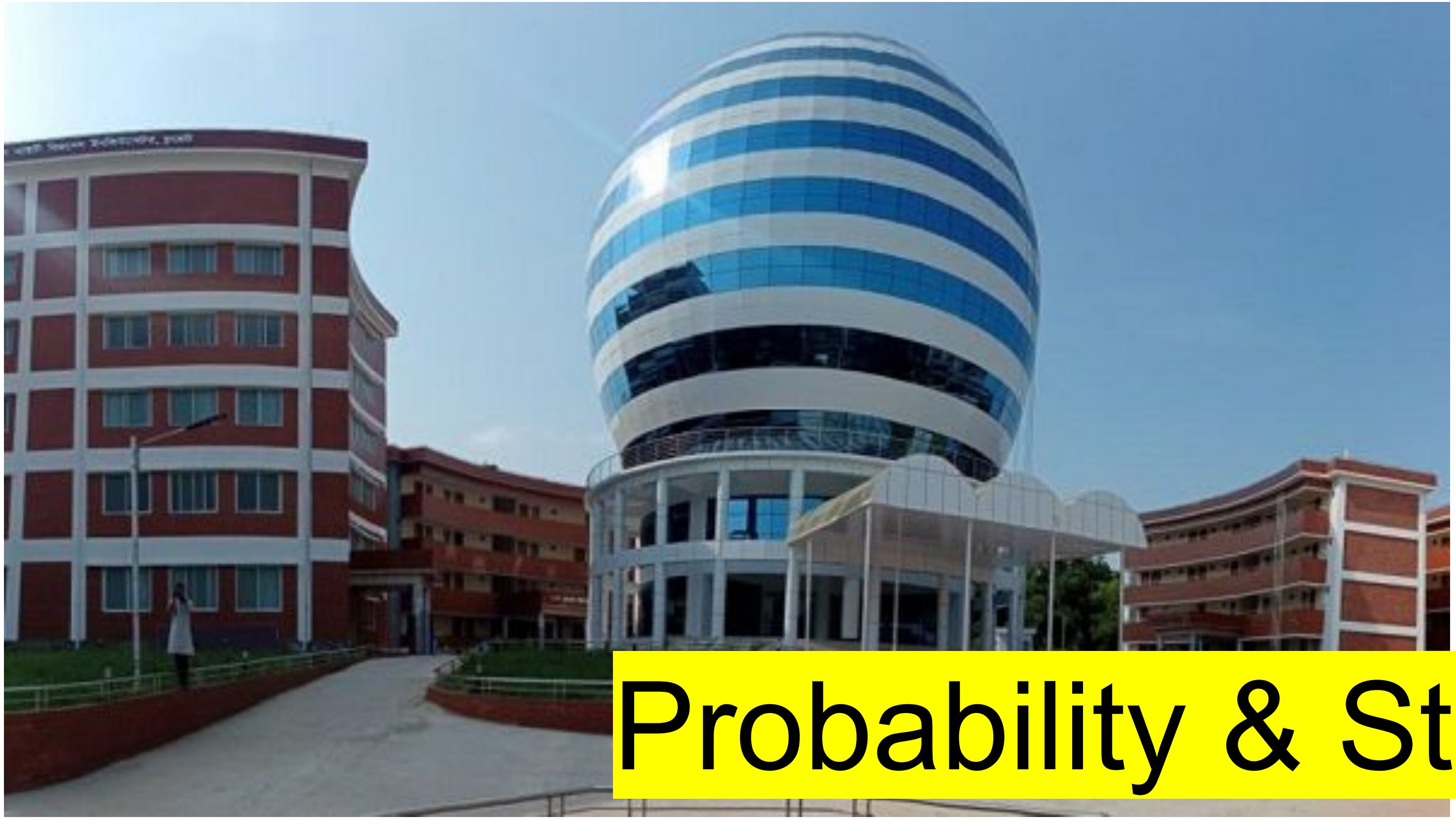

Probability & Statistics

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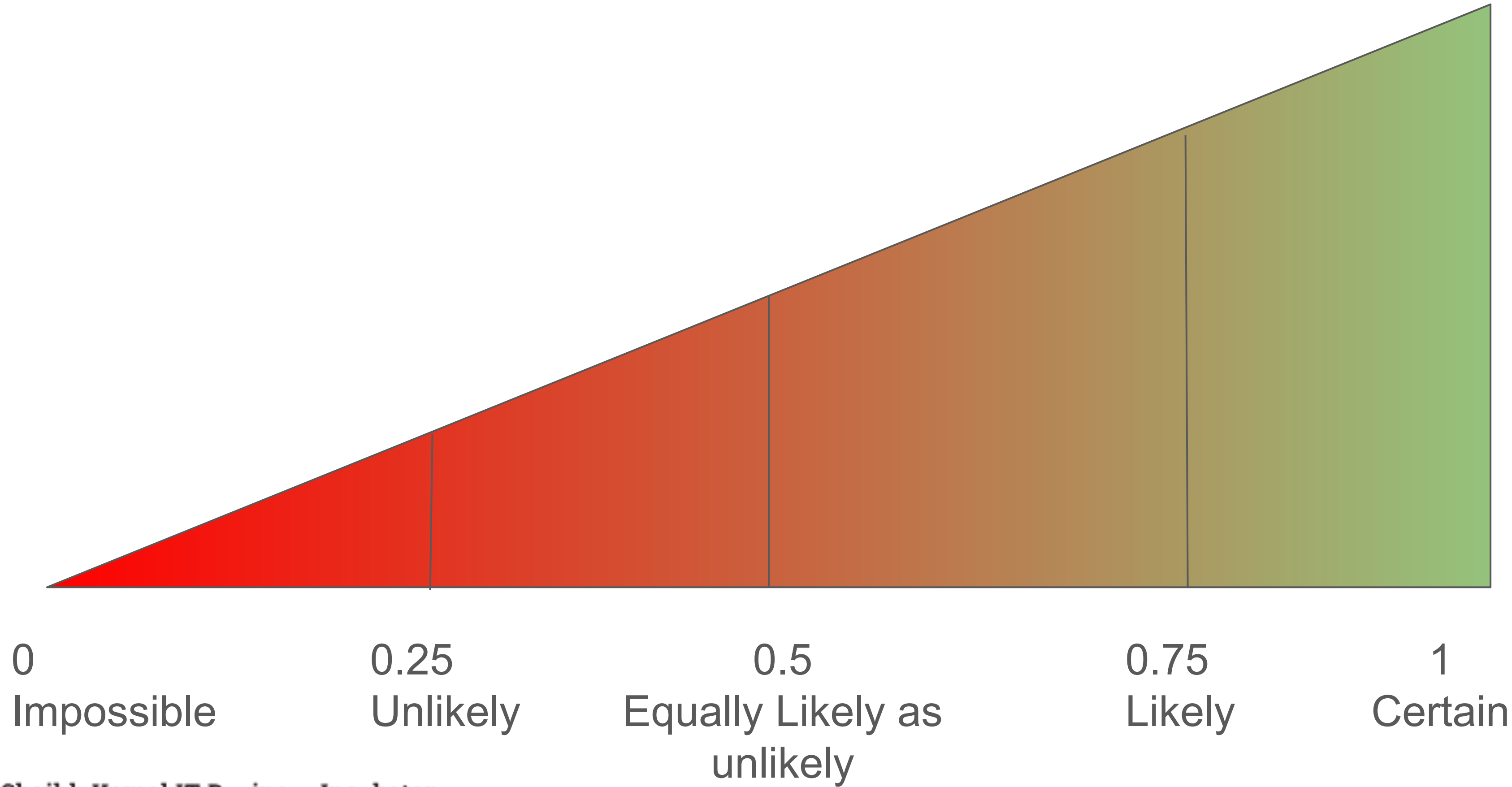
What is Probability?

- Probability denotes the possibility of the outcome of any random event.

- It is the ratio of the number ways a certain event can occur to the number of possible outcome.

Head **Tail**

Probability in Numerical Measure



Probability Terms

An "experiment" is a process or procedure that generates a set of data or outcomes.

The "sample space" in probability and statistics is the set of all possible outcomes of an experiment.

An experimental outcome is also called a “sample point”.

An “event” is a collection of sample points

The probability of any event is equal to the sum of the probabilities of the sample points in the event.

Example of Experiment

Think about a coin..

Head

Tail

Example of Experiment

Think about a dice..

Let's think about a simple event

Probability measures how likely an event is to occur.

The probability formula for a simple event is given by:

$$\mathrm{P(A)} = \frac{\text{Number of times A occurs}}{\text{Total number of possible outcomes}}$$

Let’s think about a single dice

What is our Expected Outcome?

What is our Probability of the Expected outcome happening?

Let’s think about a two dices

What is our Expected Outcome?

What is our Probability of the Expected outcome happening?

More example…

Experiment: Drawing one card from a standard 52-card deck.

Sample Space: S={52 cards}
Event E: Drawing an Ace.
Probability:

Experiment: Picking a marble from a bag containing 5 red and 5 blue marbles.

