Homework #10

Math 131 - Spring 2017

DUE on 5/5/17 at midnight (online submission through Catcourses).

NOTE: *Your answers will be graded for correctness as well as comprehensiveness, completeness, and legibility of your solution.*

1. For each of the following one-step time methods

   (a) Euler's method

   (b) Runge-Kutta method of order 2

   (c) Runge-Kutta method of order 4

   write a function that solves the IVP

   $$y'(t) = f(t, y) \text{ for } a \leqslant t \leqslant b, \text{ with } y(a) = \alpha.$$

   In each case, the function header should look something like

   ```
   function w = method_name(f,a,b,alpha,N)
   ```

   where N is the number of intervals used, so that $h = \frac{b-a}{N}$

2. Write a function that solves the IVP using a 2-step Adams-Bashforth method that computes w_1 using a second order Runge–Kutta method. The header should have the same format as the headers in problem 1.

3. Consider the IVP

   $$y'(t) = \frac{\sin(2t) - 2ty}{t^2} \text{ for } 1 \leqslant t \leqslant 2, \text{ with } y(1) = 2.$$

   (a) Use all 4 methods that you have developed in problems 1 and 2 to solve the IVP given with $N = 10; 10^2; 10^3; 10^4$.

   (b) Make a `loglog` plot of absolute error at $t = 2$ versus the number of intervals for all four methods on the same plot. Make sure to create a legend, showing which plot corresponds to which method. Comment on your results: which method works best, which method works the worst? Why?
   Hint: To compute the absolute error, you need to find the exact/true/actual solution of this IVP. One can show that it is $y(t) = \frac{4+\cos(2)-\cos(2t)}{2t^2}$ (using the knowledge you learned from Math 24).