Name - Sajal Namdeo Section - C

Roll No. - 2022 UG 3022

Maths Assignment V

1. Form the P.D.E for the given relations.

(i) $z = f(x^2+y^2)$

Sol. $z_x = f'(x^2+y^2) \cdot 2x$ — (i)

$z_y = f'(x^2+y^2) \cdot 2y$ — (ii)

$y \times$ eqn(i) $- x \times$ eqn(ii) $\Rightarrow \boxed{y z_x - x z_y = 0}$

(ii) $f(xyz, x^2+y^2+z^2) = 0$

Sol. $U = xyz \qquad V = x^2+y^2+z^2$

$$P = J\left(\frac{U,V}{y,z}\right) = \begin{vmatrix} \frac{\partial U}{\partial y} & \frac{\partial V}{\partial y} \\ \frac{\partial U}{\partial z} & \frac{\partial V}{\partial z} \end{vmatrix} = \begin{vmatrix} xz & 2y \\ xy & 2z \end{vmatrix} \Rightarrow 2xz^2 - 2xy^2 \Rightarrow 2x(z^2-y^2)$$

$$Q = J\left(\frac{U,V}{z,x}\right) = \begin{vmatrix} \frac{\partial U}{\partial z} & \frac{\partial V}{\partial z} \\ \frac{\partial U}{\partial x} & \frac{\partial V}{\partial x} \end{vmatrix} = xy \cdot 2x - yz \cdot 2z = 2y(x^2-z^2)$$

$$R = J\left(\frac{U,V}{x,y}\right) = \begin{vmatrix} \frac{\partial U}{\partial x} & \frac{\partial V}{\partial x} \\ \frac{\partial U}{\partial y} & \frac{\partial V}{\partial y} \end{vmatrix} = yz \cdot 2y - 2x \cdot xz \Rightarrow 2z(y^2-x^2)$$

The P.D.E is given by

$$Pp + Qq = R$$

$$\boxed{2x(z^2-y^2)p + 2y(x^2-z^2)q = 2z(y^2-x^2)}$$

(iii) $ax^2 + by^2 + z^2 = 1$

$2ax + 2z z_x = 0$ — (i) $\Rightarrow z_x = -\frac{ax}{z}$

$2by + 2z z_y = 0$ — (ii) $\Rightarrow z_y = -\frac{by}{z}$

eqn (i) $\times by$ $-$ eqn(ii) $\times ax \Rightarrow by z_x - ax z_y = -\frac{\cancel{abxy}}{z} + \frac{\cancel{abxy}}{z}$

$\Rightarrow \boxed{by z_x - ax z_y = 0}$

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(iv) $(x-h)^2 + (y-k)^2 + z^2 = a^2$

Sol. $2(x-h) + 2z z_x = 0$ —(i) $\qquad z_x = \frac{x-h}{z}$

$2(y-k) + 2z z_y = 0$ —(ii) $\qquad z_y = \frac{y-k}{z}$

equ(i) $\times (y-k)$ − equ(ii) $\times (x-h)$ = $\boxed{(y-k)z_x - (x-h)z_y = 0}$

(v) $xyz = g(x+y+z)$

Sol. ~~$y z_x = g'(x+y+z)$~~

$z = \frac{g(x+y+z)}{xy}$

$z_x = xy(g'(x+y+z)z_x) - g(x+y+z)y \Rightarrow z_x(1 - xy\, g'(x+y+z)) = g(x+y+z)y$

$z_y = xy(g'(x+y+z)\cdot z_y) - g(x+y+z)x \Rightarrow z_y(1 - xy\, g'(x+y+z)) = g(x+y+z)x$

equ(i) $\times x$ − equ(ii) $\times y$ =

$\Rightarrow \boxed{x z_x - y z_y = 0}$

(vi) $g\left(\frac{1}{x} - \frac{1}{y}, \frac{xy}{z}\right) = 0$

$u = \frac{1}{x} - \frac{1}{y} \qquad v = \frac{xy}{z}$

$$P = \frac{J(u,v)}{(y,z)} = \begin{vmatrix} \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} \end{vmatrix} = \begin{vmatrix} \frac{1}{y^2} & \frac{x}{z} \\ 0 & -\frac{xy}{z^2} \end{vmatrix} \Rightarrow \boxed{-\frac{x}{yz^2}}$$

$$Q = \frac{J(u,v)}{(z,x)} = \begin{vmatrix} \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \end{vmatrix} = \begin{vmatrix} 0 & -\frac{xy}{z^2} \\ \frac{-1}{x^2} & \frac{y}{z} \end{vmatrix} \Rightarrow \boxed{-\frac{y}{xz^2}}$$

