

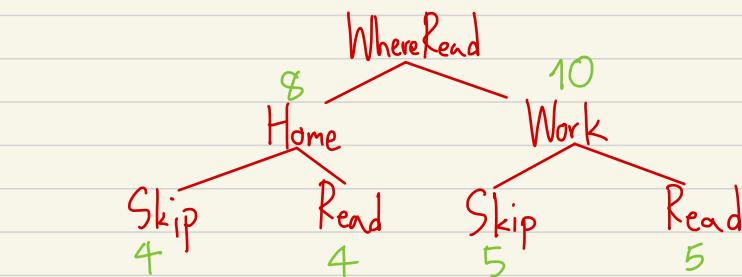
1.

Instructions: Students can use an electronic calculator and a ~~book or notes~~ pen or paper during a closed-book examination.

1. (4 marks) Consider the supervised learning dataset in **Figure 1**. Calculate the **log loss** of splitting the input attribute "WhereRead" in 'UserAction' (Show all your calculation steps).

| Example | Author | Thread | Length | WhereRead | UserAction |
|---------|---------|-----------|--------|-----------|------------|
| e1 | known | new | long | home | reads |
| e2 | unknown | new | short | work | skips |
| e3 | unknown | follow Up | long | work | skips |
| e4 | known | follow Up | long | home | skips |
| e5 | known | new | short | home | reads |
| e6 | known | follow Up | long | work | skips |
| e7 | unknown | follow Up | short | work | skips |
| e8 | unknown | new | short | work | reads |
| e9 | known | follow Up | long | home | skips |
| e10 | known | new | long | work | skips |
| e11 | unknown | follow Up | short | home | skips |
| e12 | known | new | long | work | reads |
| e13 | known | follow Up | short | home | reads |
| e14 | known | new | short | work | reads |
| e15 | known | new | short | home | reads |
| e16 | known | follow Up | short | work | reads |
| e17 | known | new | short | home | reads |
| e18 | unknown | new | short | work | reads |
| e19 | unknown | new | long | work | ? |
| e20 | unknown | follow Up | long | home | ? Unseen |

Figure 1. When the linear function has only one variable, show that its graph is a hyper-plane of dimension K (show the linear equation of the graph).



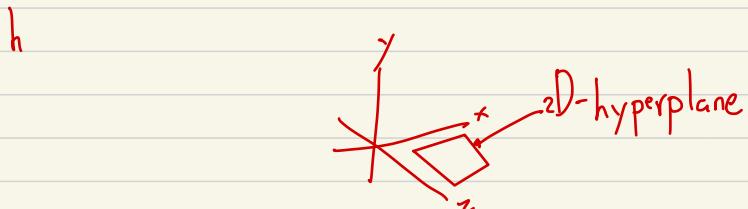
$$\text{log-loss} = - \frac{(4 \times \log_2(\frac{4}{8}) + 4 \times \log_2(\frac{4}{8}) + 5 \times \log_2(\frac{5}{10}) + 5 \times \log_2(\frac{5}{10}))}{18}$$

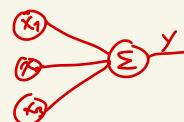
$$= \frac{4+4+5+5}{18} = 1$$

2.

2. (2 marks) When the linear function has only one variable, show that its graph is a hyper-plane of dimension K (show the linear equation of the graph).

Linear Function with one variable $f(x_1) = a_1x_1 + b$
 the graph is hyperplane of dimension 1





- dimension **B** (show me linear equations)
3. (1 mark) Check whether the following statement is **True** or **False**: In unsupervised learning, the Neural Network (NN) tries to "understand" and generate the output structure of the provided input data set "on its own" in the absence of its output pattern. **True**
 4. (1 mark) Check whether the following statement is **True** or **False**: Gradient descent is an iterative method to find the maximum of a function. **False**
 5. (1 mark) Check whether the following statement is **True** or **False**: Overfitting occurs if the model or algorithm shows high bias but low variance. **False**

| x_1 | x_2 | x_3 | y |
|-------|-------|-------|-----|
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

6. or algorithm shows high bias but low variance.
6. (2 marks) A three-input, one-output **parity detector** outputs a one if the number of "1" inputs is **even**; otherwise, it outputs a **0**. Can a Perceptron represent this function? If so, construct a Perceptron that does it; if not, argue why not.

