Practice Questions (week 9)

Semester 2, 2019

These questions are about partitions, recurrence, axioms of probability, probability of two or more events, inclusion–exclusion principle, disjoint events. Good places to go for further questions on this topic include the exercises in:

- Ross, *A first course in Probability* (6th Ed.), Chapter 2.

1. A coin is flipped and a dice is rolled

   (a) Describe the sample space

   (b) What is the size of the sample space

   (c) Describe the event in which an even number is rolled

   (d) Describe the event in which tails is flipped and a number greater then 2 is rolled.

   Solution:

   (a) Writing each outcome in the form ({outcome of coin flip},{outcome of dice roll}) we get the sample space

   $$S = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),\\ (T,1),(T,2),(T,3),(T,4),(T,5),(T,6)\}$$

   (It is also perfectly fine to write the tuples in the opposite order)

   (b) $|S| = 12$

   (c) The event of rolling an even number is

   $$E = \{(H,2),(H,4),(H,6),(T,2),(T,4),(T,6)\}$$

   (d) The event of flipping tails and rolling a number greater than 2 is

   $$F = \{(T,3),(T,4),(T,5),(T,6)\}\,.$$

---

2. Suppose we are monitoring the twitter accounts of prominent politicians and each day record whether or not they have sent a tweet.

   (a) Describe the sample space if we check on three politicians on a given day.

(b) Describe the sample space if we observe two politicians over two consecutive days

Solution:

(a) Let T denote the event that a politician sends a tweet that day, and N be the event they do not. Then each outcome can be written as tuple (each entry corresponding to a particular politician) to obtain the sample space

$$\begin{aligned} S = \{&(N,N,N),(T,N,N),(N,T,N),(N,N,T), \\ &(T,T,N),(T,N,T),(N,T,T),(T,T,T)\}. \end{aligned}$$

(b) Again let T denote the event that a politician sends a tweet that day, and N be the event they do not. On a single day the possible outcomes are $\{(N,N),(T,N),(N,T),(T,T)\}$, and over two days we can write each outcome as a tuple of tuples to obtain the sample space

$$\begin{aligned} S = \{&((N,N),(N,N)),((T,N),(N,N)),((N,T),(N,N)),((T,T),(N,N)), \\ &((N,N),(T,N)),((T,N),(T,N)),((N,T),(T,N)),((T,T),(T,N)), \\ &((N,N),(N,T)),((T,N),(N,T)),((N,T),(N,T)),((T,T),(N,T)), \\ &((N,N),(T,T)),((T,N),(T,T)),((N,T),(T,T)),((T,T),(T,T))\}. \end{aligned}$$

There are potentially many more ways this could be expressed, the important thing is that we capture all 16 possible outcomes.

---

3. Let S be a sample space and $E, F \subset S$ be events.

(a) Draw a Venn diagram representing each of the following sets/events.

i. E^c

ii. $E \cup F^c$

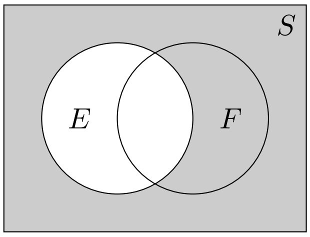
iii. $E^c \cap F^c$

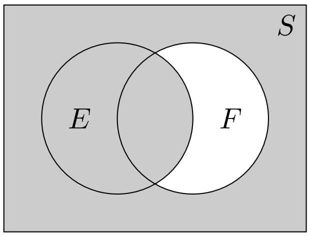
(b) Consider now having a third event $G \subset S$. Draw a Venn diagram representing each of the following.

