Quant II

TSCS Data Estimation

Ye Wang

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Outline

- Roadmap for TSCS data estimation
- Sequential experiments
- Semi-parametric models
- Trajectory balancing

Roadmap for TSCS data estimation

- What is the difference between TSCS and panel data?

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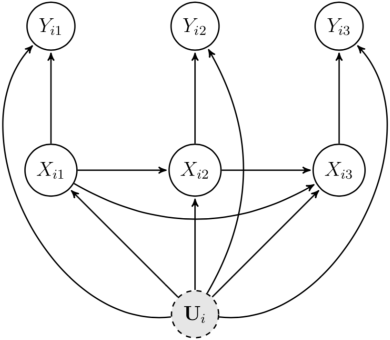

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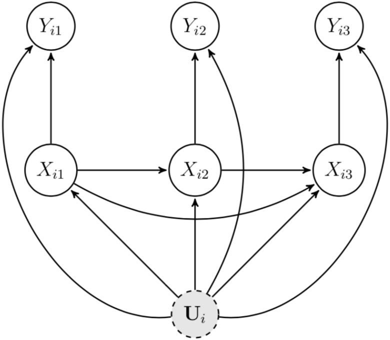

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  Doesn't exist. Only one observation exists for unit i, period t (Imai and Kim, 2019).

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  How to estimate the effect of both treatments?

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  Omitted variable bias for $X_{i,t-1}$.

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- SNMMs:
  - Regress Y_{it} on $(Y_{i,t-1}, X_{it}, X_{i,t-1})$ and get the coefficient of X_{it}, $\hat{\gamma}_0$.
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- MSMs with IPTWs: estimate the propensity score for each X_{it}, and use IPTW for ATT.

$$\widehat{SW}_{it} = \prod_{t=1}^{t} \frac{\widehat{\Pr}[X_{it} \mid X_{i,t-1}; \widehat{\gamma}]}{\widehat{\Pr}[X_{it} \mid Z_{it}, Y_{i,t-1}, X_{i,t-1}; \widehat{\alpha}]}.$$

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  IFE and MC are equivalent when MC directly penalizes the matrix dimension r (hard-impute) rather than the magnitude of eigen values (soft-impute).

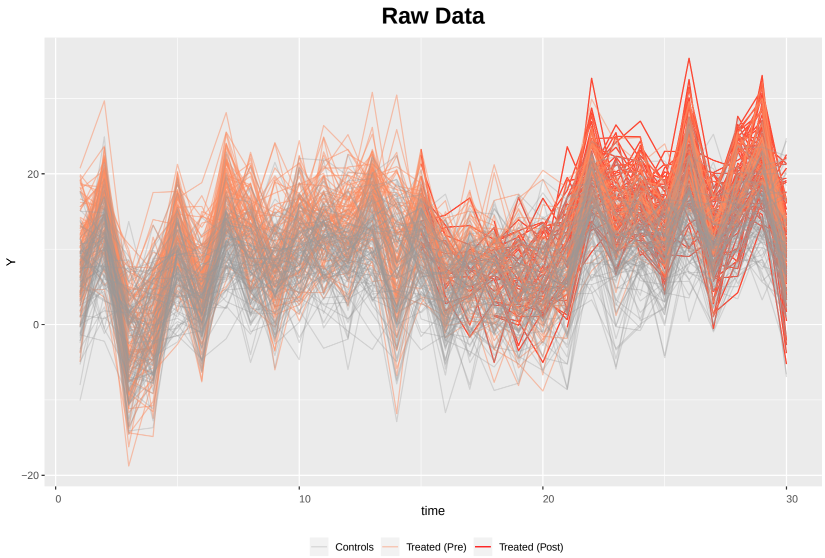
Semi-parametric models: estimation

- IFE relies on the singular value decomposition (SVD).
- We start from initial parameters, estimate β via OLS, and obtain λ_i and $\mathbf{f}_t$ via SVD of the residuals.
- r is selected by cross-validation.
- MC (soft-impute) uses the direct sum decomposition: the residual at each round can be decomposed into two orthogonal parts.
- MC (soft-impute) is biased in finite samples.
- Both work well in large samples (relative performance depends on the strength of factors).
- We can add other parts to the model such as the lagged dependent variable or ensemble methods (Athey et al., 2019)

