4. (Employment dynamics). A researcher is interested in modelling the employment dynamics of young people using a Markov chain. She determines that at age 18 a person is either a student with probability 0.9 or an intern with probability 0.1. Each year that passes is denoted as one time period. After that she estimates the following transition probabilities:

$$\begin{array}{cc} \begin{array}{cccc} \text{Student} & \text{Intern} & \text{Employed} & \text{Unemployed} \end{array} & \\ \begin{pmatrix} 0.8 & 0.5 & 0 & 0 \\ 0.1 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0.9 & 0.4 \\ 0 & 0 & 0.1 & 0.6 \end{pmatrix} & \begin{array}{c} \text{Student} \\ \text{Intern} \\ \text{Employed} \\ \text{Unemployed} \end{array} \end{array}$$

(a)

$$p_0 = \begin{pmatrix} 0.9 \\ 0.1 \\ 0 \\ 0 \end{pmatrix}$$

and

$$T = \begin{pmatrix} 0.8 & 0.5 & 0 & 0 \\ 0.1 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0.9 & 0.4 \\ 0 & 0 & 0.1 & 0.6 \end{pmatrix}$$

(b) The first two states, student and intern are called transient states. Show that the probability of revisiting those states after visiting the third state, employed is zero.

Suppose we start with the first state (student), thus it could be represented as the vector $(1, 0, 0, 0)^T$. Suppose that after some time period, k, we make it into the third state (employment). Then the probability to move back to first state is now zero because we would have to multiply by the vector $(1, 0, 0, 0)$ with $(0, 0, 1, 0)$ which is zero.

(c) Calculate the probability that if someone starts off as a student (state 1) then after 4 years they will be employed.

We start with a probability of $p_0 = (0.9, 0.1, 0, 0)$. Then after four years, this is T^4. Thus the calculated probability is

$$T^4 p_0 = 0.26367.$$