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#### **Ensemble Learning**

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Some slides borrowed from Rama Ramakrishnan and Rob Schapire etc.

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seek LIGHT

#### **Outlines**

- Ensemble methods overview
- Random forest
- Bagging
- Boosting

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## What is Ensemble learning?

According to the dictionary, the word 'ensemble' means

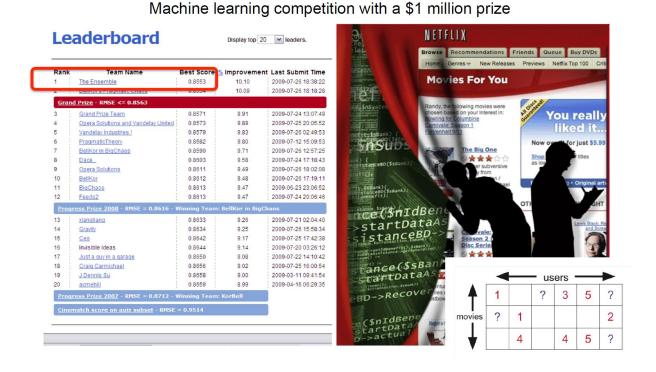
'a group of items viewed as a whole rather than individually'

- In machine learning, ensemble learning is a learning technique to learn and combine multiple predictive models
  - The models can be anything
  - More diverse the better.
  - Combine the predictions:
    - Majority voting/Weighted Majority voting
    - Average/weighted average

• ...

## Ensemble learning is powerful

 Ensemble approach is often the essential strategy for wining machine learning competition!



## Ensemble learning

- The key is to ensure the diversity of predictive method
- Let's consider an extreme case
  - Binary classification problem
  - We ensemble 1000 classifiers, the accuracy of each classifier is 51%, that is, slightly better than random guessing
  - Those classifiers are statistically independent
  - We use majority voting to make final prediction
    - If most classifier predict class 1, then the prediction is class 1, vice versa

What is the classification accuracy of the final system?

## Ensemble learning example

- If the prediction is correct, it means that more than 500 classifiers should give correct prediction
- This probability can be calculated by binomial cumulative distribution
  - What is a cumulative Binomial probability? Cross Validated (stackexchange.com)

$$P = binocdf(499,1000,1-0.51) = 0.73$$

- After calculation, the final prediction accuracy can go from 51% to over 70%!
  - If the individual classifier got slightly higher accuracy, say 55%,
     the ensembled classifier can reach 100% accuracy

## Ensemble learning

- We can see that by using a rather weak classifier, the ensemble classifier can significantly outperform the original classifier
- The key is the independence assumption
  - In practice, ensuring independence is challenging
  - we usually use different methods to ensure the individual predictors are diverse/independent as much as possible.
- How to ensure the diversity of classifiers
  - The predictor/classifier, e.g., different types of classifiers, or classifiers with different parameters
  - The dataset to train the model, e.g., classifier trained on different subset of data

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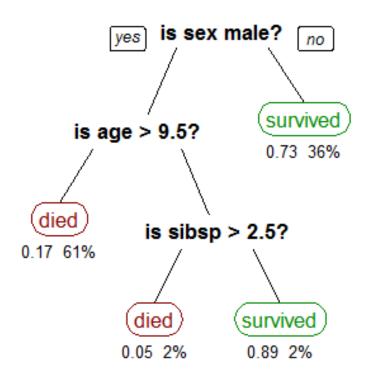
#### Random forest

- Basic idea: ensemble a set of simple classifier, called decision tree, as the final classifier
- Simple but effective approach
  - Especially good for features with diverse meanings
  - Efficient in training and testing

#### Decision Tree: An introduction

#### A classic predictive model

- A decision tree is drawn upside down with its root at the top.
- Each non-leaf node represents a test, usually it involves only one feature
- The leaf node represents the decision/prediction outcome
- Example