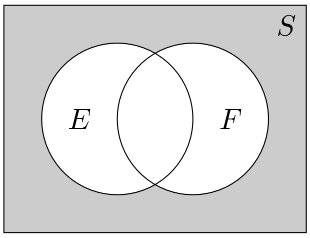
i. $(E \cap F) \cup G$

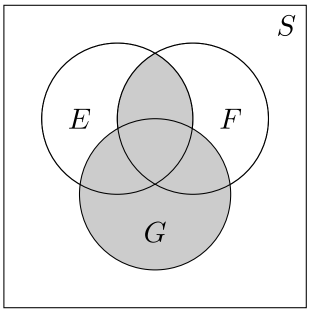
ii. $E \cap (F \cup G)$

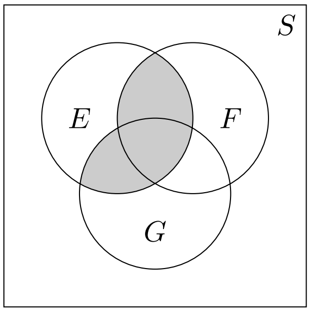
Solution:

(a) i. 

ii. 

iii. 

(b) i. 

ii. 

---

4. A small country hospital having 6 beds in the emergency section, keeps an hourly record consisting of the number of occupied beds, and the number of patients in those beds in a critical condition.

   (a) Describe the sample space S

(b) What is the size of the sample space?

(c) Describe the event E in which at least half of the beds are occupied by patients in critical condition.

(d) Describe the event F in which at most 4 beds are occupied.

(e) Describe the event G in which all current patients are critical (and there is at least one patient total).

(f) Describe the event $F \cap G^c$ and its interpretation.

(g) Describe the event $E \cup (F \cap G^c)$ and give its size.

(h) Apply De Morgan's laws to show that $(E \cup (F \cap G^c))^c = E^c \cap (F^c \cup G)$

Solution:

(a) The sample space can be denoted by

$$S = \{(a, b) : a, b \in \{0, 1, 2, \ldots, 6\} \text{ and } b \leq a\}.$$

Or in full:

$$S = \{(0,0), (1,0), (1,1), (2,0), (2,1), (2,2), (3,0), \ldots, (3,3), \\ (4,0), \ldots, (4,4), (5,0), \ldots, (5,5), (6,0), \ldots, (6,6)\}.$$

(b) One has $|S| = 1 + 2 + 3 + \cdots + 7 = 28$.

(c) $E = \{(3,3), (4,3), (4,4), (5,3), (5,4), (5,5), (6,3), (6,4), (6,5), (6,6)\}$

(d) $F = \{(0,0), (1,0), (1,1), (2,0), (2,1), (2,2), (3,0), \ldots, (3,3), (4,0), \ldots, (4,4)\}$

(e) $G = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

(f) $F \cap G^c = \{(0,0), (1,0), (2,0), (2,1), (3,0), \ldots, (3,2), (4,0), \ldots, (4,3)\}$ is the event that at most 4 occupied beds and not all patients are critical.

(g) $E \cup (F \cap G^c) = S \setminus \{(1,1), (2,2), (5,0), (5,1), (5,2), (6,0), (6,1), (6,2)\}$ and $|E \cup (F \cap G^c)| = 8$.

(h) Applying the law first to the union we obtain $(E \cup (F \cap G^c))^c = E^c \cap (F \cap G^c)^c$. Now applying the law to the second intersection we obtain $E^c \cap (F \cap G^c)^c = E^c \cap (F^c \cup G)$.

---

5. Let S be a sample space and suppose $E, F \subset S$ are events in that space. For each of the following, indicate if it is possible (True or False), and give a brief reason.

(a) $\Pr(E) = -0.1$
(b) $\Pr(F) = 0.8$
(c) $\Pr(S) = 1.2$
(d) $\Pr(E) = 0.5$ and $\Pr(F) = 0.6$
(e) $\Pr(E \cup F) = 1.1$
(f) $\Pr(E) = 0.5$ and $\Pr(F) = 0.6$ when E, F are mutually exclusive
(g) $\Pr(E) = 0.5$ and $\Pr(E \cap F) = 0.6$
(h) $\Pr(E) = 0.5$, $\Pr(F) = 0.6$ and $\Pr(E \cap F) = 0.2$

Solution:

(a) False, since $0 \leq \Pr(E) \leq 1$ (axiom 1)
(b) True, since $0 \leq \Pr(F) \leq 1$ is allowed (axiom 1)
(c) False, since $\Pr(S) = 1$ (axiom 2)
(d) True, since $0 \leq \Pr(E) \leq 1$ and $0 \leq \Pr(F) \leq 1$ are allowed
(e) False, let $G = E \cup F$, then $G \subset S$ and $0 \leq \Pr(G) \leq 1$ (axiom 1)
(f) False, let $G = E \cup F$, then $G \subset S$ and $\Pr(G) = \Pr(E) + \Pr(F)$ (axiom 3) which implies $\Pr(G) > 1$ which is not possible (axiom 1)
(g) False, let $G = E \cap F$, then $G \subset E$ but we have $\Pr(G) > \Pr(E)$ (whereas it must be that $\Pr(G) \leq \Pr(E)$)
(h) True, letting $G = E \cup F$ then $\Pr(G) = \Pr(E) + \Pr(F) - \Pr(E \cap F) = 0.9$ so we have $0 \leq \Pr(G) \leq 1$ as required.

---

6. Let S be a sample space and suppose $E, F \subset S$ are events in that space. Prove each of the following:

(a) $\Pr(E^c \cap F) = \Pr(F) - \Pr(E \cap F)$
(b) $\Pr(E \cap F) \geq \Pr(E) + \Pr(F) - 1$
(c) The probability of E or F occurring, but not both, is $2 - \Pr(E \cup F^c) - \Pr(F \cup E^c)$
(d) Based on the result of part (c), explain why $\Pr(E \cup F^c) + \Pr(F \cup E^c) \geq 1$

Solution:

(a) The event F can be partitioned (or broken up) into the two mutually exclusive events 'F and E' and 'F and E^c'. It follows that

$$\Pr(F) = \Pr(F \cap E) + \Pr(F \cap E^c)\,,$$

and the desired result follows by subtracting $\Pr(E \cap F)$ from both sides.

(b) By the inclusion/exclusion principle

$$\Pr(E \cap F) = \Pr(E) + \Pr(F) - \Pr(E \cup F)$$

and since $\Pr(E \cup F) \leq 1$ then the result follows.
(This result is also known as Bonferroni's inequality)

(c) The event of E but not F occurring is $E \cap F^c$ and by the inclusion/exclusion principle

$$\Pr(E \cap F^c) = \Pr(E) + \Pr(F^c) - \Pr(E \cup F^c)\,.$$

Similarly the event of F but not E occurring is $F \cap E^c$ and by the inclusion/exclusion principle

$$\Pr(F \cap E^c) = \Pr(F) + \Pr(E^c) - \Pr(F \cup E^c)$$

Since 'E but not F occurring' and 'F but not E occurring' are mutually exclusive (draw a Venn diagram for each to convince yourself this is true) then

$$\begin{aligned}\Pr((E \cap F^c) \cup (F \cap E^c)) &= \Pr(E) + \Pr(F^c) - \Pr(E \cup F^c) \\ &\quad + \Pr(F) + \Pr(E^c) - \Pr(F \cup E^c) \\ &= 2 - \Pr(E \cup F^c) - \Pr(F \cup E^c)\,.\end{aligned}$$

(d) From the first axiom we must have $0 \leq \Pr((E \cap F^c) \cup (F \cap E^c)) \leq 1$ from which it follows that

$$2 - \Pr(E \cup F^c) - \Pr(F \cup E^c) \leq 1\,.$$

Re-arranging gives the desired result.

---

7. In each of the following 2 events are described. Give a brief explanation whether or not the events are mutually exclusive.

(a) It rains in Adelaide tomorrow, and, it is stormy in Adelaide tomorrow.

(b) Politician A wins the next election, and, politician B wins the next election (both in the same electorate).

(c) You ace the next assignment, and, you pass the final exam.

Solution:

(a) These events are not mutually exclusive since it is possible for both to occur (i.e. their intersection is not zero).

(b) These are mutually exclusive since both cannot win (i.e. their intersection is zero).

(c) These are not mutually exclusive since it is possible for both to occur (and highly correlated!).

