Mathematics for Data Science Tutorial 4 (week 8)

Semester 2, 2019

1. Consider the matrix

$$A = \begin{pmatrix} 3 & 2 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}.$$

   (a) 2 is an eigenvalue of A. Use the trace and determinant to determine the other eigenvalue(s).

   (b) Find the eigenspace corresponding to 2.

2. Consider the matrices

$$A_1 = \begin{bmatrix} -12 & 7 \\ -7 & 2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -1 & 3 & 9 \\ 0 & -7 & -18 \\ 0 & 2 & 5 \end{bmatrix}.$$

   For each matrix A_i above

   (a) Determine the eigenvalues and eigenvectors.

   (b) For each eigenvalue, state its multiplicity and give the dimension of its associated eigenspace.

   (c) Hence determine whether the matrix A_i is diagonalisable, stating the reason for your answer.

   (d) For each diagonalisable matrix A_i, determine a matrix P such that $P^{-1}A_iP = D$, where D is a diagonal matrix. What is D? (For the purposes of this exercise, order your eigenvalues from smallest to largest i.e. $\lambda_1 \leq \lambda_2 \ldots \leq \lambda_n$.)

   (e) Determine if each matrix A_i is invertible and, where possible, use your results from (d) to write the inverse matrix A_i^{-1} in the form $P\Delta P^{-1}$, where Δ is a diagonal matrix. (You do not need to find A^{-1} if not possible by this method.)