1 Solutions Workshop 6

1.1 Exercise 1:

KP is in NP: Guess (construct in a nondeterministic way) a solution $y \in \{0,1\}^n$ and check whether $\sum_{i=1}^n p_i y_i \geq P$ and $\sum_{i=1}^n w_i y_i \leq B$ holds. Checking can be done in polynomial time which implies that the decision variant of KP is in NP.

KP is NP-hard: To show that KP is NP-hard, we reduce PARTITION to KP. Let $a_1, \ldots, a_n$ be a given input for PARTITION. We transform the input for PARTITION in into a knapsack instance by setting $p_i = w_i = a_i$, $1 \leq i \leq n$, and $B = P = \frac{1}{2} \cdot \sum_{i=1}^n a_i$.. This transformation can be done in polynomial time. If there is a solution $y \in \{0,1\}^n$ to PARTITION, then setting $x_i = y_i$, $1 \leq i \leq n$ gives solution to knapsack for profit P and weight W. If x is a solution to knapsack, then $y \in \{0,1\}^n$, with $y_i = x_i$, $1 \leq i \leq n$, is a solution to partition as $\sum_{i=1}^n p_i x_i = P = W = \sum_{i=1}^n w_i x_i$ and therefore $\sum_{i=1}^n a_i x_i = \sum_{i=1}^n a_i y_i = \frac{1}{2} \cdot \sum_{i=1}^n a_i = \sum_{i=1}^n a_i(1 - y_i)$.

1.2 Exercise 3:

Bin Packing is NP-complete: Let $f\colon \{1, \ldots, n\} \to \{1, \ldots, k\}$ be an assignment of the n items to k bins. We guess (construct in a nondeterministic way) such an assignment. Given such an assignment f we can check in polynomial time whether the sum of the size of the items in each bin is at most b.

Bin Packing is NP-hard: To show that Bin Packing is NP-hard, we reduce PARTITION to Bin Packing. Let $a_1, \ldots, a_n$ be a given input for PARTITION. We construct a Bin Packing instance by setting $s_i = a_i$, $1 \leq i \leq n$, $k = 2$, and $b = \lfloor \frac{1}{2} \cdot \sum_{i=1}^n s_i \rfloor$. Clearly this transformation can be done in polynomial time. If $\sum_{i=1}^n s_i$ is odd, then PARTITION does not have a solution and the objectives do not fit into 2 bins of size $b < \frac{1}{2} \cdot \sum_{i=1}^n s_i$. If $\sum_{i=1}^n s_i$ is even, then the objects fit into into two bins of size $b = \frac{1}{2} \cdot \sum_{i=1}^n s_i$ if and only if they can be partitioned into two subsets of equal size, i.e. if PARTITION has a solution.