

# **PSY 503: Foundations of Statistical Methods in Psychological Science**

## **Interactions, Factorial Designs**

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# Dec 3rd

- Remaining project presentation
- Lecture – Interactions & Factorial ANOVA
  - Interactions (Two-way ANOVA)
  - Factorial ANOVA
  - A look at ANCOVA

# Interactions

# Questions we've asked so far

- **t-test (independent)**: Is there a difference between two groups?
- **t-test (paired)**: Is there a difference between two conditions? (same people)
- **One-way ANOVA (between)**: Is there a difference somewhere among k groups?
- **One-way ANOVA (within / repeated measures)**: Is there a difference somewhere among k conditions? (same people)

*All are variants of “Is there an effect that exists?”*

# But there are often other questions

- Does the effect of A depend on B?

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- Does the effect of A depend on B?
- Examples
  - Does the effect of distraction on memory depend on age?
  - Does the bystander effect depend on group size?
  - Does stereotype threat depend on task difficulty?
  - Does parenting style affect outcomes differently for boys vs girls?
  - Does CBT work better for anxiety vs depression?
  - ...

# Does the effect of A depend on B?

- Equivalent to:
  - "**Is there an interaction between A and B?**"
    - Are A and B acting alone?  
Or are they working together?
    - "**Does B moderate the relationship between A and the outcome/DV?**"
- Depends on there being more than 1 (predictor / X) variable.  
Answered by:
  - Two-way anova or higher (factorial ANOVA)
  - ANCOVA / regression with interaction
  - Regression

# Methods to assess interaction

- Y is continuous

A	B	Method
Categorical	Categorical	Factorial ANOVA
Categorical	Continuous	ANCOVA / regression with interaction
Continuous	Categorical	Moderated regression
Continuous	Continuous	Moderated regression

- Y is binary: logistic regression with interaction term
- Y is ordinal: ordinal regression with interaction term

# Visualizing interactions

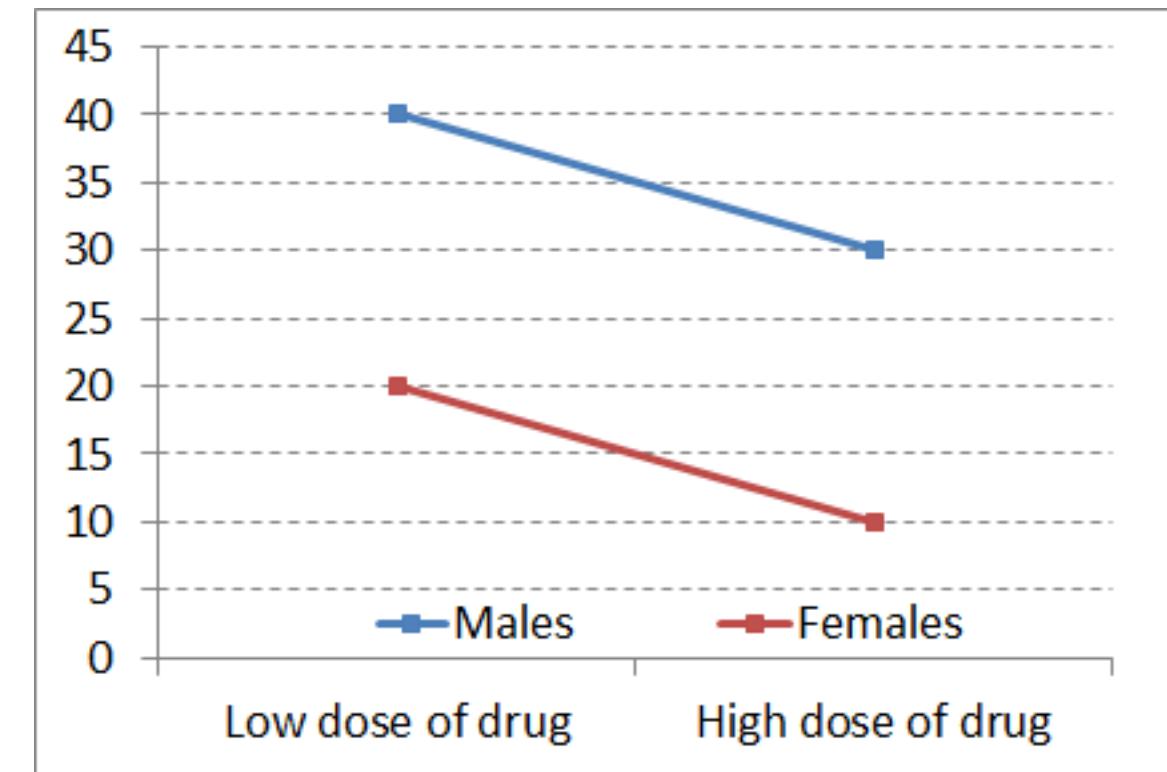
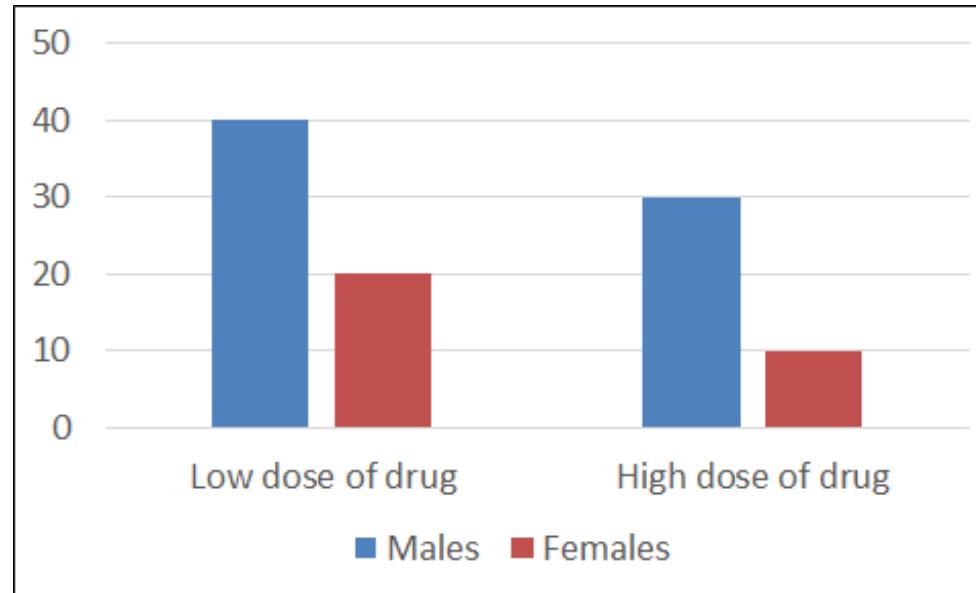
- **Golden Rule of Plots:** Look at the lines.
  - **Parallel Lines = No Interaction**
    - The effect of Variable A is the same, regardless of Variable B.
    - They are additive (independent).
  - **Non-Parallel Lines = Interaction**
    - The lines diverge, converge, or cross.
    - The effect of Variable A *changes* based on the level of Variable B.
    - This is the "It Depends" effect.

