Propositional Logic

What is Proposition?

- *A* statement that is either true or false

$$2 + 3 = 5$$

What is Proposition?

- A statement that is either true or false

$2 + 3 = 5$: true

What is Proposition?

- *A* statement that is either true or false

$$1 + 1 = 3$$

What is Proposition?

- A statement that is either true or false

  $1 + 1 = 3$: false

What is Proposition?

- A statement that is either true or false

  Dhaka is the capital of Bangladesh

What is Proposition?

- A statement that is either true or false

Dhaka is the capital of Bangladesh : true

What is Proposition?

- A statement that is either true or false

Chittagong is the capital of Bangladesh : false

What is Proposition?

- *A* statement that is either true or false

Give me an *A*

What is Proposition?

- A statement that is either true or false

Give me an A

Neither true or false

Not a proposition

What is Proposition?

- A statement that is either true or false

Would there be a third world war?

Neither true or false

Not a proposition

Compound Proposition

All humans are mortal and 2 + 3 =5

Compound Proposition

All humans are mortal and 2 + 3 =5 :
Hard to depict whether true / false

Compound Proposition

All humans are mortal

and

2 + 3 =5

Compound Proposition

All humans are mortal : true

and

2 + 3 =5 : true

Compound Proposition

All humans are mortal : true

and

2 + 3 =5 : true

Logical connector

Propositional Variable

All humans are mortal

Propositional Variable

All humans are mortal: p

- p can either be true / false

Propositional Variable

All humans are mortal

and

2 + 3 =5

p and q

Logical Connector

- Not
- And
- Or
- Implies
- Xor
- Iff

Logical Connector: NOT

- Notation: ¬
- Truth table:

p	¬ p
0	1
1	0

Logical Connector: AND

- Notation: $\wedge$
- Truth table:

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Logical Connector: OR

- Notation: V
- Truth table:

p	q	p V q
0	0	0
0	1	1
1	0	1
1	1	1

Logical Connector: IMPLIES

- Notation: $\rightarrow$
- Truth table:

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Logical Connector: XOR

- Notation: $\oplus$
- Truth table:

p	q	p xor q
0	0	0
0	1	1
1	0	1
1	1	0

Logical Connector: IFF

- Notation: $\leftrightarrow$
- Truth table:

p	q	p IFF q
0	0	1
0	1	0
1	0	0
1	1	1

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS: Example1

$$P \wedge q \oplus s$$

No of propositional variables = 3

No of rows in truth table = 2^3 = 8

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS

$$P \wedge q \oplus s$$

p	q	s
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS

$$P \land q \oplus s$$

Which operation to perform first?

$p \land q$ / $q \oplus s$

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS

$$P \wedge q \oplus s$$

Which operation to perform first?

$p \wedge q$ / $q \oplus s$:

See Precedence Table

Precedence Table

Connector	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$ $\oplus$	5

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS

$P \wedge q \oplus s$

$= (p \wedge q) \oplus s$

$= a \oplus s$

$= b$

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS

$P \wedge q \oplus s = (p \wedge q) \oplus s = a \oplus s = b$

p	q	s	$p \wedge q = a$	$a \oplus s = b$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	0

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS: Example 2

$(\neg p \leftrightarrow \neg q) \land p \rightarrow r$

$= a \land p \rightarrow r$

$= (a \land p) \rightarrow r$

$= b \rightarrow r$

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS: Example 2

p	q	r	¬p	¬q	(¬p ↔ ¬q) = a	a ∧ p = b	b → r
0	0	0	1	1	1	0	1
0	0	1	1	1	1	0	1
0	1	0	1	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	0	1
1	1	0	0	0	1	1	0
1	1	1	0	0	1	1	1

Propositional Equivalences

Tautology

$$[\neg p \land (p \lor q)] \rightarrow q$$

p	q				$[\neg p \land (p \lor q)] \rightarrow q$
0	0				1
0	1				1
1	0				1
1	1				1

Contradiction

$$\neg([\neg p \wedge (p \vee q)] \rightarrow q)$$

p	q				$\neg([\neg p \wedge (p \vee q)] \rightarrow q)$
0	0				0
0	1				0
1	0				0
1	1				0

Contingency

$p \rightarrow q$

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Logical Equivalences

The compound propositions p and q are called *logically equivalent* if $p \leftrightarrow q$ is a tautology.
Notation: $\equiv$

Logical Equivalences

Compound propositions that have the same truth values in all possible cases are called **logically equivalent**

Proving Logical Equivalences: Example 1

Prove that: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Proving Logical Equivalences: Example 1

Truth table of $p \vee (q \wedge r)$

p	q	r	$(q \wedge r)$	$p \vee (q \wedge r)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Proving Logical Equivalences: Example 1

Truth table of $(p \vee q) \wedge (p \vee r)$

p	q	r	$(p \vee q)$	$(p \vee r)$	$(p \vee q) \wedge (p \vee r)$
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

Proving Logical Equivalences: Example 1

Prove that: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
0	0
0	0
0	0
1	1
1	1
1	1
1	1
1	1

Proving Logical Equivalences: Example 2

Prove that: $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

Logical Equivalences

$p \rightarrow q \equiv \neg p \vee q$

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$p \oplus q \equiv (p \vee q) \wedge (\neg p \vee \neg q)$

Translating into propositional logic

Translating into propositional logic

$$p \rightarrow q$$

if *p*, then *q*
p implies *q*
if *p*, *q*
p only if *q*
p is sufficient for *q*

q if *p*
q whenever *p*
q when *p*
q is necessary for *p*
q follows from *p*

Translating into propositional logic

$$p \leftrightarrow q$$

p is necessary and sufficient for *q*
if *p* then *q*, and conversely
p iff *q*

Translating into propositional logic

if Aang is not in the path of totality during the solar eclipse, Aang won't defeat the firelord.

a: Aang is in the path of totality

b: Aang will defeat the firelord.

$$\neg a \rightarrow \neg b$$

Translating into propositional logic

if you are a computer science major or you are not a freshman, you can access the Internet from campus

a: you are a computer science major
b: you are a freshman
c: you can access the Internet from campus

$$a \vee \neg b \rightarrow c$$

Translating into propositional logic

you can access the Internet from campus if you are a computer science major or you are not a freshman,

a: you are a computer science major
b: you are a freshman
C: you can access the Internet from campus

$$a \lor \neg b \rightarrow c$$

Translating into propositional logic

you can access the Internet from campus only if you are a computer science major or you are not a freshman,

a: you are a computer science major
b: you are a freshman
C: you can access the Internet from campus

$$c \rightarrow a \lor \neg b$$

Translating into propositional logic

you can access the Internet from campus only if you are a computer science major but you are not a freshman,

a: you are a computer science major
b: you are a freshman
c: you can access the Internet from campus

$$c \to a \wedge \neg b$$

Translating into propositional logic

you can access the Internet from campus iff you are a computer science major but you are not a freshman,

a: you are a computer science major
b: you are a freshman
C: you can access the Internet from campus

$$c \leftrightarrow a \land \neg b$$

Negation of propositional logic

De Morgan's Las

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Negating propositions

Find the negation of $(p \vee (\neg p \wedge q))$

$\neg (p \vee (\neg p \wedge q))$

$\equiv \neg p \wedge \neg (\neg p \wedge q)$

$\equiv \neg p \wedge (\neg (\neg p) \vee \neg q))$

$\equiv \neg p \wedge (p \vee \neg q)$

Negating propositions

Find the negation of $p \leftrightarrow q$

$\neg (p \leftrightarrow q)$

$\equiv \neg (p \rightarrow q \wedge (q \rightarrow p))$