Time Series

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August 23, 2022

1 Prediction Intervals - What are they good for?

- On August 18, the weather forecast for Adelaide was for showers with 5-8 mm. rainfall.
- The forecast was not for a precise amount, but for a probable range.
- To put things in perspective, we can look at systolic blood pressure. The mean is 128.4 and the standard deviation is 19.6.
- In addition, it is assumed that systolic blood pressure readings are normally distributed.
- Therefore we can say that a 90% confidence interval for systolic blood pressure is given by $(128.4 - 1.645 \times 19.6, 128.4 + 1.645 \times 19.6)$ or $(96.2, 160.6)$.
- In a similar way, if at time t we forecast the value of a variable for time $t + 1$, it would be useful to be able to put error bounds or an interval around that forecast.

2 What about a prediction interval around the forecast?

- In some situations, constructing such an interval is easy.
- Remember the definition of stationarity - unchanging mean and variance over time. When we are able to fit an ARMA model, this ensures that the mean is not changing over time.
- What extra conditions must hold to ensure unchanging variance?
- Not only do the z_t have to be uncorrelated, they must be independent. How can we test this? One way is to look at a proxy for the variance z_t^2. They also be uncorrelated.
- If the z_t are normally distributed, this condition automatically holds. Let's look at examples where the z_t^2 are independent and where they are not, and examine what we do in each case.

3 Prediction Intervals for a Random Variable with Normally Distributed Noise

4 Example - AR(2) model with Normal noise

- In keeping with the idea of training and testing sets, I constructed 2000 values of a series with the following characteristics.
- $x_t = 50 + 1.2x_{t-1} - 0.3x_{t-2} + z_t$. with $z_t \sim N(0, 9)$.
- I then found the $AR(2)$ model for the data which had coefficients $1.189, -0.289$.
- I used this model to produce forecasts for a second set of data similarly constructed.
- I then added upper and lower bounds to the forecasts, using the standard deviation of the noise term, 3.011.

The governing forecast model is given by

$$
\begin{aligned}
\hat{x}_t &= 1.189x_{t-1} - 0.289x_{t-2} \\
LB &= \hat{x}_t - Z_\alpha \times 3.011 \\
UB &= \hat{x}_t + Z_\alpha \times 3.011
\end{aligned}
$$

