

PSY 503: Foundations of Statistical Methods in Psychological Science

Interactions, Factorial Designs

Suyog Chandramouli

311 PSH (Princeton University)

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Dec 3rd

- Remaining project presentation
- Lecture – Interactions & Factorial ANOVA
 - Interactions (Two-way ANOVA)
 - Factorial ANOVA
 - A look at ANCOVA

Interactions

Questions we've asked so far

- **t-test (independent):** Is there a difference between two groups?
- **t-test (paired):** Is there a difference between two conditions?
(same people)
- **One-way ANOVA (between):** Is there a difference somewhere among k groups?
- **One-way ANOVA (within / repeated measures):** Is there a difference somewhere among k conditions? (same people)

All are variants of “Is there an effect that exists?”

But there are often other questions

- Does the effect of A depend on B?

But there are often other questions

- Does the effect of A depend on B?
- Examples
 - Does the effect of distraction on memory depend on age?
 - Does the bystander effect depend on group size?
 - Does stereotype threat depend on task difficulty?
 - Does parenting style affect outcomes differently for boys vs girls?
 - Does CBT work better for anxiety vs depression?
 - ...

Does the effect of A depend on B?

- Equivalent to:
 - **"Is there an interaction between A and B?"**
 - Are A and B acting alone?
Or are they working together?
 - **"Does B moderate the relationship between A and the outcome/DV?"**
- Depends on there being more than 1 (predictor / X) variable.
Answered by:
 - Two-way anova or higher (factorial ANOVA)
 - ANCOVA / regression with interaction
 - Regression

Methods to assess interaction

- Y is continuous

A	B	Method
Categorical	Categorical	Factorial ANOVA
Categorical	Continuous	ANCOVA / regression with interaction
Continuous	Categorical	Moderated regression
Continuous	Continuous	Moderated regression

- Y is binary: logistic regression with interaction term
- Y is ordinal: ordinal regression with interaction term

Visualizing interactions

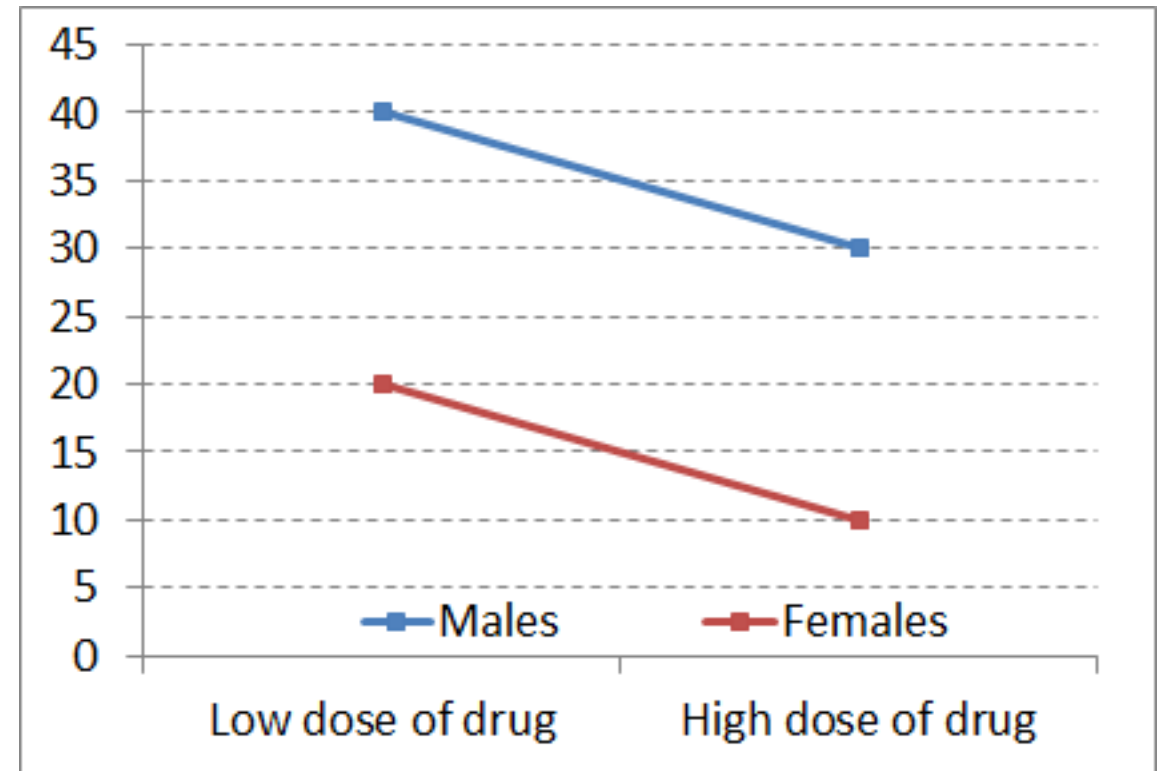
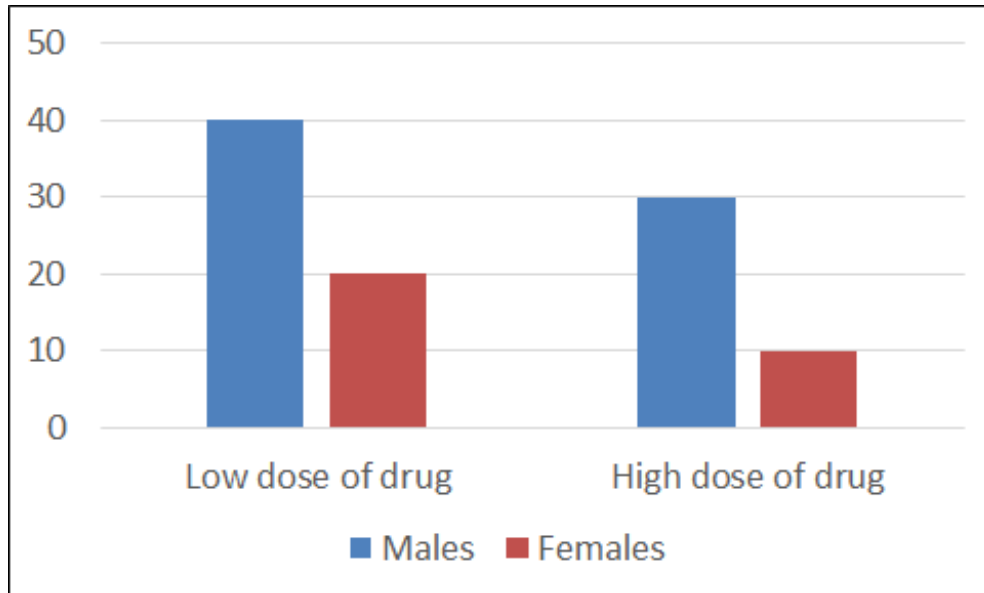
- **Golden Rule of Plots:** Look at the lines.
 - **Parallel Lines = No Interaction**
 - The effect of Variable A is the same, regardless of Variable B.
 - They are additive (independent).
 - **Non-Parallel Lines = Interaction**
 - The lines diverge, converge, or cross.
 - The effect of Variable A *changes* based on the level of Variable B.
 - This is the "It Depends" effect.