---

8. South Australia currently has 10 MPs in the house of representatives, 5 from the Labor party, 4 from the Liberal party and 1 from the Centre Alliance. Additionally, exactly one MP from each of the three parties is female. Consider choosing one of the 10 MPs at random. What is the probability the chosen MP is

   (a) female?

   (b) from the Liberal party?

   (c) from the Labor party or female?

   (d) from the Centre Alliance party or male?

   Solution:

   (a) Since three of the ten MPs are female the probability is 3/10, or 0.3.

   (b) Since four of the ten MPs are from the Liberal party the probability is 4/10, or 0.4.

   (c) Since there are five MPs from the Labor party, and three female MPs, but one of those in the Labor party, the probability is 7/10, or 0.7

   (d) The one Centre Alliance candidate is female, and additionally there are seven male MPs, thus the probability is 8/10, or 0.8.

---

9. A box contains 8 red blocks and 4 blue blocks. Consider an experiment in which three of the blocks are chosen at random (one at a time). What is the probability that

(a) the first chosen block is blue?

(b) two of the three chosen blocks are red?

(c) at least one blue block is chosen?

Solution:

(a) Since there are a total of 12 blocks, and 4 of them are blue, the probability of choosing a blue block first is 4/12 0.333.

(b) We can approach this two ways. First, if we consider the order to be important there are $12 \cdot 11 \cdot 10 = 1320$ possible outcomes. There are $8 \cdot 7 \cdot 4 = 224$ outcomes where the first two are red, $8 \cdot 4 \cdot 7 = 224$ outcomes where the first and last are red, and $4 \cdot 8 \cdot 7 = 224$ outcomes where the last two are red. Thus a total of 672 outcomes have two red blocks out of the 1320 total outcomes. Therefore the probability of choosing two red blocks out of three is $672/1320 \approx 0.509$.
The second approach is to disregard the order. There are $\binom{12}{3} = 220$ possible combinations of balls that can be chosen. The number of ways in which two red blocks and one blue block can be chosen is $\binom{8}{2}\binom{4}{1} = 28 \cdot 4 = 112$. Therefore the probability is $112/220 \approx 0.509$.

(c) The event of choosing at least one blue block is equivalent to the complement of choosing all red blocks. There are $\binom{8}{3} = 56$ ways in which just red blocks can be chosen, and therefore the probability of choosing at least one blue block is $1 - 56/220 = 164/220 \approx 0.745$.
(Again, there are other approaches to solve this. For example, we could add the probability of drawing one, two and three blue blocks, which are $\binom{8}{2}\binom{4}{1} = 112$, $\binom{8}{1}\binom{4}{2} = 48$ and $\binom{4}{3} = 4$ respectively, thus leading to 164/220 as above.)

---

10. Suppose we have collected data on 1000 Facebook users and their membership status of 3 different Facebook groups (call them groups A, B, C say). From this data we know that

- 56% are members of (at least) group A
- 43% are members of (at least) group B
- 37% are members of (at least) group C
- 23% are members of (at least) groups A and B
- 26% are members of (at least) groups A and C
- 19% are members of (at least) groups B and C
- 82% are members of at least one of the three groups

(a) How many people are in none of the three groups?
(b) How many people are in all three groups?
(c) How many people are in exactly one of the three groups?
(d) How many people are in at least two of the three groups?

Solution:

(a) We'll use A, B, C denote the events that a randomly chosen user is in the respective groups. Being in none of the three groups is the event $(A \cup B \cup C)^c$ and therefore

$$\Pr((A \cup B \cup C)^c) = 1 - \Pr(A \cup B \cup C) = 0.18 \,.$$

It follows that $0.18 \times 1000 = 180$ people are in none of the three groups.

