Mathematics for Data Science Tutorial 3 (week 6)

Semester 2, 2019

1. State whether or not the following matrix operations are possible. Perform the operations where possible.

   (a)
   $$\begin{bmatrix} 2 & 1 & 8 & 6 \\ 3 & 5 & 3 & 2 \end{bmatrix} + \begin{bmatrix} -11 & -1 & 9 \\ 2 & 9.4 & -2 \end{bmatrix}$$

   (b)
   $$\begin{bmatrix} 2 & 2 \\ 1 & -5.5 \\ 2.5 & 10 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 1 & 3 \\ 0 & -7 \\ -1 & 3 \end{bmatrix}$$

   (c)
   $$\begin{bmatrix} -3 & -2 & 9 & 1 \\ 2 & 3 & -1 & 0 \end{bmatrix} + 2 \begin{bmatrix} 4 & -8 \\ 2 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}^T$$

   (d)
   $$\begin{bmatrix} 2 & 5 \\ 0 & -3 \\ 7 & 16 \end{bmatrix} - \begin{bmatrix} 2 & -5 & 0 \\ 2 & 9 & 0 \\ 1 & 3 & 0 \end{bmatrix}$$

2. In lectures we declared that $A(B+C) = AB+AC$, but ran away from doing an example. Verify this result for the matrices
   $$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 1 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 2 \\ 3 & 1 \\ 1 & 1 \end{bmatrix}.$$

3. In lectures we also declared that $(AB)^T = B^T A^T$, without giving you any proof. Verify that this result is true for 2×2 matrices A and B.

4. Solve the linear system
   $$\begin{aligned} x_1 + x_2 - 2x_3 &= 2 \\ -2x_1 - x_2 + x_3 &= -1 \\ x_2 - 3x_3 &= 3 \\ -x_1 - x_3 &= 1 \end{aligned}$$
   by first bringing its augmented matrix into reduced row echelon form.

   Start by looking at the system and try to guess how many solutions it has. Once you've solved it, see if your guess was correct.