The University of Adelaide
School of Computer Science

Artificial Intelligence
Tutorial 5

Question 1 (Based on Question 15.2 of Russell and Norvig, 3ed)
In this exercise, we examine what happens to the probabilities in the umbrella world in the limit of long time sequences.

1. Suppose we observe an unending sequence of days on which the umbrella appears. Show that, as the days go by, the probability of rain on the current day increases monotonically toward a fixed point. Calculate this fixed point.

2. Now consider forecasting further and further into the future, given just the first two umbrella observations. First, compute the probability $P(R_{2+k} \mid U_1, U_2)$ for $k = 1, \dots, 20$ and plot the results. You should see that the probability converges towards a fixed point. Prove that the exact value of this fixed point is 0.5.

Question 2 (Based on Question 15.3 of Russell and Norvig, 3ed)
This exercise develops a space-efficient variant of the forward?backward algorithm described in the lectures. We wish to compute $P(\mathbf{X}_k \mid \mathbf{e}_{1:t})$ for $k = 1, \dots, t$. This will be done with a divide-and-conquer approach.

1. Suppose, for simplicity, that t is odd, and let the halfway point be $h = (t+1)/2$. Show that $P(\mathbf{X}_k \mid \mathbf{e}_{1:t})$ can be computed for $k = 1, \dots, h$ given just the initial forward message $\mathbf{f}_{1:0}$, the backward message $\mathbf{b}_{h+1:t}$, and the evidence $\mathbf{e}_{1:h}$.

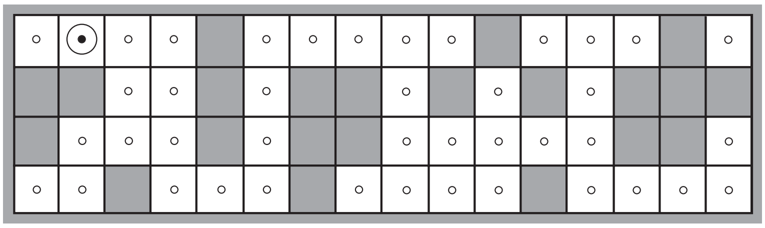
2. Show a similar result for the second half of the sequence.

3. Given the results of 1 and 2 above, a recursive divide-and-conquer algorithm can be constructed by first running forward along the sequence and then backward from the end, storing just the required messages at the middle and the ends. Then the algorithm is called on each half. Write out the algorithm in detail.

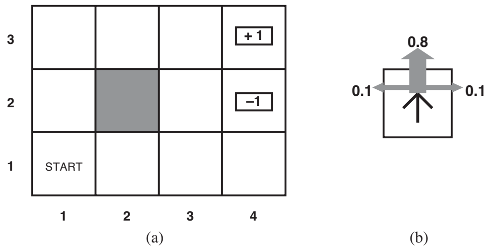
4. Compute the time and space complexity of the algorithm as a function of t, the length of the sequence. How does this change if we divide the input into more than two pieces?

Question 3 (Based on Question 15.6 of Russell and Norvig, 3ed)
Consider the following map for 2D robot localisation:

Suppose that the robot receives an observation sequence such that, with perfect sensing, there is exactly one possible location it could be in. Is this location necessarily the most probable location under noisy sensing for sufficiently small noise probability ε? Prove your claim or find a counterexample.



Question 4 (Based on Question 17.1 of Russell and Norvig, 3ed)



[redacted] (a) A simple 4×3 environment that presents the agent with a sequential decision problem. (b) Illustration of the transition model of the environment: the "intended" outcome occurs with probability 0.8, but with probability 0.2 the agent moves at right angles to the intended direction. A collision with a wall results in no movement. The two terminal states have reward +1 and –1, respectively, and all other states have a reward of –0.04.

For the 4×3 world shown above, calculate which squares can be reached from (1,1) by the action sequence [Up,Up,Right,Right,Right] and with what probabilities. Explain how this computation is related to the prediction task for a hidden Markov model.

Question 5 (Based on Question 17.8 of Russell and Norvig, 3ed)

Consider the 3 3 world shown below, where the transition model is the same as in the 4×3 world in Question 4 above: 80% of the time the agent goes in the direction it selects; the rest of the time it moves at right angles to the intended direction.

r	-1	$\boxed{+10}$
-1	-1	-1
-1	-1	-1

Implement value iteration for this world for each value of r below. Use discounted rewards with a discount factor of 0.99. Show the policy obtained in each case. Explain intuitively why the value of r leads to each policy.

1. $r = 100$
2. $r = -3$
3. $r = 0$
4. $r = +3$