Sample Space:
Event E: Picking a red marble.
Probability:

More example…

Experiment: Drawing one card from a standard 52-card deck.

Sample Space: S={52 cards}
Event E: Drawing an Ace.
Probability: P(E)= Number of Aces (4) / Total cards (52) = 4/52 = 1/13

Experiment: Picking a marble from a bag containing 5 red and 5 blue marbles.

Sample Space: S={10 marbles}
Event E: Picking a red marble.
Probability: P(E)= Number of red marbles (5) / Total marbles (10) = 5/10 = 1/2

Let’s flip a coin twice:

First Flip: We get Head

Second Flip: We get Tail

Are they dependent on each other?

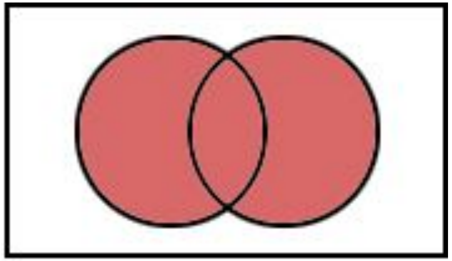
Independent Events

The outcome of one event does not affect the outcome of another. For example, flipping a coin and rolling a die are independent events because the result of the coin flip does not affect the result of the die roll.

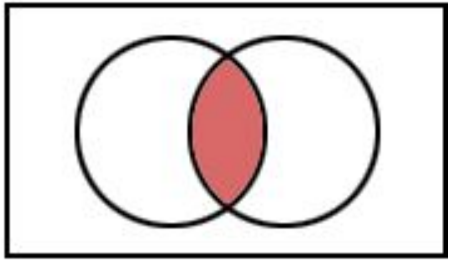
In Such case, $P(A \cap B) = P(A).P(B)$

Event Relations

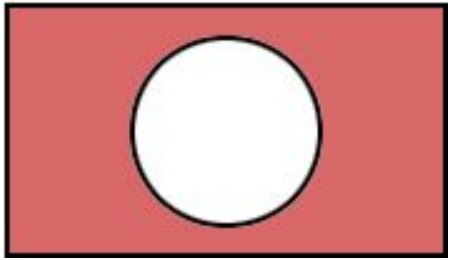
$A \cup B$



$A \cap B$



A'



Additive Rule for Union

The probability rule for unions helps us calculate the probability that at least one of two events will occur. The formula for the probability of the union of two events, A and B, is given by:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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Example:

If one card is drawn from a standard 52-card deck, the probability of drawing a heart or a king is calculated as follows:

$P(\text{Heart})=13/52$,

$P(\text{King})=4/52$, and

$P(\text{Heart} \cap \text{King})=1/52$.

