Predicate Logic

Predicates

Proposition with variables.

Example:

$P(x) ::= [x > 3]$

$P(x, y) ::= [x + 2 = y]$

Quantifiers

$\forall x$ For ALL x
$\exists y$ There EXISTS some y

∀ is like AND

Let x range over set {1, 2, 3, 4, 5, 6}
P(x) ::= [x > 3]

$$\forall x.\ P(x)$$

same as P(1) AND P(2) AND P(3) AND P(4) AND P(5) AND P(6)

∃ is like OR

Let x range over set {1, 2, 3, 4, 5, 6}
P(x) ::= [x > 3]

$$\exists x.\ P(x)$$

same as P(1) OR P(2) OR P(3) OR P(4) OR P(5) OR P(6)

Nested Quantifier

$x, y \in \mathbb{R}$

$Q(x, y) ::= [x + y = 0]$

$\forall x \exists y \; Q(x, y) = ?$

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{N} = \{1, 2, \dots\}$

$Q(x, y) ::= [x + y = 0]$

$\forall x \exists y\ Q(x, y) = ?$

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{N} = \{1, 2, \dots\}$

$Q(x, y) ::= [x + y = 0]$

$\exists x \forall y \; Q(x, y) = ?$

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{N} = \{1, 2, \dots\}$

$Q(x, y) ::= [x * y = 0]$

$\exists x \forall y \; Q(x, y) = ?$

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{R}$

$Q(x, y) ::= [(x * y)^2 \geq 0]$

$\forall x \forall y\ Q(x, y)$ = ?

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{R}$

$Q(x, y) ::= [x^2 + y^2 = 13]$

$\exists x \exists y\ Q(x, y)$ = ?

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{R}$

$Q(x, y) ::= [x^2 + y^2 = -6]$

$\exists x \exists y\ Q(x, y)$ = ?

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{R}$, $z \in \mathbb{R}$

$Q(x, y, z) ::= [x + y = z]$

$\forall x \, \forall y \, \exists z \; Q(x, y, z) = ?$

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{R}$, $z \in \mathbb{R}$

$Q(x, y, z) ::= [x + y = z]$

$\exists x \, \forall y \forall z \; Q(x, y, z) = ?$

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{R}$, $z \in \mathbb{R}$

$Q(x, y, z) ::= [x = yz]$

$\forall x \exists y \exists z \; Q(x, y, z) = ?$

Negating Nested Quantifiers

Negating nested quantifiers

Find the negation of $\forall x \exists y \exists z\ Q(x, y, z)$, where $Q(x, y, z) ::= [x = yz]$

$\neg (\forall x \exists y \exists z\ Q(x, y, z))$

$\equiv \exists x\ \neg(\exists y \exists z\ Q(x, y, z))$

$\equiv \exists x \forall y\ \neg(\exists z\ Q(x, y, z))$

$\equiv \exists x \forall y \forall z\ \neg(Q(x, y, z))$

Negating nested quantifiers

$\equiv \exists x \forall y \forall z \neg(x = yz)$

$\equiv \exists x \forall y \forall z\ (x \neq yz)$

Negating nested quantifiers

Find the negation of $\forall x \exists y \exists z\ Q(x, y, z) \vee P(x, y, z)$, where

$Q(x, y, z) ::= [x = yz]$

$P(x, y, z) ::= [x + y > z]$

$$\exists x \forall y \forall z\ (x \neq yz) \wedge (x + y \leq z)$$