Visualization: no-interaction

RQ: Understanding connection between :

Drug dosage & Gender

with respect to:
Y (some score)

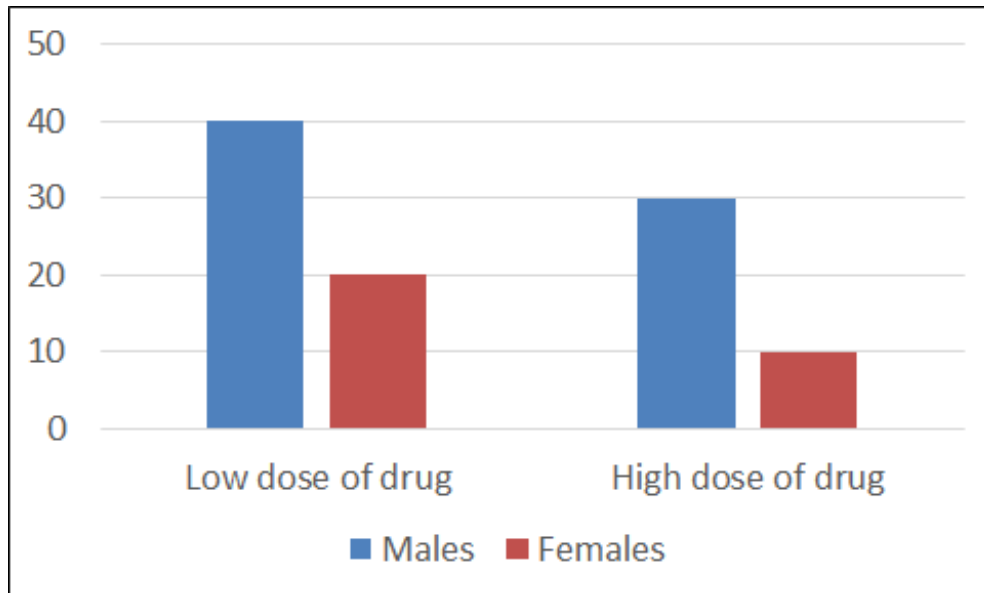


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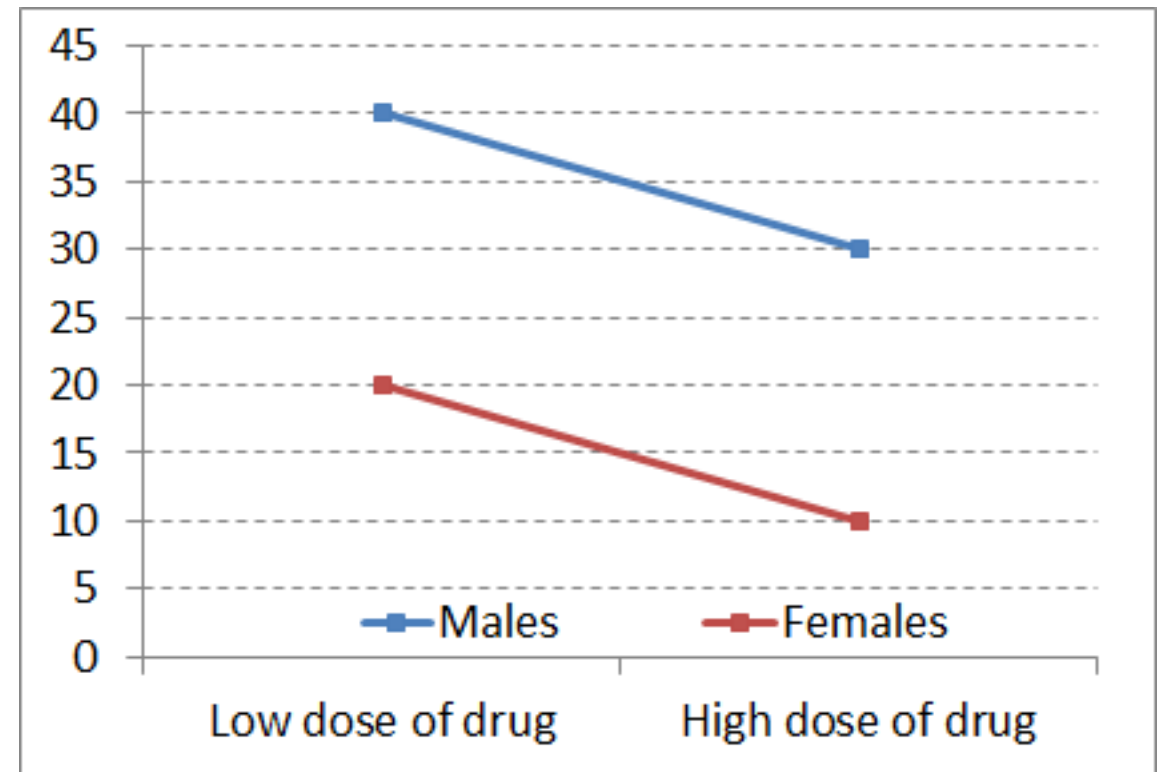
Drug dosage & Gender

with respect to:
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What's the effect of Gender, averaging over drug doses?

What's the effect of Dosage levels, averaging over Gender?

