Interpolation

Newton's Divided-Difference Interpolating Polynomials

$$f_n(x) = b_0 + b_1(x - x_0) + \cdots + b_n(x - x_0)(x - x_1)\cdots(x - x_{n-1})$$

$$\begin{aligned} b_0 &= f(x_0) \\ b_1 &= f[x_1, x_0] \\ b_2 &= f[x_2, x_1, x_0] \\ &\vdots \\ b_n &= f[x_n, x_{n-1}, \cdots, x_0] \end{aligned}$$

$$f[x_i, x_j] = \frac{f(x_i) - f(x_j)}{x_i - x_j}$$

$$f[x_i, x_j, x_k] = \frac{f[x_i, x_j] - f[x_j, x_k]}{x_i - x_k}$$

Newton's Divided-Difference Interpolating Polynomials

$$f[x_i, x_j] = \frac{f(x_i) - f(x_j)}{x_i - x_j}$$

First Divided Difference (finite)

$$f[x_i, x_j, x_k] = \frac{f[x_i, x_j] - f[x_j, x_k]}{x_i - x_k}$$

Second Divided Difference (finite)

$$f[x_n, x_{n-1}, \cdots, x_0] = \frac{f[x_n, x_{n-1}, \cdots, x_1] - f[x_{n-1}, \cdots, x_0]}{x_n - x_0}$$

Nth Divided Difference (finite)

Find $f[x_3, x_2, x_1, x_0]$

Step 1:

$$f(x_3) = ?$$
$$f(x_2) = ?$$
$$f(x_1) = ?$$
$$f(x_0) = ?$$

Step 2:

$$f[x_3, x_2\ ] = ?$$
$$f[x_2, x_1\ ] = ?$$
$$f[x_1, x_0\ ] = ?$$

Find $f[x_3, x_2, x_1, x_0]$

Step 3:

$$f[x_3, x_2, x_1\ ] = ?$$
$$f[x_2, x_1, x_0\ ] = ?$$

Step 4:

$$f[x_3, x_2, x_1, x_0] = ?$$

Lagrange Interpolating Polynomial

$$f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

$$L_i(x) = \prod_{j=0, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}$$

Find$L_2(x)$

$$L_i(x) = \prod_{j=0, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}$$

$$L_2(x) = \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

$$L_3(x) = \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)}$$