

7. The random variable X has probability mass function X as given in the table below. Let the random variable $Y = X^2$. This means that, e.g., $\{Y = 1\} = \{X = -1\} \cup \{X = 1\}$ and so on.

x	-2	-1	0	1	2
$p_X(x)$	0.1	0.3	0.2	0.3	0.1

- (a) Tabulate the range of Y and its probability mass function $p_Y(y)$.

First note the equivalence of the following events involving Y and X in the table below.

$\{Y = 0\}$	$\{X = 0\}$
$\{Y = 1\}$	$\{X = -1\} \cup \{X = 1\}$
$\{Y = 4\}$	$\{X = -2\} \cup \{X = 2\}$

Hence we calculate the probability mass function of Y as in the table below.

y		$p_Y(y)$
0	$P\{Y = 0\} = P\{X = 0\}$	0.2
1	$P\{Y = 1\} = P\{X = -1\} + P\{X = 1\}$	0.6
4	$P\{Y = 4\} = P\{X = -2\} + P\{X = 2\}$	0.2

- (b) Computer $E[Y]$ and $\text{Var}[Y]$.

$$E[Y] = 0 \times 0.2 + 1 \times 0.6 + 4 \times 0.2 = 1.4.$$

$$E[Y^2] = 0^2 \times 0.2 + 1^2 \times 0.6 + 4^2 \times 0.2 = 3.8.$$

Thus

$$\text{Var}[Y] = 3.8 - (1.4)^2 = 1.84.$$