4. **Continuous Assessment Item from Tutorial 9**. Suppose the waiting times (in hours) for a customer arriving in a queue shop (timed immediately after the previous customer or after time 0 in the case of the very 1st customer) is distributed as exponential Exp(6). Use **R** to simulate the arrival times of the customers within a 2 hour period. Suppose you ended up with k customers arriving in the 2 hour period, use **R** to calculate $P[S_k \leq 2]$, where S_k is the sum of the k waiting times.

   Solution. We have $S_k = \sum_{i=1}^{k} X_i$ where X_i is the inter-arrival time for the ith customer after the previous customer has arrived. We need k so that S_k is just before the 2 hour period and S_{k+1} goes beyond the 2 hour period.

```
> x <- rgamma(16,1,6)
> x
 [1] 0.59167531 0.08647379 0.04389924 0.10699758 0.29643131 0.07937847
 [7] 0.02615994 0.33352886 0.03324224 0.14549770 0.08301197 0.48229953
[13] 0.13082738 0.12944352 0.08818443 0.01123021
> cumsum(x)
 [1] 0.5916753 0.6781491 0.7220484 0.8290459 1.1254772 1.2048557 1.2310156
 [8] 1.5645445 1.5977867 1.7432844 1.8262964 2.3085959 2.4394233 2.5688669
[15] 2.6570513 2.6682815
```

   So in my case $k = 11$. The 12th arrival arrives after the 2 hour mark.

   Now from the section on exponential and gamma random variables, S_k is distributed as Gamma(k,λ). So in my case, I need to find $P[S_{11} \leq 2]$. Using **R** ,

```
> pgamma(2,11,6)
[1] 0.6527706
```

   that is, $P[S_{11} \leq 2] = 0.6527706$.