Solving ODE

Euler’s Method (Euler-Cauchy Method)

$$y_{i+1} = y_i + f(x_i, y_i) * h$$

$f(x_i, y_i) =$ Differential equation evaluated at x_i and y_i

$h =$ Step size

Runge-Kutta Method (2nd order)

$$y_{i+1} = y_i + (a_1 k_1 + a_2 k_2) * h$$

$$k_1 = f(x_i, y_i)$$
$$k_2 = f(x_i + p_1 h, y_i + q_{11} k_1 h)$$

Heun Method ($a_2 = {}^1/_2$)

$$y_{i+1} = y_i + (\frac{1}{2}k_1 + \frac{1}{2}k_2) * h$$

$$k_1 = f(x_i, y_i)$$
$$k_2 = f(x_i + h, y_i + k_1 h)$$

Midpoint Method ($a_2 = 1$)

$$y_{i+1} = y_i + k_2 h$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right)$$

Ralston's Method ($a_2 = {}^2/_3$)

$$y_{i+1} = y_i + (\frac{1}{3}k_1 + \frac{2}{3}k_2) * h$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f\left(x_i + \frac{3}{4}h, y_i + \frac{3}{4}k_1 h\right)$$

Runge-Kutta Method (4th order)

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) * h$$

$$k_1 = f(x_i, y_i)$$
$$k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right)$$
$$k_3 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h\right)$$
$$k_4 = f(x_i + h, y_i + k_3 h)$$