$$R = \frac{J(u,v)}{(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} -\frac{1}{x^2} & \frac{y}{z} \\ \frac{1}{y^2} & \frac{x}{z} \end{vmatrix} \Rightarrow \frac{-1}{zx} - \frac{1}{zy} \Rightarrow \frac{-1}{z}\left(\frac{1}{x} + \frac{1}{y}\right)$$

$$= \frac{-(x+y)}{xyz}$$

$Pp + Qq = R$

$\frac{\cancel{-}x}{yz^2}p + \frac{\cancel{-}y}{xz^2}q = \frac{\cancel{-}(x+y)}{xz^2} \Rightarrow \frac{x}{y}p + \frac{y}{x}q = \frac{x+y}{x}$

$\Rightarrow \boxed{x^2 p + y^2 q = y(x+y)}$

Classify the following PDE's as Linear, Semilinear and Quasilinear.

(i) $p + q = xyz + x$

Sol. Linear, Semilinear, Quasilinear

(ii) $xp + yq = zx + yx$

Sol. Linear, Semilinear, Quasilinear

(iii) $xp + y^2 q = xz^2 + xy$

Sol. Semilinear, Quasilinear

(iv) $xzp + yq = xyz + y^2$

Sol. Quasilinear

2) y.

Classify the following PDE's as Hyperbolic, Parabolic and Elliptic.

i) $x^2 \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = u$

Sol. $B = 0$, $A = x^2$, $C = -1$

$B^2 - 4AC = 0 - 4(x^2)(-1) = 4x^2 > 0$

Hyperbolic

ii) $tU_{tt} + 2U_{xt} + xU_{xx} + U_x = 0$

Sol. $B = 2$, $A = t$, $C = x$

$B^2 - 4ac = 4 - 4xt = 4(1 - xt)$

Hyperbolic if $xt < 1$

Parabolic if $xt = 1$

Elliptic if $xt > 1$

iii) $xU_{tt} + tU_{xt} + U_{tt} = 0$

Sol. $A = x$ $B = t$ $C = 1$

$B^2 - 4ac = t^2 - 4x$

Hyperbolic if $t^2 - 4x > 0$

Parabolic if $t^2 = 4x$

Elliptic if $t^2 - 4x < 0$

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(iv) $x^2 U_{tt} + 3U_{xt} + xU_{xx} + 1+U_t = 100\ldots$

Sol. $A = x^2$, $B = 3$, $C = x$

$B^2 - 4ac = 9 - 4x^3$

Hyperbolic if $9 - 4x^3 > 0$

Parabolic if $9 - 4x^3 = 0$

Elliptic if $9 - 4x^3 < 0$

(v) $U_{tt} + tU_{xt} + xU_{xx} + 2U_t + U_x + 6u = 0$

Sol. $A = 1$ $B = t$ $C = x$

$B^2 - 4ac = t^2 - 4x$

Hyperbolic if $t^2 - 4x > 0$

Parabolic if $t^2 - 4x = 0$

Elliptic if $t^2 - 4x < 0$

4. Solve the first order Quasilinear PDE's :

(i) $p\tan x + q\tan y = \tan z$

Sol $\dfrac{dx}{\tan x} = \dfrac{dy}{\tan y} = \dfrac{dz}{\tan z}$

$\int \dfrac{dx}{\tan x} = \int \dfrac{dy}{\tan y}$

$\Rightarrow \int \dfrac{\cos x}{\sin x} dx = \int \dfrac{\cos y}{\sin y} dy + c_1$

