BUS 243

Lecture 6: Introduction to Neural Net

GENSIM MODEL REVIEW

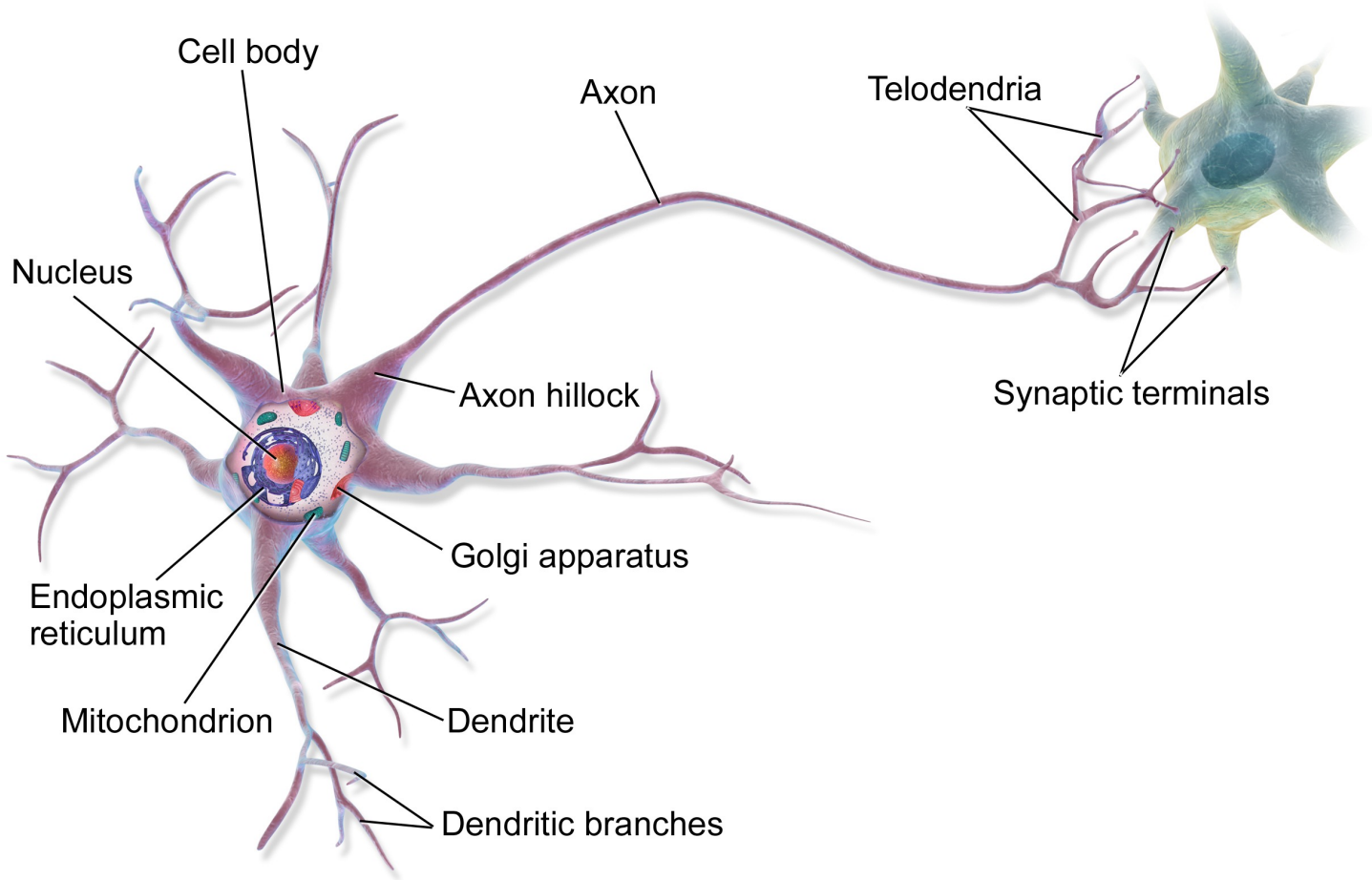
NEURAL NET FRENZY

- You may hear "AI, Deep learning, Neural net" almost everywhere
- Why?
  - Is it better than any type of traditional approach?
- Neural Net is just a class of models
  - Input, output, and parameters

NEURAL NET VS. LOGISTIC REGRESSION

- Experience and domain knowledge are essential for effective feature engineering
  - It is costly…

What is a better model?



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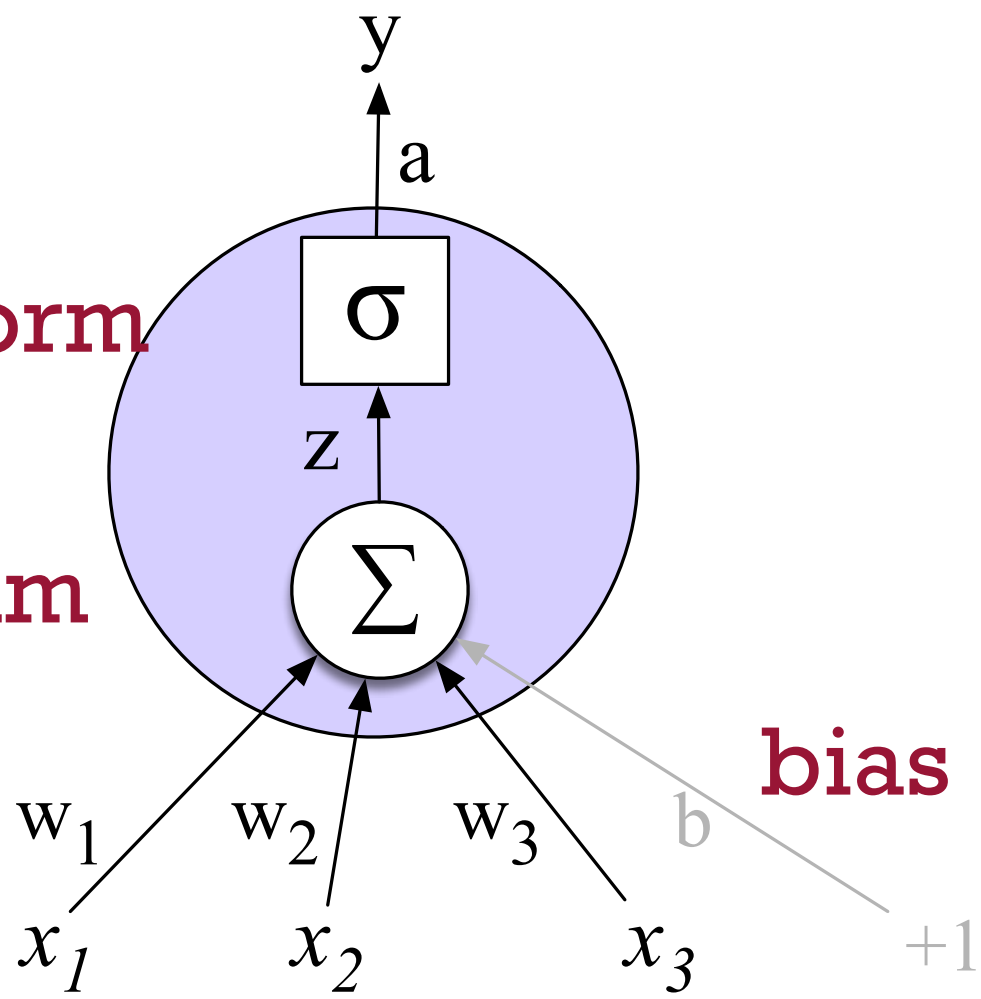
Output value

Non-linear transform

Weighted sum

Weights

Input layer



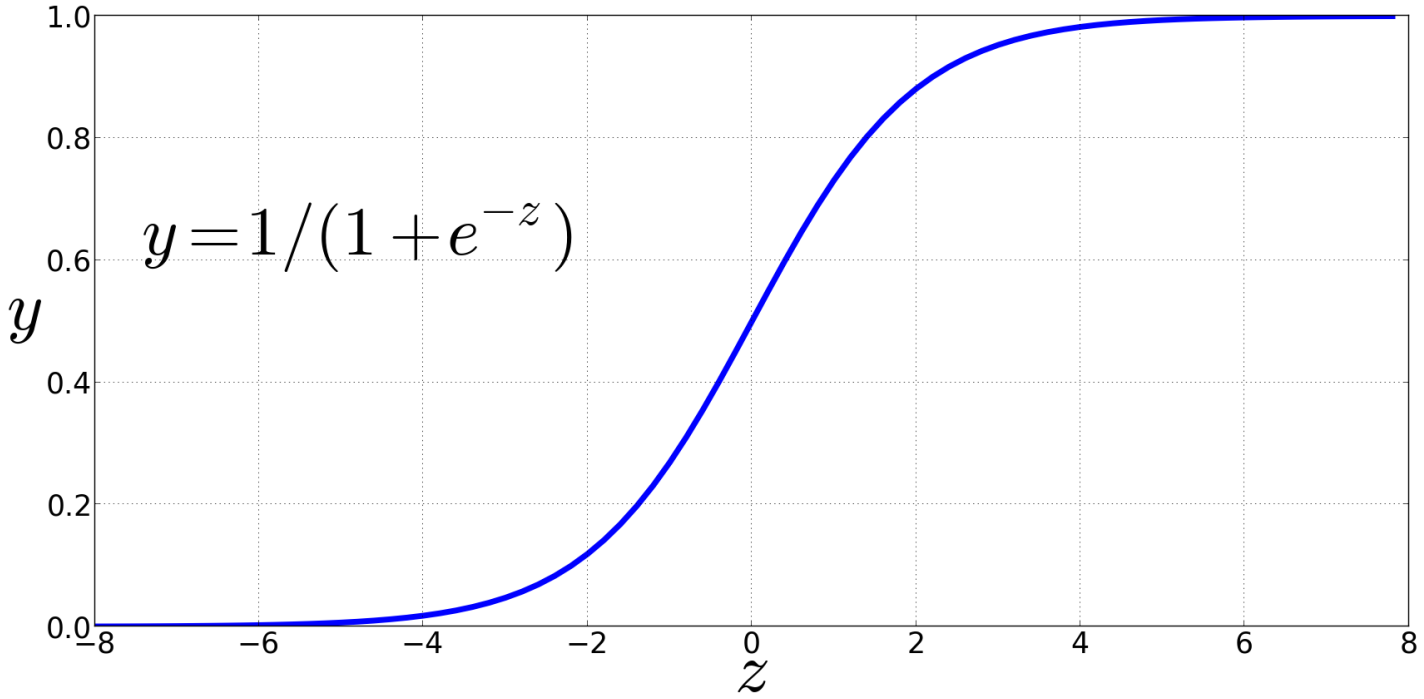
A NEURAL UNIT

- Take weighted sum of inputs, plus a bias
  - $z = b + \sum_i w_i x_i = b + w \cdot x$
- Then, we apply a nonlinear activation function *f*:
  - $y = f(z)$
- There are a range of f