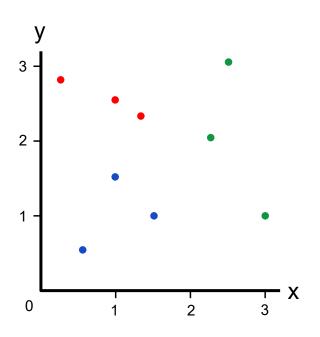


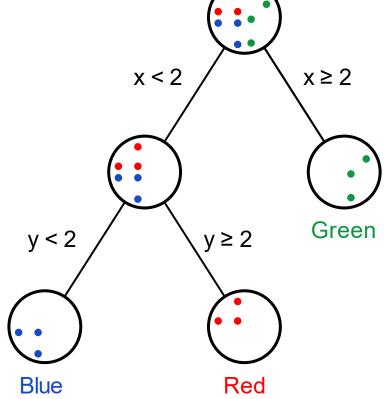
For this let's consider a very basic example that uses titanic data set for predicting whether a passenger will survive or not. Below model uses 3 features/attributes/columns from the data set, namely sex, age and sibsp (number of spouses or children along).

### Decision Tree: an example

• In each node, decision tree chooses one dimension and one threshold to perform a test. Data are divided according to the outcome of the test

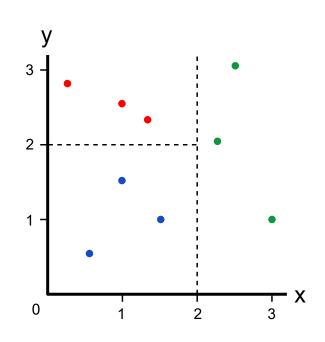
 The test ends when samples in one branch are all belong the same class.

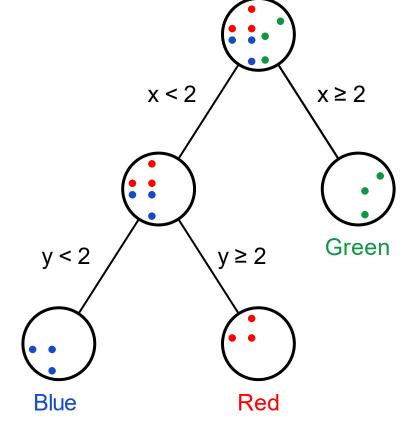




## Decision Tree: an example

It is equivalent to recursively partition the feature spaces. (by vertical or horizontal lines in the 2-D case)





#### Decision Tree: how to build

- Intuitively, we try to find a way to divide data such that the class labels are "purer" in the divided subsets
- For each node, scan all the dimensions and try all possible thresholds, find the best one
  - Best is measured by weighted average Gini impurity of the subset in the left child and right child, the lower the better

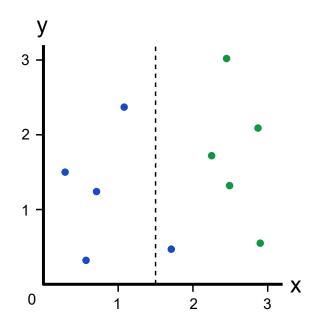
$$G = \sum_{i=1}^{C} p(i)(1 - p(i))$$

C: number of classes. p(i) percentage of the *i*-th class samples.

Weight is calculated by the number of samples in each child

We can tell G = 0, if all samples belong to the same class

## Example



$$G_{left} = 1 \times 0 = 0$$

$$G_{right} = \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} = 0.278$$

$$G_{ave} = 0 \times 0.4 + 0.278 * 0.6 = 0.167$$

#### **Decision Tree**

- By recursively apply the above rule, we could split nodes until the impurity can not be further reduced, i.e., all the samples in the children nodes belong to the same categories
- Decision Tree is fast to train and evaluate, because it only uses one feature each time
- Decision Tree can easily overfit the training data
  - Not stable since a mistake made in the parent node will affect the decision of its children.
  - Its classification accuracy is poor

