LECTURE 15:
OUTLIER AND MISSING VALUES

BUS 211A-3

GROUP WORK

Any help?

Sometimes a dataset can contain extreme values that are outside the range of what is expected and unlike the other data

We will discover outliers and how to identify and remove them from your dataset

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4. High Leverage Points

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- Nevertheless, we can use statistical methods to identify observations that appear to be rare or unlikely given the available data

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  - e.g. within one s.d. of the mean will cover 68 percent of the data

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- A value that falls outside of 3 standard deviations is rare event at approximately 1 in 370 samples

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  - for larger samples, perhaps a value of 4 s.d. (99.9 percent) can be used

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  - common k: 1.5

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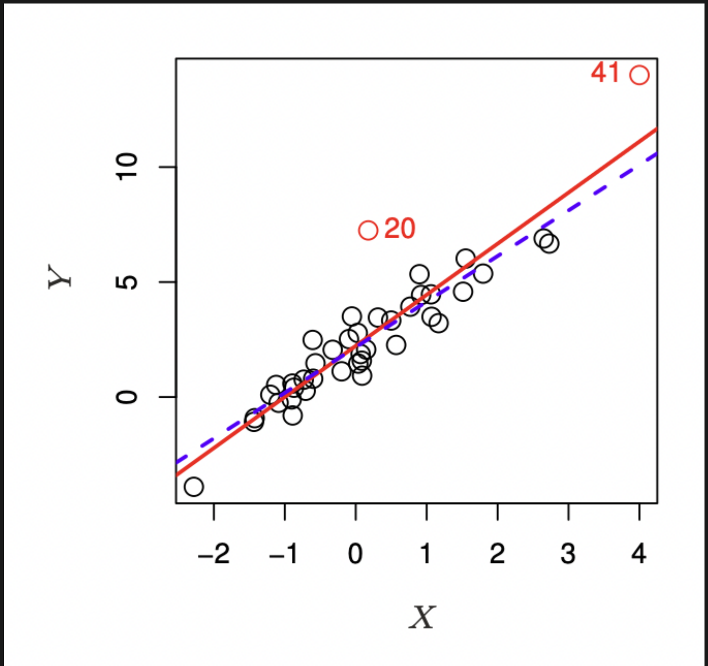
41
20
10
5
0
Y
−2
−1
0
1
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3
4
X

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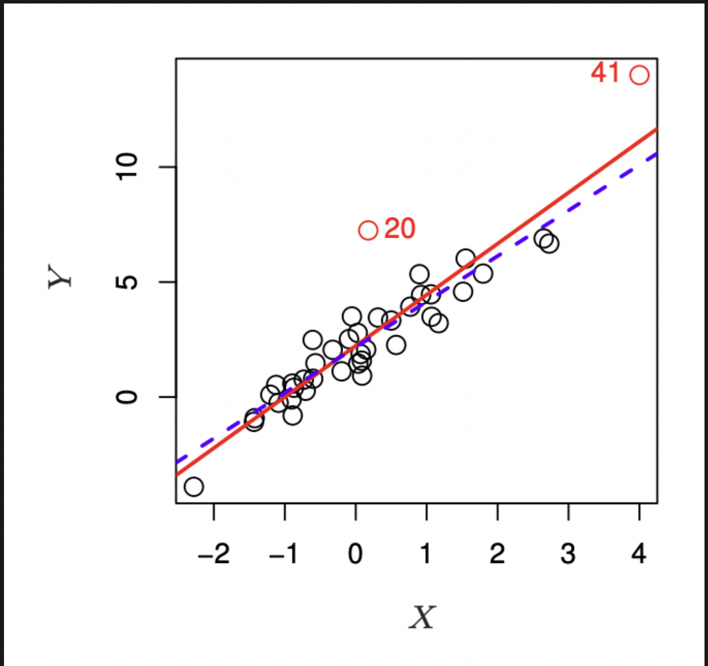