NON-LINEAR ACTIVATION FUNCTIONS

Sigmoid

$$y = \sigma(z) = \frac{1}{1 + e^{-z}}$$



FINAL FUNCTION THE UNIT IS COMPUTING

$$y = \sigma(w \cdot x + b) = \frac{1}{1 + \exp(-(w \cdot x + b))}$$

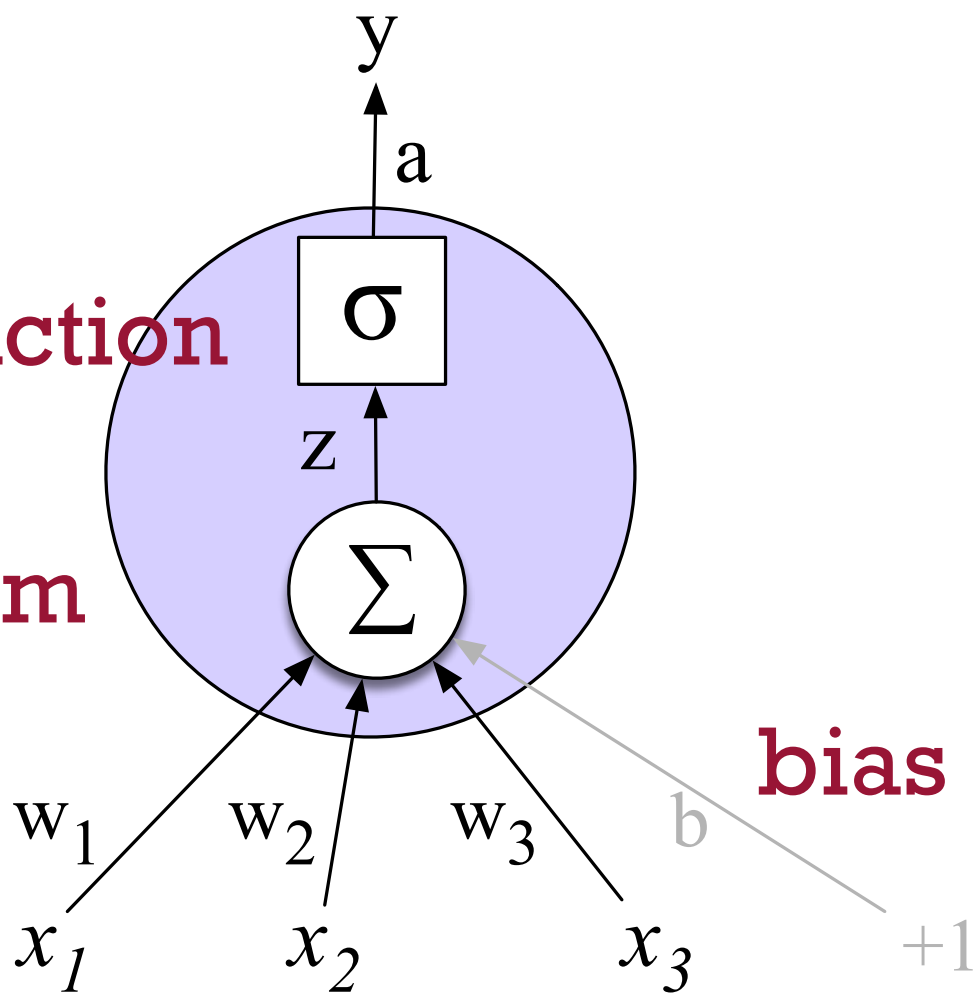
Output value

Non-linear activation function

Weighted sum

Weights

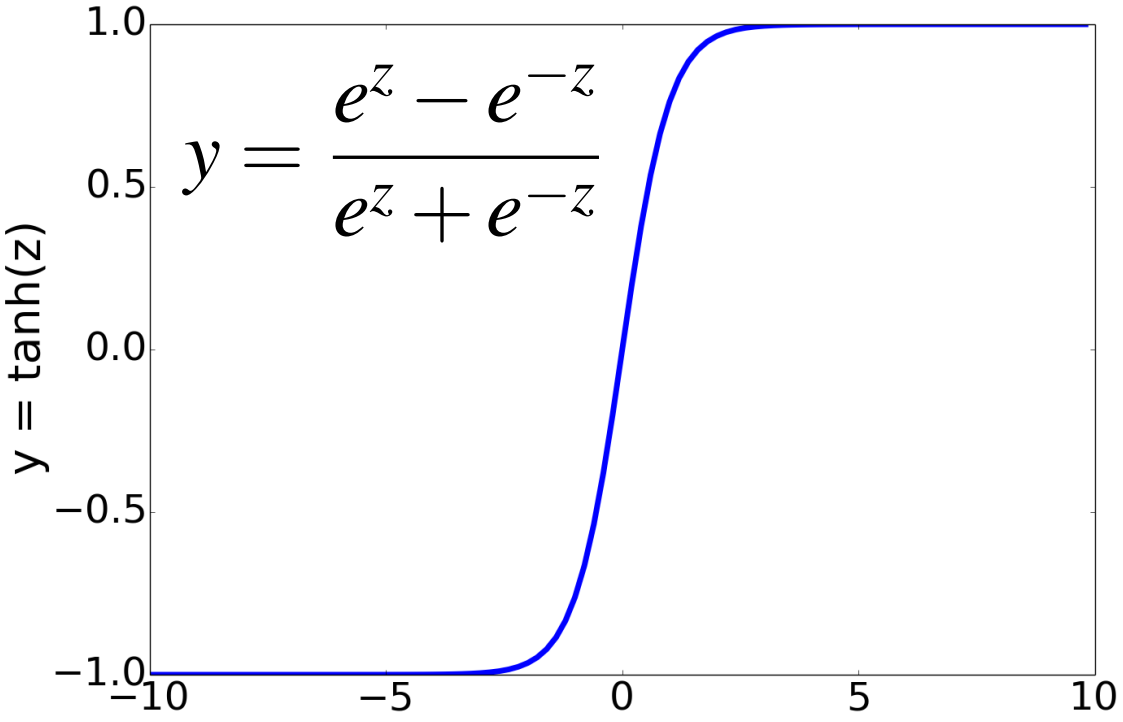
Input layer



EXAMPLE

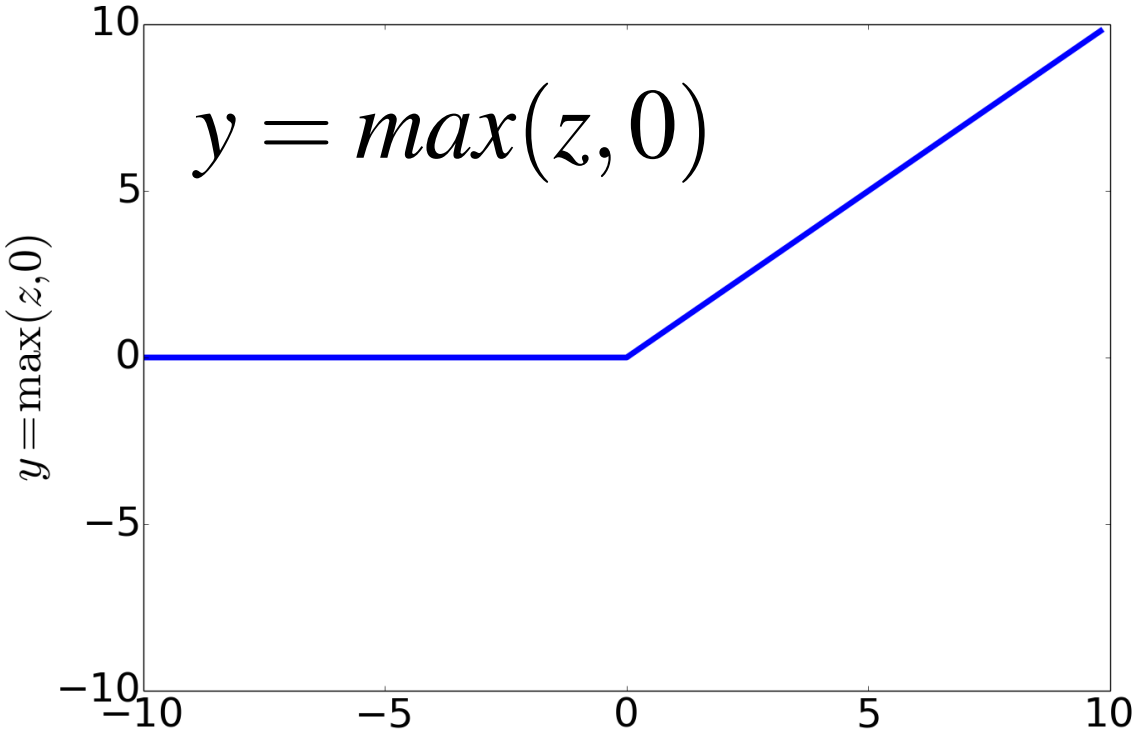
- Suppose a unit has:
  - $w = [0.2, 0.3, 0.9]$
  - $b = 0.5$
- What happens with input x:
  - $x = [0.5, 0.6, 0.1]$
  - $y = \sigma(w \cdot x + b) = \frac{1}{1+e^{-0.87}} = 0.70$

OTHER NON-LINEAR ACTIVATION FUNCTIONS



tanh

Most Common:



ReLU

Rectified Linear Unit

ACTIVATION FUNCTIONS

- Each functions have different properties
  - The tanh() is smoothly differentiable and maps outliers toward the mean
    - Preferred to the sigmoid
  - ReLU produces linear-like results
- The choice of activation function can indeed be influenced by the tasks the neural network is designed to perform
  - But **no** linear activation function!