So, $P(\text{Heart} \cup \text{King})= 13/52 + 4/52 - 1/52= 4/13$.

If Events are Mutually Exclusive

For two mutually exclusive events, A and B, the probability that either A or B occurs is given by the sum of their probabilities:

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Example:
If a card is drawn from a standard deck, the probability of drawing an Heart or a King is

P(Heart)+P(King)= 4/52 + 4/52 = 8/52 = 2/13

Assuming the draw is done without replacement, making the events mutually exclusive within the context of a single draw.

Probability for Complements

The probability of the complement of event A, written as $P(A')$, is defined as:

$$P(A') = 1 - P(A)$$

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Why subtracting from 1?

The sum of the probabilities of an event and its complement is always 1, because between them, they cover the entire sample space without overlap.

And,
Mutually Exclusive and Exhaustive: An event and its complement are mutually exclusive (they cannot both occur) and exhaustive (one of them must occur).

Probability for Complements

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Example:

If A is the event "rolling a number greater than 4" when rolling a fair six-sided die, $P(A)= 2/6$
because the favorable outcomes are 5 and 6.

Therefore, the probability of not rolling a number greater than 4, $P(A')$, is $1-2/6=4/6$ or $2/3$.

Conditional Probabilities

The conditional probability of an event A, given that event B has occurred, is denoted by and defined as:

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provided that P(B)>0.

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You May find this formula as follow:

P(A∩B) = P(A/B) * P(B)

Example of Conditional Probabilities

If a card is drawn from a standard 52-card deck, what is the probability it's an ace given that it's a spade? Here, A is the event of drawing an ace, and B is the event of drawing a spade.

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$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/52}{13/52} = \frac{1}{13}$$

because there is only one ace of spades and 13 spades in total.

Another variant of Conditional Probability

The conditional probability of an event A, given that event B has occurred, is denoted by and defined as:

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Can we write P(A∩B) = P(B\A) * P(A)?

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And This is the **Bayes Theorem**

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayes Theorem

Bayes' theorem determine the probability of an event with uncertain knowledge.

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Cause

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$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Effect

Effect

Bayes Theorem

Bayes' theorem determine the probability of an event with uncertain knowledge.

Likelihood

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Posterior Probability

Prior Probability

Bayes Theorem

Bayes' theorem determine the probability of an event with uncertain knowledge.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

This theorem is the basic of modern AI systems for probabilistic inference.

Solve with Bayes Theorem - Example 1

A doctor has medical data showing:

1) 80% patient having stiff neck has the disease meningitis.
2) Probability of patient has meningitis disease is 1/30,000
3) Probability of patient has stiff neck is 1/50

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$$P(\text{Meningitis}|\text{Stiff Neck}) = \frac{P(\text{Stiff Neck}|\text{Meningitis}) \times P(\text{Meningitis})}{P(\text{Stiff Neck})}$$

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P(Meningitis/Stiff Neck) = 0.00133

Hence, The doctor can assume that 1 patient out of 750 patients has meningitis disease with a stiff neck.

Bayes Theorem

Let's back to the Bayes' Theorem.

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$$P(A\backslash B) = \frac{P(B\backslash A)P(A)}{P(B)}$$

Or,

$$P(A\backslash E) = \frac{P(E\backslash A)P(A)}{P(E)}$$

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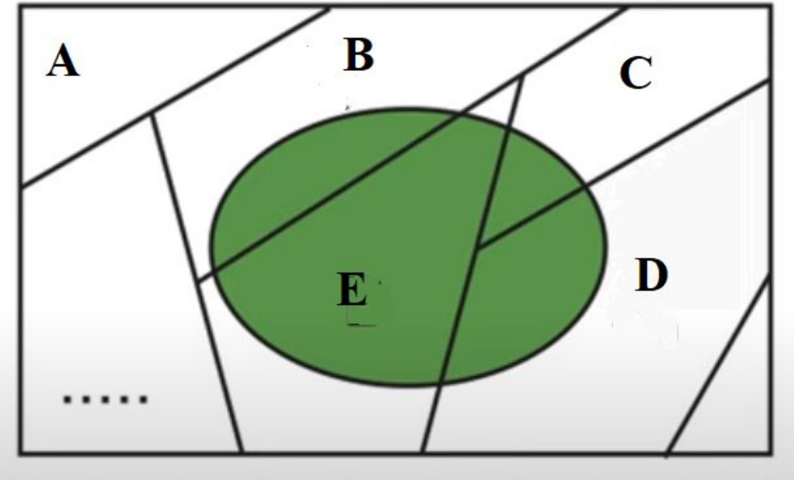
And, Any arbitrary event E.



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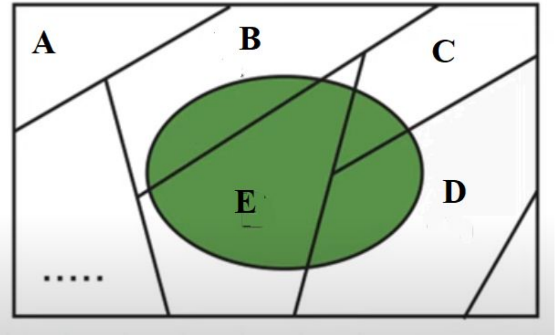
Now, E = E ∩ U
E = E ∩ (A U B U C U D)



Bayes Theorem

What happen if we have multiple events A, B, C, D and they are mutually exclusive and exhaustive events.
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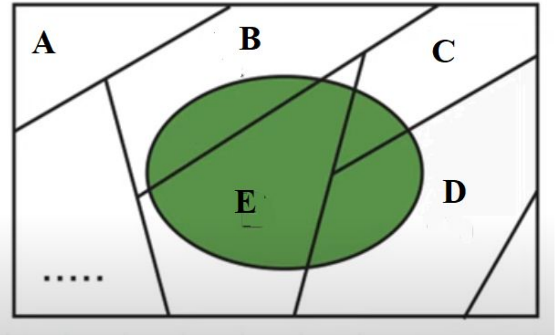
Now, $E = E \cap U$
$E = E \cap (A \cup B \cup C \cup D)$
$E = (E \cap A) \cup (E \cap B) \cup (E \cap C) \cup (E \cap D)$

