Tutorial 1: Basics

Tutorial 1 will take place in week 2. You should prepare solutions, but you don't have to hand them in and they won't get marked. We will discuss the following questions.

Exercise 1 *Induction Proofs*

Recall the principle of doing proofs by mathematical induction.

1. Prove by mathematical induction that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

   holds for every positive integer n.

2. The nth Fibonnaci number for a given non-negative integer n is defined as

$$fib(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ fib(n-1) + fib(n-2) & \text{if } n \geq 2 \end{cases}$$

   Prove by mathematical induction that $\mathrm{fib}(n) \leq 2^n$ for all $n \geq 0$.

3. Let a and $r \neq 1$ be real numbers. Prove by mathematical induction the geometric series, i. e. that

$$\sum_{i=0}^{n} a \cdot r^i = \frac{a(1 - r^{(n+1)})}{1 - r}$$

   holds for all natural numbers n.

Exercise 2 *Complexity Notation*

Prove that the following rule applies for O-notation: $f(n) + g(n) = O(f(n))$ if $g(n) = O(f(n))$.

Exercise 3 *Complexity Notation*

Prove that $n^k = o(c^n)$ for any fixed integer k and any $c > 1$.