#### Random forest

- Idea: build T trees instead of one, (that's why it is called forest), and introduce randomness for each tree
- Injecting randomness
  - Instead of trying all features every time we make a new decision node, we only try a random subset of the features.
- Random forest works surprisingly well in many applications
- Why use decision tree?
  - Easy to train
  - Fast to evaluate
  - Easy to inject randomness

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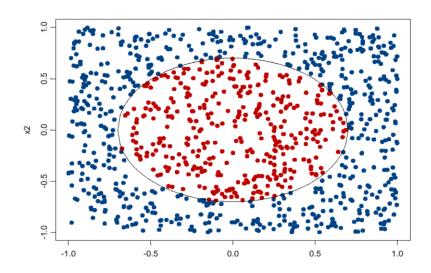
## **Bagging**

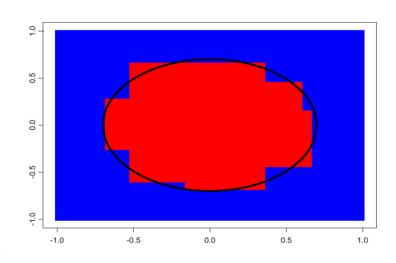
- A general method to train t diverse models
- Idea:
- given a dataset of n points:
  - Sample, with replacement, n training examples from the dataset.
  - Train a model on the n samples.
  - Repeat t times.

With replacement -> chance to repeat sample same samples. So each "n-sample" training set will be different

## **Example of Bagging**

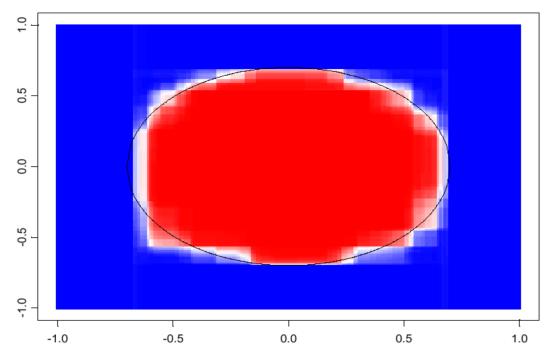
- Bagging a set of decision trees
  - Each tree is built by using decision tree algorithm, no randomness is introduced in the tree construction stage
- Decision boundary of individual trees





## Example of Bagging

#### Use T trees



shades of blue/red indicate strength of vote for particular classification

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### Boosting

- Bagging can be understood as a way to assign each sample different weights
- Recall that in general a machine learning algorithm try to minimize a loss function with the form

$$J = \sum_{i=1}^{N} l(\mathbf{x}_i, y_i)$$

if 
$$(\mathbf{x}_i, y_i) = (\mathbf{x}_j, y_j)$$
, then  $l(\mathbf{x}_i, y_i) + l(\mathbf{x}_j, y_j) = 2l(\mathbf{x}_i, y_i)$ 

If a sample is not selected, it is equivalent to set its weight to 0

• In bagging, the weights are randomly assigned and can only take limited number of values

## Boosting

- Boosting follows a similar procedure to build a bundle of predictors
  - Build a model with a given predictor with initial weights
  - Adjust weights
  - Build a model with the updated weights
  - Adjust weights
  - Repeat T times
- The weighted is updated to make the classifier focuses on samples that are most often misclassified by previous rules
- The results are combined by weighted majority voting

### Boosting

- Given a weak classifier with prediction accuracy slightly better than random, say, 55% in binary classification case, a boosting algorithm can provably construct single classifier with very high accuracy, say, 99%, given sufficient training data.
  - The weak classifier is also called weak learner
- Has many variants
  - Adaboost
  - Anyboost
  - Graident boost

