Simulation Design——Ye

$$\begin{cases} \dot{x}_1 = \theta(t)x_1 + x_2 \\ \dot{x}_2 = b(t)u \end{cases} \tag{0.1}$$

Ye's Controller

$$\beta(t) = (\beta(0) - \beta_\infty)e^{-lt} + \beta_\infty, \tag{0.2}$$

$$z_1(x_1, \beta(t)) = \tan\left(\frac{\pi}{2}\frac{x_1}{\beta(t)}\right) \tag{0.3}$$

$$z_1 = \frac{\beta(t)\tanh(x_1)}{\beta^2(t) - \tanh^2(x_1)} \tag{0.4}$$

$$z_2 = x_2 - \alpha_1 \tag{0.5}$$

$$\dot{z}_1 = \Pi_1(x_1, \beta)\dot{x}_1 + \Psi_1(x_1, \beta, \dot{\beta}) \tag{0.6}$$

$$\begin{aligned} \Pi_1 &= \frac{(\beta^2 - \tanh^2(x_1))\beta\mathrm{sech}^2(x_1) + 2\beta\tanh^2(x_1)\mathrm{sech}^2(x_1)}{(\beta^2 - \tanh^2(x_1))^2} \\ \Psi_1 &= \frac{\dot{\beta}\tanh(x_1) - 2\beta^2\dot{\beta}\tanh(x_1)}{(\beta^2 - \tanh^2(x_1))^2}. \end{aligned} \tag{0.7}$$

$$w_1 = x_1 = W_1 z_1, \quad W_1 = \frac{x_1}{z_1} \tag{0.8}$$

$$w_2 = -\frac{\partial\alpha_1}{\partial x_1}x_1 = W_2\underline{z}_2, \quad W_2 = \left[-\frac{\partial\alpha_1}{\partial x_1}W_1; 0\right] \tag{0.9}$$

$$\begin{aligned} \psi =& \Pi_1 z_1 - \frac{\partial\alpha_1}{\partial x_1}x_2 - \frac{\partial\alpha_1}{\partial\beta}\dot{\beta} - \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} - \frac{\partial\alpha_1}{\partial x_1}x_1\hat{\theta} \\ =& \Pi_1 z_1 - \frac{\partial\alpha_1}{\partial x_1}(z_2 + \alpha_1) - \frac{\partial\alpha_1}{\partial\beta}\dot{\beta} - \frac{\partial\alpha_1}{\partial\hat{\theta}}\gamma_\theta\left(\Pi x_1 z_1 - \frac{\partial\alpha_1}{\partial x_1}x_1 z_2\right) - \frac{\partial\alpha_1}{\partial x_1}x_1\hat{\theta} \\ =& \bar{\psi}\underline{z}_2 \end{aligned} \tag{0.10}$$

$$\bar{\psi} = \left[\Pi + \frac{\partial\alpha_1}{\partial x_1}\kappa - \frac{\partial\alpha_1}{\partial\beta}\frac{\dot{\beta}}{z_1} - \frac{\partial\alpha_1}{\partial\hat{\theta}}\gamma_\theta\Pi x_1 - \frac{\partial\alpha_1}{\partial x_1}W_1\hat{\theta};\ -\frac{\partial\alpha_1}{\partial x_1} + \frac{\partial\alpha_1}{\partial\hat{\theta}}\gamma_\theta\frac{\partial\alpha_1}{\partial x_1}x_1\right] \tag{0.11}$$

$$V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2\gamma_\theta}(\ell_\theta - \hat{\theta})^2 + \frac{|\ell_b|}{2\gamma_\rho}\left(\frac{1}{\ell_b} - \hat{\rho}\right)^2. \tag{0.12}$$