# Visualization: no-interaction

RQ: Understanding connection between :

Drug dosage & Gender

with respect to:  
Y (some score)

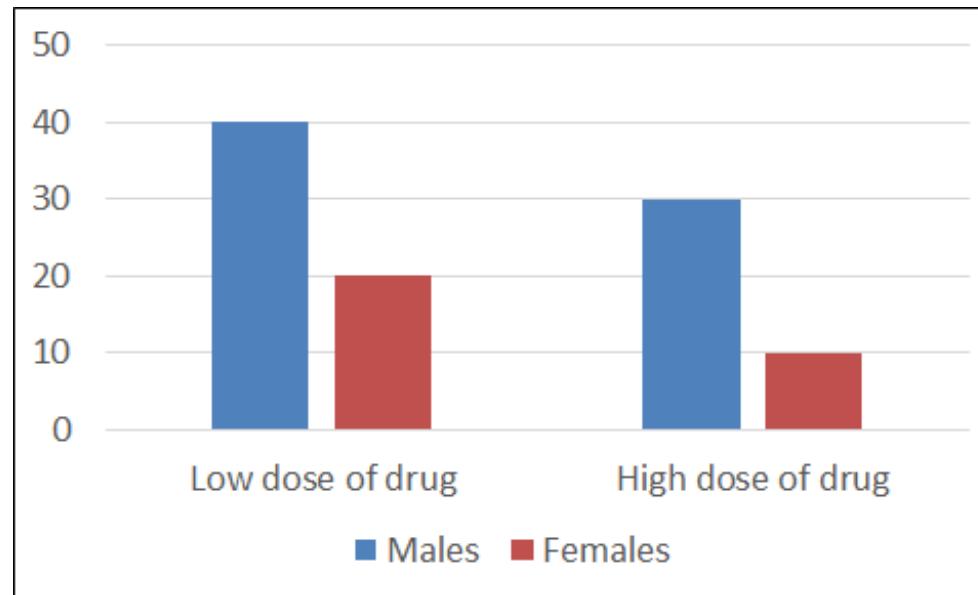


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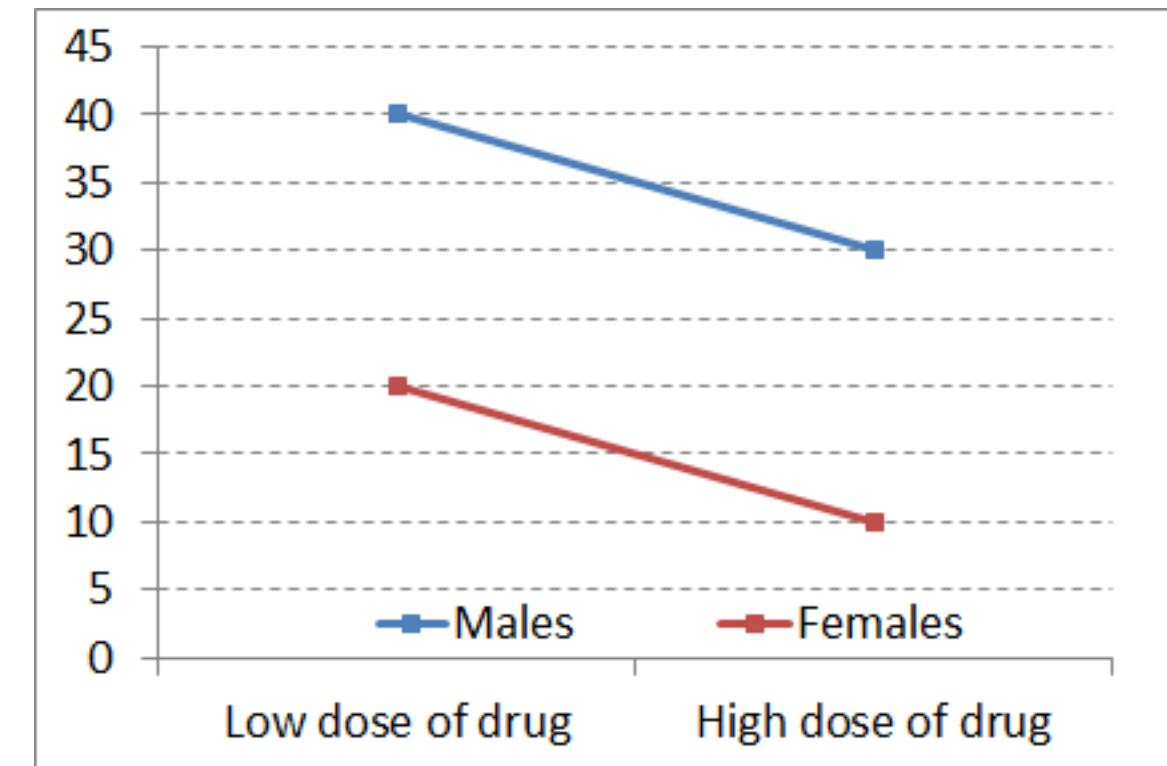
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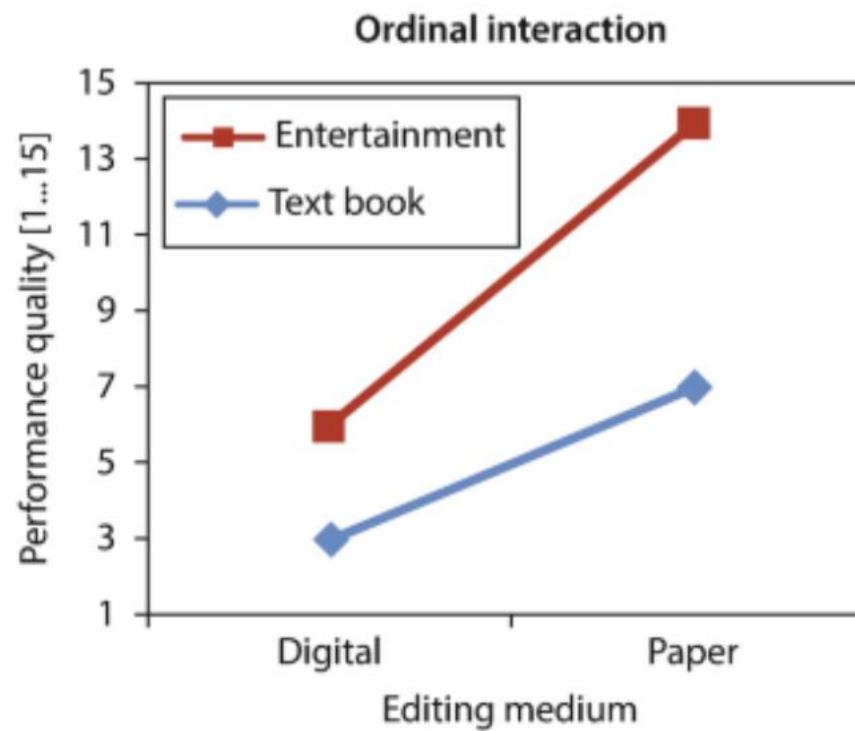


What's the effect of Gender, averaging over drug doses?