**–** ...

# A formal definition of Adaboost (binary case)

- given training set  $(x_1, y_1), \dots, (x_m, y_m)$
- $y_i \in \{-1, +1\}$  correct label of instance  $x_i \in X$
- for t = 1, ..., T:
  - construct distribution  $D_t$  on  $\{1, \ldots, m\}$
  - find weak classifier ("rule of thumb")

Weight for each sample

$$h_t: X \to \{-1, +1\}$$

with error  $\epsilon_t$  on  $D_t$ :

Weighted error rate

$$\epsilon_t = \Pr_{i \sim D_t}[h_t(x_i) \neq y_i]$$

output final/combined classifier H<sub>final</sub>

$$Pr_{i \sim D_t}[h_t(x_i) \neq y_i] = \frac{1}{N} \sum_i D_t(i)[h_t(x_i) \neq y_i]$$

# A formal definition of Adaboost (binary case)

- constructing  $D_{t+1}$ 
  - $D_1(i) = 1/m$
  - given  $D_t$  and  $h_t$ :

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} \times \begin{cases} e^{-\alpha_t} & \text{if } y_i = h_t(x_i) \\ e^{\alpha_t} & \text{if } y_i \neq h_t(x_i) \end{cases}$$
$$= \frac{D_t(i)}{Z_t} \exp(-\alpha_t y_i h_t(x_i))$$

where 
$$Z_t =$$
 normalization factor  $\alpha_t = \frac{1}{2} \ln \left( \frac{1 - \epsilon_t}{\epsilon_t} \right) > 0$ 

• final classifier:

• 
$$H_{\text{final}}(x) = \operatorname{sign}\left(\sum_{t} \alpha_{t} h_{t}(x)\right)$$

# A formal definition of Adaboost (binary case)

Weight is updated based on previous round error

Decrease

• given  $D_t$  and  $h_t$ :

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} \times \begin{cases} e^{-\alpha_t} & \text{if } y_i = h_t(x_i) \\ e^{\alpha_t} & \text{if } y_i \neq h_t(x_i) \end{cases}$$

$$= \frac{D_t(i)}{Z_t} \exp(-\alpha_t y_i h_t(x_i)) - \lim_{t \to \infty} \text{Increase}$$

where 
$$Z_t = \text{normalization factor}$$
  $\alpha_t = \frac{1}{2} \ln \left( \frac{1 - \epsilon_t}{\epsilon_t} \right) > 0$ 

## Putting together: an example procedure

- Start training with initial weight  $D_1$
- Try to find a weak classifier minimizing

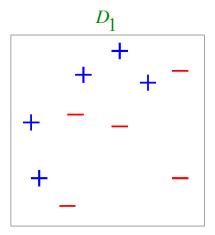
$$Pr_{i \sim D_t}[h_t(x_i) \neq y_i] = \frac{1}{N} \sum_i D_t(i)[h_t(x_i) \neq y_i]$$

- Calculate the weighted classification error  $\epsilon_1$  and calculate  $\alpha_1$
- Update weight to  $D_2$  by using  $D_1$  and  $\alpha_1$
- Try to find a weak classifier minimizing

$$Pr_{i \sim D_t}[h_t(x_i) \neq y_i] = \frac{1}{N} \sum_i D_t(i)[h_t(x_i) \neq y_i]$$

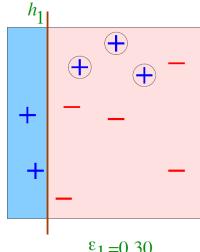
- Calculate the weighted classification error  $\epsilon_2$  and calculate  $\alpha_2$
- Repeat...

