6. For a test for a certain type of cancer, there is a probability of 0.02 that the test result is positive if it is randomly applied to a randomly chosen person off the street. The probability that a person actually has that type of cancer if tested positive is 0.98 and the probability that the person actually does not have that type of cancer if tested negative is 0.95. If I randomly pick a person and apply the test, what is the probability that the person actually has that particular cancer?

   Solution. Let $R = \{$ test result is positive$\}$.
   Let $C = \{$ The tested person actually has cancer$\}$. Of course we need to assume that each tested person has the same probabilities.

   From the information given,

   $$P(R) = 0.02,\ P(C|R) = 0.98,\ P(\bar{C}|\bar{R}) = 0.95.$$

   So we also have

   $$P(\bar{R}) = 0.98,\ P(C|\bar{R}) = 1 - P(\bar{C}|\bar{R}) = 1 - 0.95 = 0.05 \text{ (using Q4 for the latter).}$$

   Making used of Method 3,

   $$\begin{aligned} P(C) &= P(C \cap R) + P(C \cap \bar{R}) \\ &= P(C|R)P(R) + P(C|\bar{R})P(\bar{R}) \\ &= 0.98 \times 0.02 + 0.05 \times 0.98 = 0.0686. \end{aligned}$$

   N.B. Suppose we are interested in the probability that the test result is positive given that the tested person has cancer, then we are interested in $P(R|C)$. So in this case we use Baye's Rule to get

   $$\begin{aligned} P(R|C) &= \frac{P(C|R)P(R)}{P(C|R)P(R) + P(C|\bar{R})P(\bar{R})} \\ &= \frac{0.98 \times 0.02}{0.98 \times 0.02 + 0.05 \times 0.98} = \frac{2}{7}. \end{aligned}$$