Sets: Sets Definition

What is a Set?

Informally: A set is a collection of mathematical objects or elements.

A set of 4 things

$$\{7, \text{"Albert R."}, \pi/2, T\}$$

A set with 4 elements: two numbers, a string, and a Boolean.

Same as

$$\{T, \text{"Albert R."}, \pi/2, 7\}$$

-- order doesn't matter

In or not in

An element is either in a set or not in a set

{7, 2} is same as {7, 2, 7}

--No notion of being in
the set more than once

Membership

x is a member of A: $x \in A$
{7, "Albert R.", $\pi/2$, T}

$T \in$ {7, "Albert R.", $\pi/2$, T}
$14/2 \in$
$9 \notin$

Subset(⊆)

A ⊆ B : A is a subset of B

Every element of A is also an element of B:
∀x [x ∈ A implies x ∈ B]

$\mathbb{Z}$ ⊆ $\mathbb{R}$, {3} ⊆ {5,7,3} A ⊆ A, ∅ ⊆ every set

Why ∅ ⊆ every set?

∅ ⊆ B :

∀x [x ∈ ∅ implies x ∈ B]

false

true

Cartesian Product

A x B = {(a, b) | a ∈ A and b ∈ B}

(a , b) : ordered 2-tuple
(a1, a2, a3, ..., an): ordered n-tuple

Cartesian Product

$A = \{1, 2, 3\}$

$B = \{a, b\}$

$A \times B = \{1, 2, 3\} \times \{a, b\}$

$\quad = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

Cartesian Product

A = {1, 2, 3}
B = {a, b}
C = {p, q}
$A \times B \times C$ = {1, 2, 3} x {a, b} x {p, q}
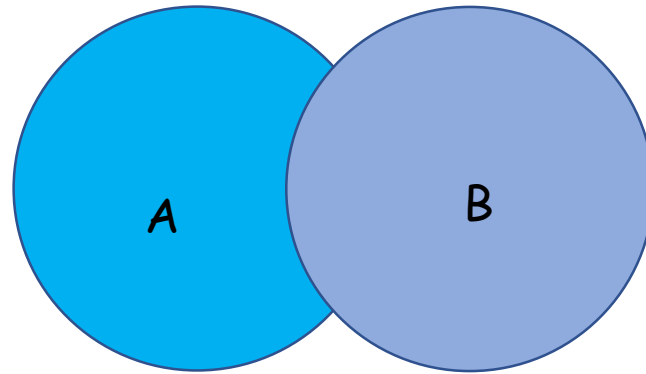
Cartesian Product

$A = \{1, 2, 3\}$
$B = \{a, b\}$
$C = \{p, q\}$
$A \times (B \times C) = \{1, 2, 3\} \times (\{a, b\} \times \{p, q\})$

Sets: Set Operations

New sets from old

Venn diagram for 2 sets

Set Operations: union

Union

$A \cup B = \{x \mid x \in A \lor x \in B\}$

$A = \{1, 2, 3, 4\}$

$B = \{1, 2, 2, 2, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

Set Operations: intersection

Intersection
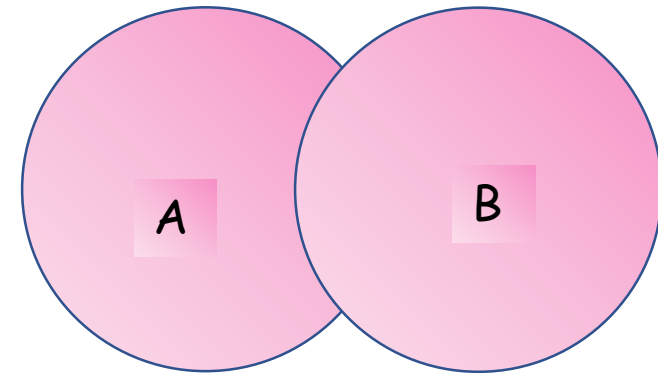
$A \cap B = \{x \mid x \in A \wedge x \in B\}$

$A = \{1, 2, 3, 4\}$

$B = \{1, 2, 2, 2, 5, 6\}$

$A \cap B = \{1, 2\}$

Set Operations: difference

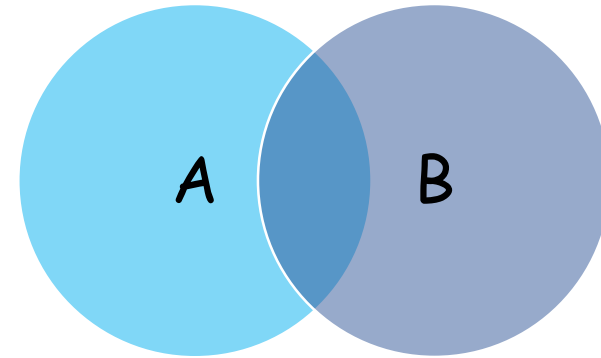Difference

$A - B = \{x \mid x \in A \land x \notin B\}$

$A = \{1, 2, 3, 4\}$

$B = \{1, 2, 2, 2, 5, 6\}$

$A - B = \{3, 4\}$
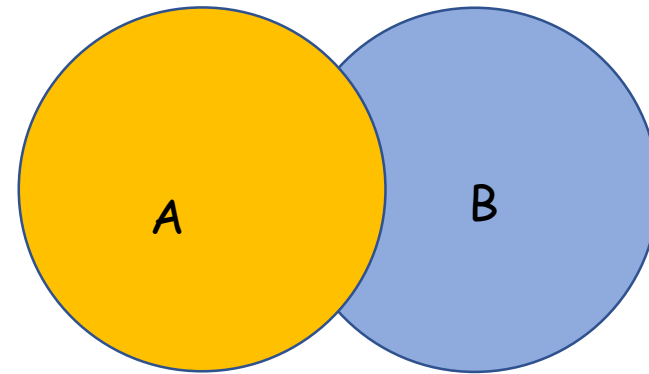
Set Operations: complement