7. Perceptron that does it, if not, argue why not.

7. (5 marks) The Neural Network (NN) with its complete input, weight, and bias values are shown in **Figure 2**. Calculate the outputs of $h_1(2)$, $h_2(2)$, $h_3(2)$, and $h_{1(3)}^{out}$ based on the following values: **Figure 2**. $w_{11}(1) = w_{12}(1) = w_{13}(1) = 0.4$, $w_{21}(1) = w_{22}(1) = w_{23}(1) = 0.5$, $w_{31}(1) = w_{32}(1) = w_{33}(1) = 0.3$, $w_{11}(2) = w_{12}(2) = w_{13}(2) = 0.2$, $b_1(1) = b_2(1) = b_3(1) = 0.7$, $b_{1(2)} = 0.3$, $x_1 = 2.5$, $x_2 = 1.5$, and $x_3 = 3.5$. Use the **sigmoid** function as its output activation function.

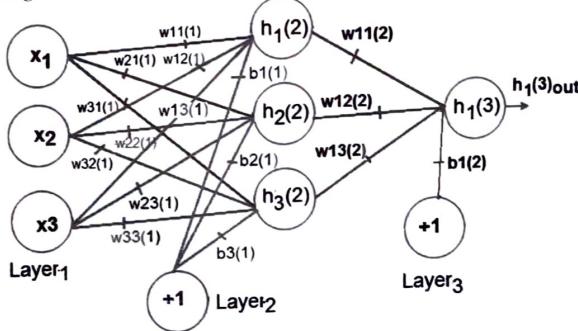


Figure 2.

$$a). H_1^{(2)} = 0.4(2.5) + 0.4(1.5) + 0.4(3.5) + 0.7 = 3.7 \\ \frac{1}{1+e^{-3.7}} = 0.9958$$

$$b) H_2^{(2)} = 0.5(2.5) + 0.5(1.5) + 0.5(3.5) + 0.7 = 4.45 \\ \frac{1}{1+e^{-4.45}} = 0.9885$$

$$c) H_3^{(2)} = 0.3(2.5) + 0.3(1.5) + 0.3(3.5) + 0.7 = 2.95 \\ \frac{1}{1+e^{-2.95}} = 0.9502$$

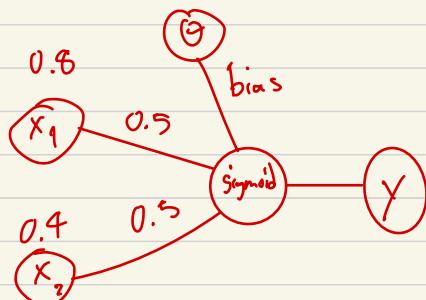
$$d) H_{1(3)}^{out} = 0.2(0.9958) + 0.2(0.9885) + 0.2(0.9502) + 0.3 = 0.8829 \\ \frac{1}{1+e^{-0.8829}} = 0.7074$$

17e

8. (4 marks) Consider a perceptron with two real-valued inputs and an output unit with a sigmoid activation function. All the initial weights and the bias(threshold) equal 0.5. Assume that the output should be 0.8 for the inputs $x_1 = 0.8$ and $x_2 = -0.4$. Show how the delta rule would alter this neural net upon processing (learning rate = 0.5). Show at least 1 iteration.

9. (3 marks) Show

$$\begin{array}{lllllll} & & \text{actual } y & & \text{predict } y & & \\ X_1 & X_2 & Y_d & w_1 & w_2 & e & w_1 \text{ new } w_2 \text{ new} \\ 0.8 & -0.4 & 0.8 & 0.5 & 0.5 & & \end{array}$$