$$
\begin{aligned}
\dot{V} =& z_1\dot{z}_1 + z_2\dot{z}_2 - \frac{1}{\gamma_\theta}(\ell_\theta - \hat{\theta})\dot{\hat{\theta}} - \frac{|\ell_b|}{\gamma_\rho}\left(\frac{1}{\ell_b} - \hat{\rho}\right)\dot{\hat{\rho}} \\
=& z_1\left(\Pi_1(\alpha_1 + z_2 + \theta(t)x_1) + \Psi_1\right) \\
& + z_2\left(b(t)u - \frac{\partial\alpha_1}{\partial x_1}(x_2 + \theta(t)x_1) - \frac{\partial\alpha_1}{\partial\hat{\theta}}\gamma_\theta\left(\Pi x_1 z_1 - \frac{\partial\alpha_1}{\partial x_1}x_1 z_2\right) - \frac{\partial\alpha_1}{\partial\beta}\dot{\beta}\right) \\
& - \frac{1}{\gamma_\theta}(\ell_\theta - \hat{\theta})\dot{\hat{\theta}} - \frac{|\ell_b|}{\gamma_\rho}\left(\frac{1}{\ell_b} - \hat{\rho}\right)\dot{\hat{\rho}} \\
=& z_1\Pi_1\alpha_1 + z_1\Pi_1 z_2 + z_1\Pi_1\hat{\theta}x_1 + z_1\Psi_1 + z_1\Pi_1(\ell_\theta - \hat{\theta})x_1 + z_1\Pi_1(\theta(t) - \ell_\theta)x_1 \\
& + z_2\bar{u} + z_2(b(t) - \ell_b)\hat{\rho}\bar{u} - z_2\ell_b\left(\frac{1}{\ell_b} - \hat{\rho}\right)\bar{u} + z_2\left(-\frac{\partial\alpha_1}{\partial x_1}x_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\gamma_\theta\left(\Pi x_1 z_1 - \frac{\partial\alpha_1}{\partial x_1}x_1 z_2\right) - \frac{\partial\alpha_1}{\partial\beta}\dot{\beta}\right) \\
& - z_2\frac{\partial\alpha_1}{\partial x_1}\hat{\theta}x_1 - z_2\frac{\partial\alpha_1}{\partial x_1}(\theta(t) - \ell_\theta)x_1 - z_2\frac{\partial\alpha_1}{\partial x_1}(\ell_\theta - \hat{\theta})x_1 - \frac{1}{\gamma_\theta}(\ell_\theta - \hat{\theta})\dot{\hat{\theta}} - \frac{|\ell_b|}{\gamma_\rho}\left(\frac{1}{\ell_b} - \hat{\rho}\right)\dot{\hat{\rho}} \\
\leq& z_1\Pi_1\alpha_1 + z_2\bar{u} \\
& + z_1\Pi_1 z_2 + z_1\Pi_1\hat{\theta}x_1 + z_1\Psi_1 + z_1^2\Pi_1\delta_{\Delta_\theta}W_1 \\
& + z_2\Delta_b\hat{\rho}\bar{u} + z_2\left(-\frac{\partial\alpha_1}{\partial x_1}x_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\gamma_\theta\left(\Pi x_1 z_1 - \frac{\partial\alpha_1}{\partial x_1}x_1 z_2\right) - \frac{\partial\alpha_1}{\partial\beta}\dot{\beta}\right) \\
& - z_2\frac{\partial\alpha_1}{\partial x_1}\hat{\theta}x_1 - z_2\frac{\partial\alpha_1}{\partial x_1}\Delta_\theta x_1 \\
& - \frac{1}{\gamma_\theta}(\ell_\theta - \hat{\theta})\left(\dot{\hat{\theta}} - \gamma_\theta\Pi_1 x_1 z_1 + z_2\frac{\partial\alpha_1}{\partial x_1}x_1\right) - \frac{|\ell_b|}{\gamma_\rho}\left(\frac{1}{\ell_b} - \hat{\rho}\right)\left(\gamma_\rho z_2\mathrm{sign}(\ell_b)\bar{u} + \dot{\hat{\rho}}\right) \\
\leq& z_1\Pi_1\alpha_1 + z_2\bar{u} + z_2\psi + z_1\Pi_1\hat{\theta}x_1 + z_1\Psi_1 + z_1^2\Pi_1\delta_{\Delta_\theta}W_1 + z_2\Delta_b\hat{\rho}\bar{u} - z_2\frac{\partial\alpha_1}{\partial x_1}\Delta_\theta x_1 \\
& \underline{-\frac{1}{\gamma_\theta}(\ell_\theta - \hat{\theta})\left(\dot{\hat{\theta}} - \gamma_\theta\Pi_1 x_1 z_1 + \gamma_\theta z_2\frac{\partial\alpha_1}{\partial x_1}x_1\right) - \frac{|\ell_b|}{\gamma_\rho}\left(\frac{1}{\ell_b} - \hat{\rho}\right)\left(\gamma_\rho z_2\mathrm{sign}(\ell_b)\bar{u} + \dot{\hat{\rho}}\right)}
\end{aligned}
\tag{0.13}
$$

