Predicate Logic

Predicates

Proposition with variables.

Example:

$P(x) ::= [x > 3]$

$P(x, y) ::= [x + 2 = y]$

Quantifiers

$\forall x$ For ALL x
$\exists y$ There EXISTS some y

∀ is like AND

Let x range over set {1, 2, 3, 4, 5, 6}
P(x) ::= [x > 3]

$$\forall x.\ P(x)$$

same as P(1) AND P(2) AND P(3) AND P(4) AND P(5) AND P(6)

∃ is like OR

Let x range over set {1, 2, 3, 4, 5, 6}
P(x) ::= [x > 3]

$$\exists x.\ P(x)$$

same as P(1) OR P(2) OR P(3) OR P(4) OR P(5) OR P(6)

Nested Quantifier1

$x, y \in \mathbb{R}$

$Q(x, y) ::= [x + y = 0]$

$\forall x \exists y \; Q(x, y) = ?$

Nested Quantifier2

$x \in \mathbb{R}$, $y \in \mathbb{N} = \{1, 2, \dots\}$

Q(x, y)::= [x + y = 0]

$\forall x \exists y$ Q(x, y) = ?

Nested Quantifier3

$x \in \mathbb{R}$, $y \in \mathbb{N} = \{1, 2, \dots\}$

Q(x, y)::= [x + y = 0]

$\exists x \forall y$ Q(x, y) = ?

Nested Quantifier4

$x \in \mathbb{R}$, $y \in \mathbb{N} = \{1, 2, \dots\}$

$Q(x, y) ::= [x * y = 0]$

$\exists x \forall y \; Q(x, y) = ?$

Nested Quantifier5

$x \in \mathbb{R}$, $y \in \mathbb{R}$

$Q(x, y) ::= [(x * y)^2 \geq 0]$

$\forall x \forall y\ Q(x, y)$ = ?

Nested Quantifier6

$x \in \mathbb{R}$, $y \in \mathbb{R}$

$Q(x, y) ::= [x^2 + y^2 = 13]$

$\exists x \exists y\ Q(x, y)$ = ?

Nested Quantifier7

$x \in \mathbb{R}$, $y \in \mathbb{R}$

$Q(x, y) ::= [x^2 + y^2 = -6]$

$\exists x \exists y\ Q(x, y)$ = ?

Nested Quantifier8

$x \in \mathbb{R}$, $y \in \mathbb{R}$, $z \in \mathbb{R}$

$Q(x, y, z) ::= [x + y = z]$

$\forall x \, \forall y \, \exists z \; Q(x, y, z) = ?$

$$\forall x \forall y \exists z [x + y = z]$$

$x = 1$

$y = 1$

$z = 2, 1 + 1 = 2$: T

$\forall x \forall y \exists z [x + y = z]$

x = 1

y = 1

z = 2, 1 + 1 = 2: T

What's the quantifier before z?

$\exists z$

$\forall x \forall y \exists z [x + y = z]$

$x = 1$

$y = 1$

$z = 2, 1 + 1 = 2$: T

What's the quantifier before z?

