Sets: Sets Definition

What is a Set?

Informally: *A* set is a collection of mathematical objects or elements.

A set of 4 things

$\{7, \text{“Albert R.”}, \pi/2, T\}$

A set with 4 elements: two numbers, a string, and a Boolean.

Same as

$\{T, \text{“Albert R.”}, \pi/2, 7\}$

-- order doesn't matter

In or not in

An element is either in a set or not in a set

{7, 2} is same as {7, 2, 7}

--No notion of being in
the set more than once

Membership

x is a member of A: $x \in A$

{7, "Albert R.", π/2, T}

T $\in$ {7, "Albert R.", π/2, T}

14/2 $\in$

9 $\notin$

Subset($\subseteq$)

$A \subseteq B$: A is a subset of B

Every element of A is also an element of B:
$\forall x$ [$x \in A$ implies $x \in B$]

$\mathbb{Z} \subseteq \mathbb{R}$, $\{3\} \subseteq \{5,7,3\}$ $A \subseteq A$, $\emptyset \subseteq$ every set

Why $\emptyset \subseteq$ every set?

$\emptyset \subseteq B$:

$$\forall x\ [\ \underbrace{\underbrace{x \in \emptyset}_{\text{false}} \text{ implies } x \in B}_{\text{true}}\ ]$$

Cartesian Product

$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$

(a , b) : ordered 2-tuple
(a1, a2, a3, ..., an): ordered n-tuple

Cartesian Product

$A = \{1, 2, 3\}$

$B = \{a, b\}$

$$A \times B = \{1, 2, 3\} \times \{a, b\}$$
$$= \{ (1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$$

