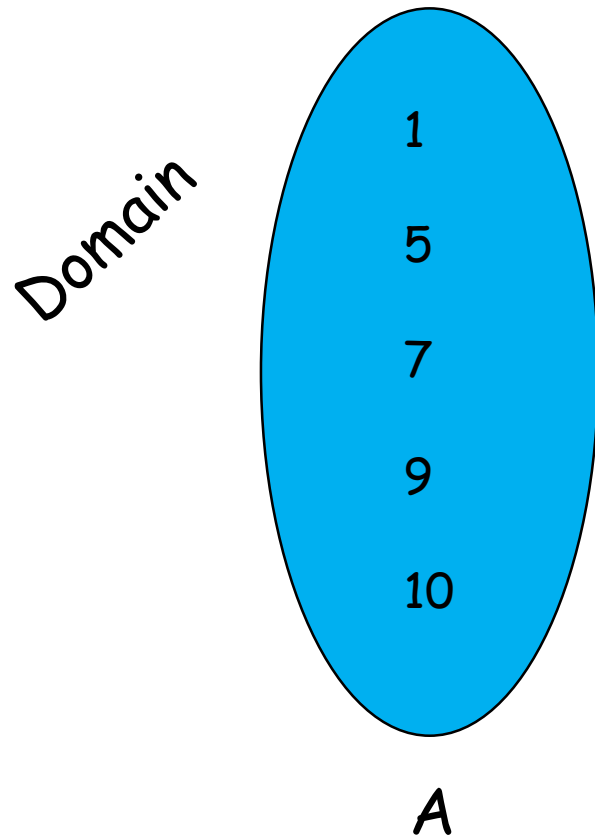


Relations

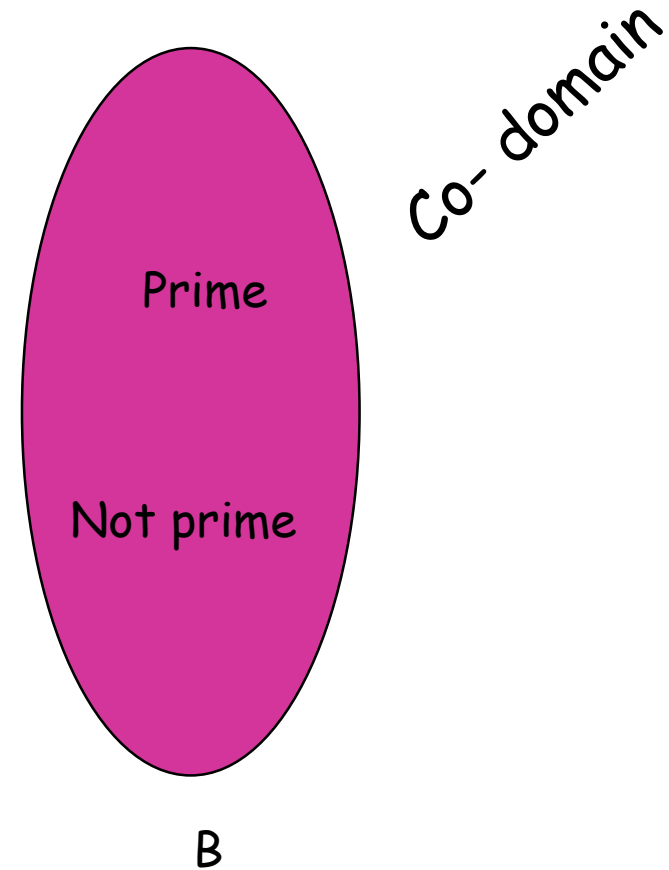
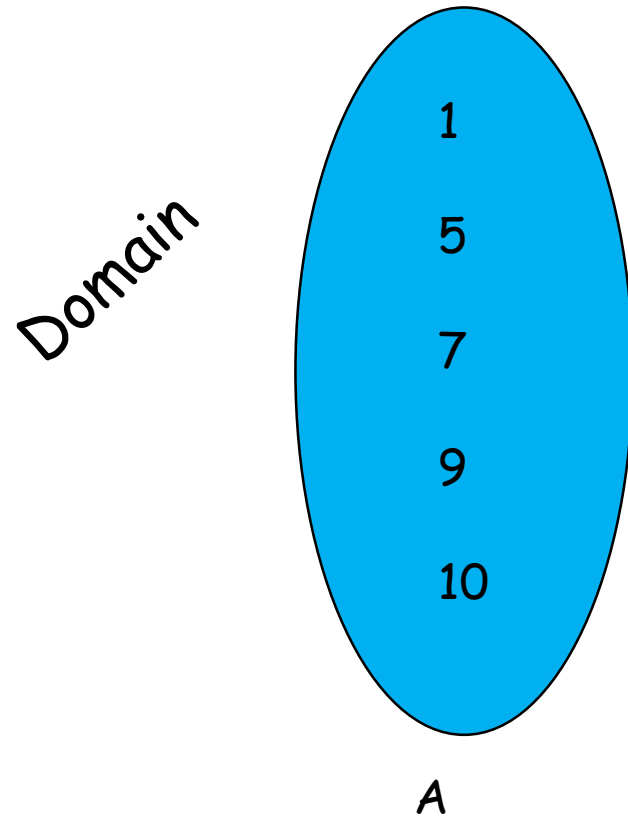
Binary Relation

