Propositional Logic

What is Proposition?

- A statement that is either true or false

$$2 + 3 = 5$$

What is Proposition?

- A statement that is either true or false

  $2 + 3 = 5$: true

What is Proposition?

- *A* statement that is either true or false

$$1 + 1 = 3$$

What is Proposition?

- *A* statement that is either true or false

$$1 + 1 = 3 : \text{false}$$

What is Proposition?

- A statement that is either true or false

  Dhaka is the capital of Bangladesh

What is Proposition?

- A statement that is either true or false

Dhaka is the capital of Bangladesh : true

What is Proposition?

- A statement that is either true or false

Chittagong is the capital of Bangladesh : false

What is Proposition?

- *A* statement that is either true or false

Give me an *A*

What is Proposition?

- A statement that is either true or false

Give me an A

Neither true or false

Not a proposition

What is Proposition?

- *A* statement that is either true or false

Would there be a third world war?

Neither true or false

Not a proposition

Compound Proposition

All humans are mortal and 2 + 3 =5

Compound Proposition

All humans are mortal and 2 + 3 =5 :

Hard to depict whether true / false

Compound Proposition

All humans are mortal

and

$2 + 3 = 5$

Compound Proposition

All humans are mortal : true

and

2 + 3 =5 : true

Compound Proposition

All humans are mortal : true

and

$2 + 3 = 5$: true

Logical connector

Propositional Variable

All humans are mortal

Propositional Variable

All humans are mortal: p

- p can either be true / false

Propositional Variable

All humans are mortal

and

2 + 3 =5

p and q

Logical Connector

- Not
- And
- Or
- Implies
- Xor
- Iff

Logical Connector: NOT

- Notation: ¬
- Truth table:

p	¬ p
0	1
1	0

Logical Connector: AND

- Notation: $\wedge$
- Truth table:

p	q	p $\wedge$ q
0	0	0
0	1	0
1	0	0
1	1	1

Logical Connector: OR

- Notation: $\vee$
- Truth table:

p	q	p $\vee$ q
0	0	0
0	1	1
1	0	1
1	1	1

Logical Connector: IMPLIES

- Notation: $\rightarrow$
- Truth table:

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Logical Connector: XOR

- Notation: $\oplus$
- Truth table:

p	q	p xor q
0	0	0
0	1	1
1	0	1
1	1	0

Logical Connector: IFF

- Notation: $\leftrightarrow$
- Truth table:

p	q	p IFF q
0	0	1
0	1	0
1	0	0
1	1	1

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS: Example1

$$P \land q \oplus s$$

No of propositional variables = 3

No of rows in truth table = $2^3 = 8$

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS

$$P \wedge q \oplus s$$

p	q	s
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS

$$P \wedge q \oplus s$$

Which operation to perform first?

$p \wedge q$ / $q \oplus s$

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS

$$P \wedge q \oplus s$$

Which operation to perform first?

$p \wedge q$ / $q \oplus s$:

See Precedence Table

Precedence Table

Connector	Precedence
¬	1
∧	2
∨	3
→	4
↔ ⊕	5

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS

$P \wedge q \oplus s$

$= (p \wedge q) \oplus s$

$= a \oplus s$

$= b$

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS

$P \wedge q \oplus s = (p \wedge q) \oplus s = a \oplus s = b$

p	q	s	$p \wedge q = a$	$a \oplus s = b$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	0

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS: Example 2

$(\neg p \leftrightarrow \neg q) \wedge p \rightarrow r$

$= a \wedge p \rightarrow r$

$= (a \wedge p) \rightarrow r$

$= b \rightarrow r$

FINDING TRUTH TABLE OF COMPOUND PROPOSTIONS: Example 2

p	q	r	¬p	¬q	(¬p ↔ ¬q) = a	a ∧ p = b	b → r
0	0	0	1	1	1	0	1
0	0	1	1	1	1	0	1
0	1	0	1	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	0	1
1	1	0	0	0	1	1	0
1	1	1	0	0	1	1	1

Propositional Equivalences

Tautology

$[\neg p \land (p \lor q)] \rightarrow q$

p	q				$[\neg p \land (p \lor q)] \rightarrow q$
0	0				1
0	1				1
1	0				1
1	1				1

Contradiction

$$\neg([\neg p \wedge (p \vee q)] \rightarrow q)$$

p	q				$\neg([\neg p \wedge (p \vee q)] \rightarrow q)$
0	0				0
0	1				0
1	0				0
1	1				0

Contingency

$p \rightarrow q$

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

Logical Equivalences

The compound propositions p and q are called *logically equivalent* if $p \leftrightarrow q$ is a tautology.
Notation: $\equiv$

Logical Equivalences

Compound propositions that have the same truth values in all possible cases are called **logically equivalent**

Proving Logical Equivalences: Example 1

Prove that: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Proving Logical Equivalences: Example 1

Truth table of $p \vee (q \wedge r)$

p	q	r	$(q \wedge r)$	$p \vee (q \wedge r)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Proving Logical Equivalences: Example 1

Truth table of $(p \vee q) \wedge (p \vee r)$

p	q	r	$(p \vee q)$	$(p \vee r)$	$(p \vee q) \wedge (p \vee r)$
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

Proving Logical Equivalences: Example 1

Prove that: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
0	0
0	0
0	0
1	1
1	1
1	1
1	1
1	1

Proving Logical Equivalences: Example 2

Prove that: $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

Logical Equivalences

$p \rightarrow q \equiv \neg p \vee q$

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$p \oplus q \equiv (p \vee q) \wedge (\neg p \vee \neg q)$