$$Y : 0.5(0.8) + 0.5(-0.4) - 0.5 = -0.3$$

$$y = \frac{1}{1 + e^{-0.3}} = 0.43$$

$$e = Y_d - y = 0.37$$

$$w_1 \text{ new} = w_1 + \alpha(e) x_1$$

$$= 0.5 + 0.5(0.37)(0.8)$$

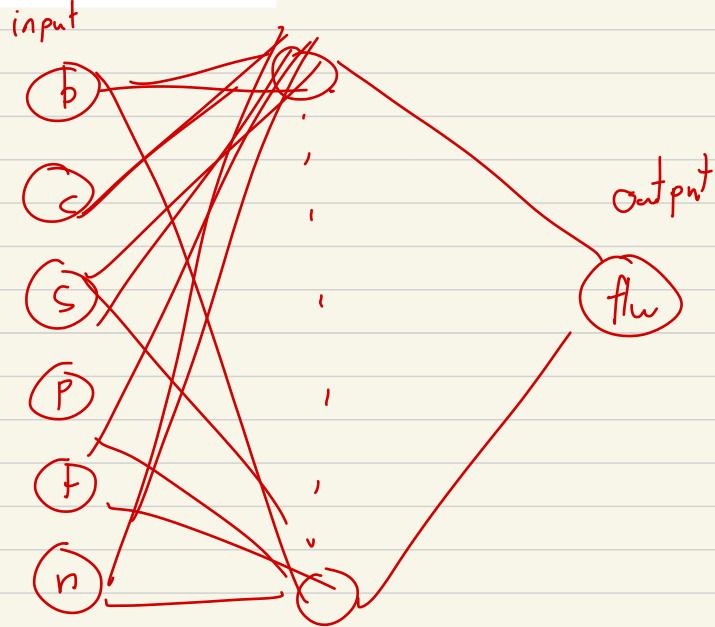
$$= 0.648$$

$$w_2 \text{ new} = w_2 + \alpha(e) x_2$$

$$= 0.5 + 0.5(0.37)(-0.4)$$

$$= 0.43$$

9. (3 marks) Suppose you use a backpropagation neural network (BPNN) for recognizing flu based on a patient dataset consisting of six input attributes (*body temperature*, *cough*, *body shivering*, *body pain*, *tastelessness*, and *nausea*) and *one* output value. All the values in the dataset are real numbers. Briefly describe how you will implement the prediction system (your description should include the topology of the BPNN, sample seen data, network training, and implementation procedure of the BPNN, etc.).



| | | | | |
|------|-----|-----|-----|---|
| 37.9 | 0.6 | ... | 0.5 | X |
| 36.9 | 0.8 | - | 0.3 | 1 |

Weight [0.1, 0.2 ... 0.3]

BackPNN

find new weight work back

10

the BPNN, etc.).

10. (3 marks) Calculate the following based on the training examples for a regression task shown in **Figure 3** (where X is input data, Y_d is desired output, Y is the predicted/calculated output, and E is the error value that must be calculated):

- (1 mark) 0/1 Error.
- (1 mark) The sum of squares error (SOSE).
- (1 mark) Root mean square error (RMSE).

| Example | X | Y_d | Y | E |
|---------|-----|-------|-----|---|
| e_1 | 0.7 | 1.7 | 1.7 | 0 |
| e_2 | 1.1 | 2.4 | 2.0 | 1 |
| e_3 | 1.3 | 2.5 | 2.5 | 0 |
| e_4 | 1.9 | 1.5 | 1.5 | 0 |
| e_5 | 2.6 | 2.1 | 1.2 | 1 |
| e_6 | 3.1 | 2.3 | 2.3 | 0 |
| e_7 | 3.9 | 3.2 | 2.8 | 1 |
| e_8 | 2.9 | 1.8 | 1.8 | 0 |
| e_9 | 5.0 | 3.4 | 3.2 | 1 |

$$\begin{array}{l} \text{0/1} \quad e \quad e^2 \\ \hline 0 & 0.4 & 0.16 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0.9 & 0.81 \\ 0 & 0.4 & 0.16 \\ 1 & 0.4 & 0.16 \\ 0 & 0 & 0 \\ 1 & 0.2 & 0.04 \end{array}$$

Figure 3.

11. (4 marks) Suppose a company wants to group the visitors to a website using just their age (one-dimensional space) as follows: 21, 22, 28, 35, 60, 61, 65, 40, 5, 15, 16, 19, 19, 20, 20, 41. Assume that the random initial centroids are $M_1 = 16$ and $M_2 = 22$. Based on the given parameters, show the iterations (at least 2) used by the k-means clustering algorithm for $k = 2$.

a. 0/1 Error = $\sum_{i=0/1} i = 1+1+1+1 = 4$

b. SOSE = $\sum e^2 = 0.16 + 0.81 + 0.76 + 0.04 = 0.117$

c. RMSE = $\sqrt{\frac{SOSE}{n}} = \sqrt{\frac{0.117}{9}} \approx 0.114$

11

11. (4 marks) Suppose a company wants to group the visitors to a website using just their age (one-dimensional space) as follows: 21, 22, 28, 35, 60, 61, 65, 40, 5, 15, 16, 19, 19, 20, 20, 41. Assume that the random initial centroids are $M_1 = 16$ and $M_2 = 22$. Based on the given parameters, show the iterations (at least 2) used by the k-means clustering algorithm for $k = 2$. *Since k=2*

