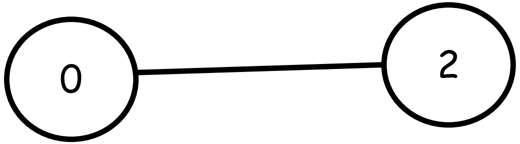
Graphs

Graphs

A graph $G = (V ,E)$ consists of
V , a nonempty set of vertices (or nodes) and E, a set of edges.

Undirected Graph

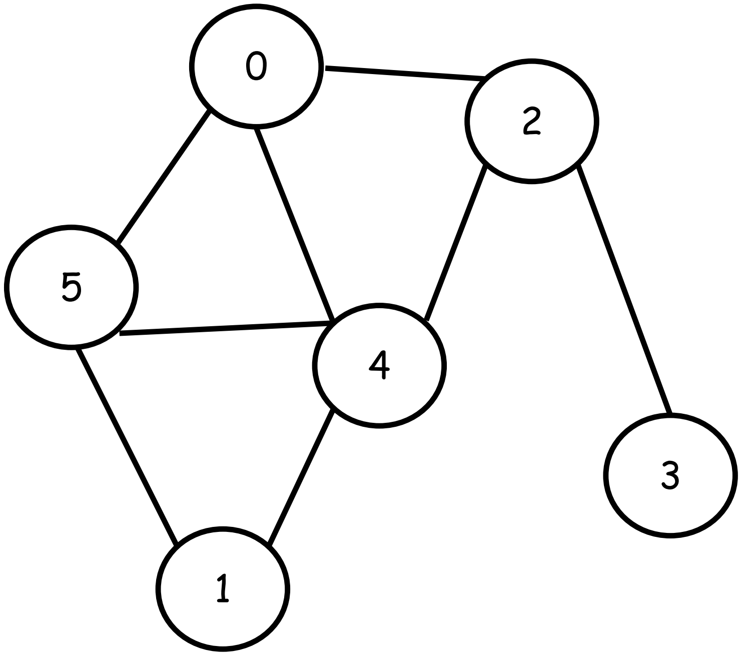


No Direction

Representation: 02

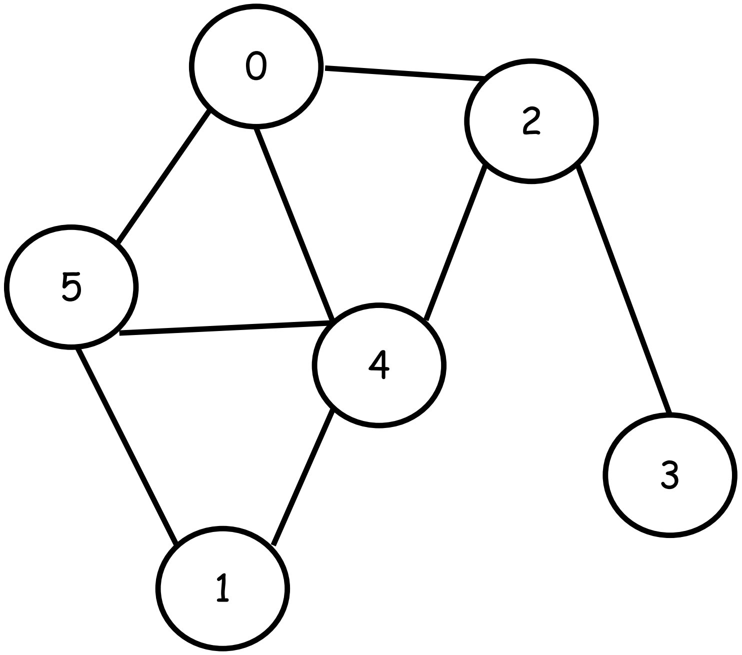
20

Undirected Graph



Vertices = {0, 1, 2, 3, 4, 5}

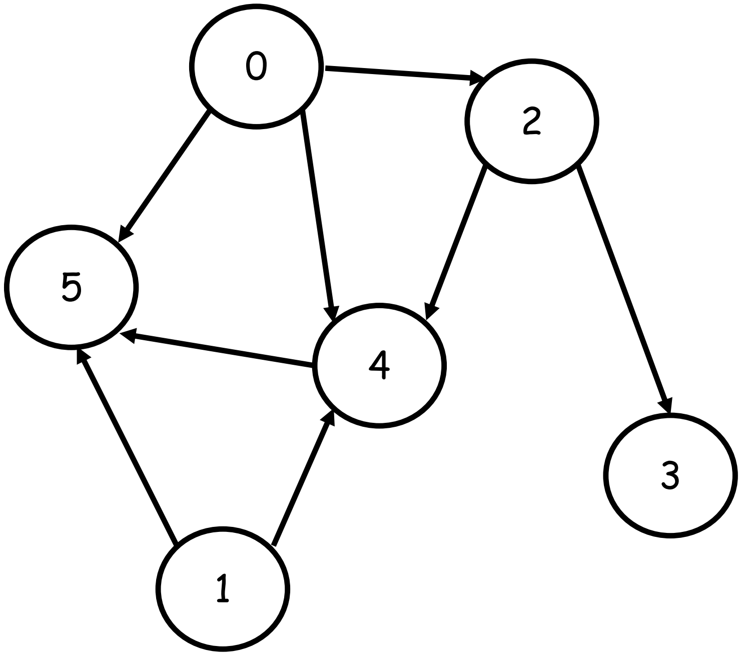
Undirected Graph

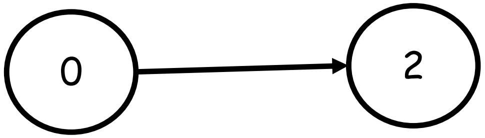


Edges = {02, 05, 04, 15, 14, 24, 23, 45 }

No convention
Which vertex to write first.

Directed Graph



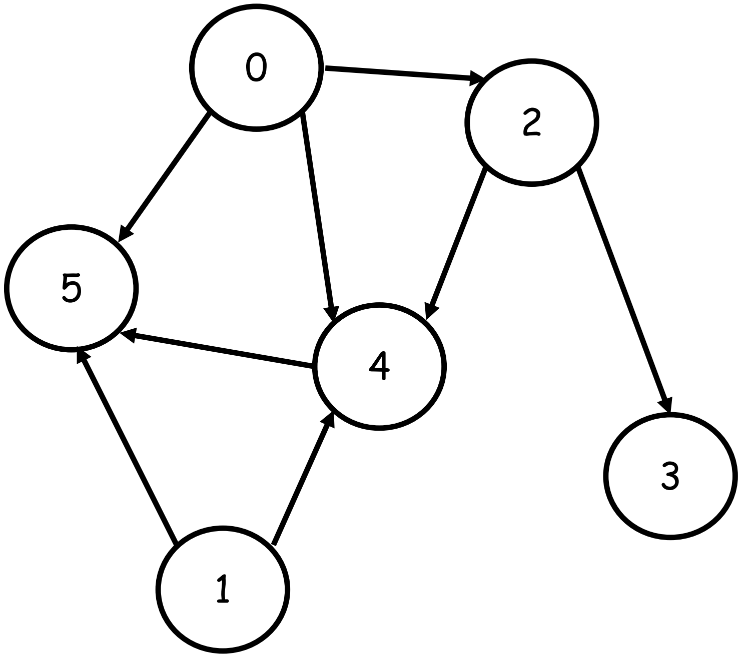


Direction: From Node 0 - Node 2
Node 0: Initial Vertex
Node 2: Terminal Vertex
Direction: Initial Vertex - Terminal Vertex