MULTILAYER NETWORKS

- The strength of neural networks lies in their ability to combine individual units into larger networks
  - Called multi-layer networks (rarely used)
- Linear models including a single neural unit cannot compute XOR problmes

AND			OR			XOR		
x1	x2	y	x1	x2	y	x1	x2	y
0	0	0	0	0	0	0	0	0
0	1	0	0	1	1	0	1	1
1	0	0	1	0	1	1	0	1
1	1	1	1	1	1	1	1	0

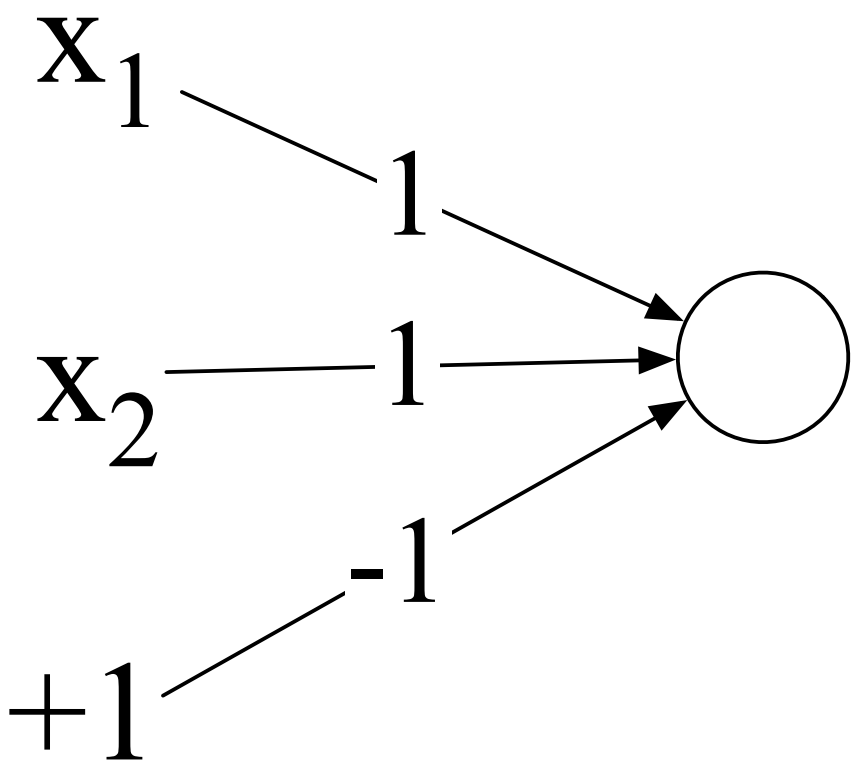
PERCEPTRON

- Consider the perceptron, a very simple neural unit that has a binary output with a step activation function

$$y = \begin{cases} 0, & \text{if } w \cdot x + b \leq 0 \\ 1, & \text{if } w \cdot x + b > 0 \end{cases}$$

- Easy to build AND or OR with perceptrons

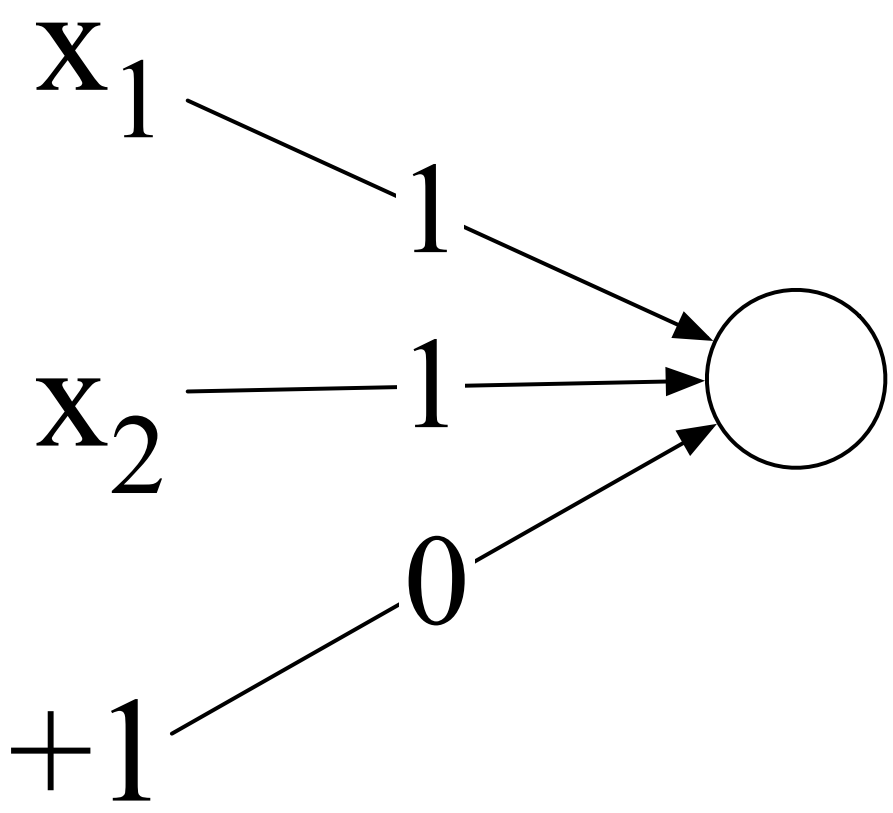
$$y = \begin{cases} 0, & \text{if } w \cdot x + b \leq 0 \\ 1, & \text{if } w \cdot x + b > 0 \end{cases}$$



AND

x1	x2	y
0	0	0
0	1	0
1	0	0
1	1	1

$$y = \begin{cases} 0, & \text{if } w \cdot x + b \leq 0 \\ 1, & \text{if } w \cdot x + b > 0 \end{cases}$$



OR

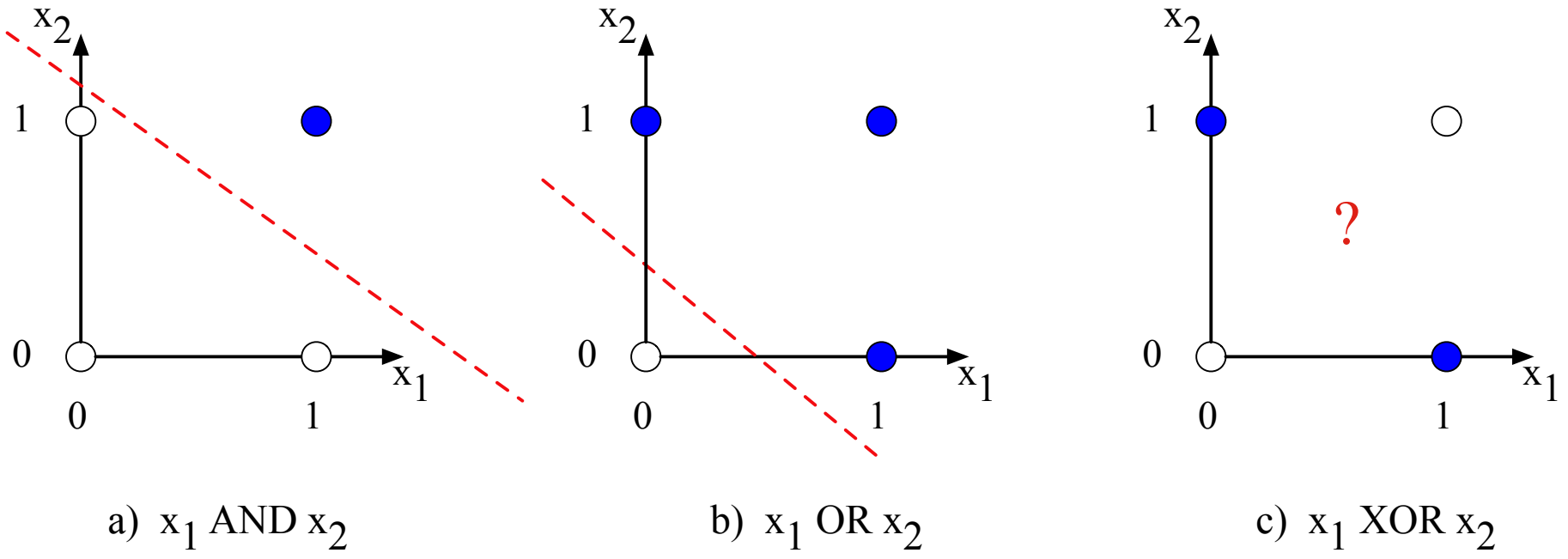
x1	x2	y
0	0	0
0	1	1
1	0	1
1	1	1

XOR PROBLEM

- Not possible to capture XOR with perceptrons
  - Try it!
- Given x_1, x_2, perceptron equation is a line
  - $w_1 x_1 + w_2 x_2 + b = 0$
- This line acts as a decision boundary

