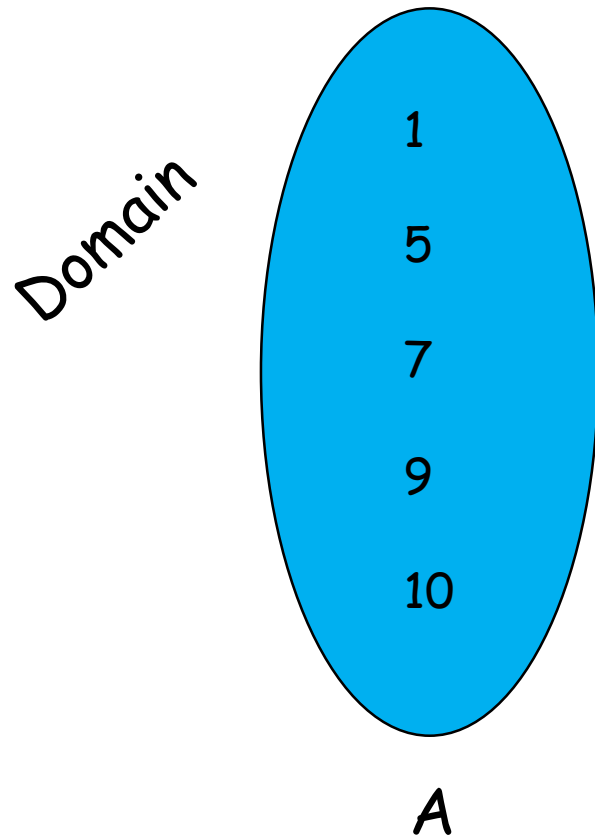


Relations

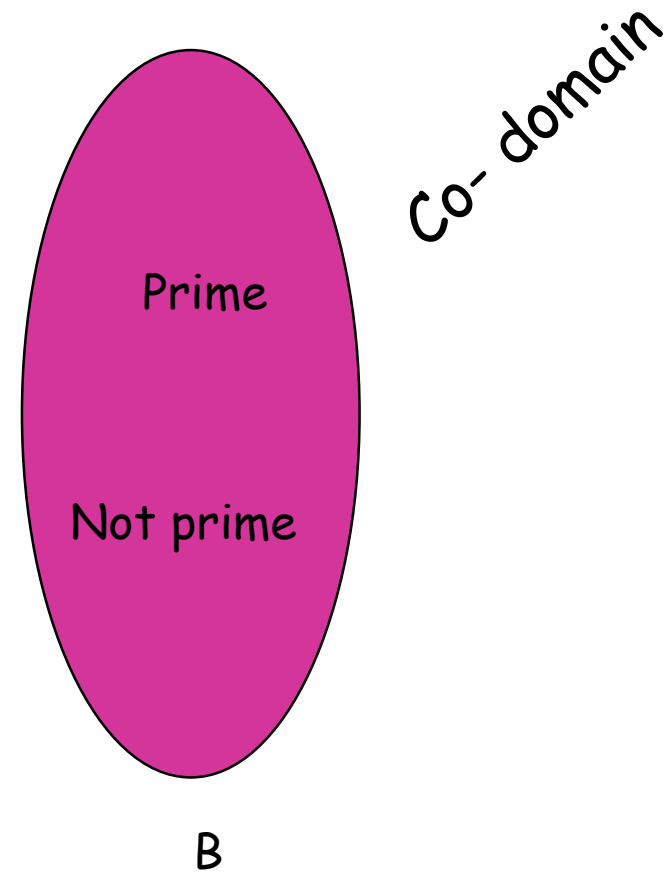
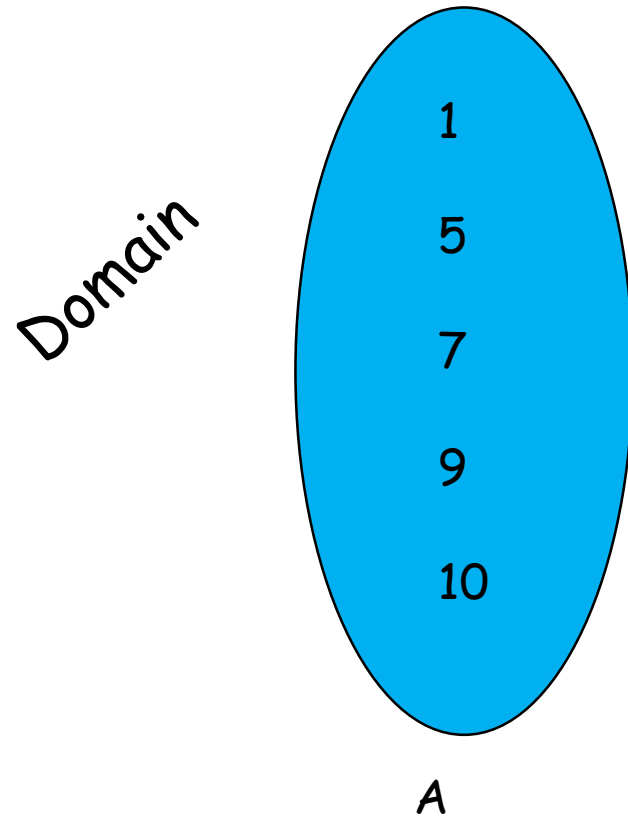
Binary Relation

