Mathematical Foundations of Data Science
Tutorial 2 (week 4)

Semester 2, 2019

1. Find $\sum_{i=2}^{11}(i+1)(i+2)$ using the results from lectures.

2. For the two series

$$\text{(a)} \sum_{k=2}^{\infty} \frac{2^k}{k!} \qquad \text{and} \qquad \text{(b)} \sum_{n=1}^{\infty} 3^{n+1}4^{-n},$$

what method would be appropriate for deciding whether each series in convergent? Apply them.

3. A smooth function $f(x)$ is such that its derivatives at $x = 1$ alternate in sign between ± 2, i.e., $f(1) = 2$, $f'(1) = -2$, $f''(1) = 2$, $f^{(3)}(1) = -2$, $f^{(4)}(1) = 2$, $f^{(5)}(1) = -2$, etc.

   (a) Write down the Taylor series of f.

   (b) Find the exact value of $f(1/2)$.

   (c) * What is the function $f(x)$?

4. (a) Calculate the degree 3 MacLaurin Polynomial, $P_3(x)$, for $\cosh x$. (Note that for $f(x) = \cosh x$, $f'(x) = \sinh x$, and $f''(x) = f(x) = \cosh x$.)

   (b) Evaluate $P(1)$.

   (c) Use the remainder term to estimate the error in using $P(1)$ to estimate $\cosh(1)$ (to put an upper bound on the error you should use the fact that $2 < e < 3$ to find a bound on $\cosh z$ for $0 < z < 1$).