And,
$P(E)\ P((E \cap A) \cup (E \cap B) \cup (E \cap C) \cup (E \cap D))$



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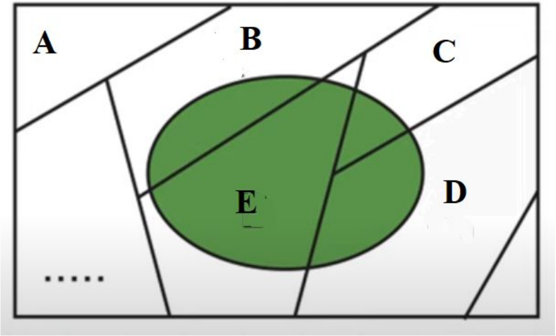

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And,

$P(E) = P((E \cap A) \cup (E \cap B) \cup (E \cap C) \cup (E \cap D))$

As A,B C, D are mutually exclusive events.

$P(E) = P(E \cap A) + P(E \cap B) + P(E \cap C) + P(E \cap D)$

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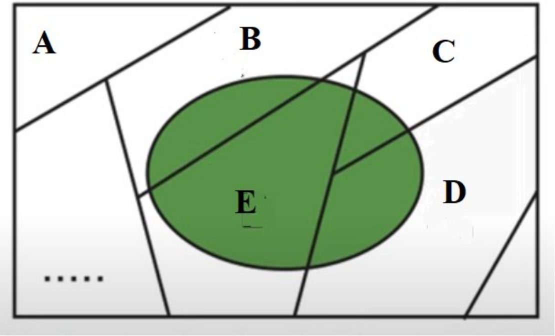

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And, A,B C, D are exhaustive events, $P(E) = 1$

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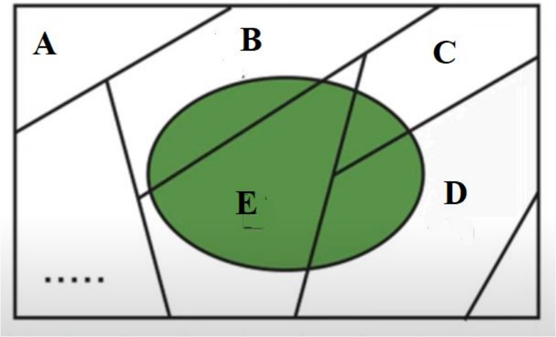

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$P(E) = P(E \cap A) + P(E \cap B) + P(E \cap C) + P(E \cap D)$

And, A,B C, D are exhaustive events, $P(E) = 1$

As we know, $P(A \cap B) = P(A \backslash B) * P(B)$

So, $P(E) = P(E \backslash A).P(A) + P(E \backslash B).P(B) + P(E \backslash C).P(C) + P(E \backslash D).P(D)$

Bayes Theorem

Now, Put everything together,

$$P(A\backslash E) = \frac{P(E\backslash A)P(A)}{P(E)}$$

$$P(A|E) = \frac{P(E|A)\cdot P(A)}{P(E|A)\cdot P(A) + P(E|B)\cdot P(B) + P(E|C)\cdot P(C) + \cdots}$$

Example

A certain disease affects about 1 out of 10,000 people. There is a test to check whether the person has the disease. The test is quite accurate. In particular, we know that

- the probability that the test result is positive (suggesting the person has the disease), given that the person does not have the disease, is only 2 percent;
- the probability that the test result is negative (suggesting the person does not have the disease), given that the person has the disease, is only 1 percent.

A random person gets tested for the disease and the result comes back positive. What is the probability that the person has the disease?

Solution

Let D be the event that the person has the disease, and let T be the event that the test result is positive. We know

$P(D)=1/10,000$
$P(T|D')=0.02$
$P(T'|D)=0.01$

What we want to compute is $P(D|T)$. Again, we use Bayes' rule:
$P(D|T)=P(T|D).P(D) / (P(T|D)P(D)+P(T|D')P(D'))$
$=0.0049$

This means there is less than half a chance that the person has the disease.

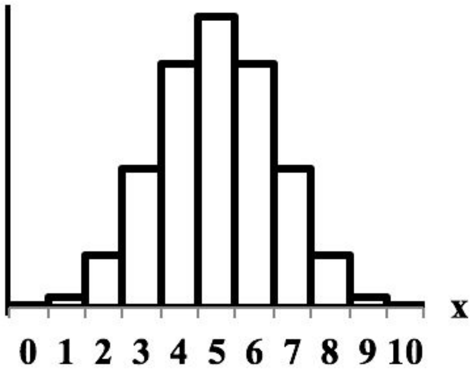
Another Example

Let's talk about Statistics

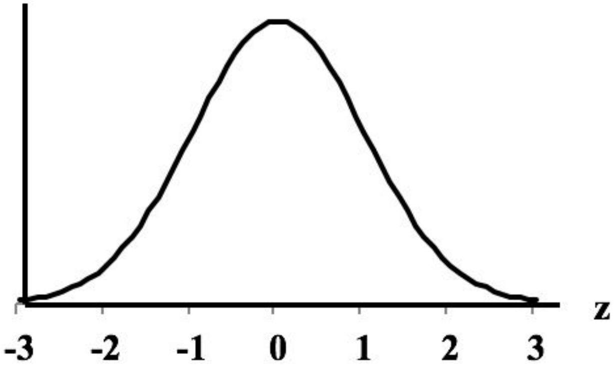
Probability Distribution

We have two type of Probability Distribution.