$A = \{1, 5, 7, 9, 10\}$: Domain

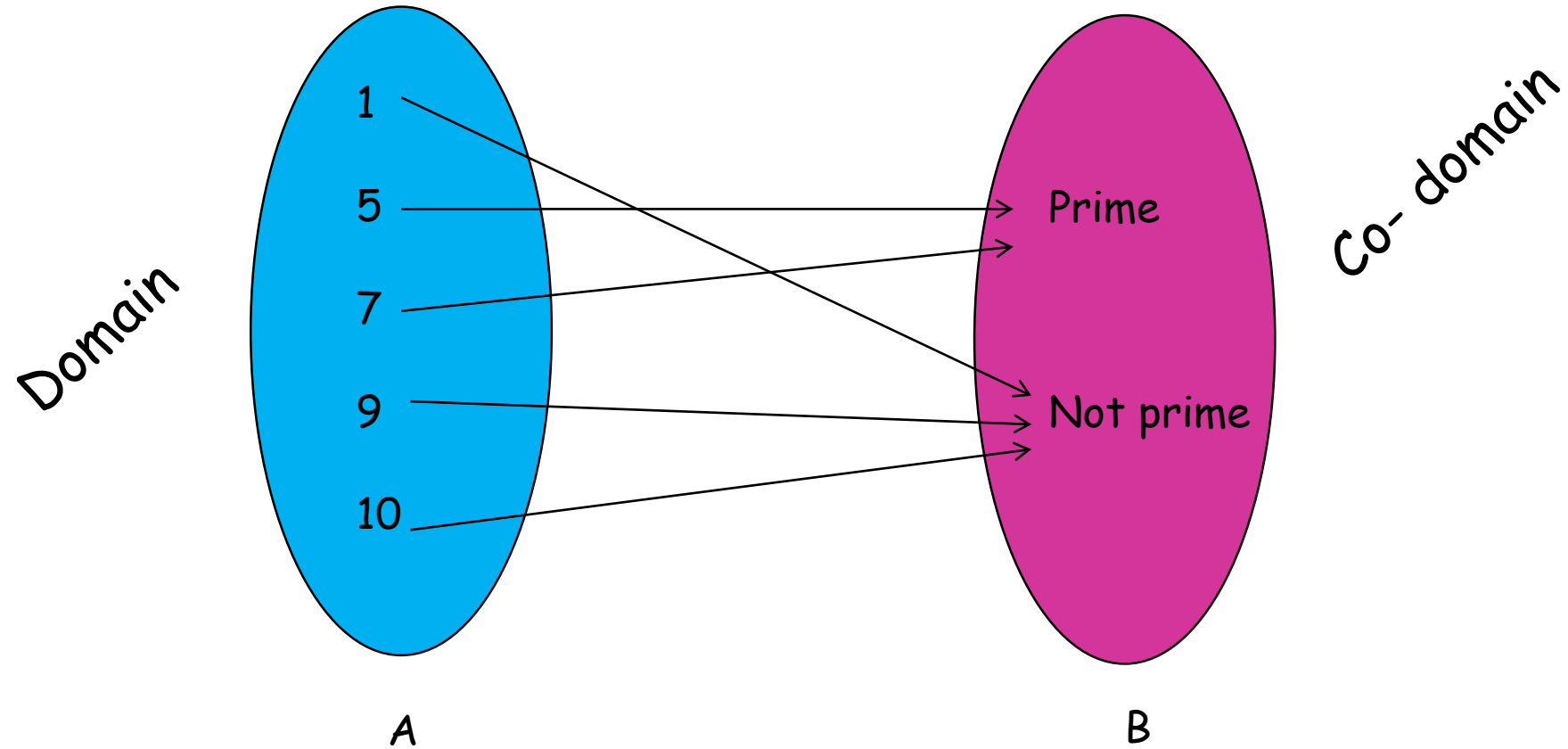


Binary Relation

$B = \{\text{prime, not prime}\}$: Co-domain

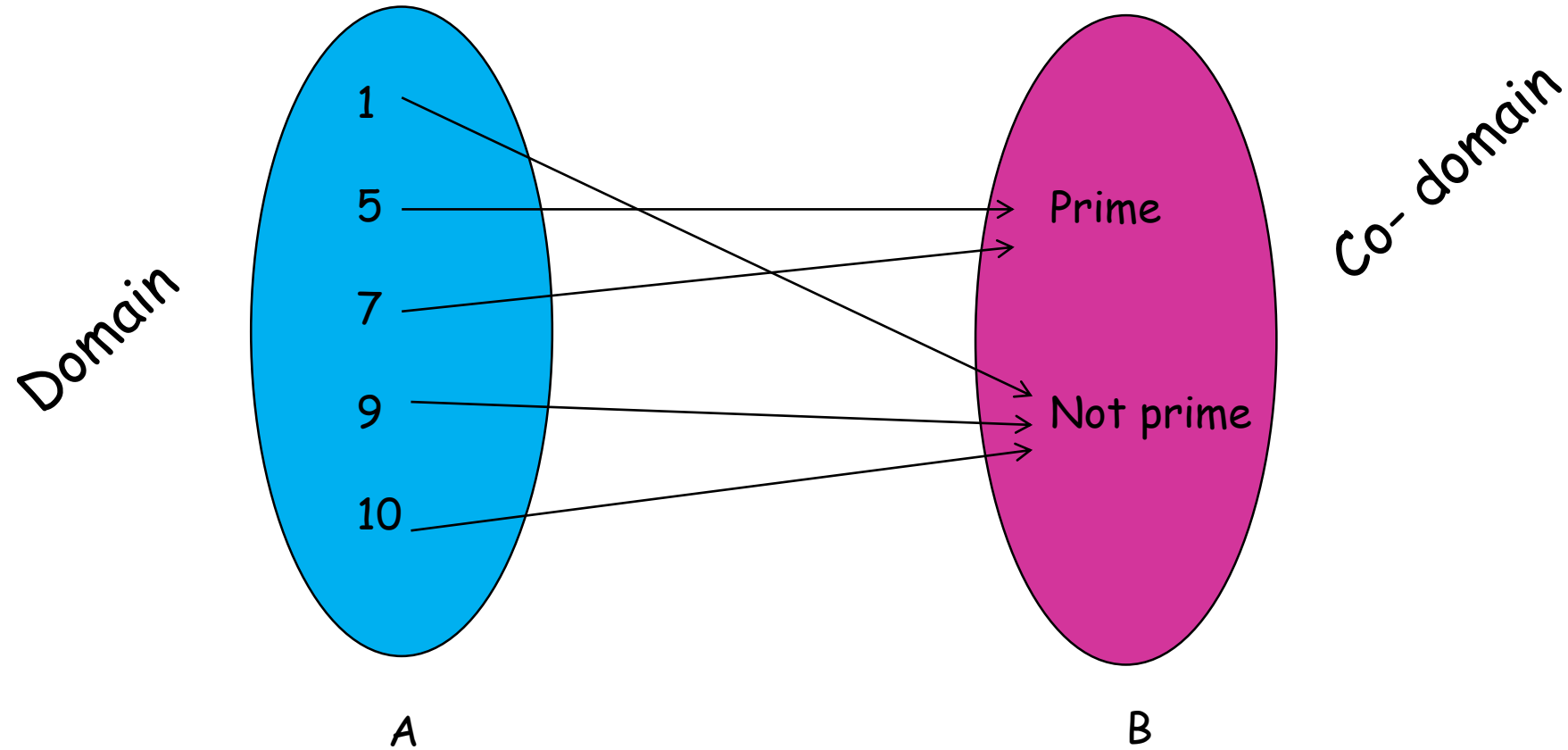