Iteration #1, $M_1 = 16, M_2 = 22$

$$C_1 = \{5, 15, 16, 19, 19\}$$

$$C_2 = \{21, 22, 28, 35, 60, 61, 65, 40, 41\}$$

$$\text{New } M_1 = 17.44 \quad \& \quad M_2 = 47.143$$

Iteration #2

$$C_1 = \{5, 15, 16, 19, 19, 21, 22, 28\}$$

$$C_2 = \{35, 60, 61, 65, 40, 41\}$$

12

12. (3 marks) Based on the performance indicators of a Boolean predictor in **Figure 4**, calculate the following: a) [1 mark] *Precision*. b) [1 mark] *Recall*. c) [1 mark] *False-Positive Rate*.

| | | |
|-----------|-----------|-----------|
| | <i>an</i> | <i>dn</i> |
| <i>pp</i> | 90 | 200 |
| <i>pn</i> | 50 | 500 |

Figure 4.

$$\text{precision} = \frac{tp}{tp + fp} = \frac{90}{90 + 200}$$

$$\text{recall} = \frac{tp}{tp + fn} = \frac{90}{90 + 50}$$

$$fn\text{-rate} = \frac{fn}{fn + tp} = \frac{200}{200 + 50}$$

13

13. (5 marks) Suppose a spinning wheel game with nine numbers [1,2,3,4,5,6,7,8,9], each with equal probability. Let S be the outcome of a spin. Based on these:
- (2 marks) Find the information (entropy) $H(S)$ of a spin.
 - (2 marks) Suppose an **odd sensor** O is connected and detects only the odd value from a spin. Then, find $H(S|O)$.
 - (1 mark) Find the **information gain (IG)** of the spin with the odd sensor O .

$$\begin{aligned} a) H(S) &= - \sum_{x=1}^9 P(X=x) \log_2 P(X=x) \\ &= - \sum_{x=1}^9 \frac{1}{9} \log_2 \left(\frac{1}{9}\right) \\ &= -9 \times \frac{1}{9} \times \frac{\log_2(1/9)}{\log_2(2)} \\ &= -(-3.167) = 3.167 \end{aligned}$$

$$\begin{aligned} b) H(S|O) &= - \frac{5}{9} \log_2 \frac{1}{5} - \frac{4}{9} \log_2 \frac{1}{4} \\ &= 2.194 \end{aligned}$$

$$c) IG = H(S) - H(S|O)$$

14.

14. (4 marks) You believe 1% of sports members in your group use steroids. You have arranged a blood test that detects steroids. There is a 99% chance you will be caught positive to steroids when you use steroids. There is a 0.5% chance you will be positive if you do not use steroids. What is the chance that a positive blood analysis identifies a steroid user? (Hints: Let A is "use steroids," and B is "positive to steroids"). Show all of your calculation steps.

$$P(A|B)$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\neg A)P(\neg A)}$$

$$P(S) = 0.01$$

$$P(+|S) = 0.99$$

$$P(+|\neg S) = 0.005$$

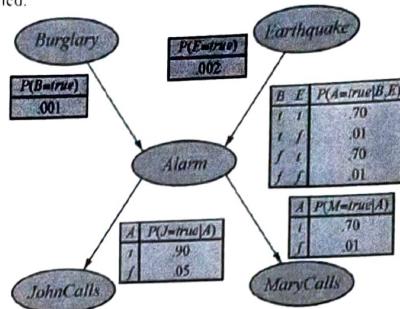
$$\begin{aligned} P(S|+) &= \frac{P(+|S)P(S)}{P(+|S)P(S) + P(+|\neg S)P(\neg S)} \\ &= \frac{0.99(0.01)}{0.99(0.01) + (0.005)(0.99)} \end{aligned}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

15

15. (3 marks) Calculate the probability of the following event based on the Bayes net shown in Figure 2:

"The alarm (A) has sounded, but no burglary (B) has occurred, but a minor earthquake (E) has occurred, and John (J) called and Mary (M) not called.



$$\begin{aligned} P(A \wedge \neg B \wedge E \wedge J \wedge \neg M) &= P(B) \times P(E) \times P(A|B \wedge E) \times P(J|A) \times P(\neg M|A) \\ &= 0.99 \times 0.02 \times 0.7 \times 0.9 \times 0.3 \end{aligned}$$

16

Figure 5.

