Solutions:

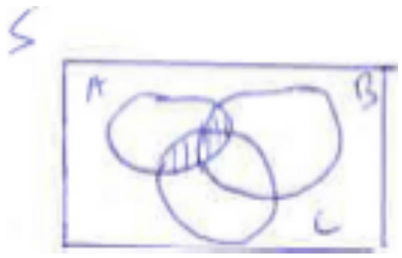
1. *Bounded below by*

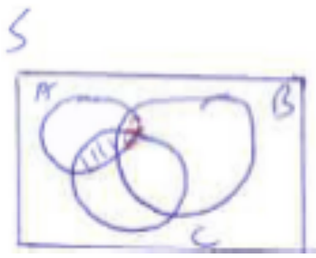
$$P(M \cup B) = \frac{25}{50}$$

and above by

$$P(M \cup B) = \frac{45}{50}.$$

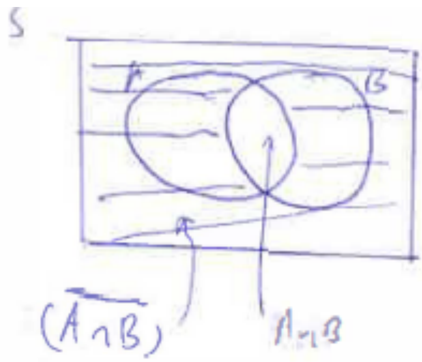
2. Part (a). Looking at the LHS, first we mark out (BU C). Then we mark out the part of $(B \cup C)$ that is also contained in A to get the region below. For the RHS, shade the regions $A \cap B$ and $A \cap C$ together. Observe that we end up with the same region as above.

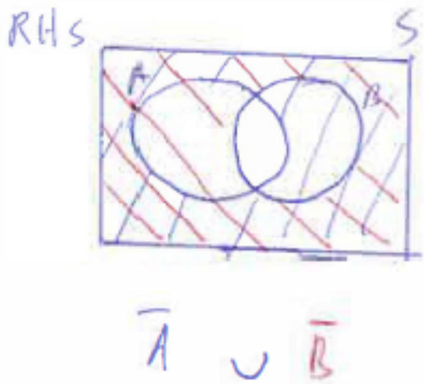




Part (b). Looking at the LHS, first we look at $A \cap B$ and shade what is outside of this region.

For the RHS, first look at what is outside of A. Then separately look at what is outside of B. Shade the region where these two sets A and B coincide. Observe that it is the same as that of the LHS.





3. The left hand side says that we need to arrange r distinct objects from n distinct objects. By definition, the number of such arrangements is P_r^n for the RHS. One way to arrange distinct objects from n distinct objects is to do so in two steps. First we select r out of the n objects - that gives us C_r^N;. Next after having selected the r objects, there are r! ways of arranging them, so by the multiplication principle, there are $C_r^n \times r!$ Ways.

Since these are simply two different ways of counting the same thing. The number of ways from the LHS must be the same as that obtained from the counting method for the RHS, thus LHS=RHS. This is a common technique used to establish equations in combinatorial mathe-matics using a counting method. First, count in such a way so as to obtain the expression given in the LHS, then do so another way to get the expression for the RHS.

4.
(a) How many ways can we select the committee?
Note that it actually does not matter whether we select the President first, then the 2 VPs, then the treasurer or do location in order.
From 10 members: there are firstly 10 ways to select the President. After the VP is selected: C_2^9 for the 2 VP positions,. Finally there are 7 choices from the remaining 7 members for the treasurer position. So that gives 2520 ways.
(b) We assume that the two VPs do not mind sitting to the left or right of each other as long as the positions am marked VP. Here we have two objects which are identical so we can factor out 2!, thus, 4!/2! = 12 ways.

c) We arranged committee in a straight row, the numher of ways is 12 from part (b). Now we arrange them around in a circle. Note that for arrangements around a circle, any particular arrangement which is a rotation of another arrangement is considered identical to that arrangement. So in particular, since there are 4 positions around a circle, then if we use the answer 12 from part (b), we are over counting by a multiple of 4 times, so the correct answer is 3.

5. Draw a Venn diagram, start by thinking about the RHS first with the goal to obtain $A \cup B \cup C$; thus if I start with P(A) + P(B) + P(C), I instantly see that I have double counted the intersections between each set. Thus, the intersections need to be taken away, however, this leaves me with taking away $A \cap B \cap C$ one too many times. Thus, we add it in at the end. This forms the proof for this question.

6. If $A \subset B$ then every possible event in A must be in B, so the $P(A) \leq P(B)$.

7. Here for each toss P(H) = P(T) = 0.5
The sample space S consists of all arrangements of H and T in five tosses.
Total number of sample points in S is N = 2 x 2 x 2 x 2 x 2 = 2^5
Let A be the event of getting 3 T and 2 H. To select any 3 positions to get T and let the remaining 2 be H, so C_2^5, thus the probability is P$P(A) = C_2^5(0.5)^5 = \frac{5}{16}$.

Of course you can also do this problem as the probability of getting 2 heads in 5 tosses and you should also get the same answer since $C_2^5 = C_3^5$.

Solutions.

8.

Outcomes	3	6	2	5	1	4
	Red die		White die		Green die	
Probability	5/6	1/6	3/6	3/6	1/6	5/6

- The 2×2 tables show pairs of dice.
- Each entry is the probability of seeing the pair of numbers corresponding to that entry.
- The color gives the winning die for that pair of numbers. (We use black instead of white when the white die wins.)

		White 2	White 5	Green 1	Green 4
Red	3	15/36	15/36	5/36	25/36
	6	3/36	3/36	1/36	5/36
Green	1	3/36	3/36		
	4	15/36	15/36		

The three comparisons are:

$$\begin{aligned} \text{P(red beats white)} &= 21/36 = 7/12 \\ \text{P(white beats green)} &= 21/36 = 7/12 \\ \text{P(green beats red)} &= 25/36 \end{aligned}$$

Thus: red is better than white is better than green is better than red.

There is no best die: the property of being 'better than' is non-transitive.