

Graphs

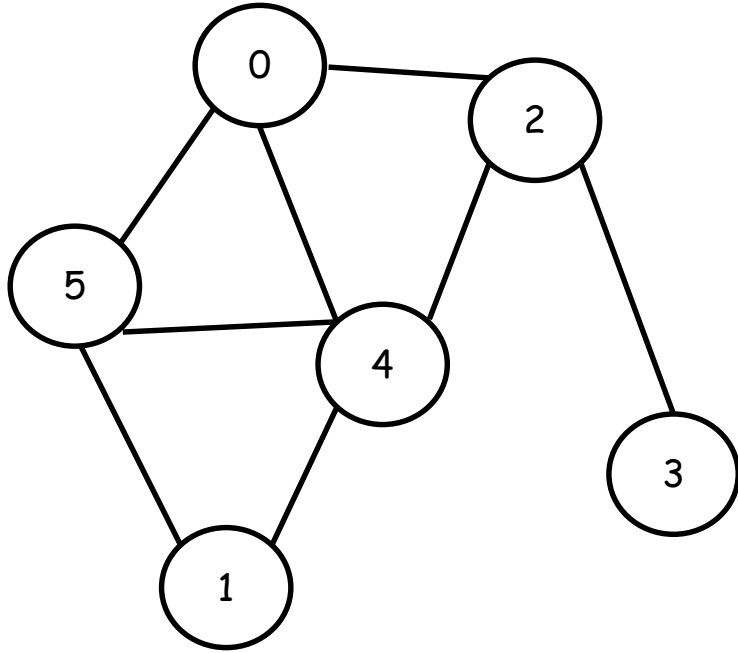
Undirected Graph: Vertices and edges



No Direction

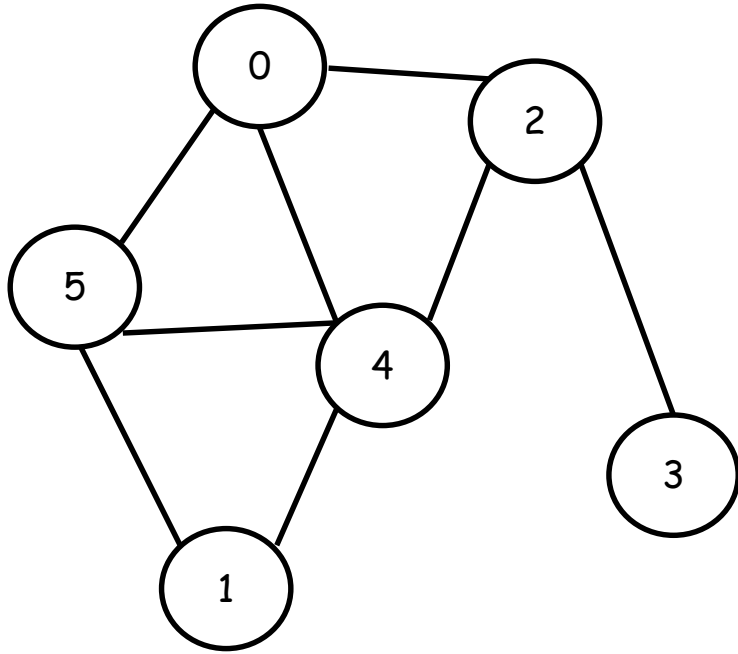
Representation: 02
20

Undirected Graph: Vertices and edges



Vertices = {0, 1, 2, 3, 4, 5}

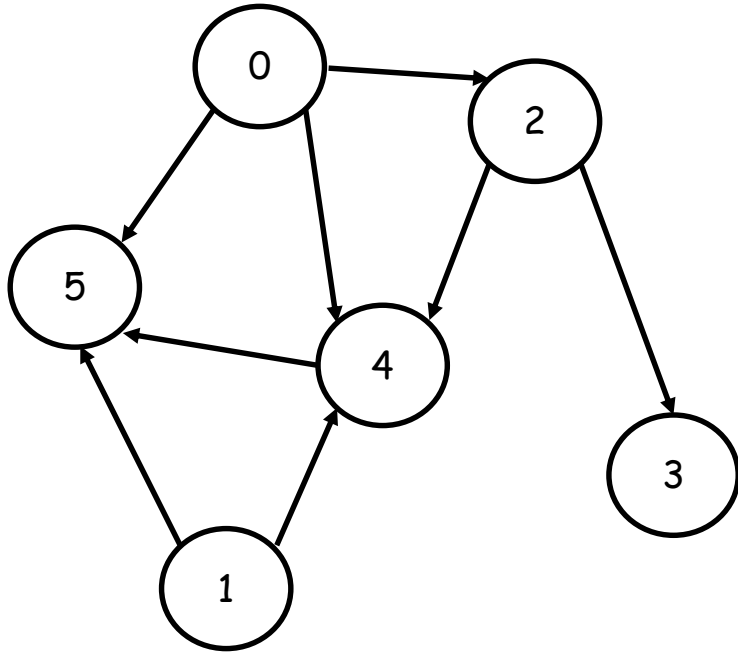
Undirected Graph: Vertices and edges



Edges = {02, 05, 04, 15,
14, 24, 23, 45 }

No convention
Which vertex to write first.