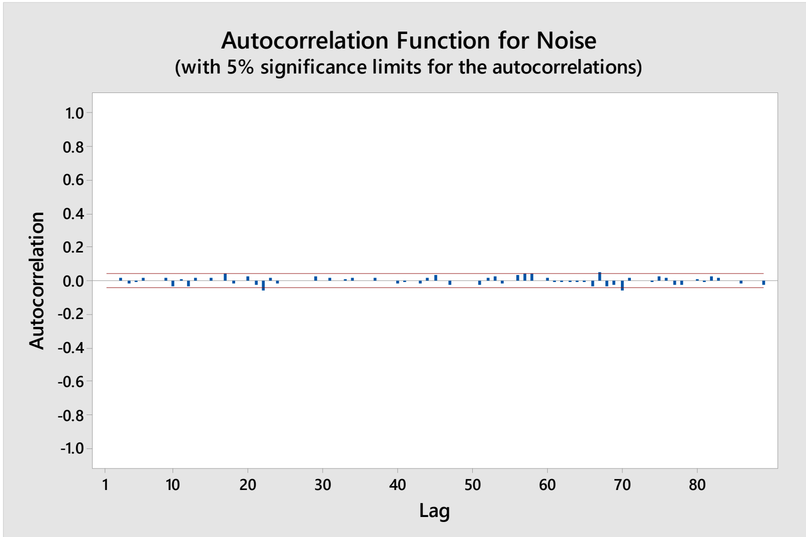

Figure 1: SACF of noise

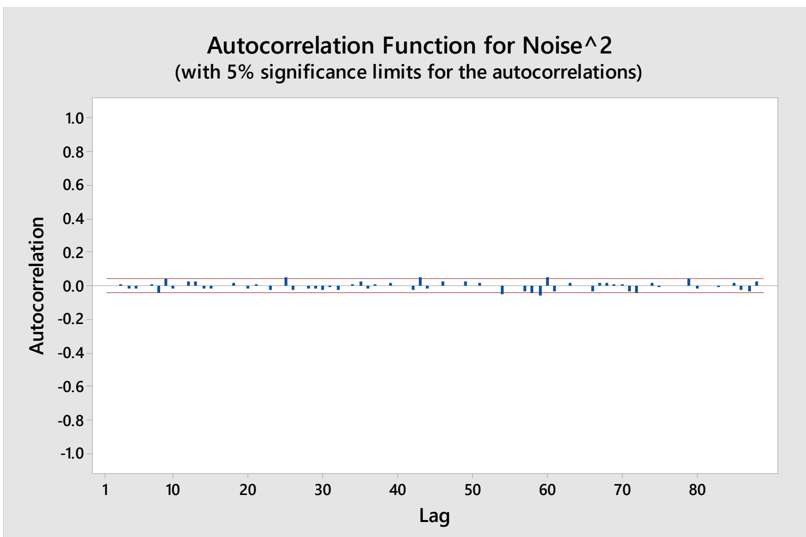

Figure 2: SACF of noise squared

The primary method of seeing if the prediction interval estimation is sound if by calculating **Coverage**. If one constructs a 95% prediction interval, then close to 95% of the observed values must lie in that interval over time. The condition is the same for all levels of probability. For this example, I calculated the following coverage levels.