What's the effect of Dosage levels, averaging over Gender?



# Visualization: interaction

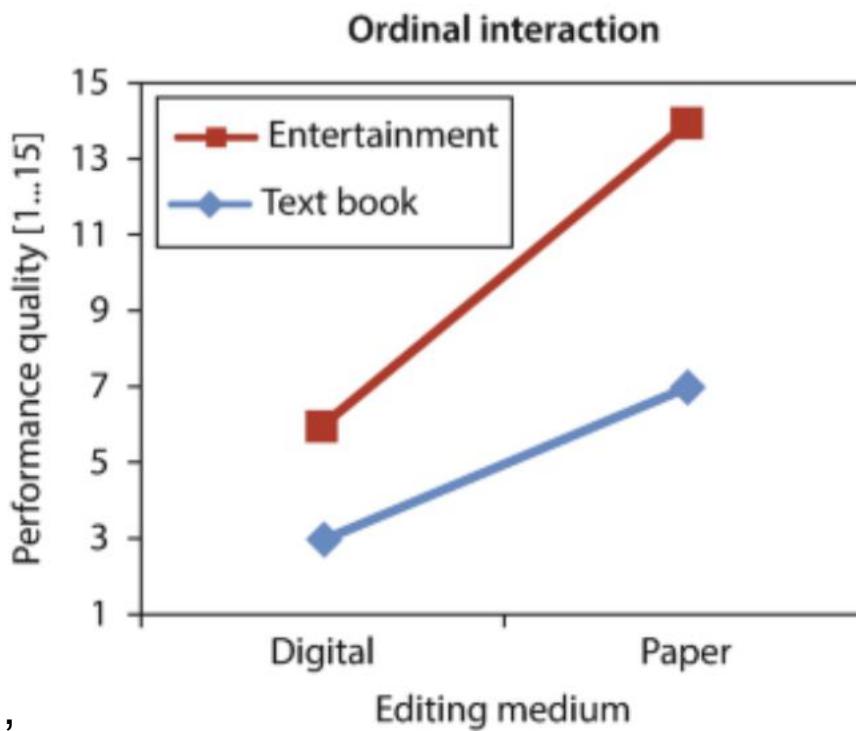


RQ: Understanding connection  
between :

editing medium & content type

with respect to:  
Y (performance quality)

# Visualization: interaction

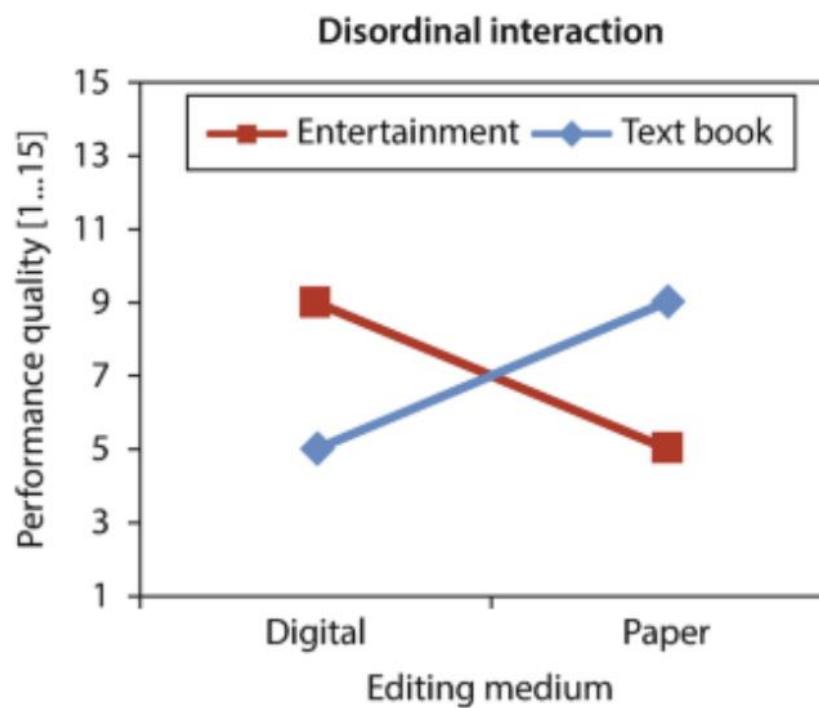


What's the effect of Editing medium,  
averaging over Content Type?

What's the effect of Content Type,  
averaging over Editing mediums?

RQ: Understanding connection  
between :  
editing medium & content type  
with respect to:  
Y (performance quality)

# Visualization: interaction



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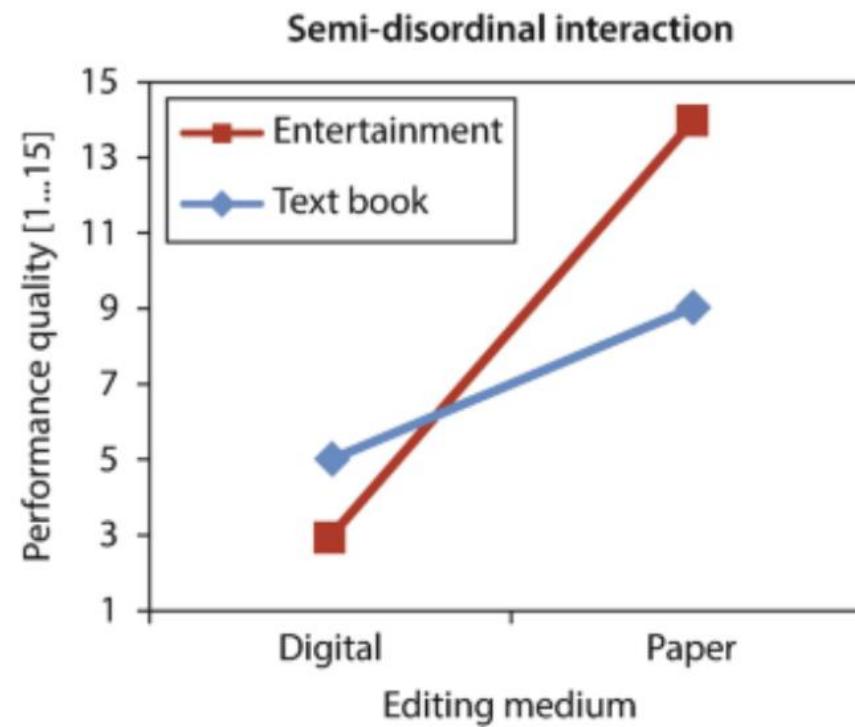
# How Interactions Affect Main Effects

<b>Interaction Type</b>	<b>Visual Pattern</b>	<b>Main Effects</b>	<b>Interpretation</b>
None	Parallel lines	Clean	"A is better."
Ordinal	Diverge, no cross	Incomplete	"A is better—depends how much."
Disordinal	Lines cross	Misleading	"A is better for some, worse for others."