16. (5 marks) The joint probability distribution of three variables, flu (f), allergy(a), and sinus (s), is shown in **Figure 6**. By applying the direct computation of the joint probability distribution, check whether $P(f^1|s^1) < P(f^1|a^1)$.

| | | | | |
|---|-------|-------|-------|--------|
| 1 | f^1 | a^1 | s^1 | 0.0270 |
| 2 | f^1 | a^1 | s^0 | 0.0030 |
| 3 | f^1 | a^0 | s^1 | 0.1620 |
| 4 | f^1 | a^0 | s^0 | 0.1080 |
| 5 | f^0 | a^1 | s^1 | 0.0140 |
| 6 | f^0 | a^1 | s^0 | 0.0560 |
| 7 | f^0 | a^0 | s^1 | 0.0063 |
| 8 | f^0 | a^0 | s^0 | 0.6237 |

Figure 6.

17. (6 marks) Consider the Bayesian Network with three Boolean variables shown in **Figure 7**.

$$P(f^1|s^1) = \frac{P(f^1 \cap s^1)}{P(s^1)} = \frac{\textcircled{1} + \textcircled{3}}{\textcircled{1} + \textcircled{2} + \textcircled{5} + \textcircled{7}} = \frac{0.189}{0.189 + 0.6203} = \frac{0.189}{0.2093} = 0.903$$

$$P(f^1|a^1) = \frac{P(f^1 \cap a^1)}{P(a^1)} = \frac{\textcircled{1} + \textcircled{2}}{\textcircled{1} + \textcircled{2} + \textcircled{5} + \textcircled{6}} = \frac{0.03}{0.03 + 0.07} = \frac{0.03}{0.1} = 0.3 \quad \text{QED}$$

17

Figure 6.

17. (6 marks) Consider the Bayesian Network with three Boolean variables shown in **Figure 7**. Compute a) [2 marks]. $P(\neg A | W, H)$. b) [2 marks] $P(\neg A, W, H)$. c) [2 marks] $P(\neg A | H)$.

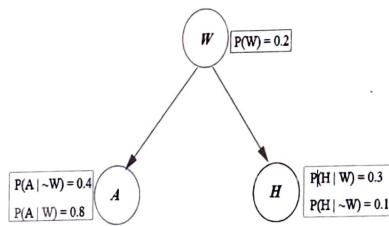


Figure 7.

$$\text{a. } P(\neg A | W, H) = \frac{P(\neg A \wedge W \wedge H)}{P(W \wedge H)} = \frac{0.2 \times 0.2 \times 0.3}{0.2 \times 0.3} = 0.2$$

$$\text{b. } P(\neg A \wedge W \wedge H) = P(W) \times P(\neg A | W) \times P(H | W) \\ = 0.2 \times 0.2 \times 0.3$$

$$c. P(\sim A | H) = \frac{P(\sim A \wedge H)}{P(H)} = \frac{P(H \wedge \sim A \wedge W) + P(H \wedge A \wedge \sim W)}{P(H \wedge \sim A \wedge W) + P(H \wedge A \wedge W) + P(H \wedge A \wedge \sim W) + P(H \wedge \sim A \wedge \sim W)}$$

18.

Figure 7.

18. (4 marks) **Figure 8** shows the two factors, `go_out`, and `get_coffee`, from a probabilistic inference function. Show summing out the variable `Rain` from the product of `get_coffee` and `go_out`.

| | <code>Full</code> | <code>Wet</code> | <code>Prob</code> |
|-------------------------|-------------------|------------------|-------------------|
| <code>get_coffee</code> | <code>t</code> | <code>t</code> | 0.6 |
| | <code>t</code> | <code>f</code> | 0.4 |
| | <code>f</code> | <code>t</code> | 0.3 |
| | <code>f</code> | <code>f</code> | 0.7 |

| | <code>Rain</code> | <code>Wet</code> | <code>Prob</code> |
|---------------------|-------------------|------------------|-------------------|
| <code>go_out</code> | <code>t</code> | <code>t</code> | 0.8 |
| | <code>t</code> | <code>f</code> | 0.2 |
| | <code>f</code> | <code>t</code> | 0.1 |
| | <code>f</code> | <code>f</code> | 0.9 |

Figure 8.

