Course outline

1. Fundamentals
   - Notation
   - Functions
   - Approximation
2. Series
   - Summation
   - Taylor series
3. Linear algebra
   - Representing big, complex, data
   - Systems of equations
   - Dimension reduction
4. Probability
   - Discrete random variables
   - Continuous random variables & integration
5. ~~Optimisation~~
6. Revision

But firstly...

Congratulations!

And thank you.

Housekeeping

Tutorial changes this week

- TU03 will be in Hughes 323, Weds 12-1pm (usually Weds 3pm)
- TU04 will be in Hughes 322, Thurs 2-3pm (usually Weds 2pm)

(If you're in these tutorials and can't make these times, please feel free to join another tutorial this week)

* Course notes: 95% complete

SELTs

- I really really appreciate your feedback!
- You will help create change in this course.
- Particular topics:
  - Python labs
  - Ordering of material (+ volume)
  - Connections to real data science

Exam-writing philosophy

- Proportion of exam marks per topic $\approx$ proportion of lectures per topic
- $\approx$ 70-80% "core" marks, $\approx$ 20-30% "advanced" marks

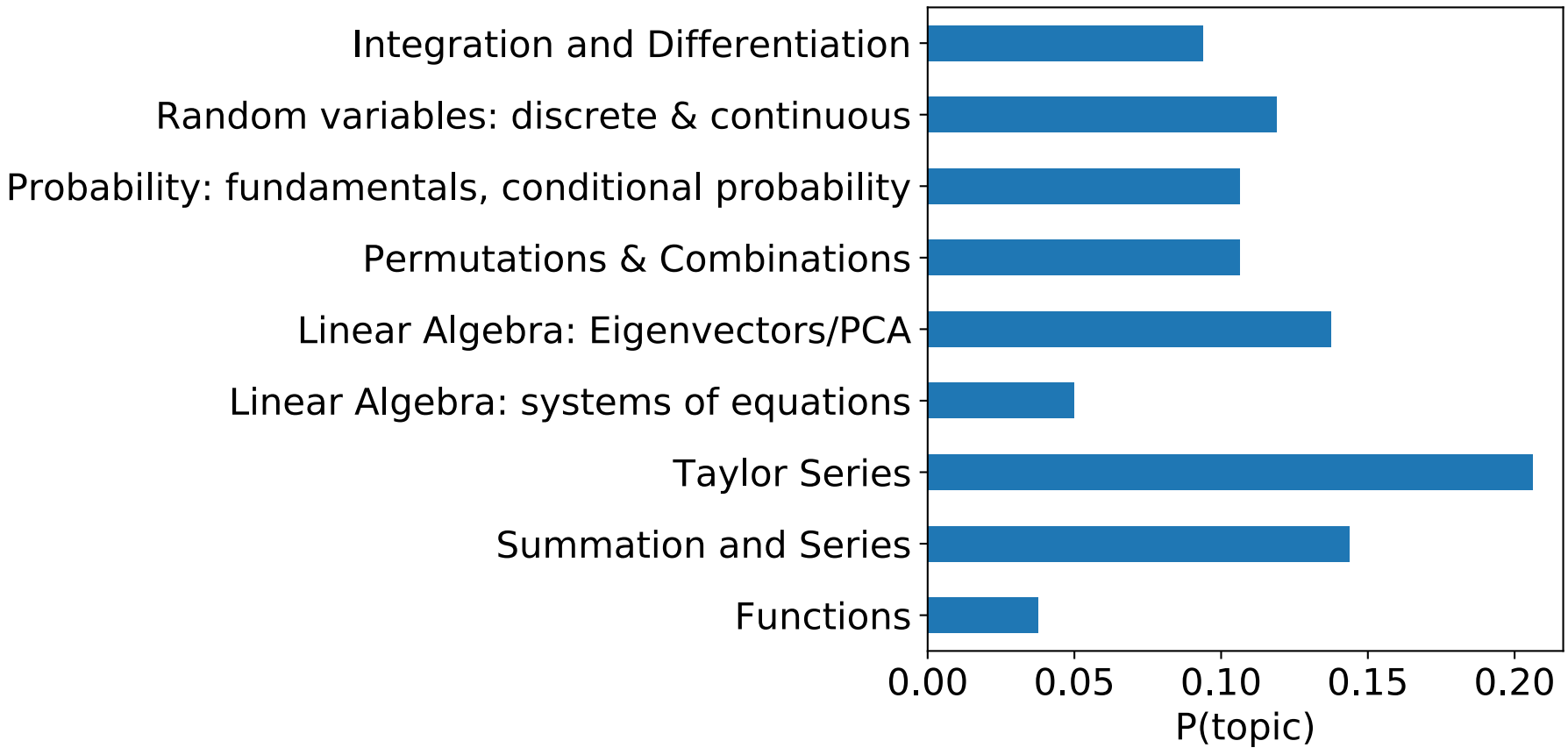
Preparing for the exam

Make sure you can do (in roughly this order):

- Examples from lectures
- Tutorial questions
- All assignments
- The practice exam
- Practice questions
- Problems from the Sacred Texts/course readings

Look for extra problems on the areas you feel weakest!

Survey results



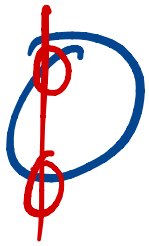
Preparing for the exam

Pre-exam consultation times:

- Tue 5 Nov, 1-3pm (Sophie IW 6.33)
- ~~Wed 5 Nov, 1-3pm (Lewis IW 6.46)~~
- Tue 12 Nov, 11am-1pm (Lewis IW 6.46)
