PSY 503: Foundations of Statistical Methods in Psychological Science

Common statistical tests, connections to lm()

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Previously

- There is a commonality to statistical models
  - We are predicting an outcome
  - We have one or more measured variables

- Linear model
  - The form
    - $\text{outcome}_i = (\text{model}_i) + \text{error}_i$
    - $\widehat{\text{outcome}}_i = \widehat{b}_0 + \widehat{b}_1 \cdot \text{predictor}_i + \cdots + \text{error}_i$

  - Familiar models are a variant of a linear model

Previously

- What we've learned:
  - Y: an outcome
  - b_0: Intercept (baseline)
  - b's: Slopes (effects)
  - X's: Our predictors
  - ε: The stuff we can't explain (residuals)

$$\text{outcome}_i = (\text{model}_i) + \text{error}_i$$

$$Y = b_0 + b_1X_1 + b_2X_2 + \ldots + \varepsilon$$

The Regression Family Tree

$$\text{outcome}_i = (\text{model}_i) + \text{error}_i$$

$$Y = b_0 + b_1X_1 + b_2X_2 + \dots + \varepsilon$$

- Multivariate Regression
  - Multiple Y's (MANOVA, etc.)
- Univariate Regression
  - Multiple regression: $Y = b_0 + b_1X_1 + b_2X_2 + \dots + \varepsilon$
    - Multiple predictors
    - Each b tells us about unique contribution
  - Simple linear regression: $Y = b_0 + b_1X_1 + \varepsilon$
    - Single predictor
    - b_1 is related to Pearson's correlation (r)
  - Simplest model: $Y = b_0 + \varepsilon$
    - Just the intercept
    - Average with deviation

•Questions we can ask:
- Is b_0 different from some value?
- Is b_1 different from 0?
- How many predictors do we really need?

Comparing Two Means

Z-test

- You know these:
  - population mean – μ_0
  - population SD – σ
  - You have a sample
    - mean – $\bar{X}$
    - Sample size - N
- Question: Is $\bar{X}$ different from μ_0?

Z-test

You know these:

- population mean – μ_0
- population SD – σ
- You have a sample
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Question: Is $\bar{X}$ different from μ_0?

Remember z-score?

$z = (X - \mu) / \sigma$

Standardized a single observation

Test-statistic

- Deals with a sample

  $z = (\bar{X} - \mu) / (\sigma/\sqrt{N})$

  $z = (\bar{X} - \mu) / SE$

- "How many SEs is this sample from expected?"

Z-test

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- population mean – μ_0
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Observed difference / Typical variability

- "How many SEs is this sample from expected?"

Z-test

“one of the most useless tests in all of statistics: the z-test. Seriously – this test is almost never used in real life.” – Danielle Navarro

T-tests

- Three kinds of t-tests
  - One-sample
  - Two-sample
    (independent sample)
  - Paired-sample

T-tests

- Three kinds of t-tests
  - One-sample - compare one group to a constant
  - Two-sample (***independent*** sample) - compare two ***independent*** groups
  - Paired-sample - compare matched observations

One-sample t-test

- Do we differ from a known value?
  - Is the average height of the class different from 6ft?

One sample t-test

You know these:

- population mean – μ_0
- You have a sample
  - mean – $\bar{X}$
  - Sample size – N
  - Sample SD – **s**

Question: **Is $\bar{X}$ different from μ_0?**

Test-statistic

- Earlier

  $z = (\bar{X} - \mu) / (\sigma/\sqrt{N})$

  $z = (\bar{X} - \mu) / SE$

- Now, we are estimating σ
  - $t = (\bar{X} - \mu) / (s/\sqrt{N})$

- "How many SEs is this sample from expected?"

One sample t-test

You know these:

- population mean – μ_0
- You have a sample
  - mean – $\bar{X}$
  - Sample size – N
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  - $t = (\bar{X} - \mu) / (s/\sqrt{N})$

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Observed difference /
***Estimated** Typical variability*

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- Do we differ from a known value?
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z-test vs t-test

z-test: know population **SD**
t-test: estimate **SD** from sample

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CLT guarantees

- Means are normal.
- Proportions are normal (large N).

