Assignment #2

ME135-02L-Spring2018

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Problem 1.

1.

The conservation of heat can be used to develop a heat balance for a long, thin rod (Fig. 27.2). If the rod is not insulated along its length and the system is at a steady state, the equation that results is

$$\frac{d^2T}{dx^2} + h'(T_a - T) = 0 \tag{27.1}$$

where h' is a heat transfer coefficient (m^{-2}) that parameterizes the rate of heat dissipation to the surrounding air and T_a is the temperature of the surrounding air (°C).

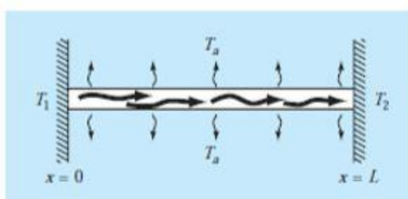
To obtain a solution for Eq. (27.1), there must be appropriate boundary conditions. A simple case is where the temperatures at the ends of the bar are held at fixed values. These can be expressed mathematically as

$$T(0) = T_1$$
$$T(L) = T_2$$

With these conditions. Eq. (27.1) can be solved analytically using calculus. For a 10-m rod with $T_a = 20$, $T_1 = 40$, $T_2 = 200$, and $h' = 0.01$, the solution is

$$T = 73.4523e^{0.1x} - 53.4523e^{-0.1x} + 20 \tag{27.2}$$

FIGURE 27.2
A noninsulated uniform rod positioned between two bodies of constant but different temperature. For this case $T_1 > T_2$ and $T_2 > T_a$.