$\exists z$

$x = 1$

T

$y = 1$

$z = 2, 1 + 1 = 2$: T

$$\forall x \forall y \exists z [x + y = z]$$

What's the quantifier before y? $\forall y$

$$\forall x \, \forall y \, \exists z \, [x + y = z]$$

What's the quantifier before y? $\forall y$

$x = 1$

T $y = 1$ → $z = 1, 1 + 1 = 2$: T

T $y = 2$ → $z = 3, 1 + 2 = 3$: T

$$\forall x \, \forall y \, \exists z \, [x + y = z]$$

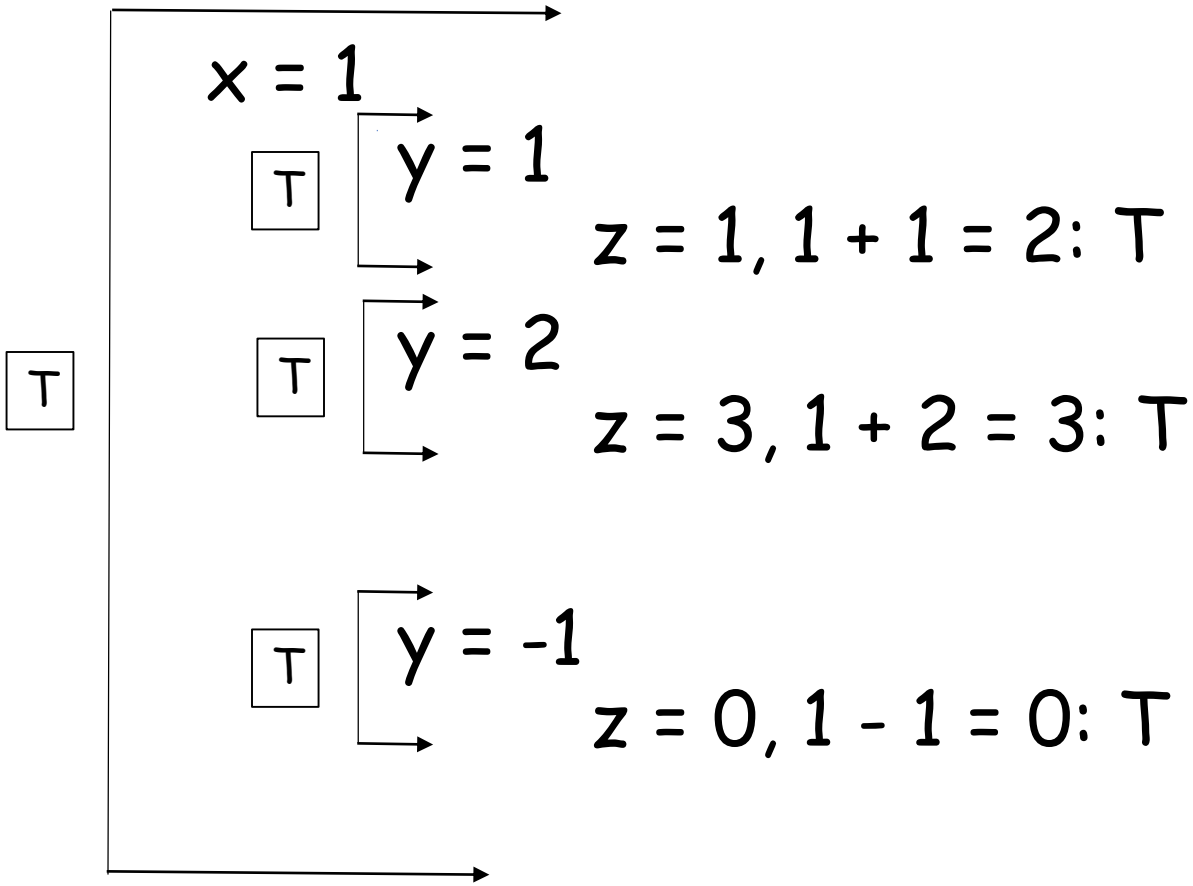
$x = 1$

T — $y = 1$ → $z = 1, 1 + 1 = 2$: T

T — $y = 2$ → $z = 3, 1 + 2 = 3$: T

T — $y = -1$ → $z = 0, 1 - 1 = 0$: T

$$\forall x \, \forall y \, \exists z \, [x + y = z]$$

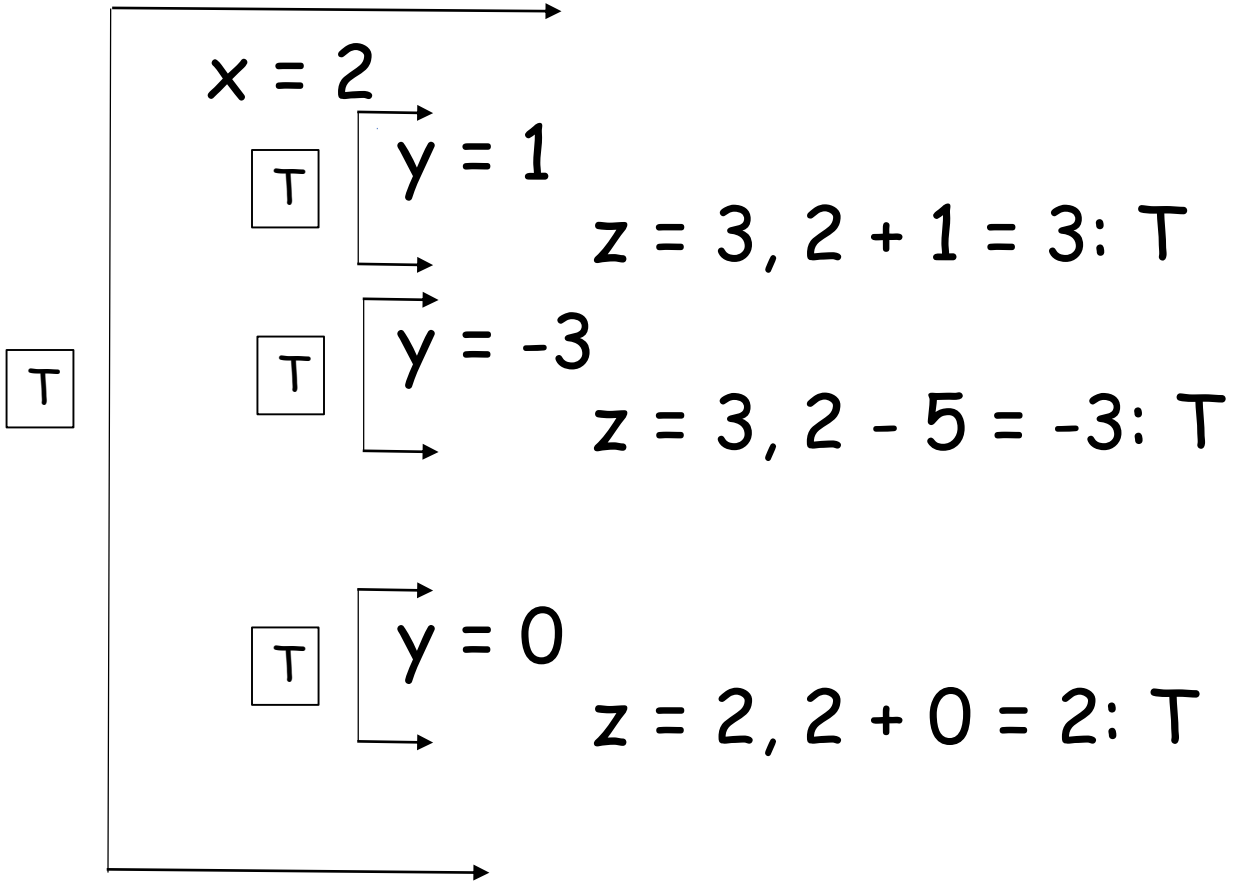


$$\forall x \, \forall y \, \exists z \, [x + y = z]$$

What's the quantifier before x? $\forall x$

$\forall x \forall y \exists z [x + y = z]$

What's the quantifier before x? $\forall x$



Nested Quantifier9

$x \in \mathbb{R}$, $y \in \mathbb{R}$, $z \in \mathbb{R}$

$Q(x, y, z) ::= [x + y = z]$

$\exists x \, \forall y \forall z \; Q(x, y, z) = ?$

$$\exists x \, \forall y \forall z \, [x + y = z]$$

$x = 1$

$y = 1$

$z = 2, 1 + 1 = 2$: T

$\exists x \forall y \forall z [x + y = z]$

$x = 1$

$y = 1$

$z = 2, 1 + 1 = 2$: T

What's the quantifier before z?

$\forall z$

$$\exists x \, \forall y \forall z \, [x + y = z]$$

$x = 1$

$y = 1$

$z = 2, 1 + 1 = 2$: T

$z = 1, 1 + 1 = 1$: F

$\exists x \forall y \forall z [x + y = z]$

$x = 1$

$y = 1$

$z = 2, 1 + 1 = 2$: T

$z = 1, 1 + 1 = 1$: F

What's the quantifier before z?