One-sample t-test

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test-statistic, sampling distributions

z-test: know population **SD**

$z = (\bar{X} - \mu) / (\sigma/\sqrt{N})$

→ use normal

t-test: estimate **SD** from sample

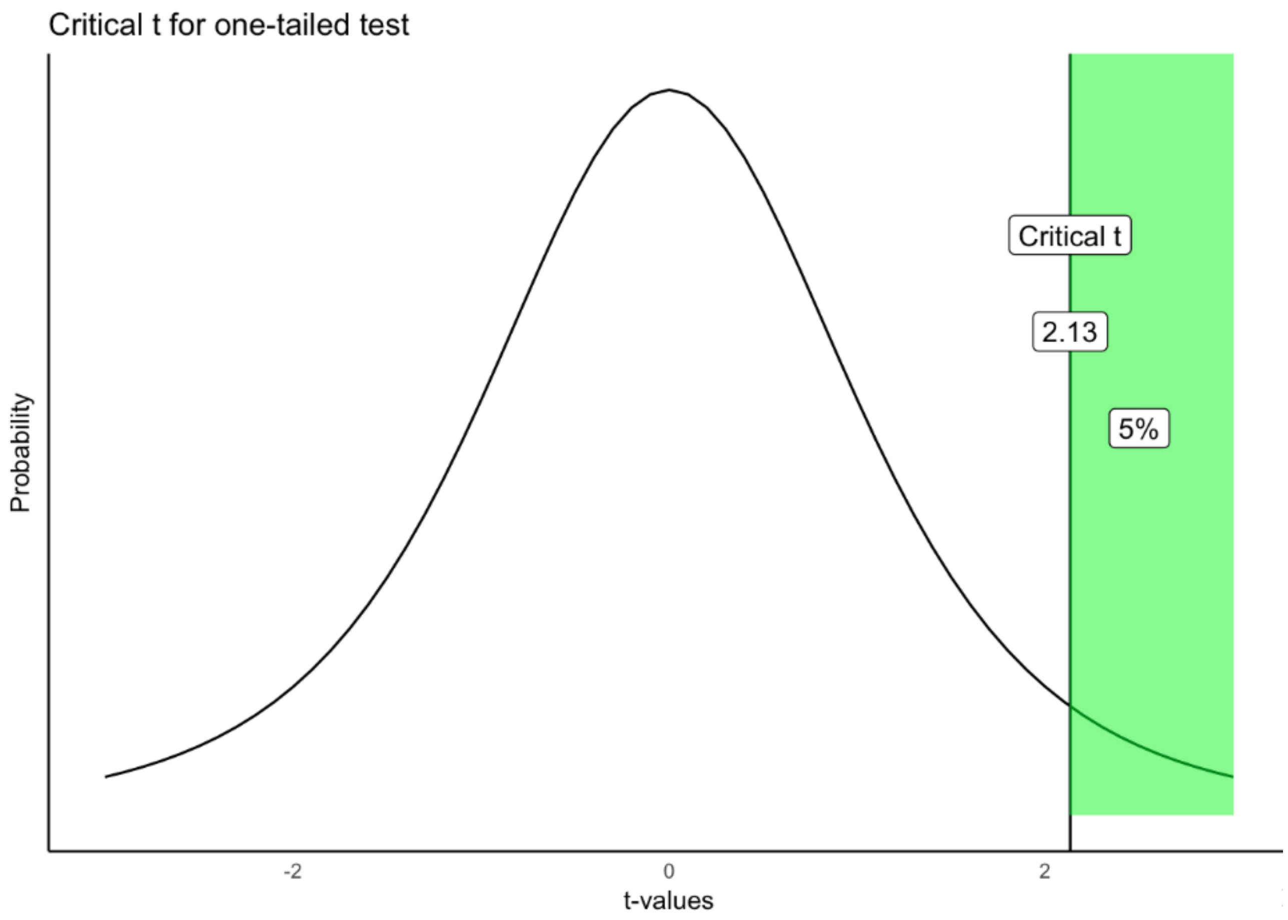
$t = (\bar{X} - \mu) / (\mathbf{s}/\sqrt{(N-1)})$