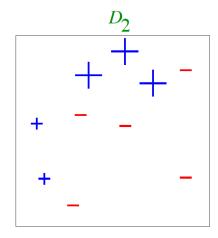
## Toy example



weak classifiers = vertical or horizontal half-planes

#### Round 1



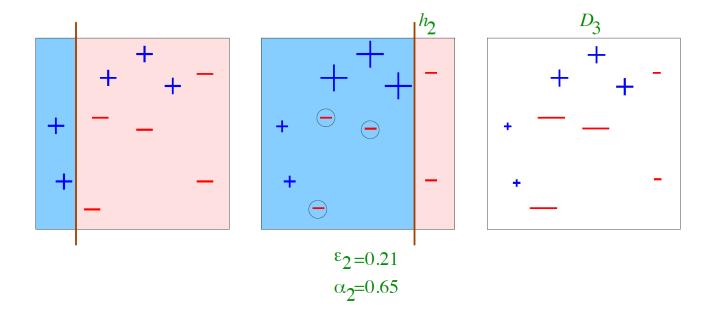


$$\epsilon_1 = 0.30$$
  
 $\alpha_1 = 0.42$ 

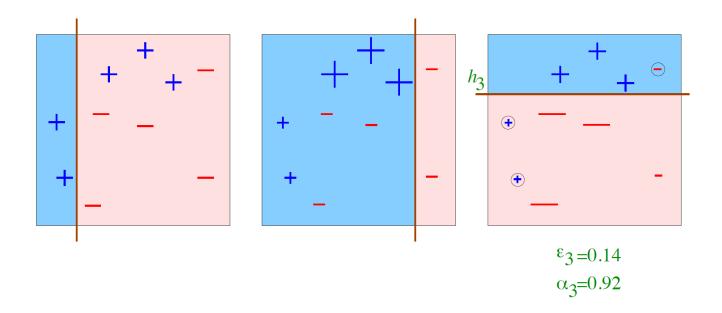
$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} \times \begin{cases} e^{-\alpha_t} & \text{if } y_i = h_t(x_i) \\ e^{\alpha_t} & \text{if } y_i \neq h_t(x_i) \end{cases}$$
$$= \frac{D_t(i)}{Z_t} \exp(-\alpha_t y_i h_t(x_i))$$

where 
$$Z_t =$$
 normalization factor  $\alpha_t = \frac{1}{2} \ln \left( \frac{1 - \epsilon_t}{\epsilon_t} \right) > 0$ 

## Round 2

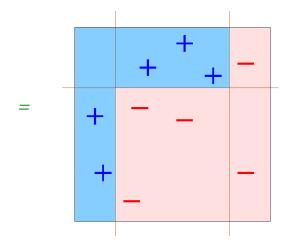


## Round 3



#### Final classifier

$$H_{\text{final}} = \text{sign} \left( 0.42 \right) + 0.65 + 0.92$$



• final classifier:

• 
$$H_{\text{final}}(x) = \operatorname{sign}\left(\sum_{t} \alpha_{t} h_{t}(x)\right)$$

#### Voted combination of classifiers

- The general problem here is to try to combine many simple "weak" classifiers into a single "strong" classifier
- $\bullet$  We consider voted combinations of simple binary  $\pm 1$  component classifiers

$$h_m(\mathbf{x}) = \alpha_1 h(\mathbf{x}; \theta_1) + \ldots + \alpha_m h(\mathbf{x}; \theta_m)$$

where the (non-negative) votes  $\alpha_i$  can be used to emphasize component classifiers that are more reliable than others

where 
$$Z_t=$$
 normalization factor  $\alpha_t=rac{1}{2}\ln\left(rac{1-\epsilon_t}{\epsilon_t}
ight)>0$  Smaller error, larger weight

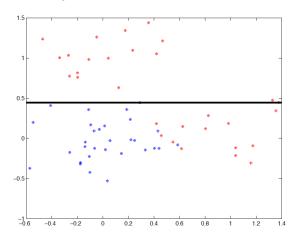
#### Example of weak learner

• Consider the following simple family of component classifiers generating  $\pm 1$  labels:

$$h(\mathbf{x};\theta) = \operatorname{sign}(w_1 x_k - w_0)$$

where  $\theta = \{k, w_1, w_0\}$ . These are called *decision stumps*.