Directed Graph: Vertices and edges



Direction: From Node 0 - Node 2

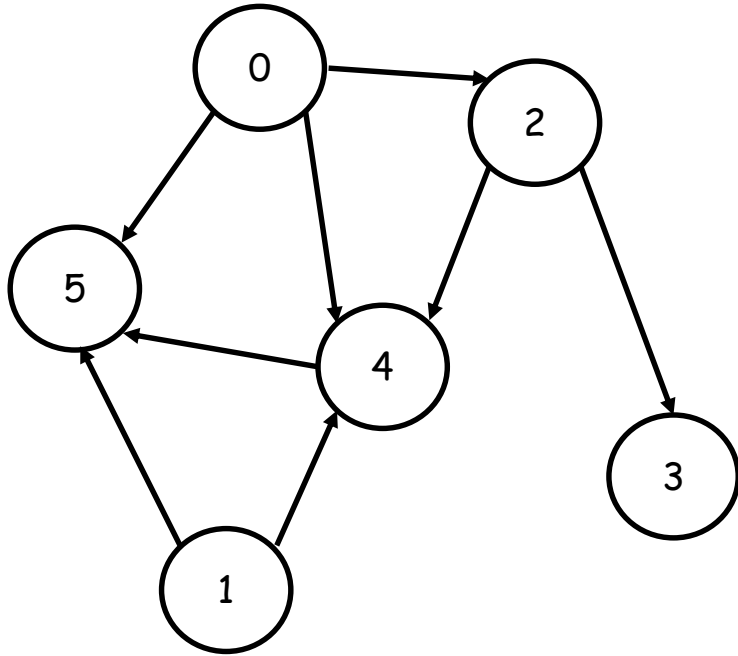
Node 0: Initial Vertex

Node 2: Terminal Vertex

Direction: Initial Vertex - Terminal Vertex

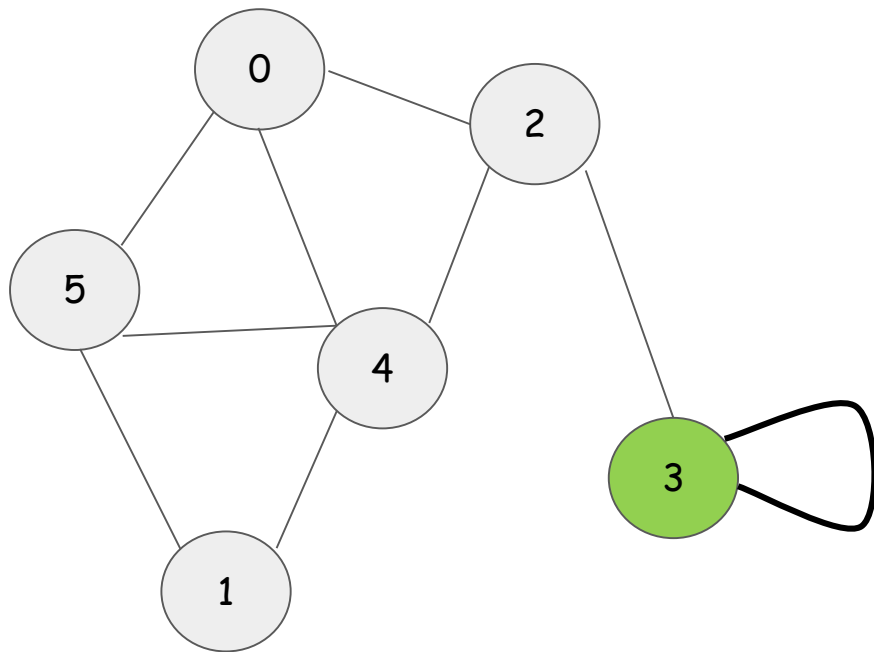
Representation: $\begin{matrix} 02 \\ \hline 20 \end{matrix}$

Directed Graph: Vertices and edges

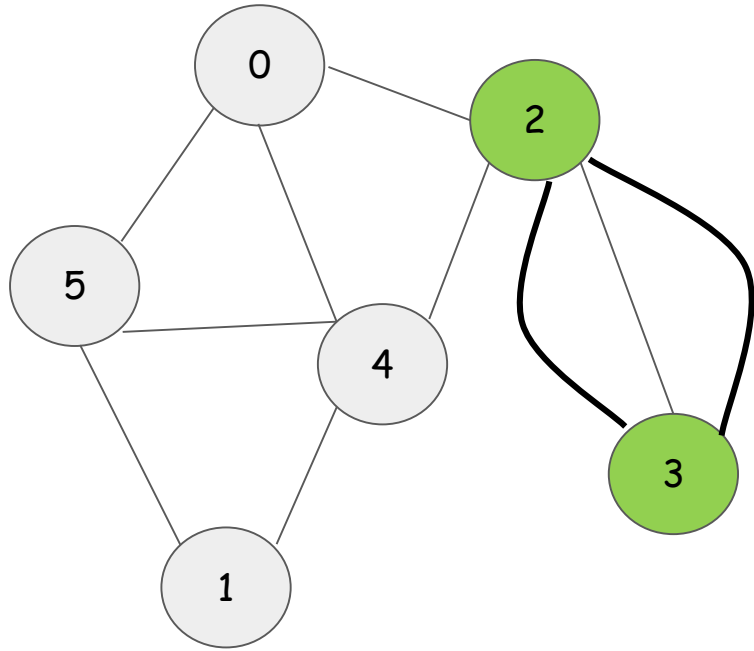


Edges = {05, 04, 02,
24, 45, 15, 14, 23}

Loops in Graphs

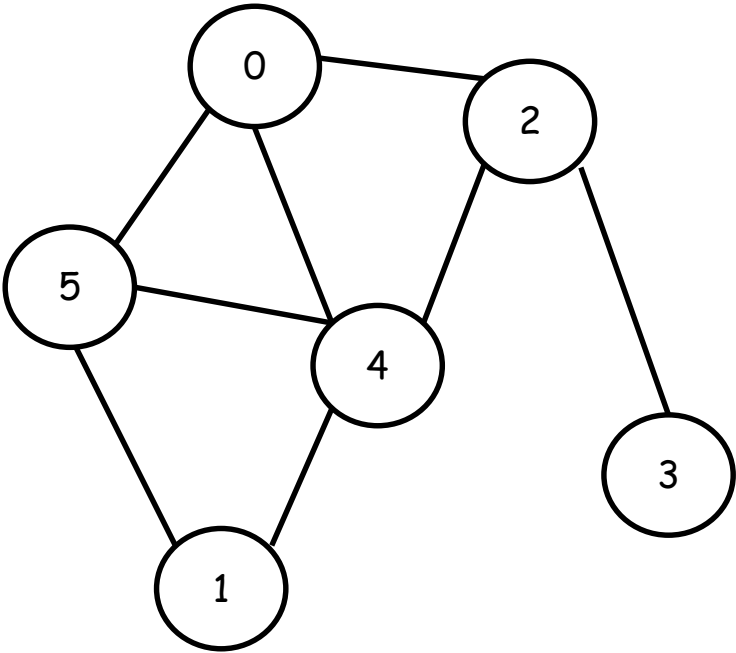


Multiple edges in Graphs



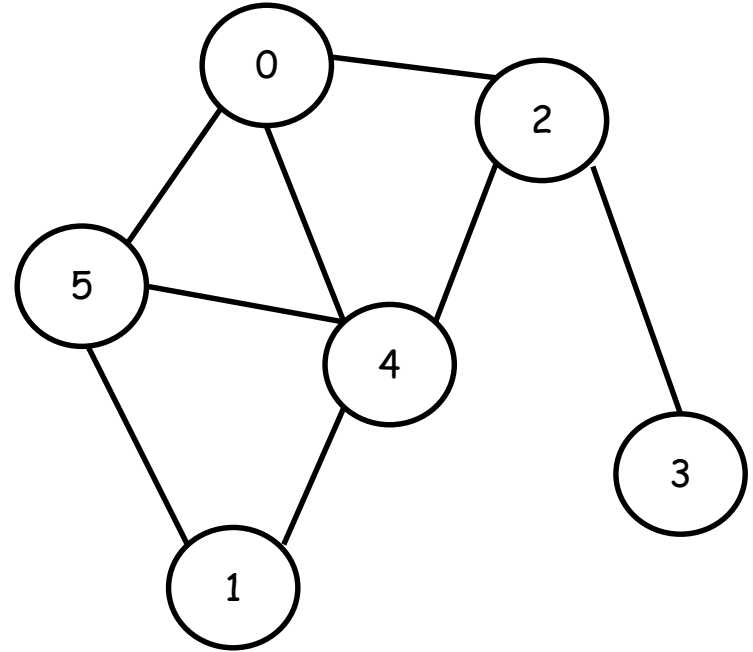
Adjacency Matrix Representation: Undirected Graph

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						



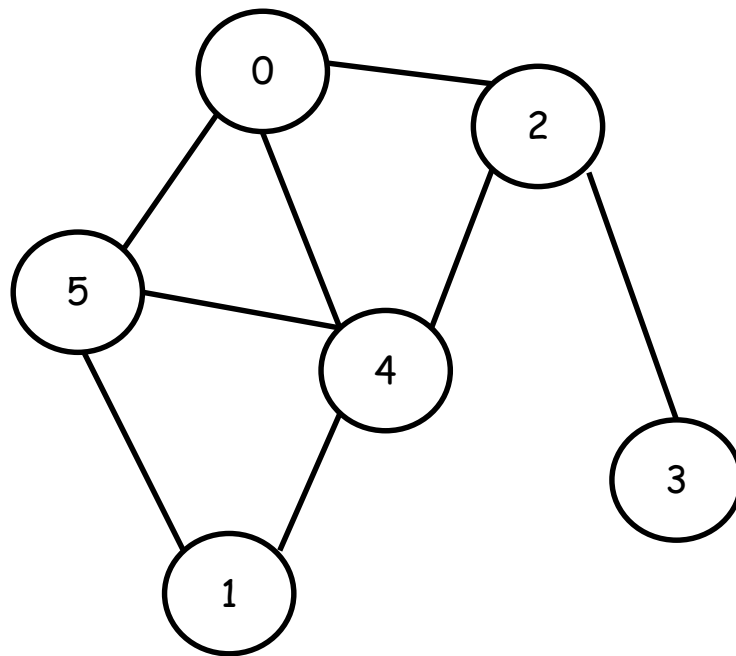
Adjacency Matrix Representation: Undirected Graph