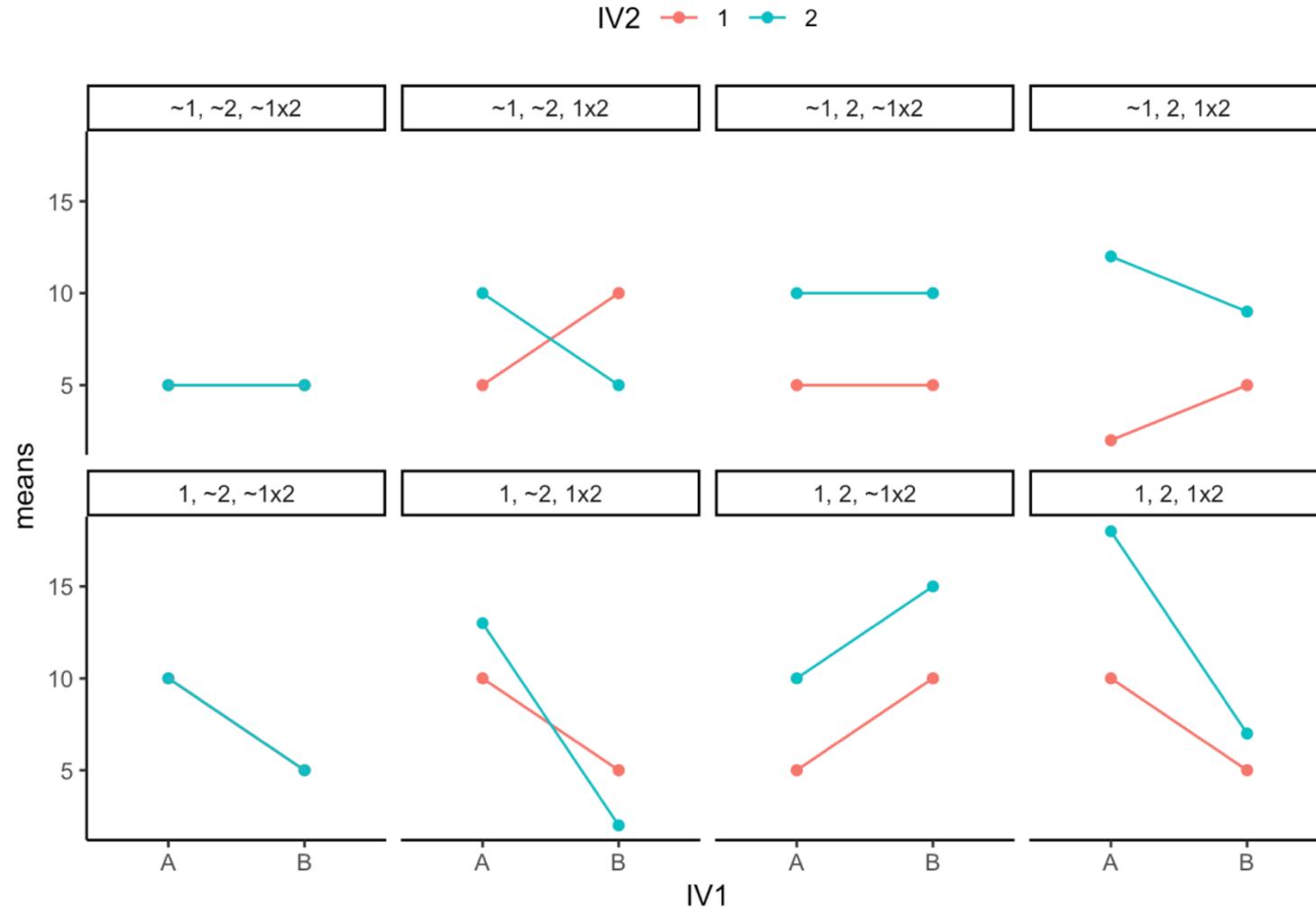
# How Interactions Affect Main Effects

<b>Interaction Type</b>	<b>Visual Pattern</b>	<b>Main Effects</b>	<b>Interpretation</b>	<b>Reporting Strategy</b>
None	Parallel lines	Clean	"A is better."	Report main effects. Done.
Ordinal	Diverge, no cross	Incomplete	"A is better—depends how much."	Report main effects + note interaction. Or: simple effects.
Disordinal	Lines cross	Misleading	"A is better for some, worse for others."	Skip main effects. Report simple effects only.