$u = \sin x$ $\quad v = \sin y$

$du = \cos x\, dx$ $\quad dv = \sin y\, dv$

$\Rightarrow \int \dfrac{du}{u} = \int \dfrac{dv}{v} + c_1$

$\Rightarrow \ln|\sin x| = \ln|\cos y| + c_1$

$\Rightarrow c_1 = \ln\left|\dfrac{\sin x}{\cos y}\right|$

$c_1 = \cancel{\ln|\tan x|}\ \dfrac{\sin x}{\sin y}$

$\int \dfrac{dy}{\tan y} = \int \dfrac{dz}{\tan z} + c_2$

$c_2 = \ln|\sin y| - \ln|\sin z|$

$c_2 = \dfrac{\sin y}{\sin z}$

$u = c_1 = \dfrac{\sin x}{\sin y}$ $\quad v = c_2 = \dfrac{\sin y}{\sin z}$

$\phi\left(\dfrac{\sin x}{\sin y}, \dfrac{\sin y}{\sin z}\right) = 0$

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$(y-z)p + (x-y)q = z-x$

$\frac{dx}{y-z} = \frac{dy}{x-y} = \frac{dz}{z-x}$

$\frac{dx+dy+dz}{y-z+x-y+z-x} = 0$

$dx+dy+dz = 0$

$\boxed{x+y+z = c_1}$

$\frac{xdy + zdy + ydz}{xy-zx+xz-yz+zy-xy} = 1$

$xd\cancel{y} + zdy + ydz = 0$

$\frac{x^2}{2} + zy + zy = c_2$

$\boxed{x^2 + 4yz = c_2}$

$\phi(x+y+z, x^2+4yz) = 0$

ii) $p+3q = 5z + \tan(y-3x)$

$\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{5+\tan(y-3x)}$

$dx = \frac{dy}{3}$

$3x = y + c_1$

$\boxed{c_1 = y-3x}$

$\frac{dx}{1} = \frac{dz}{5+\tan(c_1)}$

$5+\tan(c_1)\,dx = dz$

$5x + \tan(y-3x)x = z + c_2$

$c_2 = \cancel{5x}\ x(5+\tan(y-3x)) - z$

$\phi\left(y-3x,\ x(5+\tan(y-3x))-z\right) = 0$

i) $xp - yq + x^2 - y^2 = 0$

$x^2 + px = y^2 + qy = a$

$p = \frac{a-x^2}{x}$ $\quad q = \frac{a-y^2}{y}$

$dz = pdx + qdy$

$z = \int\left(\frac{a-x^2}{x}\right)dx + \int\frac{a-y^2}{y}dy$

$z = \int\frac{a}{x}dx \int x dx + \int\frac{a}{y}dy - \int y dy$

$z = a\ln x - \frac{x^2}{2} + a\ln y - \frac{y^2}{2}$

$z = a\ln\left(\frac{x}{y}\right) - \frac{1}{2}(x^2+y^2)$

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(v) $(z^2-2yz-y^2)p + (xy+zx)q = xy - zx$

Sol. $\frac{dx}{z^2-2yz-y^2} = \frac{dy}{xy+zx} = \frac{dz}{xy-zx}$

$\frac{xdx + ydy + zdz}{xz^2-2xyz-xy^2+xy^2+xyz+xyz} = 1$

$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = c_1$

$\boxed{c_1 = x^2+y^2+z^2}$

$\frac{dy-dz}{\cancel{x}2z\cancel{x}} = \frac{dy+dz}{\cancel{x}2y}$

$\frac{dy-dz}{z} = \frac{dy+dz}{y}$

$\int ydy - \int ydz = \int zdy + \int zdz$

$\frac{y^2}{2}$ $\int ydy - \int zdz = \int zdy - ydz$

$\Rightarrow \frac{y^2}{2} - \frac{z^2}{2} = \int d(yz)$

$\Rightarrow y^2 - z^2 = 2yz + c_2$

$\Rightarrow y^2 - z^2 - 2yz = c_2$

$\phi(x^2+y^2+z^2,\ y^2-z^2-2yz) = 0$

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Solve the first order non-linear PDEs :

1) $q - p + x - y = 0$

Sol. $p - x = q - y = a$

$p = a + x, \quad q = a + y$

$\therefore \; dz = p\,dx + q\,dy$

$dz = (a+x)dx + (a+y)dy$

$\therefore \; \boxed{z = ax + \frac{x^2}{2} + ay + \frac{y^2}{2} + C}$

2) $\sqrt{p} + \sqrt{q} = 2x$

$\sqrt{p} - 2x = -\sqrt{q} + 0 = a$

$\sqrt{p} = a + 2x \quad \sqrt{q} = -a$

$p = (a+2x)^2 \quad q = a^2$

$dz = p\,dx + q\,dy$

$dz = \cancel{a+2x}\,(a^2 + 4x^2 + 4ax)dx + a^2 dy$

$\boxed{z = a^2x + \frac{4x^3}{3} + 2ax^2 + a^2y + C}$

3) $p^2 + q^2 = z(x+y)$

Sol. $zx + zy = p^2 + q^2$

$z = \frac{p^2}{x+y} + \frac{q^2}{x+y}$

$p = a, \; q = b$

$\boxed{z = \frac{a^2 + b^2}{x+y}}$

4) $z = px + qy + p^2 + q^2$

Sol. This is clairaut's form.