Representation: 02

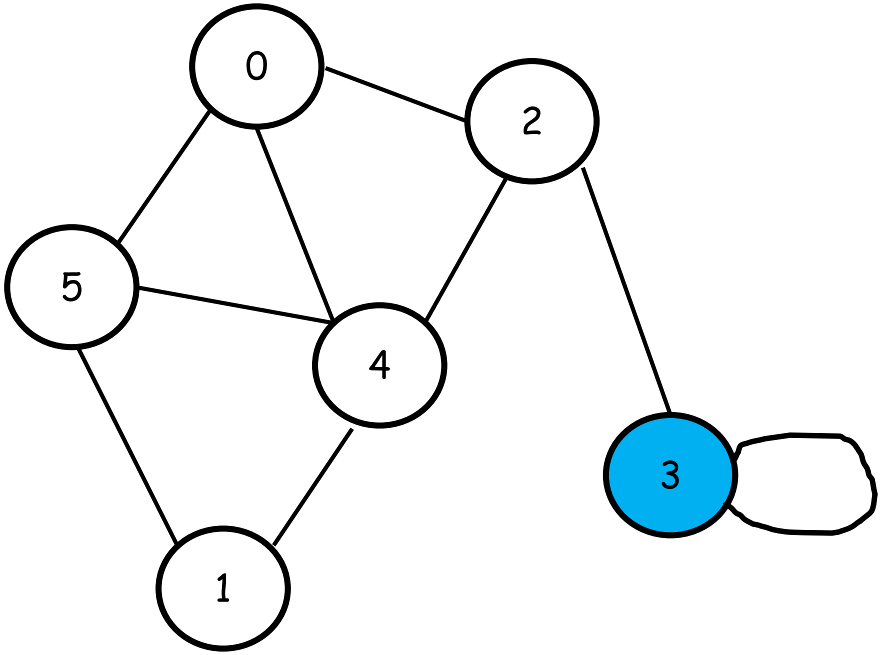
~~20~~

Directed Graph

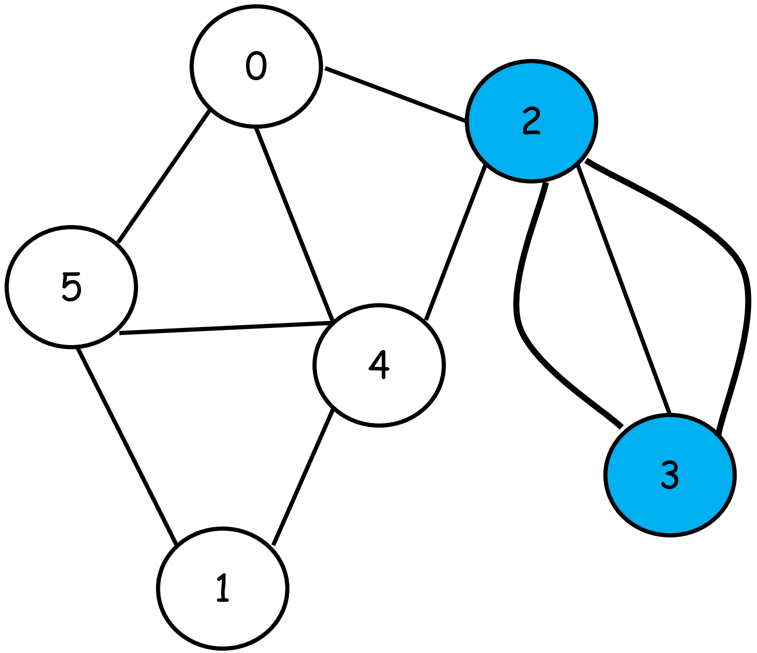


Edges = {05, 04, 02, 24, 45, 15, 14, 23}

Loops in Graphs

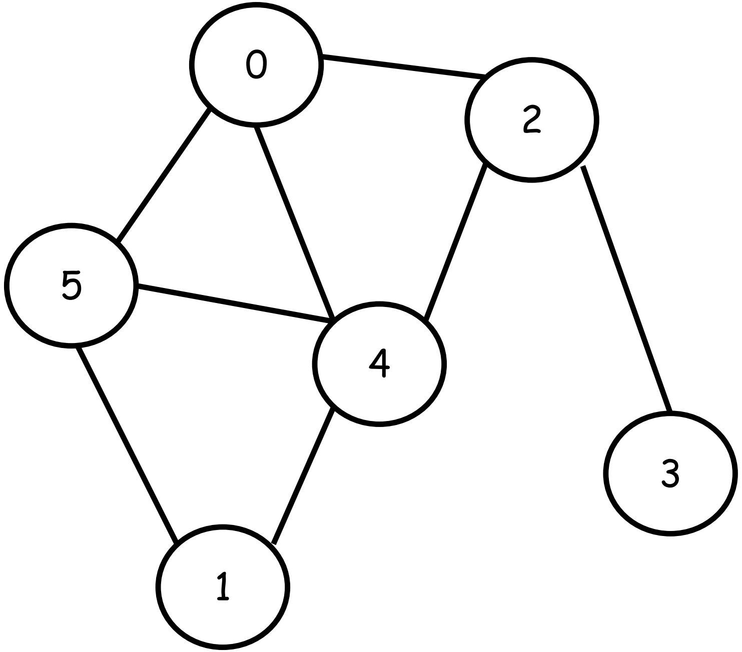


Multiple edges in Graphs



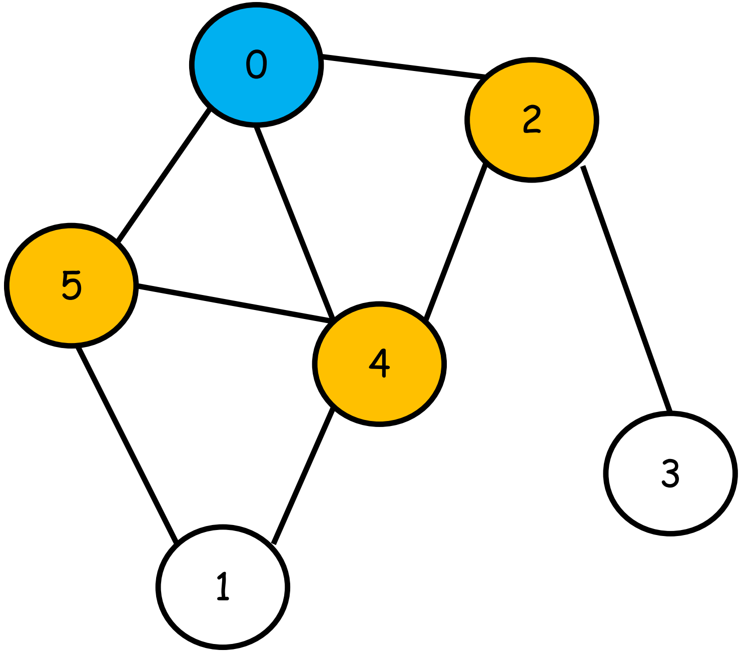
Undirected Graph: Adjacency Matrix Representation

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						



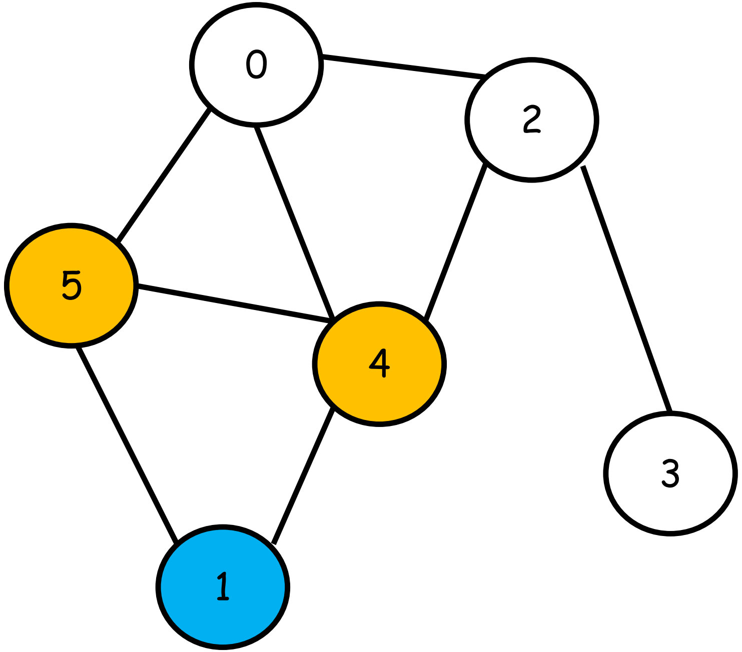
Undirected Graph: Adjacency Matrix Representation