- Thu 21 Nov, 10am-1pm (Sophie IW 6.33)

*Now finalised, apart from rescheduled 5 Nov session TBD

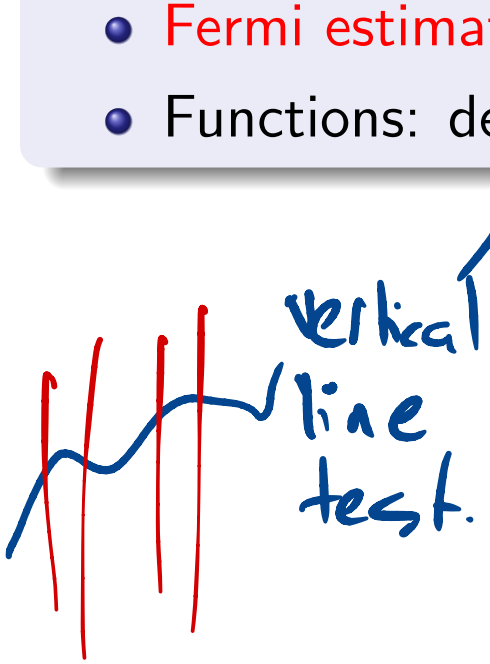
Course topics



"Advanced" content in red

Fundamentals (≈ 3 lectures)

- Notation: sets, functions, etc
- Fermi estimation
- Functions: definition, composition, 1-1 functions, inverse functions



$f(g(x))$

horizontal
line
test

$f^{-1}(f(x)) = x$

Course topics

Summation and series (≈ 3 lectures)

- Notation & manipulation
- Proof by induction
- Multiple summation
- Infinite series & limits
- Convergence of infinite series: ratio test

Course topics

Taylor series (≈ 3 lectures)

- Deriving Taylor polynomials
- Common Maclaurin series
- Error theorem & error bounds
- Intervals of convergence — ratio test again!
- Gradient descent and Taylor series

Course topics

Matrices (≈ 3 lectures)

- Matrices & vectors — ways of representing "big" data.
- Special matrices (e.g., identity)
- Matrix operations:
  - addition & subtraction
  - transposition
  - scalar multiplication
  - matrix multiplication

Course topics

Linear algebra (≈ 6 lectures)

- Systems of equations are the basis of linear regression
- Gauss-Jordan elimination
- (Reduced) row echelon form
- Inverse matrices $A^{-1}A = AA^{-1} = I$
- Determinants (and their relation to the topics above!)

$|A| = 0 \Rightarrow A$ is non-invertible.

$|A| \neq 0 \Rightarrow$ system of equations has a unique solution.

Independent if $a_1\underset{\sim}{v}_1 + a_2\underset{\sim}{v}_2 + \ldots + a_n\underset{\sim}{v}_n = \underset{\sim}{0}$ has only trivial solution $a_1 = a_2 = \ldots = a_n = 0$.

