

## Workshop 2: Discussion Questions

### **Exercise 1** *Assignment 1*

Summarize your approach and results for Assignment 1. Present your implementation and point out difficulties that you faced with the assignment.

### **Exercise 2** *Induction Proofs*

Let  $N = \{0, 1, 2, \dots\}$  be the set of natural numbers.

1. (from Melhorn 2.10). Access to data structures is often governed by the recurrence  $T(1) = a, T(n) = c + T(n/2)$ . Prove by induction that  $T(n) \in O(\log n)$ . Do not attempt to use the Master Theorem for this proof.
2. Can the statement above be proven by the Master Theorem? If so, show your proof. If not then explain why not.
3. Let  $F(n)$  the  $n$ -th Fibonacci number. We have that  $F(1) = F(2) = 1$  and  $F(n) = F(n-1) + F(n-2)$ . Find an  $a$  value so that  $F(n) \in O(a^n)$ . We want  $a$  to be as small as possible.

### **Exercise 3** *Insertion Sort*

Prove that the Insertion Sort algorithm given in Algorithm 1 is correct, i.e. it sorts a given array  $A[1, \dots, N]$  of size  $N$  in increasing order. Show that appropriate invariants hold during the execution of the algorithm in order to prove correctness.

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**Algorithm 1** Insertion Sort( $A$ )

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1. for  $j=2$  to  $N$  do
  2.  $key = A[j]$
  3.  $i = j - 1$
  4. while  $i > 0$  AND  $A[i] > key$  do
    - $A[i + 1] = A[i]$
    - $i = i - 1$
  5. end while
  6.  $A[i+1] = key$
  7. end for
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