Level	*Coverage*
80	80.2
90	90.2
95	94.9

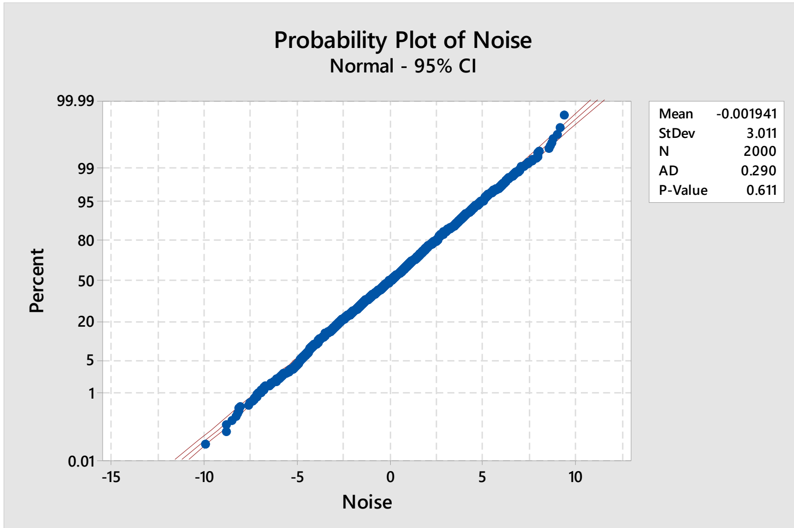

Figure 3: Checking normality

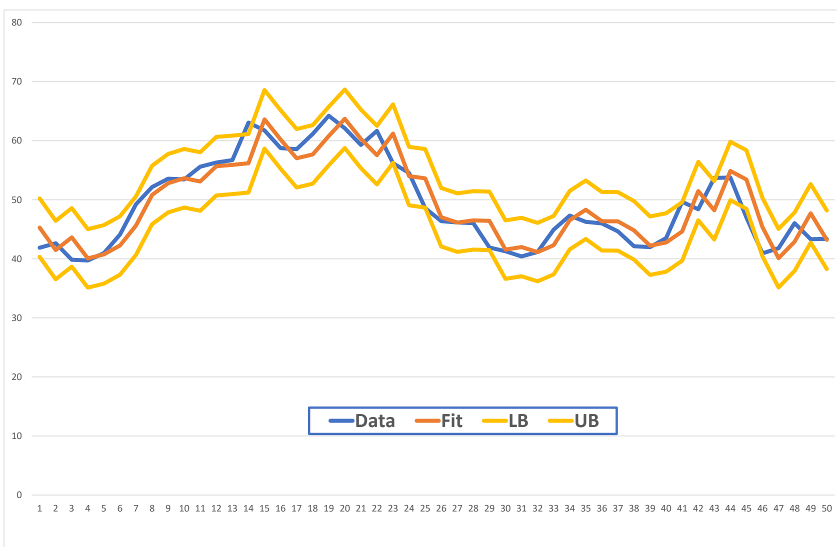

Figure 4: One step ahead forecast

5 First Complication - non Normal noise

We will look at the daily solar irradiation data that we used in the forecasting practical session. One can see that there are greater differences in a negative direction than positive in Figure 5. This is confirmed in the histogram with a normal curve overlaid in Figure 6.

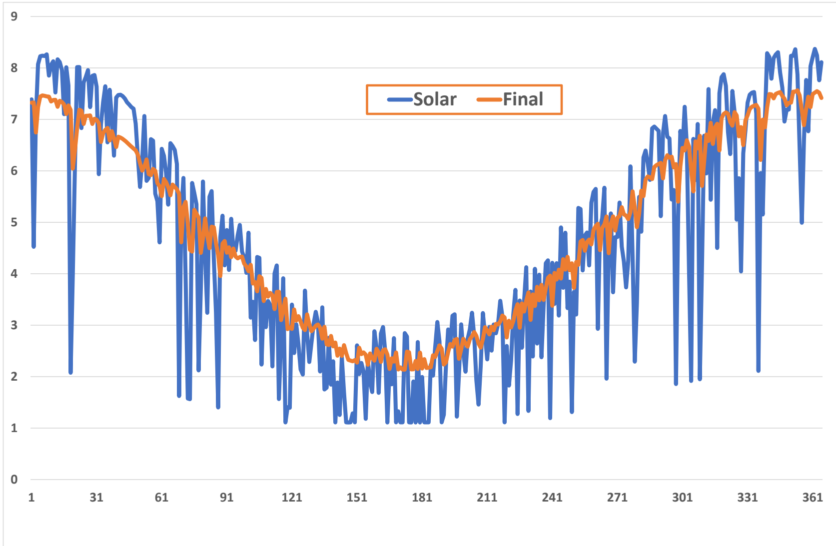

Figure 5: One step ahead forecast for daily solar irradiation

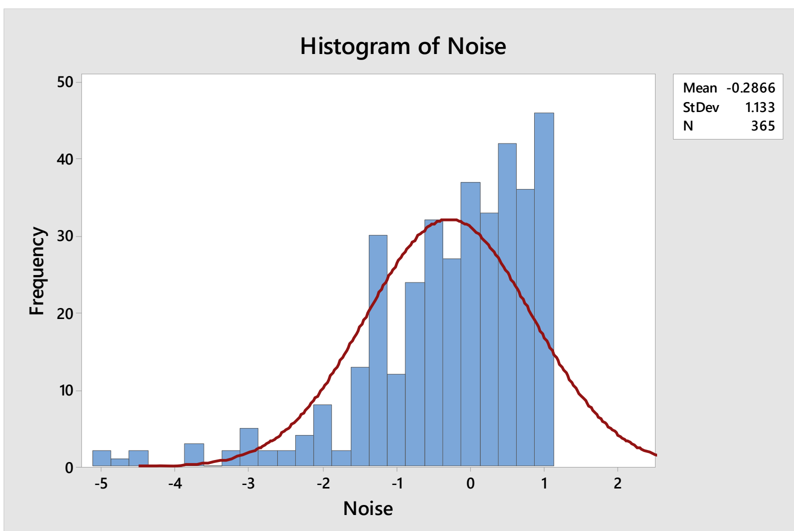

Figure 6: Histogram of the noise

So, what happens if we blindly ignore the fact that the noise is negatively skewed and try to construct prediction intervals with the standard approach of using $\hat{x}_t \pm z_\alpha s$ where s is the standard deviation of the noise? We get the following coverage values for the three levels of probability.