$A = \{1, 5, 7, 9, 10\}$: Domain

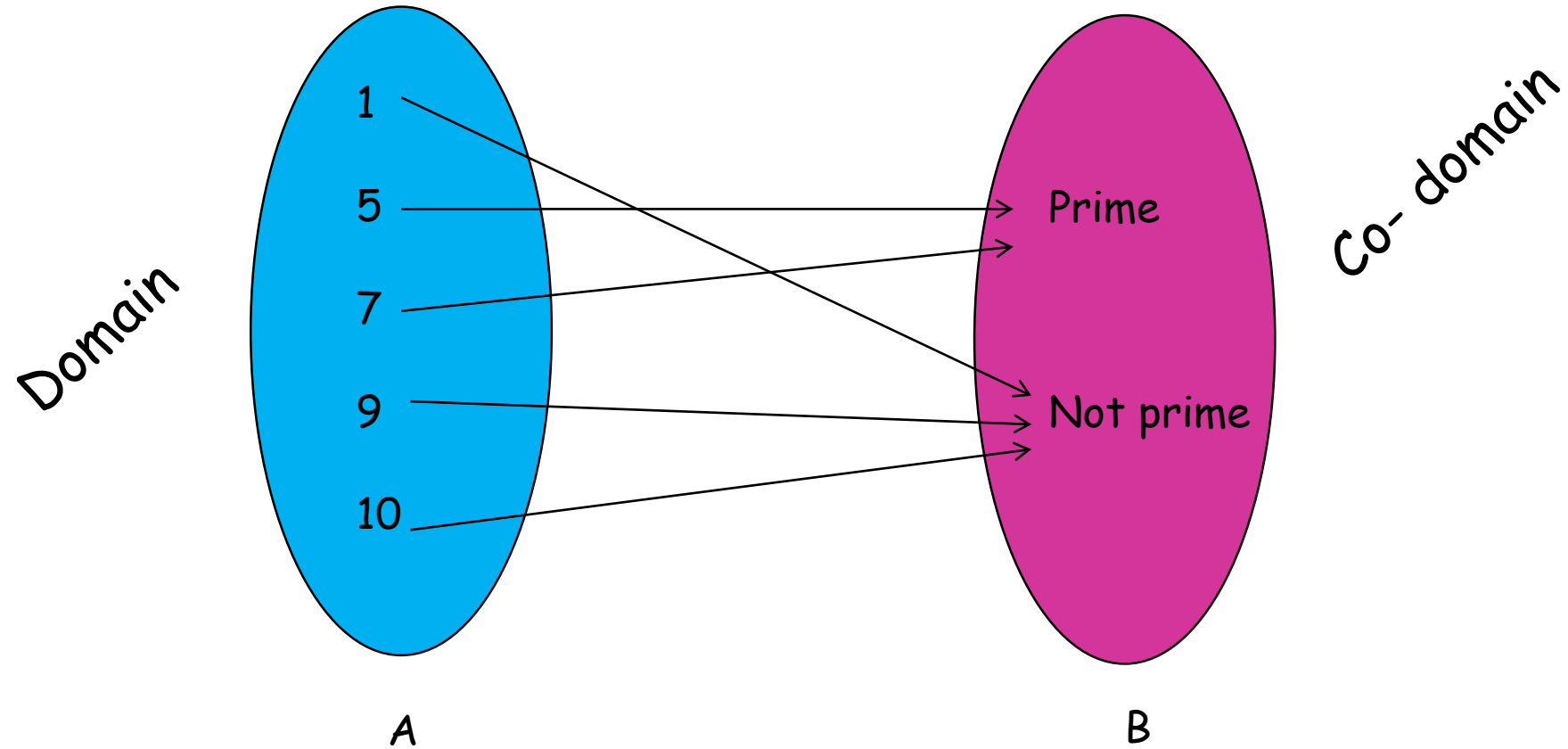


Binary Relation