1) Discrete
2) Continuous



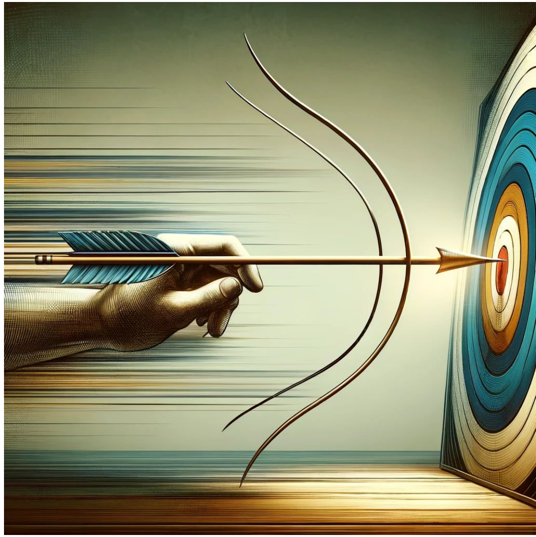
Binomial Distribution



Standard Normal Distribution

Binomial distribution

- In many situations, an experiment has only two outcomes: **success** and **failure**.
    - Such outcome is called **dichotomous** outcome.



- An experiment when consists of repeated trials
  Each with **dichotomous** outcome is called **Bernoulli process**.
  Each trial in it is called a **Bernoulli** trial.
- Example: Getting Head or Tails

Binomial distribution

Suppose, in a Bernoulli process, we define a random variable

X =the number of successes in trials.

Binomial distribution

Suppose, in a Bernoulli process, we define a random variable

X =the number of successes in trials.
X obeys the binomial probability distribution, if the experiment satisfies the following conditions:

- The experiment consists of n identical trials.
- Each trial results in one of two mutually exclusive outcomes, Success/Failure, Head/Tail, Yes/No, Do/Don't
- The probability of a success on a single trial is equal to **p**.
- The value of **p** remains constant throughout the experiment.
- The trials are independent.

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Binomial distribution

- The function for computing the probability for the binomial probability distribution is given by

$$P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$ for x = 0, 1, 2, ...,n

Where,
n is the number of trail
x is the number of success desired
p is the probability of getting a success in one trial
q is the probability of getting a failure in one trial

Binomial distribution

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for x = 0, 1, 2, ...,n

- The trials are independent.
- If sampling is done without replacement, draws are not independent, so resulting distribution is a **Hypergeometric distribution** and not **Binomial**.

Example.

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👉 Find the probability of getting at least 5 times head-on tossing an unbiased coin for 6 times by using the binomial distribution.

Example.

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👉 Find the probability of getting at least 5 times head-on tossing an unbiased coin for 6 times by using the binomial distribution.

💡 Here,

p = P(getting an head in a single toss) = ½

q = P(not getting an head in a single toss) = ½

X = successfully getting a head

$P(X \geq 5)$ = P(getting at least 5 heads) = $P(X = 5) + P(X = 6)$

$= {}^6C_5 (1/2)^5 (1/2)^{(6-5)} + {}^6C_6 (1/2)^6 (1/2)^{(6-6)}$

$= 6 \times (1/2)^6 + 1 \times (1/2)^6 = 7/24.$

Hence, the probability of getting at least 5 heads is 7/24.

Example.

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Negative Binomial Experiments

Consider an experiment where the properties are the same as those listed for a Binomial Experiment.

Negative Binomial Experiments

Consider an experiment where the properties are the same as those listed for a Binomial Experiment.

Only exception here is that the **trials will be repeated until a fixed number of successes occur.**

For Example, You flip a coin repeatedly and count the number of times the coin lands on heads. You continue flipping the coin until it has landed 5 times on heads.

This is known as **Negative Binomial Experiments**

Difference between Binomial Distribution and Negative Binomial Distribution

Binomial Distribution	Negative Binomial Distribution
Used to compute probability of x successes in n trials, where n is fixed.	Used to compute probability of k^{th} success occurs on the x^{th} trial

Negative Binomial Formula

If repeated independent trials can result in a success with probability p and a failure with probability q (= 1 − p), then the probability distribution of the random variable X, the number of the trial on which the kth success occurs, is

$$P(X=n) = {}^{(r-1)}C_{(n-1)} p^r q^{n-r} = {}^{(r-1)}C_{(n-1)} p^r (1-p)^{n-r}$$

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Is the formula seems **familiar**??

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Yes, It's a variant of Binomial Formula.
But **Why**?

Negative Binomial Formula

$P(X=n) = {}^{(n-1)}C_{(r-1)} p^r (1-p)^{n-r}$

Let’s say, We need to find the probability of getting 3 Heads in 5 turn.

Negative Binomial Formula

$P(X=n) = {}^{(n-1)}C_{(r-1)} p^r (1-p)^{n-r}$

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So,

				Head

This one is Fixed

Negative Binomial Formula

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Let's say, We need to find the probability of getting 3 Heads in 5 turn.