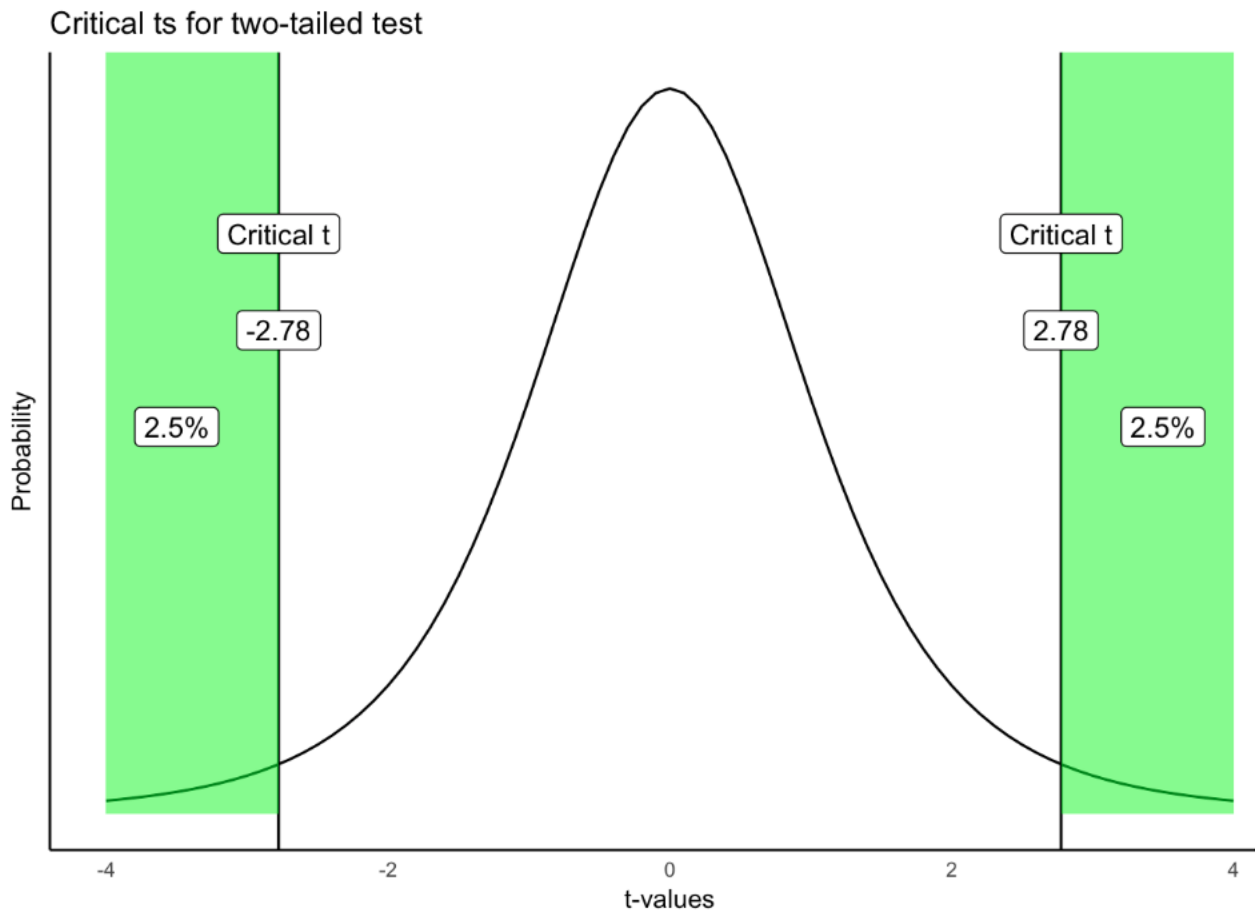
→ use t-distribution

T-distribution vs Gaussian

- A Shiny demo: https://shiny.abdn.ac.uk/Stats/apps/app_distn/

- Key differences
  - T-distribution has heavier tails
    - When n (or df) is small, the tail is heavier
    - As n increases, t-distribution resembles the Gaussian more
  - Implications?


Critical t for one-tailed test
Critical t
2.13
5%
Probability
-2
0
2
t-values


Critical ts for two-tailed test
Critical t
-2.78
2.5%
Critical t
2.78
2.5%
Probability
-4
-2
0
2
4
t-values

Sampling distributions and connections to Normal

- χ^2 – sum of squared normals.
- t – ratio of normal and $\sqrt{\chi^2}$
- F – ratio of two χ^2.

````
```{r}
set.seed(123)
data <- rnorm(30, mean=1, sd=2)

# One-sample t-test
t_model <- t.test(data, mu=0)
print(t_model)
```
````

```
        One Sample t-test

data:  data
t = 2.5286, df = 29, p-value = 0.01715
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.1731467 1.6384383
sample estimates:
mean of x
0.9057925
```

t-test and linear model

- **Use Case:** Comparing a single group's mean to a known value/reference point

- **As a linear model:**
  - $Y = b_0 + \varepsilon$
    - *Model without predictors*

  - NHST
    - Traditional form: $H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu$
    - lm() equivalent: $H_0: b_0 = \mu_0$ vs $H_1: b_0 \neq \mu_0$

But where is the t-statistic in lm()?

- Linear Model
  - $y = \beta_0 + \varepsilon$
  - Estimate: $b_0 = \bar{x}$
  - Standard Error: $SE(b_0) = s/\sqrt{n}$
  - t-statistic: $(b_0 - \beta_0)/(s/\sqrt{n})$
- T-test
  - β_0 in linear model = μ in t-test
  - Both estimate central tendency
