

Propositional Logic

What is Proposition?

- A statement that is either true or false

$$2 + 3 = 5$$

What is Proposition?

- A statement that is either true or false
 $2 + 3 = 5$: true

What is Proposition?

- A statement that is either true or false

$$1 + 1 = 3$$

What is Proposition?

- A statement that is either true or false
 $1 + 1 = 3$: false

What is Proposition?

- A statement that is either true or false
Dhaka is the capital of Bangladesh

What is Proposition?

- A statement that is either true or false
Dhaka is the capital of Bangladesh : true

What is Proposition?

- A statement that is either true or false
Chittagong is the capital of Bangladesh :
false

What is Proposition?

- A statement that is either true or false
Give me an A

What is Proposition?

- A statement that is either true or false

Give me an A

Neither true or false

Not a proposition

What is Proposition?

- A statement that is either true or false

Would there be a third world war?

Neither true or false

Not a proposition

Compound Proposition

All humans are mortal and $2 + 3 = 5$

Compound Proposition

All humans are mortal and $2 + 3 = 5$:

Hard to depict whether true / false

Compound Proposition

All humans are mortal
and

$$2 + 3 = 5$$

Compound Proposition

All humans are mortal : true

and

$2 + 3 = 5$: true

Compound Proposition

All humans are mortal : true

and

$2 + 3 = 5$: true



Logical connector

Propositional Variable

All humans are mortal

Propositional Variable

All humans are mortal: p

- p can either be true / false

Propositional Variable

All humans are mortal
and

$$2 + 3 = 5$$

p and q

Logical Connector

- Not
- And
- Or
- Implies
- Xor
- Iff

Logical Connector: NOT

- Notation: \neg
- Truth table:

p	$\neg p$
0	1
1	0

Logical Connector: AND

- Notation: \wedge
- Truth table:

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Logical Connector: OR

- Notation: \vee
- Truth table:

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Logical Connector: IMPLIES

- Notation: \rightarrow
- Truth table:

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Logical Connector: XOR

- Notation: \oplus
- Truth table:

p	q	p xor q
0	0	0
0	1	1
1	0	1
1	1	0

Logical Connector: IFF

- Notation: \leftrightarrow
- Truth table:

p	q	p IFF q
0	0	1
0	1	0
1	0	0
1	1	1

FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS: Example1

$$P \wedge q \oplus s$$

No of propositional variables = 3

No of rows in truth table = $2^3 = 8$

FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS

$$P \wedge q \oplus s$$

p	q	s
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS

$$P \wedge q \oplus s$$

Which operation to perform first?

$$p \wedge q / q \oplus s$$

FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS

$$P \wedge q \oplus s$$

Which operation to perform first?

$$p \wedge q / q \oplus s:$$

See Precedence Table

Precedence Table

Connector	Precedence
\neg	1
\wedge	2
\vee	3
\rightarrow	4
$\leftrightarrow \oplus$	5

FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS

$$P \wedge q \oplus s$$

$$= (p \wedge q) \oplus s$$

$$= a \oplus s$$

$$= b$$

FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS

$$P \wedge q \oplus s = (p \wedge q) \oplus s = a \oplus s = b$$

p	q	s	$p \wedge q = a$	$a \oplus s = b$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	0

FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS: Example 2

$$(\neg p \leftrightarrow \neg q) \wedge p \rightarrow r$$

$$= a \wedge p \rightarrow r$$

$$= (a \wedge p) \rightarrow r$$

$$= b \rightarrow r$$

FINDING TRUTH TABLE OF COMPOUND PROPOSITIONS: Example 2

p	q	r	$\neg p$	$\neg q$	$(\neg p \leftrightarrow \neg q)$ = a	$a \wedge p = b$	$b \rightarrow r$
0	0	0	1	1	1	0	1
0	0	1	1	1	1	0	1
0	1	0	1	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	0	1
1	1	0	0	0	1	1	0
1	1	1	0	0	1	1	1

Propositional Equivalences

Tautology

$$[\neg p \wedge (p \vee q)] \rightarrow q$$

p	q				$[\neg p \wedge (p \vee q)] \rightarrow q$
0	0				1
0	1				1
1	0				1
1	1				1

Contradiction

$$\neg([\neg p \wedge (p \vee q)] \rightarrow q)$$

p	q				$\neg([\neg p \wedge (p \vee q)] \rightarrow q)$
0	0				0
0	1				0
1	0				0
1	1				0

Contingency

$$p \rightarrow q$$

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Logical Equivalences

The compound propositions p and q are called *logically equivalent* if $p \leftrightarrow q$ is a tautology.

Notation: \equiv

Logical Equivalences

Compound propositions that have the same truth values in all possible cases are called **logically equivalent**

Proving Logical Equivalences: Example 1

Prove that: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Proving Logical Equivalences: Example 1

Truth table of $p \vee (q \wedge r)$

p	q	r	$(q \wedge r)$	$p \vee (q \wedge r)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Proving Logical Equivalences: Example 1

Truth table of $(p \vee q) \wedge (p \vee r)$

p	q	r	$(p \vee q)$	$(p \vee r)$	$(p \vee q) \wedge (p \vee r)$
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

Proving Logical Equivalences: Example 1

Prove that: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
0	0
0	0
0	0
1	1
1	1
1	1
1	1
1	1

Proving Logical Equivalences: Example 2

Prove that: $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

Logical Equivalences

$$p \rightarrow q \equiv \neg p \vee q$$

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$p \oplus q \equiv (p \vee q) \wedge (\neg p \vee \neg q)$$

Translating into
propositional logic

Translating into propositional logic

if Aang is not in the path of totality during the solar eclipse, Aang won't defeat the firelord.

a: Aang is in the path of totality

b: Aang will defeat the firelord.

$$\neg a \rightarrow \neg b$$

Translating into propositional logic

if you are a computer science major or you are not a freshman, you can access the Internet from campus

a: you are a computer science major

b: you are a freshman

c: you can access the Internet from campus

$$a \vee \neg b \rightarrow c$$

Translating into propositional logic

you can access the Internet from campus
only if you are a computer science major or you
are not a freshman,

a: you are a computer science major

b: you are a freshman

c: you can access the Internet from campus

$$a \vee \neg b \rightarrow c$$

Translating into propositional logic

you can access the Internet from campus
only if you are a computer science major but you
are not a freshman,

a: you are a computer science major

b: you are a freshman

c: you can access the Internet from campus

$$a \wedge \neg b \rightarrow c$$

Translating into propositional logic

you can access the Internet from campus
iff you are a computer science major but you
are not a freshman,

a: you are a computer science major

b: you are a freshman

c: you can access the Internet from campus

$$a \wedge \neg b \leftrightarrow c$$

Translating into propositional logic

you can access the Internet from campus
if and only if you are a computer science major
but you are not a freshman,

a: you are a computer science major

b: you are a freshman

c: you can access the Internet from campus

$$a \wedge \neg b \leftrightarrow c$$

Negation of propositional logic

De Morgan's Las

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Negating propositions

Find the negation of $(p \vee (\neg p \wedge q))$

$$\neg (p \vee (\neg p \wedge q))$$

$$\equiv \neg p \wedge \neg (\neg p \wedge q)$$

$$\equiv \neg p \wedge (\neg (\neg p) \vee \neg q)$$

$$\equiv \neg p \wedge (p \vee \neg q)$$

Negating propositions

Find the negation of $p \leftrightarrow q$

$$\neg (p \leftrightarrow q)$$

$$\equiv \neg (p \rightarrow q \wedge (q \rightarrow p))$$