Numerical Integration

Newton-Cotes Formula

Find the integration: $\int_a^b f(x)dx$

Replace $f(x)$ with $f_n(x)$, an approximating function. Why?

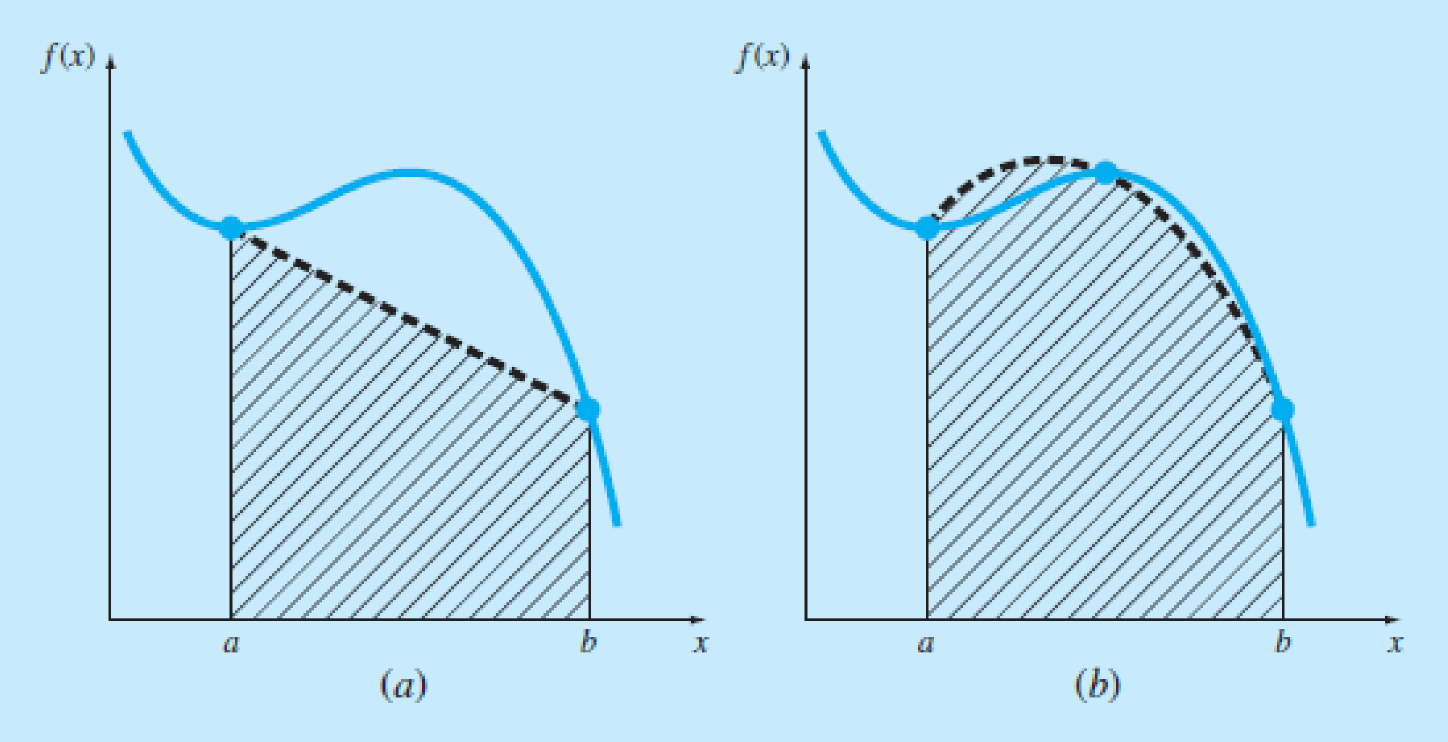
$$f(x) = \text{complicated function, hard to integrate}$$

$$f_n(x) = \text{easy to integrate}$$

$$f_n(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n,$$

Polynomial of order n

Newton-Cotes Formula



The approximation of an integral by the area under (*a*) a single straight line and (*b*) a single parabola.

