Homework #4

Math 131 - Spring 2017

DUE on 2/21/17 at 1:30pm (online submission through Catcourses).

NOTE: *Your answers will be graded for correctness as well as comprehensiveness, completeness, and legibility of your solution.*

1. Implement the fixed-point iteration for the following root-finding problems up to accuracy of 10^{-6}. For each of these problems use the graphical method described in class to find the rate of convergence of the iteration.

   (a) $f(x) = x^5 + 5x^3 - x^2 + 1$ on interval $[-1, 2]$.

   (b) $f(x) = x^4 - 8x^3 + 24x^2 - 32x + 16$ on interval $[-1, 2]$.

   Hint: design function $g(x)$ such that solving the corresponding fixed-point problem $x = g(x)$ and use the fixed-point theorem to check whether the fixed point problems you designed has a unique solution and whether the corresponding fixed-point iteration will converge if the initial guess is picked from the given interval. If the solution to the root finding problem is not unique, try changing the interval to construct a fixed-point-problem with a unique solution.

2. Implement the Newton and Secant methods and solve the following two problems to within 10^{-5}. Plot the logarithms of the errors appropriately to generate a graph that will help you find the order of convergence of the algorithms. Display the order of convergence of either of the techniques for each of the problems:

   (a) $f(x) = \cos(x + \sqrt{(2)}) + x(x/2 + \sqrt{(2)}) = 0$ for $-2 \leq x \leq -1$;

   (b) $f(x) = e^{6x} + 3(\ln 2)^2 e^{2x} - (\ln 8)e^{4x} - (\ln 2)^3 = 0$ for $-1 \leq x \leq 0$.

   Hint: Since you don't know the exact solution before you begin solving the problem and thus cannot find the exact error, you can use the difference between successive estimates $|p_{n+1} - p_n|$ as an approximation to the error ϵ_n.

3. Given your results above explain what advantages does Newton method have over Secant method and what advantages Secant method has over Newton method.