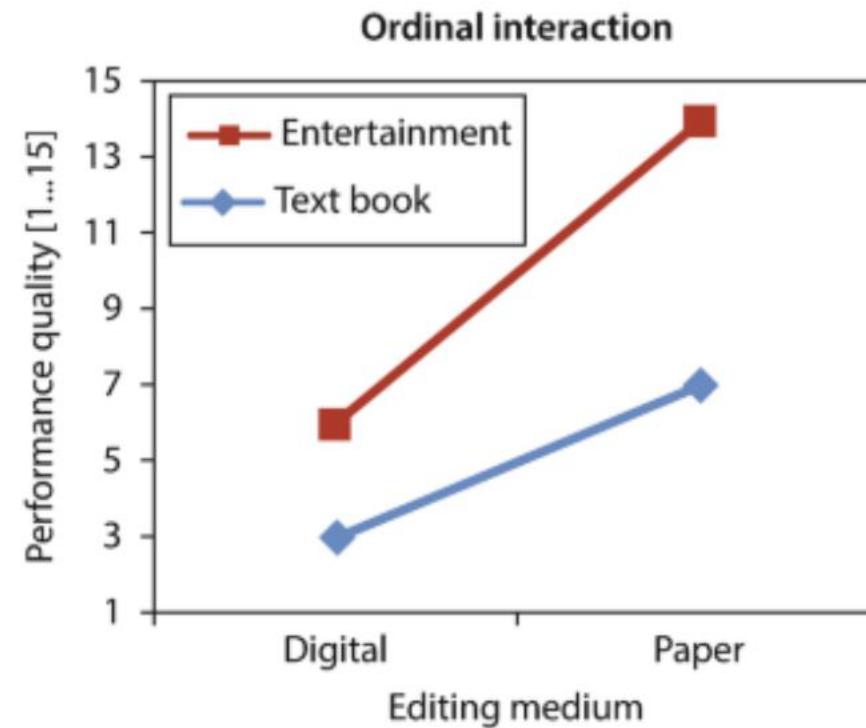


Visualization: interaction

RQ: Understanding connection between :

editing medium & content type

with respect to:
Y (performance quality)

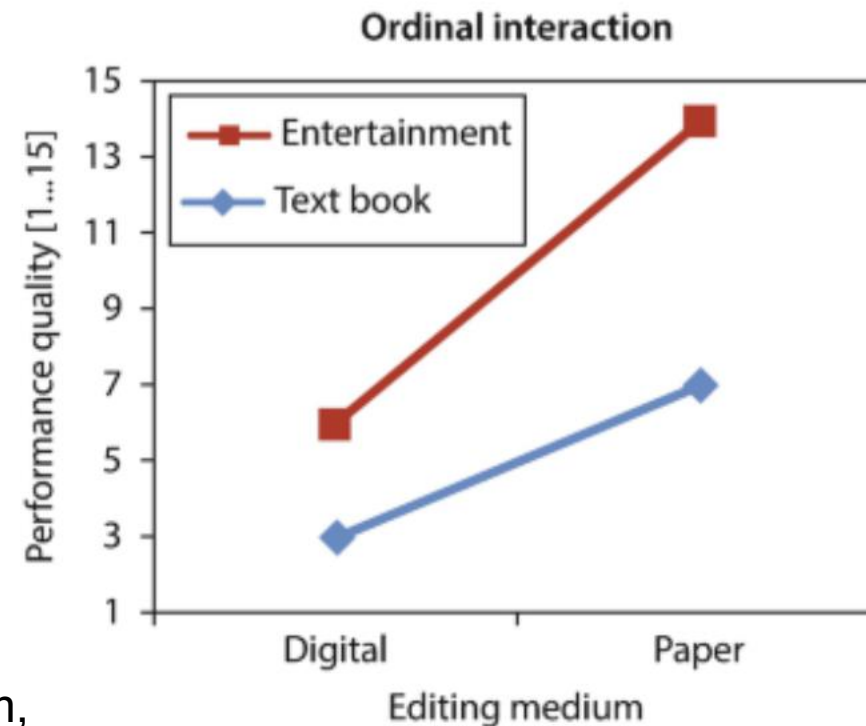


Visualization: interaction

RQ: Understanding connection between :

editing medium & content type

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What's the effect of Editing medium, averaging over Content Type?

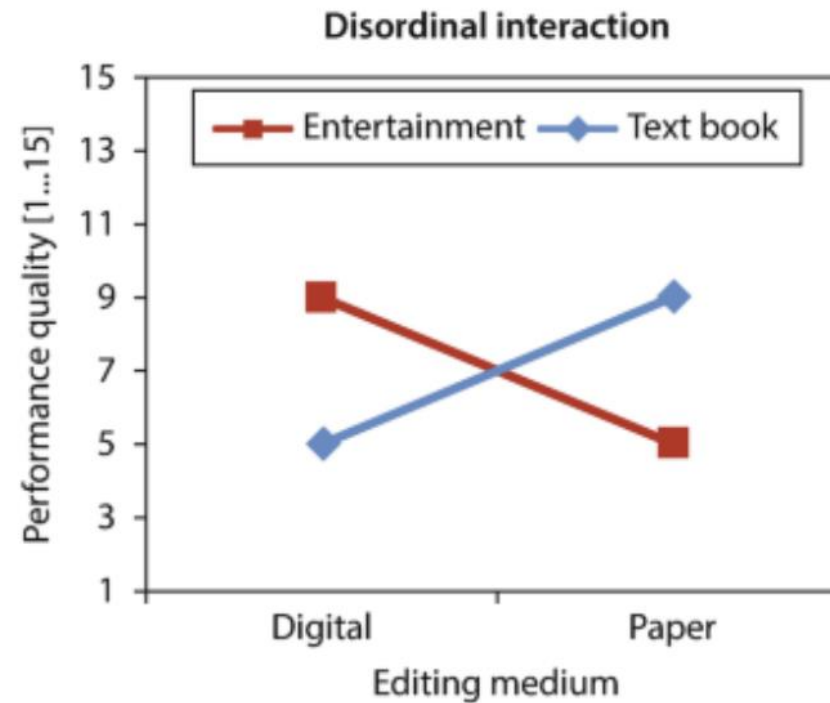
What's the effect of Content Type, averaging over Editing mediums?

Visualization: interaction

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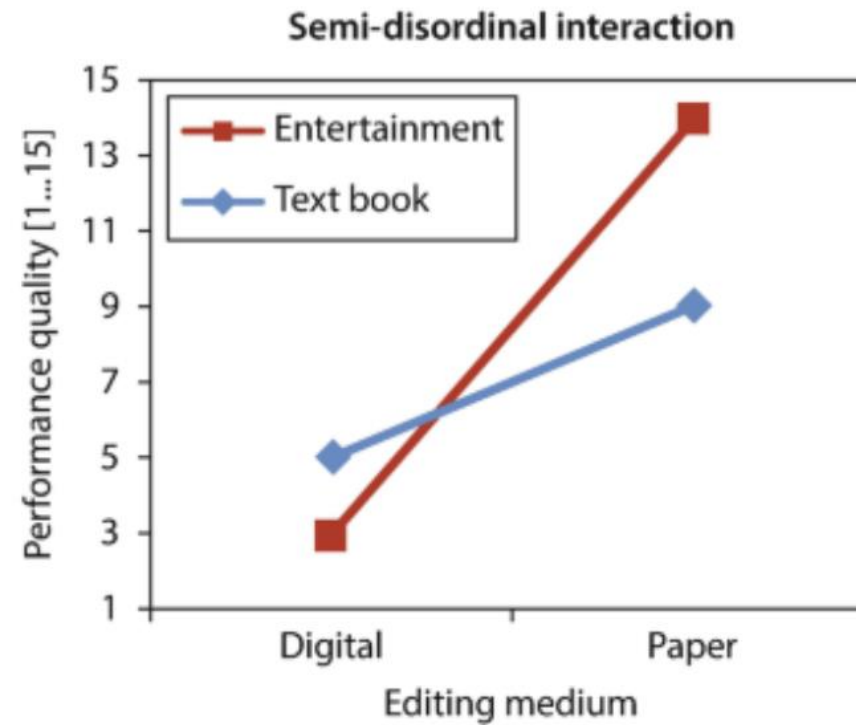
How Interactions Affect Main Effects