The Trapezoidal Rule

$$I = \int_a^b f(x)dx \cong \int_a^b f_1(x)dx$$

First order polynomial (Straight Line)

$$I = \frac{h}{2}[sth]$$

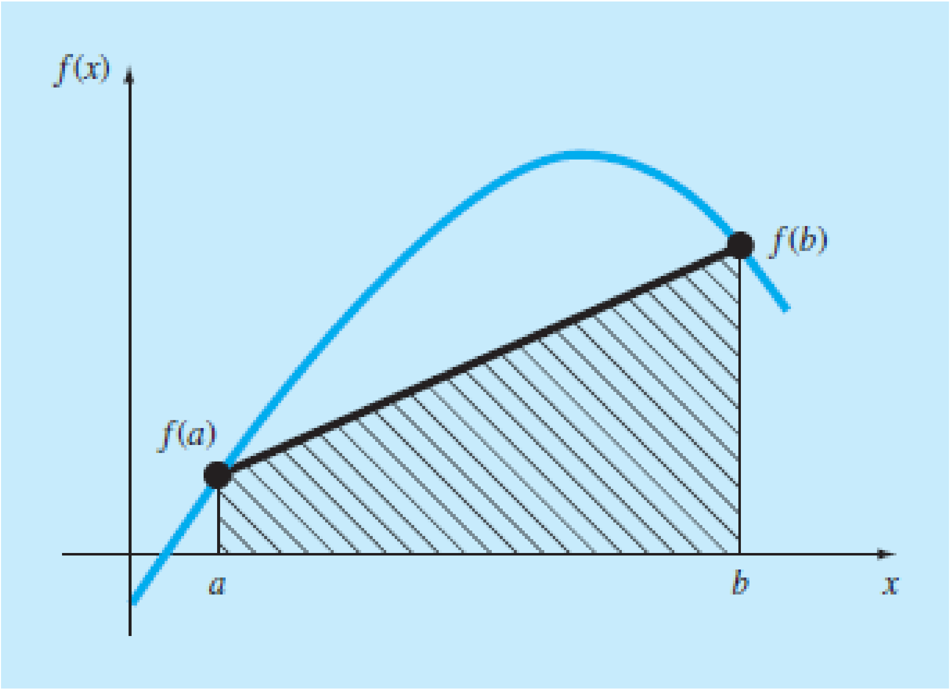
$$h = \frac{b-a}{n}$$

The Trapezoidal Rule: Single Segment

$$I = (b - a)\,\frac{f(a)+f(b)}{2}$$

$$I = \frac{h}{2}\left[f(a) + f(b)\right]$$

$$h = \frac{b-a}{1}$$



The Trapezoidal Rule: Multi-Segment

$$I = \frac{h}{2}\left[f(x_0)+2\sum_{i=1}^{n-1} f(x_i)+f(x_n)\right]$$

$$h = \frac{b-a}{n}$$

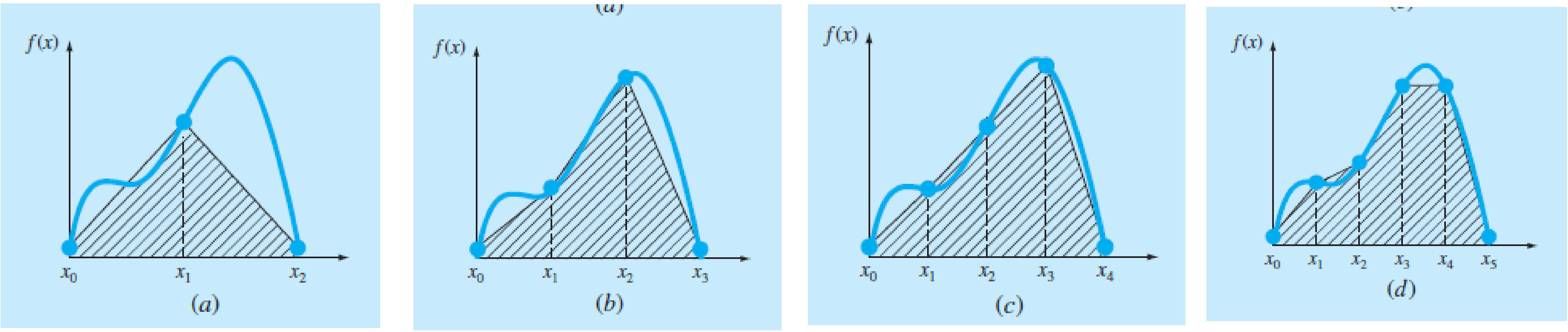


Illustration of the multiple-application trapezoidal rule. (*a*) Two segments, (*b*) three segments, (*c*) four segments, and (*d*) five segments.