$\forall z$

$\exists x \forall y \forall z [x + y = z]$

x = 1

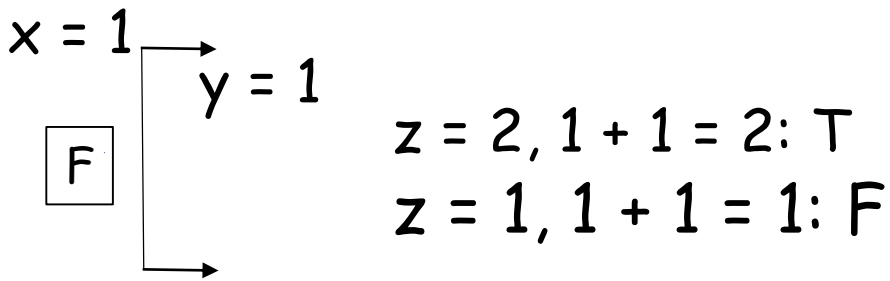
y = 1

z = 2, 1 + 1 = 2: T

z = 1, 1 + 1 = 1: F

What's the quantifier before z?

$\forall z$

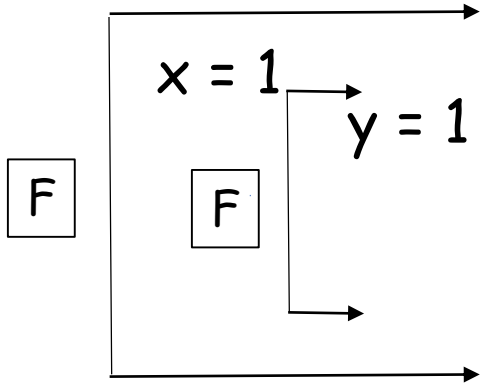


$\exists x \forall y \forall z [x + y = z]$

What's the quantifier before y? $\forall y$

$\exists x \forall y \forall z [x + y = z]$

What's the quantifier before y? $\forall y$



z = 2, 1 + 1 = 2: T

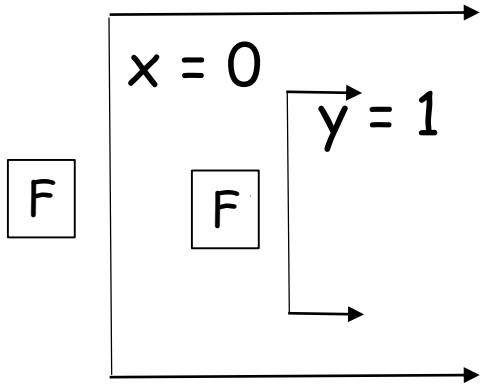
z = 1, 1 + 1 = 1: F

$\exists x \forall y \forall z [x + y = z]$

What's the quantifier before x? $\forall x$

$\exists x \forall y \forall z [x + y = z]$

What's the quantifier before x? $\forall x$



z = 1, 0 + 1 = 1: T

z = 2, 0 + 1 = 2: F

Nested Quantifier10

$x \in \mathbb{N}, y \in \mathbb{N}, z \in \mathbb{N}$

$Q(x, y, z) ::= [x = yz]$

$\forall x \exists y \exists z \; Q(x, y, z) = ?$

$$\forall x \exists y \exists z \, [x = yz]$$

Negating Nested Quantifiers

Negating nested quantifiers

Find the negation of $\forall x \exists y \exists z\ Q(x, y, z)$, where $Q(x, y, z) ::= [x = yz]$

$\neg (\forall x \exists y \exists z\ Q(x, y, z))$

$\equiv \exists x\ \neg(\exists y \exists z\ Q(x, y, z))$

$\equiv \exists x \forall y\ \neg(\exists z\ Q(x, y, z))$

$\equiv \exists x \forall y \forall z\ \neg(Q(x, y, z))$

Negating nested quantifiers

$\equiv \exists x \forall y \forall z \neg(x = yz)$

$\equiv \exists x \forall y \forall z\ (x \neq yz)$

Negating nested quantifiers

Find the negation of $\forall x \exists y \exists z\ Q(x, y, z) \vee P(x, y, z)$, where

$Q(x, y, z) ::= [x = yz]$

$P(x, y, z) ::= [x + y > z]$

$$\exists x \forall y \forall z\ (x \neq yz) \wedge (x + y \leq z)$$