  - Both use same standard error formula

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```{r}
lm_model<- lm(data ~ 1)  # intercept-only model
summary(lm_model)
```

```
Call:
lm(formula = data ~ 1)

Residuals:
    Min      1Q  Median      3Q     Max
-3.8390 -1.2486 -0.0533  1.0714  3.6680

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.9058     0.3582   2.529   0.0172 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.962 on 29 degrees of freedom
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Two-sample t-test
(Independent samples t-test)

- Do two groups differ (between-subjects design)?
    - Treatment vs. control
        - Did the manipulation (A vs. B) cause a difference in the measure?

subjects_A	A	subjects_B	B
1	1	6	4
2	4	7	8
3	3	8	7
4	6	9	9
5	5	10	10

Two sample t-test

You know these:

- You have sample from Group 1
  - mean – $\bar{X}_1$
  - Sample size – N_1
  - Sample SD – s_1
- You have sample from Group 2
  - mean – $\bar{X}_2$
  - Sample size – N_2
  - Sample SD – s_2

Question: Is $\bar{X}_1$ different from $\bar{X}_2$?
(Are the two groups different?)

Test-statistic

- Earlier

  $z = (\bar{X} - \mu) / (\sigma/\sqrt{N})$

  $t = (\bar{X} - \mu) / (s/\sqrt{N})$

- Now,
  - $t = (\bar{X}_1 - \bar{X}_2)$ / SE_diff
    - SE_diff = $\sqrt{(s_1^2/N_1 + s_2^2/N_2)}$
- "How many SEs apart are these two samples?"

Two sample t-test

You know these:

- You have sample from Group 1
  - mean – $\bar{X}_1$
  - Sample size – N_1
  - Sample SD – s_1
- You have sample from Group 2
  - mean – $\bar{X}_2$
  - Sample size – N_2
  - Sample SD – s_2

Question: Is $\bar{X}_1$ different from $\bar{X}_2$?
(Are the two groups different?)