	0	1	2	3	4	5
0	0	0	1	0	1	1
1						
2						
3						
4						
5						



Undirected Graph: Adjacency Matrix Representation

	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2						
3						
4						
5						



Undirected Graph: Adjacency Matrix Representation

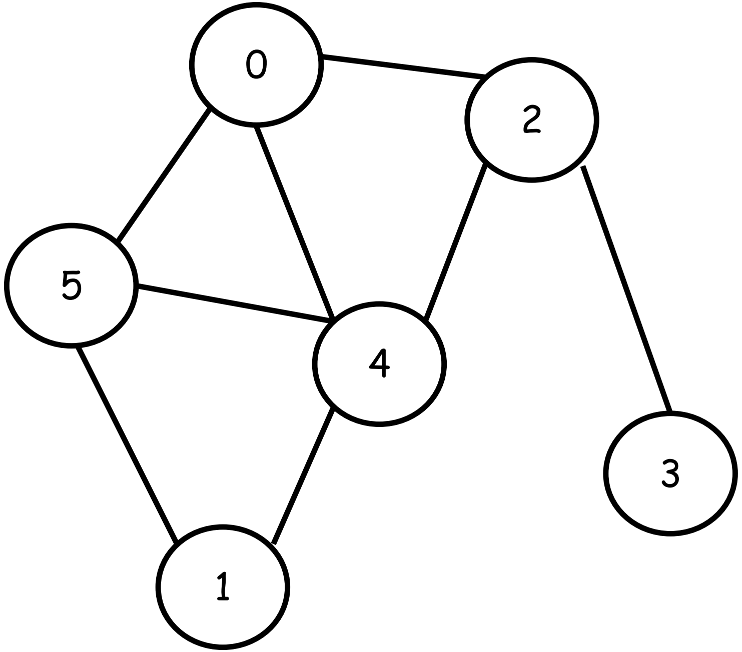
	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2	1	0	0	1	1	0
3	0	0	1	0	0	0
4	1	1	1	0	0	1
5	1	1	0	0	1	0

A 2D array:
a[Number of vertices]
[Number of vertices]

a[i][j] = a[j][i]