	0	1	2	3	4	5
0	0	0	1	0	1	1
1						
2						
3						
4						
5						



Adjacency Matrix Representation: Undirected Graph

	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2						
3						
4						
5						



Adjacency Matrix Representation: Undirected Graph

	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2	1	0	0	1	1	0
3	0	0	1	0	0	0
4	1	1	1	0	0	1
5	1	1	0	0	1	0



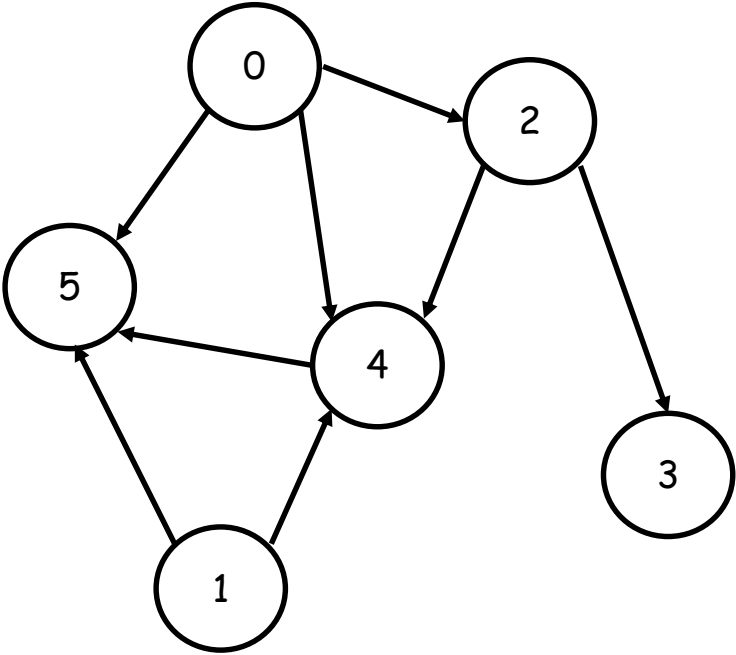
A 2D array:
 $a[\text{Number of vertices}]$
 $[\text{Number of vertices}]$

$$a[i][j] = a[j][i]$$

Adjacency Matrix Representation: Directed Graph

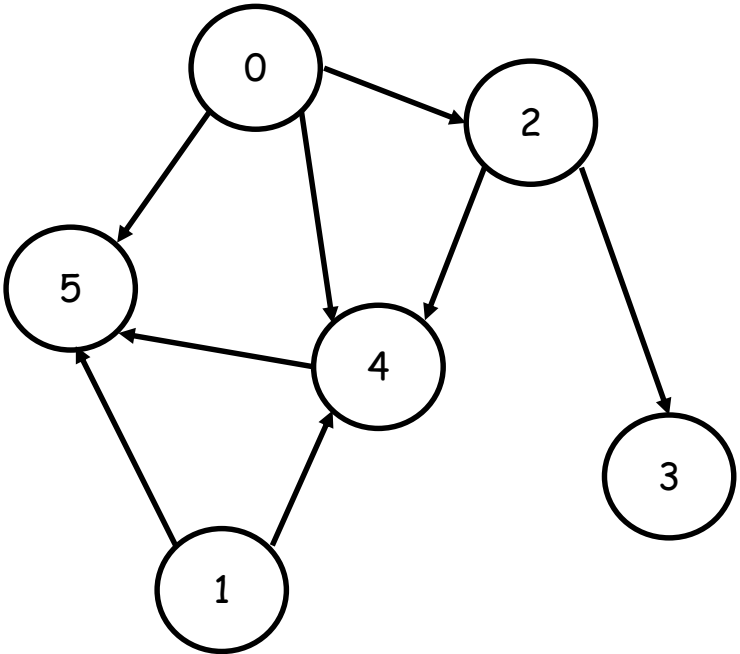
Initial Vertex

	Terminal Vertex					
	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						



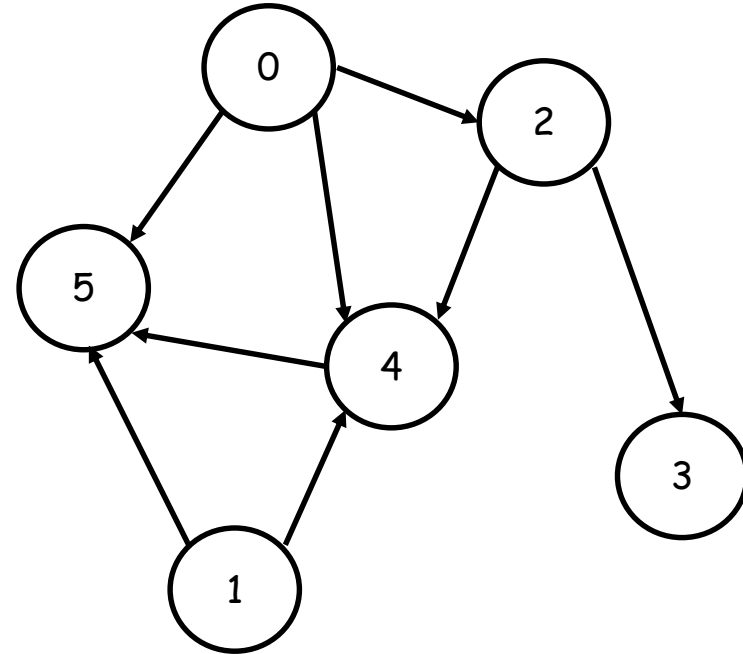
Adjacency Matrix Representation: Directed Graph

		Terminal Vertex					
Initial Vertex		0	1	2	3	4	5
	0	0	0	1	0	1	1
	1						
	2						
	3						
	4						
	5						



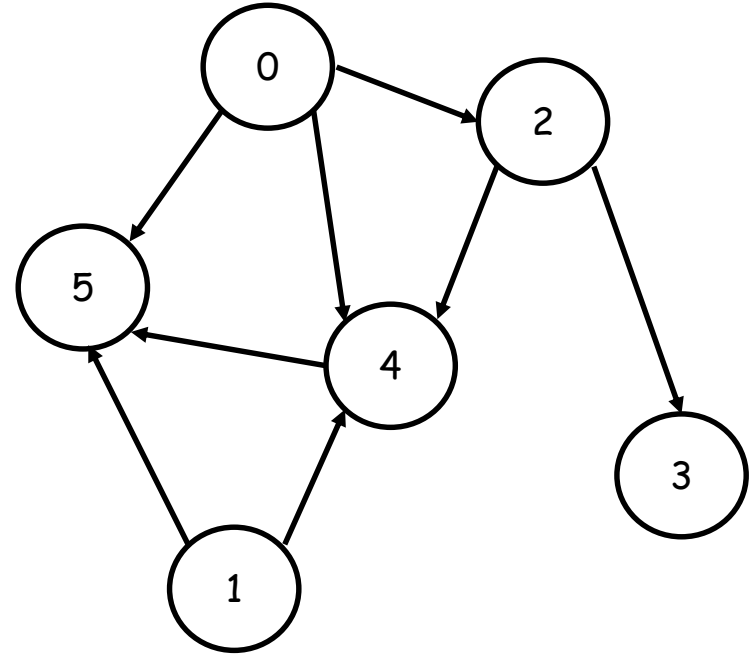
Adjacency Matrix Representation: Directed Graph

		Terminal Vertex					
Initial Vertex		0	1	2	3	4	5
	0	0	0	1	0	1	1
	1	0	0	0	0	1	1
	2						
	3						
	4						
	5						



