Tutorial 11 – MATH 4043 SP2 2019

1. Refer to Question 3, Tutorial Week 8.
   The number of visitors clicking an online shopping website is distributed as Poisson with a mean visit rate of 7 per hour. Any individual visitor who goes onto the website has a 65% chance of making a purchase. Assume each individual visitor acts independently of each other, and assume that the visitor only makes 1 click to the website within that hour. If the ith visitor makes a purchase, the amount spent Y_i is independently distributed as exponential $\text{Exp}(\lambda_e)$ with $\lambda_e = 0.05$. Simulate the number of visitors in a two hour period, the number from these visitors who make a purchase, and if they make a purchase, their purchase amounts.

2. Refer to Q1 above.
   Suppose now that if the ith visitor makes a purchase, the log of the amount spent $\ln(Y_i)$ is normally distributed as $N(3, 2)$. Simulate the number of visitors in a two hour period, the number from these visitors who make a purchase, and if they make a purchase, their purchase amounts.

3. Consider a drunk man walking on a straight line. He starts off at point 0. For each step that he takes, the length X_i of his ith stride is distributed as continuous uniform $U[-1, 1]$. We assume that each stride is independent of each other and a negative value means a step backwards. Let $S_t = \sum_{i=1}^{t} X_i$ be his position at time t. Simulate his first 30 steps and plot his path.

4. Suppose the daily log returns of a stock is distributed as normal with annual drift 0.2 and annual volatility 75. Using the Black-Scholes-Merton Geometric Brownian motion model for stock price
   $$S_t = S_0 e^{\mu t + \sigma B_t},$$
   with $S_0 = 2$, simulate a stock price path for a 3 year period (356×3 days).

5. Suppose $B(t)$ is a standard Brownian Motion.
   (a) Find the probability that $0 < B(t) < 1$.
   (b) Find the probability that $B(6) \leq 4$ given that $B(1) = 1$.

6. Let Z be a Normally distributed random variable with mean 0 and variance 1, such that $Z \sim N(0, 1)$. Consider the stochastic process $G(t)$ $\sqrt{t}Z$.
   (a) By using **R**, generate values for $G(t)$ for $t = 1, 2, ..., 5$.

(b) Show that $G(t)$ is not a Brownian Motion by using the formal definition.

7. Suppose you are high frequency trading of stocks, if the current price of the stock is 5% higher than the purchase price then you will buy. Suppose that a stock follows a Standard Geometric Brownian Motion lifted by the current market price of \$20? What is the probability that the stock price is ready to buy after T units of time later? Evaluate for for multiple values of T and the plot the distribution with respect to T.