$$
\underline{\dot{\hat{\theta}} = \gamma_\theta\left(\Pi x_1 z_1 - \frac{\partial\alpha_1}{\partial x_1}x_1 z_2\right)} \tag{0.14}
$$

$$
\underline{\dot{\hat{\rho}} = -\gamma_\rho\mathrm{sign}(\ell_b)z_2\bar{u}} \tag{0.15}
$$

$$
z_2\psi = z_2\bar{\psi}\underline{z}_2 \leq \frac{1}{2}\bar{\psi}^\top\bar{\psi}z_2^2 + \frac{1}{2}z_2^2 + \frac{1}{2}z_1^2 \tag{0.16}
$$

$$
-z_2\frac{\partial\alpha_1}{\partial x_1}\Delta_\theta x_1 = z_2 w_2\Delta_\theta \leq \frac{\delta_{\Delta_\theta}}{2}\left(W_2^2 + 1\right)z_2^2 + \frac{\delta_{\Delta_\theta}}{2}z_1^2 \tag{0.17}
$$

$$
\begin{aligned}
\dot{V} \leq& \mathbf{z_1\Pi_1}\alpha_1 + \mathbf{z_1\Pi_1}\hat{\theta}\mathbf{x_1} + \mathbf{z_1\Psi_1} + \mathbf{z_1^2\Pi_1}\delta_{\mathbf{\Delta}_\theta}\mathbf{W_1} + \frac{\delta_{\mathbf{\Delta}_\theta}}{\mathbf{2}}\mathbf{z_1^2} + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{z_1^2} \\
& + z_2\bar{u} + \frac{1}{2}\bar{\psi}^\top\bar{\psi}z_2^2 + \frac{1}{2}z_2^2 + \frac{\delta_{\Delta_\theta}}{2}(W_2^2 + 1)z_2^2 + \mathbf{z_2\Delta_b\hat{\rho}\bar{u}}
\end{aligned}
\tag{0.18}
$$

$$
\alpha_1 = -\frac{k_1}{\Pi_1}z_1 - \hat{\theta}x_1 - \frac{\Psi_1}{\Pi} - z_1 W_1\delta_{\Delta_\theta} - \frac{\delta_{\Delta_\theta}}{2\Pi_1}z_1 - \frac{z_1}{2\Pi} \tag{0.19}
$$

$$
\alpha_1 = -\kappa_1 z_1, \quad \kappa_1 = \left(\frac{k_1}{\Pi_1} + \hat{\theta}W_1 + \frac{\Psi}{\Pi z_1} + W_1\delta_{\Delta_\theta} + \frac{\delta_{\Delta_\theta}}{2\Pi_1} + \frac{1}{2\Pi}\right) \tag{0.20}
$$

$$
\bar{u} = -\kappa_2 z_2 = -\left(k_2 + \frac{1}{2}\left(\delta_{\Delta_\theta}(W_2^2 + 1) + 1 + \bar{\psi}^\top\bar{\psi}\right)\right)z_2 \tag{0.21}
$$

$$
\kappa_2 = k_2 + \frac{1}{2}\left(\delta_{\Delta_\theta}(W_2^2 + 1) + 1 + \bar{\psi}^\top\bar{\psi}\right) \tag{0.22}
$$

$$
u = \hat{\rho}\bar{u} \tag{0.23}
$$

Chen's Controller

$$z_1 = x_1 \tag{0.24}$$

$$z_2 = x_2 - \alpha_1 \tag{0.25}$$

$$w_1 = x_1 = W_1 z_1, \quad W_1 = 1 \tag{0.26}$$

$$w_2 = -\frac{\partial \alpha_1}{\partial x_1} x_1 = W_2 \underline{z}_2, \quad W_2 = \left[-\frac{\partial \alpha_1}{\partial x_1} W_1; 0 \right] \tag{0.27}$$

$$\begin{aligned}
\psi =& z_1 - \frac{\partial \alpha_1}{\partial x_1} x_2 - \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}} - \frac{\partial \alpha_1}{\partial x_1} x_1 \hat{\theta} \\
=& z_1 - \frac{\partial \alpha_1}{\partial x_1} (z_2 + \alpha_1) - \frac{\partial \alpha_1}{\partial \hat{\theta}} \gamma_\theta \left(x_1 z_1 - \frac{\partial \alpha_1}{\partial x_1} x_1 z_2 \right) - \frac{\partial \alpha_1}{\partial x_1} x_1 \hat{\theta} \\
=& \bar{\psi} \underline{z}_2
\end{aligned} \tag{0.28}$$

