Tutorial 3 – MATH 4043

1. Would you accept a gamble that offers a 10% chance to win \$95 and a 90% chance of losing \$5?

2. Would you pay \$5 to participate in a lottery that offers a 10% chance to win \$100 but a 90% chance to win nothing?

3. (Insurance companies) Loss aversion and cost versus loss sustain the insurance industry: people pay more in premiums than they get back in claims on average (otherwise the industry wouldn't be sustainable), but they buy insurance anyway to protect themselves against substantial losses.

   (a) Suppose you pay \$1 each year to protect yourself against a 1 in 1000 chance of losing \$100 that year. Calculate the expected value of this scheme and interpret your answer?

4. We toss a fair coin 3 times. Let X be a random variable that counts the number of heads in 3 coin tosses.

   (a) Write down the sample space S for this experiment.

   (b) Show the mapping from the sample space S to the various values of X and tabulate the probability mass function of X.

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5. The random variable X has probability mass function X as given in the table below.

x	-1	1	2
$p_X(x)$	k	2k	k

   (a) Find the value of k.

   (b) Tabulate and sketch the cumulative distribution function $F_X(x)$ of X.

   (c) Compute $E[X]$ and $\text{Var}[X]$.

6. Two fair dice (with 6 faces each) are tossed together. Let Y denote the sum of the numbers on the top faces of each die.

   (a) Write down the sample space S. You may use a short cut notation for this.

   (b) Tabulate all the values in the range of Y and the corresponding probability mass function $p_Y(y)$.

(c) Compute $E[Y]$ and $\mathrm{Var}[Y]$.

7. The random variable X has probability mass function X as given in the table below. Let the random variable $Y = X^2$. This means that, e.g., $\{Y = 1\} = \{X = -1\} \cup \{X = 1\}$ and so on.

x	-2	-1	0	1	2
$p_X(x)$	0.1	0.3	0.2	0.3	0.1

(a) Tabulate the range of Y and its probability mass function $p_Y(y)$.

(b) Computer $E[Y]$ and $\mathrm{Var}[Y]$.