DECISION BOUNDARIES

- XOR is not linearly separable function



a) x_1 AND x_2

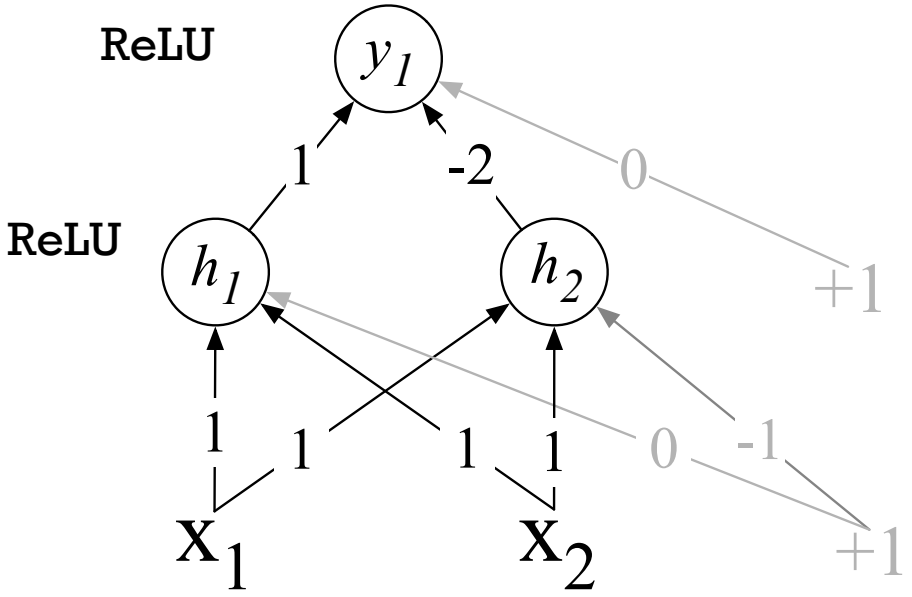
b) x_1 OR x_2

c) x_1 XOR x_2

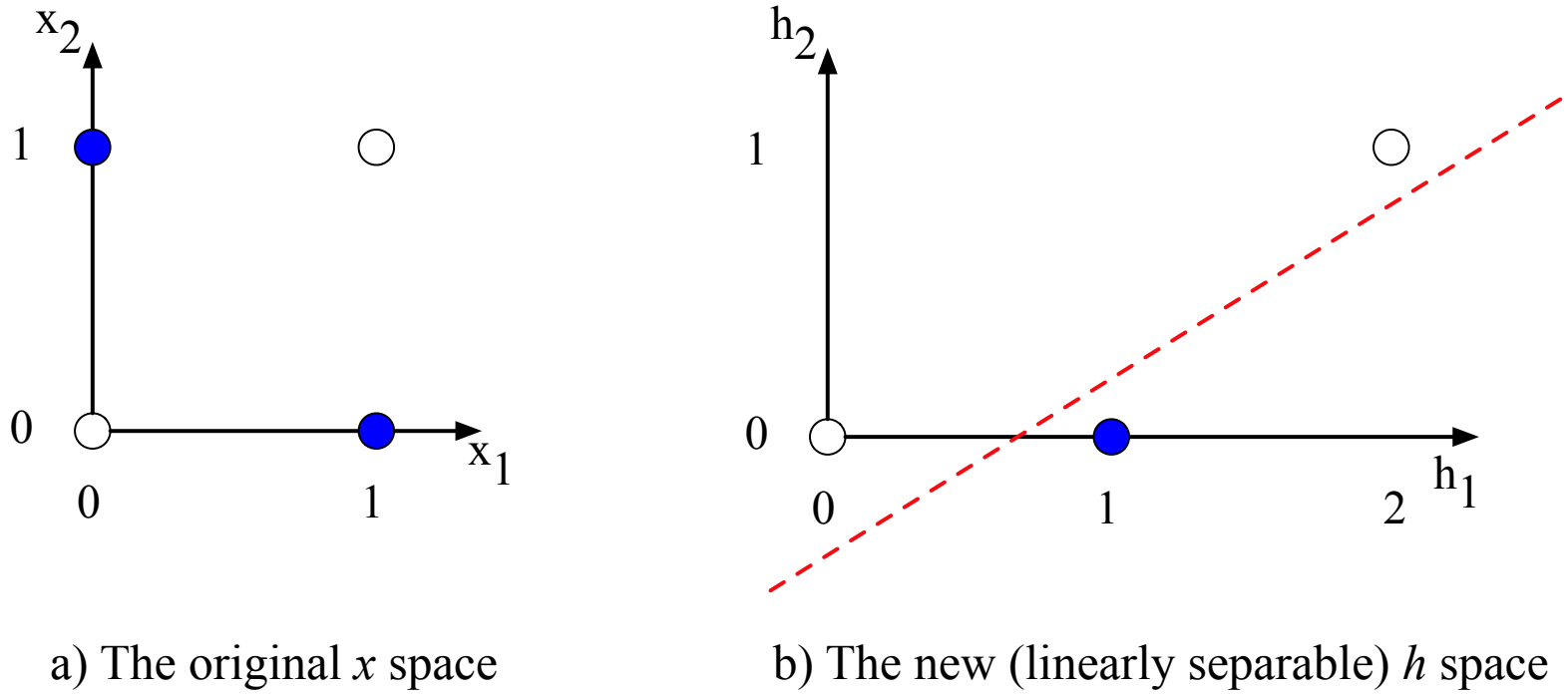
SOLUTION TO THE XOR PROBLEM

- XOR can't be calculated by a single perceptron
- XOR can be calculated by a layered network of units

XOR		
x1	x2	y
0	0	0
0	1	1
1	0	1
1	1	0



- We can view the hidden layer of the network as forming a representation of the input



a) The original *x* space

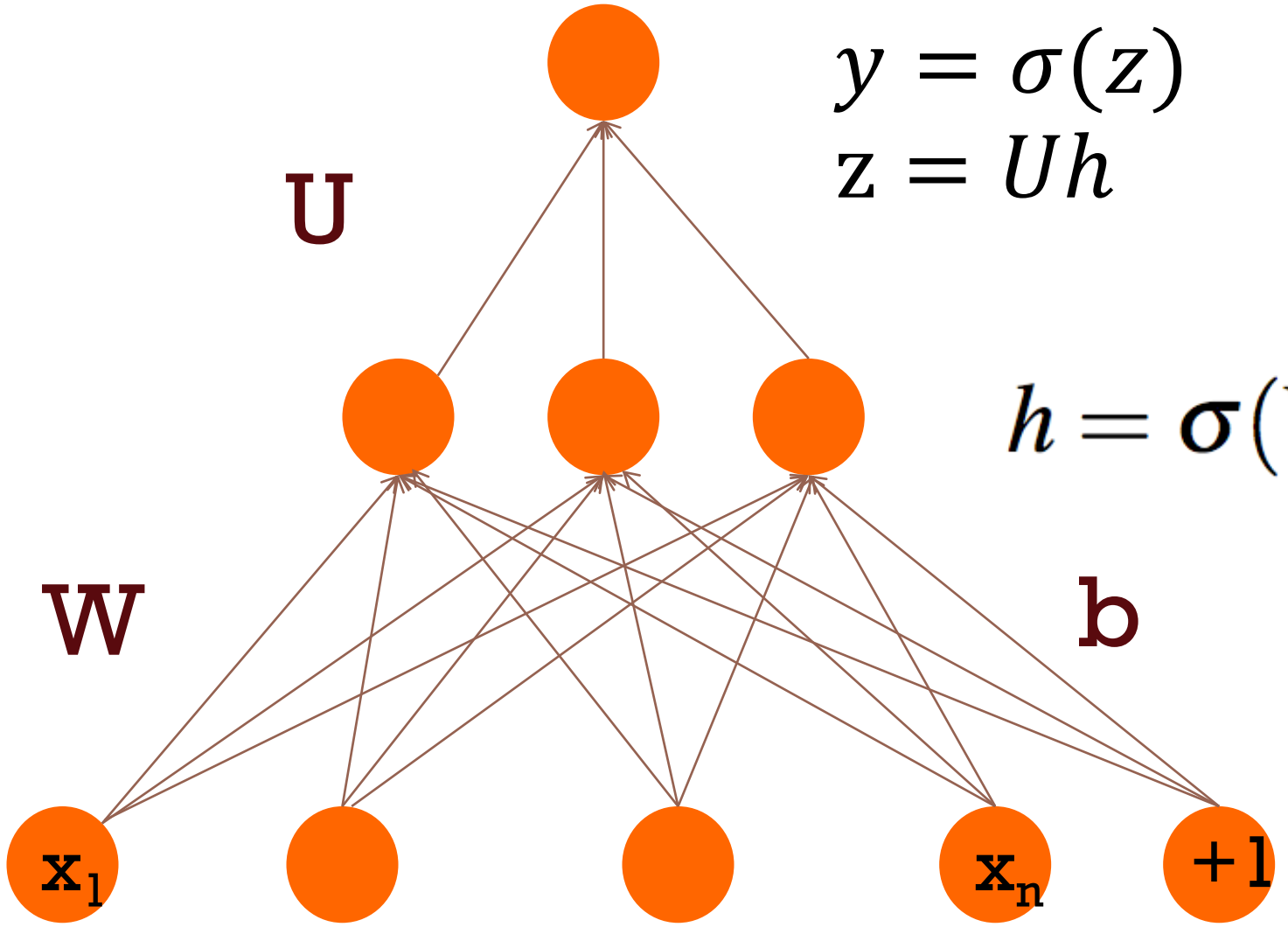
b) The new (linearly separable) *h* space

- The hidden layers will learn to form useful representations

Output layer (σ node)

hidden units (σ node)