Adjacency Matrix Representation: Directed Graph

		Terminal Vertex					
Initial Vertex		0	1	2	3	4	5
	0	0	0	1	0	1	1
	1	0	0	0	0	1	1
	2	0	0	0	1	1	0
	3						
	4						
	5						



Adjacency Matrix Representation: Directed Graph

Graph

Terminal Vertex

	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2	0	0	0	1	1	0
3	0	0	0	0	0	0
4	0	0	0	0	0	1
5	0	0	0	0	0	0

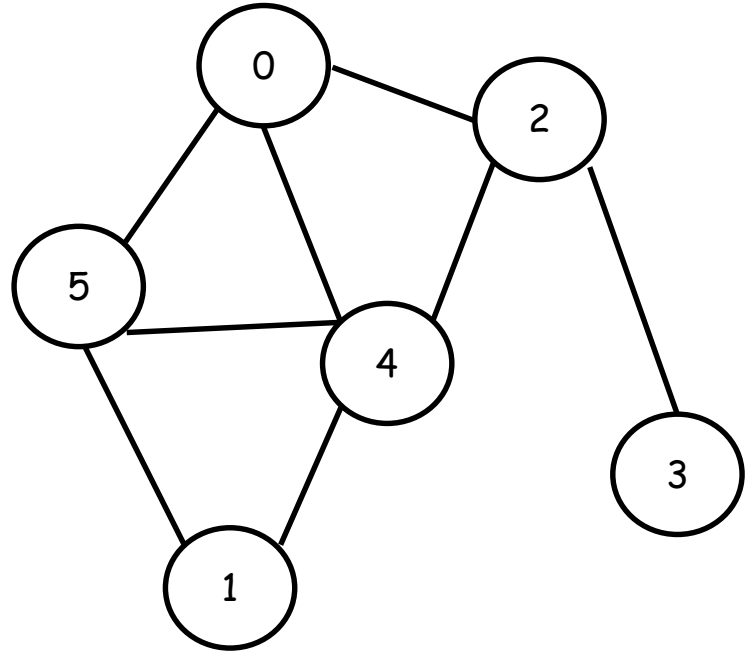


A 2D array:
 $a[\text{Number of vertices}][\text{Number of vertices}]$

$a[i][j] = a[j][i]$
 $a[i][j] = a[j][i]$
No Restriction

Degree of a vertex: Undirected Graph

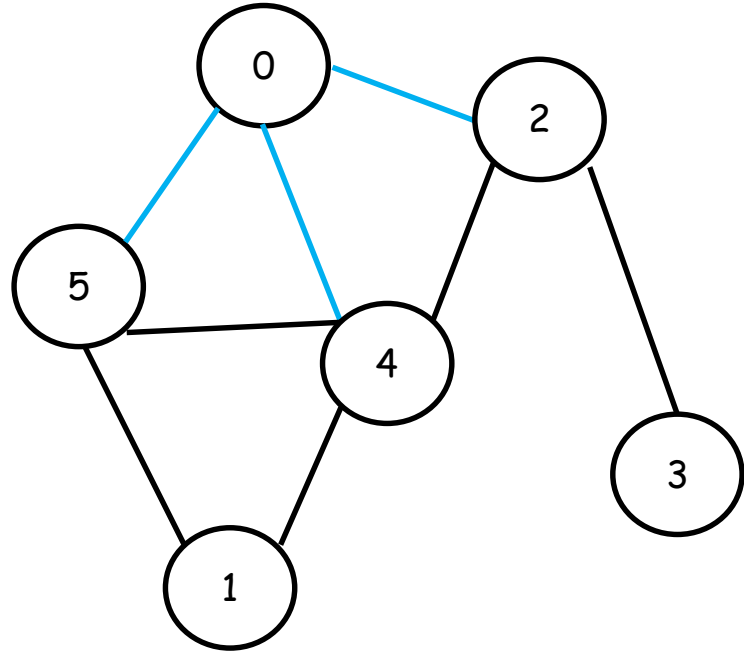
- Number of edges incident with it
- In case of Loop add +2