Complement

$A' = U - A = \{x \mid x \notin A\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 3, 4\}$

$A' = \{5, 6, 7, 8, 9\}$
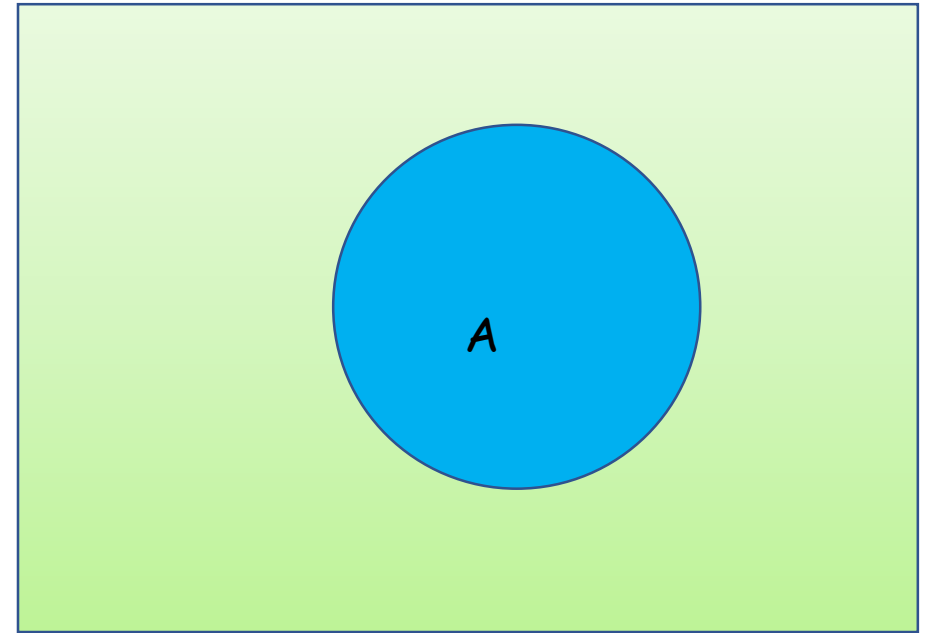
Computer representation of sets

U = {1, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17}

U =

1	5	6	7	8	9	10	11	12	13	14	15	16	17

U =

1	1	1	1	1	1	1	1	1	1	1	1	1	1

(Bit set
Representation)

Computer representation of sets

$$A = \{1, 6, 8, 11, 12, 13, 16\}$$

1	5	6	7	8	9	10	11	12	13	14	15	16	17

U =

1	1	1	1	1	1	1	1	1	1	1	1	1	1

A =

Computer representation of sets

A = {1, 6, 8, 11, 12, 13, 16}

1	5	6	7	8	9	10	11	12	13	14	15	16	17

U =

1	1	1	1	1	1	1	1	1	1	1	1	1	1

A =

1													

Computer representation of sets

$$A = \{1, \textcolor{green}{6}, 8, 11, 12, 13, 16\}$$

1	5	6	7	8	9	10	11	12	13	14	15	16	17

U =

1	1	1	1	1	1	1	1	1	1	1	1	1	1

A =

1		1											

Computer representation of sets

$A = \{1, 6, 8, 11, 12, 13, 16\}$

1	5	6	7	8	9	10	11	12	13	14	15	16	17

U =

1	1	1	1	1	1	1	1	1	1	1	1	1	1

A =

1		1		1									

Computer representation of sets

$A = \{1, 6, 8,$ $11$$, 12, 13, 16\}$

1	5	6	7	8	9	10	11	12	13	14	15	16	17

U =

1	1	1	1	1	1	1	1	1	1	1	1	1	1

A =

1		1		1			1						

Computer representation of sets

$A = \{1, 6, 8, 11, 12, 13, 16\}$

1	5	6	7	8	9	10	11	12	13	14	15	16	17

U =

1	1	1	1	1	1	1	1	1	1	1	1	1	1

A =

1		1		1			1	1	1			1	

Computer representation of sets

A = {1, 6, 8, 11, 12, 13, 16}

1	5	6	7	8	9	10	11	12	13	14	15	16	17

U =

1	1	1	1	1	1	1	1	1	1	1	1	1	1

A =

1	0	1	0	1	0	0	1	1	1	0	0	1	0

Computer representation of sets

B = {5, 7, 8}

1	5	6	7	8	9	10	11	12	13	14	15	16	17

U =

1	1	1	1	1	1	1	1	1	1	1	1	1	1

B =

0	1	0	1	1	0	0	0	0	0	0	0	0	0

Set operations: Union

U = | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

A = | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |

B = | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

A ∪ B = | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |

Perform OR operation

$$A \cup B = \{x \mid x \in A \lor x \in B\}$$

Set operations: Intersection

U =

| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

A =

| 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |

B =

| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

A ∩ B =

| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Perform AND operation

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

Set operations: Complement

U = | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

A = | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |

B = | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

A '= | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |

Toggle the bits

Set operations: Difference

U = | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

A = | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

B = | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

$A - B =$

$A - B = \{x \mid x \in A \wedge x \notin B\}$

$A - B = \{x \mid x \in A \wedge x \in B'\}$

Set operations: Difference

U =

| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

A =

| 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |

B =

| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

B' =

| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

A − B =

| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |

$A - B = \{x \mid x \in A \land x \in B'\}$

{1, 4} {2, 3, 4}