Predicate Logic

Predicates

Proposition with variables.

Example:

$$P(x) ::= [x > 3]$$
$$P(x, y) ::= [x + 2 = y]$$

Quantifiers

$\forall x$ For ALL x
$\exists y$ There EXISTS some y

$\forall$ is like AND

Let x range over set $\{1, 2, 3, 4, 5, 6\}$
$P(x) ::= [x > 3]$

$$\forall x\, P(x)$$

same as P(1) AND P(2) AND P(3) AND P(4) AND P(5) AND P(6)

$\exists$ is like OR

Let x range over set $\{1, 2, 3, 4, 5, 6\}$
$P(x) ::= [x > 3]$

$$\exists x.\ P(x)$$

same as $P(1)$ OR $P(2)$ OR $P(3)$ OR $P(4)$ OR $P(5)$ OR $P(6)$

Nested Quantifier1

$x, y \in \mathbb{R}$

$Q(x, y) ::= [x + y = 0]$

$$\forall x \exists y \; Q(x, y) = ?$$

Nested Quantifier2

$x \in \mathbb{R}$, $y \in \mathbb{N} = \{1, 2, \ldots\}$

$Q(x, y) ::= [x + y = 0]$

$$\forall x \exists y \; Q(x, y) = ?$$

Nested Quantifier3

$x \in \mathbb{R}$, $y \in \mathbb{N} = \{1, 2, \ldots\}$

$Q(x, y) ::= [x + y = 0]$

$$\exists x \forall y \; Q(x, y) = ?$$

Nested Quantifier4

$x \in \mathbb{R}$, $y \in \mathbb{N} = \{1, 2, \ldots\}$

$Q(x, y) ::= [x * y = 0]$

$$\exists x \forall y \; Q(x, y) = ?$$

Nested Quantifier5

x ∈ $\mathbb{R}$, y ∈ $\mathbb{R}$

Q(x, y)::= [$(x * y)^2$≥ 0]

$\forall x \forall y$ Q(x, y) = ?

Nested Quantifier6

$x \in \mathbb{R}$, $y \in \mathbb{R}$

$Q(x, y) ::= [x^2 + y^2 = 13]$

$\exists x \exists y\, Q(x, y) = ?$

Nested Quantifier7

$x \in \mathbb{R}$, $y \in \mathbb{R}$

$Q(x, y) ::= [x^2 + y^2 = -6]$

$\exists x \exists y\, Q(x, y) = ?$

Nested Quantifier8

$x \in \mathbb{R}$, $y \in \mathbb{R}$, $z \in \mathbb{R}$

$Q(x, y, z) ::= [x + y = z]$

$$\forall x \, \forall y \, \exists z \; Q(x, y, z) = ?$$

$$\forall x \, \forall y \, \exists z \, [x + y = z]$$

$x = 1$

$y = 1$

$z = 2$, $1 + 1 = 2$: T

$\forall x \, \forall y \, \exists z \, [x + y = z]$

$x = 1$

$y = 1$

$z = 2$, $1 + 1 = 2$: T

What's the quantifier before z?

$\exists z$

$$\forall x \, \forall y \, \exists z \, [x + y = z]$$

$x = 1$

$y = 1$

$z = 2, 1 + 1 = 2$: T

What's the quantifier before z?