Binary Relation



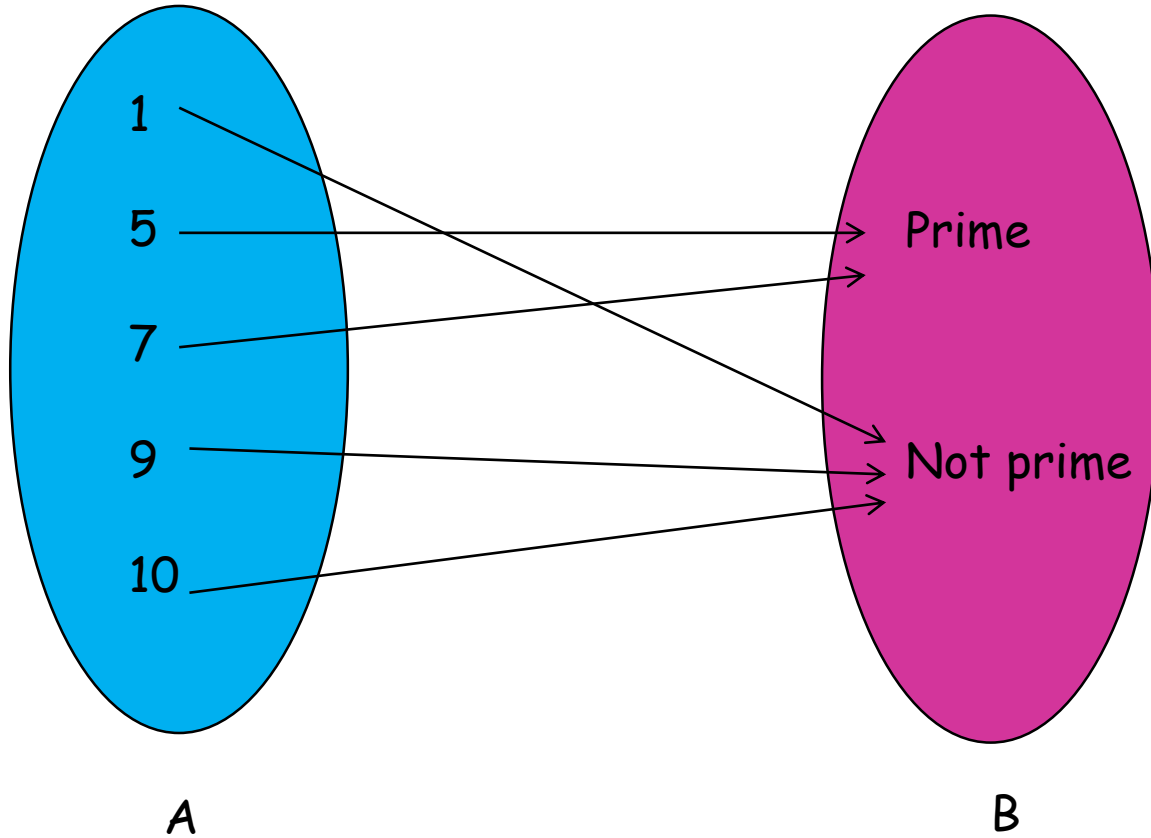
Binary Relation

$R = \text{Is } a \in A \text{ prime or not?}$



Binary Relation

