Assignment 2 – MATH 4043
SP5 2022

INSTRUCTIONS

1. This assignment is due by 23:00, on Friday 28th October, Adelaide time.
2. The assignment is a total of 75 marks.
3. An online submission link will be available for the Assignment in the course website about 1 week before the deadline.
4. Please submit a .docx or .pdf version of your Assignment online.
5. You may discuss with another person in the class, or in the relevant online forums, regarding the assignment, but the final write-up must be your own work.

QUESTIONS

1. (20 marks) Let the random variable X be whether a customer buys a specific product. It is roughly known that when a customer walks into the store that the probability of buying is 0.15 for every 100 customers. The mean and standard deviation of X can be represented by μ_X and σ_X respectively. You may use **R** for some of the calculations, but please also write out the relevant formulas in order to get the full marks for this question.

   (a) (2 marks) What is the distribution of X and μ_X?

   (b) (4 marks) Calculate $P[\mu_X - \sigma_X \leq X \leq \mu_X + \sigma_X]$.

   (c) (4 marks) Calculate $P[\mu_X - \sigma_X \leq X \leq \mu_X + \sigma_X]$ using the Poisson approximation.

   (d) (4 marks) Calculate $P[\mu_X - \sigma_X \leq X \leq \mu_X + \sigma_X]$ using the Normal approximation. Verify your answer using the Standard Normal table.

   (e) (3 marks) Show that the probability for values of X which lie outside of

   $$(\mu_X - \sqrt{2}\sigma_X, \mu_X + \sqrt{2}\sigma_X)$$

   is less than 1/2 without the use of **R** .

   (f) (3 marks) If X happens to be distributed as Binomial(100,0.1) then would the Normal approximation be appropriate? Give reasons.

2. (20 marks) An analytical chemist uses a balance device to produce measurements that are Normally distributed with mean equal to the true mass of the sample and standard deviation of 0.2 mg that has been estimated by the manufacturer's balance and confirmed against calibration standards. Suppose the chemist wants to measure the mass of a sample. By looking at the sample that was taken, the mass is about 10 milligrams and based on previous experience in estimating masses, the guess for the standard deviation is 2 mg. It is decided that the prior for the mass of the sample has a Normal distribution with mean, 10 milligrams, and standard deviation, 2 milligrams. **Show your working out by hand but use R to code and calculate the following probabilities.**

   (a) (5 marks) Suppose a sample of five observations is taken and measures 10.5 for the mean. Calculate the new posterior distribution, the mean and standard deviation.

   (b) (3 marks) What is the impact from the chemist point of view?

   (c) (5 marks) Use **R** to simulate the mass of another four items. Calculate the mean and sample standard deviation.

   (d) (5 marks) By using the mean of the simulated sample in part (c) and the posterior distribution. Calculate the new posterior distribution, the mean and standard deviation.

   (e) (2 marks) What is the new conclusion of the chemist? Explain and give reasons.

3. (30 marks) Brownian motion is a widely-used random process. It has been used in engineering, finance, and physical sciences. It is a Gaussian random process and it has been used to model motion of particles suspended in a fluid, percentage changes in the stock prices, integrated white noise, etc. Denote a Brownian motion B_t over $[0, 365]$:

$$B_t = \sum_{i=1}^{t} Y_i,$$

   where Y_1, Y_2, $\cdots$, Y_n are independently and identically distributed Normal $N(0, 1)$.

   You are interested in purchasing a particular share which has a daily changing Stock Price ($). The historical data for the share's Stock Price ($) over the last 365 days (including today) is contained in the data file *Stock.csv*. It is assumed that the change in Stock Price ($) is independent, so there is no prior knowledge of the price influencing a future price change. Suppose you purchase a share at Stock Price s dollars plus a trading fee applies of b dollars, where $b = 10$. You decide to hold the stock for a period of 90 days before selling the stock.

   (a) (4 marks) Show that the historical data for the Stock Price ($) of a share follows a Brownian motion process. **Note:** The condition of independence has already been assumed.

(b) (2 marks) By using the dataset, Stock Price ($), what is the value s of your stock?

(c) (8 marks) Use **R** to simulate the future Brownian motion of the Stock Price ($) for the next 90 days **five times**. Compare each future path of the Stock Price ($) and whether each path makes a loss or a profit. Include the point $B_0 = 0$ on your line plot. Provide your **R** code for full marks.

(d) (6 maks) By using your simulate data in part (c), average (day to day) the five Brownian motions. Plot in **R**, the new average distribution and provide the mean profit and variation (standard deviation) summaries.

(e) (8 marks) What is the probability that your Stock Price ($) does not reach $s + b$ over period of 90 days?

(f) (5 marks) If the share is held for longer than 90 days then what would be the effect on the probability that your stock price does not reach $s + b$? Give reasons.

Instructions to simulate B_t. Divide the desired interval $[0, 90]$ into a grid from $0 = t_0 < t_1 < \cdots < t_{N-1} < t_N = 90$. Set $i = 1$ and $B_0 = 0$ and iterate as follows:

i. Generate a new random number z from the Standard Normal Distribution.

ii. Set i to $i + 1$

iii. Set $B(t_i) = B(t_{i-1}) + z\sqrt{t_{i+1} - t_i}$

iv. If $i < N$, then repeat this process.