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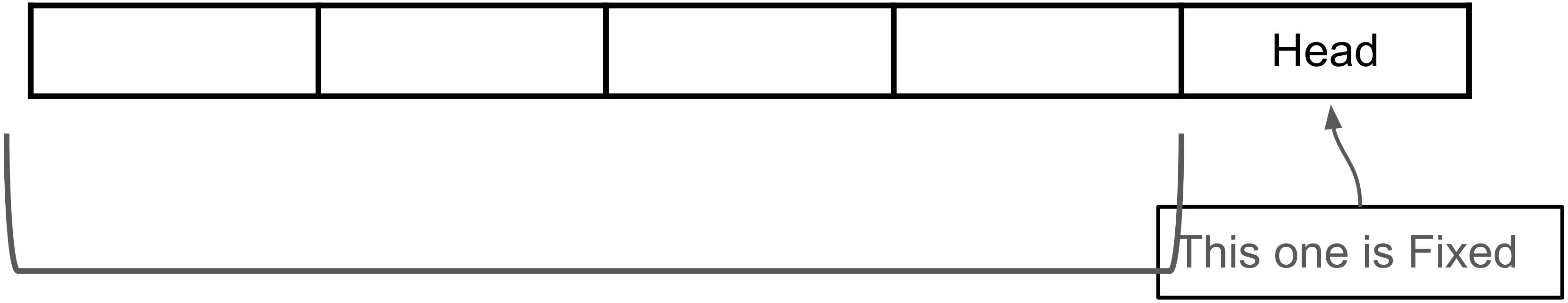

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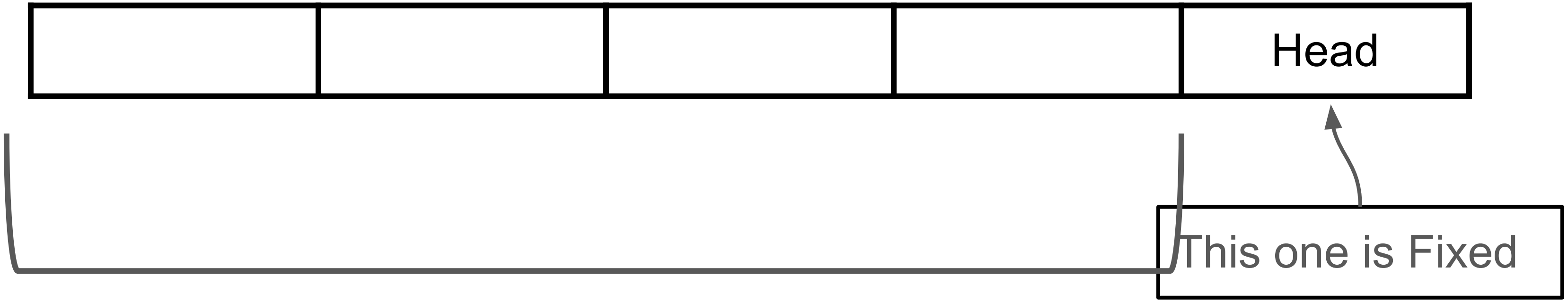

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Probability of success P(s) = 60% = 0.6, Probability of failure P(f) = 40% = 0.4.

And, we have $x = 5$, $r = 3$, $P = 0.6$, $q = 0.4$

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$= {}^{(5-1)}C_{(3-1)}. (0.6)^3.(0.4).$

$= {}^{4}C_{2}.(0.6)^3.(0.4)^2 = 0.020736$

Therefore the probability of Jim giving the third correct answer for his fifth attempted question is 0.02.

Geometric Distribution

Again, Consider an experiment where the properties are the same as those listed for a Binomial Experiment.

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And, The probability of getting first success occurs after k^{th} number of trials.

This is called **Geometric Distribution**

Geometric Distribution

If repeated independent trials can result in a success with probability p and a failure with a probability $q = 1 - p$, then the probability distribution of the random variable X, the number of the trial on which the first success occurs, is

$$g(x;p) = pq^{x-1}; \; x = 1,2,3....$$

Problem

$P(X=n) = pq^{x-1}$

👉 For a certain manufacturing process, it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found?

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Using the geometric distribution with

$x = 5$, and $p = 0.01$, we have

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Problem

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$$\begin{aligned} g(5;0.01) &= pq^{x-1} \\ &= 0.01(1-0.01)^{5-1} \\ &= 10^{-10} \end{aligned}$$

Poisson Distribution

There are some experiments, which involve the occurring of the number of outcomes during a given time interval (or in a region of space).

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For example, How many telephone call arrive at a reception?

Poisson Distribution

The probability of the Poisson distribution is:

$P(X = k) = e^{-\lambda t}.(\lambda t)^{x} /x!$, $x = 0,1,2,...$

Where:

- X is a random variable following a Poisson distribution
- k is the number of times an event occurs
- P(X = k) is the probability that an event will occur k times
- e is Euler's constant (approximately 2.718)
- λ is the average number of times an event occurs

Problem

$P(X = k) = e^{-\lambda t}.(\lambda t)^x / x!$

👉 Electrical power failure at a particular location occurs with an average frequency of 1 in 24 hours. What is the probability that there will be 2 or more failures in 24 hours?

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Given, $\lambda = 1/24$ and $t=24$.

Now, $Pr(X \geq 2) = 1 - Pr(X \leq 1) = = 1 - Pr(X = 0) - Pr(X = 1)$

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So, $Pr(X = 0) = 0.368$, and $Pr(X=1)=0.368$

Therefore, $Pr(X \geq 2) = 1 - Pr(X \leq 1)$

$= 1 - Pr(X = 0) - Pr(X = 1)$

$= 1 - 0.368 - 0.368$

$= 0.264$

Descriptive Measures

Given a random variable X in an experiment, denoted function f(x)

Properties of discrete probability distribution:

- $0 \leq f(x) \leq 1$
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- Variance = $\sigma^2 = \sum (x - \mu)^2 f(x)$