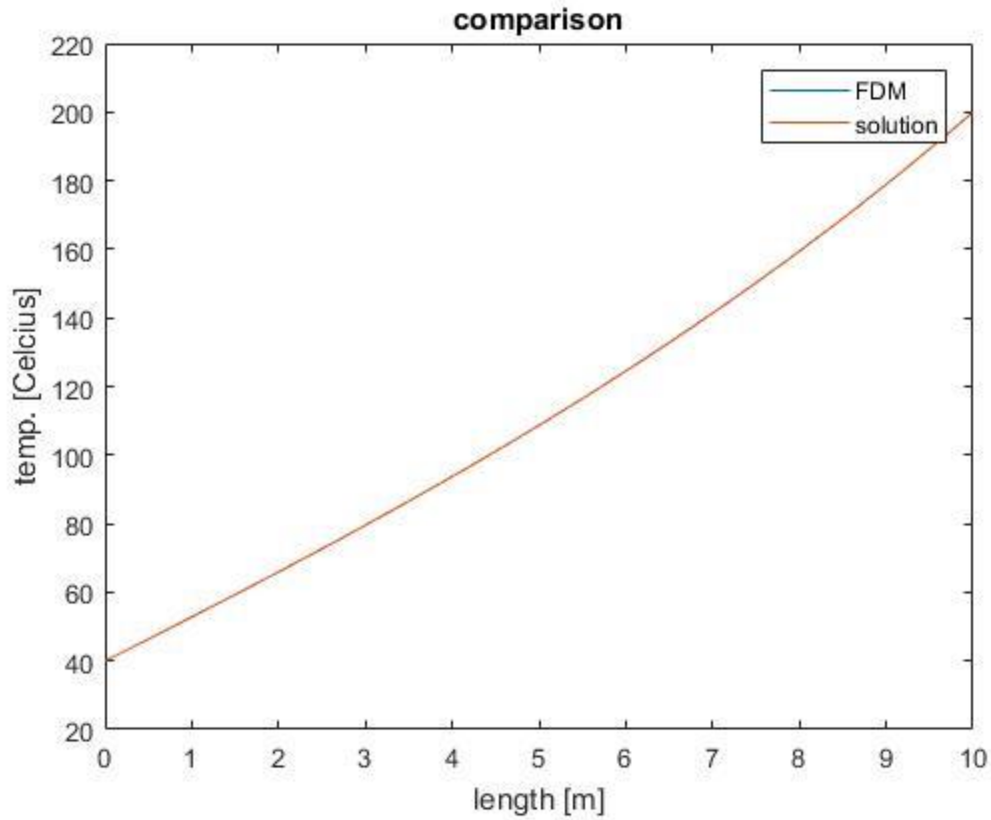


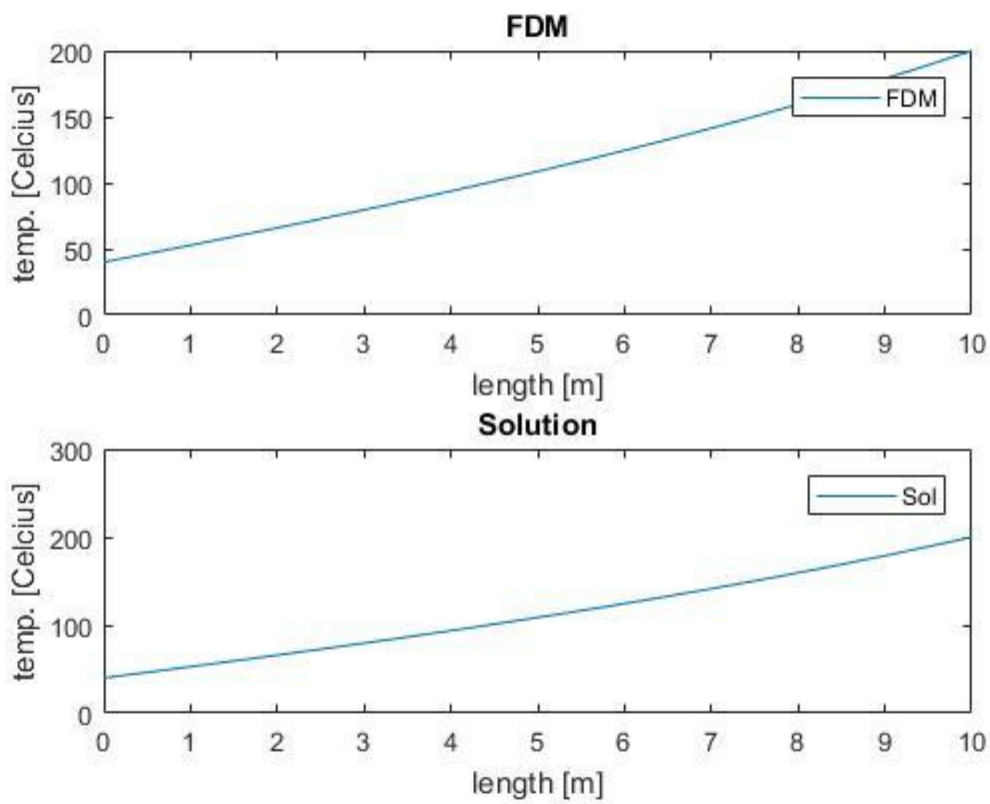
Use the information above to find the solution of the governing equation using finite difference method and compare it with the analytical solution above by plotting both solutions on the same graph.

To solve this problem, we needed to use the finite difference method. To get a better comparison between the method and solution I created two figures one with both the graphs on the same figure and one with both of them side by side. The code is available the appendix and on GitHub [https://github.com/Yeash96/Eng135.FEA/tree/master/Assigment_2].

Matlab input and output

>>pr1


comparison
FDM
solution
temp. [Celcius]
length [m]


FDM
FDM
temp. [Celcius]
length [m]
Solution
Sol
temp. [Celcius]
length [m]

Problem 2.

2. The basic differential equation of the elastic curve for a uniformly loaded beam (Fig. P28.27) is given as:

$$EI\frac{d^2y}{dx^2} = \frac{wLx}{2} - \frac{wx^2}{2}$$

where E = the modulus of elasticity and I = the moment of inertia.

Solve for the deflection of the beam using the finite-difference approach. Parameter values apply: E = 30,000 ksi, I = 800 in4, ω = 1 kip/ft, L = 10 ft.
Compare your numerical results to the analytical solution: (by plotting the two solutions on the same graph)