Undirected Graph: Adjacency List Representation

vertices	Adjacent vertices
0	2, 4, 5
1	4, 5
2	0, 3, 4
3	
4	
5	

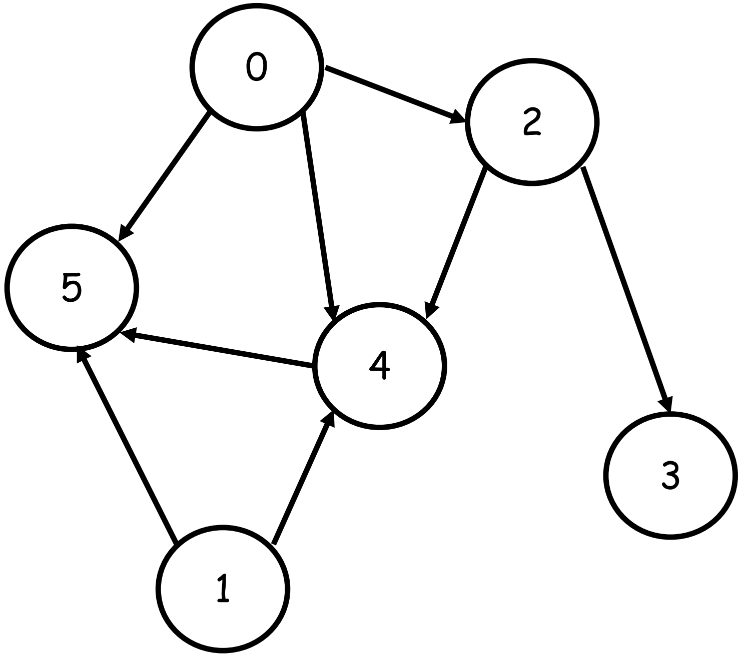


Directed Graph: Adjacency Matrix Representation

Terminal Vertex

Initial Vertex

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

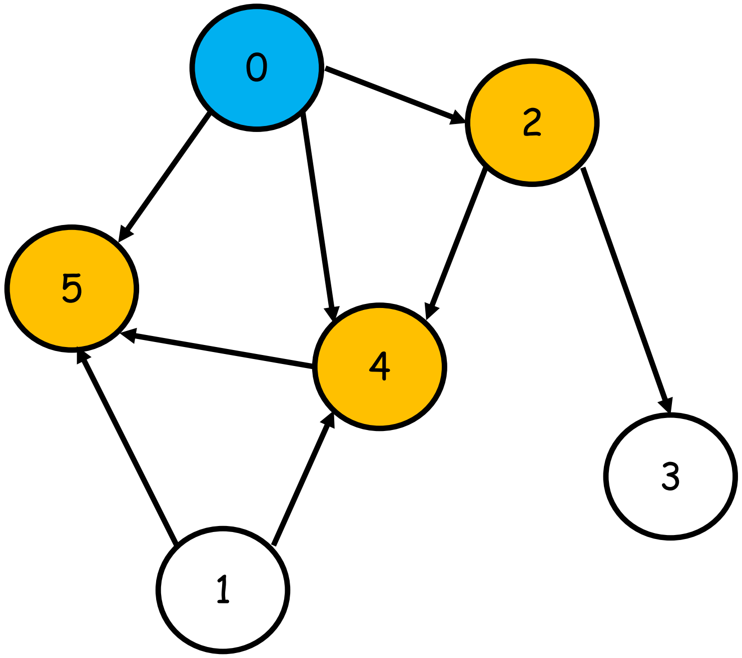


Directed Graph: Adjacency Matrix Representation

Terminal Vertex

Initial Vertex

	0	1	2	3	4	5
0	0	0	1	0	1	1
1						
2						
3						
4						
5						

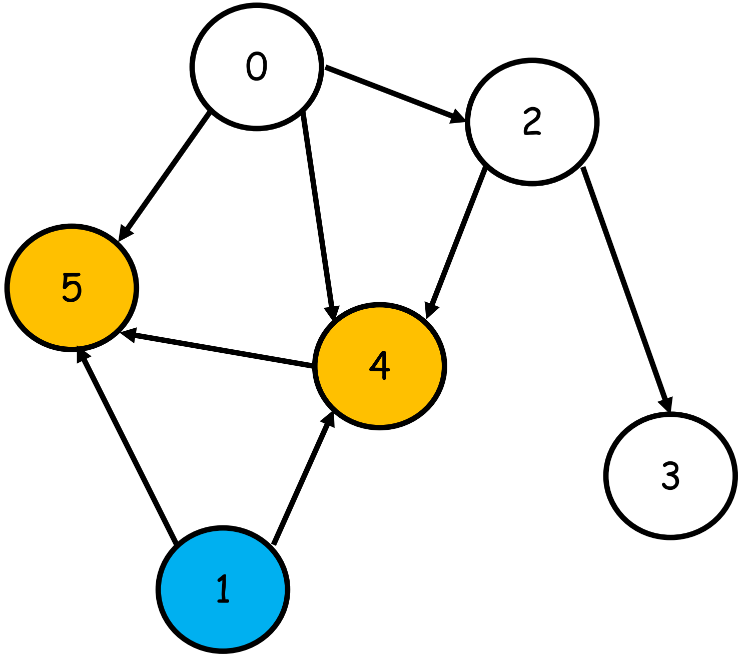


Directed Graph: Adjacency Matrix Representation

Terminal Vertex

Initial Vertex

	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2						
3						
4						
5						

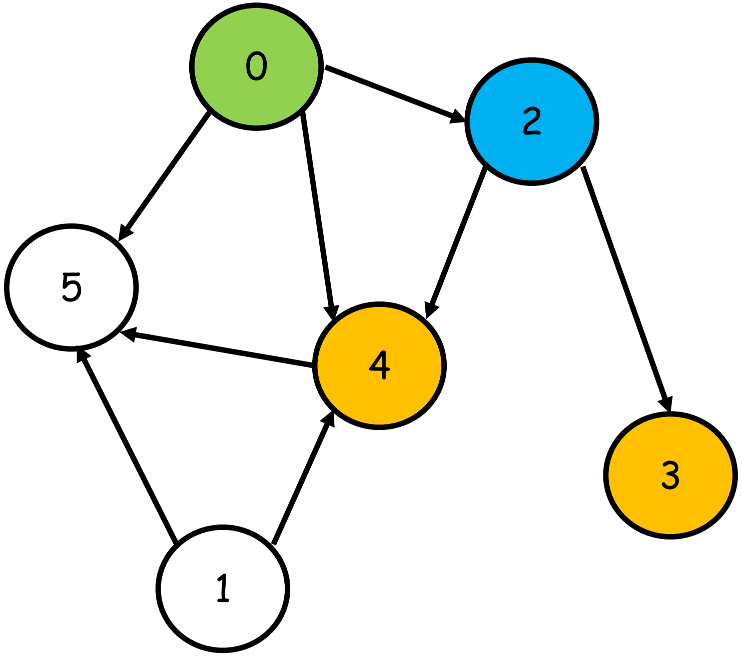


Directed Graph: Adjacency Matrix Representation

Terminal Vertex

Initial Vertex

	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2	0	0	0	1	1	0
3						
4						
5						



Directed Graph: Adjacency Matrix Representation

Terminal Vertex

Initial Vertex

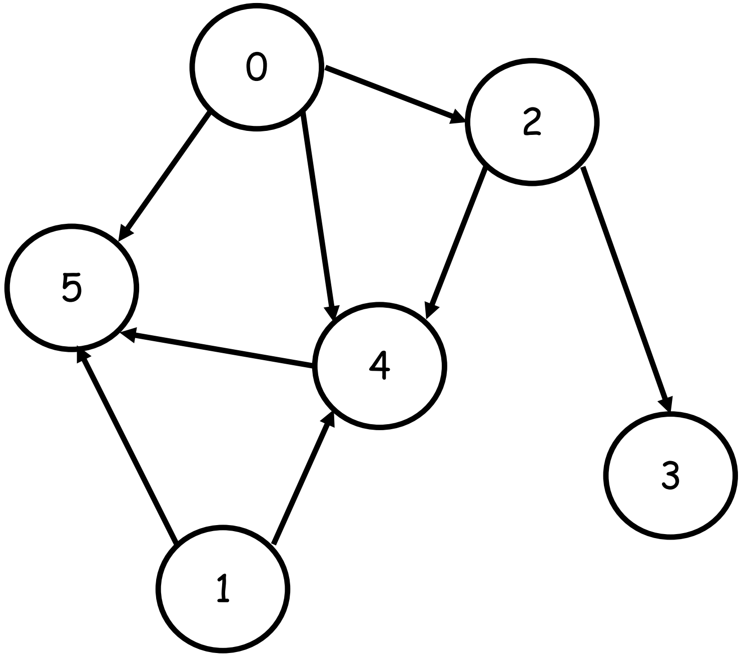
	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2	0	0	0	1	1	0
3	0	0	0	0	0	0
4	0	0	0	0	0	1
5	0	0	0	0	0	0

A 2D array:
a[Number of vertices]
[Number of vertices]

a[i][j] = a[j][i]/
a[i][j] = a[j][i]
No Restriction

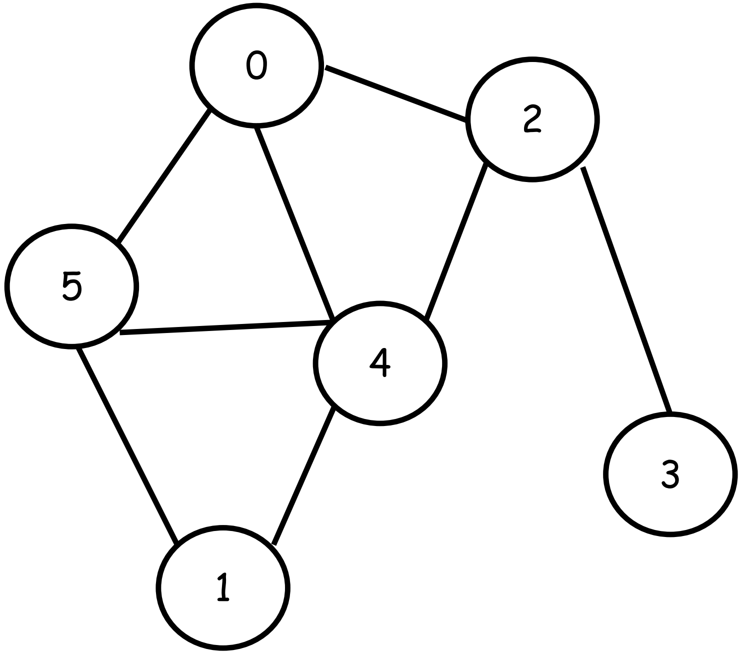
Directed Graph: Adjacency List Representation

Initial vertex	Terminal vertex
0	2, 4, 5
1	4, 5
2	3, 4
3	
4	
5	



Undirected Graph: Degree of a vertex

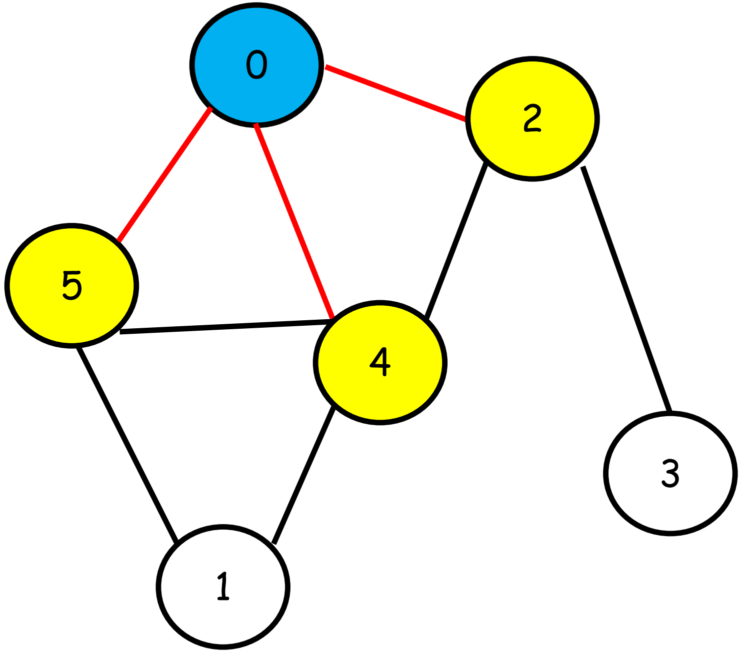
- Number of edges incident with it
- In case of Loop add +2