Interaction Type	Visual Pattern	Main Effects	Interpretation
None	Parallel lines	Clean	"A is better."
Ordinal	Diverge, no cross	Incomplete	"A is better—depends how much."
Disordinal	Lines cross	Misleading	"A is better for some, worse for others."

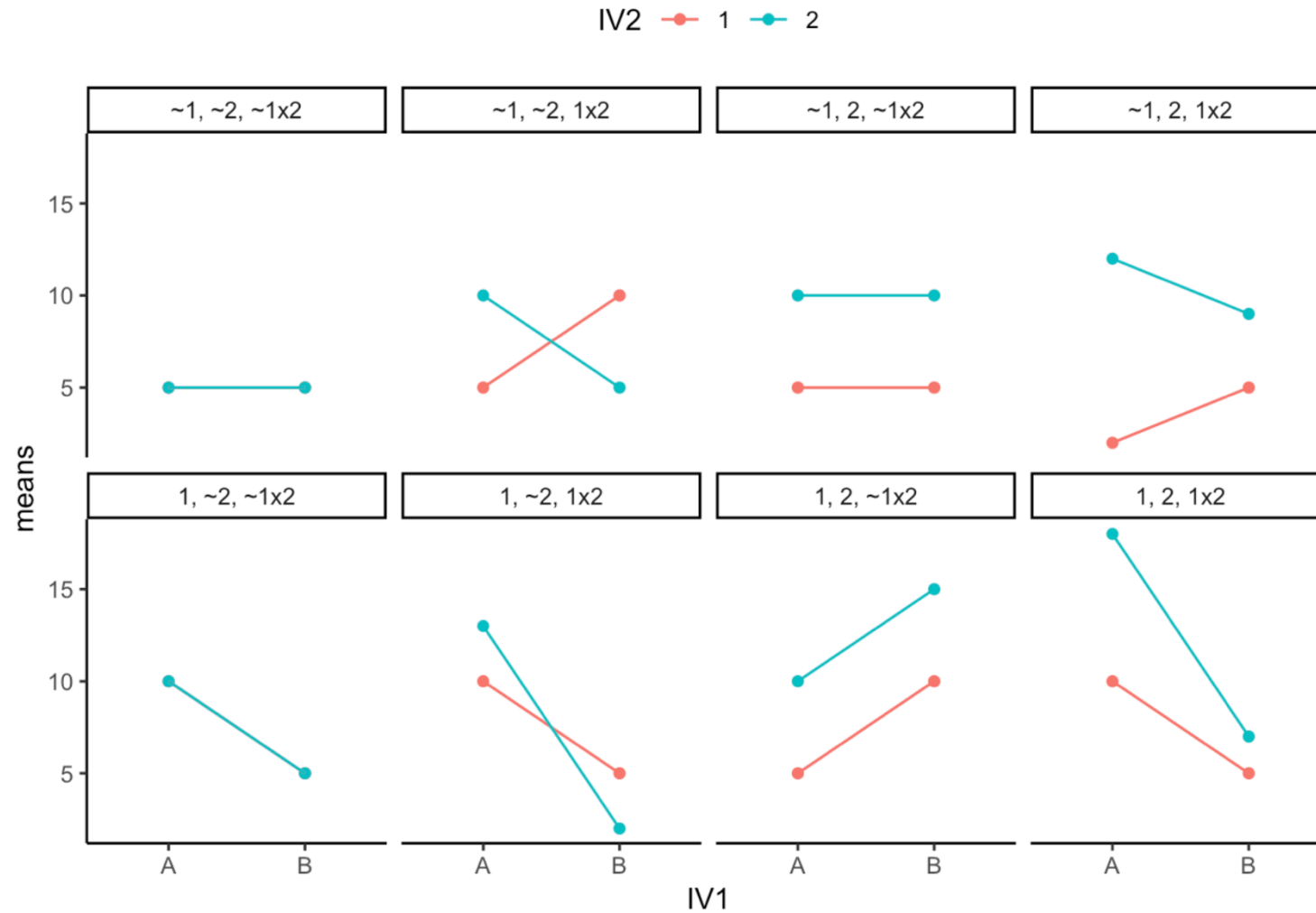
How Interactions Affect Main Effects

Interaction Type	Visual Pattern	Main Effects	Interpretation	Reporting Strategy
None	Parallel lines	Clean	"A is better."	Report main effects. Done.
Ordinal	Diverge, no cross	Incomplete	"A is better—depends how much."	Report main effects + note interaction. Or: simple effects.
Disordinal	Lines cross	Misleading	"A is better for some, worse for others."	Skip main effects. Report simple effects only.