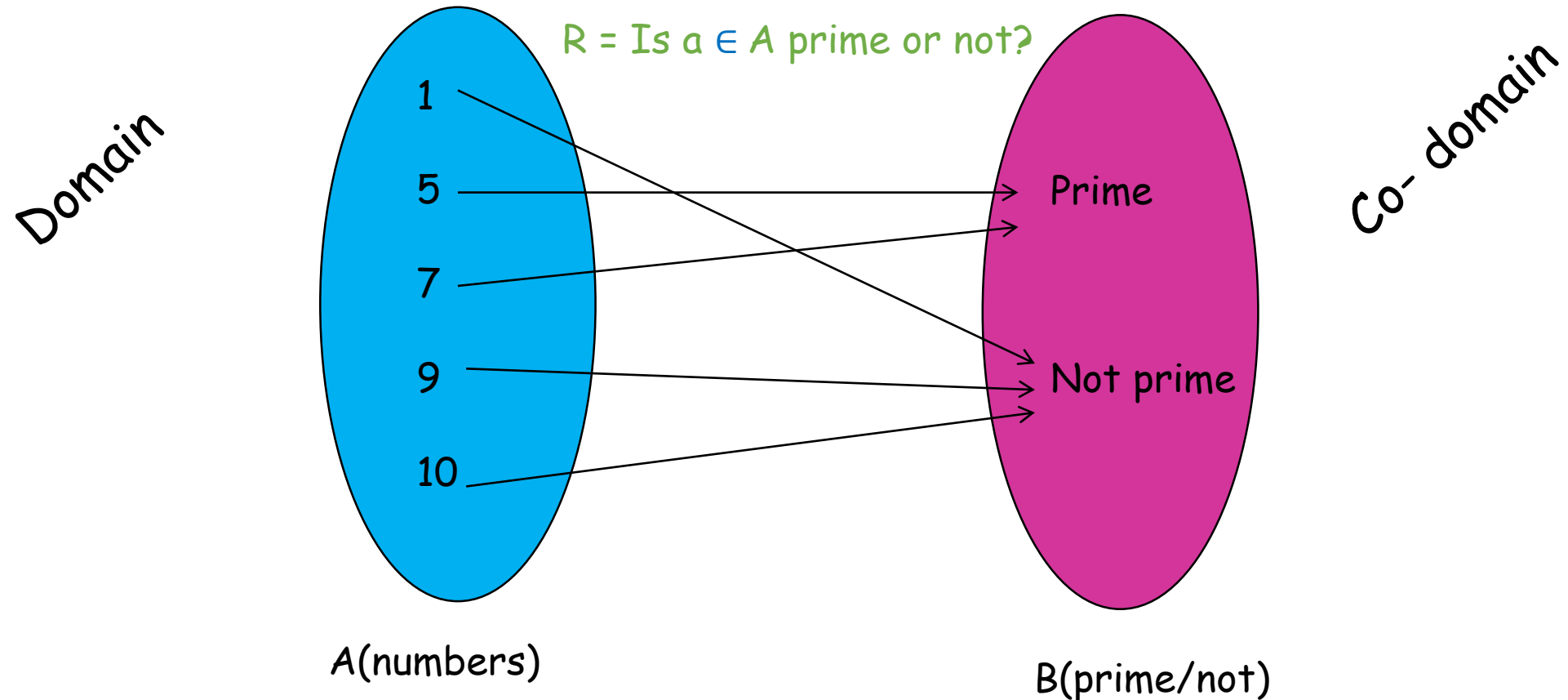
$R = \text{Is } a \in A \text{ prime or not?}$



$R = \{(1, \text{Not prime}),$
 $(5, \text{prime}),$
 $(7, \text{prime}),$
 $(9, \text{Not prime}),$
 $(10, \text{Not prime})\}$

Binary Relation

A binary relation associates elements of one set called domain, with element of another set called co-domain

