NP-Completeness

Exercise 1 *Knapsack Problem*

The decision variant of the knapsack problem is stated as follows. Given n items where each item has a positive integer weight w_i and a positive integer profit p_i. Is there a selection of items such that the sum of its profits is at least P and the sum of its weights is at most B, i.e. is there an $x \in \{0,1\}^n$ such that $\sum_{i=1}^{n} p_i x_i \geq P$ and $\sum_{i=1}^{n} w_i x_i \leq B$ holds?

The partition problem is stated as follows. Given n positive integers $a_1, \ldots a_n$, is there a partitioning of these number into two disjoint subsets such the sum of the numbers is the same, i.e. is there an $x \in \{0,1\}^n$ such that $\sum_{i=1}^{n} a_i x_i = \sum_{i=1}^{n} a_i(1-x_i)$. The partition problem is NP-complete.

Show that the knapsack problem is NP-complete. To show that the knapsack problem is NP-complete reduce the partition problem to the knapsack problem.

Exercise 2 *Dynamic programming for the knapsack problem*

- The knapsack problem is NP-complete. However, it can be solved in time $O(n^2W)$ using dynamic programming, where $W = \max_{i=1}^{n} w_i$. Does this imply P=NP? Justify your answer.
- Give a dynamic programming approach that solve the knapsack problem in time $O(n^2W)$.

Exercise 3 *Bin Packing*

The decision variant of the bin packing problem is stated as follows. Given n objects $1, \ldots, n$ with sizes $s_1, \ldots, s_n$, can they be placed into k bins of size b such that each bin contains objects whose sum of the sizes is at most b. Show that the decision variant of bin packing is NP-complete. You can use that partition is NP-complete.