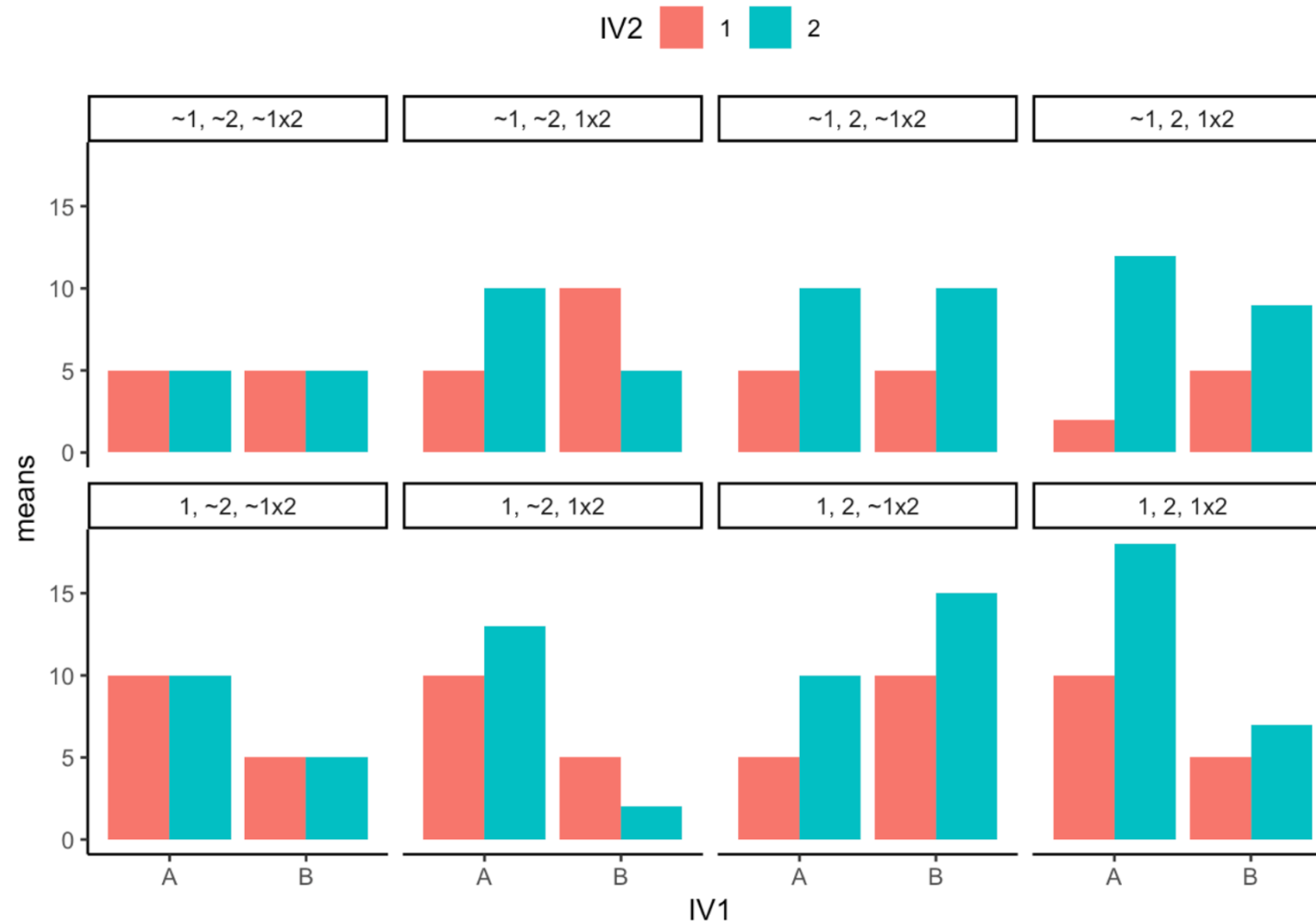
Visualization: interaction



Visualization: Possible outcomes for a 2x2 design



Visualization: Possible outcomes for a 2x2 design



Possible outcomes for a 2x2 design

1. no IV1 main effect, no IV2 main effect, no interaction
2. IV1 main effect, no IV2 main effect, no interaction
3. IV1 main effect, no IV2 main effect, interaction
4. IV1 main effect, IV2 main effect, no interaction
5. IV1 main effect, IV2 main effect, interaction
6. no IV1 main effect, IV2 main effect, no interaction
7. no IV1 main effect, IV2 main effect, interaction
8. no IV1 main effect, no IV2 main effect, interaction

Jargon

- "Marginal" = lives in the margins.
 - Collapse across one factor → get marginal mean for the other.
- Main effects compare marginal means.

	LOW DOSE	HIGH DOSE	MARGIN
Male	40	5	22.5
Female	15	30	22.5
Margin	27.5	17.5	Marginal Means

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Main effect of
Dosage = Δ = 10

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Simple effect of dosage = simple effects =
−35 and +15

(Stay in one row or column)

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This Δ = **interaction effect**

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Main effect of
Dosage = $\Delta = 10$

Factorial designs

- Specification
 - Number of factors
 - Number of levels
- 2 x 3 design

	B1	B2	B3
A1	.	.	.
A2	.	.	.

- Example: Gender (2) × Dose (low, medium, high)

Factorial designs

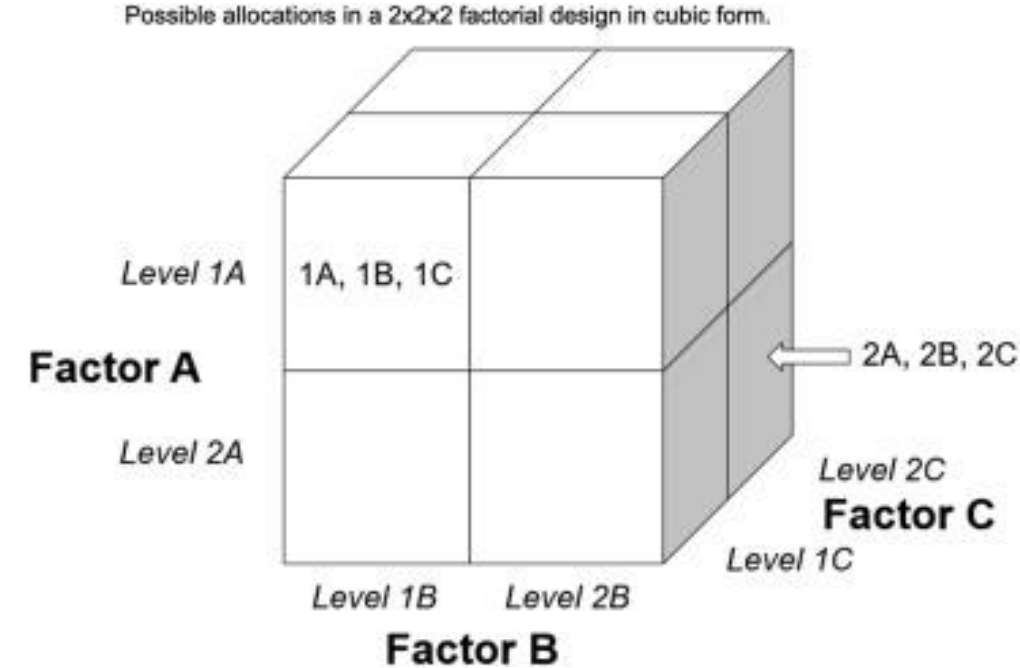
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2 x 2 x 2 design