Test-statistic

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  $z = (\bar{X} - \mu) / (\sigma/\sqrt{N})$

  $t = (\bar{X} - \mu) / (s/\sqrt{N})$

- Now,
  - $t = (\bar{X}_1 - \bar{X}_2) / \text{SE\_diff}$
    - $\text{SE\_diff} = \sqrt{(s_1^2/N_1 + s_2^2/N_2)}$
- "How many SEs is this sample from expected?"

Observed difference /
***Estimated** Typical variability*

Two sample t-test

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  - mean – $\bar{X}_1$
  - Sample size – N_1
  - Sample SD – s_1
- You have sample from Group 2
  - mean – $\bar{X}_2$
  - Sample size – N_2
  - Sample SD – s_2

Question: Is $\bar{X}_1$ different from $\bar{X}_2$? (Are the two groups different?)

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- Now,
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    - $SE\_diff = \sqrt{(s_1^2/N_1 + s_2^2/N_2)}$
- "How many SEs is this sample from expected?"

Df depends on assumptions:

$\sigma_1^2 = \sigma_2^2$ (equal variances)

$\Rightarrow$ *Pooled t-test*

$\sigma_1^2 \mathrel{!}= \sigma_2^2$ (equal variances)

$\Rightarrow$ *Welch's t-test*

Observed difference /
Estimated *Typical variability*

Pooled t-test vs Welch's t-test

- Pooled t-test
  - **Assumption**
    $\sigma_1^2 = \sigma_2^2$ (equal variances)
  - **Estimation**
    Pool both samples to estimate one common variance
    - df = $N_1 + N_2 - 2$
    - SE_diff = s_pooled × $\sqrt{}(1/N_1 + 1/N_2)$

- Welch's t-test
  - **Assumption**
    $\sigma_1^2 \mathrel{!=} \sigma_2^2$ (unequal variances)
  - **Estimation**
    Use separate variance estimates
    - df = complex formula (Welch-Satterthwaite)
      - Often not an integer
      - df is always ≤ $N_1 + N_2 - 2$
      - More conservative when variances differ

Pooled t-test vs Welch's t-test

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      - Often not an integer
      - df is always $\leq N_1 + N_2 - 2$
      - More conservative when variances differ

Pro Tip: Use Welch's by default. It's safer and performs well even when variances are equal.

Fyi, df for Welch's t-test

$$df = \frac{\left(\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}\right)^2}{\frac{\left(\frac{s_1^2}{N_1}\right)^2}{N_1 - 1} + \frac{\left(\frac{s_2^2}{N_2}\right)^2}{N_2 - 1}}$$

Two-sample t-test

- **Use Case:** Comparing means between two independent groups

- **As a Linear Model:**
  - $Y = b_0 + b_1X_1 + \varepsilon$
    - where X_1 is dummy coded (0,1)

  - NHST
    - Traditional Form: $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$
    - lm() equivalent: $H_0: b_1 = 0$ vs $H_1: b_1 \neq 0$

Two-sample: t.test () vs lm()

```
#Careful of defaults!

#default: Welch's
t.test(Y ~ X1, data = df)

#Change default to pool!
t.test(Y ~ X, var.equal = TRUE)
```

Two-sample: t.test () vs lm()

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#Change default to pool!
t.test(Y ~ X, var.equal = TRUE)
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```
#default: Pooled
lm(Y ~ X1, data = df)
#NOTE: lm() not equipped for welch's
```

From: https://steverxd.github.io/Stat_tests/two-means-independent-samples.html

Two-sample: t.test () vs lm()

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#default: Pooled
lm(Y ~ X1, data = df)
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```