(b) From the inclusion/exclusion principle we know that

$$\begin{aligned}\Pr(A \cup B \cup C) = {} & \Pr(A) + \Pr(B) + \Pr(C) \\ & - \Pr(A \cap B) - \Pr(A \cap C) - \Pr(B \cap C) + \Pr(A \cap B \cap C)\end{aligned}$$

which we can re-arrange to obtain

$$\begin{aligned}\Pr(A \cap B \cap C) &= \Pr(A \cup B \cup C) - \Pr(A) - \Pr(B) - \Pr(C) \\ &\quad + \Pr(A \cap B) + \Pr(A \cap C) + \Pr(B \cap C) \\ &= 0.82 - 0.56 - 0.43 - 0.37 + 0.23 + 0.26 + 0.19 \\ &= 0.14 \,.\end{aligned}$$

It follows that $0.14 \times 1000 = 140$ people are in all three groups.

(c) We know that $0.82 \times 1000 = 820$ people are in at least one group. If we subtract the number of people in A and B, then subtract the number of people in A and C, and then subtract the number of people in B and C we obtain $820 - (0.23 + 0.26 + 0.19) \times 1000 = 140$. However, we have subtracted the number of people in all three groups three times and therefore need to add it back in twice (since we only want to subtract it once), and then obtain $140 + 2 \times 0.14 \times 1000 = 420$ people.
Alternatively, being only in group A is the event $A \cap (B \cup C)^c$ and has probability

$$\begin{aligned}\Pr(A \cap (B \cup C)^c) &= \Pr(A) - \Pr(A \cap (B \cup C)) \\ &= \Pr(A) - (\Pr(A \cap B) + \Pr(A \cap C) - \Pr(A \cap (B \cap C))) \\ &= 0.56 - 0.23 - 0.26 + 0.14 = 0.21 \,.\end{aligned}$$

Similarly one can obtain $\Pr(B \cap (A \cup C)^c) = 0.43 - 0.23 - 0.19 + 0.14 = 0.15$ and $\Pr(C \cap (A \cup B)^c) = 0.37 - 0.26 - 0.19 + 0.14 = 0.06$ and since the three separate events are mutually exclusive we can add them together to obtain the probability of being in exactly one of the three groups is 0.42, corresponding to 420 people.

(d) There are 820 in at least one group and 420 in exactly one group, so there must be $820 - 420 = 400$ in two or more groups. Alternatively, the number of people in at least two of the three groups is given by

$$\begin{aligned} &1000 \times (\Pr(A \cap B) + \Pr(A \cap C) + \Pr(B \cap C) - 2\Pr(A \cap B \cap C)) \\ &\qquad = 230 + 260 + 190 - 280 = 400\,. \end{aligned}$$

Alternatively, we could approach each of these by first converting each $\Pr(E)$ to $|E| = 1000 \times \Pr(E)$ and then apply the inclusion/exclusion principle to the sizes of the relevant subsets.

---

11. Consider dealing a three card hand from a standard 52 card deck (which has been shuffled). What is the probability of each of the following events:

(a) the hand contains a queen?

(b) the hand contains a spade?

(c) the hand does not contain a clubs, heart nor ace card?

(d) the hand contains a diamond and a king?

Solution:

(a) There are $52 \times 51 \times 50 = 132600$ possible hands. Consider hands which do not contain a queen. Since there are 48 cards which are not a queen then the hands which do not contain a queen are $48 \times 47 \times 46 = 103776$. Therefore $132600 - 103776 = 28824$ hands must contain a queen so the probability of such a hand is $28824/132600 \approx 0.217$.
(Alternatively one could work out the number of hands with one, two or three queens, then add them up.)

(b) Again, it is simpler to approach this from the complement. The number of hands which do not contain a spade are $39 \times 38 \times 37 = 54834$. Thus, the probability of dealing a hand containing a spade is $(132600 - 54834)/132600 = 77766/132600 \approx 0.586$.

(c) The number of cards which are neither a clubs, heart nor ace is 24. Thus there are $24 \times 23 \times 22 = 12144$ possible hands that can be made with these cards. Therefore, the probability of dealing such a hand is $12144/132600 \approx 0.0916$.