$B = \{\text{prime, not prime}\}$: Co-domain

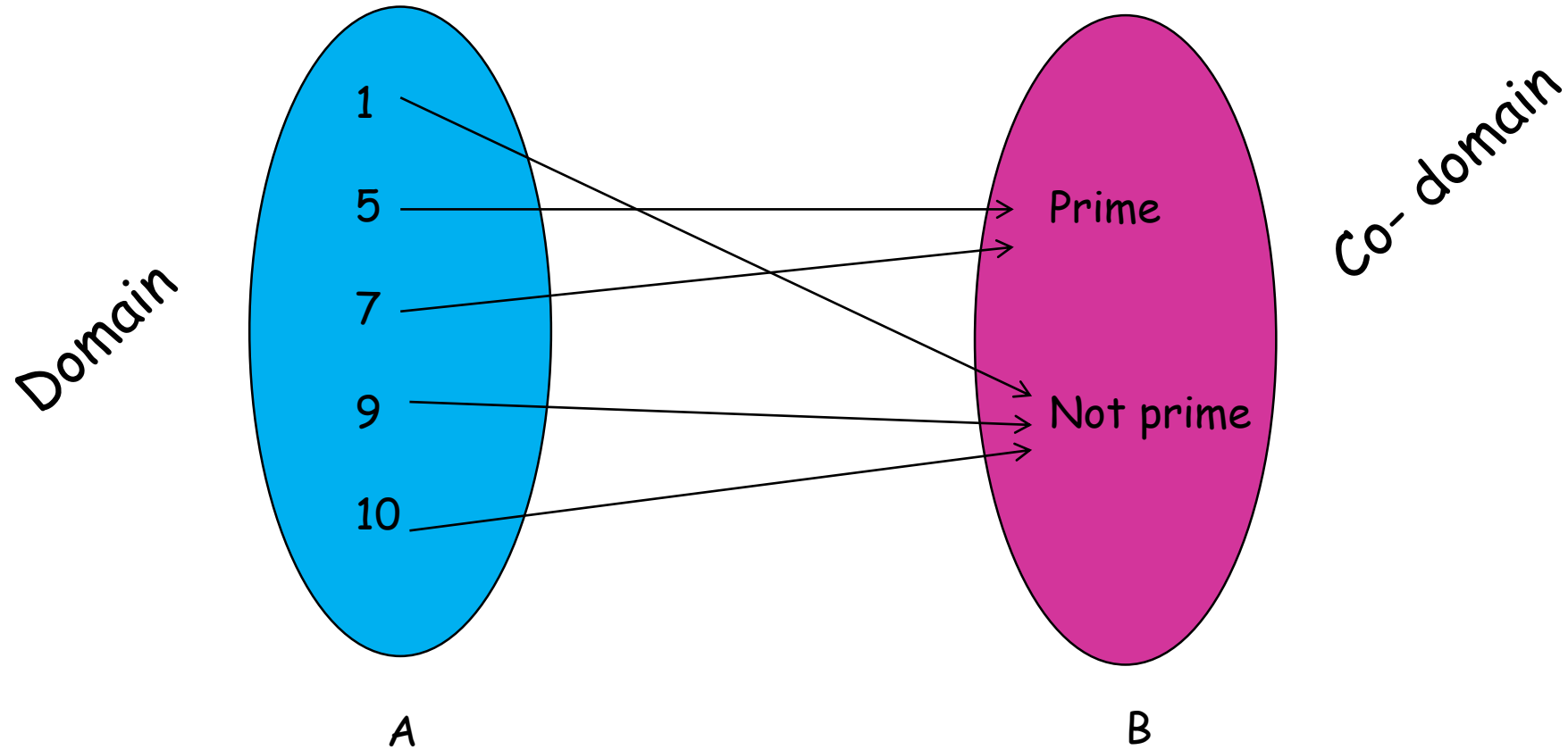


Binary Relation