$$y = \frac{wLx^3}{12EI} - \frac{wx^4}{24EI} - \frac{wL^3x}{24EI}$$

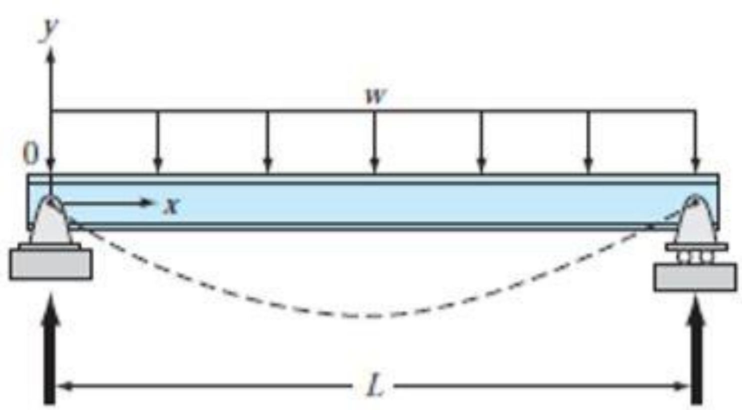
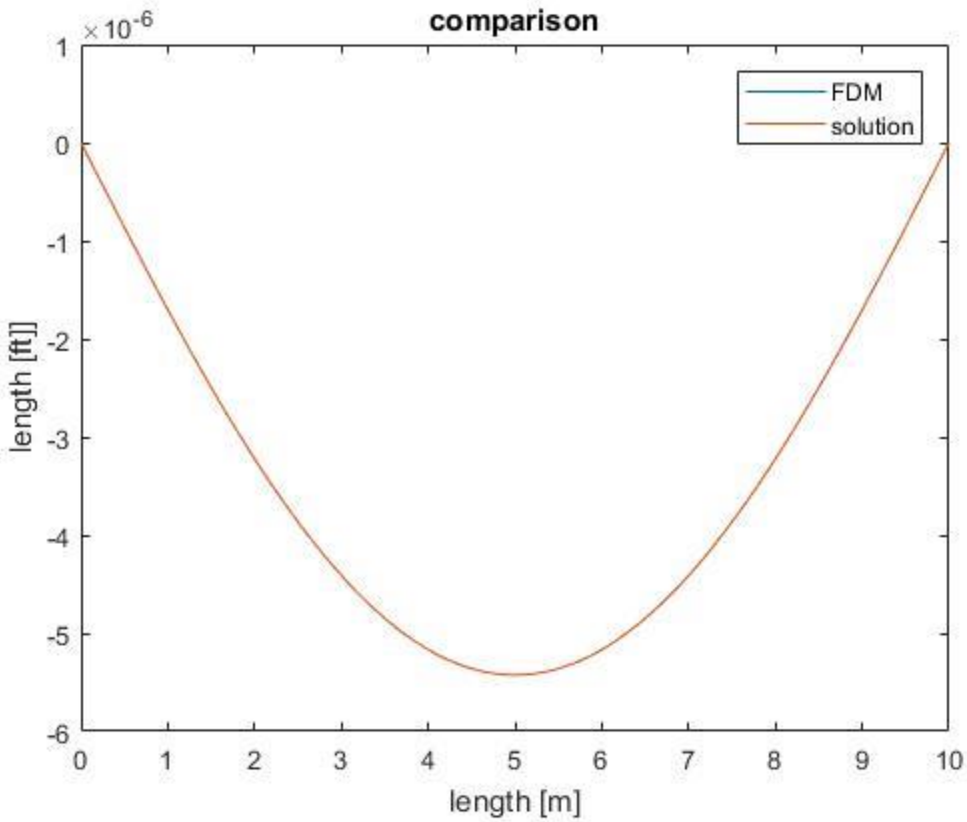