1. times & Natural Join

| <code>Full</code> | <code>Wet</code> | <code>Rain</code> | <code>Prob</code> |
|-------------------|------------------|-------------------|-------------------|
| <code>t</code> | <code>t</code> | <code>t</code> | 0.48 |
| <code>t</code> | <code>t</code> | <code>f</code> | 0.06 |
| <code>t</code> | <code>f</code> | <code>t</code> | 0.08 |
| <code>t</code> | <code>f</code> | <code>f</code> | 0.36 |
| <code>f</code> | <code>t</code> | <code>t</code> | 0.24 |
| <code>f</code> | <code>t</code> | <code>f</code> | 0.03 |
| <code>f</code> | <code>f</code> | <code>t</code> | 0.14 |
| <code>f</code> | <code>f</code> | <code>f</code> | 0.63 |

2. Summing

| <code>Full</code> | <code>Wet</code> | <code>Prob</code> |
|-------------------|------------------|-------------------|
| <code>t</code> | <code>t</code> | 0.54 |
| <code>t</code> | <code>f</code> | 0.44 |
| <code>f</code> | <code>t</code> | 0.27 |
| <code>f</code> | <code>f</code> | 0.97 |

e.

$$\begin{array}{c} A \longrightarrow F \\ \swarrow B \end{array}$$

4) $P(C|D) = P(C|D \cap C)P(C) + P(C|D \cap \neg C)P(\neg C)$

$$\begin{aligned} P(C) &= P(C \cap A \cap B) + P(C \cap A \cap \neg B) + P(C \cap \neg A \cap B) + P(C \cap \neg A \cap \neg B) \\ &= P(C|A, B)P(A \cap B) + P(C|A, \neg B)P(A \cap \neg B) + P(C|\neg A, B)P(\neg A \cap B) \\ &\quad + P(C|\neg A, \neg B)P(\neg A \cap \neg B) \end{aligned}$$

Assumption University
 Vincent Mary School of Science and Technology
 CS3425/CSX4209/ITX4209 ISD
Quiz II Semester 2/2022 (60 marks)

Instructions: Students can use an electronic calculator and a **black** or **blue** pen for answering. This is a **closed-book** examination.

- (4 marks) Consider the supervised learning dataset in **Figure 1**. Calculate the **log loss** of splitting the input attribute "**WhereRead**" in '**UserAction**' (Show all your calculation steps).

| Example | Author | Thread | Length | WhereRead | UserAction |
|-----------------|---------|-----------|--------|-----------|------------|
| e ₁ | known | new | long | home | skips |
| e ₂ | unknown | new | short | work | reads |
| e ₃ | unknown | follow Up | long | work | skips |
| e ₄ | known | follow Up | long | home | skips |
| e ₅ | known | new | short | home | reads |
| e ₆ | known | follow Up | long | work | skips |
| e ₇ | unknown | follow Up | short | work | skips |
| e ₈ | unknown | new | short | work | reads |
| e ₉ | known | follow Up | long | home | skips |
| e ₁₀ | known | new | long | work | skips |
| e ₁₁ | unknown | follow Up | short | home | skips |
| e ₁₂ | known | new | long | work | skips |
| e ₁₃ | known | follow Up | short | home | reads |
| e ₁₄ | known | new | short | work | reads |
| e ₁₅ | known | new | short | home | reads |
| e ₁₆ | known | follow Up | short | work | reads |
| e ₁₇ | known | new | short | home | reads |
| e ₁₈ | unknown | new | short | work | reads |
| e ₁₉ | unknown | new | long | work | ? Unseen |
| e ₂₀ | unknown | follow Up | long | home | ? Unseen |

Figure 1.

- (2 marks) When the linear function has only **one** variable, show that its graph is a hyper-plane of dimension **K** (show the *linear equation of the graph*).
- (1 mark) Check whether the following statement is **True** or **False**: In unsupervised learning, the Neural Network (NN) tries to "understand" and generate the output structure of the provided input data set "on its own" in the absence of its output pattern. **X T?**
- (1 mark) Check whether the following statement is **True** or **False**: Gradient descent is an iterative method to find the maximum of a function. **F**
- (1 mark) Check whether the following statement is **True** or **False**: Overfitting occurs if the model or algorithm shows high bias but low variance. **F**
- (2 marks) A three-input, one-output **parity detector** outputs a one if the number of "1" inputs is **even**; otherwise, it outputs a **0**. Can a Perceptron represent this function? If so, construct a Perceptron that does it; if not, argue why not.
- (5 marks) The Neural Network (NN) with its complete input, weight, and bias values are shown in **Figure 2**. Calculate the outputs of $h_1(2)$, $h_2(2)$, $h_3(2)$, and $h_{1(3)}^{out}$ based on the following values: $w_{11}(1) = w_{12}(1) = w_{13}(1) = 0.4$, $w_{21}(1) = w_{22}(1) = w_{23}(1) = 0.5$, $w_{31}(1) = w_{32}(1) = w_{33}(1) = 0.3$, $w_{11}(2) = w_{12}(2) = w_{13}(2) = 0.2$, $b_{1(1)} = b_{2(1)} = b_{3(1)} = 0.7$, $b_{1(2)} = 0.3$, $x_1 = 2.5$, $x_2 = 1.5$, and $x_3 = 3.5$. Use the **sigmoid** function as its output activation function.

