Tutorial 4 – MATH 4043

1. Suppose a balanced die is tossed once. Consider the bivariate random variable $Y = (Y_1, Y_2)$ where $Y_1 = 0$ if the number on the top face is even and $Y_1 = 1$ if odd, and $Y_2 = 0$ if the number on the top face is 1, 2, or 3, and $Y_2 = 1$ if 4, 5, or 6.

   (a) What is the sample space?

   (b) Show the mapping of the sample points to the various possible values of $Y = (Y_1, Y_2)$.

   (c) Tabulate the joint probability mass function of Y and the marginal probability mass function of its individual components.

   (d) Are Y_1 and Y_2 independent?

   (e) Tabulate the conditional probability mass function $p_{Y_2|Y_1}(y_2|1) = P[Y_2 = y_2|Y_1 = 1]$ over the range of Y_2.

   (f) Calculate $\text{Cov}[Y_1, Y_2]$.

2. The bivariate random variable $X = (X_1, X_2)$ have joint probability mass function given in the table below

$X_2 \setminus X_1$	0	1	2
0	0.1	0.3	0.1
1	0.2	0.1	0.2

   (a) Calculate the marginal probability mass function of each of the components X_1 and X_2.

   (b) Are the components independent?

   (c) Tabulate the conditional probability mass function

$$p_{X_1|X_2}(x_1|0) = P[X_1 = x_1|X_2 = 0]$$

   over the range of X_1.

   (d) Calculate $\text{Cov}[X_1, X_2]$.

3. Consider a coin tossing experiment where a biased coin is tossed 2 times. Assume that the tosses are independent of one another and that the probability of getting a head is 0.75. Let $X = (X_1, X_2)$ where X_1 counts the number of heads in the 2 tosses and X_2 counts the number of tails in the 2 tosses.

(a) Write down the sample space and show the mappings of the sample points to the various values of $X = (X_1, X_2)$.

(b) Find the marginal probability mass function of the individual components.

(c) Are they independent?

(d) Calculate $\mathrm{Cov}[X_1, X_2]$.

4. (Employment dynamics). A researcher is interested in modelling the employment dynamics of young people using a Markov chain. She determines that at age 18 a person is either a student with probability 0.9 or an intern with probability 0.1. Each year that passes is denoted as one time period. After that she estimates the following transition probabilities:

$$
\begin{array}{cc}
\begin{array}{cccc} \text{Student} & \text{Intern} & \text{Employed} & \text{Unemployed} \end{array} & \\
\begin{pmatrix} 0.8 & 0.5 & 0 & 0 \\ 0.1 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0.9 & 0.4 \\ 0 & 0 & 0.1 & 0.6 \end{pmatrix} & \begin{array}{c} \text{Student} \\ \text{Intern} \\ \text{Employed} \\ \text{Unemployed} \end{array}
\end{array}
$$

(a) Write down the initial probability vector and the transition matrix for the problem.

(b) The first two states, student and intern are called transient states. Show that the probability of revisiting those states after visiting the third state, employed is zero.

(c) Calculate the probability that if someone starts off as a student (state 1) then after 4 years they will be employed.