 Each decision stump pays attention to only a single component of the input vector



In practice, it is OK to assume  $w_1$ = 1 or -1

How to train a decision stump for a set of given weights?

- How to train a decision stump for a set of given weights?
  - Try different model parameters and pick the one given the minimal weighted error

$$Pr_{i \sim D_t}[h_t(x_i) \neq y_i] = \frac{1}{N} \sum_i D_t(i)[h_t(x_i) \neq y_i]$$

- Efficient method could be used to fast search the optimal parameter
- Example: consider choosing the k-th dimension and  $w_1$ = 1, how to choose  $w_0$

Feature values at the k-th dimension [0.5, 0.2, 0.3,-0.4, 0.1] Weights for each sample [0.2,0.1, 0.3, 0.2, 0.2]

- How to train a decision stump for a set of given weights?
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– Example: consider choosing the k-th dimension and  $w_1$ = 1, how to choose  $w_0$ 

```
Feature values at the k-th dimension [0.5, 0.2, 0.3,-0.4, 0.1] Weights for each sample [0.2,0.1, 0.3, 0.2, 0.2]
```

- Rank the feature values and try threshold  $\frac{(v_t+v_{t+1})}{2}$
- Calculate weighted classification error for each threshold and find the best threshold

```
[0.5, 0.3, 0.2, 0.1, -0.4]
[0.2, 0.3, 0.1, 0.2, 0.2]
```

#### Questions

- Although intuitively reasonable, why should we choose the update equation in Adaboost?
- What is the objective function of Adaboost?

#### Loss of Adaboost

• We need to define a loss function for the combination so we can determine which new component  $h(\mathbf{x}; \theta)$  to add and how many votes it should receive

$$h_m(\mathbf{x}) = \alpha_1 h(\mathbf{x}; \theta_1) + \ldots + \alpha_m h(\mathbf{x}; \theta_m)$$

 While there are many options for the loss function we consider here only a simple exponential loss

$$\exp\{-y\,h_m(\mathbf{x})\}$$

Note that 
$$y \in \{1, -1\}, h_m(\mathbf{x}) \in \{1, -1\}$$

For correct classification,  $loss = \exp(-1)$ ;

incorrect classification  $loss = \exp(1)$ 

 $\sum_{i} exp(-y_i h_m(\mathbf{x}_i))$  is smaller if classification error is smaller

• Consider adding the  $m^{th}$  component:

$$\sum_{i=1}^{n} \exp\{-y_i[h_{m-1}(\mathbf{x}_i) + \alpha_m h(\mathbf{x}_i; \theta_m)]\}$$

$$= \sum_{i=1}^{n} \exp\{-y_i h_{m-1}(\mathbf{x}_i) - y_i \alpha_m h(\mathbf{x}_i; \theta_m)\}$$

Note that Adaboost sequentially add weak learner, Adaboost can be seen as a greedy way of optimizing the loss function: fixed previously found classifiers and weights and only optimize the current round of classifier and weight

• Consider adding the  $m^{th}$  component:

$$\sum_{i=1}^{n} \exp\{-y_i[h_{m-1}(\mathbf{x}_i) + \alpha_m h(\mathbf{x}_i; \theta_m)]\}$$

$$= \sum_{i=1}^{n} \exp\{-y_i h_{m-1}(\mathbf{x}_i) - y_i \alpha_m h(\mathbf{x}_i; \theta_m)\}$$

Variables to be optimized at the m round

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$$= \sum_{i=1}^{n} \exp\{-y_i h_{m-1}(\mathbf{x}_i) - y_i \alpha_m h(\mathbf{x}_i; \theta_m)\}$$

$$= \sum_{i=1}^{n} \underbrace{\exp\{-y_i h_{m-1}(\mathbf{x}_i)\}}_{\text{fixed at stage } m} \exp\{-y_i \alpha_m h(\mathbf{x}_i; \theta_m)\}$$