Course topics

Eigenvalues & eigenvectors (≈ 5 lectures)

- Eigenvalues/vectors satisfy $A\boldsymbol{x} = \lambda\boldsymbol{x}$
- Characteristic equation $|\lambda I - A| = 0$ to find eigenvalues
- Gauss-Jordan to find eigenvectors
- Linear (in)dependence of vectors eg. $3\underset{\sim}{v}_1 + 2\underset{\sim}{v}_2 = \underset{\sim}{0} \Rightarrow \underset{\sim}{v}_2 = \frac{-3\underset{\sim}{v}_1}{2}$ dependent.
- Eigenspaces (the set of all eigenvectors for a particular λ)
- Diagonalisation $P^{-1}AP = D$
- Applications:
  - Dynamical systems (Spotted Owl aww so cute)
  - Principal component analysis — eigenvectors of covariance matrix.
  - Google's PageRank

Course topics

Probability: fundamentals (≈ 5 lectures)

- Counting:
  - Permutations without replacement
  - Combinations without replacement
  - Permutations with replacement
  - Combinations with replacement
- Binomial coefficient and theorem
- Axioms of probability
- Conditional probability
- Bayes theorem & naive Bayes classifiers
- Law of Total Probability

- Independent events.

Course topics

Probability: random variables (≈ 6 lectures)

- Discrete random variables:
  - Definition & properties
  - Expectation & variance
  - Bernoulli, binomial, Poisson
  - Ranking items with ratings
- Continuous random variables:
  - Probability density functions, probabilities are integrals
  - Expectation & variance
  - Integration recap:
    - Improper integrals
    - Integration by parts

Where to from here?

If you enjoyed:

Statistics-y/Data Science-y examples

- Linear regression (Boston housing)
- Logistic regression (Titanic dataset)
- PCA
- Wisconsin breast cancer dataset

Then you should check out:

STATS 2107 Statistical Modelling & Inference

Where to from here?

If you enjoyed:

Probability & random variables

- Google's PageRank
- Text generation (from labs)
- Markov chains
- Ranking products by rating

Then you should check out:

MATHS 2013 Probability & Statistics II

Where to from here?

If you enjoyed:

Linear algebra applications

- The spotted owl (so cute)
- Optimisation
- Programming

Then you should check out:

APP MTH 2105 Optimisation & Operations Research II

Where to from here?

If you enjoyed:

Gradient descent

- Machine learning
- Optimisation (e.g. `http://fa.bianp.net/teaching/2018/eecs227at/`)
- Programming, `sklearn`
- (Multivariate) calculus

Then you should check out:

APP MTH 3104 Optimisation III

And a shameless plug

If you enjoyed:

The things I like to research

- Online social networks: Reddit, Twitter, ...
- Text data/natural language processing/sentiment analysis
- hedonometer.org lab example

Then you should check out:

