The University of Adelaide
School of Computer Science

Artificial Intelligence
Tutorial 3

Question 1

You have three baskets of fruit: the first one contains two apples, the second one contains two oranges, and the third one contains one apple and one orange. Assume that a basket is selected randomly and that a piece of fruit is picked randomly from that basket. Let B be the random variable corresponding to the basket number selected (B can have as value 1, 2 or 3) and let F be the random variable corresponding to the type of fruit picked (F can have as value *apple* or *orange*).

1. What is the distribution $P(B)$ and what are the conditional distributions $P(F|B)$? Write your answers in tabular form.
2. What is the joint probability of selecting the first basket and picking an apple, i.e. what is $P(B = 1, F = apple)$?
3. If we observe that the picked fruit is an apple, what is the conditional probability that the chosen basket is the basket containing two apples, i.e. what is $P(B = 1|F = apple)$?

Question 2

A CS student may or may not show up in class (C) on a given day. If they do not show up ($C = \neg c$), there two possible explanations. It could be that the student was abducted by aliens ($A = a$). It could also be that the student has the stomach bug ($B = b$) that is going around. The possible outcomes are listed in a joint probability table $P(A,B,C)$ in Table 1:

a	b	c	0.00
a	b	$\neg c$	0.01
a	$\neg b$	c	0.00
a	$\neg b$	$\neg c$	0.02
$\neg a$	b	c	0.01
$\neg a$	b	$\neg c$	0.04
$\neg a$	$\neg b$	c	0.90
$\neg a$	$\neg b$	$\neg c$	0.02

Table 1: $P(A,B,C)$

1. What is the distribution $P(A,B)$? Your answer should be in the form of a table.

2. Are A and B independent? Justify your answer using the actual probabilities computed in part 1.

3. We would like to reason about whether or not a certain student has the bug. What is the marginal distribution over B given no evidence (sometimes this is called the prior distribution)? Is it most likely that the student does or does not have the bug?

4. If we observe that the student does not come to class ($C = \neg c$), what is the conditional distribution $P(B|C = \neg c)$ (sometimes this is called a posterior distribution)? Is it most likely that the student did or did not have the bug given the evidence? Does it make intuitive sense how the numbers have shifted from part 3 given the new evidence that has been observed?

5. If we further discover that the student has been abducted by aliens ($A = a$), what is the new posterior distribution over B, i.e. $P(B|C = \neg c, A = a)$? Is it most likely that the student did or did not have the bug given all the evidence? Does it make intuitive sense how the numbers have shifted from part 4 given the new evidence that has been observed? The general phenomenon where the discovery that one possible cause is true decreases belief in other possible causes is called explaining-away.

6. Are A and B conditionally independent given C? Justify your answer using specific probabilities (hint: you should already have sufficient probabilities computed).

Question 3

Consider the following network, in which a mouse is reasoning about the behaviour of a cat. The mouse really wants to know whether the cat will attack (A), which depends on whether the cat is hungry (H) and whether the cat is sleepy (S). The mouse can observe two things, whether the cat is sleepy (S) and whether the cat has a collar (C). The cat is more often sleepy (S) when it's either full (f) or starved (v) than when it is peckish (p) and the collar (C) tends to indicate that the cat is not starved. Note that entries are omitted, such as $P(C = \neg c)$, when their complements are given.

1. Draw the Bayesian network corresponding to the above joint probability distribution on C, H, S and A.

2. If the conditional probability tables for the network are as follows,

   compute the following probabilities:

   (a) $P(A = \text{a}, C = c, S = s, H = f)$

   (b) $P(A = \text{a}, C = c, S = s)$

P(C)	
C	P(C)
c	0.4

P(H|C)		
H	C	P
f	c	0.70
v	c	0.10
p	c	0.20
f	¬c	0.20
v	¬c	0.50
p	¬c	0.30

P(S|H)		
S	H	P
s	f	0.90
s	v	0.60
s	p	0.30

P (A|H,S)			
A	H	S	P
a	f	s	0.01
a	f	¬s	0.10
a	v	s	0.40
a	v	¬s	0.90
a	p	s	0.20
a	p	¬s	0.70

(c) $P(C = c, S = s)$

Question 4

A CS student in the AI class notices that people who drive 4WDs vehicles (S) consume large amounts of gas (G) and are involved in more accidents than the national average (A). They have constructed the Bayesian network in Figure 1 (here t implies "true" and f implies "false"):