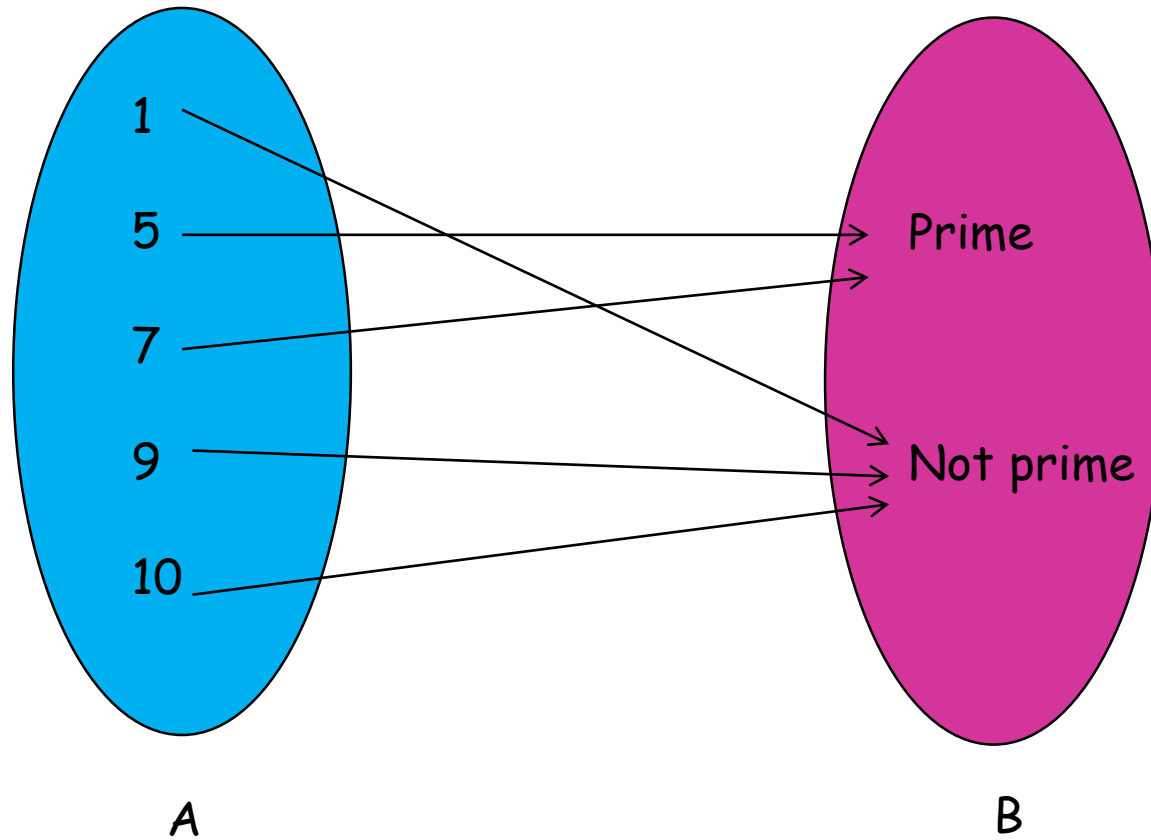
Binary Relation

Is $a \in A$ prime or not?