Simpson’s 1/3 rule

$$I = \int_a^b f(x)dx \cong \int_a^b f_2(x)dx$$

Second order polynomial (Parabola)

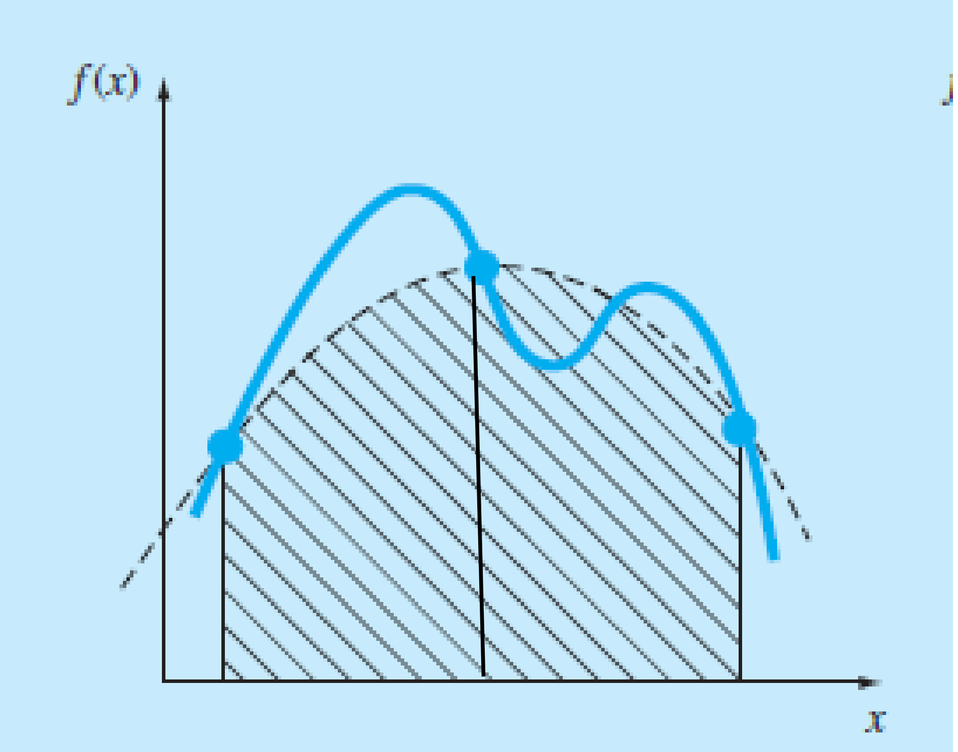
$$I = \frac{h}{3}[sth]$$

$$h = \frac{b-a}{n}$$

Simpson's 1/3 rule: Single Application

$$I = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)]$$

$$h = \frac{b-a}{2}$$



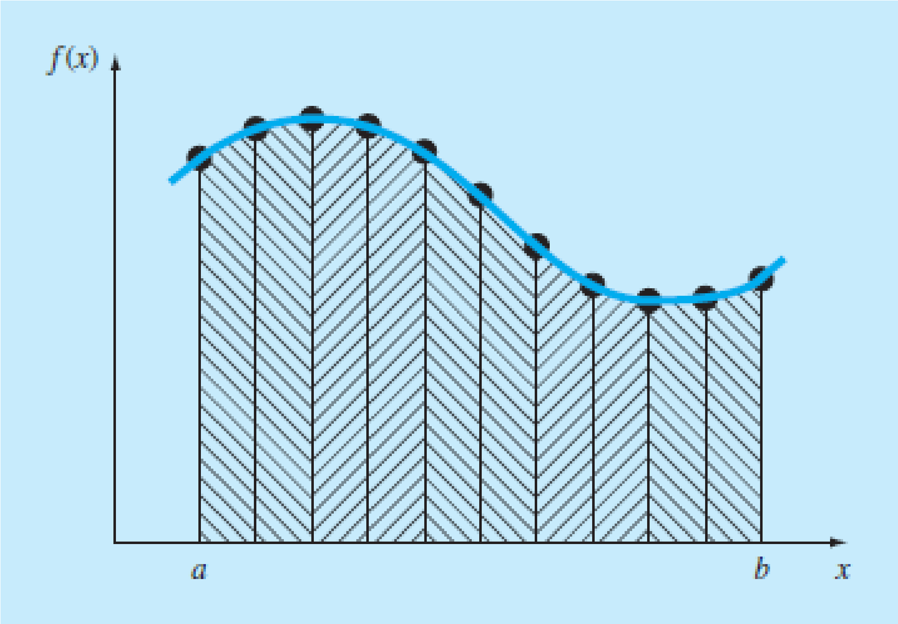
(a)

Simpson’s 1/3 rule: Multi-application

$$I = \frac{h}{3}\left[f(x_0) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + 2\sum_{j=2,4,6}^{n-2} f(x_j) + f(x_n)\right]$$

$$h = \frac{b-a}{n}$$

No of segments must be even to imply this method



Simpson's 3/8 rule

$$I = \int_a^b f(x)dx \cong \int_a^b f_3(x)dx$$

Third order polynomial (Cubic Equation)

$$I = \frac{3h}{8}[sth]$$

$$h = \frac{b-a}{n}$$

Simpson's 3/8 rule: Single Application

$$I = \frac{3h}{8}[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)]$$

$$h = \frac{b-a}{3}$$