```
# Linear model (GLS)
lm <- nlme::gls(value ~ 1 + group_y2, weights = nlme::varIdent(form=~1|group), method="ML"
  lm %>% summary() %>% print(digits = 8) # show summary output
  confint(lm) # show confidence intervals
```

From: https://steverxd.github.io/Stat_tests/two-means-independent-samples.html

Paired Sample t-test

- Do two samples differ (within-subjects design)?
  - Same participants
    - Compare
      - Before Treatment vs. After Treatment

subjects	level_A	level_B
1	1	4
2	4	8
3	3	7
4	6	9
5	5	10

Paired sample t-test

You know these:

- You have paired observations
  - Before/after for each subject
  - Or matched pairs
- From this you can construct a difference distribution/ sample
  - $D = X\_after - X\_before$
    - mean difference – $\bar{D}$
    - SD of differences – s_D
  - Sample size – N

Question: Is $\bar{D}$ different from 0? (Did the treatment?)

Test-statistic

- Earlier

$$z = (\bar{X} - \mu) / (\sigma/\sqrt{N})$$
$$t = (\bar{X} - \mu) / (s/\sqrt{N})$$
$$t = (\bar{X}_1 - \bar{X}_2) / SE\_diff$$

- Now, treats differences as one sample
  - $t = \bar{D} / (s\_D/\sqrt{N})$ • $t = \bar{D} / SE\_D$
- " How many SEs is the mean difference from zero?"

Paired sample t-test

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- You have paired observations
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- From this you can construct a difference distribution/ sample
  - $D = X\_after - X\_before$
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- Now, treats differences as one sample
  - $t = \bar{D} / (s\_D/\sqrt{N})$ • $t = \bar{D} / SE\_D$
- " How many SEs is the mean difference from zero?"

Observed difference /
***Estimated** Typical variability*

Paired t-test

- **Use Case:** Comparing means between two related/matched measurements (e.g., before vs. after)

- **As a Linear Model:**
  - $D = b_0 + \varepsilon$
    - where $D = Y_2 - Y_1$ (differences)

  - NHST
    - Traditional Form: H_0: $\mu D = 0$ vs H_1: $\mu D \neq 0$
    - lm() equivalent: H_0: $b_0 = 0$ vs H_1: $b_0 \neq 0$

Paired t-test – t.test() vs lm()

