Homework #8

Math 131 - Spring 2017

DUE on 4/18/17 at 4:30pm (online submission through Catcourses).

NOTE: *Your answers will be graded for correctness as well as comprehensiveness, completeness, and legibility of your solution.*

1. Consider the function $f(x) = e^{-x^2}\sin(x)$. Estimate the derivative of the function at the point $x = 0$ using each of the four formulas listed below for all of the following values of $h = 10^{-n},\ n = 1, 2, \cdots, 6$.

   $$f'(x_0) \approx \frac{f(x_0+h) - f(x_0)}{h} \quad \text{(forward difference)}$$

   $$f'(x_0) \approx \frac{f(x_0+h) - f(x_0-h)}{2h} \quad \text{(3-point centered difference)}$$

   $$f'(x_0) \approx \frac{-f(x_0+2h) + 4f(x_0+h) - 3f(x_0)}{2h} \quad \text{(3-point one-side difference)}$$

   $$f'(x_0) \approx \frac{-f(x_0+2h) + 8f(x_0+h) - 8f(x_0-h) + f(x_0-2h)}{2h} \quad \text{(5-point centered difference)}$$

   Plot your error verses h for all methods on the same graph. You may have cause to use `loglog` plot with the commands:
   `loglog(x1,y1,-k,x2,y2,-b,x3,y3,-g,x4,y4,-m,x5,y5,-r,linewidth,2);`
   `legend(FD,3pt CD,3pt 1SD,5pt CD,5pt 1SD);`
   Explain your findings. Is the error decreasing with h? Why or why not? Which method has the largest error? Which method has the smallest error? Why?

2. Create a function file called `trap_int.m` that inputs a function f, a pair of endpoints, $a; b$, and a number n of subintervals, and outputs the approximation to the integral of f from a to b using the trapezoid rule on n points. Your function header should look like this
   `function I = trap_int(f,a,b,n)`

3. Create a function file called `Simp_int.m` that inputs a function f, a pair of endpoints, $a; b$, and a number n of subintervals, and outputs the approximation to the integral of f from a to b using Simpsons rule on n points. Your function header should look like this
   `function I = Simp_int(f,a,b,n)`

4. Consider the integral

   $$I = \int_{-2}^{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx$$

   .

   (a) Use the code you developed in Problems 2 and 3 to compute the integral for the number of points n = 10; 100; 1000; 10000.

   (b) Compute the error in your computation for each n and for each method. Note that the 'exact' answer to the given integral can be computed by using the error function `erf` in MATLAB, namely,

   $$I_{\text{exact}} = \frac{\texttt{erf}(\sqrt{2}) - \texttt{erf}(-\sqrt{2})}{2}.$$

   (c) Make a `loglog` plot of error verses the number of points for all methods on the same plot. Make sure to create a legend, showing which plot corresponds to which method. Comment on your results: which method works better? Why?