$p = a, \; q = b$

$\boxed{z = ax + by + a^2 + b^2}$

(v) $x^2p^2 + y^2q^2 = z^2$

Sol. $z^2 - x^2p^2 + y^2q^2 = 0$

$q = ap$

$\Rightarrow x^2p^2 + a^2y^2p^2 = z^2$

$p = \frac{z}{\sqrt{x^2 - a^2y^2}}$

$dz = p(dx + a\,dy)$

$\frac{dz}{z} = \sqrt{x^2 - a^2y^2}\,(x + ay)$

$\Rightarrow \log z = (x + ay)(\sqrt{x^2 - a^2y^2})$

$\Rightarrow z = e^{x+ay} \cdot e^{\sqrt{x^2 - a^2y^2}}$

(vi) $pq = 1$

$p = a, \quad q = g(p)$

$\Rightarrow q = \frac{1}{p} = g(p)$

$dz = p\,dx + q\,dy$

$z = \int a\,dx + g(a)\,dy + c$

$$\boxed{z = ax + \frac{y}{a} + c}$$

(vii) $p^2 + q^2 = 1$

Sol. Let $p = a, \quad q = \pm\sqrt{1 - a^2}$

$dz = p\,dx + q\,dy$

$z = \int a\,dx \pm \sqrt{1 - a^2}\,dy$

$$\boxed{z = ax \pm \sqrt{1 - a^2}\,y + c}$$

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Solve the higher order PDEs :

$(D^3 - 4D^2D' + 3DD'^3)z = 0$

It is non homogeneous -

$F(D, D')$ is not reducible to linear factors

C.F. $= \Sigma A e^{hx+ky}$

where $h^3 - 4h^2k + 3hk^3 = 0$

$(D^4 - 2D^3D' + 2DD'^3 - D'^4)z = 0$

Putting $D \to m$, $D' \to 1$

A.E. $m^4 - 2m^3 + 2m - 1$

$m = 1$ is one solution

$\Rightarrow (m-1)(m^3 - m^2 - m + 1)$

$\Rightarrow (m-1)(m+1)(m^2 - 2m + 1)$

$\Rightarrow (m-1)(m+1)(m-1)^2$, $\Rightarrow$ $m = 1, 1, 1$, $m = -1$

C.F. $= \phi_1(y-x) + \phi_2(y+x) + x\phi_3(y+x) + x^2\phi_4(y+x)$

$(D^4 + D'^4)z = 0$

Putting $D \to m$, $D' \to 1$

A.E. $m^4 + 1 = 0$ $\Rightarrow$ $(m^2+1)^2 - 2m^2$

$m^4 = -1$ $\Rightarrow$ $(m^2 + 1 - \sqrt{2}m)(m^2 + 1 + \sqrt{2}m) = 0$

$m^2 = \pm i$ $\Rightarrow$ $m^2 + 1 - \sqrt{2}m = 0$, $m^2 + \sqrt{2}m + 1 = 0$

$m = \dfrac{\sqrt{2} \pm \sqrt{2-4}}{}$, $m = \dfrac{-\sqrt{2} \pm \sqrt{2-4}}{}$

$m = \sqrt{2} \pm \sqrt{2}i$, $m = -\sqrt{2} \pm \sqrt{2}i$

$m = \sqrt{2}(1+i)$, $\sqrt{2}(1-i)$, $\sqrt{2}(-1+i)$, $\sqrt{2}(-1, -i)$

C.F. $= \phi_1(y + \sqrt{2}(1+i)x) + \phi_2(y + \sqrt{2}(1-i) + \phi_3(y + \sqrt{2}(-1+i)x) + \phi_4(y + \sqrt{2}(-1-i)x)$

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(iv) $(D^4 + D'^4 - 2D^2D'^2)z = 0$