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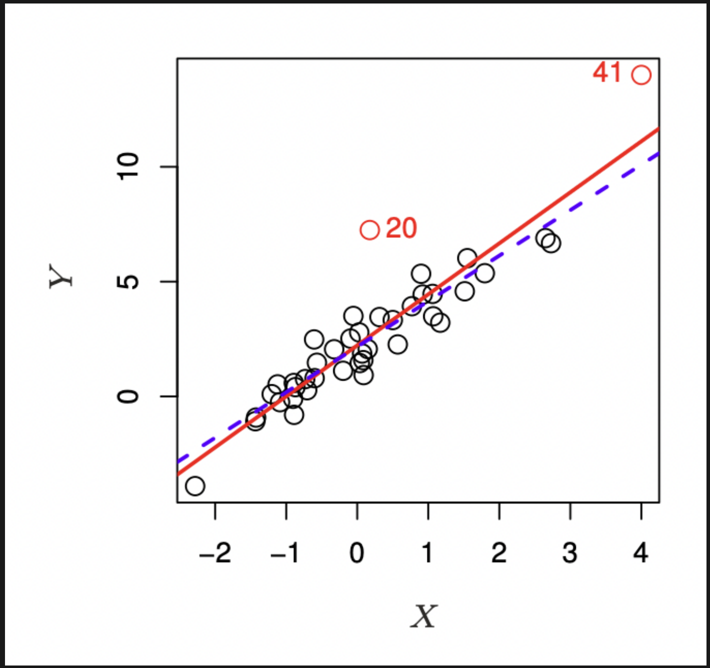

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- The red solid line is the least squares fit to the data, while the blue dashed line is the fit produced when observation 41 is removed

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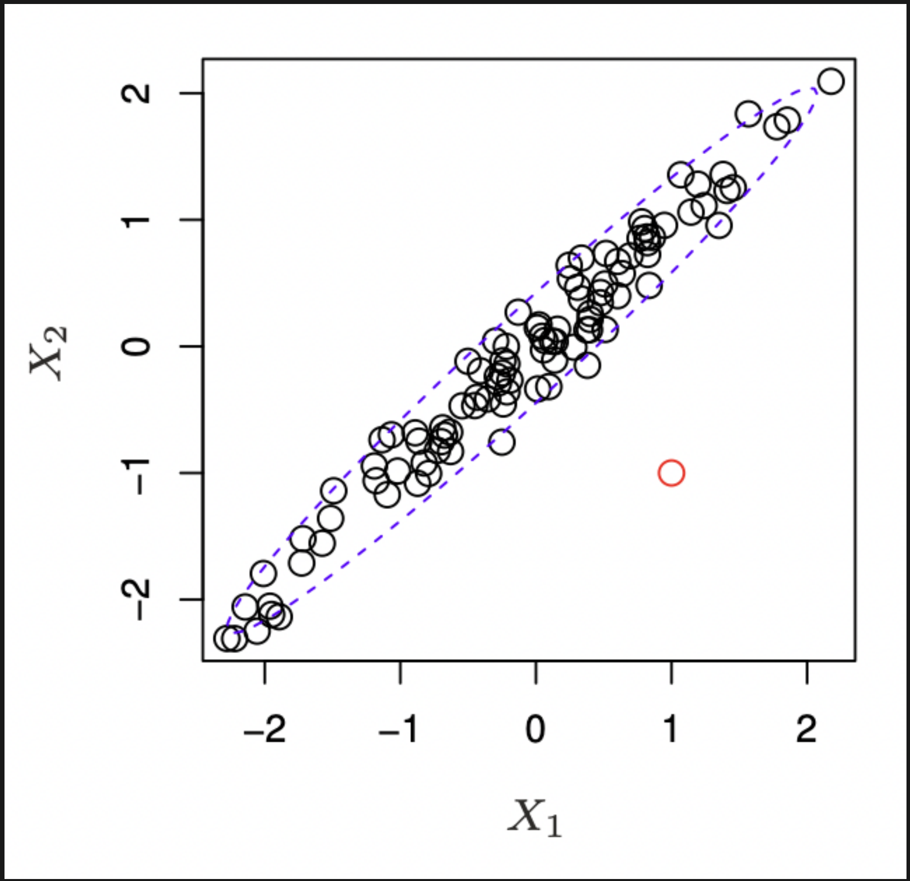
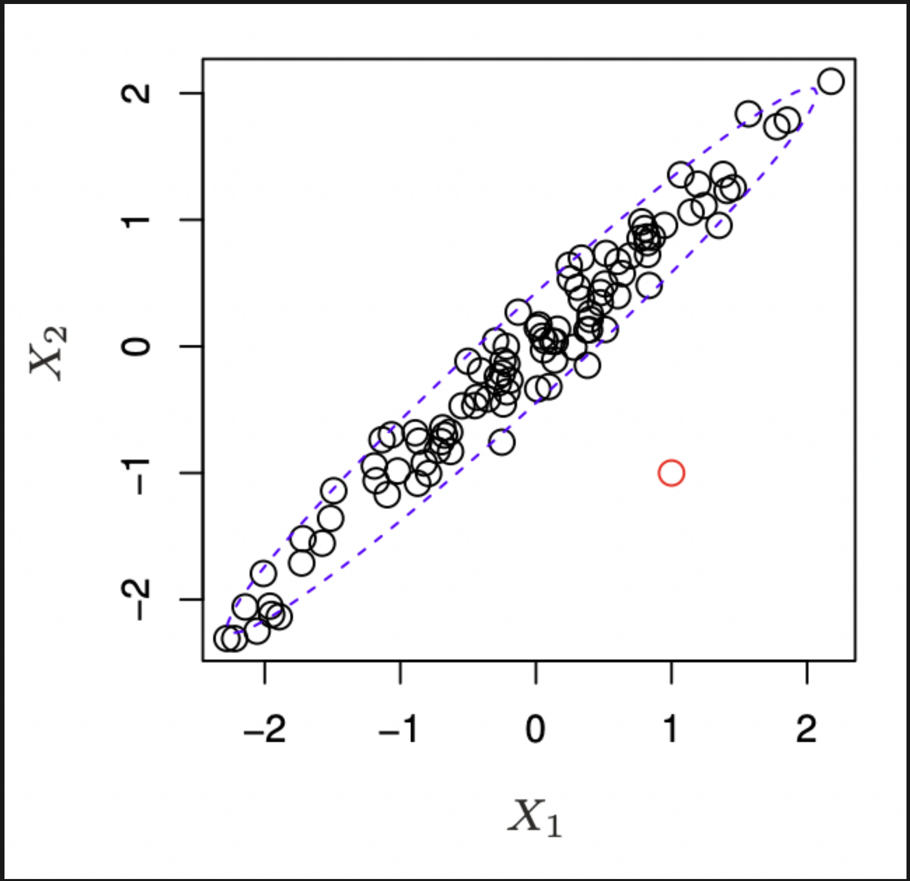
- In a simple linear regression, high leverage observations are fairly easy to identify