Descriptive Measures of Distributions

Binomial Distribution

Mean: $\mu = n.p$
Variance: $\sigma^2 = n\,p\,(1-p)$

Poisson Distribution

Mean: $\mu = \lambda$
Variance: $\sigma^2 = \lambda$

Descriptive Measures

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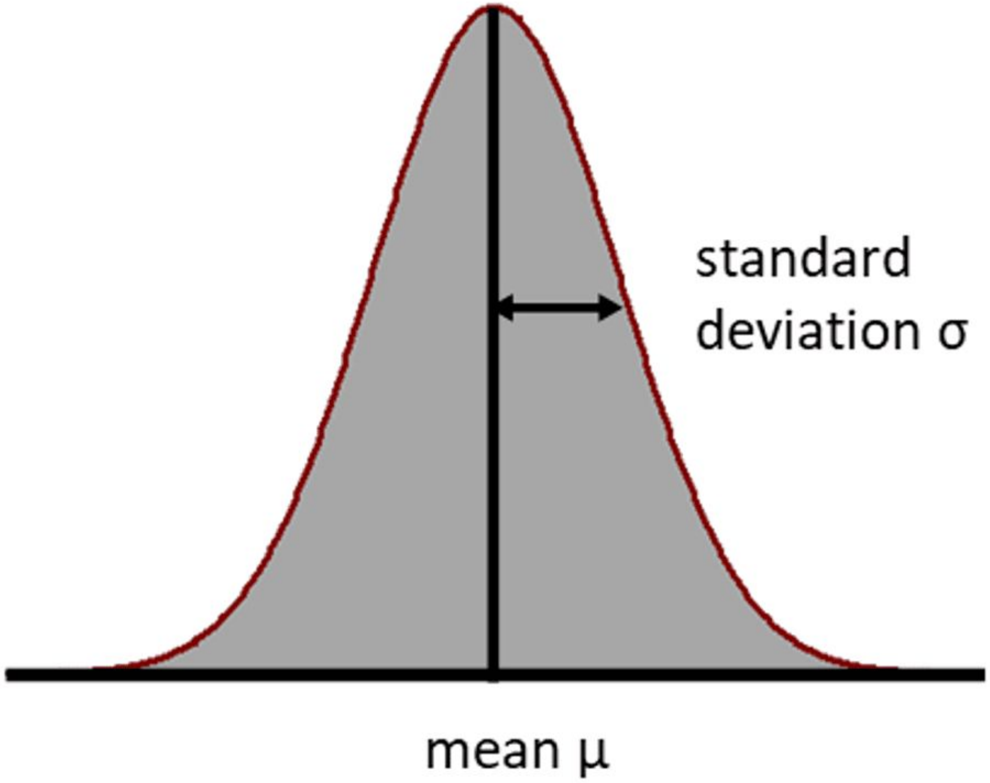
Continuous Probability Distributions



Normal Distribution

- The most often used continuous probability distribution is the **Normal distribution** or **Gaussian distribution**.

- Its graph called the normal curve is the bell-shaped curve.

- Describes many phenomenon that occur in nature, industry and research.

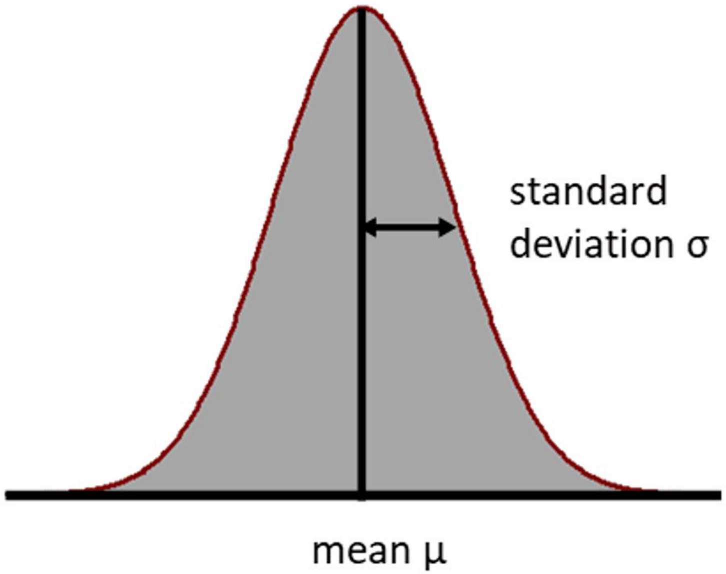
- Example: Meteorological experiments, rainfall studies and measurement of manufacturing parts etc.

Properties of Normal Distribution

- The curve is symmetric about a vertical axis through the mean μ.

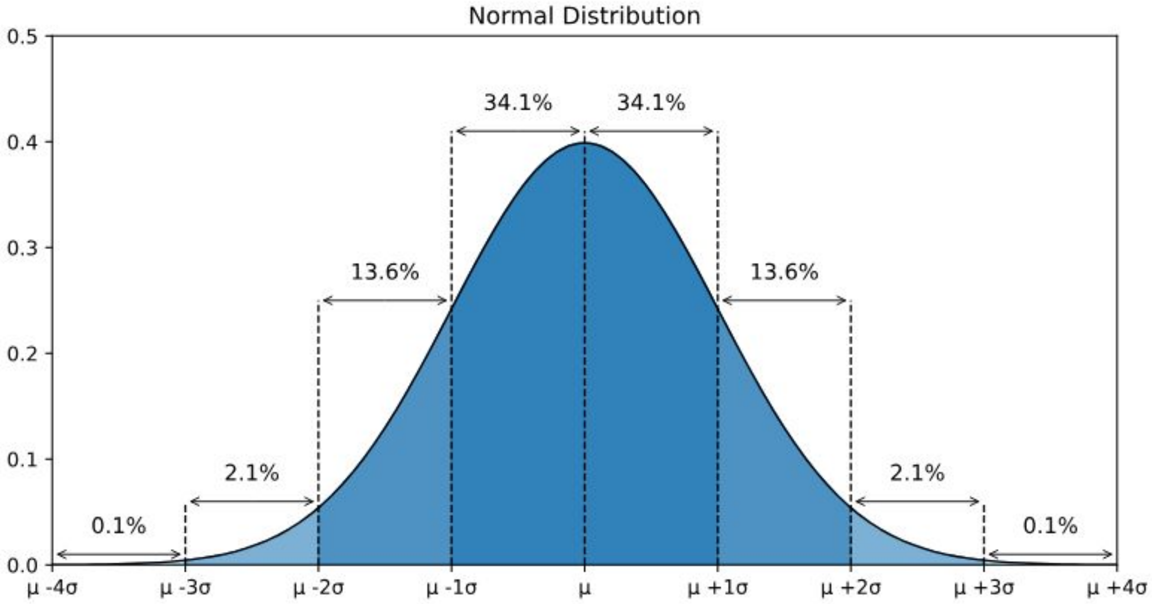
- The mode, which is the point on the horizontal axis where the curve is a maximum occurs at $x = \mu$

- The total area under the curve and above the horizontal axis is equal to 1.