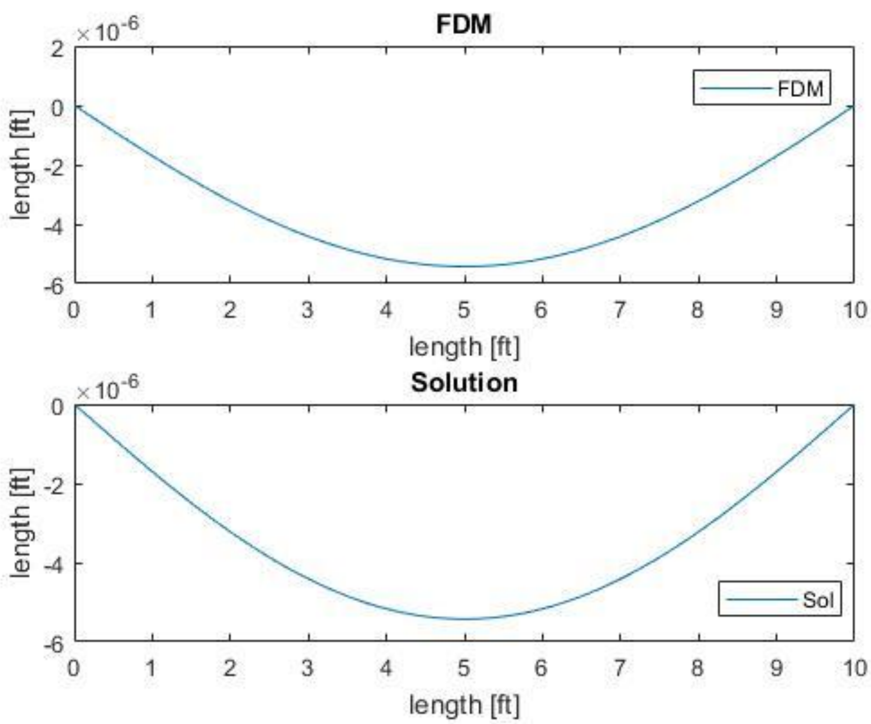
Figure P28.27

To solve this problem, we needed to use the finite difference method. To get a better comparison between the method and solution I created two figures one with both the graphs on the same figure and one with both of them side by side. The code is available the appendix and on GitHub [https://github.com/Yeash96/Eng135.FEA/tree/master/Assigment_2].

Matlab input and output

>>pr2


comparison
× 10^-6
FDM
solution
length [ft]]
length [m]
1
0
-1
-2
-3
-4
-5
-6
0 1 2 3 4 5 6 7 8 9 10


FDM
× 10^-6
FDM
length [ft]
2
0
-2
-4
-6
0 1 2 3 4 5 6 7 8 9 10
length [ft]
Solution
× 10^-6
Sol
length [ft]
0
-2
-4
-6
0 1 2 3 4 5 6 7 8 9 10
length [ft]

Appendix

Pr.1.m

```
clc
clear all
close all
format long

%given constants
hco=0.01;
ta=20;
%ODE using finite difference
% parameters
x_i=0;
x_f=10;

dx=0.01;

x=x_i:dx:x_f;

%initializing the matrices

A=zeros(length(x),length(x));
c(1:length(x),1)=-hco*ta;

%Boundary conditons

A(1,1)=1;
c(1,1)=40;


A(length(x),length(x))=1;
c(length(x),1)=200;

%internal points
for i=2:length(x)-1

    A(i,i+1)=1/(dx^2);
    A(i,i)=(-2/dx^2)+(-hco);
    A(i,i-1)=1/dx^2;

end

U=A\c; %A inverse *c

figure(1)
plot(x,U,x,73.4523*exp(0.1*x)-53.4523*exp(-0.1*x)+20);
legend('FDM','solution')
xlabel('length [m]')
ylabel('temp. [Celcius]')
title('comparison')
```

```
figure(2)
subplot(2,1,1)
plot(x,U)
legend('FDM','solution')
xlabel('length [m]')
ylabel('temp. [Celcius]')
title('FDM')
subplot(2,1,2)
plot(x,73.4523*exp(0.1*x)-53.4523*exp(-0.1*x)+20);
legend('Sol','solution')
xlabel('length [m]')
ylabel('temp. [Celcius]')
title('Solution')
```

Pr2.m

```
clc
clear all
close all
format long

%given constants
E=30000;
I=800;
w=1;
l=10;
%ODE using finite difference
% parameters
x_i=0;
x_f=l;

dx=0.01;

x=x_i:dx:x_f;

%initializing the matrices

A=zeros(length(x),length(x));
c(1:length(x),1)=(w*l*x./2)-(w*x.^2/2);

%Boundary conditons

A(1,1)=1;
c(1,1)=0;


A(length(x),length(x))=1;
c(length(x),1)=0;

%internal points
for i=2:length(x)-1

    A(i,i+1)=E*I/(dx^2);
    A(i,i)=(-2*E*I/dx^2);
```

```
    A(i,i-1)=E*I/dx^2;

end

U=A\c; %A inverse *c

figure(1)
plot(x,U,x,(w*l*x.^3./(12*E*I))-(w*x.^4./(24*E*I))-(w*l^3*x./(24*E*I)));
legend('FDM','solution')
legend('FDM','solution')
xlabel('length [m]')
ylabel('length [ft]]')
title('comparison')

figure(2)
subplot(2,1,1)
plot(x,U)
legend('FDM','solution')
xlabel('length [ft]')
ylabel('length [ft]')
title('FDM')
subplot(2,1,2)
plot(x,(w*l*x.^3./(12*E*I))-(w*x.^4./(24*E*I))-(w*l^3*x./(24*E*I)));
legend('Sol','solution')
xlabel('length [ft]')
ylabel('length [ft]')
title('Solution')
```