Practice Questions (week 9)

Semester 2, 2019

These questions are about partitions, recurrence, axioms of probability, probability of two or more events, inclusion–exclusion principle, disjoint events. Good places to go for further questions on this topic include the exercises in:

- Ross, *A first course in Probability* (6th Ed.), Chapter 2.

1. A coin is flipped and a dice is rolled
   (a) Describe the sample space
   (b) What is the size of the sample space
   (c) Describe the event in which an even number is rolled
   (d) Describe the event in which tails is flipped and a number greater then 2 is rolled.

2. Suppose we are monitoring the twitter accounts of prominent politicians and each day record whether or not they have sent a tweet.
   (a) Describe the sample space if we check on three politicians on a given day.
   (b) Describe the sample space if we observe two politicians over two consecutive days

3. Let S be a sample space and $E, F \subset S$ be events.
   (a) Draw a Venn diagram representing each of the following sets/events.
      i. E^c
      ii. $E \cup F^c$
      iii. $E^c \cap F^c$
   (b) Consider now having a third event $G \subset S$. Draw a Venn diagram representing each of the following.
      i. $(E \cap F) \cup G$
      ii. $E \cap (F \cup G)$

4. A small country hospital having 6 beds in the emergency section, keeps an hourly record consisting of the number of occupied beds, and the number of patients in those beds in a critical condition.
   (a) Describe the sample space S
   (b) What is the size of the sample space?
   (c) Describe the event E in which at least half of the beds are occupied by patients in critical condition.

(d) Describe the event F in which at most 4 beds are occupied.

(e) Describe the event G in which all current patients are critical (and there is at least one patient total).

(f) Describe the event $F \cap G^c$ and its interpretation.

(g) Describe the event $E \cup (F \cap G^c)$ and give its size.

(h) Apply De Morgan's laws to show that $(E \cup (F \cap G^c))^c = E^c \cap (F^c \cup G)$

5. Let S be a sample space and suppose $E, F \subset S$ are events in that space. For each of the following, indicate if it is possible (True or False), and give a brief reason.

   (a) $\Pr(E) = -0.1$

   (b) $\Pr(F) = 0.8$

   (c) $\Pr(S) = 1.2$

   (d) $\Pr(E) = 0.5$ and $\Pr(F) = 0.6$

   (e) $\Pr(E \cup F) = 1.1$

   (f) $\Pr(E) = 0.5$ and $\Pr(F) = 0.6$ when E, F are mutually exclusive

   (g) $\Pr(E) = 0.5$ and $\Pr(E \cap F) = 0.6$

   (h) $\Pr(E) = 0.5$, $\Pr(F) = 0.6$ and $\Pr(E \cap F) = 0.2$

6. Let S be a sample space and suppose $E, F \subset S$ are events in that space. Prove each of the following:

   (a) $\Pr(E^c \cap F) = \Pr(F) - \Pr(E \cap F)$

   (b) $\Pr(E \cap F) \geq \Pr(E) + \Pr(F) - 1$

   (c) The probability of E or F occurring, but not both, is $2 - \Pr(E \cup F^c) - \Pr(F \cup E^c)$

   (d) Based on the result of part (c), explain why $\Pr(E \cup F^c) + \Pr(F \cup E^c) \geq 1$

7. In each of the following 2 events are described. Give a brief explanation whether or not the events are mutually exclusive.

   (a) It rains in Adelaide tomorrow, and, it is stormy in Adelaide tomorrow.

   (b) Politician A wins the next election, and, politician B wins the next election (both in the same electorate).

   (c) You ace the next assignment, and, you pass the final exam.

8. South Australia currently has 10 MPs in the house of representatives, 5 from the Labor party, 4 from the Liberal party and 1 from the Centre Alliance. Additionally, exactly one MP from each of the three parties is female. Consider choosing one of the 10 MPs at random. What is the probability the chosen MP is

(a) female?
(b) from the Liberal party?
(c) from the Labor party or female?
(d) from the Centre Alliance party or male?

9. A box contains 8 red blocks and 4 blue blocks. Consider an experiment in which three of the blocks are chosen at random (one at a time). What is the probability that

   (a) the first chosen block is blue?
   (b) two of the three chosen blocks are red?
   (c) at least one blue block is chosen?

10. Suppose we have collected data on 1000 Facebook users and their membership status of 3 different Facebook groups (call them groups A, B, C say). From this data we know that

    - 56% are members of (at least) group A
    - 43% are members of (at least) group B
    - 37% are members of (at least) group C
    - 23% are members of (at least) groups A and B
    - 26% are members of (at least) groups A and C
    - 19% are members of (at least) groups B and C
    - 82% are members of at least one of the three groups

    (a) How many people are in none of the three groups?
    (b) How many people are in all three groups?
    (c) How many people are in exactly one of the three groups?
    (d) How many people are in at least two of the three groups?

11. Consider dealing a three card hand from a standard 52 card deck (which has been shuffled). What is the probability of each of the following events:

    (a) the hand contains a queen?
    (b) the hand contains a spade?
    (c) the hand does not contain a clubs, heart nor ace card?
    (d) the hand contains a diamond and a king?

12. Consider the tiler problem from the week 8 practice questions (Q14). That is a tiler has m black tiles and n white tiles with $m > n$ and wants to lay them out so that no two white tiles are next to each other. If every possible valid pattern is equally likely, what is the probability that the pattern starts with a black tile?

13. A game is played in which a player repeatedly rolls a dice until they roll a 6 and their score is determined by the number of dice rolls.

    (a) Describe the sample space for the possible sequence of dice rolls that may occur.

    (b) Let E_n be the event the game has not ended after n rolls (alternatively, the event that the final score is greater than n). Explain why $E_{n+1} \subset E_n$.

    (c) What is the event $\left(\bigcap_{n=4}^{\infty} E_n\right)^c$?

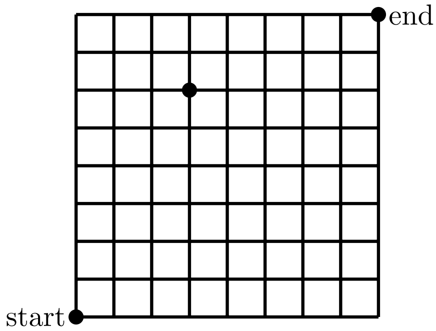
    (d) Explain why $\Pr(E_{n+1}) = \frac{5}{6}\Pr(E_n)$.

    (e) The event that the game never ends is $\lim_{n\to\infty} E_n$, what is the probability of this event?

    (f) What is the probability of obtaining a score of at most 5?

14. Suppose we randomly choose one letter from each of the words ADELAIDE and KARRAWIRRA so that the chance of each occurrence of each letter in each word is equiprobable (e.g. so that D has two chances to be chosen out of the eight letters in ADELAIDE). What is the probability that the same letter is chosen from the two words?

15. Consider an ant that must traverse the grid shown on the right from start to end only moving up or right along each edge. Suppose that every possible path is equally likely.



    (a) What is the probability that the ant passes through the point marked in the grid?

    (b) What is the probability that the ant remains below the point marked on the grid (without passing through)?