Binary Relation

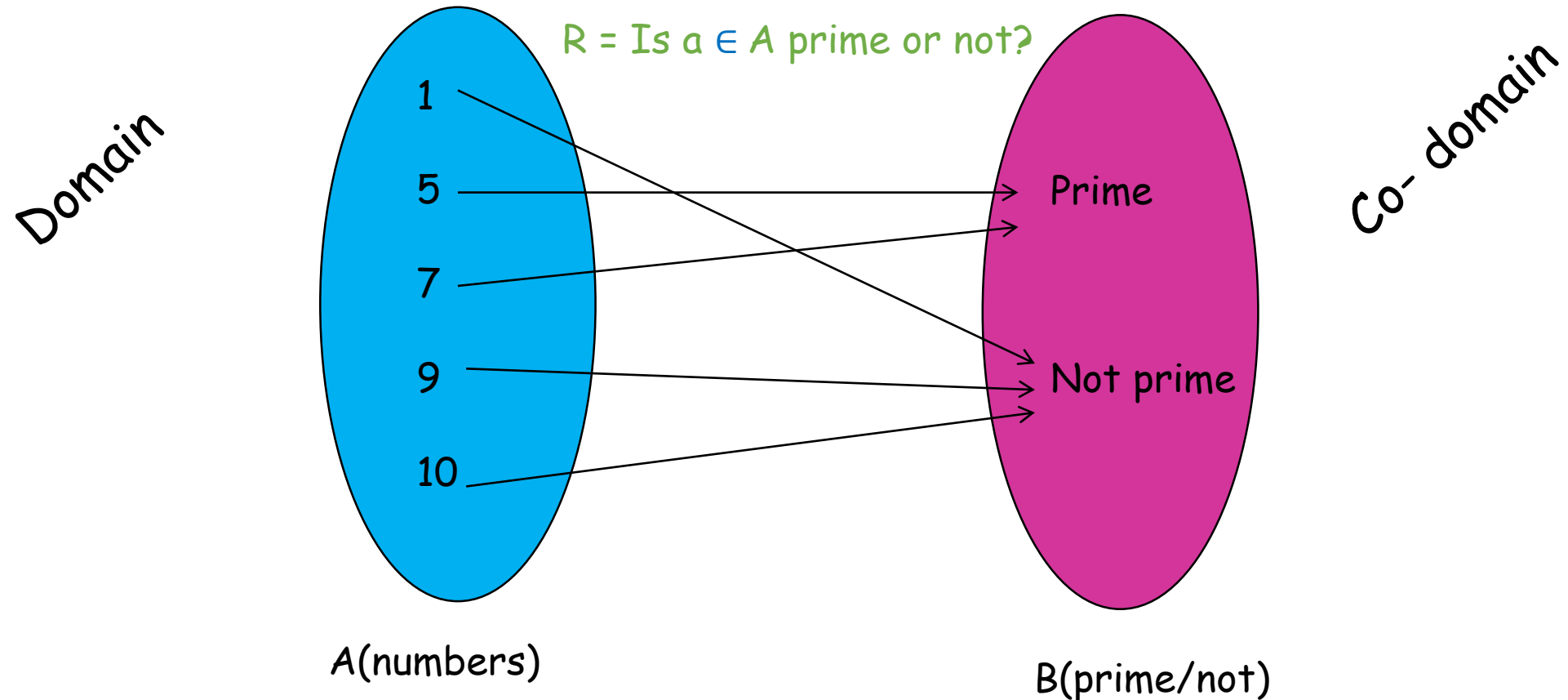
Is $a \in A$ prime or not?



$R = \{(1, \text{Not prime}), (5, \text{prime}), (7, \text{prime}), (9, \text{Not prime}), (10, \text{Not prime})\}$

Binary Relation

A binary relation associates elements of one set called domain, with element of another set called co-domain