Factorial designs

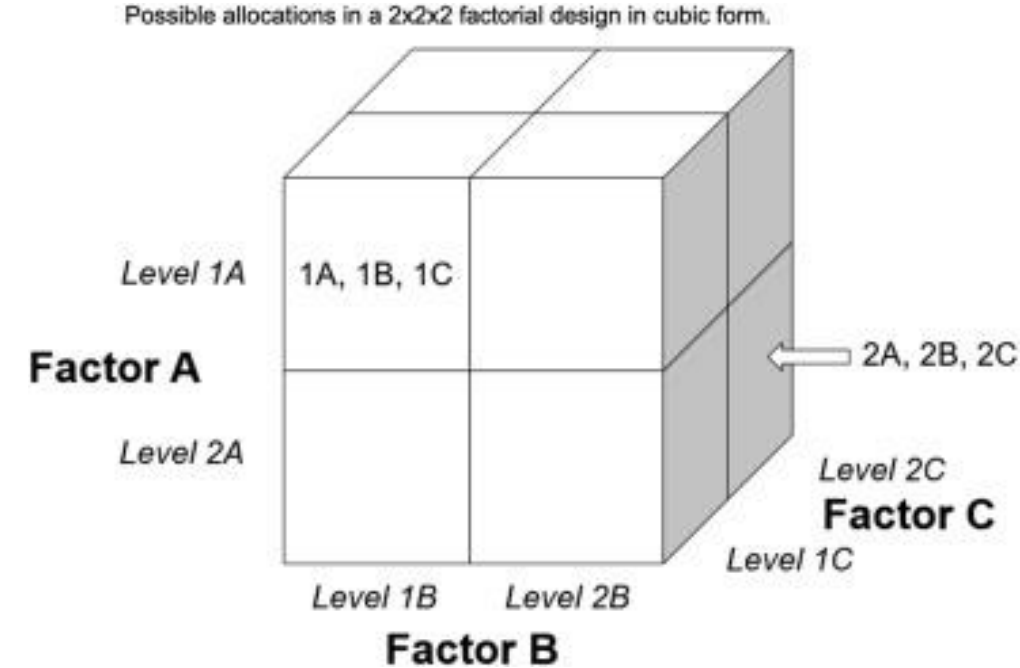
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- 3 main effects (A, B, C)
- 3 two-way interactions (A×B, A×C, B×C)
- 1 three-way interaction (A×B×C)
- Gets complicated fast

Fully Factorial

- A design is fully factorial when the levels of each variable are fully crossed with the levels of every other variable.
 - e.g., the levels of IV1 are manipulated across the levels of IV2 (in a 2x2 design)
- in other words, there are no missing cells

Two-way ANOVA

Two-way ANOVA

- Two factors. One outcome.
- Questions:
 - Main effect of A?
 - Main effect of B?
 - Interaction: Does A depend on B?

Two-way ANOVA

You know these:

- Two factors – A and B
 - A has i levels
 - B has j levels
- that result in $i*j$ cells
 - Cell means
 - $\bar{X}_{11}, \bar{X}_{12}, \dots, \bar{X}_{ij}$
 - Cell sizes
 - $n_{11}, n_{12}, \dots, n_{ij}$
 - Within group variation

Question: **Are there main effects?**
Is there an interaction?

Test-statistic

- Earlier
$$z = (\bar{X} - \mu) / (\sigma / \sqrt{N})$$
$$z = (\bar{X} - \mu) / SE$$
$$t = (\bar{X} - \mu) / (s / \sqrt{N})$$

Now, comparing k means

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}}$$

*Between group variation /
variation within groups*

(comparing k means)

“Does Factor A matter? Factor B?
***Do they interact?*”**

Two-way ANOVA

- **Use Case:** Comparing means across groups while considering two different categorical factors and their interaction
- **As a Linear Model:**
 - $Y = b_0 + b_1X_1 + b_2X_2 + b_3(X_1 X_2) + \varepsilon$
 - where X's are dummy coded
 - NHST
 - Traditional Form:
 - H_0_A : No main effect of factor A
 - H_0_B : No main effect of factor B
 - $H_0_A \times B$: No interaction between A and B
 - `lm()` equivalent:
 - H_0_A : $b_1 = 0$
 - H_0_B : $b_2 = 0$
 - $H_0_A \times B$: $b_3 = 0$

Each gets its own F-ratio in the ANOVA table.

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Two-way ANOVA steps

1. Run the model (3 F-tests)
2. Check if the interaction term is significant
 - If Yes:
 - don't interpret main effects
 - run simple effects (i.e. not collapsing => subsetting)
 - If no interaction
 - interpret main effects
 - A is significant → Post-hoc on A (to compare levels of A)
 - B significant? → Post-hoc on B

Implementation in R (Example)

```
```{r}
NR_D <- c(3,2,4,1)
NR_ND <- c(8,7,8,6)
R_D <- c(6,7,5,7)
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DV <- c(NR_D,NR_ND,R_D,R_ND)
Reward <- rep(c("NoReward", "Reward"),each=8)
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|--------------------|----|--------|---------|---------|----------|-----|
| Reward | 1 | 25.00 | 25.00 | 19.355 | 0.000866 | *** |
| Distraction | 1 | 49.00 | 49.00 | 37.935 | 4.88e-05 | *** |
| Reward:Distraction | 1 | 6.25 | 6.25 | 4.839 | 0.048162 | * |
| Residuals | 12 | 15.50 | 1.29 | | | |
| --- | | | | | | |
| Signif. codes: | 0 | '***' | 0.001 | '**' | 0.01 | '*' |
| | | | | 0.05 | '.' | 0.1 |
| | | | | | ' ' | 1 |

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```

```

```{r}
summary(aov(DV~Reward*Distraction,df))
```

```

↑equivalent
↓

```

```{r}
summary(aov(DV~Reward + Distraction + Reward:Distraction,df))
```

```

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Note

- Which to use when?
 - * -- almost always. You want to test the interaction.
 - + -- rare. Only if you have theoretical reason to assume no interaction (unusual).

| Formula | What it fits |
|---------|--|
| $A * B$ | $A + B + A:B$ (full factorial)

main effects + interaction |
| $A + B$ | main effects only |
| $A : B$ | interaction only |

Factorial ANOVA

- Two ***or more*** factors. One outcome.
- Questions:
 - Main effect of A?
 - Main effect of B?
 - Main effect of C?
 - A × B interaction?
 - A × C interaction?
 - B × C interaction?
 - A × B × C interaction?
 - ..