```r
#handles pairing automatically
t.test(after, before, paired = TRUE)
```

```r
# lm works with the difference distribution
D <- after - before
lm(D ~ 1)  # intercept-only
```

Non-parametric t-tests

- Parametric tests still make some strong assumptions

- Non-parametric tests can be desirable when
  - Data is not continuous
  - Skewed generating (population) distribution
  - Heavy outliers

- Independent groups: **Mann-Whitney U test**
- Paired samples: **Wilcoxon signed-rank test**.

Non-parametric t-tests

- Previously
  - **Bootstrapping:**
    - Sample *with replacement*. Estimates sampling variability.
- Similarly,
  - **Permutation (for Mann-Whitney/Wilcoxon):**
    - Shuffle group labels. Tests null hypothesis directly.

Mann-Whitney algorithm

Sampling distribution:

1. Calculate observed U statistic
2. Randomly shuffle group labels
3. Recalculate U
4. Repeat 10,000+ times

Now, where does observed U fall?

Mann-Whitney algorithm

Sampling distribution:

1. Calculate observed U statistic
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Now, where does observed U fall?

- **U statistic:**
  - Count pairwise comparisons.
  - For each observation in Group 1, count how many in Group 2 it beats.
  - Sum those counts = U
  - Example:
    - Group 1: [3, 7]
    - Group 2: [1, 4, 5]
      - 3 beats 1 → +1
3 loses to 4, 5 → +0
      - 7 beats 1, 4, 5 → +3
      - U = 4

Wilcoxon signed-rank via permutation

Sampling distribution:

1. Calculate signed ranks of differences
2. Randomly flip signs of differences
3. Recalculate test statistic
4. Repeat

Now, where does observed stat fall?

Wilcoxon signed-rank via permutation

Sampling distribution:

1. Calculate signed ranks of differences
2. Randomly flip signs of differences
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4. Repeat

Now, where does observed stat fall?

- **Signed ranks:**
  - Take differences.
  - Ignore sign, rank by magnitude.
  - Put signs back.
  - Example:
    - Differences: [+5, -2, +8, -1]
    - Absolute: [5, 2, 8, 1]
    - Ranks: [3, 2, 4, 1]
    - Signed ranks: [+3, -2, +4, -1]
    - Sum positive ranks.
    - Sum negative ranks.
    - Compare.

Comparing Several Means

One-way ANOVA

- **Use Case:** Comparing means across three or more independent groups

- **As a Linear Model:**
  - $Y = b_0 + b_1X_1 + b_2X_2 + \ldots + \varepsilon$
    - where X's are dummy coded for k-1 groups

  - NHST
    - Traditional Form: $H_0: \mu_1 = \mu_2 = \ldots = \mu_k$ vs H_1: not all μ_i equal
    - lm() equivalent: $H_0: b_1 = b_2 = \ldots = 0$ vs H_1: not all $b_i = 0$

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One-way ANOVA

- **Test Statistic**

$$F = \frac{\text{Between-group variability}}{\text{Within-group variability}} = \frac{\text{MS}_{\text{between}}}{\text{MS}_{\text{within}}}$$

- **Global test**
  - H_0: $\mu_1 = \mu_2 = \ldots = \mu_k$
  - Our F-test tells us if we can reject this
  - **If We Reject H_0 ($p < \alpha$):**
    - We can then investigate specific patterns through:
      - a) Planned Contrasts
      - b) post-hoc tests (Bonferroni, etc.)

Example:
Initial ANOVA: H_0: $\mu_1 = \mu_2 = \mu_3$
↓ (Reject H_0)

Specific Questions:
- Is $\mu_1 > \mu_2$? (Pairwise)
- Is $\mu_1 > (\mu_2 + \mu_3)/2$? (Contrast)

F-statistic and connection to lm()

- **From Coefficients to Sums of Squares:**
  - The coefficients (b_1, b_2) tell us how each group differs from the reference