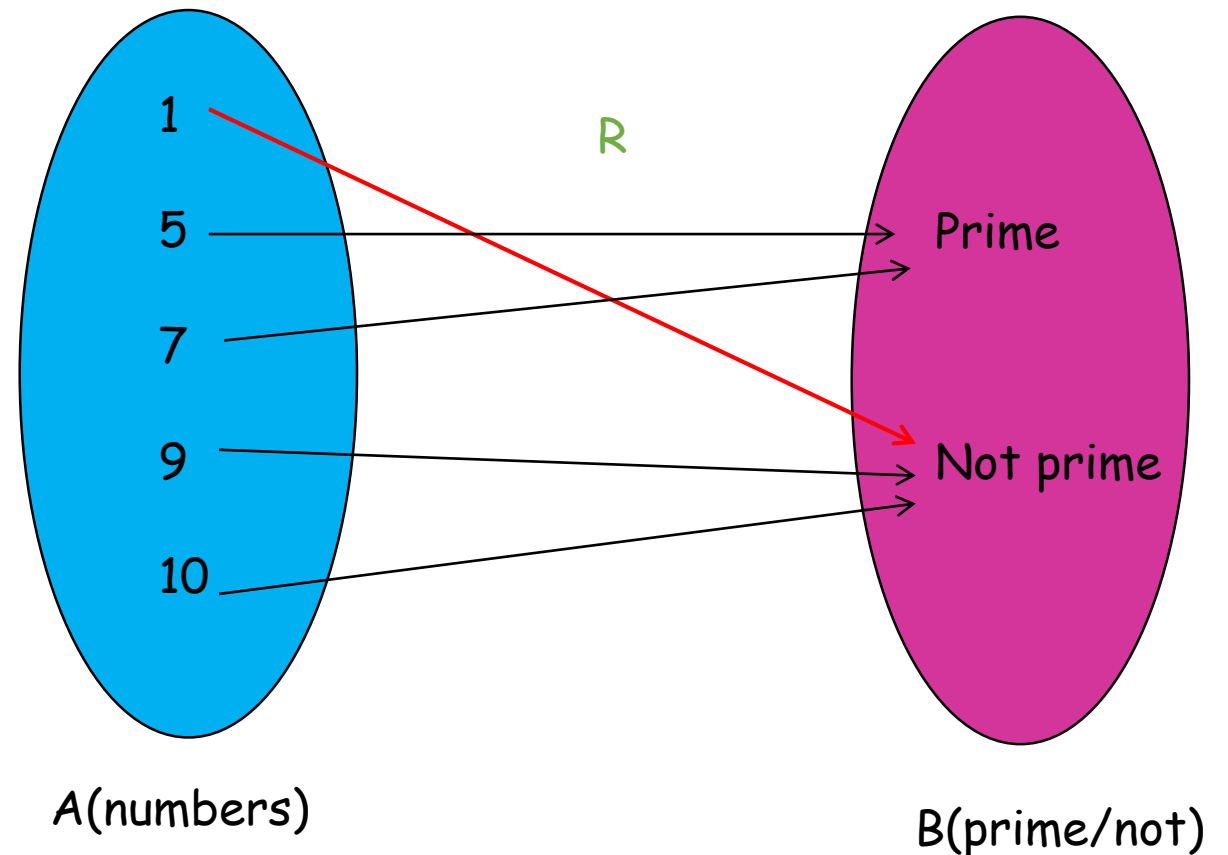


"Is prime/ not" relation R

Notation:

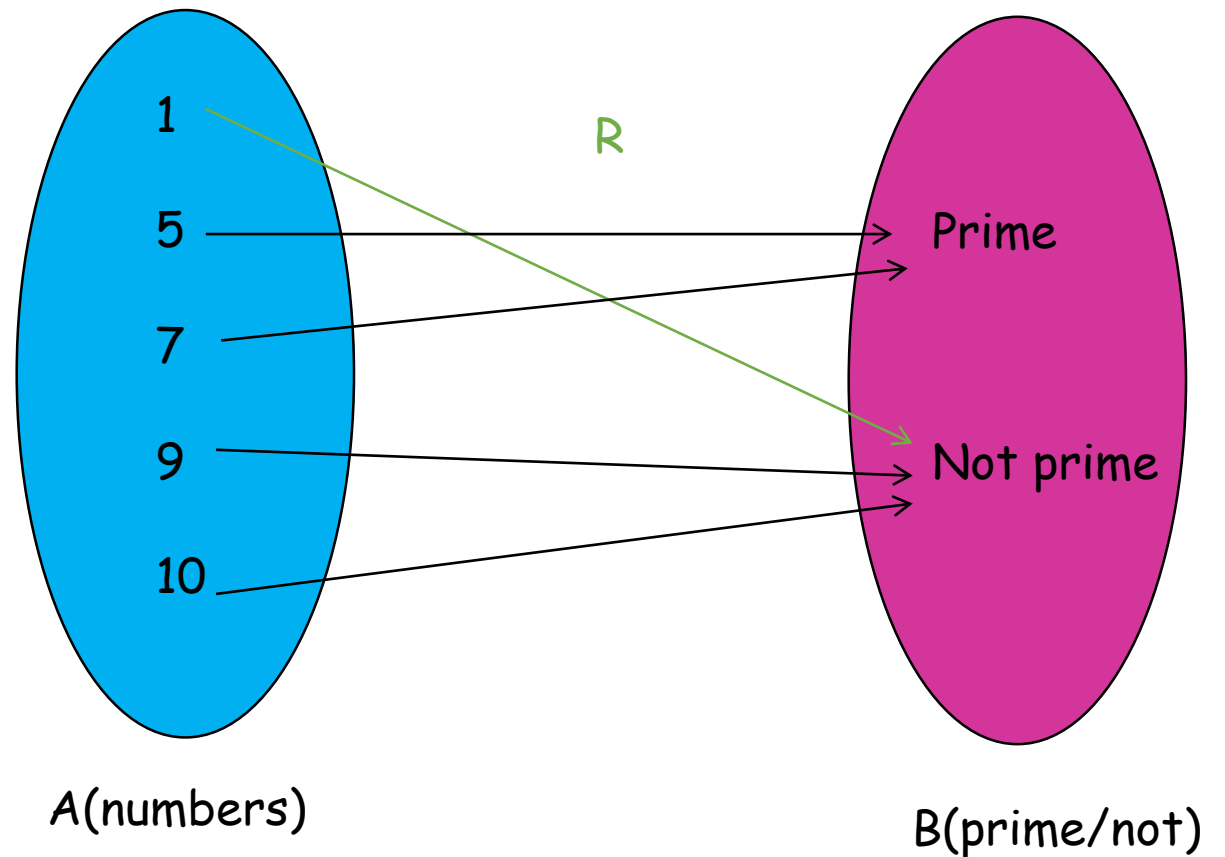
$1 \ R \text{ not prime}$

$R(1, \text{not prime})$



Images under R

$$R(1) = \{\text{not prime}\}$$



Binary Relation

A binary relation from set A (Domain) to B (Co-domain) is a subset of $A \times B$

Relation on a set

A relation on a set A is a relation from A to A .

Relation on a set

Let A be the set $\{1, 2, 3, 4\}$. Which ordered pairs are in the relation $R = \{(a, b) \mid a \text{ divides } b\}$?

$$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$$

$$R = \{ \hspace{15em} \}$$

Relation on a set

Let A be the set $\{1, 2, 3, 4\}$. Which ordered pairs are in the relation $R = \{(a, b) \mid a \text{ less equal } b\}$?

$$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$$

$$R = \{ \hspace{15em} \}$$

Relation on a set

Let A be the set $\{1, 2, 3, 4\}$. Which ordered pairs are in the relation $R = \{(a, b) \mid a = b\}$?

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$

$R = \{ \hspace{15em} \}$

Properties of Relations

Reflexive Relation

A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.

Reflexive Relation

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$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4), (5, 5)\}$,
is a relation on $A = \{1, 2, 3, 4, 5\}$

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is a relation on $A = \{1, 2, 3, 4, 5\}$

$(1, 1) \in R$

$(2, 2) \in R$

$(3, 3) \in R$

$(4, 4) \in R$

$(5, 5) \in R$

reflexive relation

Reflexive Relation

A relation R on a set A is called **reflexive** if $(a, a) \in R$ for every element $a \in A$.