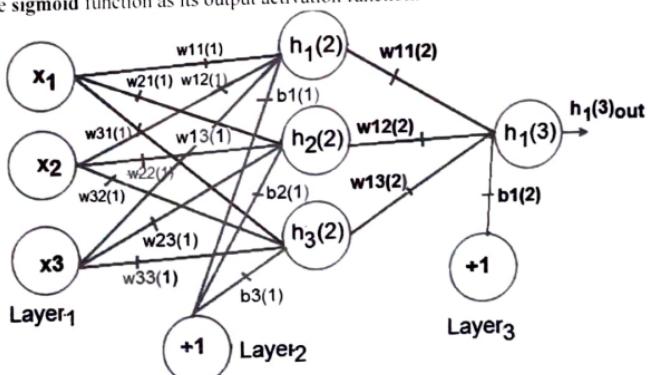


Figure 2.

$$\frac{1}{1+e^{-x}}$$

8. (4 marks) Consider a perceptron that with two real-valued inputs and an output unit with a sigmoid activation function. All the initial weights and the bias(threshold) equal 0.5. Assume that the output should be 0.8 for the inputs $x_1 = 0.8$ and $x_2 = -0.4$. Show how the delta rule would alter this neural net upon processing (learning rate = 0.5). Show at least 1 iteration.
9. (3 marks) Suppose you use a backpropagation neural network (BPNN) for recognizing flu based on a patient dataset consisting of six input attributes (*body temperature, cough, body shivering, body pain, tastelessness, and nausea*) and **one** output value. All the values in the dataset are real numbers. Briefly describe how you will implement the prediction system (your description should include the topology of the BPNN, sample seen data, network training, and implementation procedure of the BPNN, etc.).
10. (3 marks) Calculate the following based on the training examples for a regression task shown in **Figure 3** (where X is input data, Y_d is desired output, Y is the predicted/calculated output, and E is the error value that must be calculated):
- (1 mark) **0/1 Error.**
 - (1 mark) The sum of squares error (SOSE).
 - (1 mark) Root mean square error (RMSE).

| Example | X | Y_d | Y | E |
|---------|-----|-------|-----|---|
| e_1 | 0.7 | 1.7 | 1.7 | 0 |
| e_2 | 1.1 | 2.4 | 2.0 | 1 |
| e_3 | 1.3 | 2.5 | 2.5 | 0 |
| e_4 | 1.9 | 1.5 | 1.5 | 0 |
| e_5 | 2.6 | 2.1 | 1.2 | 1 |
| e_6 | 3.1 | 2.3 | 2.3 | 0 |
| e_7 | 3.9 | 3.2 | 2.8 | 1 |
| e_8 | 2.9 | 1.8 | 1.8 | 0 |
| e_9 | 5.0 | 3.4 | 3.2 | 1 |

Figure 3.

11. (4 marks) Suppose a company wants to group the visitors to a website using just their age (one-dimensional space) as follows: 21, 22, 28, 35, 60, 61, 65, 40, 5, 15, 16, 19, 19, 20, 20, 41. Assume that the random initial centroids are $M1 = 16$ and $M2 = 22$. Based on the given parameters, show the iterations (at least 2) used by the **k-means clustering** algorithm for $k = 2$. *Sum k=2*
12. (3 marks) Based on the performance indicators of a Boolean predictor in **Figure 4**, calculate the following: a) [1 mark] *Precision*. b) [1 mark] *Recall*. c) [1 mark] *False-Positive Rate*.

| | <i>ap</i> | <i>an</i> |
|-----------|-----------|-----------|
| <i>pp</i> | 90 | 200 |
| <i>pn</i> | 50 | 500 |

Figure 4.