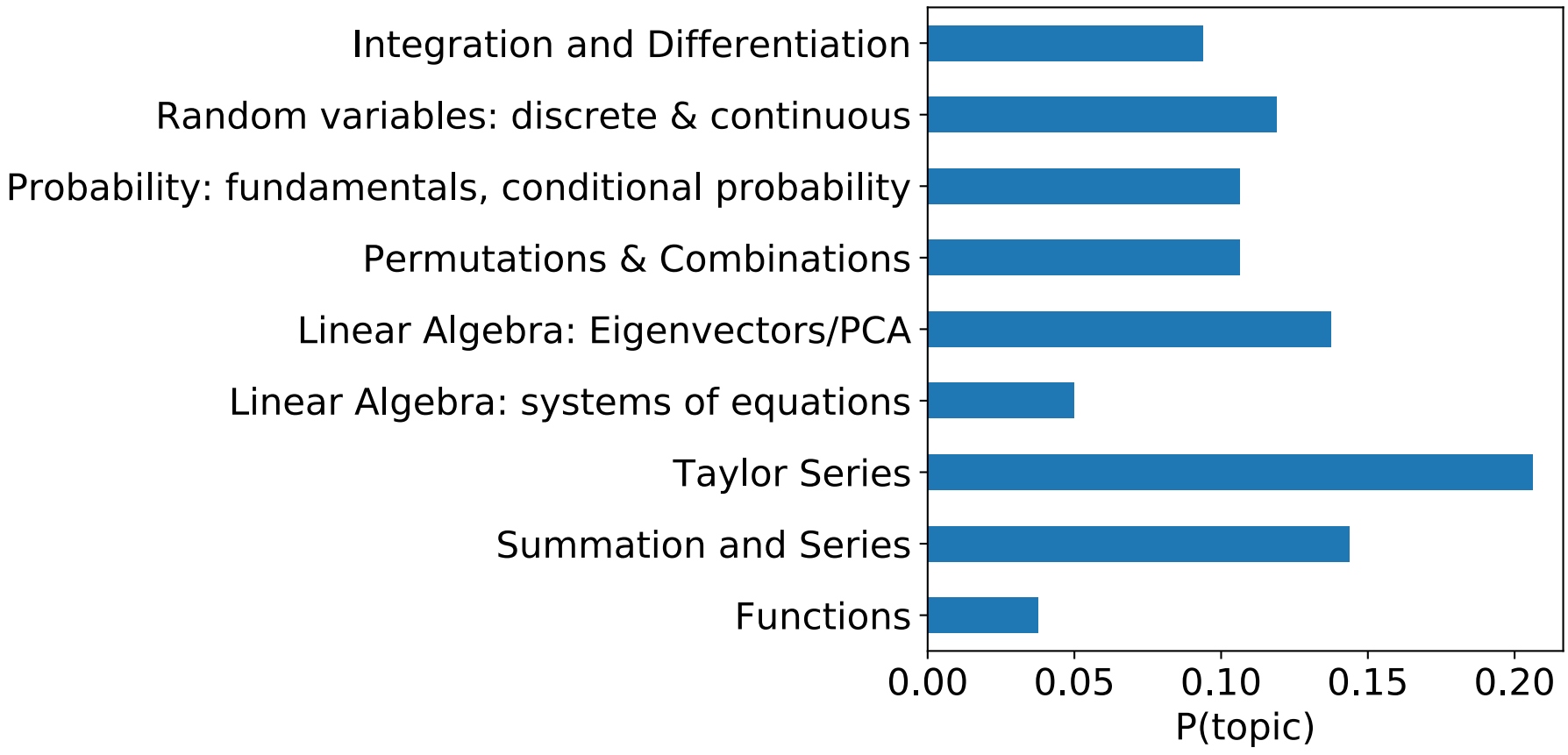
- `http://maths.adelaide.edu.au/lewis.mitchell/`
- `@lewis_math`
- Hedonometer at MOD (before Sunday)

So once again...

Congratulations! (Really.)

And thank you. (Really.)

Survey results



Taylor series revision

Example

Find the Taylor polynomial of degree 3 for the function $f(x) = \frac{1}{x}$ about the centre $a = -3$.

Taylor series revision

Example

Find the Taylor polynomial of degree 3 for the function $f(x) = \frac{1}{x}$ about the centre $a = -3$.

Example

- Find the Taylor series for $f(x) = (x+1)^k$ around the centre $a = 0$.
- What is its radius of convergence?
- What is the error when using 3 terms of this series to approximate $\sqrt{2}$?

Summation and series revision

Example

Evaluate:

- $$\sum_{i=1}^{n} i(4i^2 - 3)$$
- $$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[\left(\frac{i}{n}\right)^2 + 1\right]$$

Summation and series revision

Key formulae

$$\lim_{n\to\infty} \frac{1}{n^k} = 0, \quad k > 0$$

$$\lim_{x\to\infty} e^{-x} = 0$$

Ratio test:

$$r = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

if $r < 1$ the series converges,
if $r > 1$ the series diverges,
if $r = 1$ the ratio test is inconclusive.

Summation and series revision

Example

Apply the ratio test to each of the following series to investigate convergence.

- $$\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$$
- $$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$$

Eigenvalues/eigenvectors/PCA revision

Example

Show $(1, 1)$ is an eigenvector of

$$\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$$

and find its corresponding eigenvalue.

Example

Show 5 is an eigenvalue of

$$\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

and find its corresponding eigenspace.