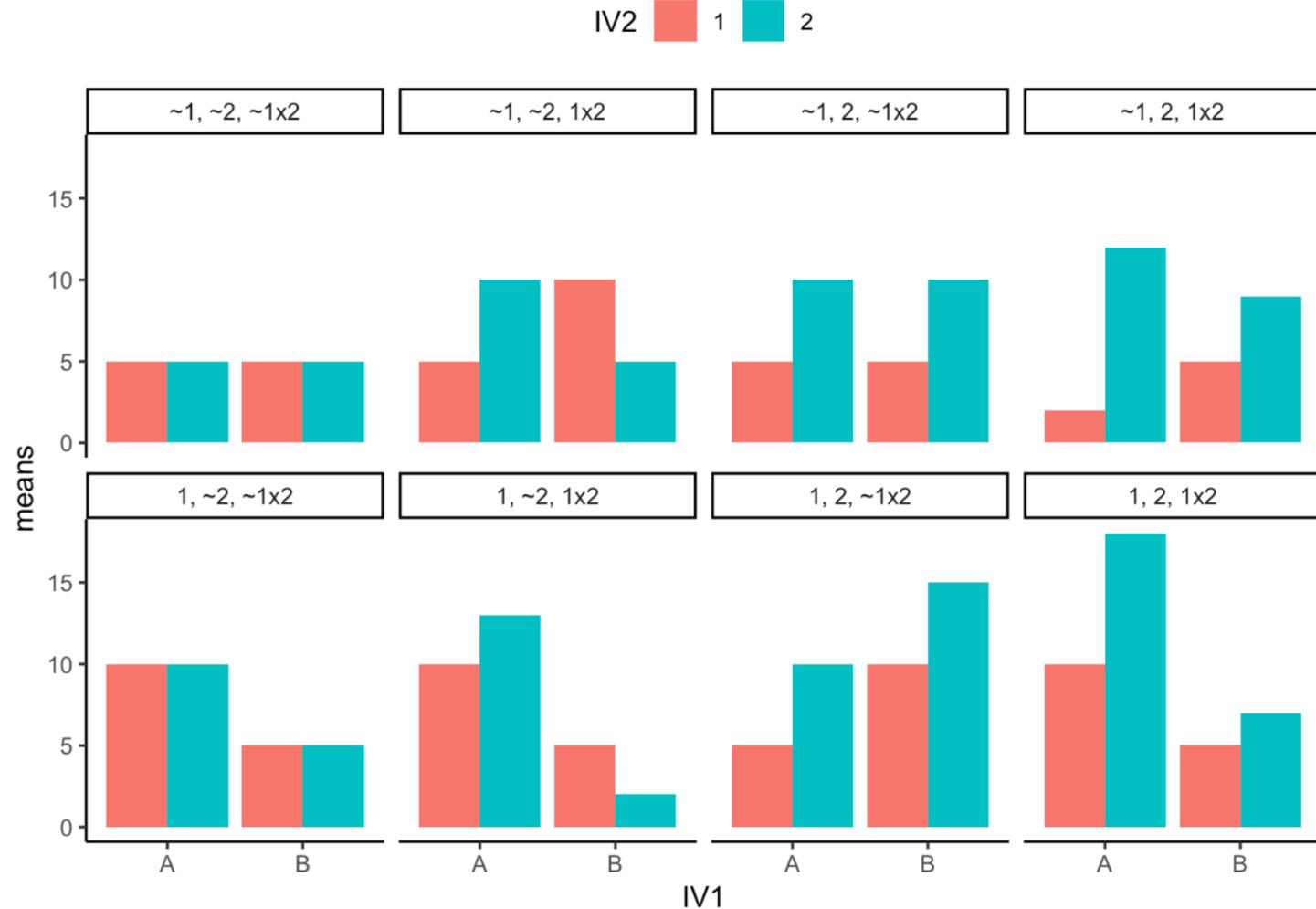
# Visualization: interaction



# Visualization: Possible outcomes for a 2x2 design



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# Possible outcomes for a 2x2 design

1. no IV1 main effect, no IV2 main effect, no interaction
2. IV1 main effect, no IV2 main effect, no interaction
3. IV1 main effect, no IV2 main effect, interaction
4. IV1 main effect, IV2 main effect, no interaction
5. IV1 main effect, IV2 main effect, interaction
6. no IV1 main effect, IV2 main effect, no interaction
7. no IV1 main effect, IV2 main effect, interaction
8. no IV1 main effect, no IV2 main effect, interaction

# Jargon

- "Marginal" = lives in the margins.
  - Collapse across one factor → get marginal mean for the other.
- Main effects compare marginal means.

	LOW DOSE	HIGH DOSE	MARGIN
Male	40	5	<b>22.5</b>
Female	15	30	<b>22.5</b>
<b>Margin</b>	<b>27.5</b>	<b>17.5</b>	<b>Marginal Means</b>

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Main effect of  
Dosage =  $\Delta = 10$

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**Simple effect** of dosage = simple effects =  
–35 and +15

(Stay in one row or column)

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This  $\Delta$  = interaction effect

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# Factorial designs

- Specification
  - Number of factors
  - Number of levels
- $2 \times 3$  design

	B1	B2	B3
A1	.	.	.
A2	.	.	.

- Example: Gender (2)  $\times$  Dose (low, medium, high)

# Factorial designs

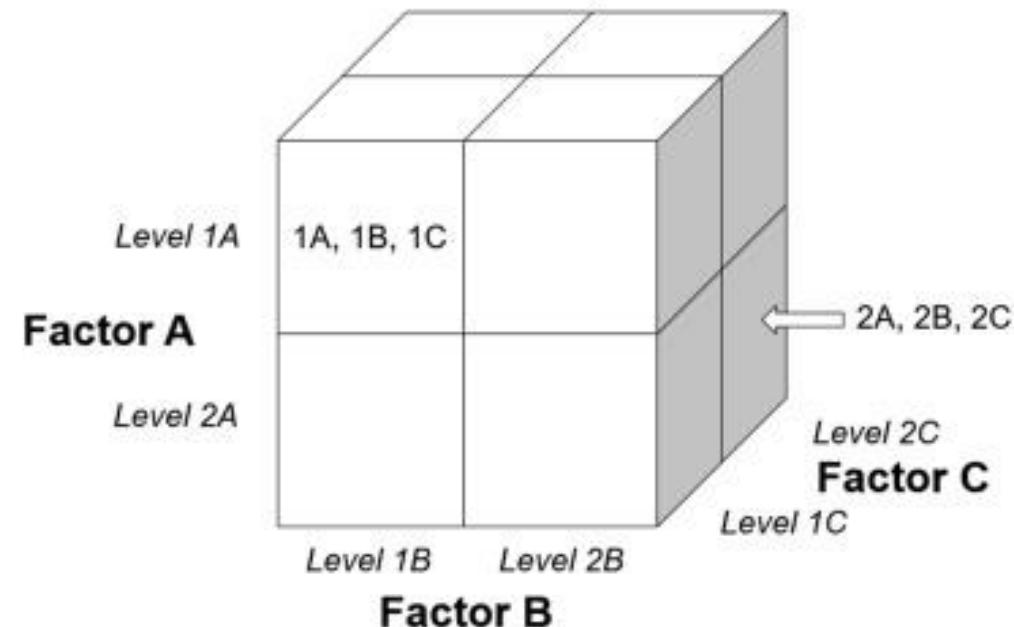
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## $2 \times 2 \times 2$ design

Possible allocations in a  $2 \times 2 \times 2$  factorial design in cubic form.