Degree of a vertex: Undirected Graph

- Number of edges incident with it
- In case of Loop add +2

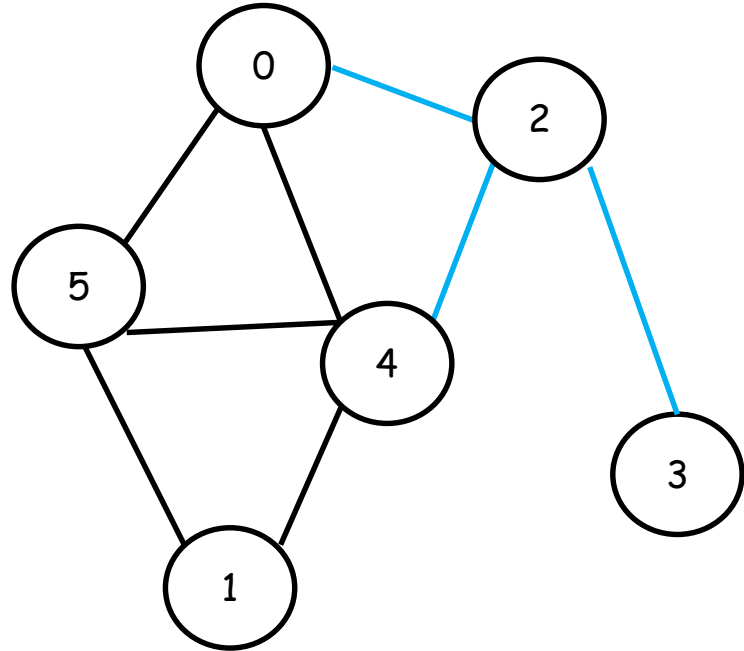
Vertex	Degree
0	3
1	
2	
3	
4	
5	



Degree of a vertex: Undirected Graph

- Number of edges incident with it
- In case of Loop add +2

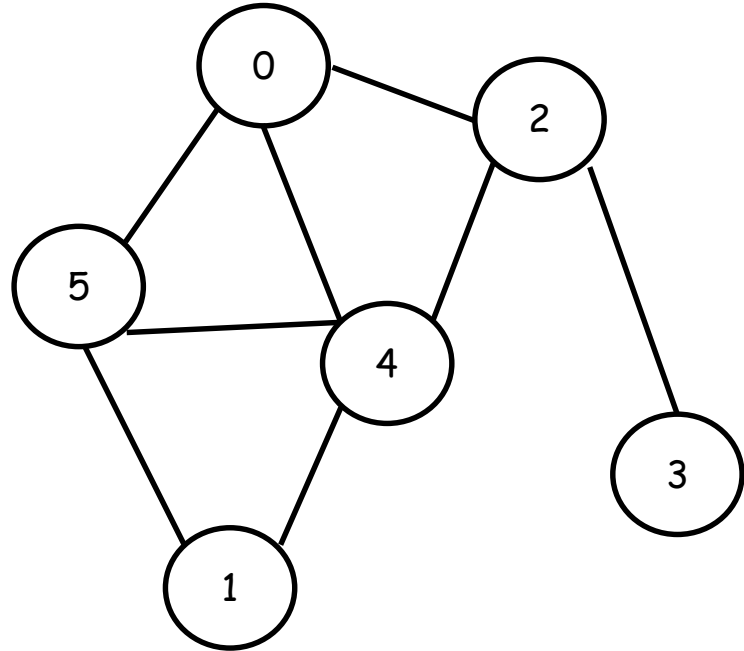
Vertex	Degree
0	3
1	2
2	
3	
4	
5	



Degree of a vertex: Undirected Graph

- Number of edges incident with it
- In case of Loop add +2

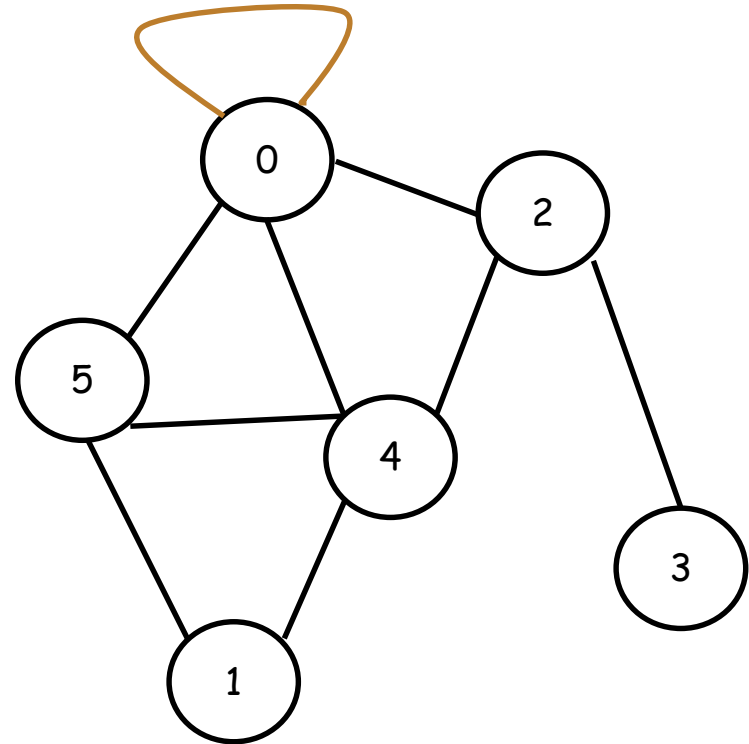
Vertex	Degree
0	3
1	2
2	3
3	1
4	4
5	3



Degree of a vertex: Undirected Graph

- Number of edges incident with it
- In case of Loop add +2

Vertex	Degree
0	5 (3 + 2)
1	2
2	3
3	1
4	4
5	3



Degree of a vertex: Undirected Graph

Vertex	Degree
0	3
1	2
2	3
3	1
4	4
5	3



Σ (1's)



Σ (1's)

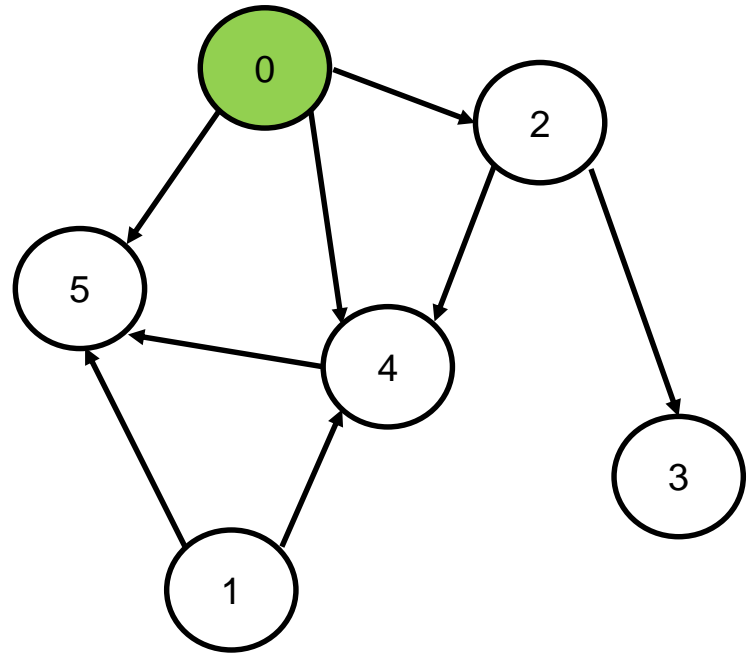
	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2	1	0	0	1	1	0
3	0	0	1	0	0	0
4	1	1	1	0	0	1
5	1	1	0	0	1	0

Degree of a vertex: Directed Graph

- In Degree : No of **INCOMING** edges
- Out Degree : No of **OUTGOING** edges

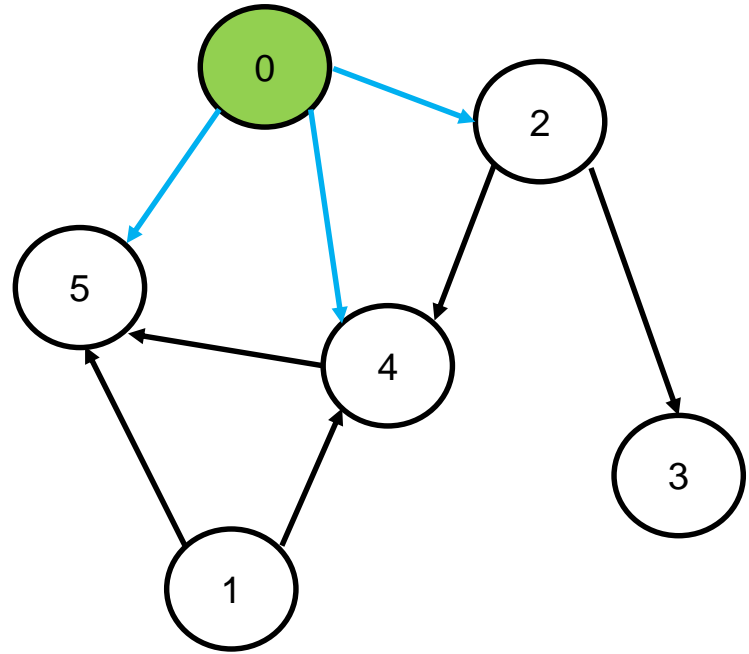
Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	
1		
2		
3		
4		
5		



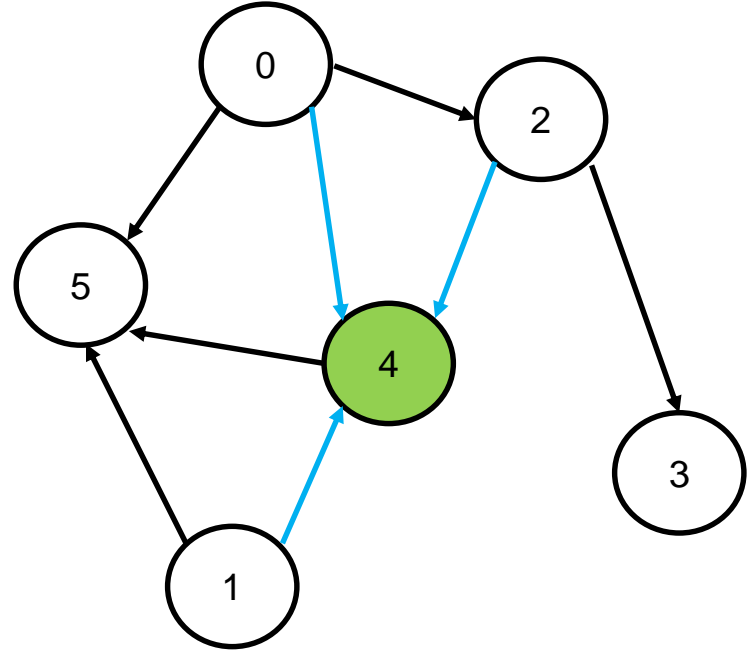
Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	3
1		
2		
3		
4		
5		