Undirected Graph: Degree of a vertex

- Number of edges incident with it
- In case of Loop add +2

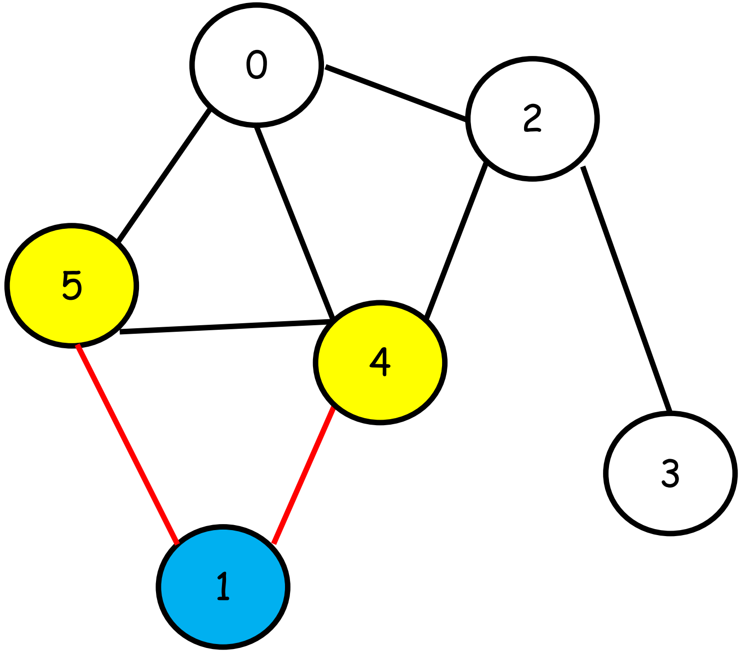
Vertex	Degree
0	3
1	
2	
3	
4	
5	



Undirected Graph: Degree of a vertex

- Number of edges incident with it
- In case of Loop add +2

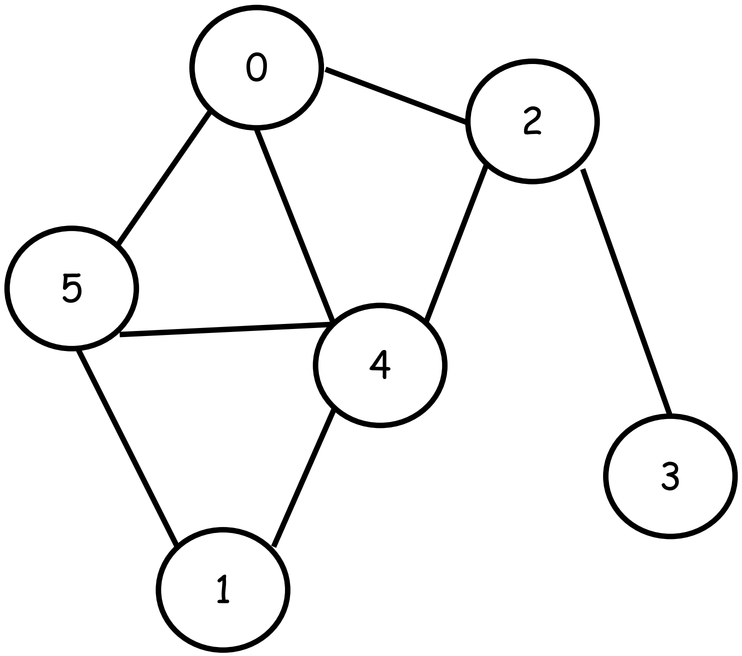
Vertex	Degree
0	3
1	2
2	
3	
4	
5	



Undirected Graph: Degree of a vertex

- Number of edges incident with it
- In case of Loop add +2

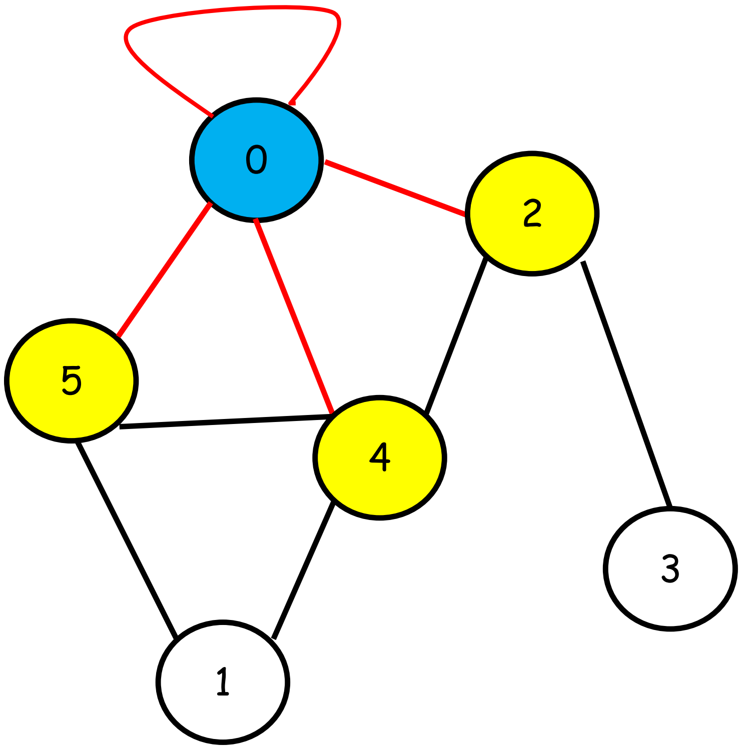
Vertex	Degree
0	3
1	2
2	3
3	1
4	4
5	3



Undirected Graph: Degree of a vertex

- Number of edges incident with it
- In case of Loop add +2

Vertex	Degree
0	5 (3 + 2)
1	2
2	3
3	1
4	4
5	3



Handshaking theorem

Let G= (V, E) be an undirected graph with e edges. Then

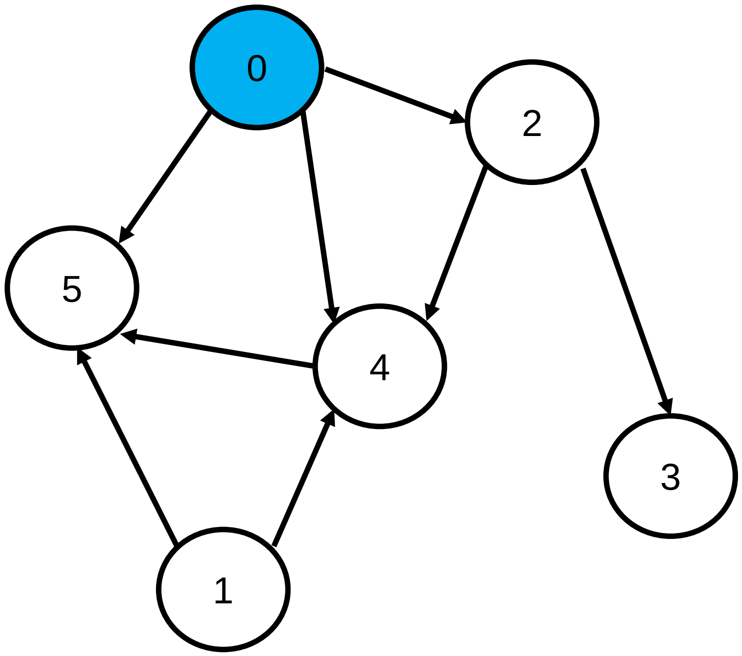
$$2\mathrm{e} = \sum_{v \in V} \deg(v)$$