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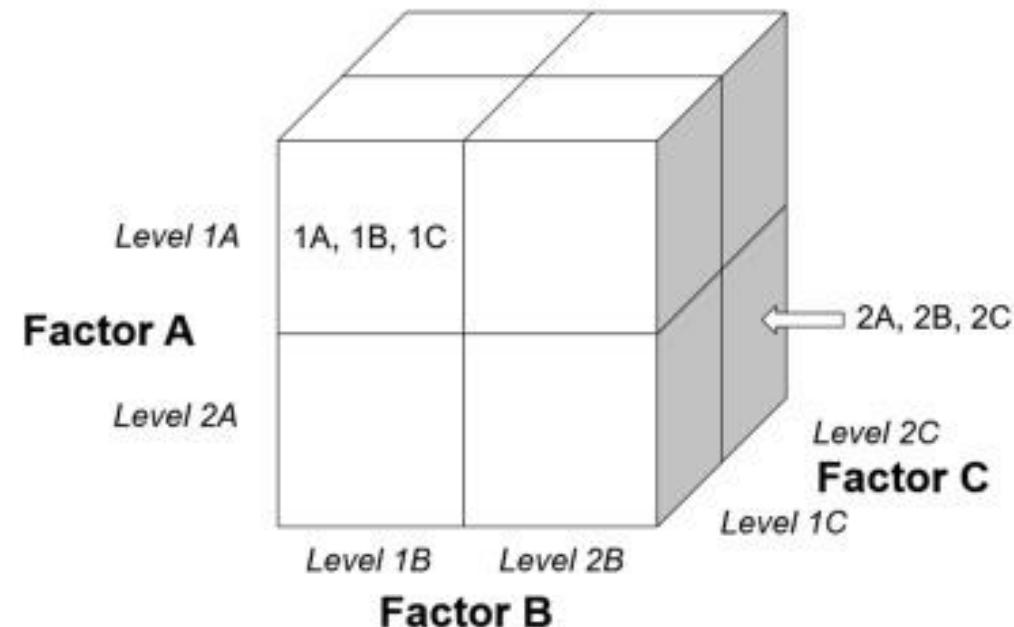
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## $2 \times 2 \times 2$ design

Possible allocations in a  $2 \times 2 \times 2$  factorial design in cubic form.



- 3 main effects (A, B, C)
- 3 two-way interactions ( $A \times B$ ,  $A \times C$ ,  $B \times C$ )
- 1 three-way interaction ( $A \times B \times C$ )
- Gets complicated fast

# Fully Factorial

- A design is fully factorial when the levels of each variable are fully crossed with the levels of every other variable.
  - e.g., the levels of IV1 are manipulated across the levels of IV2 (in a 2x2 design)
- in other words, there are no missing cells

# Two-way ANOVA

# Two-way ANOVA

- Two factors. One outcome.
- Questions:
  - Main effect of A?
  - Main effect of B?
  - Interaction: Does A depend on B?

# Two-way ANOVA

You know these:

- Two factors – A and B
  - A has i levels
  - B has j levels
- that result in  $i \times j$  cells
  - Cell means
    - $\bar{X}_{11}, \bar{X}_{12}, \dots, \bar{X}_{ij}$
  - Cell sizes
    - $n_{11}, n_{12}, \dots, n_{ij}$
  - Within group variation

Question: Are there main effects?  
Is there an interaction?

Test-statistic

- Earlier

$$z = (\bar{X} - \mu) / (\sigma/\sqrt{N})$$

$$z = (\bar{X} - \mu) / SE$$

$$t = (\bar{X} - \mu) / (s/\sqrt{N})$$

Now, comparing k means

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}}$$

(comparing k means)

“Does Factor A matter? Factor B?  
*Do they interact?*”

Between group variation /  
**variation within groups**

# Two-way ANOVA

- **Use Case:** Comparing means across groups while considering two different categorical factors and their interaction
- **As a Linear Model:**
  - $Y = b_0 + b_1X_1 + b_2X_2 + b_3(X_1 X_2) + \varepsilon$ 
    - where X's are dummy coded
  - NHST
    - Traditional Form:
      - $H_0\_A$ : No main effect of factor A
      - $H_0\_B$ : No main effect of factor B
      - $H_0\_AxB$  : No interaction between A and B
    - lm() equivalent:
      - $H_0\_A$ :  $b_1 = 0$
      - $H_0\_B$  :  $b_2 = 0$
      - $H_0\_AxB$ :  $b_3 = 0$

Each gets its own F-ratio in the ANOVA table.

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# Two-way ANOVA steps

1. Run the model (3 F-tests)
2. Check if the interaction term is significant
  - If Yes:
    - don't interpret main effects
    - run simple effects (i.e. not collapsing => subsetting)
  - If no interaction
    - interpret main effects
      - A is significant → Post-hoc on A (to compare levels of A)
      - B significant? → Post-hoc on B

# Implementation in R (Example)

```
```{r}
NR_D <- c(3,2,4,1)
NR_ND <- c(8,7,8,6)
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|                    | Df   | Sum Sq | Mean Sq | F value | Pr(>F)   |     |
|--------------------|------|--------|---------|---------|----------|-----|
| Reward             | 1    | 25.00  | 25.00   | 19.355  | 0.000866 | *** |
| Distraction        | 1    | 49.00  | 49.00   | 37.935  | 4.88e-05 | *** |
| Reward:Distraction | 1    | 6.25   | 6.25    | 4.839   | 0.048162 | *   |
| Residuals          | 12   | 15.50  | 1.29    |         |          |     |
|                    | ---  |        |         |         |          |     |
| Signif. codes:     | 0    | ***    | 0.001   | **      | 0.01     | **  |
|                    | 0.05 | .      | 0.1     | '       | 1        |     |

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df<-data.frame(Reward,Distraction,DV)
```

```

```
```{r}
summary(aov(DV~Reward*Distraction,df))
```

```

↑  
↓ equivalent

```
```{r}
summary(aov(DV~Reward + Distraction + Reward:Distraction,df))
```

```

|    | Reward   | Distraction   | DV |
|----|----------|---------------|----|
| 1  | NoReward | Distraction   | 3  |
| 2  | NoReward | Distraction   | 2  |
| 3  | NoReward | Distraction   | 4  |
| 4  | NoReward | Distraction   | 1  |
| 5  | NoReward | NoDistraction | 8  |
| 6  | NoReward | NoDistraction | 7  |
| 7  | NoReward | NoDistraction | 8  |
| 8  | NoReward | NoDistraction | 6  |
| 9  | Reward   | Distraction   | 6  |
| 10 | Reward   | Distraction   | 7  |
| 11 | Reward   | Distraction   | 5  |
| 12 | Reward   | Distraction   | 7  |
| 13 | Reward   | NoDistraction | 9  |
| 14 | Reward   | NoDistraction | 8  |
| 15 | Reward   | NoDistraction | 7  |
| 16 | Reward   | NoDistraction | 10 |

|                    | Df   | Sum Sq | Mean Sq | F value | Pr(>F)   |     |
|--------------------|------|--------|---------|---------|----------|-----|
| Reward             | 1    | 25.00  | 25.00   | 19.355  | 0.000866 | *** |
| Distraction        | 1    | 49.00  | 49.00   | 37.935  | 4.88e-05 | *** |
| Reward:Distraction | 1    | 6.25   | 6.25    | 4.839   | 0.048162 | *   |
| Residuals          | 12   | 15.50  | 1.29    |         |          |     |
|                    | ---  |        |         |         |          |     |
| Signif. codes:     | 0    | ***    | 0.001   | **      | 0.01     | *   |
|                    | 0.05 | .      | 0.1     | '       | 1        |     |