Sol. $D \to m,\ D' \to 1$

~~m~~ A.E. $m^4 + 1 - 2m^2 = 0$

$m^4 - 2m^2 + 1 = 0$

$(m^2-1)^2 = 0$

$m = \pm 1,\ m = \pm 1$

C.F $= \phi_1(y+x) + x\phi_2(y+x) + \phi_3(y-x) + x\phi_3(y-x)$

(v) $(D + D' - 2)(D + 4D' - 3)z = 0$

Sol. $D \to m,\ D' \to 1$

$(m+1-2)(m+4-3) = 0$

$(m-1)(m+1) = 0$

$m = 1,\ m = -1$

C.F. $= \phi_1(y+x) + \phi_2(y-x)$

(vi) $(D + 3D' + 4)^2 z = 0$

Sol. $D \to m,\ D' \to 1$

$(m + 3 + 4)^2 = 0$

$(m+7)^2 = 0$

$m = -7, -7$

C.F. $= \phi_1(y-7x) + x\phi_2(y-7x)$

(vii) $r - t + p - q = 0$

Sol. $D^2 - D'^2 + D - D' = 0$

$D \to m,\ D' \to 1$

$m^2 - 1 + m - 1 = 0$

$m^2 + m - 2 = 0$

$(m+2)(m-1) = 0$

$\boxed{m = 1,\ m = -2}$

C.F $= \phi_1(y+x) + \phi_2(y-2x)$

$(D^2 - 4DD' + 4D'^2)z = e^{2x+y}$

A.E. $D \to m$, $D' \to 1$

$m^2 - 4m + 4 = 0$

$(m-2)^2 = 0$

$\boxed{m = 2, 2}$

C.F. $= \phi_1(y+2x) + x\phi_2(y+2x)$

P.I. $= \frac{1}{F(D,D')} \phi(ax+by)$ $F(2,1) = 4 - 8 + 4 = 0$

$\cancel{= \frac{1}{F(2,1)} \int}$ $F'(2,1) = 2D - 4D'\big|_{2,1} = 4 - 4 = 0$

$F''(2,1) = \boxed{2}$

P.I. $= \frac{x^2}{2} e^{2x+y}$

$z =$ C.F. + P.I.

$z = \phi_1(y+2x) + x\phi_2(y+2x) + \frac{x^2}{2}e^{2x+y}$

(x) $(D^2 - D'^2 - 2D + 2D')z = e^{2x+3y}$

Sol. A.E. $D \to m$, $D' \to 1$

$m^2 - 1 - 2m + 2 = 0$

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = 1, 1$

C.F. $= \phi_1(y+x) + x\phi_2(y+x)$

P.I. $= \frac{1}{F(D,D')} e^{2x+3y}$ $F(2,3) = 4 - 9 - 4 + 6$
$= 10 - 13 = -3$

P.I. $= \frac{1}{F(2,3)} \int e^u du \Big|_{u = 2x+3y}$

P.I. $= \frac{-1}{3} e^{2x+3y}$

$z =$ C.F. + P.I.

$z = \phi_1(y+x) + x\phi_2(y+x) - \frac{e^{2x+3y}}{3}$

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(X) $(D^2 + 2DD' + D'^2)z = \sin(2x+3y)$

Sol. A.E. $D \to m$, $D' \to 1$

$m^2 + 2m + 1 = 0$

$(m+1)^2 = 1$

$m = -1, -1$

C.F. $= \phi_1(y-x) + x\phi_2(y-x)$

P.I. $= \frac{1}{F(D,D')} \iint \phi(u)\,du \Big|_{u=ax+by}$

$F(2,3) = 4 + 2\times2\times3 + 3^2$

$= 4 + 12 + 9$

$= 25$

P.I. $= \frac{1}{25} \int \sin u\,du \Big|_{u=2x+3y}$

P.I. $= -\frac{1}{25}\cos(2x+3y)$

$z = \phi_1(y-x) + x\phi_2(y-x) - \frac{1}{25}\cos(2x+3y)$

(Xi) $(D+1)(D+D'-1)z = \sin(x+2y)$

Sol. A.E. $D \to m$, $D' \to 1$

$(m+1)(m+1-1) = 0$

$m = 0$, $m = -1$

C.F. $= \phi_1(y) + \phi_2(y-x)$

P.I. $= \frac{1}{F(D,D')} \int \phi(u)\,du \Big|_{u=ax+by}$

$F(1,2) = (1+1)(1+2-1)$

$= 2(2) = 4$

P.I. $= \frac{1}{4} \int \sin u\,du \Big|_{u=x+2y}$

P.I. $= \frac{-\cos(x+2y)}{4}$

$z = \phi_1(y) + \phi_2(y-x) - \frac{\cos(x+2y)}{4}$

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ii) $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y) + e^{3x+y}$

A.E. $D \to m, \; D' \to 1$

$m^3 - 7m - 6 = 0$

$(m+1)(m^2 - m - 6) = 0$

$(m+1)(m+2)(m-3) = 0$

$m = -1, -2, 3$

C.F. $= \phi_1(y-x) + \phi_2(y-2x) + \phi_3(y+3x)$

P.I. $= \dfrac{1}{F(D,D')}\int\int \sin(x+2y) + \dfrac{1}{F(D,D')}\int e^{3x+y}$

$F(1,2) = 1 - 28 - 48$
$= \boxed{-75}$

$\Rightarrow \dfrac{1}{-75}\int\int \sin u \, du \Big|_{u=x+2y} + \dfrac{1 \cdot x}{11}\int e^u du \Big|_{u=3x+y}$