Directed Graph: Degree of a vertex

- In Degree : No of INCOMING edges
- Out Degree : No of OUTGOING edges

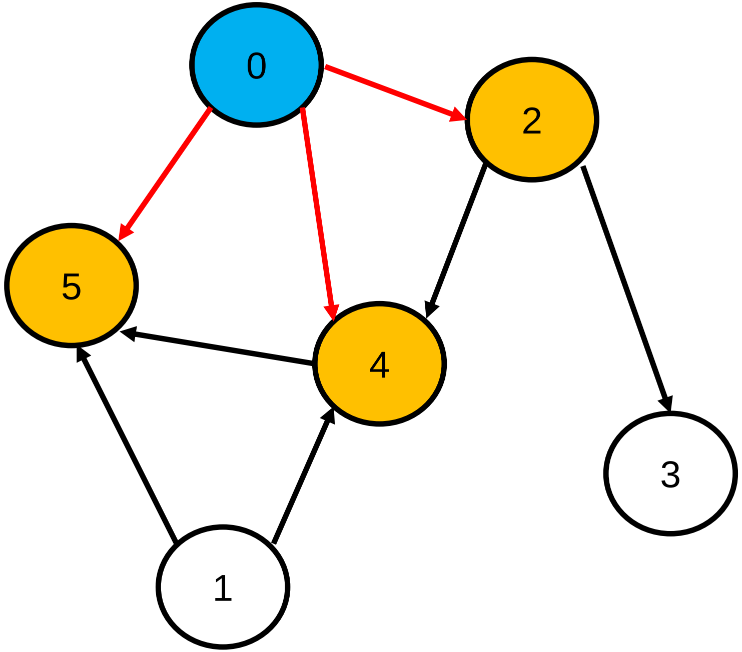
Directed Graph: Degree of a vertex

Vertex	In Degree	Out Degree
0	0	
1		
2		
3		
4		
5		



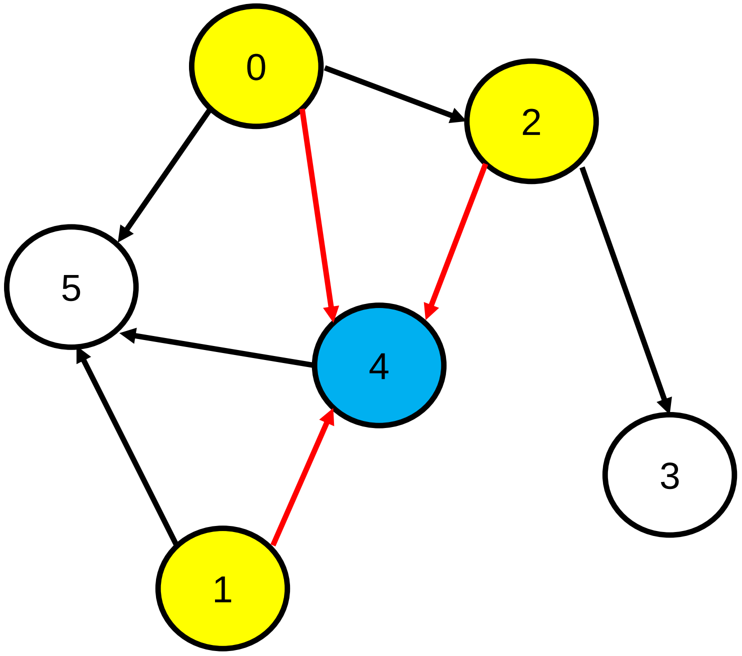
Directed Graph: Degree of a vertex

Vertex	In Degree	Out Degree
0	0	3
1		
2		
3		
4		
5		



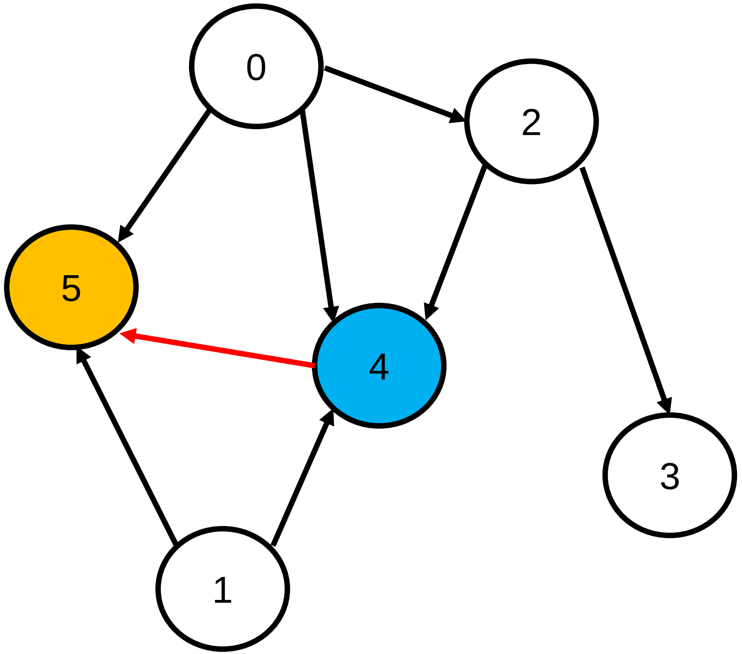
Directed Graph: Degree of a vertex

Vertex	In Degree	Out Degree
0	0	3
1		
2		
3		
4	3	
5		



Directed Graph: Degree of a vertex

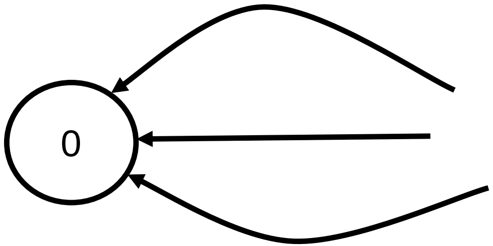
Vertex	In Degree	Out Degree
0	0	3
1		
2		
3		
4	3	1
5		



Directed Graph: Degree of a vertex

Vertex	In Degree	Out Degree
0	0	3

In degree = number of incoming edges
In degree of vertex 0 = number of incoming edges to 0



So, for counting in degrees of vertex 0
0 must be a **TERMINAL VERTEX**

Let G= (V, E) be a directed graph.

$deg^{-}(v)$ = in degree of v

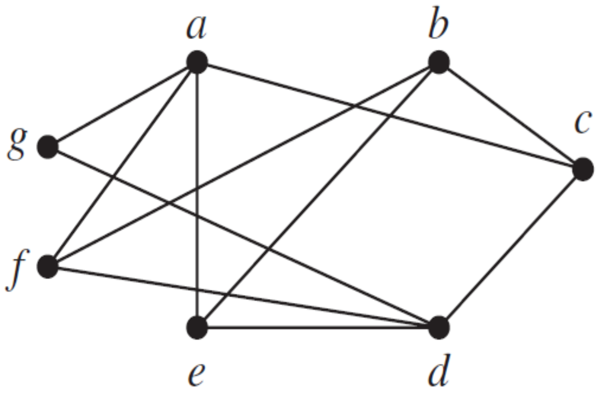
$deg^{+}(v)$ = out degree of v

$$\sum_{v \in V} deg^{-}(v) = \sum_{v \in V} deg^{+}(v) = |E|$$

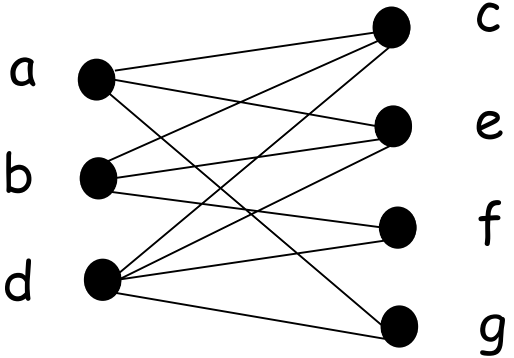
Bipartite Graph

if its vertex set V can be partitioned into two disjoint sets V1 and V2 such that no edge in G connects either two vertices in V1 or two vertices in V2

