

## Solutions.

8.

|             | Red die |     | White die |     | Green die |     |
|-------------|---------|-----|-----------|-----|-----------|-----|
| Outcomes    | 3       | 6   | 2         | 5   | 1         | 4   |
| Probability | 5/6     | 1/6 | 3/6       | 3/6 | 1/6       | 5/6 |

- The  $2 \times 2$  tables show pairs of dice.
- Each entry is the probability of seeing the pair of numbers corresponding to that entry.
- The color gives the winning die for that pair of numbers. (We use black instead of white when the white die wins.)

|       |   | White |       | Green |       |
|-------|---|-------|-------|-------|-------|
|       |   | 2     | 5     | 1     | 4     |
| Red   | 3 | 15/36 | 15/36 | 5/36  | 25/36 |
|       | 6 | 3/36  | 3/36  | 1/36  | 5/36  |
| Green | 1 | 3/36  | 3/36  |       |       |
|       | 4 | 15/36 | 15/36 |       |       |

The three comparisons are:

$$\begin{aligned}
 P(\text{red beats white}) &= 21/36 = 7/12 \\
 P(\text{white beats green}) &= 21/36 = 7/12 \\
 P(\text{green beats red}) &= 25/36
 \end{aligned}$$

Thus: red is better than white is better than green is better than red.

There is no best die: the property of being 'better than' is non-transitive.