Factorial ANOVA

You know these:

- **k** factors – A, B, C,....
 - A has a levels
 - B has b levels
 - C has c levels
 - ..
 - Design: a x b x c... (k terms)
- that result in a*b*c... cells
 - Cell means
 - Cell sizes
 - Within cell variation

Question: **Are there main effects?**
Is there an interaction?

Test-statistic

- Same logic

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}}$$

*(one F for each Main effect,
one F for each interaction)*

*Between group variation /
variation within groups*

“Question: Are there main effects? Do factors interact? Do interactions interact?”

| Source | df | SS | MSE | F | p |
|--------|---------------|-----------------|--|------------------------------------|--------------------------------|
| A | $a - 1$ | $SS_{EffectA}$ | $MS_{EffectA} = \frac{SS_{EffectA}}{a - 1}$ | $\frac{MS_{EffectA}}{MS_{Error}}$ | Calculated from F-distribution |
| B | $b - 1$ | $SS_{EffectB}$ | $MS_{EffectB} = \frac{SS_{EffectB}}{b - 1}$ | $\frac{MS_{EffectB}}{MS_{Error}}$ | Calculated from F-distribution |
| A*B | $(a-1)*(b-1)$ | $SS_{EffectAB}$ | $MS_{EffectAB} = \frac{SS_{EffectAB}}{(a - 1)(b - 1)}$ | $\frac{MS_{EffectAB}}{MS_{Error}}$ | Calculated from F-distribution |
| Error | $N - (a * b)$ | SS_{Error} | $MS_{Error} = \frac{SS_{Error}}{N - (a * b)}$ | | |

a = number of groups in A; b = number of groups in B; N = number of subjects

$$SS_{Total} = SS_{EffectA} + SS_{EffectB} + SS_{EffectAB} + SS_{Error}$$

$$SS_{Effect} = \sum_{i=1}^k n_i (X_i - \bar{X})^2$$

$$SS_{interaction} = n \text{ in cells } \sum_{i=1}^n (X_{ij} - X_i - X_j + \bar{X})^2$$

Notes: \bar{X} = Grand Mean, X_i = condition mean (SS effect)

ANCOVA (controlling for covariates)

- **ANOVA** : Analysis of Variance
- **ANCOVA**: Analysis of Covariance (referring to the covariate)
- Example questions:
 - Do CBT vs medication differ in depression scores — controlling for baseline severity?
 - Do sleep conditions differ in memory recall — controlling for IQ?
 - Do exercise interventions differ in anxiety — controlling for baseline fitness?
 - ...
- **"Do groups differ in Y — after removing variance due to C?"**

ANCOVA

You know these:

- One factors – A (with i levels)
- One or more continuous covariate – B, C..
- Cell means
- Within group variation

Question: Does A matter **after accounting for the covariate?**

Test-statistic

- Same logic

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}}$$

(but variance explained by B, C... removed first)

“After C takes its share of variance, does A still explain anything?”

ANCOVA

- **Use Case:** Comparing group means while statistically controlling for a continuous variable.
- **As a Linear Model:**
 - $Y = b_0 + b_1X_1 + b_2C + \varepsilon$
 - where X's are dummy coded
 - C is continuous covariate
 - NHST
 - Traditional Form:
 - H_{0_A} : No main effect of factor A (adjusted for covariate)
 - H_{0_C} : Covariate doesn't predict Y
 - lm() equivalent:
 - H_{0_A} : $b_1 = 0$
 - H_{0_C} : $b_2 = 0$

ANCOVA steps

1. Check assumptions
 - covariate and factor are independent
 - Homogeneity of regression slopes (no interaction between factor and covariate)
2. Run the model (2 F-tests)
3. Interpret covariate effect
 - If significant:
 - Interpretation: Covariate matters
 - Good. You're controlling for real variance.
 - If not:
 - Keep it anyway if theoretically justified (e.g., pre-registered)
 - Drop C -- run regular ANOVA instead

Interpretation: "Covariate doesn't explain outcome variance. Adjusting for it doesn't change group comparison."

A wrap

This semester

- We have covered
 - Statistical Concepts
 - Useful tools

This semester

- Statistics
 - Modeling, data generating processes
 - Exploratory Data analysis
 - Descriptive stats, visualization
 - Null Hypothesis Significance Tests, p-values
 - Sampling Distributions, Estimation and Confidence Intervals
 - Effect size, power
 - Checking assumptions
 - Basic Model Selection
 - Regression (unifying framework)
 - Continuous predictors (correlation, R-squared, etc.)
 - Categorical predictors – comparing means (t tests, Anovas, F-statistic, etc.)
 - Multiple predictors (cursory)

This semester

- Deployment / Practical tools
 - Base R
 - Tidyverse
 - Data Wrangling
 - GGplot
 - R-markdown
 - Papaja
 - Github

This semester

- Hands-on work
 - Assignments
 - Lab
 - Project

Aside: Project

- Guidelines
 - See earlier slides from class for tldr
 - <https://princeton.instructure.com/courses/18979/files/4156597?wrap=1>
 - More comprehensive :
 - <https://docs.google.com/document/d/1sw534y3XU7reNnp1dK0bY9ATjQwjLk3Z2KP3Kh-i58s/edit?usp=sharing>
- If there is any contradiction between what we've discussed (1:1) and what's in the general guidelines, the former applies.

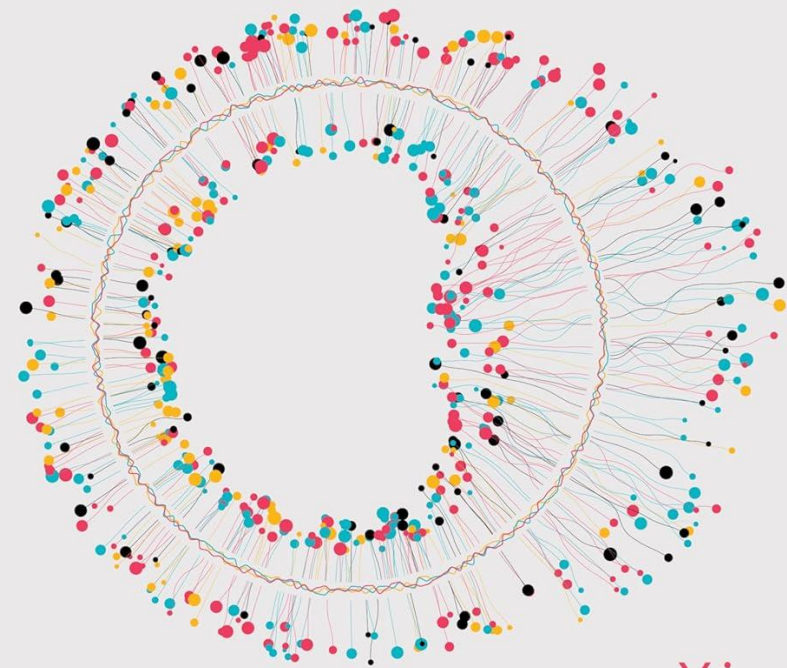
But please still get in touch with if there is still any confusion!

