1}{2}} \\ &= \frac{\lambda}{2\pi\varepsilon_0 R}\end{aligned}$$

所以 O 点处的合场强为：0

9-25

1)
$$U_o = \frac{q}{4\pi\varepsilon_0 l} + \left(-\frac{q}{4\pi\varepsilon_0 (l)}\right) = 0$$

$$U_c = \frac{q}{4\pi\varepsilon_0 \cdot 3l} + \left(-\frac{q}{4\pi\varepsilon_0 l}\right) = -\frac{q}{6\pi\varepsilon_0 l}$$

$$A_{oa} = q_0(U_c - U_\infty) = \frac{qq_0}{6\pi\varepsilon_0 l}$$

2)
$$A_{c\infty} = -q_0(U_c - U_\infty) = \frac{qq_0}{6\pi\varepsilon_0 l}$$

9-28

a)
$$U_a = \int_{0.3}^{\infty} \frac{Q_1+Q_2}{4\pi\varepsilon_0 r_2^2} dr + \int_{0.2}^{0.3} \frac{Q_1}{4\pi\varepsilon_0 r_1^2 dr}$$
$$= 900V$$

b)
$$U_b = \int_{0.5}^{\infty} \frac{Q_1+Q_2}{4\pi\varepsilon_0 r_2^2} dr$$
$$= 500V$$

9-29

$$U_{AB} = U_{CD} = \int_R^{2R} \frac{\lambda dx}{4\pi\varepsilon_0 x} = \frac{\lambda}{4\pi\varepsilon_0}\ln 2$$

$$U_{\overset{\frown}{BC}} = \int_0^{\pi} \frac{\lambda d\theta}{4\pi\varepsilon_0 R} = \frac{\lambda}{4\varepsilon_0}$$

$$U_O = U_{AB} + U_{\overset{\frown}{BC}} + U_{CD} = \frac{\lambda}{4\pi\varepsilon_0}(2\ln 2 + \pi)$$

9-31

1)
$$U_A = \int_{-l}^{l} \frac{\frac{Q}{2l}dx}{4\pi\varepsilon_0\sqrt{x^2+y^2}}$$
$$= \frac{Q}{4\pi\varepsilon_0 l}\int_0^l \frac{1}{\sqrt{x^2+y^2}}dx$$
$$= \frac{Q}{4\pi\varepsilon_0 l}\left[\ln\left(x+\sqrt{x^2+y^2}\right)\right]_0^l$$
$$= \frac{Q}{4\pi\varepsilon_0 l}\ln\left(\frac{l+\sqrt{l^2+y^2}}{y}\right)$$

$$E_A = -\frac{dU_A}{dy} = \frac{Q}{4\pi\varepsilon_0 l}\left(\frac{\frac{y}{\sqrt{y^2+l^2}}}{l+\sqrt{l^2+y^2}} - \frac{1}{y}\right)$$
$$= \frac{Q}{4\pi\varepsilon_0\sqrt{l^2+y^2}y}$$

2)
$$U_B=\int_{x-l}^{x+l}\frac{\frac{Q}{2l}dr}{4\pi\varepsilon_0 r}=\frac{Q}{8\pi\varepsilon_0 l}\ln\left(\frac{x+l}{x-l}\right)$$
$$E_B=-\frac{dU_B}{dx}=-\frac{Q}{8\pi\varepsilon_0 l}\left(\frac{1}{x+l}-\frac{1}{x-l}\right)$$
$$=\frac{Q}{4\pi\varepsilon_0(x+l)(x-l)}$$

9-32

$$dq=\sigma 2\pi l\sin\theta dl$$
$$U=\int_{\frac{R_1}{\sin\theta}}^{\frac{R_2}{\sin\theta}}\frac{\sigma 2\pi l\sin\theta dl}{4\pi\varepsilon_0 l}=\frac{\sigma}{2\varepsilon_0}(R_2-R_1)$$

9-33

$$\sigma=\frac{Q}{\pi(b^2-a^2)}$$
$$dq=\sigma 2\pi r dr$$
$$U=\int_a^b\frac{\sigma 2\pi r dr}{4\pi\varepsilon_0 r}=\frac{Q}{2\pi\varepsilon_0(b^2-a^2)}(b-a)$$
$$=1.8\times10^{10}\times6\times10^{-6}\div1.2=9\times10^3(V)$$

10-1

$$U_o=\frac{q}{4\pi\varepsilon_0 r}+\frac{-q}{4\pi\varepsilon_0 a}+\frac{Q+q}{4\pi\varepsilon_0 b}$$

10-4

1)
$$U_O=U_q+U_{感}=\frac{q}{4\pi\varepsilon_0 l}+\frac{1}{4\pi\varepsilon_0 R}\int q'=\frac{q}{4\pi\varepsilon_0 l}$$

2)
$$U_{感}=\frac{1}{4\pi\varepsilon_0 R}\int q'=\frac{q'}{4\pi\varepsilon_0 R}=U_O-U_q=0-\frac{q}{4\pi\varepsilon_0 l}=-\frac{q}{4\pi\varepsilon_0 l}$$
$$\therefore q'=-q\frac{R}{l}$$