Semi-parametric models: tools

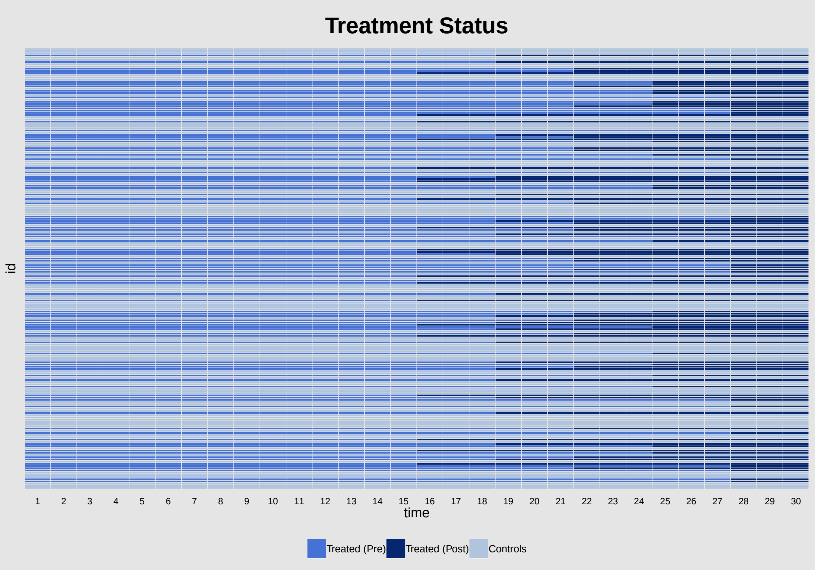
- A series of packages provided by Yiqing Xu at UCSD and his collaborators
- panelView for displaying the basic patterns
- fastplm for estimating the two-way FE models fast
- gsynth for estimating the interactive FE models
- fect to rule them all

Example



Raw Data
20
0
−20
Y
0
10
20
30
time
Controls
Treated (Pre)
Treated (Post)

Example

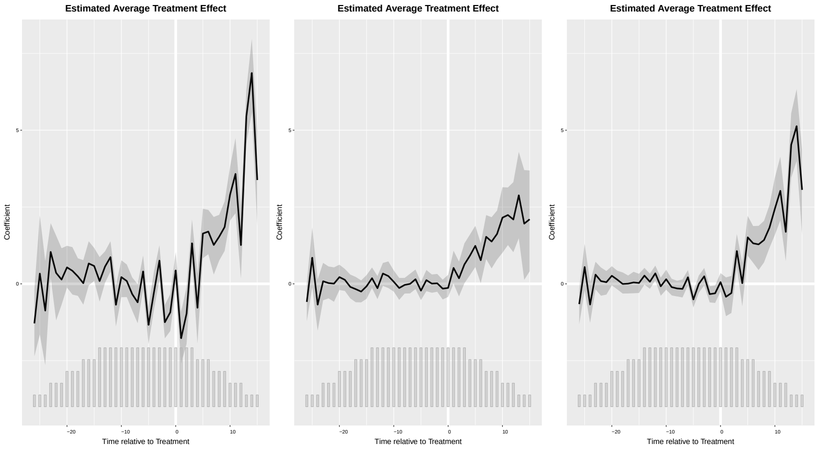


Treatment Status
id
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
time
Treated (Pre)
Treated (Post)
Controls

Example

```
##
##      Coef Std. Error t value Pr(>|t|) CI_lower CI_upper
## D  0.617      0.166   3.727        0    0.293    0.942
## X1 0.987      0.042  23.569        0    0.905    1.069
## X2 3.008      0.042  72.460        0    2.926    3.089
##
## ---
##
## Residual standard error: 3.205 on 5768 degrees of freedo
## Multiple R-squared(full model): 0.817    Adjusted R-squa
## Multiple R-squared(proj model): 0.502    Adjusted R-squa
## F-statistic(full model): 111.640 on 231 and 5768 DF, p-v
## F-statistic(proj model): 1936.160 on 3 and 5768 DF, p-va
```

Example



Estimated Average Treatment Effect
Coefficient
Time relative to Treatment
5
0
−20
−10
0
10
Estimated Average Treatment Effect
Coefficient
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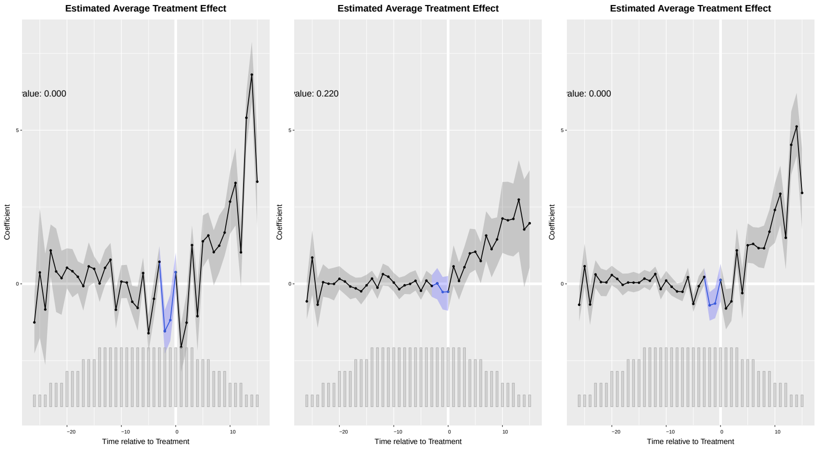
- Liu, Wang, and Xu (2019): dynamic treatment effects, equivalence test and placebo test.

