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School of Computer Science

COMP SCI 1103/2103 Algorithm Design & Data Structure

Recursion 2

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seek LIGHT

Review

- Recursion
- If a problem can be solved recursively, we can always do that iteratively as well.
- Sometimes, from a design or conceptual view, recursion is superior.
- There will always be efficiency issues in using recursion.

Overview

- In this lecture we will discuss:
 - More details of recursion.
 - Think recursively
 - How to improve the efficiency of recursion
 - Tail recursion
 - Helper function

Checklist for Recursion

- Three properties to be checked for a recursive algorithm:
 - There is no infinite recursion
 - Each stopping case performs the correct action
 - For all recursive cases: IF all recursive calls perform correctly, THEN the entire case performs correctly.
- To use recursion, break the problem into subproblems.
 - at least one sub-problem needs to be the same problem as the main problem, but with a smaller size (simpler).
- Why does this not work?

```
int countRow(Person i){  
    if Person i is in the leftmost seat || Person i is in the rightmost seat  
        return 1;  
    else  
        return countRow(Person i+1)+countRow(Person i-1);  
}
```

Checking $n!$

- Infinite recursion?
 - The relationship $n! = n * (n-1)!$ has n getting smaller and heading towards 0. Provided we have a base case for 0, no infinite recursion.
- Is the returned value for base correct?
 - Base case: $n = 0$, we return 1. This is correct.
- Is the relationship for recursion correct?
 - If $(n-1)!$ returns the correct result, then $n! = n * (n-1)!$ will return the correct result.
- Therefore, this is correct! (But what if $n < 0$?)

Think Recursively

- Many of the problems we talked before can be solved using recursion if we think recursively.
- Consider the palindrome problem in prac 1.

Problem Solving with Recursion

- Practice makes perfect!
- Step 1. Consider various ways to simplify inputs
 - Find sub-problems that perform the same task as the original problem, but with a simpler (or smaller) input.
- Step 2. Combine solutions with simpler inputs into a solution of the original problem.
- Step 3. Find solutions to the simplest cases.
- Step 4. Implement the solution by combining the simple cases and the reduction step.

Example

```
bool isPalindrome(string s){  
    //base  
    if(s.length()<=1)  
        return true;  
  
    //recursion  
    if(tolower(s[0]) != tolower(s[s.length()-1]))  
        return false;  
    return isPalindrome(s.substr(1,s.length()-2));  
}
```


Recursive Helper Functions

- This implementation of `isPalindrome()` is not efficient. Why?
 - It creates a new string for every recursive call
 - What about checking whether a substring is a palindrome or not?
- It is a common design technique in recursive programming to declare a second function that receives additional parameters.


```
int isPalindrome(string s, int start, int end)
```

Example

```
bool isPalindrome(string s){
    isPalindromHelper(s, 0, s.length-1);
}

bool isPalindromHelper(string s, int start, int end){
    //base
    if(end==-1 || start==end)
        return true;

    //recursion
    if(tolower(s[start]) != tolower(s[end]))
        return false;
    return isPalindromHelper(s, start+1, end-1);
}
```



No string created here. Reusing s. In previous solution, use of substr() created and returned another string.

Stack use for recursive isPalindrome

- How does it work?

Heads and Tails

- Some compilers perform optimisation to reduce your call overhead.
- If possible, the compiler will remove stack heavy operations and replace them with lighter ones.
- It's possible to do this, in recursion, using a technique called 'tail recursion'.
- A recursive function is tail recursive when recursive call is the last thing executed by the function.

Tail-recursive factorial

- How is stack used for this one?
- Here's a recursive factorial that a compiler can optimise to reduce stack overhead:

```
int fac(int n){  
    if(n < 1){  
        return 1;  
    }else{  
        return n*fac(n-1);  
    }  
}
```

```
int fac(int n, int acc){  
    if (n < 1){  
        return acc;  
    }else{  
        return fac(n-1, acc*n);  
    }  
}
```

More than one call

- Recursive functions can generate more than one call on each pass.
- Consider the Truckloads problem,
 - $\text{numTrucks}(\text{pile1}, \text{loadSize}) + \text{numTrucks}(\text{pile2}, \text{loadSize})$
- Each call potentially generates two more calls
- Explosion!!
 - Exponential growth!
- You can see how this can quickly add up in terms of space

Hanoi Tower

- The Towers of Hanoi problem can be solved easily using recursion, but is difficult to solve without using recursion.
- The problem involves moving a specified number of disks of distinct sizes from one tower to another while observing the following rules:
 - Only one more tower can be used other these two towers
 - No disk can be on top of a smaller disk at any time
 - All the disks are initially placed on one tower
 - Only one disk can be moved at a time and it must be the top disk on the tower.

Summary

- Recursion is a useful tool for understanding problems and producing solutions, but:
 - You can always solve it iteratively
 - It can be inefficient and space hungry
 - Analysing recursive code can get tricky quickly



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