Input layer (vector)



$$y = \sigma(z)$$
$$z = Uh$$

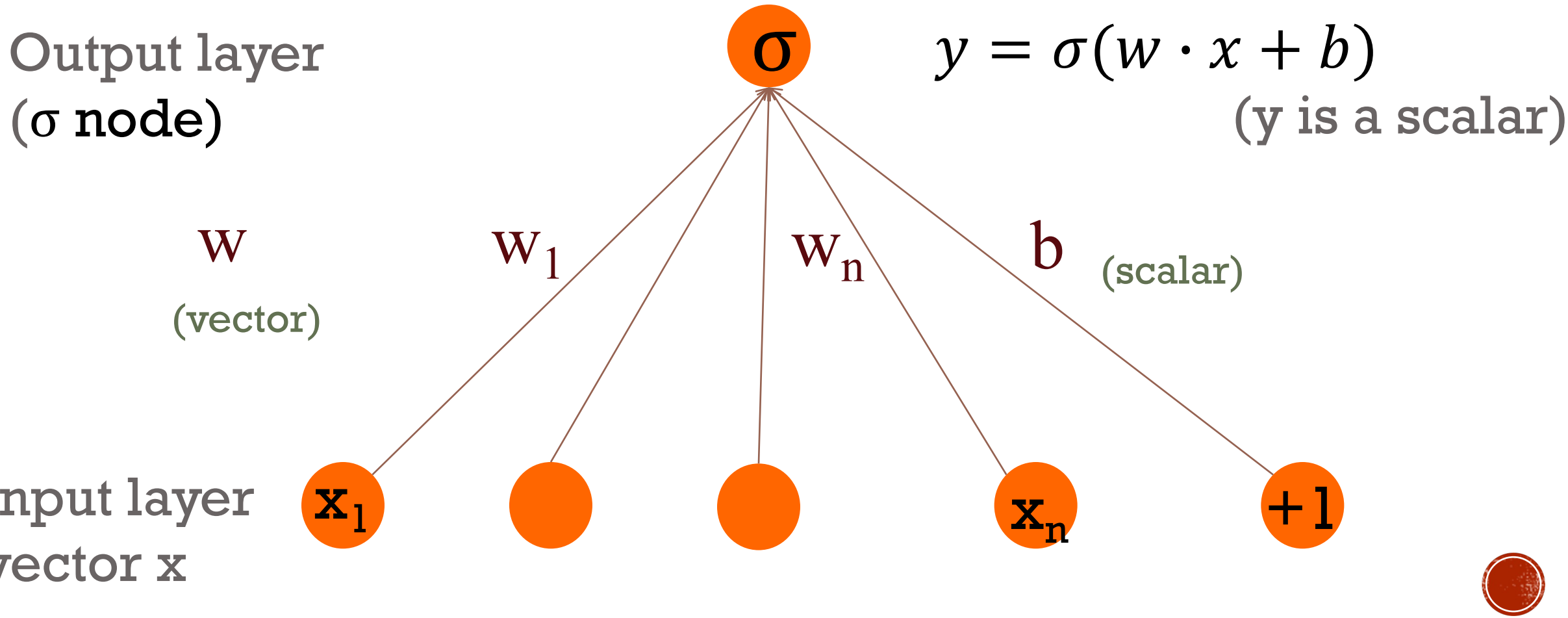
$$h = \sigma(Wx + b)$$

FEEDFORWARD NEURAL NETWORKS

- The feedforward network is the simplest NN
  - Fully connected **with no cycles**
- Minimum requirement: input units, hidden units, output units
  - Layers, nodes, units used interchangeably
- The core is the hidden layer

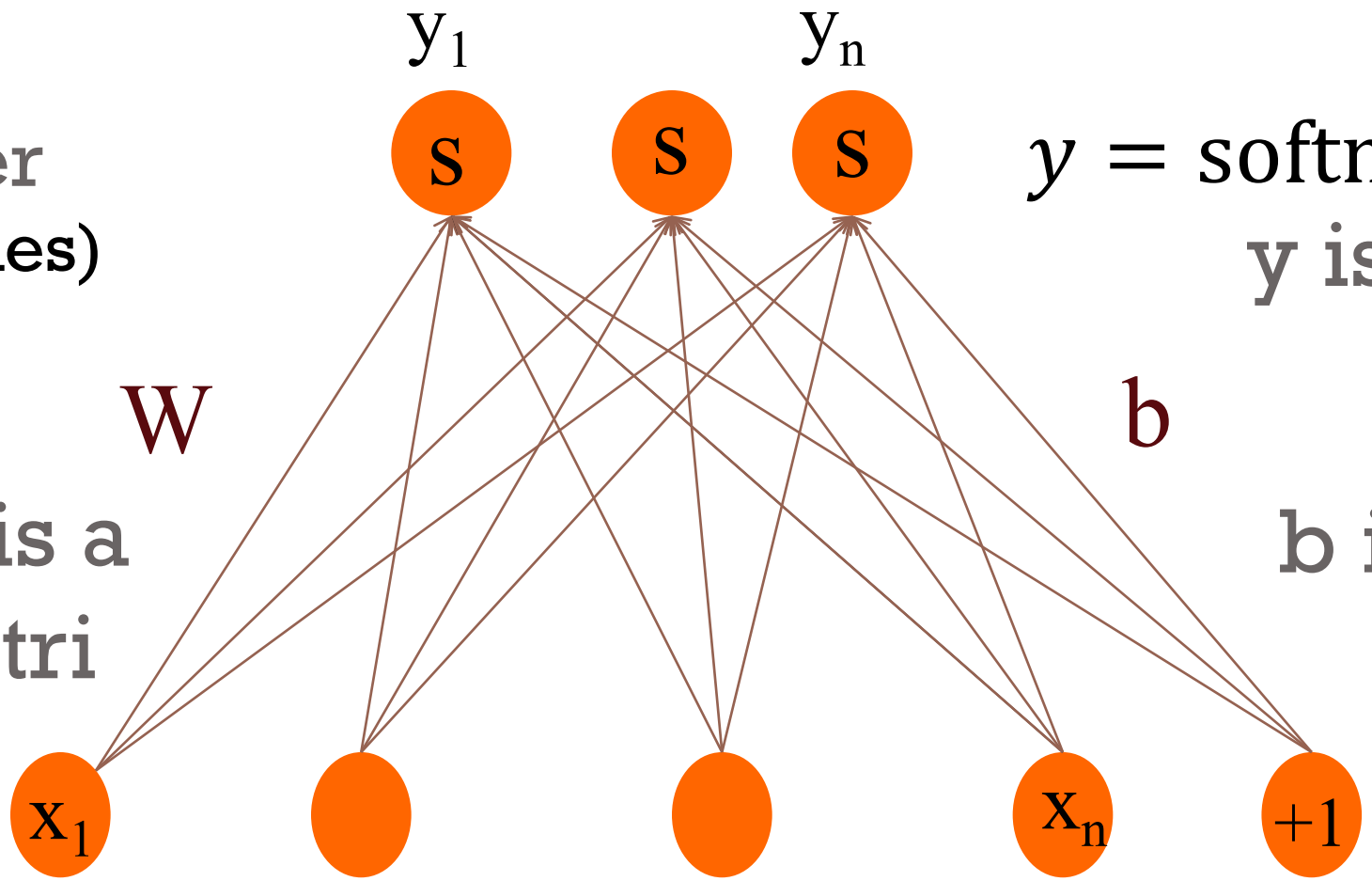
BINARY LOGISTIC REGRESSION AS A 1-LAYER NETWORK

(we don't count the input layer in counting layers!)



MULTINOMIAL LOGISTIC REGRESSION AS A 1-LAYER NETWORK

Fully connected single layer network



Output layer
(softmax nodes)

$$y = \text{softmax}(Wx + b)$$

y is a vector

W

b

W is a matri

x

b is a vector

Input layer
scalars

SOFTMAX: A GENERALIZATION OF SIGMOID

- For a vector z of dimensionality k, the softmax is:

$$\text{softmax}(z) = \left[\frac{\exp(z_1)}{\sum_{i=1}^{k} \exp(z_i)}, \frac{\exp(z_2)}{\sum_{i=1}^{k} \exp(z_i)}, \ldots, \frac{\exp(z_k)}{\sum_{i=1}^{k} \exp(z_i)} \right]$$

$$\text{softmax}(z_i) = \frac{\exp(z_i)}{\sum_{j=1}^{k} \exp(z_j)} \quad 1 \leq i \leq k$$

- Example:

$$z = [0.6, 1.1, -1.5, 1.2, 3.2, -1.1]$$

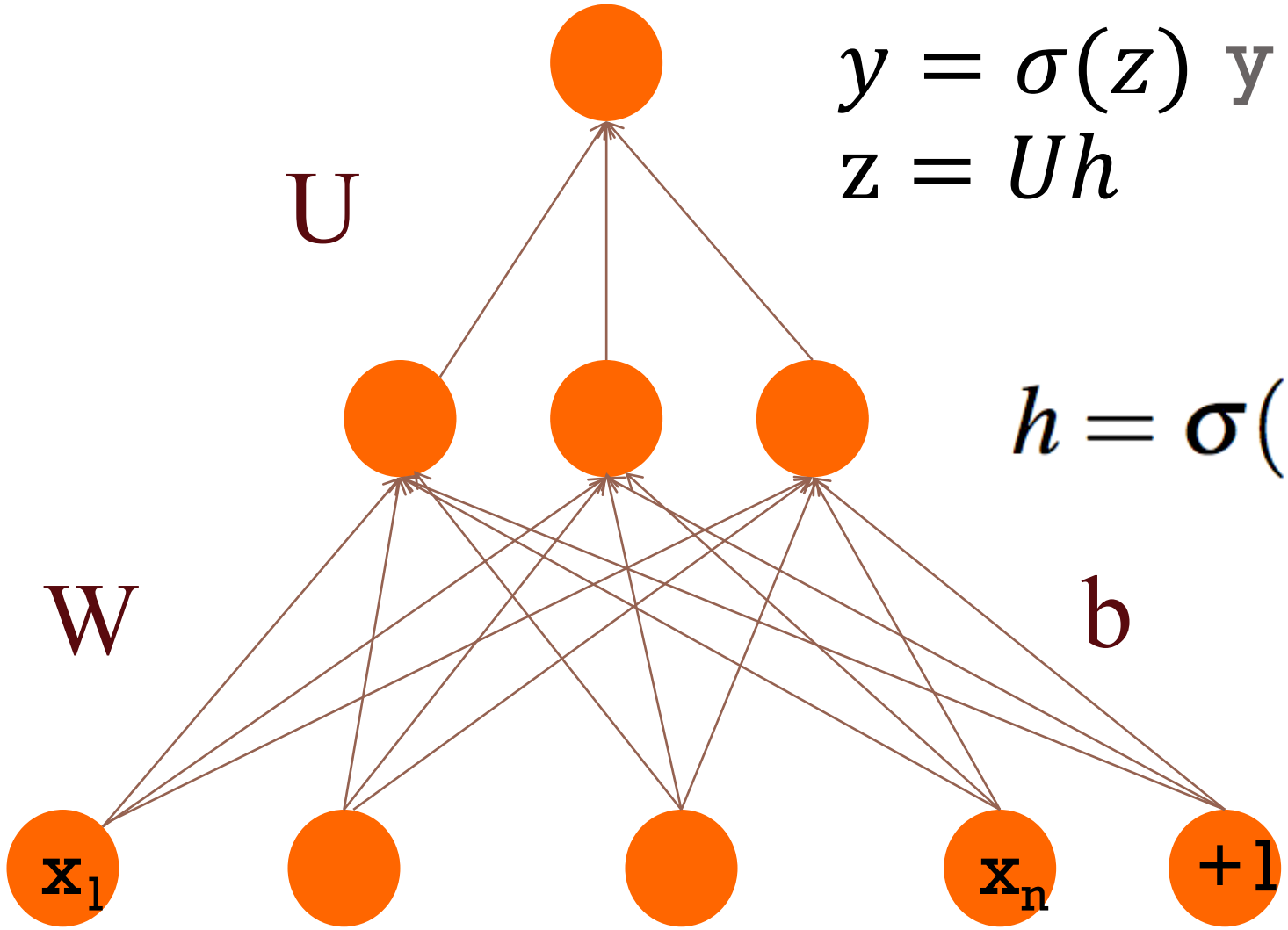
$$\text{softmax}(z) = [0.055, 0.090, 0.006, 0.099, 0.74, 0.010]$$

TWO-LAYER NETWORK WITH SCALAR OUTPUT

Output layer (σ node)

hidden units (σ node)

Input layer (vector)



$y = \sigma(z)$ y is a scala

$z = Uh$

$h = \sigma(Wx + b)$

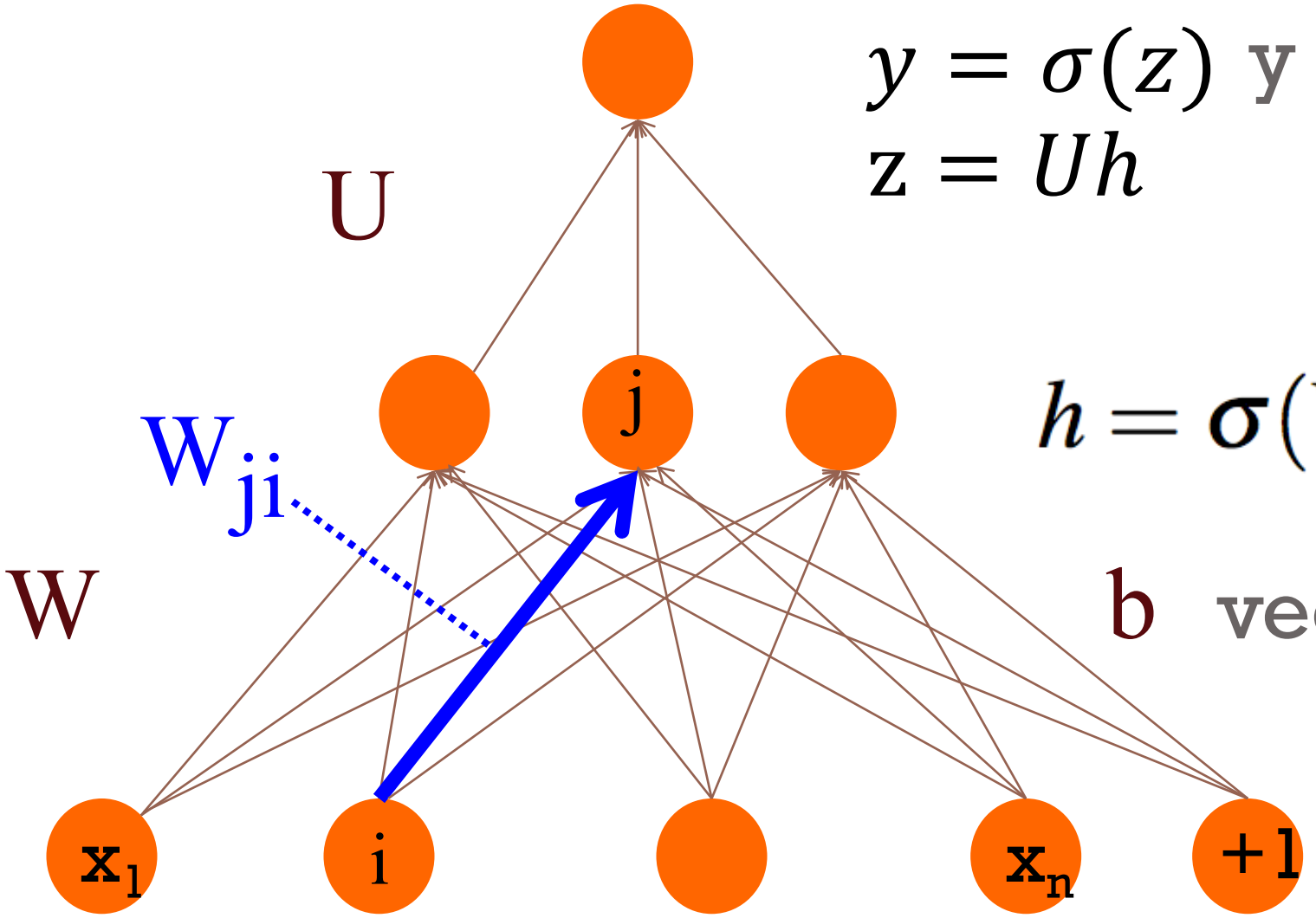
Could be ReLU
Or tanh

TWO-LAYER NETWORK WITH SCALAR OUTPUT

Output layer (σ node)

hidden units (σ node)

Input layer (vector)



$y = \sigma(z)$ y is a scala

$z = Uh$

$h = \sigma(Wx + b)$

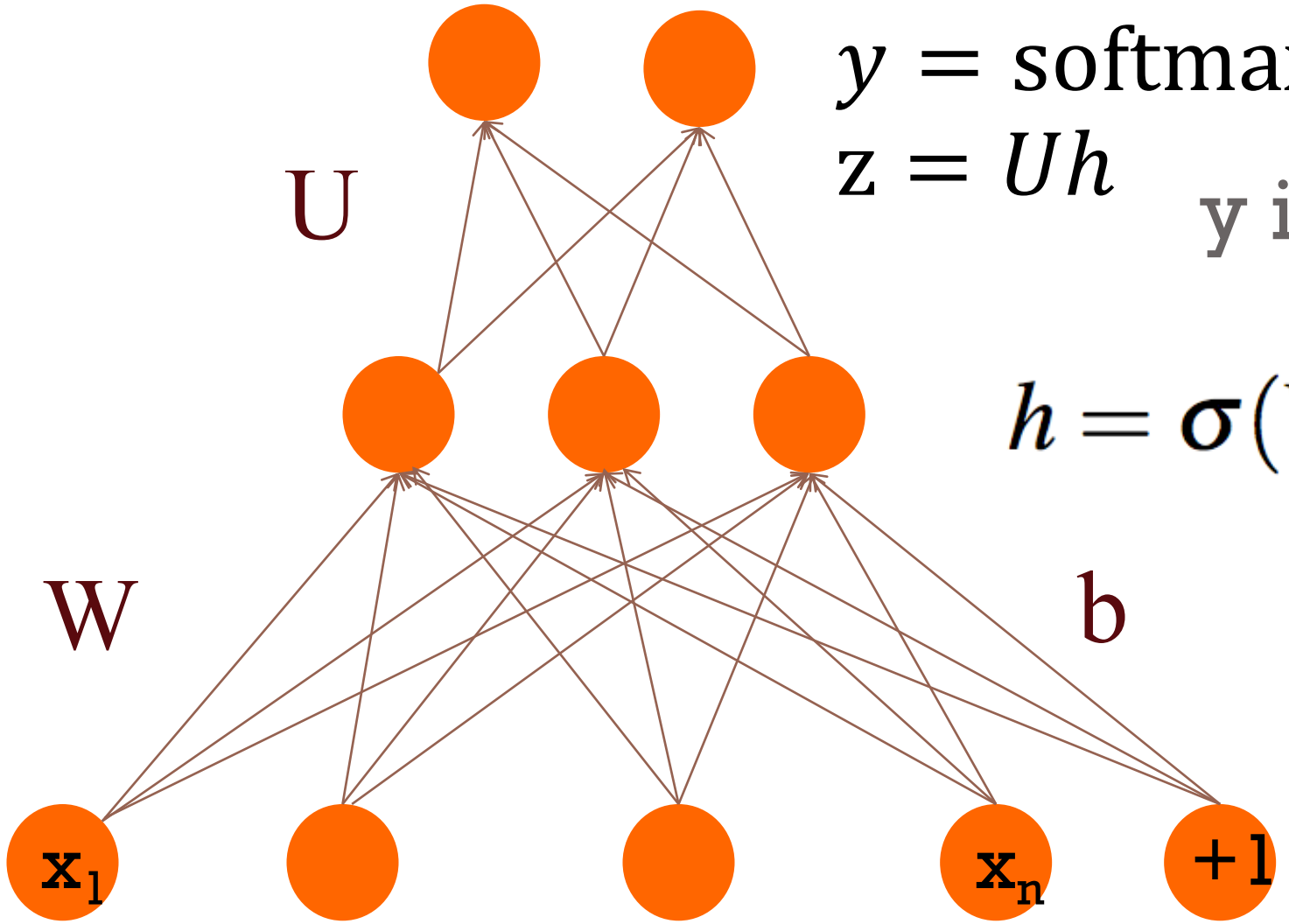
b vector

TWO-LAYER NETWORK WITH SOFTMAX OUTPUT

Output layer (σ node)

hidden units (σ node)