$\exists z$

$x = 1$

$y = 1$

$\boxed{T}$ $z = 2, 1 + 1 = 2$: T

$$\forall x \, \forall y \, \exists z \, [x + y = z]$$

What's the quantifier before y? $\forall y$

$\forall x \, \forall y \, \exists z \, [x + y = z]$

What's the quantifier before y? $\forall y$

$x = 1$

$\boxed{T}$ $y = 1$

$z = 1, 1 + 1 = 2$: T

$\boxed{T}$ $y = 2$

$z = 3, 1 + 2 = 3$: T

$$\forall x\, \forall y\, \exists z\, [x + y = z]$$

$x = 1$

$\boxed{T}$ $y = 1$

$z = 1,\ 1 + 1 = 2$: T

$\boxed{T}$ $y = 2$

$z = 3,\ 1 + 2 = 3$: T

$\boxed{T}$ $y = -1$

$z = 0,\ 1 - 1 = 0$: T

$\forall x \, \forall y \, \exists z \, [x + y = z]$

$x = 1$

$\boxed{T}$

$\boxed{T}$ $y = 1$

$z = 1, 1 + 1 = 2$: T

$\boxed{T}$ $y = 2$

$z = 3, 1 + 2 = 3$: T

$\boxed{T}$ $y = -1$

$z = 0, 1 - 1 = 0$: T

$$\forall x \, \forall y \, \exists z \, [x + y = z]$$

What's the quantifier before x? $\forall x$

$\forall x \, \forall y \, \exists z \, [x + y = z]$

What's the quantifier before x? $\forall x$

$x = 2$

$\boxed{T}$ $y = 1$

$z = 3, 2 + 1 = 3$: T

$\boxed{T}$ $y = -3$

$z = 3, 2 - 5 = -3$: T

$\boxed{T}$ $y = 0$

$z = 2, 2 + 0 = 2$: T

$\boxed{T}$

Nested Quantifier9

$x \in \mathbb{R}$, $y \in \mathbb{R}$, $z \in \mathbb{R}$

$Q(x, y, z) ::= [x + y = z]$

$$\exists x \, \forall y \forall z \; Q(x, y, z) = ?$$

$$\exists x \forall y \forall z\, [x + y = z]$$

$x = 1$

$y = 1$

$z = 2,\ 1 + 1 = 2$: T

$\exists x \forall y \forall z\,[x + y = z]$

$x = 1$

$y = 1$

$z = 2$, $1 + 1 = 2$: T

What's the quantifier before z?

$\forall z$

$$\exists x \, \forall y \forall z \, [x + y = z]$$

$x = 1$

$y = 1$

$z = 2$, $1 + 1 = 2$: T

$z = 1$, $1 + 1 = 1$: F

$$\exists x \forall y \forall z \, [x + y = z]$$

$x = 1$

$y = 1$

$z = 2$, $1 + 1 = 2$: T

$z = 1$, $1 + 1 = 1$: F

What's the quantifier before z?

$\forall z$

$\exists x \, \forall y \forall z \, [x + y = z]$

$x = 1$

$y = 1$

$z = 2$, $1 + 1 = 2$: T

$z = 1$, $1 + 1 = 1$: F

What's the quantifier before z?

$\forall z$

$x = 1$

$y = 1$

$\boxed{F}$

$z = 2$, $1 + 1 = 2$: T

$z = 1$, $1 + 1 = 1$: F

$\exists x \forall y \forall z\ [x + y = z]$

What's the quantifier before y? $\forall y$

$\exists x \, \forall y \forall z \, [x + y = z]$

What's the quantifier before y? $\forall y$

$\boxed{F}$

$x = 1$

$\boxed{F}$

$y = 1$

$z = 2$, $1 + 1 = 2$: T

$z = 1$, $1 + 1 = 1$: F

$$\exists x\, \forall y \forall z\, [x + y = z]$$

What's the quantifier before x? $\exists x$

$$\exists x \, \forall y \forall z \, [x + y = z]$$

What's the quantifier before x? $\exists x$

$x = 0$

$y = 1$

$\boxed{F}$ $\boxed{F}$

$z = 1$, $0 + 1 = 1$: T

$z = 2$, $0 + 1 = 2$: F

Nested Quantifier10

$x \in \mathbb{N}, y \in \mathbb{N}, z \in \mathbb{N}$

$Q(x, y, z) ::= [x = yz]$

$$\forall x \exists y \exists z\ Q(x, y, z) = ?$$

$\forall x \exists y \exists z \, [x = yz]$

Negating Nested Quantifiers

Negating nested quantifiers

Find the negation of $\forall x \exists y \exists z\ Q(x, y, z)$, where
$Q(x, y, z) ::= [x = yz]$

$\neg (\forall x \exists y \exists z\ Q(x, y, z))$

$\equiv \exists x\ \neg(\exists y \exists z\ Q(x, y, z))$

$\equiv \exists x \forall y\ \neg(\exists z\ Q(x, y, z))$

$\equiv \exists x \forall y \forall z\ \neg(Q(x, y, z))$

Negating nested quantifiers

$\equiv \exists x \forall y \forall z \; \neg(x = yz)$

$\equiv \exists x \forall y \forall z \; (x \neq yz)$

Negating nested quantifiers

Find the negation of $\forall x \exists y \exists z\ Q(x, y, z) \vee P(x, y, z)$, where

$Q(x, y, z) ::= [x = yz]$

$P(x, y, z) ::= [x + y > z]$

$$\exists x \forall y \forall z\ (x \neq yz) \wedge (x + y \leq z)$$