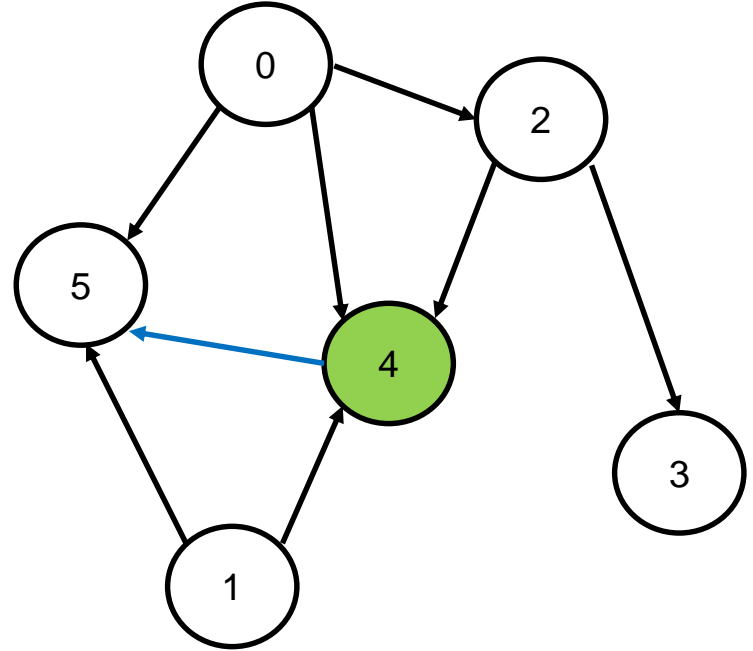
Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	3
1		
2		
3		
4	3	
5		



Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	3
1		
2		
3		
4	3	1
5		

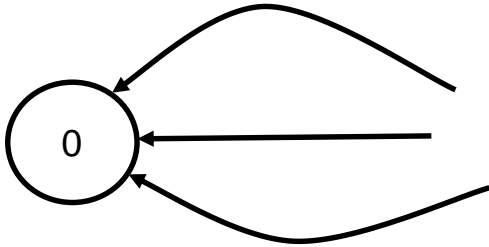


Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	3

In degree = number of incoming edges

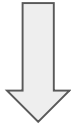
In degree of vertex 0 = number of incoming edges to 0



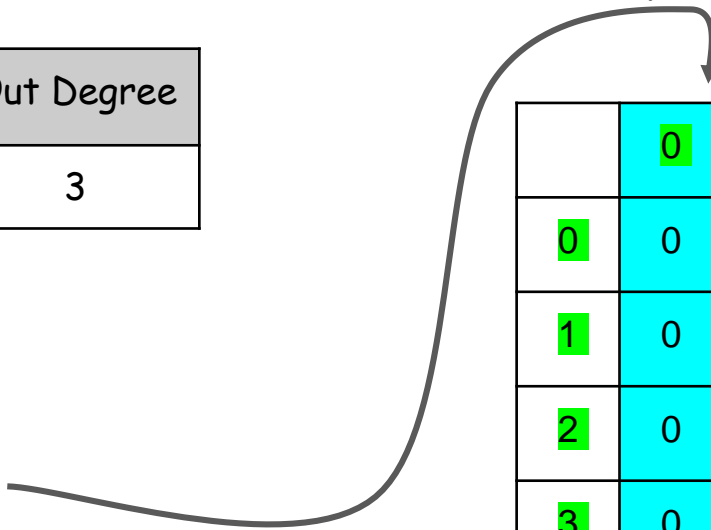
So, for counting in degrees of vertex 0
0 must be a **TERMINAL VERTEX**

Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	3



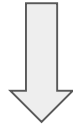
Σ
(1's)



	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2	0	0	0	1	1	0
3	0	0	0	0	0	0
4	0	0	0	0	0	1
5	0	0	0	0	0	0

Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	3



Σ
(1's)

	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2	0	0	0	1	1	0
3	0	0	0	0	0	0
4	0	0	0	0	0	1
5	0	0	0	0	0	0

Bipartite Graph

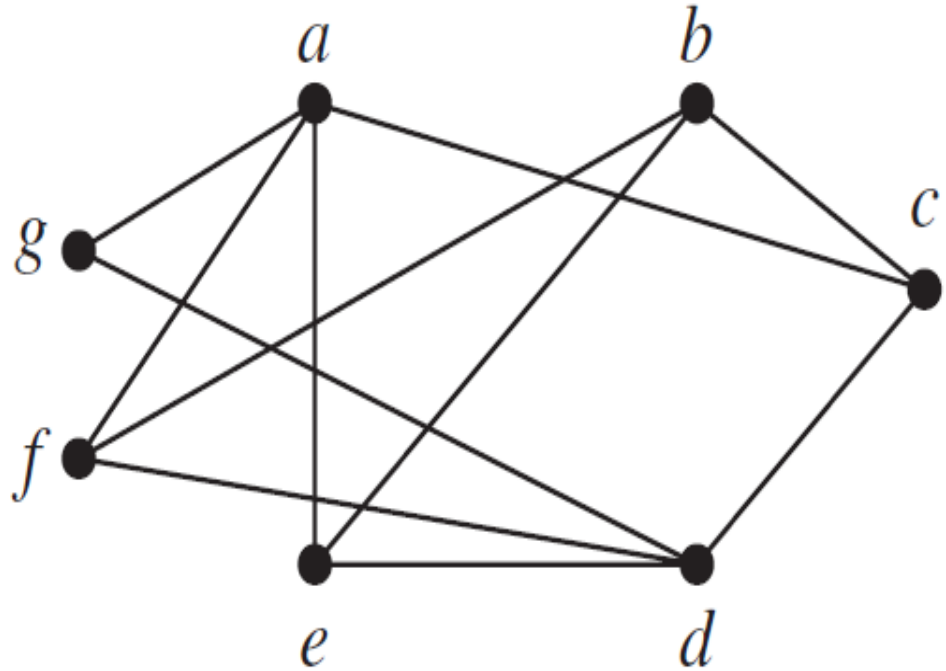
if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that