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

$(1, 1) \in R$

$(2, 2) \in R$

$(3, 3) \notin R$

$(4, 4)$

Not a reflexive relation

Symmetric Relation

A relation R on a set A is called **symmetric** if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

Symmetric Relation

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is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, a)
(1, 1)	
(1, 2)	
(2, 1)	
(2, 2)	
(3, 4)	
(4, 1)	
(4, 4)	

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is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, a)
(1, 1)	(1, 1)
(1, 2)	(2, 1)
(2, 1)	(1, 2)
(2, 2)	(2, 2)
(3, 4)	(4, 3)
(4, 1)	
(4, 4)	

$\in R$

$\in R$

$\in R$

$\in R$

$\notin R$

Not symmetric

Symmetric Relation

$R_2 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4), (5, 5)\}$, is a relation on $A = \{1, 2, 3, 4, 5\}$

(a, b)	(b, a)
----------	----------

A relation R on a set A is called **symmetric** if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

Antisymmetric Relation

A relation R on a set A such that for all $a, b \in A$, if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ is called **antisymmetric**.

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$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, a)
(1, 1)	
(1, 2)	
(2, 1)	
(2, 2)	
(3, 4)	
(4, 1)	
(4, 4)	

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is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, a)
(1, 1)	
(1, 2)	(2, 1)
(2, 1)	
(2, 2)	
(3, 4)	
(4, 1)	
(4, 4)	

pass
 $\in R$

Not antisymmetric

Antisymmetric Relation

A relation R on a set A such that for all $a, b \in A$, if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ is called **antisymmetric**.

$R_2 = \{(1, 1), (1, 2), (1, 4), (2, 2), (3, 3), (4, 4), (5, 5)\}$, is a relation on $A = \{1, 2, 3, 4, 5\}$

(a, b)	(b, a)
(1, 1)	
(1, 2)	(2, 1)
(1, 4)	(4, 1)
(2, 2)	
(3, 3)	
(4, 4)	
(5, 5)	

pass

$\notin R$

$\notin R$

pass

pass

pass

pass

antisymmetric

Transitive Relation

A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

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$R1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

[illegible]

Transitive Relation

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$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, c)	(a, c)
(1, 1)	(1, 1)	
	(1, 2)	

$(1, 1) \rightarrow (a, b)$

$a = 1, b = 1$

Find the (b, c) s \rightarrow find those tuples that start with 1s

Transitive Relation

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$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, c)	(a, c)
(1, 1)	(1, 1)	(1, 1)
	(1, 2)	(1, 2)

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A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, c)	(a, c)
$(1, 1)$	$(1, 1)$	$(1, 1)$
	$(1, 2)$	$(1, 2)$

$\in R$

$\in R$

Transitive Relation

A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, c)	(a, c)
(1, 1)	(1, 1)	(1, 1)
	(1, 2)	(1, 2)
(1, 2)	(2, 1)	(1, 1)
	(2, 2)	(1, 2)

$\in R$

$\in R$

$\in R$

$\in R$

Transitive Relation

A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, c)	(a, c)
(1, 1)	(1, 1)	(1, 1)
	(1, 2)	(1, 2)
(1, 2)	(2, 1)	(1, 1)
	(2, 2)	(1, 2)

$\in R$

$\in R$

$\in R$

$\in R$

Transitive Relation

$R1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

Not transitive

(a, b)	(b, c)	(a, c)
(1, 1)	(1, 1)	(1, 1)
	(1, 2)	(1, 2)
(1, 2)	(2, 1)	(1, 1)
	(2, 2)	(1, 2)
(2, 1)	(1, 1)	(2, 1)
	(1, 2)	(2, 1)

$\in R$

$\in R$

$\in R$

$\in R$

$\in R$

$\in R$

(a, b)	(b, c)	(a, c)
(2, 2)	(2, 1)	(2, 1)
	(2, 2)	(2, 2)
(3, 4)	(4, 1)	(3, 1)
	(4, 4)	(1, 2)

$\in R$

$\in R$

$\notin R$

Is the "divides" relation on the set of positive integers reflexive?

A relation R on a set A is called **reflexive** if $(a, a) \in R$ for every element $a \in A$.

Is the "divides" relation on the set of positive integers symmetric?

A relation R on a set A is called **symmetric** if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

Is the "divides" relation on the set of positive integers anti-symmetric?

A relation R on a set A such that for all $a, b \in A$, if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ is called **antisymmetric**.

Is the "divides" relation on the set of positive integers transitive?

A relation R on a set A is called **transitive** if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.