$F(3,1) = 27 - 21 - 6$
$= 0$

$F'(3,1) = 3D^2 - 7D'^2 \Big|_{3,1}$
$= 18 - 7 = \boxed{11}$

$\Rightarrow \dfrac{\cos(x+2y)}{75} + \dfrac{xe^{3x+y}}{11}$

$z = \text{C.F} + \text{P.I.}$

$z = \phi_1(y-x) + \phi_2(y-2x) + \phi_3(y+3x) + \dfrac{\cos(x+2y)}{275} + \dfrac{xe^{3x+y}}{11}$

xiii) $(4D^2 - 4DD' + D'^2)z = 16\log(x+2y)$

Sol. A.E. $D \to m, \; D' \to 1$

$4m^2 - 4m + 1 = 0$

$(2m-1)^2 = 0$

$m = \frac{1}{2}, \frac{1}{2}$

C.F. $= \phi_1\left(y + \frac{x}{2}\right) + x\phi_2\left(y + \frac{x}{2}\right)$

$F(1,2) = 4\times1 - 4\times1\times2 + 2^2$
$= 4 - 8 + 4 = 0$

P.I. $= \dfrac{1}{F(D,D')}\int \phi(u)\,du \Big|_{\phi = ax+by}$

$F'(1,2) = 8D - 4D'$
$= 8 - 8 = 0$

$F''(1,2) = 8$

P.I. $= \dfrac{x^2}{8}\int 16\log(u)\,du \Big|_{u=x+2y}$

P.I. $= 2x^2\left[u(\log u - 1)\right]$

P.I. $= 2x^2\left[(x+2y)(\log(x+2y) - 1)\right]$

$z = \phi_1(2y+x) + x\phi_2(2y+x) + 2x^2\left[(x+2y)(\log(x+2y) - 1)\right]$

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(xiv) $(D^2D' - 2DD'^2 + D'^3)z = x^{-2}$

Sol. A.E. $D' \to m$, $D \to 1$

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = 1, 1$

C.F. $= \phi_1(y+x) + x\phi_2(y+x)$

~~P.I. = $\frac{1}{F(D,D')}$~~ $F(D,D') = x^{-2}y^0$

P.I. $= \frac{1}{(D^2D' - 2DD'^2 + D'^3)}\{x^{-2}y^0\}$

P.I. $= \frac{1}{D'^3\left[1 - \frac{2D}{D'} + \frac{D^2}{D'^2}\right]} x^{-2}$

~~P.I. $= \frac{1}{D'^3}\left[\left(1 - \left[\frac{2D}{D'} + \frac{D^2}{D'^2}\right]\right)^{-1} x^{-2}\right)$~~

~~P.I. $= \frac{1}{D'^3}\left[\left(1 + \left(\frac{2D}{D'} - \frac{D^2}{D'^2}\right) - \left(\frac{2D}{D'} - \frac{D^2}{D'^2}\right)^2 + \left(\frac{2D}{D'} - \frac{D^2}{D'^2}\right)^3 - - \right) x^{-2}\right]$~~

~~P.I. $= \frac{1}{D'^3}\Big[$~~

P.I. $= \frac{1}{D'^3\left[1 - \frac{(2DD' + D^2)}{D'^2}\right]} x^{-2}$

P.I. $= \frac{1}{D'^3}\left[1 + \frac{2DD' + D^2}{D'^2} - \frac{(2DD' + D^2)^2}{D'^4}\right] x^{-2}$

P.I. $= \frac{1}{D'^3}\left[x^{-2} + \frac{1}{D'^2}\left[2D'x - 2x^{3} + 6x^{-4}\right]\right]$

P.I. $= \frac{1}{D'^3}\left[x^{-2} + \frac{1}{D'^2}\left[\cancel{-4x^{-3}}y^0 + 6x^{-4}\right]\right]$

P.I. $= \frac{1}{D'^3}\left[x^{-2} + 6x^{-4}y^2\right]$

P.I. $= x^{-2}y^3 + 6x^{-4}y^5$

$z = \phi_1(y+x) + x\phi_2(y+x) + x^{-2}y^3 + 6x^{-4}y^5$

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$(D^3 - D'^3)z = x^3y^3$.

