

# Propositional Logic

# What is Proposition?

- A statement that is either true or false

$$2 + 3 = 5$$

# What is Proposition?

- A statement that is either true or false  
 $2 + 3 = 5$  : true

# What is Proposition?

- A statement that is either true or false

$$1 + 1 = 3$$

# What is Proposition?

- A statement that is either true or false  
 $1 + 1 = 3$  : false

# What is Proposition?

- A statement that is either true or false  
Dhaka is the capital of Bangladesh

# What is Proposition?

- A statement that is either true or false  
Dhaka is the capital of Bangladesh : true

# What is Proposition?

- A statement that is either true or false  
Chittagong is the capital of Bangladesh :  
false



# What is Proposition?

- A statement that is either true or false  
Give me an A

# What is Proposition?

- A statement that is either true or false

Give me an A

Neither true or false

Not a proposition

# What is Proposition?

- A statement that is either true or false

Would there be a third world war?

Neither true or false

Not a proposition

# Compound Proposition

All humans are mortal and  $2 + 3 = 5$

# Compound Proposition

All humans are mortal and  $2 + 3 = 5$  :

Hard to depict whether true / false

# Compound Proposition

All humans are mortal  
and

$$2 + 3 = 5$$

# Compound Proposition

All humans are mortal : true

and

$2 + 3 = 5$  : true

# Compound Proposition

All humans are mortal : true

and

$2 + 3 = 5$  : true



Logical connector



# Propositional Variable

All humans are mortal

# Propositional Variable

All humans are mortal:  $p$

- $p$  can either be true / false

# Propositional Variable

All humans are mortal  
and

$$2 + 3 = 5$$

$p$  and  $q$

# Logical Connector

- Not
- And
- Or
- Implies
- Xor
- Iff

# Logical Connector: NOT

- Notation:  $\neg$
- Truth table:

p	$\neg p$
0	1
1	0

# Logical Connector: AND

- Notation:  $\wedge$
- Truth table:

$p$	$q$	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

# Logical Connector: OR

- Notation:  $\vee$
- Truth table:

$p$	$q$	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

# Logical Connector: IMPLIES

- Notation:  $\rightarrow$
- Truth table:

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1



# Logical Connector: XOR

- Notation:  $\oplus$
- Truth table:

p	q	p xor q
0	0	0
0	1	1
1	0	1
1	1	0

# Logical Connector: IFF

- Notation:  $\leftrightarrow$
- Truth table:

p	q	p IFF q
0	0	1
0	1	0
1	0	0
1	1	1

# FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS: Example1

$$P \wedge q \oplus s$$

No of propositional variables = 3

No of rows in truth table =  $2^3 = 8$

# FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS

$$P \wedge q \oplus s$$

p	q	s
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

# FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS

$$P \wedge q \oplus s$$

Which operation to perform first?

$$p \wedge q / q \oplus s$$

# FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS

$$P \wedge q \oplus s$$

Which operation to perform first?

$$p \wedge q / q \oplus s:$$

See Precedence Table

# Precedence Table

Connector	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow \oplus$	5

## FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS

$$P \wedge q \oplus s$$

$$= (p \wedge q) \oplus s$$

$$= a \oplus s$$

$$= b$$



# FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS

$$P \wedge q \oplus s = (p \wedge q) \oplus s = a \oplus s = b$$

p	q	s	$p \wedge q = a$	$a \oplus s = b$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	0

## FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS: Example 2

$$(\neg p \leftrightarrow \neg q) \wedge p \rightarrow r$$

$$= a \wedge p \rightarrow r$$

$$= (a \wedge p) \rightarrow r$$

$$= b \rightarrow r$$

# FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS: Example 2

p	q	r	$\neg p$	$\neg q$	$(\neg p \leftrightarrow \neg q)$ = a	$a \wedge p = b$	$b \rightarrow r$
0	0	0	1	1	1	0	1
0	0	1	1	1	1	0	1
0	1	0	1	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	0	1
1	1	0	0	0	1	1	0
1	1	1	0	0	1	1	1