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$$= \sum_{i=1}^{n} \underbrace{\exp\{-y_i h_{m-1}(\mathbf{x}_i)\}}_{\text{fixed at stage } m} \exp\{-y_i \alpha_m h(\mathbf{x}_i; \theta_m)\}$$

$$= \sum_{i=1}^{n} W_i^{(m-1)} \exp\{-y_i \alpha_m h(\mathbf{x}_i; \theta_m)\}$$

## Solving $\alpha$

$$\frac{\partial}{\partial \alpha_m} \sum_{i=1}^n W_i^{(m-1)} \exp\{-y_i \alpha_m h(\mathbf{x}_i; \theta_m)\} = \left[\sum_{i=1}^n W_i^{(m-1)} \exp\{-y_i \alpha_m h(\mathbf{x}_i; \theta_m)\} \cdot \left(-y_i h(\mathbf{x}_i; \theta_m)\right)\right] = 0$$
Define  $I_i = y_i h(\mathbf{x}_i; \theta_m) \in \{1, -1\}$ 

$$\exp(-y_i \alpha_m h(\mathbf{x}_i; \theta_m)) = \frac{(I_{i+1})}{2} \exp(-\alpha_m) - \frac{(I_{i-1})}{2} \exp(\alpha_m)$$

$$Above = -\sum_i W_i^{m-1} \left(\frac{(I_i+1)}{2} \exp(-\alpha_m) - \frac{(I_i-1)}{2} \exp(\alpha_m)\right) I_i$$

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 $= -\sum_{i} W_{i}^{m-1} \left( \frac{(I_{i}^{2} + I_{i})}{2} \exp(-\alpha_{m}) - \frac{(I_{i}^{2} - I_{i})}{2} \exp(\alpha_{m}) \right)$ 

 $= -\sum_{i} W_i^{m-1} \left( \frac{(1+I_i)}{2} \exp(-\alpha_m) - \frac{(1-I_i)}{2} \exp(\alpha_m) \right) = 0$ 

## Solving $\alpha$

$$Above = -\sum_{i} W_{i}^{m-1} \left( \frac{(I_{i}+1)}{2} \exp(-\alpha_{m}) - \frac{(I_{i}-1)}{2} \exp(\alpha_{m}) \right) I_{i}$$

$$= -\sum_{i} W_{i}^{m-1} \left( \frac{(I_{i}^{2}+I_{i})}{2} \exp(-\alpha_{m}) - \frac{(I_{i}^{2}-I_{i})}{2} \exp(\alpha_{m}) \right)$$

$$= -\sum_{i} W_{i}^{m-1} \left( \frac{(1+I_{i})}{2} \exp(-\alpha_{m}) - \frac{(1-I_{i})}{2} \exp(\alpha_{m}) \right) = 0$$

$$\exp(-\alpha_{m}) \left( \sum_{i|I_{i}=1} W_{i}^{m-1} \right) = \exp(\alpha_{m}) \left( \sum_{i|I_{i}=-1} W_{i}^{m-1} \right)$$

$$\alpha_m = \frac{1}{2} \ln \left( \frac{\sum_{i|I_i=1} W_i^{m-1}}{\sum_{i|I_i=-1} W_i^{m-1}} \right)$$

If 
$$\sum_{i} W_{i}^{m-1} = 1$$
, then  $\alpha_{m} = \frac{1}{2} \ln(\frac{1 - \epsilon_{m}}{\epsilon_{m}})$ 