Example


Estimated Average Treatment Effect
value: 0.000
Coefficient
Time relative to Treatment
Estimated Average Treatment Effect
value: 0.220
Coefficient
Time relative to Treatment
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value: 0.000
Coefficient
Time relative to Treatment

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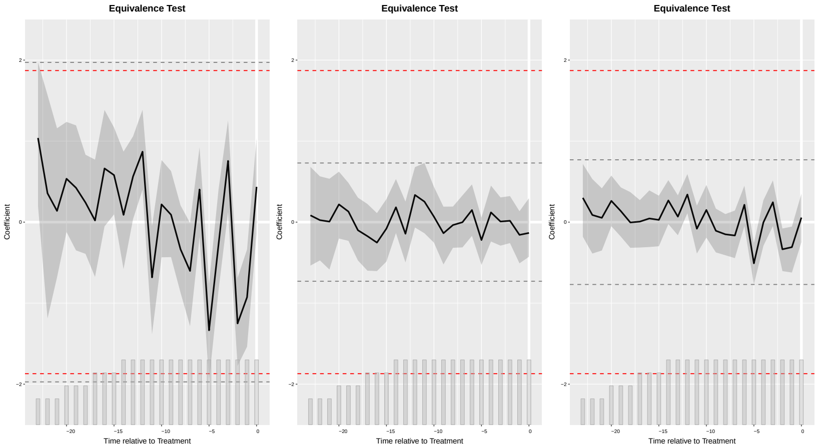
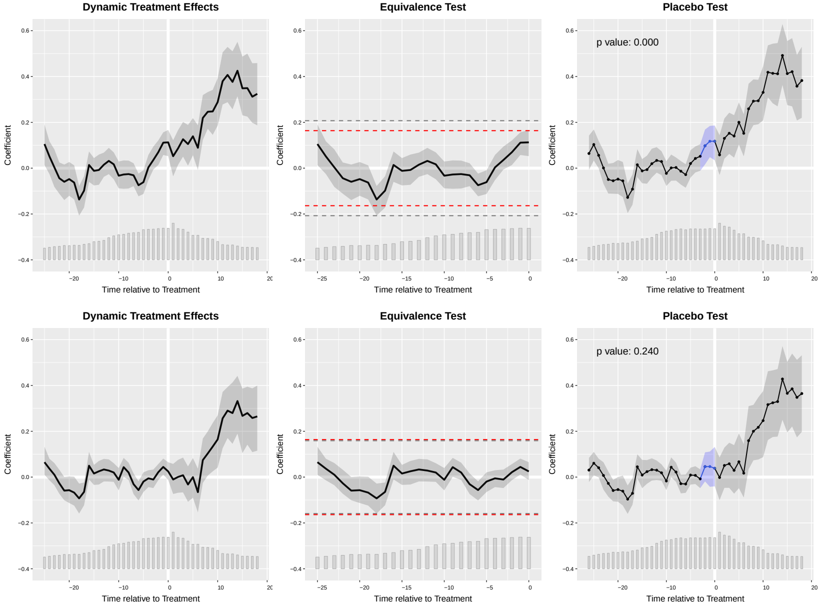

Equivalence Test
Equivalence Test
Equivalence Test
Coefficient
Time relative to Treatment
2
0
−2
−20
−15
−10
−5
0

Example: Tomz et al. (2007)



Trajectory balancing

- Outcomes in a TSCS dataset can be divided into four parts:

$$\boldsymbol{Y}^{\text{obs}} = \begin{pmatrix} \boldsymbol{Y}^{\text{obs}}_{\text{t,post}} & \boldsymbol{Y}^{\text{obs}}_{\text{c,post}} \\ \boldsymbol{Y}^{\text{obs}}_{\text{t,pre}} & \boldsymbol{Y}^{\text{obs}}_{\text{c,pre}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{Y}_{\text{t,post}}(1) & \boldsymbol{Y}_{\text{c,post}}(0) \\ \boldsymbol{Y}_{\text{t,pre}}(0) & \boldsymbol{Y}_{\text{c,pre}}(0) \end{pmatrix}$$

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- All we need to do is to predict $\mathbf{Y}_{t,post}(0)$ using the weighted sum of the other three parts.
- The weights should minimize the difference between $\mathbf{Y}_{t,pre}(0)$ and $\mathbf{Y}_{c,pre}(0)$.

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- Problems: hard to get balance with many units and periods.
- Doudchenko and Imbens (2016): Directly minimize the difference with a penalty function.
- Hazlett and Xu (2017): Balance moments of the kernelized outcomes.
- Imai, Kim, and Wang (2018): Matching on the pre-treatment outcomes.

Future

- What the TA is working on...
- Clearly there are some gaps among different branches of the literature.
- How to unify sequential experiments with semi-parametric models?
- How to think about TSCS models from the design-based perspective? (Athey and Imbens, 2018)
- What is the common ground between spatial interference and temporal interference?
- For example, what can we do to estimate the dynamic effects in a network? (Egami, 2019)