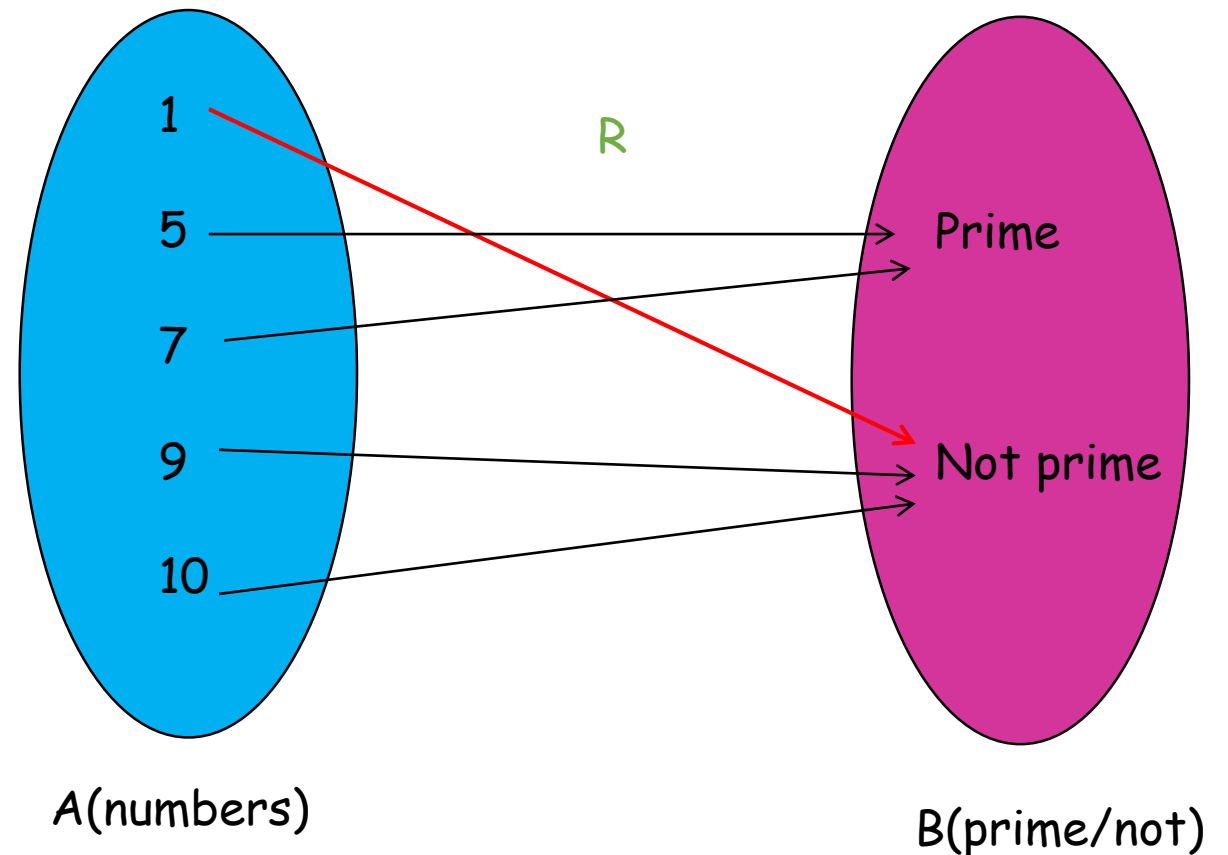


"Is prime/ not" relation R

Notation:

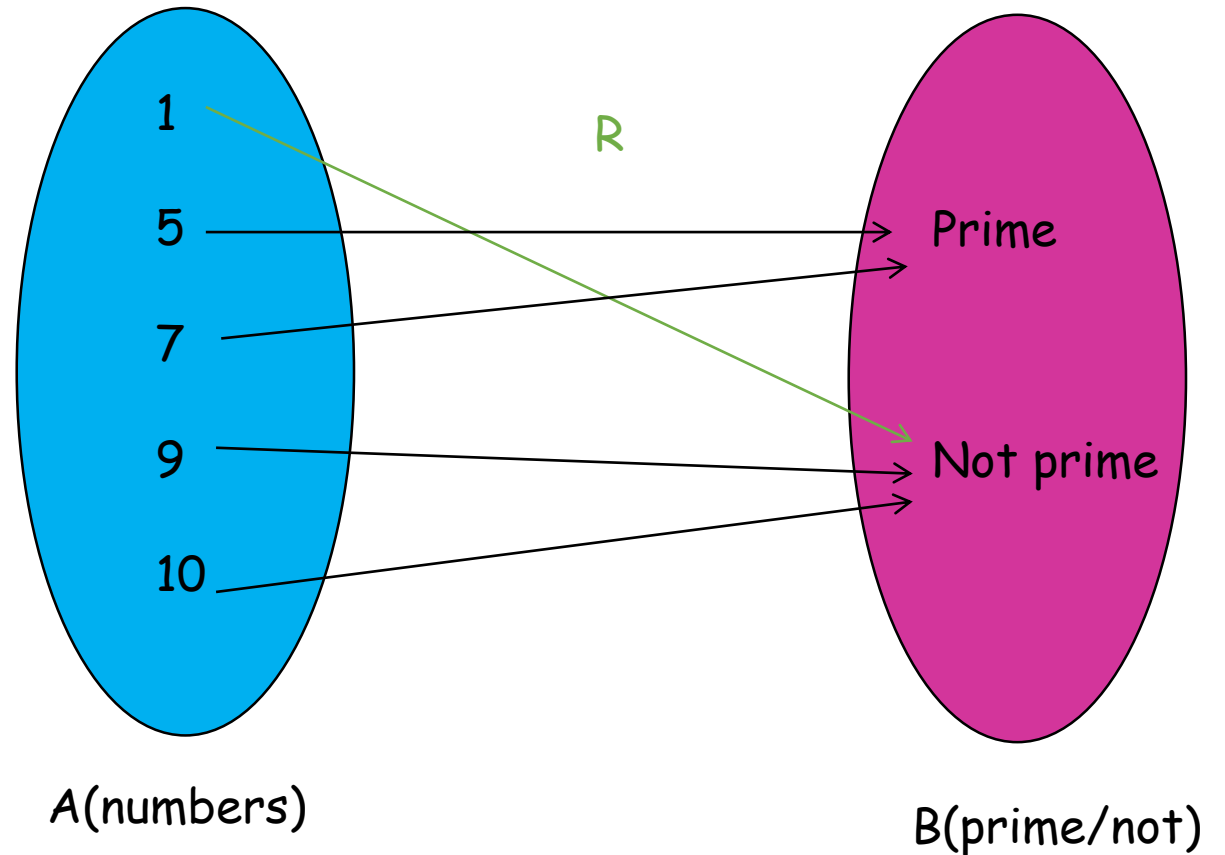
$1 R \text{ prime}$

$R(1, \text{prime})$



Images under R

$$R(1) = \{\text{not prime}\}$$



Binary Relation

A binary relation from set A (Domain) to B (Co-domain) is a subset of $A \times B$

Relation on a set

A relation on a set A is a relation from A to A .

Relation on a set

Let A be the set $\{1, 2, 3, 4\}$. Which ordered pairs are in the relation $R = \{(a, b) \mid a \text{ divides } b\}$?

$$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$$

$$R = \{ \hspace{15em} \}$$

Relation on a set

Let A be the set $\{1, 2, 3, 4\}$. Which ordered pairs are in the relation $R = \{(a, b) \mid a \text{ less equal } b\}$?

$$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$$

$$R = \{ \hspace{15em} \}$$

Relation on a set

Let A be the set $\{1, 2, 3, 4\}$. Which ordered pairs are in the relation $R = \{(a, b) \mid a = b\}$?

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$

$R = \{ \hspace{15em} \}$

Properties of Relations

Reflexive Relation

A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.

Reflexive Relation

A relation R on a set A is called **reflexive** if $(a, a) \in R$ for every element $a \in A$.

$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4), (5, 5)\}$,
is a relation on $A = \{1, 2, 3, 4, 5\}$

Reflexive Relation

A relation R on a set A is called **reflexive** if $(a, a) \in R$ for every element $a \in A$.

$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4), (5, 5)\}$,
is a relation on $A = \{1, 2, 3, 4, 5\}$

$(1, 1) \in R$

$(2, 2) \in R$

$(3, 3) \in R$

$(4, 4) \in R$

$(5, 5) \in R$

reflexive relation

Reflexive Relation

A relation R on a set A is called **reflexive** if $(a, a) \in R$ for every element $a \in A$.