(d) Again, it is simplest to approach from the complement. Let D denote the event of a hand containing a diamond card and K denote the event of a hand containing a king card. A hand avoiding both diamonds and kings has only 36 cards to choose from, thus leading to $36 \times 35 \times 34 = 42840$ possible hands. Thus, the probability of dealing a hand containing a diamond **or** a king (that is $\Pr(D \cup K)$) is $(132600 - 42840)/132600 = 89760/132600 \approx 0.677$. Now we use the inclusion/exclusion principle.

$$\Pr(D \cap K) = \Pr(D) + \Pr(K) - \Pr(D \cup K) \quad = \frac{77766}{132600} + \frac{28824}{132600} - \frac{89760}{132600} = \frac{16830}{132600} \approx 0$$

(Note: here I have used that $\Pr(D)$ is the same as the probability of a hand containing a spade, and $\Pr(K)$ is identical to the probability of a hand containing a queen.)

---

12. Consider the tiler problem from the week 8 practice questions (Q14). That is a tiler has m black tiles and n white tiles with $m > n$ and wants to lay them out so that no two white tiles are next to each other. If every possible valid pattern is equally likely, what is the probability that the pattern starts with a black tile?

    Solution: Recall that there were $\binom{m+1}{n}$ possible outcomes. If we restrict the first tile to being black, then we can follow the same arguments to show there are $\binom{m}{n}$ such outcomes. Therefore, the probability that the path/pattern starts with a black tile is

    $$\binom{m}{n} \Big/ \binom{m+1}{n} = \frac{m+1-n}{m+1} = 1 - \frac{n}{m+1}\,.$$

---

13. A game is played in which a player repeatedly rolls a dice until they roll a 6 and their score is determined by the number of dice rolls.

    (a) Describe the sample space for the possible sequence of dice rolls that may occur.

    (b) Let E_n be the event the game has not ended after n rolls (alternatively, the event that the final score is greater than n). Explain why $E_{n+1} \subset E_n$.

    (c) What is the event $\left(\bigcap_{n=4}^{\infty} E_n \right)^c$?

    (d) Explain why $\Pr(E_{n+1}) = \frac{5}{6} \Pr(E_n)$.

(e) The event that the game never ends is $\lim_{n\to\infty} E_n$, what is the probability of this event?

(f) What is the probability of obtaining a score of at most 5?

Solution:

(a) Each outcome can be represented by tuples of the form $(r_1, r_2, \ldots, r_{n-1}, 6)$ where n is a positive integer ($n \in \mathbb{N}$) and $r_i \in \{1, 2, 3, 4, 5\}$ for each $i \in \{1, 2, \ldots, n-1\}$. Thus we can write the sample space as

$$S = \{(r_1, r_2, \ldots, r_{n-1}, 6) : n \in \mathbb{N} \text{ and } r_1, \ldots, r_{n-1} \in \{1, 2, 3, 4, 5\}\} .$$

(b) E_{n+1} is the event a 6 has not been rolled in the first $n+1$ rolls. This necessarily implies that a 6 has not been rolled in the first n rolls. Therefore any outcome in E_{n+1} must also be in E_n, that is $E_{n+1} \subset E_n$.

(c) Each $(E_n)^c$ is the event the game ends on or before the n'th roll. The intersection of these for each $n \geq 4$ is that the game ends on or before the fourth roll, and the complement of this is clearly E_4.
Alternatively, using De Morgan's laws

$$\left(\bigcap_{n=4}^{\infty} (E_n)^c\right)^c = \bigcup_{n=4}^{\infty} E_n$$

but since $E_{n+1} \subset E_n$ it follows this event is equivalent to E_4, i.e. the event that the game does not end in the first 4 rolls.

(d) Each outcome in E_n has had the first n rolls be one of $\{1, 2, 3, 4, 5\}$. The outcome is also in E_{n+1} provided the $(n+1)$'th roll is not a 6. Since the probability of a single roll not being a 6 is $5/6$ then it must be that $\Pr(E_{n+1}) = \frac{5}{6}\Pr(E_n)$.