Level	*Coverage*
80	86.3
90	92.3
95	92.6

These all differ from the required values, and that would be because of our incorrect assumption of normality. So, how do we cater for the skewness of the noise? One straightforward way is to select the appropriate quantiles to suit the interval we want, that is for example $0.025, 0.975$ for a 95% prediction interval, or $0.05, 0.95$ for a 90% prediction interval - see Figure 7. Then, add these values to the noise at each time step. The result is Figure 8. The coverage figures when we do this are given in the following table. The next complication will be to look at the situation where the noise is roughly symmetrical, but the noise from time step to time step is dependent. After the mid-semester break we will look at the situation where we have dependent noise plus skewness, and even where the distributions change systematically over time.

Level	*Coverage*
80	80.2
90	90.1
95	95.1

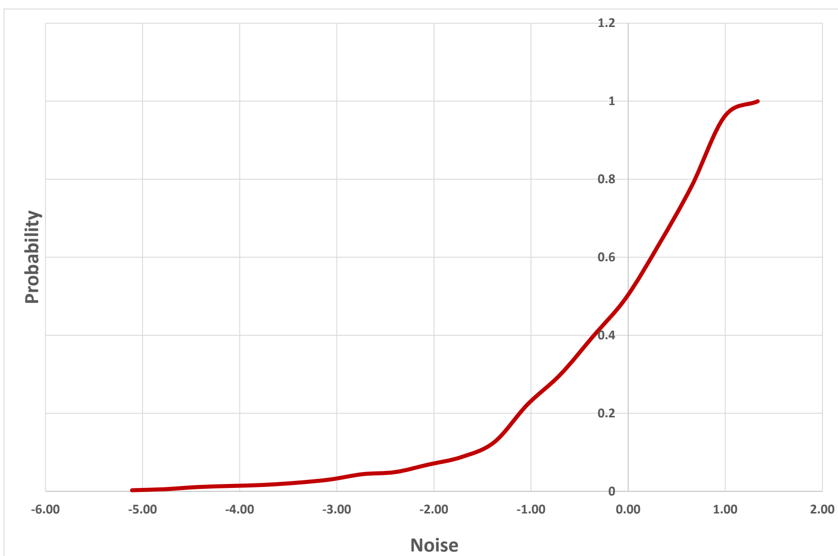

Figure 7: Cumulative Distribution Function of the Noise

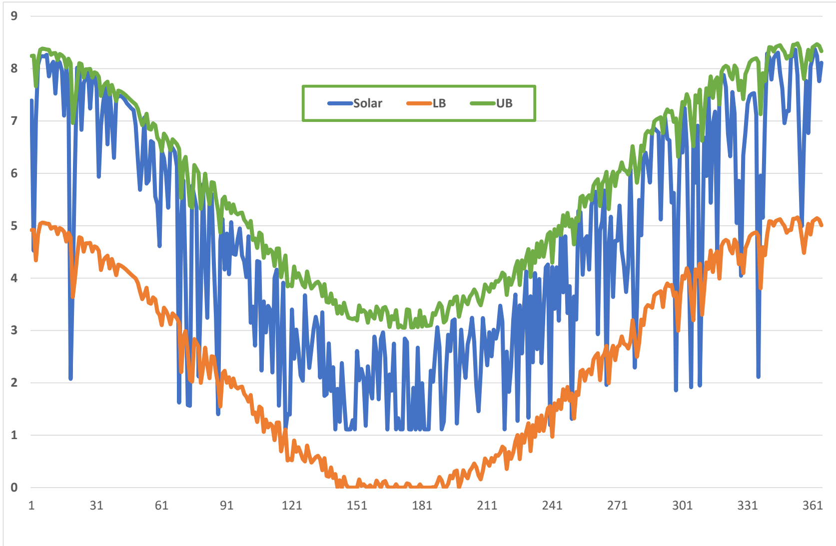

Figure 8: Forecast with Bounds using the Quantiles

6 Computer Practical

This week, I want you to duplicate what I have done for the daily solar, plus see if the assumption of normality will work for the daily temperature data. If not, repeat the solar procedure. Then, do what I do in the workshop for ARCH/GARCH examples.