$R1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

$(1, 1) \in R$

$(2, 2) \in R$

$(3, 3) \notin R$

$(4, 4)$

Not a reflexive relation

Symmetric Relation

A relation R on a set A is called **symmetric** if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

Symmetric Relation

A relation R on a set A is called **symmetric** if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, a)
(1, 1)	
(1, 2)	
(2, 1)	
(2, 2)	
(3, 4)	
(4, 1)	
(4, 4)	

Symmetric Relation

A relation R on a set A is called **symmetric** if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, a)
(1, 1)	(1, 1)
(1, 2)	(2, 1)
(2, 1)	(1, 2)
(2, 2)	(2, 2)
(3, 4)	(4, 3)
(4, 1)	
(4, 4)	

$\in R$

$\in R$

$\in R$

$\in R$

$\notin R$

Not symmetric

Symmetric Relation

$R_2 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4), (5, 5)\}$, is a relation on $A = \{1, 2, 3, 4, 5\}$

(a, b)	(b, a)
----------	----------

A relation R on a set A is called **symmetric** if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

Antisymmetric Relation

A relation R on a set A such that for all $a, b \in A$, if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ is called **antisymmetric**.

Antisymmetric Relation

A relation R on a set A such that for all $a, b \in A$, if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ is called **antisymmetric**.

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, a)
(1, 1)	
(1, 2)	
(2, 1)	
(2, 2)	
(3, 4)	
(4, 1)	
(4, 4)	

Antisymmetric Relation

A relation R on a set A such that for all $a, b \in A$, if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ is called **antisymmetric**.

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, a)
(1, 1)	
(1, 2)	(2, 1)
(2, 1)	
(2, 2)	
(3, 4)	
(4, 1)	
(4, 4)	

pass
 $\in R$

Not antisymmetric

Antisymmetric Relation

A relation R on a set A such that for all $a, b \in A$, if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ is called **antisymmetric**.

$R_2 = \{(1, 1), (1, 2), (1, 4), (2, 2), (3, 3), (4, 4), (5, 5)\}$, is a relation on $A = \{1, 2, 3, 4, 5\}$

(a, b)	(b, a)
$(1, 1)$	
$(1, 2)$	$(2, 1)$
$(1, 4)$	
$(2, 2)$	
$(3, 3)$	
$(4, 4)$	
$(5, 5)$	

pass

$\notin R$

pass

pass

pass

pass

pass

antisymmetric

Transitive Relation

A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

Transitive Relation

A relation R on a set A is called **transitive**

if whenever $(a, b) \in R$ and $(b, c) \in R$,
then $(a, c) \in R$, for all $a, b, c \in A$.

$R1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

[illegible]

Transitive Relation

A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, c)	(a, c)
(1, 1)	(1, 1)	
	(1, 2)	

$(1, 1) \rightarrow (a, b)$

$a = 1, b = 1$

Find the (b, c) s \rightarrow find those tuples that start with 1s

Transitive Relation

A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, c)	(a, c)
(1, 1)	(1, 1)	(1, 1)
	(1, 2)	(1, 2)

Transitive Relation

A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, c)	(a, c)
(1, 1)	(1, 1)	(1, 1)
	(1, 2)	(1, 2)

$\in R$

$\in R$

Transitive Relation

A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, c)	(a, c)
(1, 1)	(1, 1)	(1, 1)
	(1, 2)	(1, 2)
(1, 2)	(2, 1)	(1, 1)
	(2, 2)	(1, 2)

$\in R$

$\in R$

$\in R$

$\in R$

Transitive Relation

A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, c)	(a, c)
(1, 1)	(1, 1)	(1, 1)
	(1, 2)	(1, 2)
(1, 2)	(2, 1)	(1, 1)
	(2, 2)	(1, 2)

$\in R$

$\in R$

$\in R$

$\in R$

Transitive Relation

$R1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

Not transitive

(a, b)	(b, c)	(a, c)
(1, 1)	(1, 1)	(1, 1)
	(1, 2)	(1, 2)
(1, 2)	(2, 1)	(1, 1)
	(2, 2)	(1, 2)
(2, 1)	(1, 1)	(2, 1)
	(1, 2)	(2, 1)

$\in R$
 $\in R$
 $\in R$
 $\in R$
 $\in R$
 $\in R$

(a, b)	(b, c)	(a, c)
(2, 2)	(2, 1)	(2, 1)
	(2, 2)	(2, 2)
(3, 4)	(4, 1)	(3, 1)
	(4, 4)	(1, 2)
(2, 1)	(1, 1)	(2, 1)
	(1, 2)	(2, 1)

$\in R$
 $\in R$
 $\notin R$