Cartesian Product

$A = \{1, 2, 3\}$

$B = \{a, b\}$

$C = \{p, q\}$

$A \times B \times C = \{1, 2, 3\} \times \{a, b\} \times \{p, q\}$

Cartesian Product

$A = \{1, 2, 3\}$

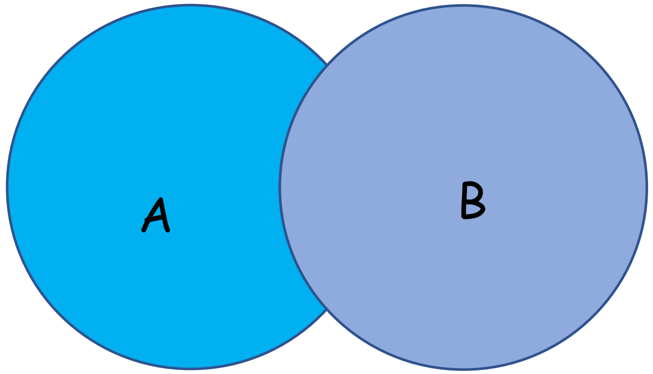
$B = \{a, b\}$

$C = \{p, q\}$

$A \times (B \times C) = \{1, 2, 3\} \times (\{a, b\} \times \{p, q\})$

Sets: Set Operations

New sets from old



Venn diagram for 2 sets

Set Operations: union

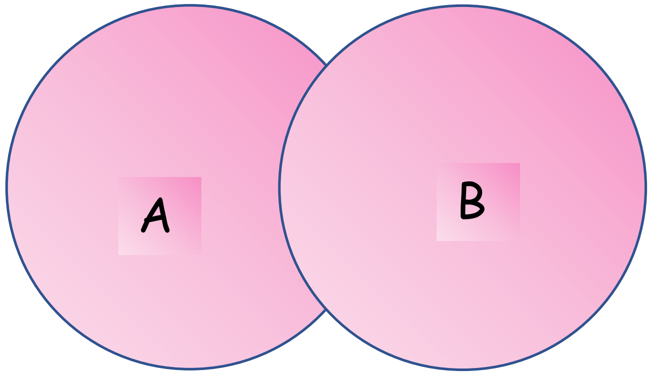
Union

$A \cup B = \{x \mid x \in A \vee x \in B\}$

$A = \{1, 2, 3, 4\}$

$B = \{1, 2, 2, 2, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$



Set Operations: intersection

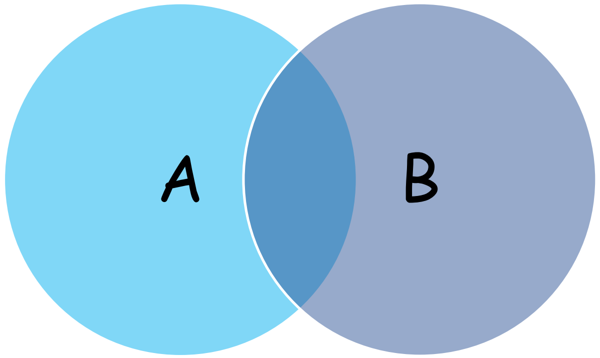
Intersection

$A \cap B = \{x \mid x \in A \land x \in B\}$

$A = \{1, 2, 3, 4\}$

$B = \{1, 2, 2, 2, 5, 6\}$

$A \cup B = \{1, 2\}$



Set Operations: difference

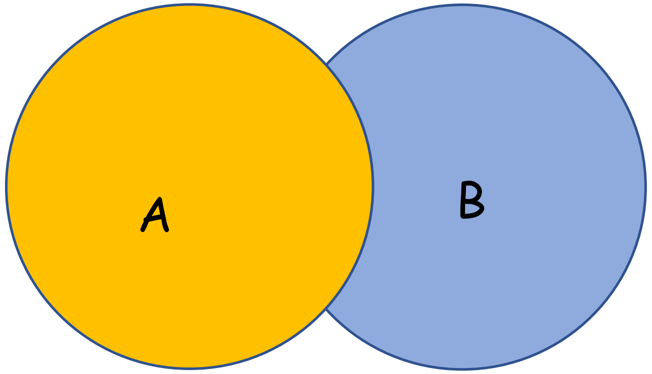
Difference

$A - B = \{x \mid x \in A \land x \notin B\}$

$A = \{1, 2, 3, 4\}$

$B = \{1, 2, 2, 2, 5, 6\}$

$A - B = \{3, 4\}$



Set Operations: complement

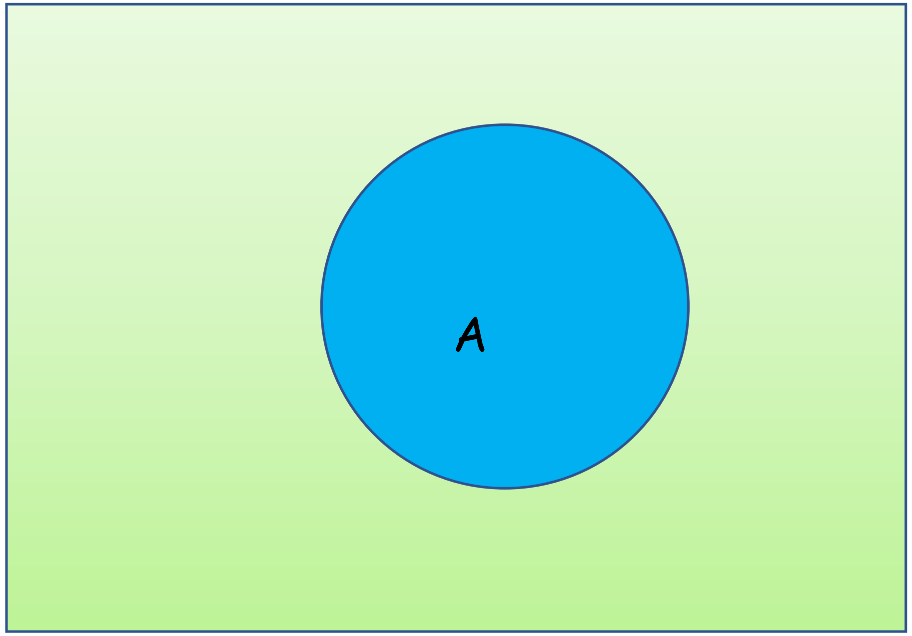
Complement

$A' = U - A = \{x \mid x \notin A\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 3, 4\}$