  - These differences contribute to the Explained Sum of Squares (ESS)
  - ESS = $\sum(\hat{y}_i - \bar{y})^2$ = sum of squared differences between fitted values and overall mean
  - RSS = $\sum(y_i - \hat{y}_i)^2$ = sum of squared residuals
- **F-statistic Formation:**
  - F={ESS/(k−1)}/ {RSS/(N−k)}
    - where: k-1 = degrees of freedom for model (number of coefficients excluding intercept)
- **Equivalence to Testing Coefficients:**
  - H_0: $b_1 = b_2 = 0$ is equivalent to H_0: $\mu_1 = \mu_2 = \mu_3$
  - This is because when coefficients are 0, all group means are equal

Two-way ANOVA

- **Use Case:** Comparing means across groups while considering two different categorical factors and their interaction

- **As a Linear Model:**
  - $Y = b_0 + b_1X_1 + b_2X_2 + b_3(X_1 \times X_2) + \varepsilon$
    - where X's are dummy coded

  - NHST
    - Traditional Form:
      - H_0^A: No main effect of factor A
      - H_0^B: No main effect of factor B
      - $H_0^{A \chi B}$: No interaction between A and B
    - lm() equivalent:
      - H_0^A: $b_1 = 0$
      - H_0^B: $b_2 = 0$
      - $H_0^{A \chi B}$: $b_3 = 0$

Key Insights

- Every test is asking about b coefficients
- The complexity comes from how we code our X variables
- The lm() approach often offers more flexibility:
  - Easy to add covariates
  - Can handle missing data more flexibly
  - Provides more detailed output
  - Easier to extend to more complex designs

Common Questions and Their Models:

- "Different from a known value?" → $Y = b_0 + \varepsilon$
- "Different between two groups?" → $Y = b_0 + b_1X_1 + \varepsilon$
- "Change over time?" → $D = b_0 + \varepsilon$
- "Different across multiple groups?" → $Y = b_0 + b_1X_1 + b_2X_2 + \ldots + \varepsilon$
- "Interaction between factors?" → $Y = b_0 + b_1X_1 + b_2X_2 + b_3(X_1 \times X_2) + \varepsilon$

Test	Goal	lm() Equivalent	Key Metric
One-Sample t-Test	Compare sample mean to a known value	lm(y ~ 1)	t-value
Two-Sample t-Test	Compare means of two groups	lm(y ~ group)	t-value
ANOVA	Compare means across multiple groups	lm(y ~ factor(group))	F-statistic
Linear Regression	Predict continuous outcomes	lm(y ~ x1 + x2)	R-squared, coefficients
Interaction Effects	Test dependent effects between vars	lm(y ~ x1 * x2)	Interaction terms
ANCOVA	Control for covariates	lm(y ~ group + covariate)	F-statistic

Multiple regression

Simple linear regression vs. multiple linear regression

$$X \to Y$$

$$\begin{tikzcd}
X_1 \arrow[dr] & \\
X_2 \arrow[r] & Y \\
\vdots & \\
X_k \arrow[uur] &
\end{tikzcd}$$

Multiple Regression - equation

- Simple as adding predictors to our linear equation
- SLR

$$\hat{Y} = b_0 + \beta_1 X$$

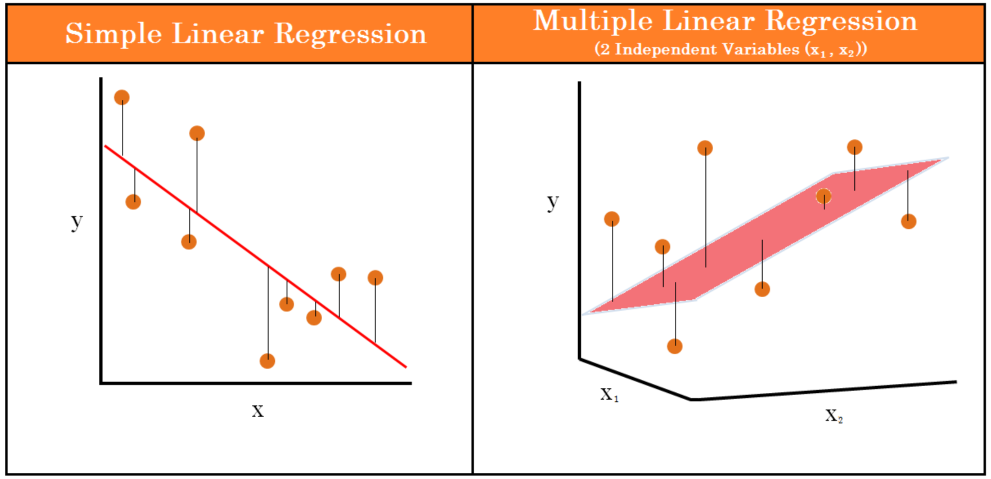
- MR

$$\hat{Y} = b_0 + b_1 X_1 + b_2 X_2 + \ldots + b_k X_k$$

- $\hat{Y}$ = predicted value on the outcome variable Y
- b_0 = predicted value on Y when all Xs = 0
- X_k = predictor variables
- b_k = unstandardized regression coefficients
- k = the number of predictor variables

Straight Line to Hyperplane

- More than two predictors (plane)
- Multi-dimensional space
- Regression coefficients are "partial" regression coefficients
- Slope for variable X_1 (b_1) predicts the change in Y per unit X_1 holding X_2 constant
- Slope for variable X_2 (b_2) predicts the change in Y per unit X_2 holding X_1 constant



Multicolinearity

- Multicolinearity
  - You want X and Y to be correlated
  - You do not want the Xs to be highly correlated