$$\bar{\psi} = \left[1 + \frac{\partial \alpha_1}{\partial x_1} \kappa - \frac{\partial \alpha_1}{\partial \hat{\theta}} \gamma_\theta x_1 - \frac{\partial \alpha_1}{\partial x_1} W_1 \hat{\theta}; \;\; -\frac{\partial \alpha_1}{\partial x_1} + \frac{\partial \alpha_1}{\partial \hat{\theta}} \gamma_\theta \frac{\partial \alpha_1}{\partial x_1} x_1 \right] \tag{0.29}$$

$$V = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2\gamma_\theta} (\ell_\theta - \hat{\theta})^2 + \frac{|\ell_b|}{2\gamma_\rho} \left(\frac{1}{\ell_b} - \hat{\rho} \right)^2 . \tag{0.30}$$

$$\begin{aligned}
\dot{V} =& z_1 \dot{z}_1 + z_2 \dot{z}_2 - \frac{1}{\gamma_\theta} (\ell_\theta - \hat{\theta}) \dot{\hat{\theta}} - \frac{|\ell_b|}{\gamma_\rho} \left(\frac{1}{\ell_b} - \hat{\rho} \right) \dot{\hat{\rho}} \\
=& z_1 (\alpha_1 + z_2 + \theta(t) x_1) \\
& + z_2 \left(b(t) u - \frac{\partial \alpha_1}{\partial x_1} (x_2 + \theta(t) x_1) - \frac{\partial \alpha_1}{\partial \hat{\theta}} \gamma_\theta \left(x_1 z_1 - \frac{\partial \alpha_1}{\partial x_1} x_1 z_2 \right) \right) \\
& - \frac{1}{\gamma_\theta} (\ell_\theta - \hat{\theta}) \dot{\hat{\theta}} - \frac{|\ell_b|}{\gamma_\rho} \left(\frac{1}{\ell_b} - \hat{\rho} \right) \dot{\hat{\rho}} \\
=& z_1 \alpha_1 + z_1 z_2 + z_1 \hat{\theta} x_1 + z_1 \Psi_1 + z_1 (\ell_\theta - \hat{\theta}) x_1 + z_1 (\theta(t) - \ell_\theta) x_1 \\
& + z_2 \bar{u} + z_2 (b(t) - \ell_b) \hat{\rho} \bar{u} - z_2 \ell_b \left(\frac{1}{\ell_b} - \hat{\rho} \right) \bar{u} + z_2 \left(-\frac{\partial \alpha_1}{\partial x_1} x_2 - \frac{\partial \alpha_1}{\partial \hat{\theta}} \gamma_\theta \left(x_1 z_1 - \frac{\partial \alpha_1}{\partial x_1} x_1 z_2 \right) \right) \\
& - z_2 \frac{\partial \alpha_1}{\partial x_1} \hat{\theta} x_1 - z_2 \frac{\partial \alpha_1}{\partial x_1} (\theta(t) - \ell_\theta) x_1 - z_2 \frac{\partial \alpha_1}{\partial x_1} (\ell_\theta - \hat{\theta}) x_1 - \frac{1}{\gamma_\theta} (\ell_\theta - \hat{\theta}) \dot{\hat{\theta}} - \frac{|\ell_b|}{\gamma_\rho} \left(\frac{1}{\ell_b} - \hat{\rho} \right) \dot{\hat{\rho}} \\
\leq& z_1 \alpha_1 + z_2 \bar{u} \\
& + z_1 z_2 + z_1 \hat{\theta} x_1 + z_1^2 \delta_{\Delta_\theta} W_1 \\
& + z_2 \Delta_b \hat{\rho} \bar{u} + z_2 \left(-\frac{\partial \alpha_1}{\partial x_1} x_2 - \frac{\partial \alpha_1}{\partial \hat{\theta}} \gamma_\theta \left(x_1 z_1 - \frac{\partial \alpha_1}{\partial x_1} x_1 z_2 \right) \right) \\
& - z_2 \frac{\partial \alpha_1}{\partial x_1} \hat{\theta} x_1 - z_2 \frac{\partial \alpha_1}{\partial x_1} \Delta_\theta x_1 \\
& - \frac{1}{\gamma_\theta} (\ell_\theta - \hat{\theta}) \left(\dot{\hat{\theta}} - \gamma_\theta x_1 z_1 + z_2 \frac{\partial \alpha_1}{\partial x_1} x_1 \right) - \frac{|\ell_b|}{\gamma_\rho} \left(\frac{1}{\ell_b} - \hat{\rho} \right) (\gamma_\rho z_2 \mathrm{sign}(\ell_b) \bar{u} + \dot{\hat{\rho}}) \\
\leq& z_1 \alpha_1 + z_2 \bar{u} + z_2 \psi + z_1 \hat{\theta} x_1 + z_1^2 \delta_{\Delta_\theta} W_1 + z_2 \Delta_b \hat{\rho} \bar{u} - z_2 \frac{\partial \alpha_1}{\partial x_1} \Delta_\theta x_1 \\
& \underline{- \frac{1}{\gamma_\theta} (\ell_\theta - \hat{\theta}) \left(\dot{\hat{\theta}} - \gamma_\theta x_1 z_1 + \gamma_\theta z_2 \frac{\partial \alpha_1}{\partial x_1} x_1 \right) - \frac{|\ell_b|}{\gamma_\rho} \left(\frac{1}{\ell_b} - \hat{\rho} \right) (\gamma_\rho z_2 \mathrm{sign}(\ell_b) \bar{u} + \dot{\hat{\rho}})}
\end{aligned} \tag{0.31}$$

