Mathematics for Data Science Tutorial 1 (week 2)

Semester 2, 2019

1. How much rain could be collected from the rooftops of houses in Adelaide in one year? How does this compare to the amount of water the population of Adelaide uses in a year?

   Discuss this question with your colleagues! Try to do it without using any calculators or Googles.

2. Consider the function
$$f(x) = \frac{6 - 10x}{8x + 7}.$$
   (a) What is the domain of f?
   (b) Find the inverse function $f^{-1}(x)$.
   (c) Verify for this function that $f(f^{-1}(x)) = x$.
   (d) What is the range of f? (Use the fact that the range of f^{-1} is equal to the domain of f.)

3. Consider the function
$$f(x) = \begin{cases} |2x - x^2| & \text{if } -1 \leq x < 2 \\ (x-3)H(x-3) & \text{if } x \geq 2 \end{cases}$$
   on the domain $\mathcal{D} = [-1, 4]$.

   Recall that $H(x)$ is the Heaviside function,
$$H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0. \end{cases}$$
   The domain can be broken into four sub-intervals such that on each sub-interval, $f(x)$ can be expressed as a polynomial. Find the sub-intervals and the corresponding polynomial functions. Hence sketch the graph of $f(x)$ and find the range, $\mathcal{R}$.

4. If $f(x) = \sinh x$, $x \in \mathbb{R}$, find the inverse function, $f^{-1}(x)$, giving your answer in terms of the natural logarithm function.
   Hint: use the definition to write $\sinh x$ *in terms of the exponential function, and obtain a quadratic equation satisfied by* e^x.

5. Find all solutions in $[0, \pi]$ of the equation
$$4\cos^2 x + 3\sin^2 x - 2\sin x = 2,$$
   giving your answers in exact form in terms of the arcsin function.
   Hint: Use one of the properties of the sin *and* cos *functions to convert the equation into a quadratic in* $\sin x$.