# Propositional Equivalences

# Tautology

$$[\neg p \wedge (p \vee q)] \rightarrow q$$

p	q				$[\neg p \wedge (p \vee q)] \rightarrow q$
0	0				1
0	1				1
1	0				1
1	1				1

# Contradiction

$$\neg([\neg p \wedge (p \vee q)] \rightarrow q)$$

p	q				$\neg([\neg p \wedge (p \vee q)] \rightarrow q)$
0	0				0
0	1				0
1	0				0
1	1				0

# Contingency

$$p \rightarrow q$$

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

# Logical Equivalences

The compound propositions  $p$  and  $q$  are called *logically equivalent* if  $p \leftrightarrow q$  is a tautology.

Notation:  $\equiv$



# Logical Equivalences

Compound propositions that have the same truth values in all possible cases are called **logically equivalent**

## Proving Logical Equivalences: Example 1

Prove that:  $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

# Proving Logical Equivalences: Example 1

Truth table of  $p \vee (q \wedge r)$

p	q	r	$(q \wedge r)$	$p \vee (q \wedge r)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

# Proving Logical Equivalences: Example 1

Truth table of  $(p \vee q) \wedge (p \vee r)$

p	q	r	$(p \vee q)$	$(p \vee r)$	$(p \vee q) \wedge (p \vee r)$
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

## Proving Logical Equivalences: Example 1

Prove that:  $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
0	0
0	0
0	0
1	1
1	1
1	1
1	1
1	1

## Proving Logical Equivalences: Example 2

Prove that:  $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

# Logical Equivalences

$$p \rightarrow q \equiv \neg p \vee q$$

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$p \oplus q \equiv (p \vee q) \wedge (\neg p \vee \neg q)$$

Translating into  
propositional logic



## Translating into propositional logic

if Aang is not in the path of totality during the solar eclipse, Aang won't defeat the firelord.

a: Aang is in the path of totality

b: Aang will defeat the firelord.

$$\neg a \rightarrow \neg b$$

## Translating into propositional logic

if you are a computer science major or you are not a freshman, you can access the Internet from campus

a: you are a computer science major

b: you are a freshman

c: you can access the Internet from campus

$$a \vee \neg b \rightarrow c$$

## Translating into propositional logic

you can access the Internet from campus  
if you are a computer science major or you  
are not a freshman,

a: you are a computer science major

b: you are a freshman

c: you can access the Internet from campus

$$a \vee \neg b \rightarrow c$$

## Translating into propositional logic

you can access the Internet from campus  
**only if** you are a computer science major or you  
are not a freshman,

**a:** you are a computer science major

**b:** you are a freshman

**c:** you can access the Internet from campus

$$c \rightarrow a \vee \neg b$$

## Translating into propositional logic

you can access the Internet from campus  
only if you are a computer science major but you  
are not a freshman,

a: you are a computer science major

b: you are a freshman

c: you can access the Internet from campus

$$a \wedge \neg b \rightarrow c$$

## Translating into propositional logic

you can access the Internet from campus  
iff you are a computer science major but you  
are not a freshman,

a: you are a computer science major

b: you are a freshman

c: you can access the Internet from campus

$$a \wedge \neg b \leftrightarrow c$$

## Translating into propositional logic

you can access the Internet from campus  
if and only if you are a computer science major  
but you are not a freshman,

a: you are a computer science major

b: you are a freshman

c: you can access the Internet from campus

$$a \wedge \neg b \leftrightarrow c$$

# Negation of propositional logic



## De Morgan's Las

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

# Negating propositions

Find the negation of  $(p \vee (\neg p \wedge q))$

$$\neg (p \vee (\neg p \wedge q))$$

$$\equiv \neg p \wedge \neg (\neg p \wedge q)$$

$$\equiv \neg p \wedge (\neg (\neg p) \vee \neg q)$$

$$\equiv \neg p \wedge (p \vee \neg q)$$

# Negating propositions

Find the negation of  $p \leftrightarrow q$

$$\neg (p \leftrightarrow q)$$

$$\equiv \neg (p \rightarrow q \wedge (q \rightarrow p))$$