$$\underline{\dot{\hat{\theta}} = \gamma_\theta \left(x_1 z_1 - \frac{\partial \alpha_1}{\partial x_1} x_1 z_2 \right)} \tag{0.32}$$

$$\underline{\dot{\hat{\rho}} = -\gamma_\rho \mathrm{sign}(\ell_b) z_2 \bar{u}} \tag{0.33}$$

$$z_2\psi = z_2\bar{\psi}\underline{z}_2 \leq \frac{1}{2}\bar{\psi}^\top\bar{\psi}z_2^2 + \frac{1}{2}z_2^2 + \frac{1}{2}z_1^2 \tag{0.34}$$

$$-z_2\frac{\partial\alpha_1}{\partial x_1}\Delta_\theta x_1 = z_2 w_2 \Delta_\theta \leq \frac{\delta_{\Delta_\theta}}{2}\left(W_2^2+1\right)z_2^2 + \frac{\delta_{\Delta_\theta}}{2}z_1^2 \tag{0.35}$$

$$\begin{aligned}\dot{V} \leq & \mathbf{z_1}\alpha_1 + \mathbf{z_1}\hat{\theta}\mathbf{x_1} + \mathbf{z_1^2}\delta_{\Delta_\theta}\mathbf{W_1} + \frac{\delta_{\boldsymbol{\Delta}_\theta}}{\mathbf{2}}\mathbf{z_1^2} + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{z_1^2} \\ & + z_2\bar{u} + \frac{1}{2}\bar{\psi}^\top\bar{\psi}z_2^2 + \frac{1}{2}z_2^2 + \frac{\delta_{\Delta_\theta}}{2}(W_2^2+1)z_2^2 + \mathtt{z_2\Delta_b\hat{\rho}\bar{u}}\end{aligned} \tag{0.36}$$

$$\alpha_1 = -k_1 z_1 - \hat{\theta}x_1 - z_1 W_1 \delta_{\Delta_\theta} - \frac{\delta_{\Delta_\theta}}{2}z_1 - \frac{z_1}{2} \tag{0.37}$$

$$\alpha_1 = -\kappa_1 z_1, \quad \kappa_1 = \left(k_1 + \hat{\theta}W_1 + W_1\delta_{\Delta_\theta} + \frac{\delta_{\Delta_\theta}}{2} + \frac{1}{2}\right) \tag{0.38}$$

$$\bar{u} = -\kappa_2 z_2 = -\left(k_2 + \frac{1}{2}\left(\delta_{\Delta_\theta}(W_2^2+1) + 1 + \bar{\psi}^\top\bar{\psi}\right)\right)z_2 \tag{0.39}$$

$$\kappa_2 = k_2 + \frac{1}{2}\left(\delta_{\Delta_\theta}(W_2^2+1) + 1 + \bar{\psi}^\top\bar{\psi}\right) \tag{0.40}$$

$$u = \hat{\rho}\bar{u} \tag{0.41}$$