Now there are even more types of questions (Next Semester)

- **Mediation:** Does A affect Y *through* M?
- **Multilevel / Mixed models:** When effects vary across clusters — people, schools, time.
- **Growth curve modeling:** Do trajectories differ across groups?
- **Bayesian analysis:** using prior knowledge in analyses, different interpretation of probability
- **Maybe**
 - **Survival analysis:** When does an event happen?
 - **SEM (Structural Equation Modeling)**
 - Multiple pathways at once.

I'll be providing copies of chapters where necessary. No need to buy.

Categorical Data Analysis and Multilevel Modeling Using R



Xing Liu



Statistical Rethinking: A Bayesian Course with Examples in R and Stan

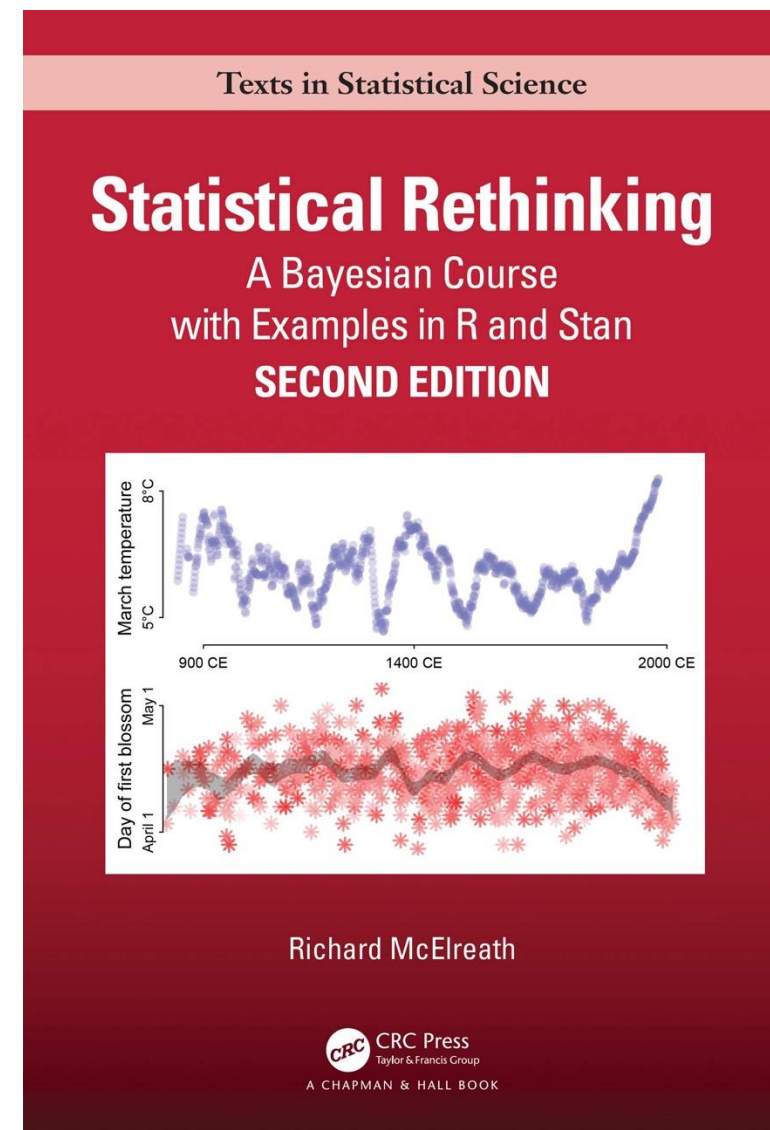
Richard McElreath

★★★★★ 4.71

480 ratings · 49 reviews

Code translations:

- R rethinking package: [github](#)
- R code: [file](#)
- Tidyverse and brms
 - Vincent Arel-Bundock [translation](#)
 - Solomon Kurz [translation](#)
- Python
 - PyMC3 [translation](#)
 - NumPyro translations: [number1](#) [number2](#)
 - TensorFlow [translation](#)
 - Pyro/Pytorch [translation](#)
 - PyMC 2023 examples [translation](#)
 - PyMC5 2023 examples [translation](#)
- Julia
 - Rethinking Julia [translation](#)
 - rethinking-2ed-julia [translation](#)
- Other
 - R-INLA [translation](#)



Spring 2026 – PSY 504

- GLM
- Intro to Bayesian Stats
- Mixed Models (Frequentist and Bayesian)
- Moderation & Mediation
- Miscellaneous methods

Spring 2026 - PSY 505

- Good companion to PSY 504, and complementary
- Seminar Series – External Speakers (6-7)
 - Plus tutorial or workshop
 - Project Runthrough – An internal speaker or two

Upcoming Events



FEB
3

**Tracy Sweet -
Associate Professor,
University of
Maryland**

Location: A03 Princeton

Neuroscience Institute

Virtual Location: [Zoom](#)

Speaker

**QMMS Psychometric
Computation and
Simulation (PCS) Lab**

[University of Maryland](#)



FEB
17

**Michael Betancourt
- Chief Research
Scientist,
Symplectomorphic,
LLC**

Location: A03 Princeton

Neuroscience Institute

Virtual Location: [Zoom](#)

Speaker

Michael Betancourt



MAR
3

**George Kachergis -
Data Scientist,
TransUnion**

Location: A03 Princeton

Neuroscience Institute

Virtual Location: [Zoom](#)

Speaker

TransUnion

[Levante Project](#)



MAR
17

**Andrew R.A. Conway
- Head of
Department of
Psychology, New
Mexico State
University**

Location: A03 Princeton

Neuroscience Institute

Virtual Location: [Zoom](#)

Speaker

Andrew R.A. Conway

[New Mexico State](#)

[University](#)



MAR
24

**Russell Alan
Poldrack - Albert
Ray Lang Professor
of Psychology,
Stanford University**

Location: A03 Princeton

Neuroscience Institute

Virtual Location: [Zoom](#)

Speaker

Poldrack Lab

[Stanford University](#)



APR
7

**Shravan Vasishth -
Professor,
University of
Potsdam**

Location: A03 Princeton

Neuroscience Institute

Virtual Location: [Zoom](#)

Speaker

Shravan Vasishth

[University of Potsdam](#)