Mathematics for Data Science I
Practice Questions (week 4)

These questions are all about matrices. Difficult questions are starred.

1. Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ -1 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 4 & 2 \\ 2 & 4 & 6 \end{bmatrix}$, $C = \begin{bmatrix} 5 & -4 \\ 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 3 & 5 & 7 \end{bmatrix}$
   and $E = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$.

   Evaluate each of the following matrices if they exist:

   (i) AB (ii) BAC (iii) $A + C$ (iv) ED (v) AED (vi) C^3

2. Let A, B, C and D be matrices which satisfy the following conditions: ABC^2D is defined and is a 3×3 matrix, A has 7 columns, and B has 5 columns. Find the size of each of the matrices A, B, C and D.

3. Let A be an $m \times n$ matrix and suppose there exist $n \times m$ matrices C and D such that $CA = I_n$ and $AD = I_m$. Prove that $C = D$.

4. * Two square matrices A and B are said to *commute* if $AB = BA$. Find all 2×2 matrices A with commute with *every* 2×2 matrix.

   Hint: let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; choose some very simple matrices B, for instance matrices with most entries equal to 0, and investigate what conditions must be satisfied by a, b, c and d in order to have $AB = BA$.

5. Let A, B and C be invertible $n \times n$ matrices. Find expressions in terms of A, B, C and their inverses for the inverses of the following matrices. (i) ABC (ii) $AB^{-1}A$ (iii) $3ABC^2$ (iv) $-BA^{-1}CA$

6. Suppose that A is an invertible $n \times n$ matrix satisfying $A^3 - 3A + 2I = 0$. Find an expression for A^{-1} in terms of A and I.