Bipartite Graph

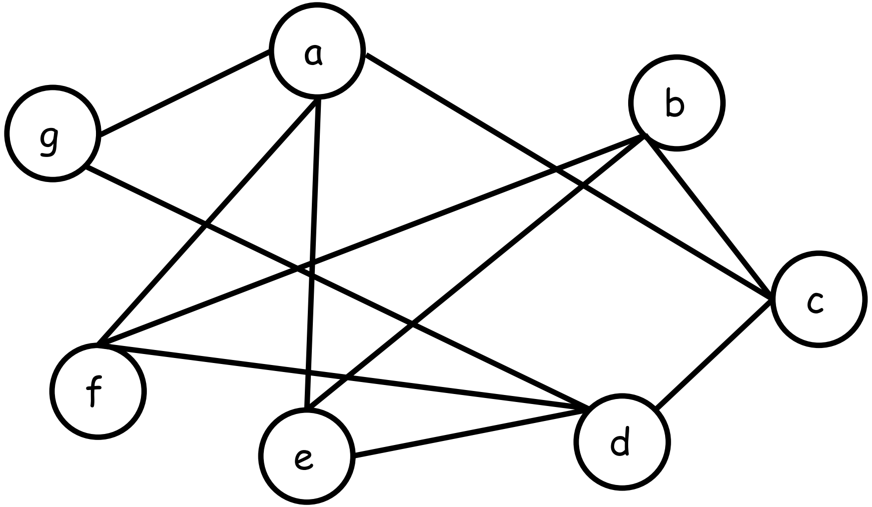


V = {a, b, c, d, e, f, g}

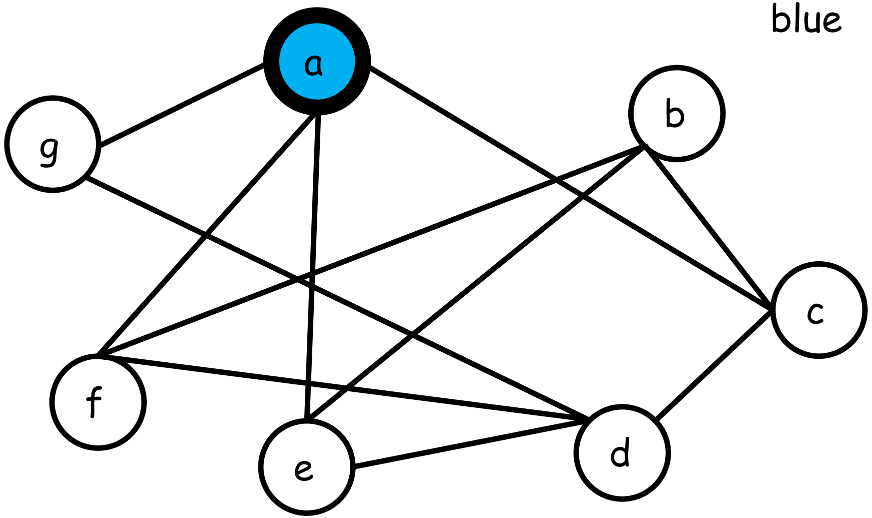


V1 = {a, b, d}
V2 = {c, e, f, g}

How to decide if a graph is bipartite or not

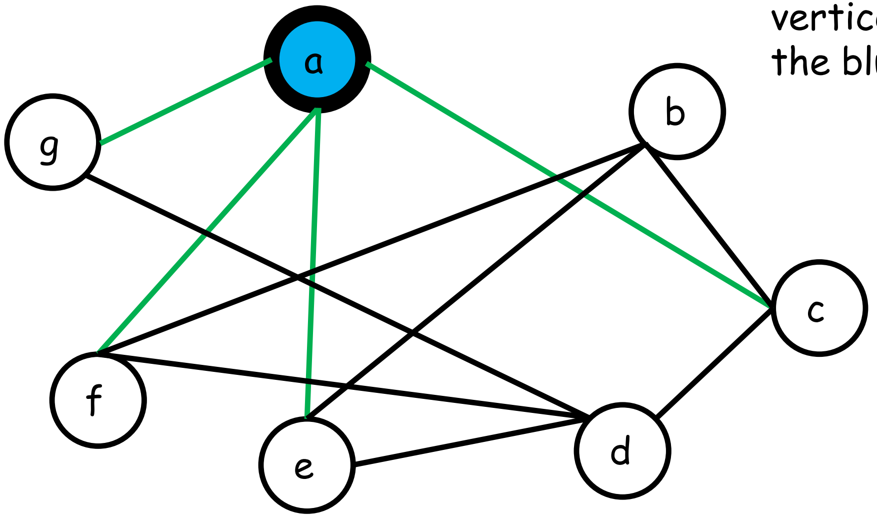


How to decide if a graph is bipartite or not



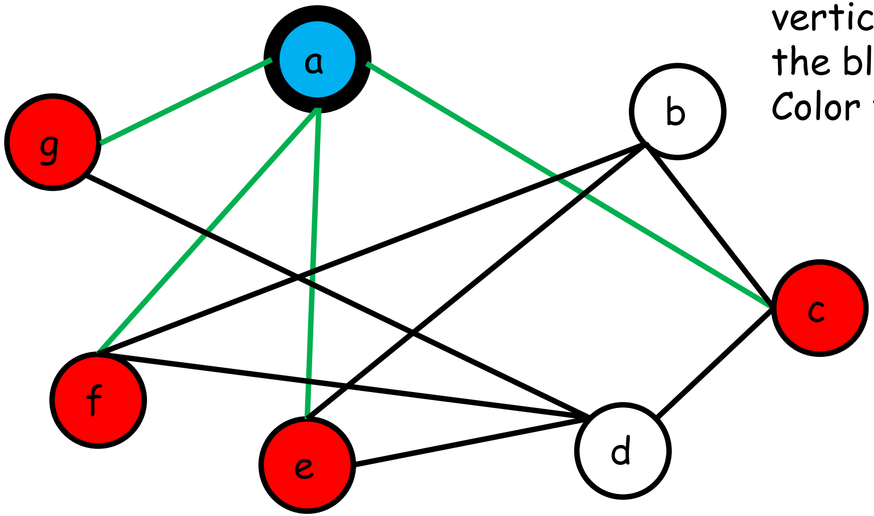
1. Color any of the vertices blue

How to decide if a graph is bipartite or not



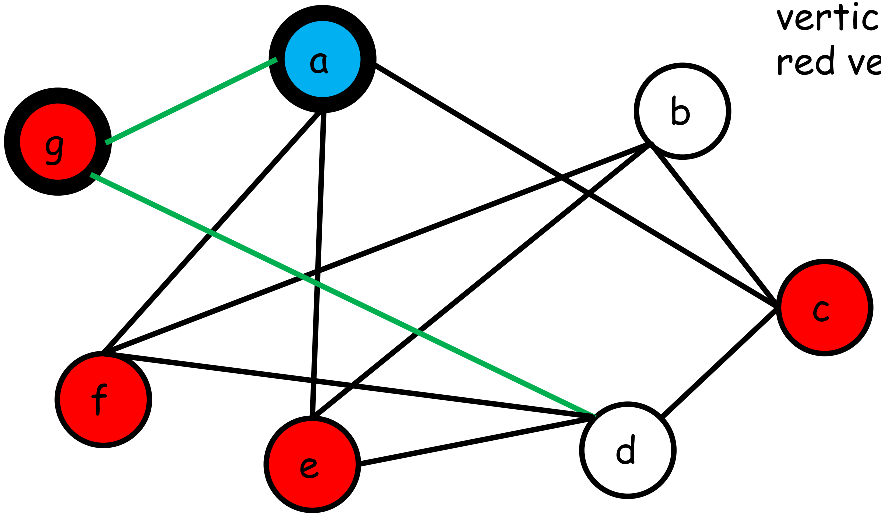
2. Identify all **uncolored** vertices that are adjacent to the blue vertex.

How to decide if a graph is bipartite or not



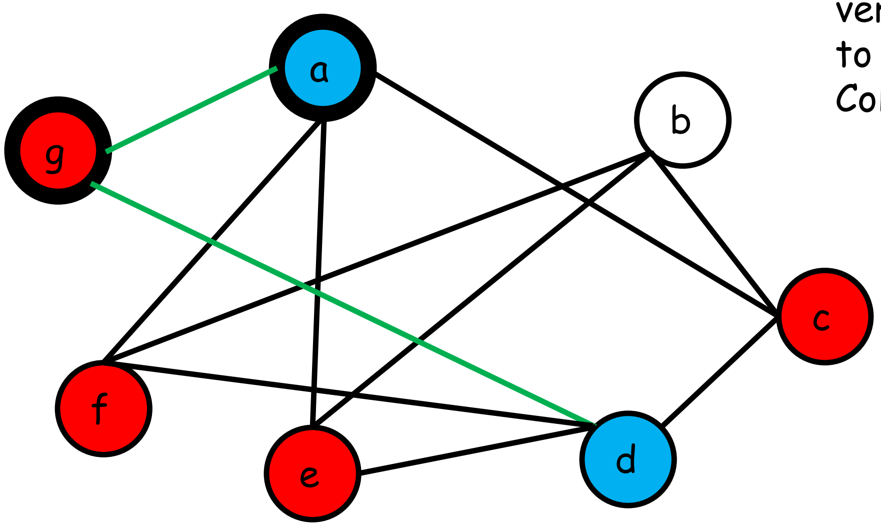
2. Identify all uncolored vertices that are adjacent to the blue vertex.
Color them red