13. (5 marks) Suppose a spinning wheel game with nine numbers [1,2,3,4,5,6,7,8,9], each with equal probability. Let S be the outcome of a spin. Based on these:
- (2 marks) Find the information (entropy) $H(S)$ of a spin.
 - (2 marks) Suppose an **odd sensor O** is connected and detects only the odd value from a spin. Then, find $H(S|O)$.
 - (1 mark) Find the **information gain (IG)** of the spin with the odd sensor **O**.
14. (4 marks) You believe **1%** of sports members in your group use steroids. You have arranged a blood test that detects steroids: There is a **99%** chance you will be caught positive to steroids when you use steroids. There is a **0.5%** chance you will be positive if you do not use steroids. What is the chance that a positive blood analysis identifies a steroid user? (Hints: Let **A** is "use steroids," and **B** is "positive to steroids"). Show all of your calculation steps.

$$P(A|B)$$

15. (3 marks) Calculate the probability of the following event based on the Bayes net shown in Figure 5:
 "The alarm (*A*) has sounded, but no burglary (*B*) has occurred, but a minor earthquake (*E*) has occurred, and John (*J*) called and Mary (*M*) not called.

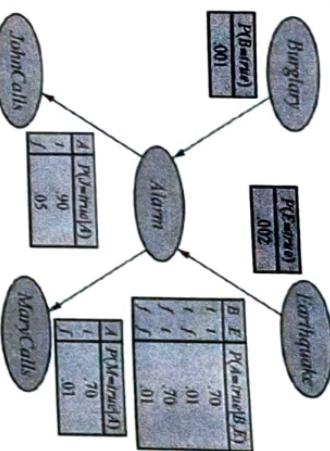


Figure 5.

16. (5 marks) The joint probability distribution of three variables, flu (*f*), allergy(*a*), and sinus (*s*), is shown in Figure 6. By applying the direct computation of the joint probability distribution, check whether $\mathbf{P}(f|s^1) < \mathbf{P}(f|a^1)$.

| | | | |
|-----------------------|-----------------------|-----------------------|--------|
| <i>f</i> ¹ | <i>a</i> ¹ | <i>s</i> ¹ | 0.0270 |
| <i>f</i> ¹ | <i>a</i> ¹ | <i>s</i> ⁰ | 0.0030 |
| <i>f</i> ¹ | <i>a</i> ⁰ | <i>s</i> ¹ | 0.1620 |
| <i>f</i> ¹ | <i>a</i> ⁰ | <i>s</i> ⁰ | 0.1080 |
| <i>f</i> ⁰ | <i>a</i> ¹ | <i>s</i> ¹ | 0.0140 |
| <i>f</i> ⁰ | <i>a</i> ¹ | <i>s</i> ⁰ | 0.0560 |
| <i>f</i> ⁰ | <i>a</i> ⁰ | <i>s</i> ¹ | 0.0063 |
| <i>f</i> ⁰ | <i>a</i> ⁰ | <i>s</i> ⁰ | 0.6237 |

Figure 6.

17. (6 marks) Consider the Bayesian Network with three Boolean variables shown in Figure 7.
 Compute a) [2 marks], $P(\neg A | W, H)$. b) [2 marks] $P(\neg A, W, H)$. c) [2 marks] $P(\neg A | H)$.

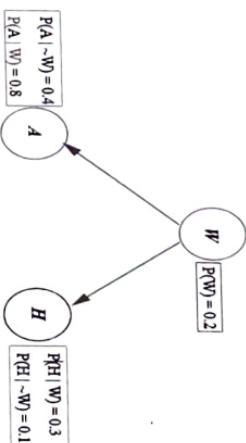


Figure 7.

18. (4 marks) Figure 8 shows the two factors, *go_out*, and *get_coffee*, from a probabilistic inference function. Show summing out the variable *Rain* from the product of *get_coffee* and *go_out*.

| Full | Wet | Prob |
|----------|----------|------|
| <i>t</i> | <i>t</i> | 0.6 |
| <i>t</i> | <i>f</i> | 0.4 |

| Rain | Wet | Prob |
|----------|----------|------|
| <i>t</i> | <i>t</i> | 0.8 |
| <i>f</i> | <i>t</i> | 0.2 |

| get_coffee | go_out | Prob |
|------------|----------|------|
| <i>f</i> | <i>t</i> | 0.3 |
| <i>f</i> | <i>f</i> | 0.7 |

Figure 8.