Input layer (vector)



$$y = \text{softmax}(z)$$

$$z = Uh$$

y is a vecto

$$h = \sigma(Wx + b)$$

Could be ReLU
Or tanh

FEEDFORWARD NEURAL NETWORKS

- The resulting value **h** forms a representation of the input
- The role of the output layer is to take this new representation **h** and compute a final output

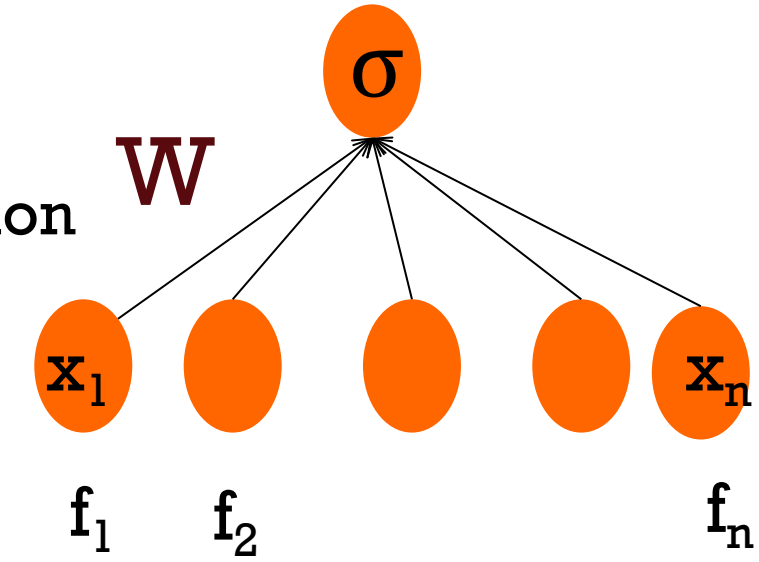
CLASSIFICATION: SENTIMENT ANALYSIS

- Consider what we did for logistic regression

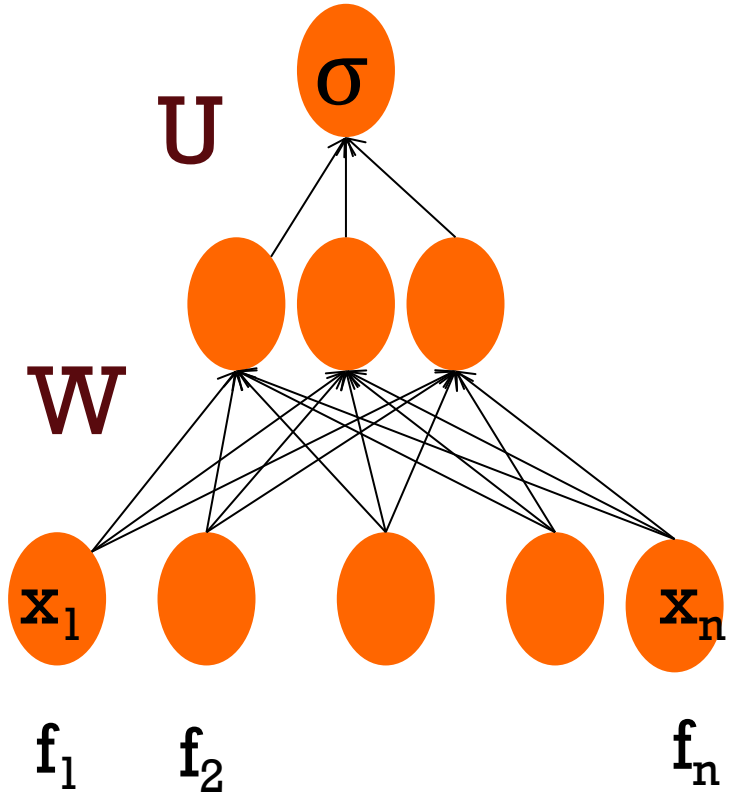
Var	Definition
x_1	count(positive lexicon) $\in$ doc)
x_2	count(negative lexicon) $\in$ doc)
x_3	$\begin{cases} 1 & \text{if } \text{“no”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$
x_4	count(1st and 2nd pronouns $\in$ doc)
x_5	$\begin{cases} 1 & \text{if } \text{“!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$
x_6	log(word count of doc)

FEEDFORWARD NETS FOR SIMPLE CLASSIFICATION

Logistic Regression



2-layer feedforward network

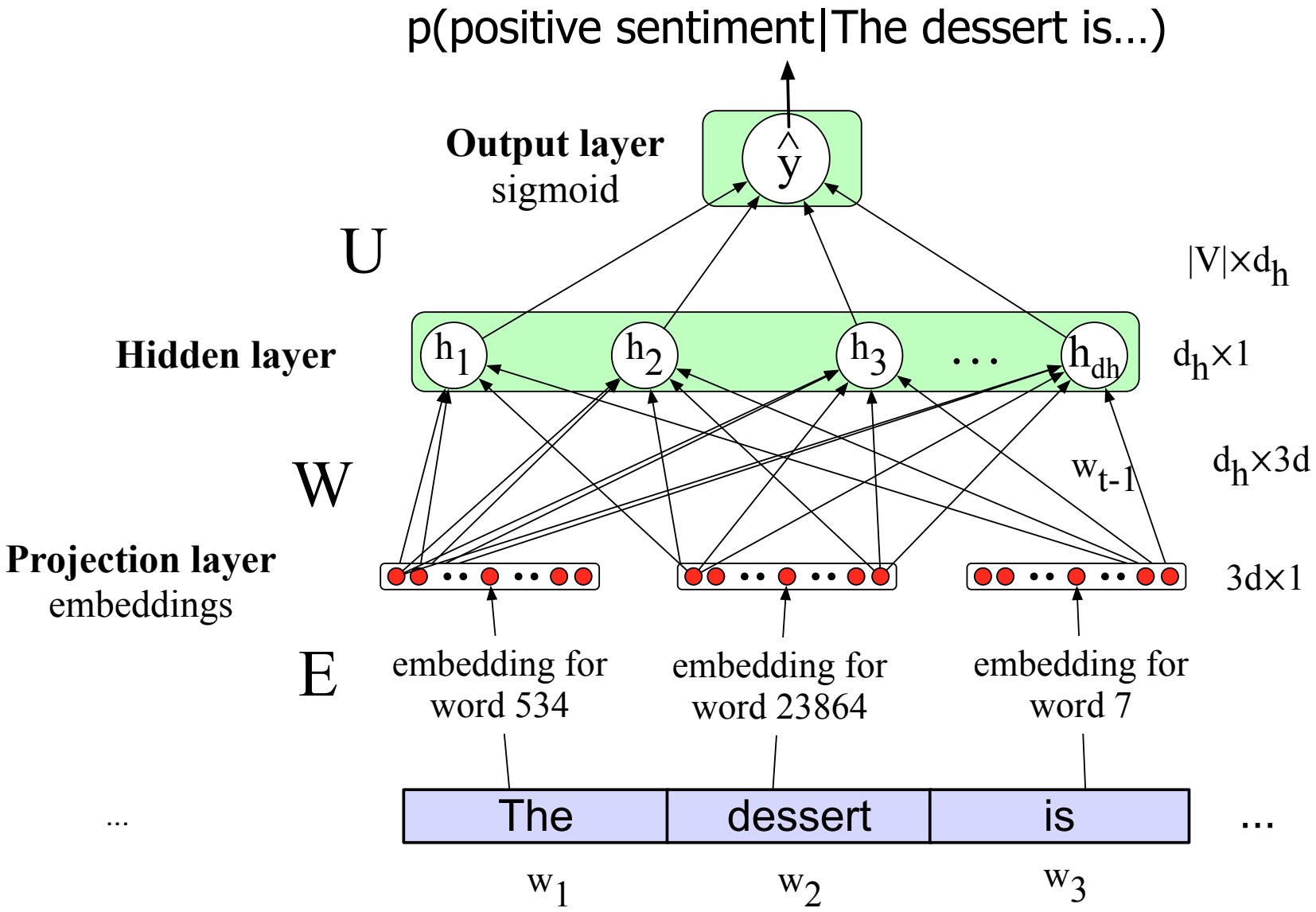


- Just adding a hidden layer to logistic regression
  - Allow the the network to use non-linear interactions between features
  - Logistic regression assume features are independent
  - May or may not to improve performance

EVEN BETTER: REPRESENTATION LEARNING

- The real power of deep learning comes from the ability to **learn** features from the data
  - Instead of using hand-built features
- Use the word embeddings

NEURAL NET CLASSIFICATION WITH EMBEDDINGS AS INPUT FEATURES!