# Solving $\theta_m$

$$\sum_{i=1}^{n} W_{i}^{m-1} \exp\{-y_{i}\alpha_{m}h(\mathbf{x}_{i};\theta_{m})\}\$$

$$= \frac{1}{2}\sum_{i} W_{i}^{m-1}I_{i}(\exp(-\alpha_{m}) - \exp(\alpha_{m})) + \frac{1}{2}\sum_{i} W_{i}^{m-1}(\exp(-\alpha_{m}) + \exp(\alpha_{m}))$$

$$= (\exp(-\alpha_{m}) - \exp(\alpha_{m}))\frac{1}{2}\sum_{i} W_{i}^{m-1}I_{i} + (\exp(-\alpha_{m}) + \exp(\alpha_{m}))\frac{1}{2}\sum_{i} W_{i}^{m-1}$$

$$= (\exp(-\alpha_{m}) - \exp(\alpha_{m}))\frac{1}{2}\sum_{i} W_{i}^{m-1}I_{i} + const$$

Equivalent to minimize the weighted error

Equivalent to minimize  $-\sum_{i} W_{i}^{m-1} I_{i}$ 

$$\exp(-y_i\alpha_m h(\mathbf{x}_i;\theta_m)) = \frac{(I_i+1)}{2}\exp(-\alpha_m) - \frac{(I_i-1)}{2}\exp(\alpha_m)$$

#### Update W

From the definition of  $W_i^{m-1}$ 

$$= \sum_{i=1}^{n} \underbrace{\exp\{-y_i h_{m-1}(\mathbf{x}_i)\}}_{\text{fixed at stage } m} \exp\{-y_i \alpha_m h(\mathbf{x}_i; \theta_m)\}$$

$$= \sum_{i=1}^{n} W_i^{(m-1)} \exp\{-y_i \alpha_m h(\mathbf{x}_i; \theta_m)\}$$

and the update equation of  $h_m$ 

$$h_m(\mathbf{x}_i) = h_{m-1} + \alpha_m h(\mathbf{x}_i; \theta_m)$$

We get

$$W_i^m = W_i^{m-1} \times \exp(-y_i \alpha_m h(\mathbf{x}_i; \theta_m))$$

Note that since we update W greedily, it is OK to divide W by a constant or normalize W to make it sum to 1

## Put the above derivation together

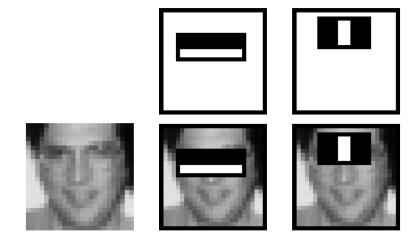
• We show that Adaboost can be seen as a greedy solution to the following optimization problem (the objective function of Adaboost)

$$\min_{\{\alpha_k\},\{\theta_k\}} \sum_i \exp(-y_i h_m(\mathbf{x}_i))$$

• It sequentially optimizes each  $\alpha_k$  and  $\theta_m$ 

## Application of Adaboost

- problem: find faces in photograph or movie
- weak classifiers: detect light/dark rectangles in image

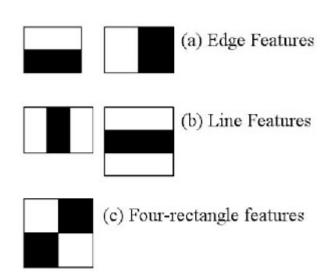


many clever tricks to make extremely fast and accurate

P. Viola; M. Jones. Rapid object detection using a boosted cascade of simple features. CVPR 2001

# Application of Adaboost: Face detection

- Face detection: scan the image with a bounding box and evaluate the decision at each candidate location
  - Very time consuming, need fast evaluation
- Haar features
  - The sum of pixel values in the black area – The sum of pixel values in the white area
  - Parameters:
    - Type of Haar features
    - Location of Haar template inside the bounding box



Combine results by Adaboost Used in OpenCV

#### Summary

- Ensemble Learning
  - Main idea.
  - Why it works
- Random forest
  - Decision tree
  - Injecting randomness to decision tree to make a random forest
- Bagging
- Boosting
  - Adaboost: procedure
  - Weak classifier
  - The objective function of Adaboost