# Note

- Which to use when?
  - \* -- almost always. You want to test the interaction.
  - + -- rare. Only if you have theoretical reason to assume no interaction (unusual).

| Formula | What it fits                                                 |
|---------|--------------------------------------------------------------|
| $A * B$ | $A + B + A:B$ (full factorial)<br>main effects + interaction |
| $A + B$ | main effects only                                            |
| $A : B$ | interaction only                                             |

# Factorial ANOVA

- Two ***or more*** factors. One outcome.
- Questions:
  - Main effect of A?
  - Main effect of B?
  - Main effect of C?
  - $A \times B$  interaction?
  - $A \times C$  interaction?
  - $B \times C$  interaction?
  - $A \times B \times C$  interaction?
  - ..

# Factorial ANOVA

You know these:

- **k** factors – A, B, C,....
  - A has a levels
  - B has b levels
  - C has c levels
  - ..
  - Design:  $a \times b \times c \dots$  (k terms)
- that result in  $a^*b^*c^*$ ... cells
  - Cell means
  - Cell sizes
  - Within cell variation

**Question: Are there main effects?  
Is there an interaction?**

Test-statistic

- Same logic

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}}$$

(one F for each Main effect,  
one F for each interaction)

*Between group variation /  
variation within groups*

**“Question: Are there main  
effects? Do factors interact? Do  
interactions interact?”**

| Source | df            | SS              | MSE                                                    | F                                  | p                              |
|--------|---------------|-----------------|--------------------------------------------------------|------------------------------------|--------------------------------|
| A      | $a - 1$       | $SS_{EffectA}$  | $MS_{EffectA} = \frac{SS_{EffectA}}{a - 1}$            | $\frac{MS_{EffectA}}{MS_{Error}}$  | Calculated from F-distribution |
| B      | $b - 1$       | $SS_{EffectB}$  | $MS_{EffectB} = \frac{SS_{EffectB}}{b - 1}$            | $\frac{MS_{EffectB}}{MS_{Error}}$  | Calculated from F-distribution |
| A*B    | $(a-1)*(b-1)$ | $SS_{EffectAB}$ | $MS_{EffectAB} = \frac{SS_{EffectAB}}{(a - 1)(b - 1)}$ | $\frac{MS_{EffectAB}}{MS_{Error}}$ | Calculated from F-distribution |
| Error  | $N - (a * b)$ | $SS_{Error}$    | $MS_{Error} = \frac{SS_{Error}}{N - (a * b)}$          |                                    |                                |

a = number of groups in A; b = number of groups in B; N = number of subjects

$$SS_{Total} = SS_{EffectA} + SS_{EffectB} + SS_{EffectAB} + SS_{Error}$$

$$SS_{Effect} = \sum_{i=1}^k n_i (X_i - \bar{X})^2$$

$$SS_{interaction} = n \text{ in cells} \sum_{i=1}^n (X_{ij} - X_i - X_j + \bar{X})^2$$

Notes:  $\bar{X}$  = Grand Mean,  $X_i$  = condition mean (SS effect)

# ANCOVA (controlling for covariates)

- **ANOVA** : Analysis of Variance
- **ANCOVA**: Analysis of Covariance (referring to the covariate)
- Example questions:
  - Do CBT vs medication differ in depression scores — controlling for baseline severity?
  - Do sleep conditions differ in memory recall — controlling for IQ?
  - Do exercise interventions differ in anxiety — controlling for baseline fitness?
  - ...
- "**Do groups differ in Y — after removing variance due to C?**"

# ANCOVA

You know these:

- One factors – A (with i levels)
- One or more continuous covariate – B, C..
- Cell means
- Within group variation

Question: Does A matter **after accounting for the covariate?**

Test-statistic

- Same logic

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}}$$

*(but variance explained by B, C... removed first)*

**“After C takes its share of variance, does A still explain anything?”**

# ANCOVA

- **Use Case:** Comparing group means while statistically controlling for a continuous variable.
- **As a Linear Model:**
  - $Y = b_0 + b_1X_1 + b_2C + \varepsilon$ 
    - where X's are dummy coded
    - C is continuous covariate
  - NHST
    - Traditional Form:
      - $H_0 - A$ : No main effect of factor A (adjusted for covariate)
      - $H_0 - C$ : Covariate doesn't predict Y
    - lm() equivalent:
      - $H_0 - A$ :  $b_1 = 0$
      - $H_0 - C$ :  $b_2 = 0$

# ANCOVA steps

1. Check assumptions
    - covariate and factor are independent
    - Homogeneity of regression slopes (no interaction between factor and covariate)
  2. Run the model (2 F-tests)
  3. Interpret covariate effect
    - If significant:
      - Interpretation: Covariate matters
      - Good. You're controlling for real variance.
    - If not:
      - Keep it anyway if theoretically justified (e.g., pre-registered)
      - Drop C -- run regular ANOVA instead
- Interpretation: “Covariate doesn't explain outcome variance. Adjusting for it doesn't change group comparison.

A wrap

# This semester

- We have covered
  - Statistical Concepts
  - Useful tools

# This semester

- Statistics
  - Modeling, data generating processes
  - Exploratory Data analysis
    - Descriptive stats, visualization
  - Null Hypothesis Significance Tests, p-values
  - Sampling Distributions, Estimation and Confidence Intervals
  - Effect size, power
  - Checking assumptions
  - Basic Model Selection
  - Regression (unifying framework)
    - Continuous predictors (correlation, R-squared, etc.)
    - Categorical predictors – comparing means (t tests, Anovas, F-statistic, etc.)
    - Multiple predictors (cursory)

# This semester

- Deployment / Practical tools
  - Base R
  - Tidyverse
    - Data Wrangling
    - GGplot
  - R-markdown
  - Papaja
  - Github

# This semester

- Hands-on work
  - Assignments
  - Lab
  - Project

# Aside: Project

- Guidelines
  - See earlier slides from class for tldr
    - <https://princeton.instructure.com/courses/18979/files/4156597?wrap=1>
  - More comprehensive :
    - <https://docs.google.com/document/d/1sw534y3XU7reNnp1dK0bY9ATjQwjLk3Z2KP3Kh-i58s/edit?usp=sharing>
- If there is any contradiction between what we've discussed (1:1) and what's in the general guidelines, the former applies.