$A' = \{5, 6, 7, 8, 9\}$



Computer representation of sets

U = {1, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17}

U =

1	5	6	7	8	9	10	11	12	13	14	15	16	17

U =

1	1	1	1	1	1	1	1	1	1	1	1	1	1

(Bit set Representation)

Computer representation of sets

$A = \{1, 6, 8, 11, 12, 13, 16\}$

	1	5	6	7	8	9	10	11	12	13	14	15	16	17
U =	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A =														

Computer representation of sets

$A = \{1, 6, 8, 11, 12, 13, 16\}$

	1	5	6	7	8	9	10	11	12	13	14	15	16	17
U =	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A =	1													

Computer representation of sets

$A = \{1, 6, 8, 11, 12, 13, 16\}$

	1	5	6	7	8	9	10	11	12	13	14	15	16	17
U =	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A =	1		1											

Computer representation of sets

$A = \{1, 6, 8, 11, 12, 13, 16\}$

1	5	6	7	8	9	10	11	12	13	14	15	16	17

U =

1	1	1	1	1	1	1	1	1	1	1	1	1	1

A =

1		1		1									

Computer representation of sets

$A = \{1, 6, 8, 11, 12, 13, 16\}$

	1	5	6	7	8	9	10	11	12	13	14	15	16	17
U =	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A =	1		1		1			1						

Computer representation of sets

$A = \{1, 6, 8, 11, 12, 13, 16\}$

	1	5	6	7	8	9	10	11	12	13	14	15	16	17
U =	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A =	1		1		1			1	1	1			1	

Computer representation of sets

$A = \{1, 6, 8, 11, 12, 13, 16\}$

	1	5	6	7	8	9	10	11	12	13	14	15	16	17
U =	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A =	1	0	1	0	1	0	0	1	1	1	0	0	1	0

Computer representation of sets

B = {5, 7, 8}

	1	5	6	7	8	9	10	11	12	13	14	15	16	17
U =	1	1	1	1	1	1	1	1	1	1	1	1	1	1
B =	0	1	0	1	1	0	0	0	0	0	0	0	0	0

Set operations: Union

U =	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A =	1	0	1	0	1	0	0	1	1	1	0	0	1	0
B =	0	1	0	1	1	0	0	0	0	0	0	0	0	0
$A \cup B$ =	1	1	1	1	1	0	0	1	1	1	0	0	1	0

Perform OR operation

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

Set operations: Intersection

U =	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A =	1	0	1	0	1	0	0	1	1	1	0	0	1	0
B =	0	1	0	1	1	0	0	0	0	0	0	0	0	0
$A \cap B$ =	0	0	0	0	1	0	0	0	0	0	0	0	0	0

Perform AND operation

$$A \cap B = \{x \mid x \in A \land x \in B\}$$

Set operations: Complement

U =	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A =	1	0	1	0	1	0	0	1	1	1	0	0	1	0
B =	0	1	0	1	1	0	0	0	0	0	0	0	0	0
A ‘=	0	1	0	1	0	1	1	0	0	0	1	1	0	1

Toggle the bits

Set operations: Difference

U =

1	1	1	1	1	1	1	1	1	1	1	1	1	1

A =

1	0	1	0	1	0	0	1	1	1	0	0	1	0

B =

0	1	0	1	1	0	0	0	0	0	0	0	0	0

$A - B =$

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

$$A - B = \{x \mid x \in A \wedge x \in B'\}$$

Set operations: Difference

$U =$	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$A =$	1	0	1	0	1	0	0	1	1	1	0	0	1	0
$B =$	0	1	0	1	1	0	0	0	0	0	0	0	0	0
$B' =$	1	0	1	0	0	1	1	1	1	1	1	1	1	1
$A - B =$	1	0	1	0	0	0	0	1	1	1	0	0	1	0

$$A - B = \{x \mid x \in A \wedge x \in B'\}$$