Predicate Logic

Predicates

Proposition with variables.

Example:

$$P(x) ::= [x > 3]$$
$$P(x, y) ::= [x + 2 = y]$$

Quantifiers

$\forall x$ For ALL x
$\exists y$ There EXISTS some y

$\forall$ is like AND

Let x range over set $\{1, 2, 3, 4, 5, 6\}$
$P(x) ::= [x > 3]$

$$\forall x.\ P(x)$$

same as P(1) AND P(2) AND P(3) AND P(4) AND P(5) AND P(6)

$\exists$ is like OR

Let x range over set {1, 2, 3, 4, 5, 6}
$P(x) ::= [x > 3]$

$$\exists x.\ P(x)$$

same as P(1) OR P(2) OR P(3) OR P(4) OR P(5) OR P(6)

Existential Quantifier

Let x, y range over $\mathbb{N} = \{0, 1, 2, \ldots\}$
$P(x, y) ::= [x + y > 0]$

$\exists x\, P(x, y) = ?$

Existential Quantifier

$P(1, 2) ::= [1 + 2 > 0] = [3 > 0]$ T

$P(2, 3) ::= [2 + 3 > 0] = [5 > 0]$ T

$P(0, 0) ::= [0 + 0 > 0] = [0 > 0]$ F

$\exists x\, P(x, y) =$ T

Existential Quantifier

Let x, y range over $\mathbb{N} = \{0, 1, 2, \dots\}$
P(x, y) ::= [x + y < 0]

$$\exists x\, P(x, y) = ?$$

Existential Quantifier

$P(1, 2) ::= [1 + 2 < 0] = [3 < 0]$ F

$P(2, 3) ::= [2 + 3 < 0] = [5 < 0]$ F

$P(0, 0) ::= [0 + 0 < 0] = [0 < 0]$ F

$\exists x\, P(x, y) =$ F

Universal Quantifier

Let x, y range over $\mathbb{N} = \{1, 2, \ldots\}$
P(x, y) ::= [x + y > 0]

$\forall x\, P(x, y) = ?$

Universal Quantifier

$P(1, 2) ::= [1 + 2 > 0] = [3 > 0]$ T

$P(2, 3) ::= [2 + 3 > 0] = [5 > 0]$ T

$P(0, 0) ::= [4 + 5 > 0] = [9 > 0]$ T

$$\forall x\, P(x, y) = T$$

Universal Quantifier

Let x, y range over $\mathbb{N} = \{0, 1, 2, \dots\}$
P(x, y) ::= [x + y > 0]

$\forall x\, P(x, y) = ?$

Universal Quantifier

$P(1, 2) ::= [1 + 2 > 0] = [3 > 0]$ T

$P(2, 3) ::= [2 + 3 > 0] = [5 > 0]$ T

$P(0, 0) ::= [0 + 0 > 0] = [0 > 0]$ F

$\forall x\, P(x, y) =$ F

Nested Quantifier

$x, y \in \mathbb{R}$

$Q(x, y) ::= [x + y = 0]$

$$\forall x \exists y\ Q(x, y) = ?$$

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{N} = \{1, 2, \ldots\}$

$Q(x, y) ::= [x + y = 0]$

$$\forall x \exists y \, Q(x, y) = ?$$

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{N} = \{1, 2, \ldots\}$

$Q(x, y) ::= [x + y = 0]$

$$\exists x \forall y \; Q(x, y) = ?$$

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{N} = \{1, 2, \ldots\}$

$Q(x, y) ::= [x * y = 0]$

$$\exists x \forall y \; Q(x, y) = ?$$

Nested Quantifier

x ∈ $\mathbb{R}$, y ∈ $\mathbb{R}$

Q(x, y)::= $[(x * y)^2 \geq 0]$

∀x∀y Q(x, y) = ?

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{R}$

$Q(x, y) ::= [x^2 + y^2 = 13]$

$$\exists x \exists y\ Q(x, y) = ?$$

Nested Quantifier

x ∈ $\mathbb{R}$, y ∈ $\mathbb{R}$

Q(x, y)::= $[x^2 + y^2 =$ -6]

∃x∃y Q(x, y) = ?

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{R}$, $z \in \mathbb{R}$

$Q(x, y, z) ::= [x + y = z]$

$$\forall x \, \forall y \, \exists z \; Q(x, y, z) = ?$$

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{R}$, $z \in \mathbb{R}$

$Q(x, y, z) ::= [x + y = z]$

$$\exists x \, \forall y \forall z \; Q(x, y, z) = ?$$

Nested Quantifier

$x \in \mathbb{R}$, $y \in \mathbb{R}$, $z \in \mathbb{R}$

$Q(x, y, z) ::= [x = yz]$

$$\forall x \exists y \exists z\ Q(x, y, z) = ?$$

Negating Nested Quantifiers

Negating nested quantifiers

Find the negation of $\forall x \exists y \exists z\ Q(x, y, z)$, where
$Q(x, y, z) ::= [x = yz]$

$\neg\ (\forall x \exists y \exists z\ Q(x, y, z))$

$\equiv \exists x\ \neg(\exists y \exists z\ Q(x, y, z))$

$\equiv \exists x \forall y\ \neg(\exists z\ Q(x, y, z))$

$\equiv \exists x \forall y \forall z\ \neg(Q(x, y, z))$

Negating nested quantifiers

$\equiv \exists x \forall y \forall z\ \neg(x = yz)$

$\equiv \exists x \forall y \forall z\ (x \neq yz)$

Negating nested quantifiers

Find the negation of $\forall x \exists y \exists z\ Q(x, y, z) \vee P(x, y, z)$, where
$Q(x, y, z) ::= [x = yz]$
$P(x, y, z) ::= [x + y > z]$

$$\exists x \forall y \forall z\ (x \neq yz) \wedge (x + y \leq z)$$