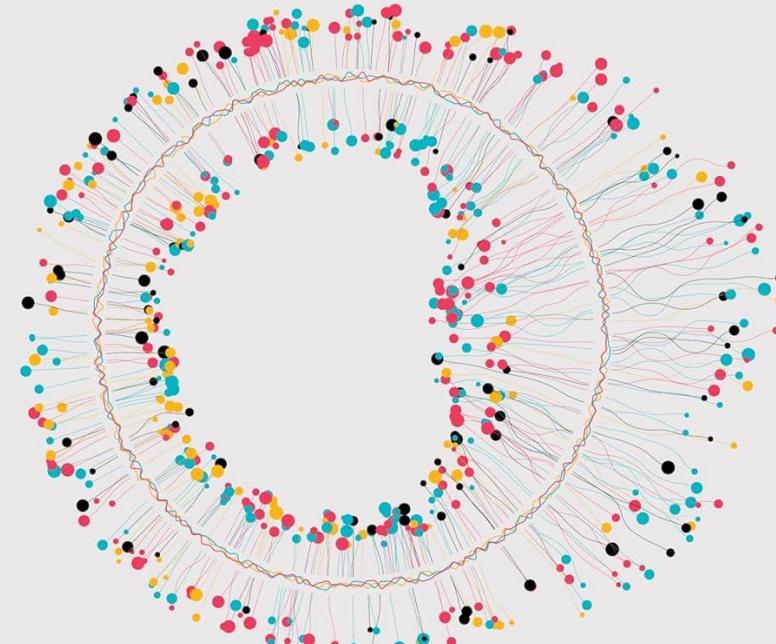
But please still get in touch with if there is still any confusion!

# Now there are even more types of questions (Next Semester)

- **Mediation:** Does A affect Y through M?
- **Multilevel / Mixed models:** When effects vary across clusters — people, schools, time.
- **Growth curve modeling:** Do trajectories differ across groups?
- **Bayesian analysis:** using prior knowledge in analyses, different interpretation of probability
- **Maybe**
  - **Survival analysis:** When does an event happen?
  - **SEM (Structural Equation Modeling)**
  - Multiple pathways at once.

I'll be providing copies of chapters where necessary. No need to buy.

# Categorical Data Analysis and Multilevel Modeling Using R



Xing Liu



# Statistical Rethinking: A Bayesian Course with Examples in R and Stan

Richard McElreath

★★★★★ 4.71

480 ratings · 49 reviews

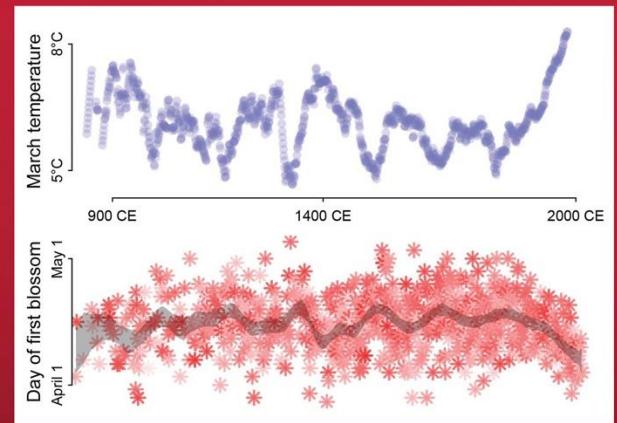
Code translations:

- R rethinking package: [github](#)
- R code: [file](#)
- Tidyverse and brms
  - Vincent Arel-Bundock [translation](#)
  - Solomon Kurz [translation](#)
- Python
  - PyMC3 [translation](#)
  - NumPyro translations: [number1](#) [number2](#)
  - TensorFlow [translation](#)
  - Pyro/Pytorch [translation](#)
  - PyMC 2023 examples [translation](#)
  - PyMC5 2023 examples [translation](#)
- Julia
  - Rethinking Julia [translation](#)
  - rethinking-2ed-julia [translation](#)
- Other
  - R-INLA [translation](#)

Texts in Statistical Science

# Statistical Rethinking

A Bayesian Course  
with Examples in R and Stan  
**SECOND EDITION**



Richard McElreath

 CRC Press  
Taylor & Francis Group  
A CHAPMAN & HALL BOOK

# Spring 2026 – PSY 504

- GLM
- Intro to Bayesian Stats
- Mixed Models (Frequentist and Bayesian)
- Moderation & Mediation
- Miscellaneous methods

# Spring 2026 - PSY 505

- Good companion to PSY 504, and complementary
- Seminar Series – External Speakers (6-7)
  - Plus tutorial or workshop
  - Project Runthrough – An internal speaker or two

## Upcoming Events



FEB  
**3** **Tracy Sweet -**  
**Associate Professor,**  
**University of**  
**Maryland**  
  
Location: A03 Princeton  
Neuroscience Institute  
Virtual Location: [Zoom](#)  
Speaker  
[QMMS Psychometric Computation and Simulation \(PCS\) Lab](#)  
[University of Maryland](#)



FEB  
**17** **Michael Betancourt -**  
**Chief Research**  
**Scientist,**  
**Symplectomorphic,**  
**LLC**  
  
Location: A03 Princeton  
Neuroscience Institute  
Virtual Location: [Zoom](#)  
Speaker  
[Michael Betancourt](#)



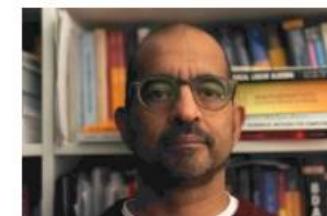
MAR  
**3** **George Kachergis -**  
**Data Scientist,**  
**TransUnion**  
  
Location: A03 Princeton  
Neuroscience Institute  
Virtual Location: [Zoom](#)  
Speaker  
[TransUnion](#)  
[Levante Project](#)



MAR  
**17** **Andrew R.A Conway -**  
**Head of**  
**Department of**  
**Psychology, New**  
**Mexico State**  
**University**  
  
Location: A03 Princeton  
Neuroscience Institute  
Virtual Location: [Zoom](#)  
Speaker  
[Andrew R.A Conway](#)  
[New Mexico State University](#)



MAR  
**24** **Russell Alan Poldrack -**  
**Albert Ray Lang Professor**  
**of Psychology,**  
**Stanford University**  
  
Location: A03 Princeton  
Neuroscience Institute  
Virtual Location: [Zoom](#)  
Speaker  
[Poldrack Lab](#)  
[Stanford University](#)



APR  
**7** **Shravan Vasishth -**  
**Professor,**  
**University of**  
**Potsdam**  
  
Location: A03 Princeton  
Neuroscience Institute  
Virtual Location: [Zoom](#)  
Speaker  
[Shravan Vasishth](#)  
[University of Potsdam](#)