- This assumes a fixed size length (3), which is unrealistic
- Some solutions
  - Fix the input sentence size
    - If shorter, then pad with zero embeddings
    - Truncate if you get longer reviews at test time
  - Create a single "sentence embedding" to represent all the words in each sentence
    - Take the mean of all the word embedding
    - Take the element-wise max of all the word embeddings

BACKPROPAGATION

- For many years, researchers struggled to find a way to train NN
- Gradient descent was the potential candidate, but it requires computing the gradients of the model's error with regard to the parameters
- Not clear at the time how to do this efficiently with such a complex model containing so many parameters
- In 1970, Linnainmaa proposed the reverse-mode autodiff
  - In just two passes through the network, able to compute the gradients of the NN's error with respect to every single model parameter

- A mini-batch enters the network through the input layer
  - Compute the output of network through every layer: forward pass
  - All intermediate results are preserved
- The algorithm computes the network's output error through a loss
- Then it computes how much each output bias and each connection to the output layer contributed to the error
  - Use chain rule
- Then the algorithm measures how much of these error contributions came from each connection in the layer below, working backward until it reaches the input layer
- Finally, the algorithm performs a gradient descent step to tweak all the connection weights in the network, using the error gradients it just computed.

$$L(a,b,c) = c(a+2b)$$

$$\begin{aligned} d &= 2*b \\ e &= a+d \\ L &= c*e \end{aligned}$$

$$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial e}\frac{\partial e}{\partial a}$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial e}\frac{\partial e}{\partial d}\frac{\partial d}{\partial b}$$

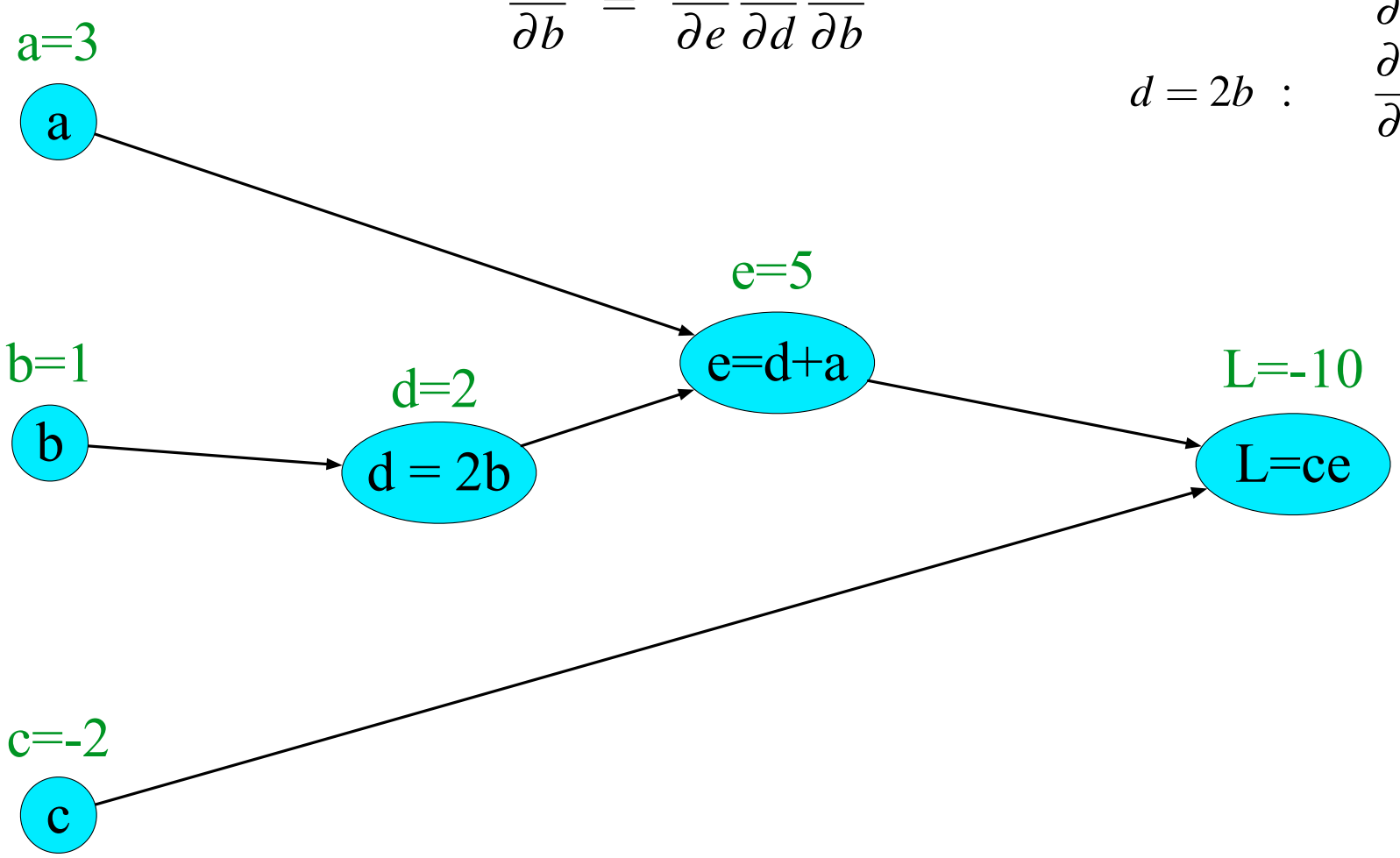
$$L = ce : \quad \frac{\partial L}{\partial e} = c, \frac{\partial L}{\partial c} = e$$

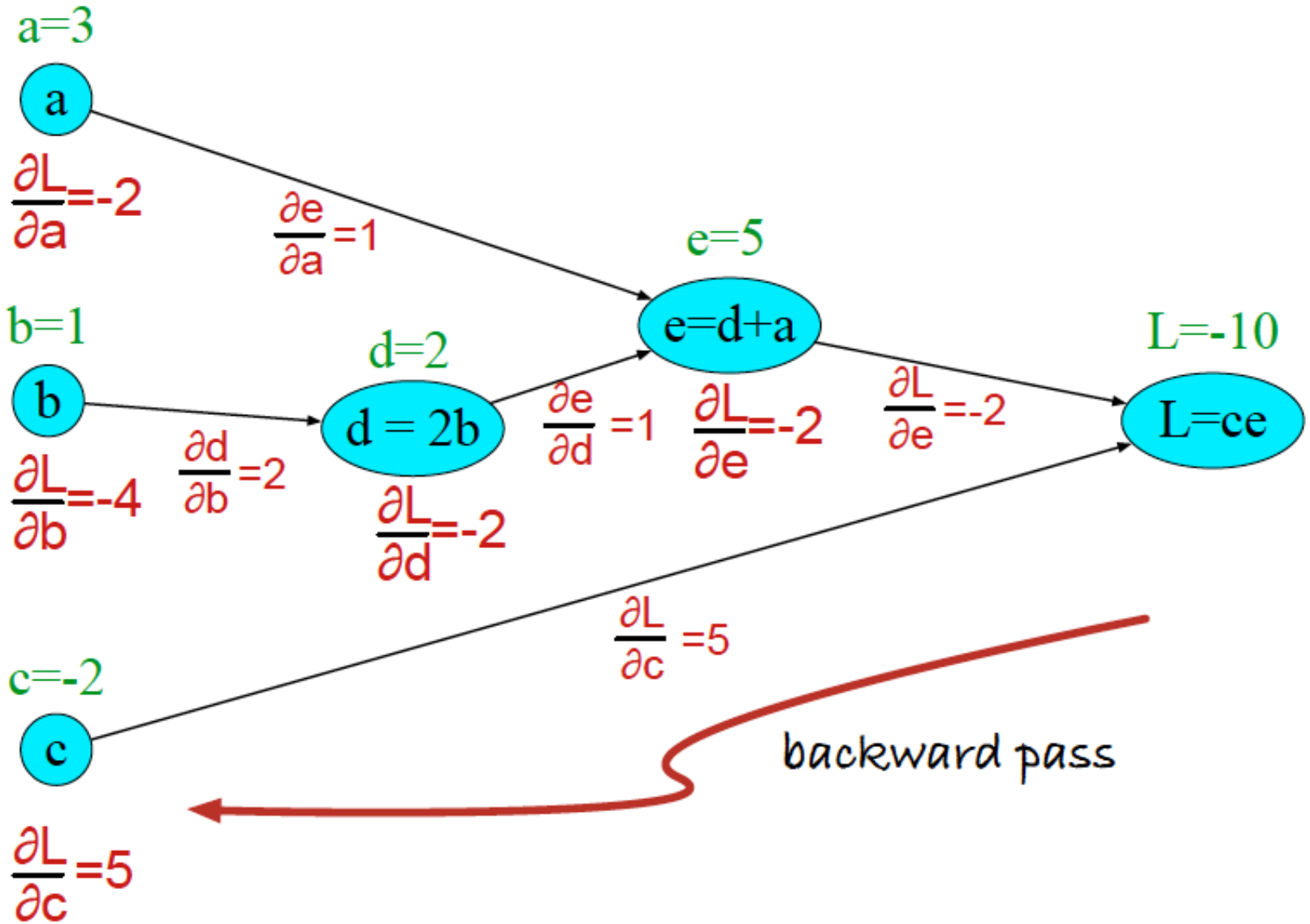
$$e = a+d : \quad \frac{\partial e}{\partial a} = 1, \frac{\partial e}{\partial d} = 1$$

$$d = 2b : \quad \frac{\partial d}{\partial b} = 2$$

$$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial e}\frac{\partial e}{\partial a}$$
$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial e}\frac{\partial e}{\partial d}\frac{\partial d}{\partial b}$$

$$L = ce : \quad \frac{\partial L}{\partial e} = c, \frac{\partial L}{\partial c} = e$$
$$e = a + d : \quad \frac{\partial e}{\partial a} = 1, \frac{\partial e}{\partial d} = 1$$
$$d = 2b : \quad \frac{\partial d}{\partial b} = 2$$




a=3
a
∂L/∂a=-2
∂e/∂a=1
e=5
e=d+a
b=1
b
d=2
d = 2b
∂d/∂b=2
∂e/∂d=1
∂L/∂e=-2
∂L/∂e=-2
L=-10
L=ce
∂L/∂b=-4
∂L/∂d=-2
∂L/∂c=5
c=-2
c
backward pass
∂L/∂c=5

SUMMARY: PROCESS

- In short, backpropagation makes predictions for a mini-batch (forward pass),
- measures the error, then goes through each layer in reverse to measure the error contribution from each parameter (reverse pass),
- and finally tweaks the connection weights and biases to reduce the error (gradient descent step).