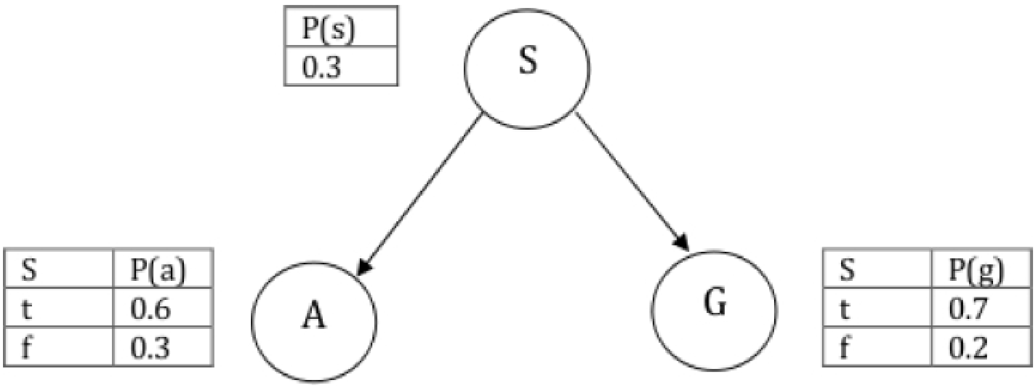
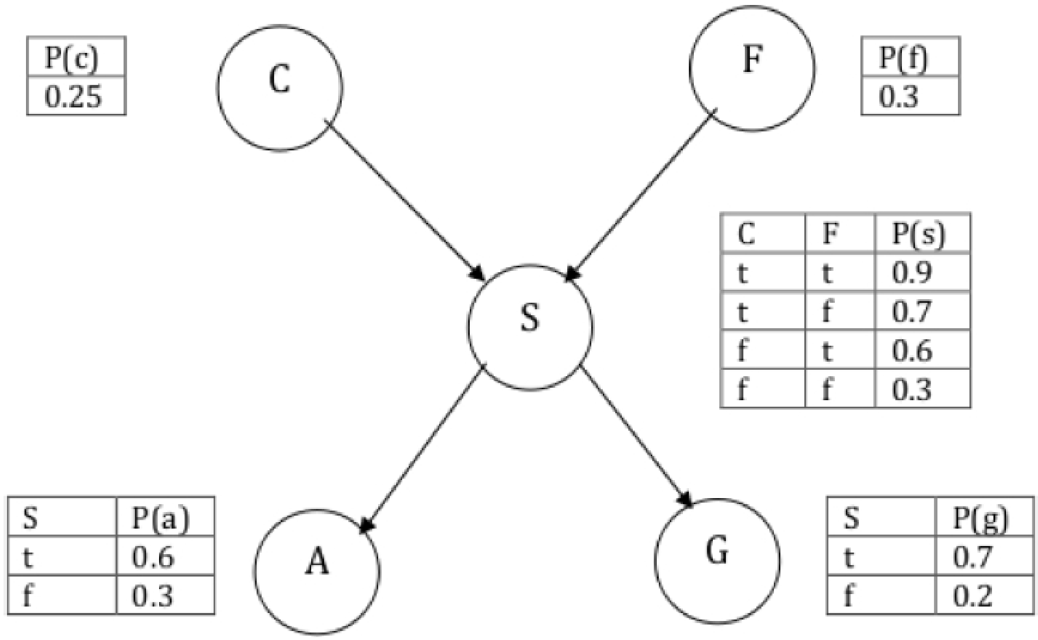


Figure 1: 4WD Bayesian network

1. Compute $P(A)$ (this is the probability distribution of A).
2. Using conditional independence, compute $P(\neg g, \mathrm{a}|s)$ and $P(\neg g, \mathrm{a}|\neg s)$. Then use Bayes' rule to compute $P(s|\neg g, \mathrm{a})$.
3. The enterprising AI student notices that there are two types of people that drive 4WDs, people from the country (C) and people with large families (F). After collecting some statistics, the student arrives at the Bayesian network in Figure 2. Using the chain rule (product rule) compute the probability $P(\neg g, \mathrm{a}, s, c, \neg f)$.
4. Using the local semantics implied by the structure of Bayesian Networks,



P(c)
0.25

P(f)
0.3

C	F	P(s)
t	t	0.9
t	f	0.7
f	t	0.6
f	f	0.3

S	P(a)
t	0.6
f	0.3

S	P(g)
t	0.7
f	0.2

Figure 2: New 4WD Bayesian network

state whether the following variables from the Bayesian network in Figure 2 are (conditionally) independent.

(a) C, G

(b) $F, A|S$

(c) C, F

(d) A, G

(e) $C, F|S$

(f) $C, F|A$

Question 5 ***(optional)***

At the nuclear reactor at the Australian Nuclear Science and Technology Organisation, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider the Boolean variables A (alarm sounds), F_A (Alarm faulty), and F_G (gauge is faulty) and the multi valued (continuous random variables) nodes G (gauge reading) and T (actual core temperature).

1. Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core temperature gets too high.

2. Suppose there are just two possible actual and measured temperatures, *NORMAL* and *HIGH*; the probability that the gauge gives the correct temperature is x when it is working, but y when it is faulty. Give the conditional probability table associated with G.

3. Suppose the alarm works correctly unless it is faulty, in which case it never sounds. Give the conditional probability table associated with A.

4. Suppose the alarm and gauge are working and the alarm sounds. Calculate an expression for the probability that the temperature of the core is too high, in terms of the various conditional probabilities in the network.