How to decide if a graph is bipartite or not



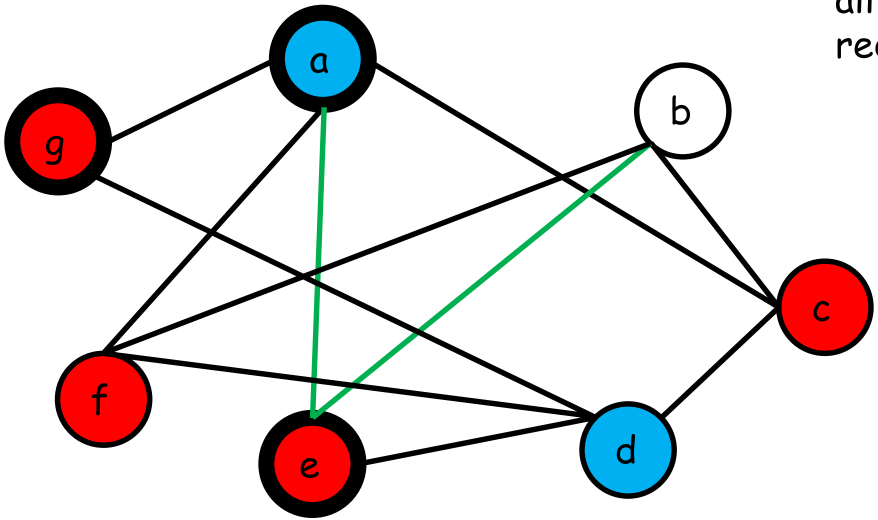
3. Identify all uncolored vertices that are adjacent to a red vertex.

How to decide if a graph is bipartite or not



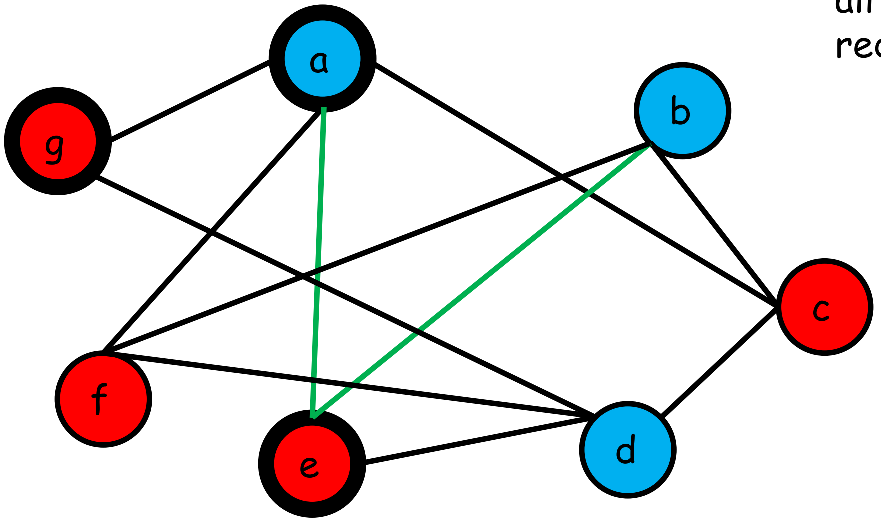
3. Identify all uncolored vertices that are adjacent to a red vertex.
Color them blue.

How to decide if a graph is bipartite or not



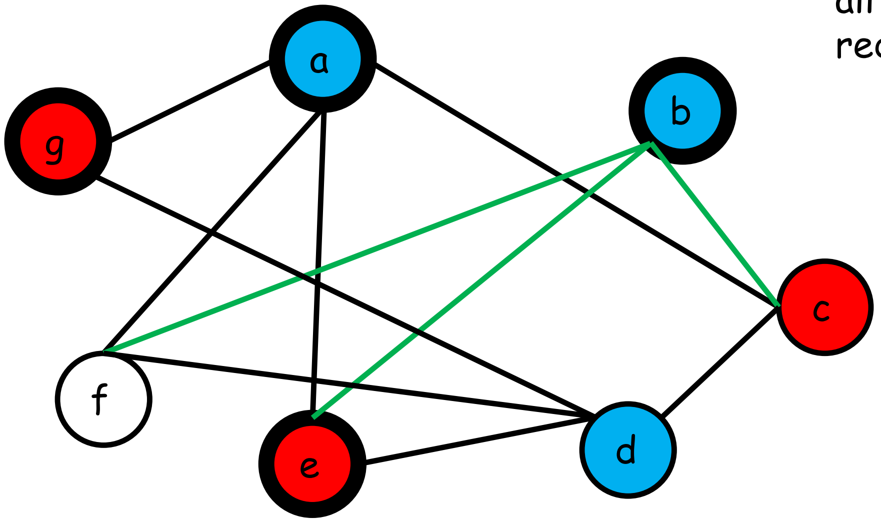
Repeat steps 2 and 3 until all the vertices are colored red or blue.

How to decide if a graph is bipartite or not



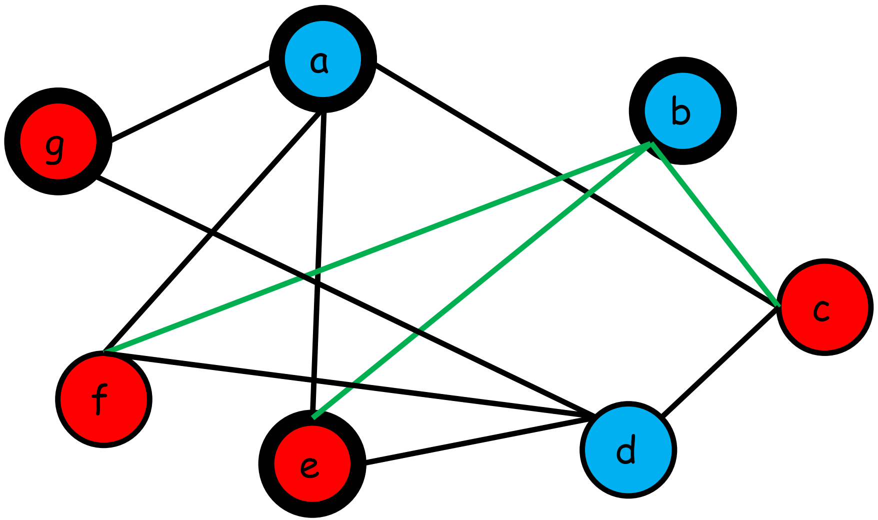
Repeat steps 2 and 3 until all the vertices are colored red or blue.

How to decide if a graph is bipartite or not



Repeat steps 2 and 3 until all the vertices are colored red or blue.

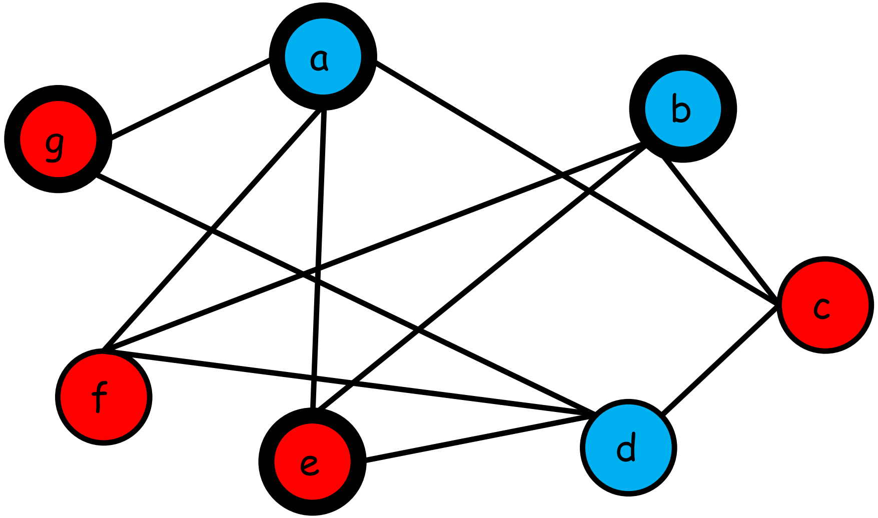
How to decide if a graph is bipartite or not



If there are any two vertices adjacent of the same color, then your graph is not bipartite, otherwise it is bipartite

∴Bipartite graph

How to decide if a graph is bipartite or not



Disjoint sets
V1 = {a, b, d}
V2 = {c, e, f, g}