Tutorial 10 – MATH 4043

This tutorial contains **a** continuous assessment item.

1. Suppose you go to a doctor for a check up to determine your health status, i.e. whether you are sick or healthy. The doctor takes a blood sample and measures a critical continuous variable B to determine how healthy you are. The probability distribution of the variable B is known to be Normally Distributed to the doctor with variance 16 but unknown mean. The doctor believes that the probability for you of your health status on average means the blood sample has a measure of 45 units to be sick and 55 units on average to be healthy with $P(\mu = 55) = 0.8$ and $P(\mu = 45) = 0.2$.

   (a) Write down the prior distribution.

   (b) Suppose the doctor takes a sample of 50 patients and measures the mean blood sample when healthy to be $\bar{x} = 50$, compute the Likelihoods for being sick and healthy, that is

   $$P(H = \bar{x} = 50 \mid \mu = 45, \sigma = 4/\sqrt{50}),$$
   $$P(H = \bar{x} = 50 \mid \mu = 55, \sigma = 4/\sqrt{50}).$$

   (c) Calculate the posterior distribution.

   (d) What is the expected value of your health status based off B.

2. The number of customers entering a certain store on a given day is assumed to be Normally distributed with unknown mean $\bar{\mu}$ and unknown variance $\bar{\sigma}^2$. As the store manager, you feel that your prior distribution $\bar{\mu}$ is Normal with mean 1000 and that $P(900 \leq \bar{\mu} \leq 1100) = 0.95$ (Credible Interval). You then take a random sample of 10 days, observing a sample mean of $m = 900$ customers and a sample variance of $s^2 = 50000$.

   (a) Find your approximate posterior distribution for $\bar{\mu}$. Aside from the usual argument concerning the applicability of the Normal model, why is your posterior distribution only approximate?

3. **Continuous Assessment** A newly developed construction material was tested for its fire resistance. Results from testing two samples show that serious fire damage after exposed to only 2 hrs of fire, whereas the other lasted for 3 hrs. Suppose the duration of fire, T, for damage of a material follows a Normal Distribution $N(\mu, 0.5)$. before the tests, the mean duration until damage, μ for this material was expected to be 3 hrs with a variance about the mean of 20% based on a study.

   (a) Determine the updated mean duration until damage for this material.

   (b) With the above observed information, what is the probability that a wall constructed using this material will be damaged in less than 1 hr of fire?