# Mathematics for Data Science I
# Practice Questions (week 3)

Semester 2, 2018

These questions are all about Taylor and Maclaurin series, and power series more generally. Difficult questions are starred. Good places to go for further questions on this topic include the exercises in:

- Stewart, *Calculus* (7th Ed.), Sections 11.10 and 11.11,
- Morris & Stark, *Fundamentals of Calculus*, Sections 9.2 and 9.3.

1. (a) Write down, *from memory*, the Taylor series for the function $\sin x$, $\cos x$ and $e^x$ around the point $x = 0$.

   (b) By manipulating the above series (rather than by actually computing derivatives), determine the power series (around $x = 0$) for each of functions (i) $x \cos x$ and (ii) $\sin(2x)$.

2. Find the Taylor series around $x = 0$ for the functions $\sinh x$ and $\cosh x$.

3. Find the leading terms (up to the fourth power) of the Taylor series for each of the following functions.

   (a) $e^x \sin x$, around $c = 0$

   (b) $\left(\tan^{-1} x\right)^2$, around $c = 0$

   (c) $\dfrac{1}{x}$, around $c = 2$

   (d) $\sin x$, around $c = \pi/4$

4. For what values of $x$ can we replace $\sin x$ by $x - x^3/3!$ with an error of magnitude no greater than $3 \times 10^{-4}$?

5. Find the radius and intervals of convergence for each of the following power series.

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2)(4)(6)\cdots(2n)} x^n$

6. We know (from the geometric series) that

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots + x^n + \cdots \quad ; \quad |x| < 1.$$

By manipulating this series in various ways, determine the power series for the following functions. Make sure you mention the domain of validity for your power series.

(a) $\dfrac{1}{1+x}$

(b) $\dfrac{1}{1-2x}$

(c) $\dfrac{1}{x+2}$

(d) $\ln(1+x)$ (Hint: integrate the geometric series)

7. The series $\displaystyle\sum_{n=0}^{\infty}(e^x-4)^{-n}$ is not, technically speaking, a power series because it is not expressed in powers of $x$. On the other hand, it *is* some sort of series. Determine the values of $x$ for which it converges.

8. * Find the power series for the function $\tan^{-1} x$. (Hint: term-by-term integration, and the geometric series, may be useful ideas.)

9. Use the Taylor series expansion for $e^x$ to determine the value of

$$\lim_{x\to 0}\frac{e^x-1-x}{x^2}.$$