no edge in G connects either two vertices in V_1 or two vertices in V_2

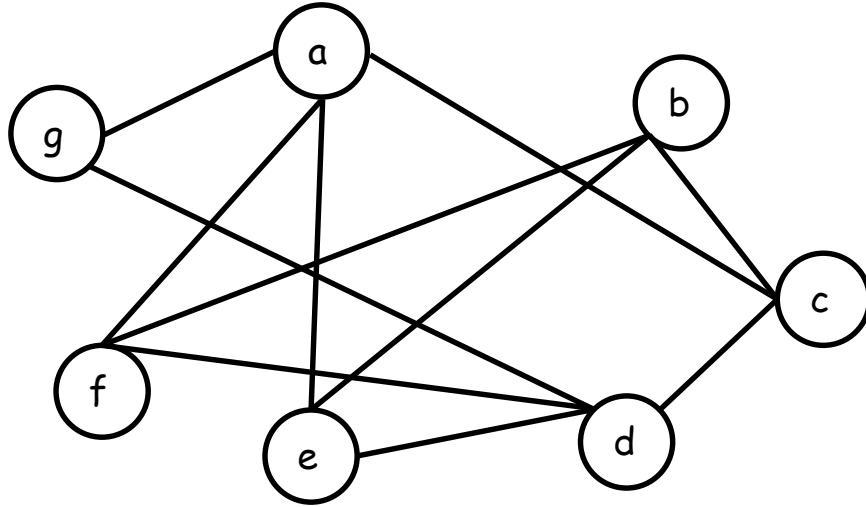
Bipartite Graph

$V1 = \{a, b, d\}$

$V2 = \{c, e, f, g\}$

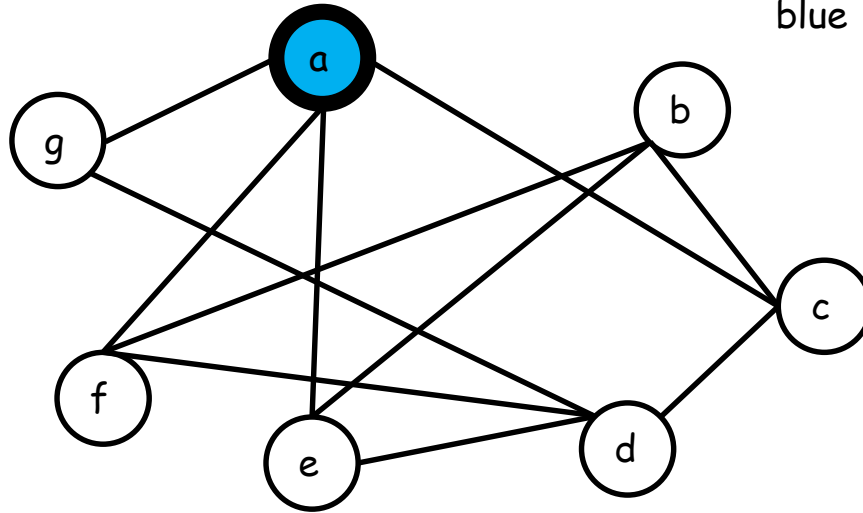


How to decide if a graph is bipartite or not

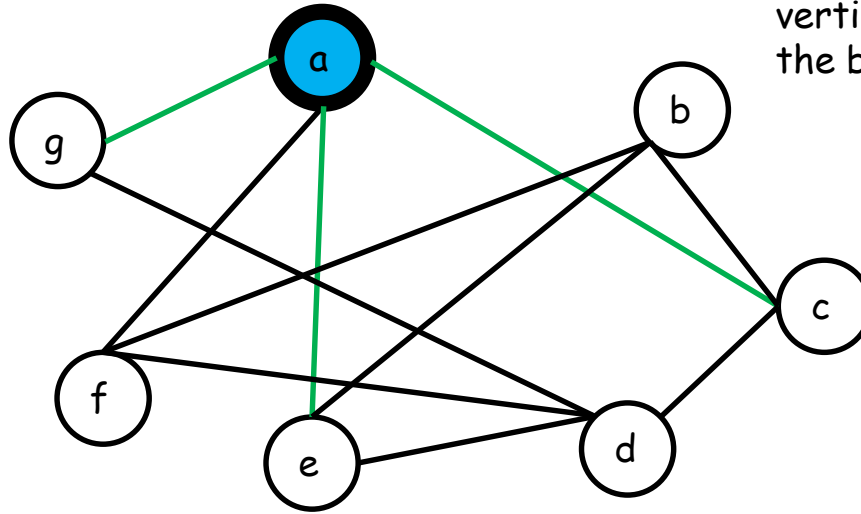


How to decide if a graph is bipartite or not

1. Color any of the vertices blue

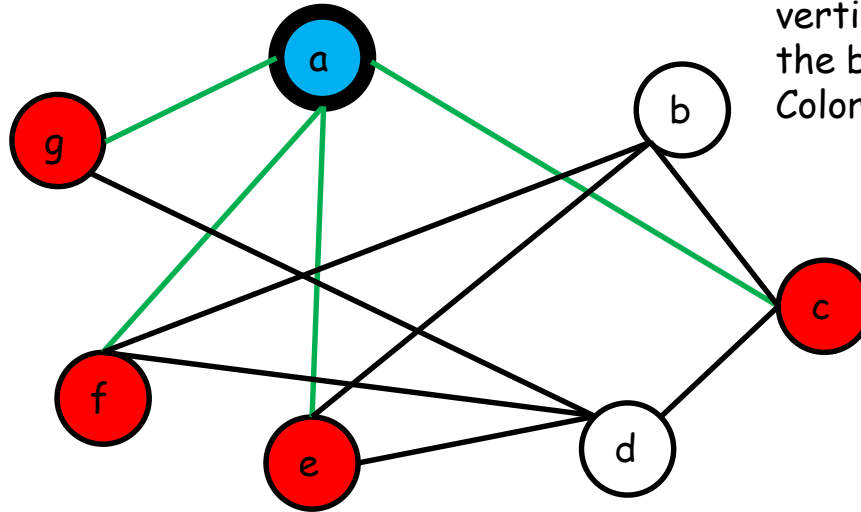


How to decide if a graph is bipartite or not



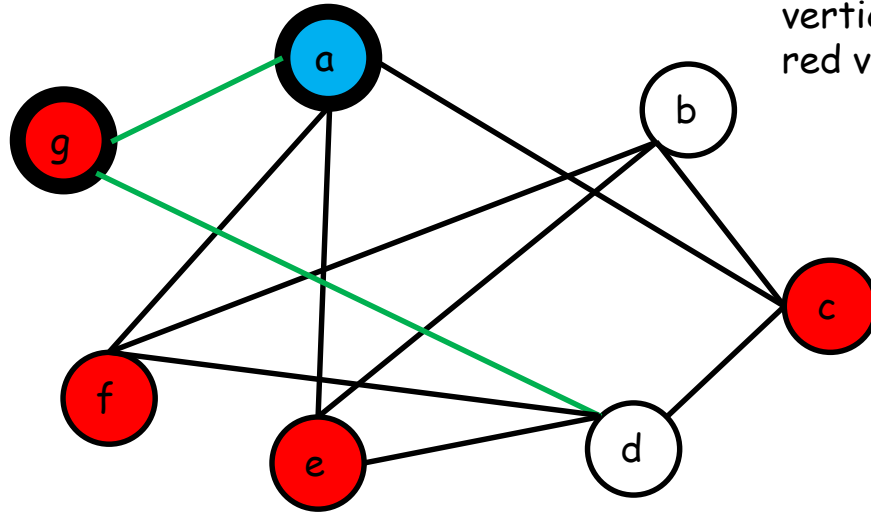
2. Identify all uncolored vertices that are adjacent to the blue vertex.

How to decide if a graph is bipartite or not



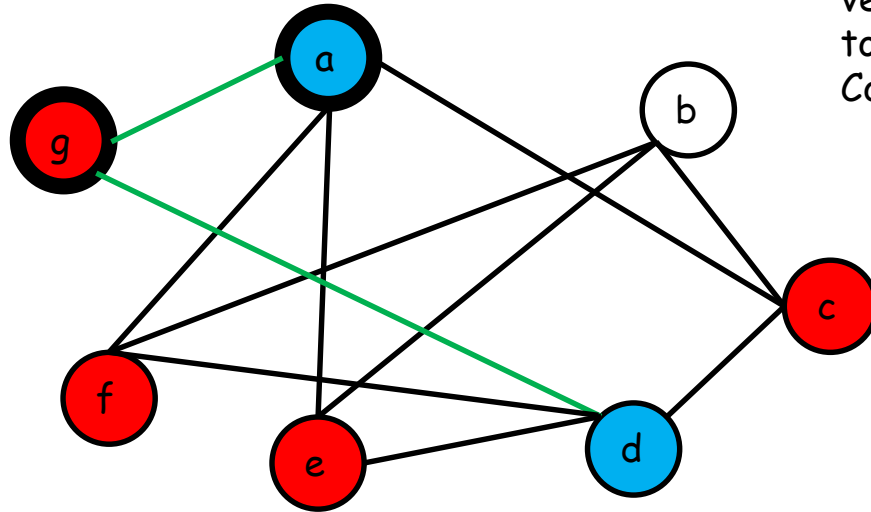
2. Identify all uncolored vertices that are adjacent to the blue vertex. Color them red