2
1
0
−1
−2
X_2
−2
−1
0
1
2
X_1



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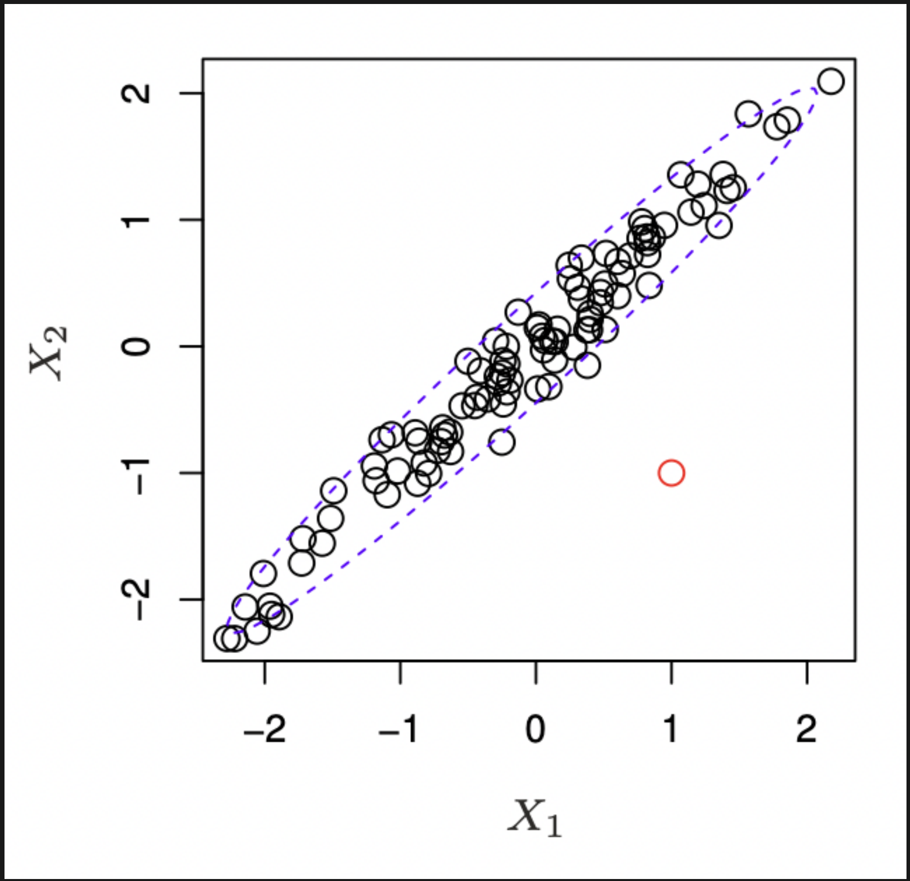

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- So if we examine just X_1 or just X_2, we will fail to notice this high leverage point

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- h_i is always between 1/n and 1, and the average leverage for all the observations is always equal to (p + 1)/n
- So if a given observation has a leverage statistic that greatly exceeds (p+1)/n, then we may suspect that the corresponding point has high leverage

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- How do you delete it?