Root of an Equation

Mathematical Background: Different Equations

Algebraic Equation

The equations of the form $f(x) = 0$ where $f(x)$ is purely a polynomial in x, e.g. $x^6 - x^2 - x - 1 = 0$

Transcendental equation

The equations of the form $f(x) = 0$ where $f(x)$ involves trigonometrical, arithmetic or exponential terms in it, e.g. $xe^x - x = 0$

Mathematical Background: Basic Properties of Algebraic eqn.

- If $f(x)$ is divisible by $(x-a)$ then a is a root of $f(x)$
- Every algebraic equation of nth degree has n and only n real or imaginary roots.
- If $f(x)$ is continuous in the interval $[a, b]$and $f(a), f(b)$ have different signs, then the equation have at least one root between $x = a$ and $x = b$ (Intermediate value theorem)

Task: Numerical Computation of roots

Given $f(x) = 0$, determine the numerical value of a **single** real root on the basis of foreknowledge of its approximate location.

Numerical value of the root = Some approximate value of the root which satisfies our need without much change in its basic characters

How to find numerical a numerical root?

generally start with rough estimate of the root and the iterate it for better approximations

Methods for numerical computation of roots

Bracketing Method:

- Two initial guesses are required to bracket the root ($f(x)$ changes sign between the estimates)
- Root essentially converges as we move closer and closer to the root in each iteration

Open Method:

- Single or Two initial guesses are required and they necessarily do not bracket the root
- Root sometimes diverge or move away from the true root as the computation progresses

  But, when it converges it is much quicker than Bracketing method