A.E. $D \to m$, $D' \to 1$

$m^3 - 1 = 0$

$m = 1, \omega, \omega^2$

C.F. $= \phi_1(y+x) + \phi_2(y+\omega x) + \phi_3(y+\omega^2 x)$

P.I. $= \frac{1}{D^3 - D'^3} x^3 \cdot y^3$

$\Rightarrow \frac{1}{D^3}\left[1 - \frac{D'^3}{D^3}\right]^{-1} \cdot x^3y^3$

$\Rightarrow \frac{1}{D^3}\left[1 + \frac{D'^3}{D^3} - \frac{D'^6}{D^6} + \cdots\right] \cdot x^3y^3$

$\Rightarrow \frac{1}{D^3}\left[x^3y^3 + \frac{1}{D^3} x^3 \cdot 6\right]$

$\Rightarrow \frac{x^6y^3}{4 \cdot 5 \cdot 6} + \frac{6x^9}{4 \times 5 \times 6 \times 7 \times 8 \times 9} \Rightarrow \frac{x^6y^3}{120} + \frac{x^9}{10080}$

$z = $ C.F. + P.I.

$= \phi_1(y+x) + \phi_2(y+\omega x) + \phi_3(y+\omega^2 x) + \frac{x^6y^3}{120} + \frac{x^9}{10080}$

(vi) $(D^2 + 2DD' + D'^2)z = x^2 + xy + y^2$

A.E. $D \to m$, $D' \to 1$

$m^2 + 2m + 1 = 0$

$(m+1)^2 = 0$

$m = -1, -1$

C.F. $= \phi_1(y-x) + \cancel{x(y-x)}\ x\phi_2(y-x)$

P.I. $= \frac{1}{F(D,D')}[x^2 + xy + y^2]$

P.I. $= \frac{1}{D^2+2DD'+D'^2} x^2 + \frac{1}{D^2+2DD'+D'^2} xy + \frac{1}{D^2+2DD'+D'^2} y^2$

P.I. $= \underbrace{\frac{1}{D^2}\left[1 + \frac{2D'}{D} + \frac{D'^2}{D^2}\right]^{-1} x^2}_{1} + \underbrace{\frac{1}{D^2}\left[1 + \frac{2D'}{D} + \frac{D'^2}{D^2}\right] xy}_{2} + \underbrace{\frac{1}{D'^2}\left[1 + \frac{2D}{D'} + \frac{D^2}{D'^2}\right] y^2}_{3}$

P.I.$_1 = \frac{1}{D^2}\left[1 - \frac{2DD' + D'^2}{D^2} + \left(\frac{2DD'+D'^2}{D^2}\right)^2\right] x^2 + \frac{1}{D^2}\left[1 + \frac{2DD'+D'^2}{D^2}\right] xy + \frac{1}{D'^2}\left[1 + \frac{2DD'+D^2}{D'^2}\right] y^2$

$\Rightarrow \frac{1}{D^2}\left[x^2 - 0\right] + \frac{1}{D^2}\left[xy + \frac{1}{D^2}[2]\right] + \frac{1}{D'^2}\left[y^2 + \frac{10}{D'^2}\right]$

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$$\Rightarrow \frac{1}{D^2}[x^2] + \frac{1}{D^2}\left[xy + x^2\right] + \frac{1}{D'^2}[y^2]$$

$$\Rightarrow \frac{x^4}{12} + \frac{x^3y}{6} + \frac{y^4}{12}$$

$$z = \phi_1(y-x) + x\phi_2(y-x) + \frac{x^4}{12} + \frac{x^3y}{6} + \frac{y^4}{12}$$

(xvii) $r - 4t = \frac{4x}{y^2} - \frac{y}{x^2}$

Sol. $D^2 - 4D' = \frac{4x}{y^2} - \frac{y}{x^2}$

$(D^2 - 4D')z = 4xy^{-2} - yx^{-2}$

A.E → $D \to m$, $D' \to 1$

$m^2 - 4 = 0$

$m = \pm 2$

C.F $= \phi_1(y+2x) + \phi_2(y-2x)$

P.I. $= \frac{1}{F(D,D')} \cancel{\frac{4x}{y}}\, 4xy^{-2} - \frac{1}{F(D,D')}\frac{y}{x^2}$

$$\Rightarrow \frac{1}{D^2 - 4D'^2} 4xy^{-2} - \frac{1}{D^2 - 4D'^2} yx^{-2}$$

$$\Rightarrow \frac{1}{D^2}\left[1 - \frac{4D'^2}{D^2}\right]^{-1} yx^{-2} - \frac{1}{(-4D'^2)}\left[1 - \frac{D^2}{4D'^2}\right]^{-1} 4xy^{-2}$$

$$\cancel{\Rightarrow \frac{1}{D^2}\left[1 + \frac{4D'^2}{D^2} - \frac{16D'^4}{D^4}\right] 4xy^{-2} + \frac{1}{4D'^2}\left[1 + \frac{D^2}{4D'^2} - \frac{D^4}{16D'^4}\right] yx^{-2}}$$