Properties of Normal Distribution

68.2% data will lie in the first variance from the mean
95.4% data will lie in the second variance from the mean
99.6% data will lie in the third variance from the mean



Standard Normal Distribution

The calculation of $P(x1 < x < x2)$ is computationally complex.

To avoid this difficulty, the concept of z −transformation is followed.

Z −Transformation is defined as $z = (x - \mu)/\sigma$

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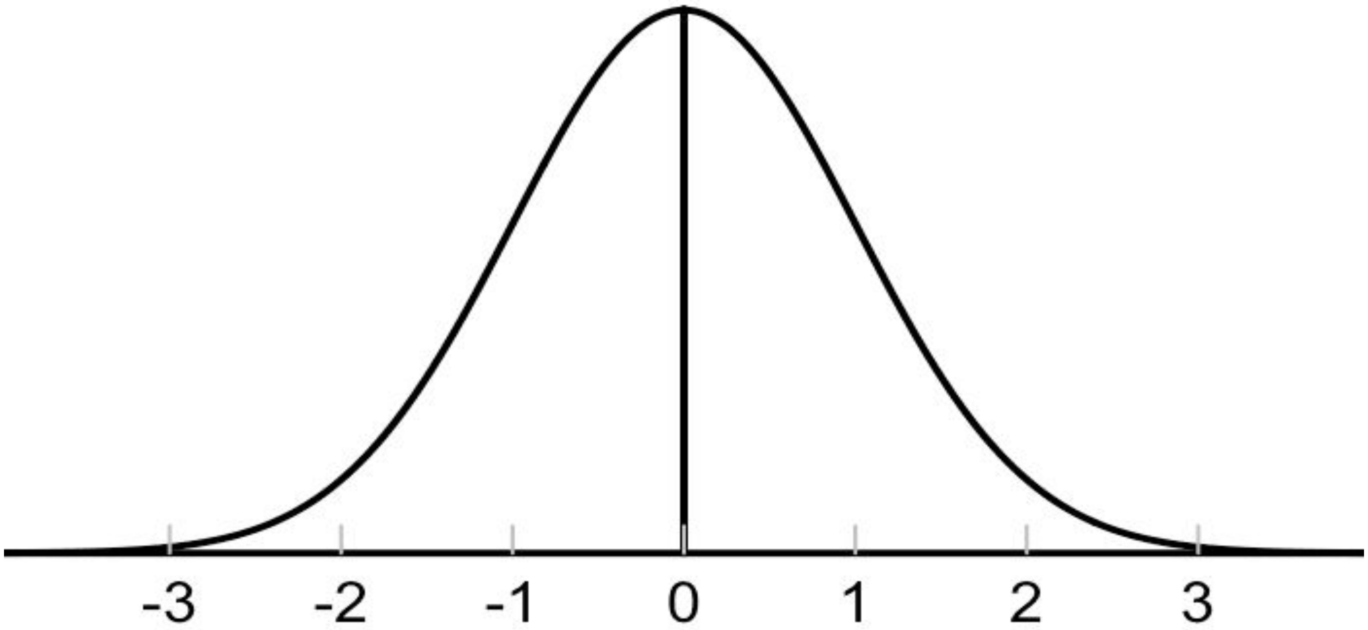
X: Normal distribution with mean μ and variance σ^2.

Z: Standard normal distribution with mean $\mu = 0$ and variance $\sigma^2 = 1$.

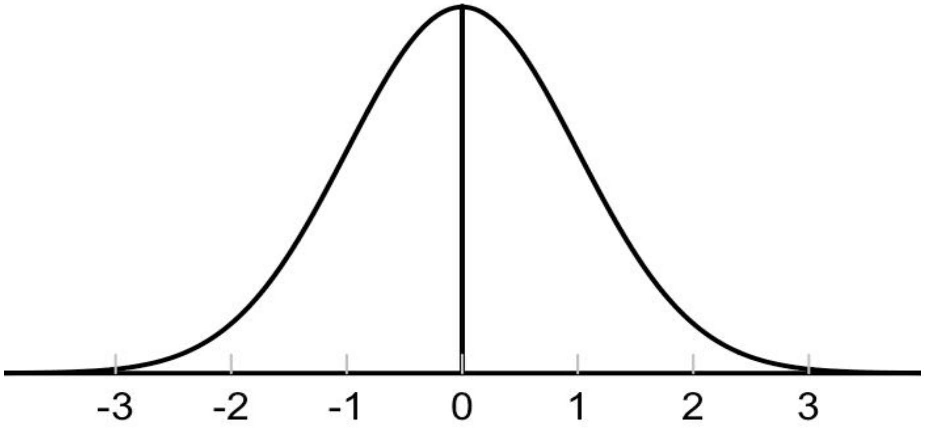
Therefore if X assumes a value x, the corresponding value of z is given by $z = (x - \mu)/\sigma$

Standard Normal Distribution



Problem

👉 Given that X has a normal distribution with $\mu = 300$ and $\sigma = 50$, find the probability that X assumes a value greater than 362.
💡 Here,
The z transformation is, $z_1 = (362 - 300)/50 = 1.24$



Final Thoughts

There are couple of distributions, but those are not important for this course.

1) Hypergeometric distribution - Discrete

2) Gamma Distribution - Continuous

3) Exponential Distribution - Continuous


THANK YOU