How to decide if a graph is bipartite or not



3. Identify all uncolored vertices that are adjacent to a red vertex.

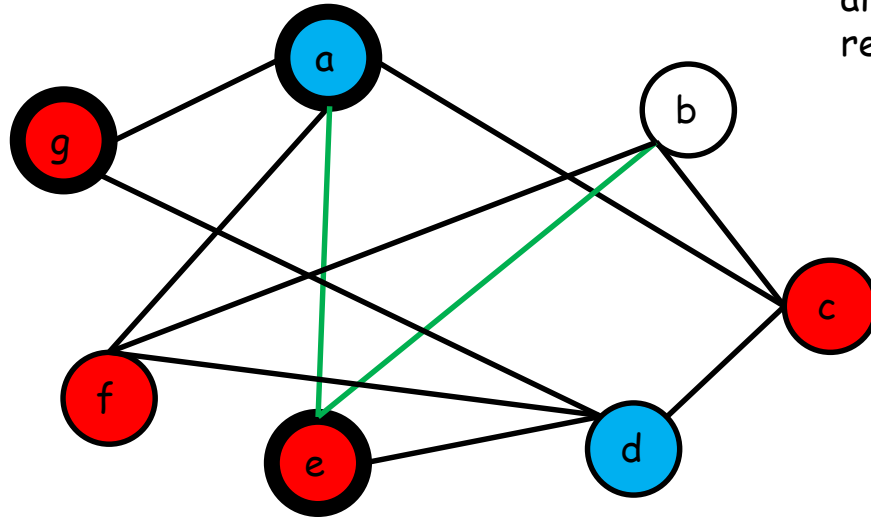
How to decide if a graph is bipartite or not



3. Identify all uncolored vertices that are adjacent to a red vertex. Color them blue.

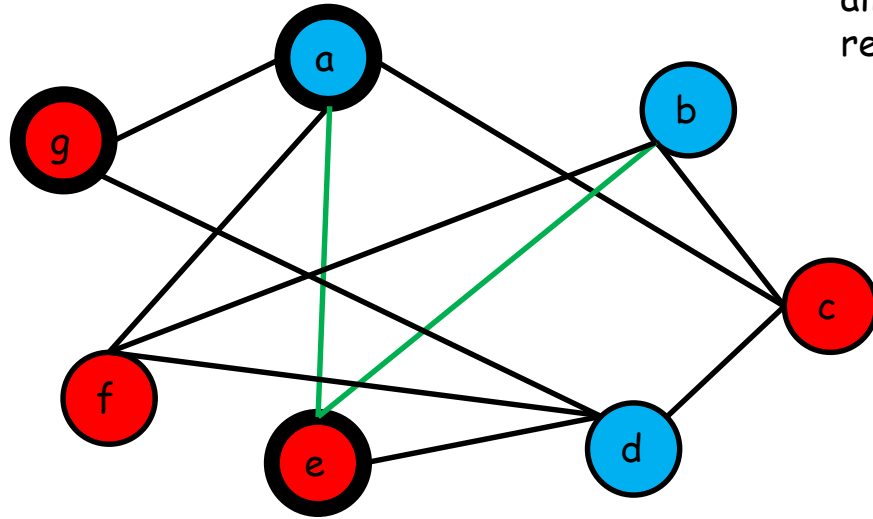
How to decide if a graph is bipartite or not

Repeat steps 2 and 3 until all the vertices are colored red or blue.



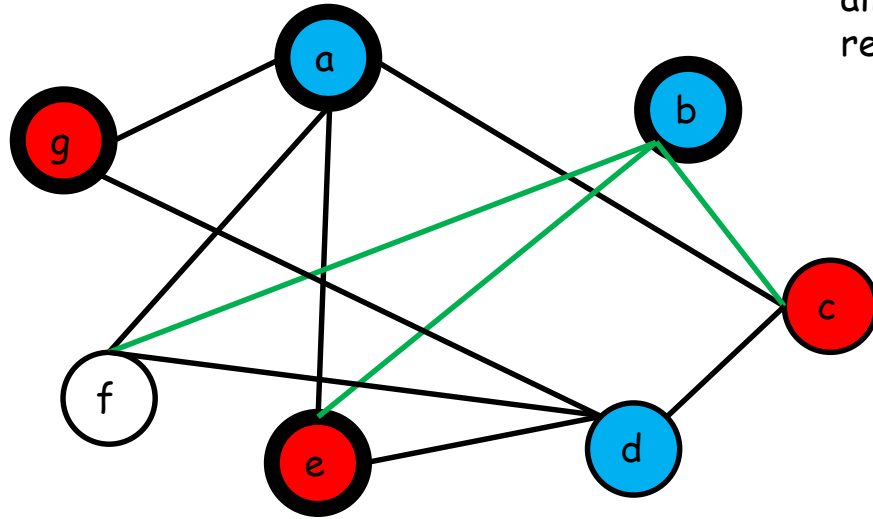
How to decide if a graph is bipartite or not

Repeat steps 2 and 3 until all the vertices are colored red or blue.

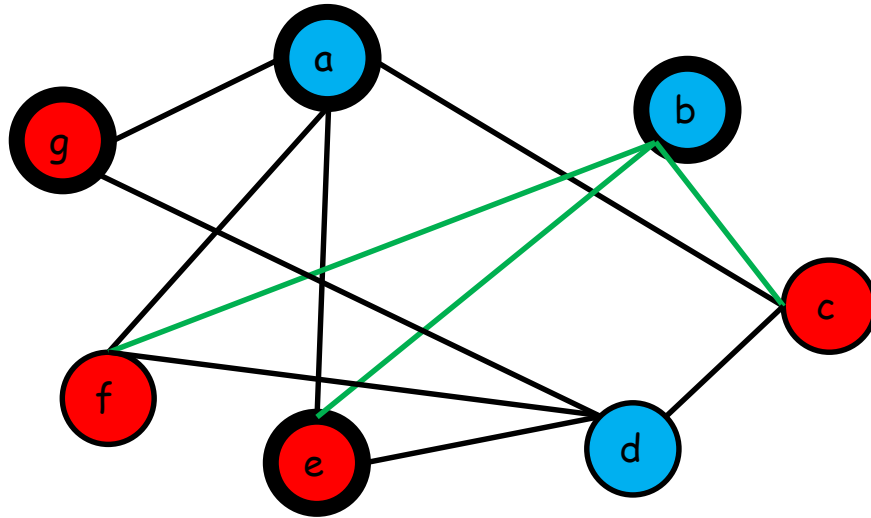


How to decide if a graph is bipartite or not

Repeat steps 2 and 3 until all the vertices are colored red or blue.



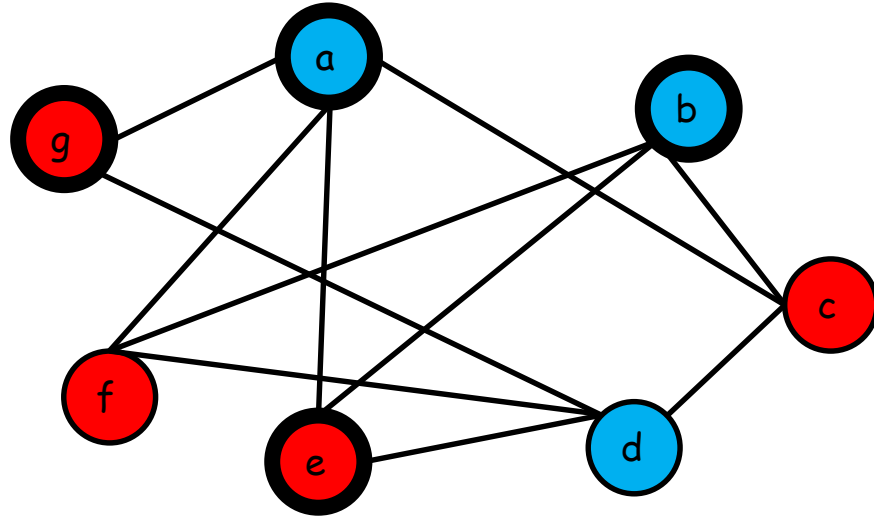
How to decide if a graph is bipartite or not



If there are any two vertices adjacent of the same color, then your graph is not bipartite, otherwise it is bipartite

∴ Bipartite graph

How to decide if a graph is bipartite or not



Disjoint sets

$V1 = \{a, b, d\}$

$V2 = \{c, e, f, g\}$