$$\Rightarrow \frac{1}{D^2}\left[1 + \frac{4D'^2}{D^2} - \frac{16D'^4}{D^4}\right] yx^{-2} + \frac{1}{4D'^2}\left[1 + \frac{D^2}{4D'^2} - \frac{D^4}{16D'^4}\right] 4xy^{-2}$$

$$\Rightarrow \frac{1}{D^2}\left[yx^{-2} + 0\right] + \frac{1}{-4D'^2}\left[4xy^{-2} + 0\right]$$

$$\Rightarrow -y\log x - x\log y$$

$$z = \phi_1(y+2x) + \phi_2(y-2x) - (y\log x + x\log y)$$

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viii) $(D^2 - D'^2 + D + 3D' - 2)z = x^2y$

A.E. $D \to m$, $D' \to 1$

$m^2 - 1 + m + 3 - 2 = 0$

$m^2 + m = 0$

$m(m+1) = 0$

$m = 0,\ m = -1$

C.F. $= \phi_1(y) + \phi_2(y-x)$

P.I. $= \frac{1}{F(D,D')} x^2y$

P.I. $= \frac{1}{D^2 - D'^2 + D + 3D' - 2} x^2y$

P.I. $= \frac{x^2y}{D^2\left[1 - \left[\frac{D'^2}{D^2} - \frac{1}{D} - \frac{3D'}{D^2} + \frac{2}{D^2}\right]\right]} \Rightarrow \frac{x^2y}{D^2\left[1 - \left(\frac{D'^2 - D - 3D' + 2}{D^2}\right)\right]}$

P.I. $= \frac{1}{D^2}\left[1 - \left(\frac{D'^2 - D - 3D' + 2}{D^2}\right)\right]^{-1} x^2y \Rightarrow$

$\Rightarrow \frac{1}{D^2}\left[1 + \frac{D'^2 - D - 3D' + 2}{D^2} - \frac{(D'^2 - D - 3D' + 2)^2}{D^4}\right] x^2y$

$\Rightarrow \frac{1}{D^2}\left[x^2y + \frac{1}{D^2}\left[0 - 2xy - 3x^2 + 2x^2y\right] - \frac{1}{D^4}\cancel{\left[0 - 2y - 6x + 4y + 6x + 6x^2\right]} \ 4xy - 6x^2 + 4x^2y\right]$

$\Rightarrow \frac{1}{D^2}\left[x^2y + \left[\frac{x^3y}{3} - \frac{x^4}{4} + \frac{x^4y}{6}\right]\right] - 0$

$\Rightarrow \frac{x^4y}{12} + \frac{x^5y}{60} - \frac{x^6}{120} + \frac{x^6y}{180}$

$z = \phi_1(y) + \phi_2(y-x) + \frac{x^4y}{12} + \frac{x^5y}{60} - \frac{x^6}{120} + \frac{x^6y}{180}$

ix) $(D^2 + DD' - 6D'^2)z = x^2\sin(x+y)$

A.E. $D \to m$, $D' \to 1$

$m^2 + m - 6 = 0$

$(m+3)(m-2) = 0$

$m = -3,\ m = 2$

C.F. $= \phi_1(y-3x) + \phi_2(y+2x)$

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P.I. $=$ Imaginary Part of $\frac{1}{F(D,D')} e^{ix+iy} x^2$ $\quad D \to D+i$

$$P.I. = \frac{1}{D^2 - 1 + 2Di + DD' + Di + D'i - 1 - 6D'^2 + 6 - 12D'i}$$

$$\Rightarrow \frac{1}{D^2 - 6D'^2 + 4 + DD' + D(2i+1) + D'(i-12i)}$$

$$\Rightarrow \frac{1}{D^2 - 6D'^2 + 4 + DD' + 3iD - 11iD'}$$

$$\Rightarrow \frac{1}{D^2}\left[1 - \left[\frac{4 - 11iD' - 6D'^2 + DD' + 3iD}{D^2}\right]\right]^{-1} x^2$$

~~Img. Part~~, $\frac{1}{D^2}\left[x^2 - \frac{1}{D^2}\left[4x^2 + 6ix\right]\right]$

$$= \frac{1}{D^2}\left[\frac{x^4}{12} - \frac{x^4}{3} - ix^3\right]$$

Imaginary Part $= \boxed{-\frac{x^5}{20} i}$

$$\boxed{z = \phi_1(y - 3x) + \phi_2(y + 2x) - \frac{x^5}{20}}$$

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