(e) First, recall the proposition: for an increasing or decreasing sequence of events

$$\Pr\left(\lim_{n\to\infty} E_n\right) = \lim_{n\to\infty} \Pr(E_n) .$$

Now, since $\Pr(E_{n+1}) = \frac{5}{6}\Pr(E_n)$ it follows that $\Pr(E_n) = \left(\frac{5}{6}\right)^{n-1}\Pr(E_1)$. Also, clearly $\Pr(E_1) = \frac{5}{6}$ and so $\Pr(E_n) = \left(\frac{5}{6}\right)^n$. Therefore we have

$$\Pr\left(\lim_{n\to\infty} E_n\right) = \lim_{n\to\infty} \Pr(E_n) = \lim_{n\to\infty} \left(\frac{5}{6}\right)^n = 0 .$$

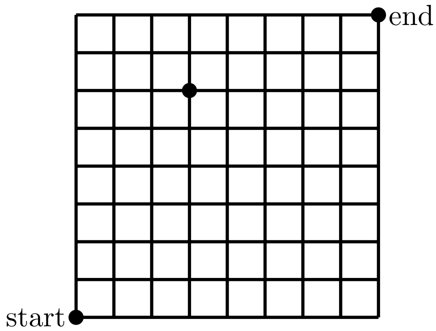
(f) Obtaining a score of at most 5 is equivalent to the event $(E_5)^c$. Since $\Pr(E_5) = \left(\frac{5}{6}\right)^5 \approx 0.402$, then $\Pr((E_5)^c) = 1 - \Pr(E_5) \approx 0.598$.

14. Suppose we randomly choose one letter from each of the words ADELAIDE and KARRAWIRRA so that the chance of each occurrence of each letter in each word is equiprobable (e.g. so that D has two chances to be chosen out of the eight letters in ADELAIDE). What is the probability that the same letter is chosen from the two words?

Solution: There are 8 letters in the first word and 10 letters in the second, therefore there are 80 possible outcomes. Only the letters A,I are common to both words. There is only one outcome where the letter I is chosen from both words. There are two ways of choosing A from ADELAIDE and three ways of choosing it from KARRAWIRRA. Therefore there are $2 \times 3 = 6$ outcomes in which A is chosen from both words. Thus, the probability of choosing the same letter from both words is $(1 + 6)/80 = 7/80 = 0.0875$.

---

15. Consider an ant that must traverse the grid shown on the right from start to end only moving up or right along each edge. Suppose that every possible path is equally likely.



(a) What is the probability that the ant passes through the point marked in the grid?

(b) What is the probability that the ant remains below the point marked on the grid (without passing through)?

Solution:

(a) We start by counting the number of possible outcomes. Each path must consist of exactly 8 movements up (U) and 8 movements right (R), it only matters what order they occur in. As such we can consider the problem in terms of how to place the R in between the eight U. Each R can be placed either at the start or after one of the 8 U. Therefore this is an unordered combination of 8 objects into 9 places with repetition, so there are $\binom{8+9-1}{8} = \binom{16}{8} = 12870$ possible outcomes.
To pass through the given point we can break the problem into

first completing 3 R and 6 U movements in any order, and then 5 R and 2 U movements in any order. Via similar arguments to above, each of these two parts can be completed in $\binom{3+7-1}{3} = 84$ and $\binom{5+3-1}{5} = 21$ different ways respectively. Thus the total number of possible outcomes which pass through the given point are $84 \times 21 = 1764$, and thus, the probability of passing through the given point is $1764/12870 \approx 0.137$.

(b) There are a couple of ways this could be approached. Here we will consider the complement, that is the probability of passing through the given point or above it. Using similar arguments to the first part, passing through the point immediately above the given one occurs in $\binom{3+8-1}{3}\binom{5+2-1}{5} = 120 \times 6 = 720$ outcomes, and passing through the point above that occurs in $\binom{3+9-1}{3}\binom{5+1-1}{5} = 165$ outcomes. Thus the probability of the complement event is $(1764+720+165)/12870 = 2649/12870 \approx 0.206$. It follows that the probability of the ants path passing below the given point is $\approx 1 - 0.206 = 0.794$.
