Open methods

Fixed Iterative method

Algorithm: Fixed Iterative Method

- Rearrange the function so that *x* is on the left side of the equation:

$$f(x) = 0$$
$$g(x) = x$$
$$x_k = g(x_{k-1}), \text{given } k = 1, 2, 3, \dots ,$$

- Now progressively estimate the value of *x* based on given termination criteria (max no of iterations or $\varepsilon_a \leq \varepsilon_s$) and an initial x (x_i)

Problem: Fixed Iterative Method

- Find the root of $f(x) = e^{-x} - x = 0$ given $x_0 = 0$ and $x_{true} = 0.56714329$

  - Rearranging the function:

$$e^{-x} - x = 0$$
$$x = e^{-x}$$
$$x_{n+1} = e^{-x_n}$$

Problem: Fixed Iterative Method

- Estimating x

Itr_no	x_i	x_{i+1}	ε_a	ε_t
1	0	1.000	1	0.763
2	1	0.368	1.718	0.351
3	0.368	0.692	0.469	0.221
4	0.692	0.500	0.383	0.118
5	0.500	0.606	0.174	0.689

Calculating ε_t, True Percent Relative Error

$$\varepsilon_t = \frac{x_{true} - x_{approximate}}{x_{true}}$$

For itr no 1:

$$x_{true} = 0.56714329$$
$$x_{approximate} = 1.000$$

$$\varepsilon_t = \frac{0.56714329 - 1}{0.56714329} = 0.763$$

Newton-Raphson method

Algorithm: Newton-Raphson method

- progressively estimate the value of *x* based on given termination criteria and an initial x (x_i)

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Problem: Newton Raphson Method

- Find the root of $f(x) = e^{-x} - x = 0$ given $x_0 = 0$ and $x_{true} = 0.56714329$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x) = e^{-x} - x$$
$$f'(x) = -e^{-x} - 1$$

$$x_{i+1} = x_i - \frac{e^{-x_i} - x_i}{-e^{-x_i} - 1}$$

Problem: Newton Raphson Method

- Estimating x

Itr_no	x_i	x_{i+1}	ε_a	ε_t

Secant method

Algorithm: Secant method

- progressively estimate the value of *x* based on given termination criteria and an two initial x (x_{i-1} and x_i)

$$x_{i+1} = x_i - \frac{f(x_i) * (x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

Problem: Secant Method

- Find the root of $f(x) = e^{-x} - x = 0$ given $x_{-1} = 0, x_0 = 1$ and $x_{true} = 0.56714329$

$$x_{i+1} = x_i - \frac{f(x_i) * (x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

Problem: Secant Method

Itr_no	x_{i-1}	x_i	x_{i+1}	ε_a	ε_t
1	0	1.0	0.61270	0.08	
2	1.0	0.61270	0.56384	0.0058	
3	0.61270	0.56834			

Modified Secant method

Algorithm: Modified Secant method

- progressively estimate the value of *x* based on given termination criteria, an initial x (x_i) and a small perturbation fraction (δ)

$$x_{i+1} = x_i - \frac{f(x_i) * \delta x_i}{f(x_i + \delta x_i) - f(x_i)}$$

Problem: Modified Secant Method

- Find the root of $f(x) = e^{-x} - x = 0$ given $x_0 = 1$, $\delta = 0.01$ and $x_{true} = 0.56714329$

$$x_{i+1} = x_i - \frac{f(x_i) * (x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

Problem: Modified Secant Method

Itr_no	x_i	x_{i+1}	ε_a	ε_t
1				
2				
3				