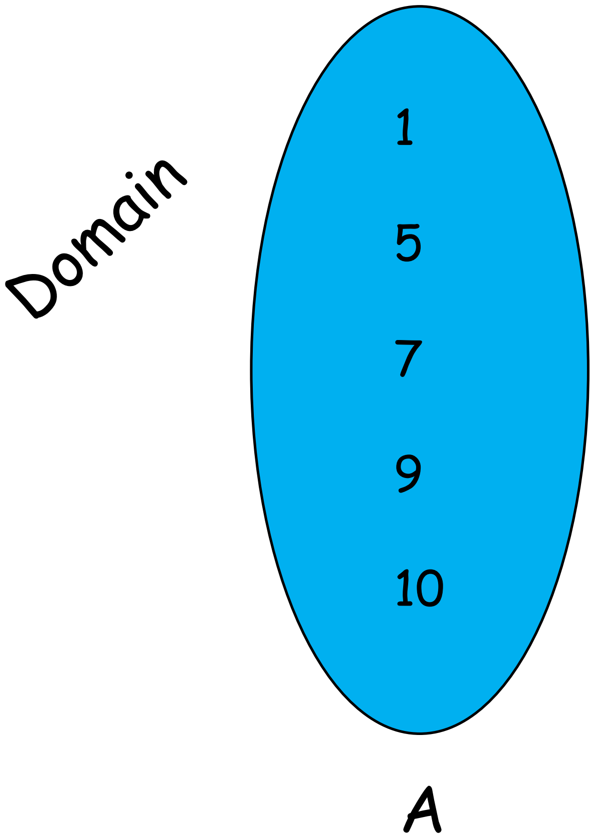
Relations

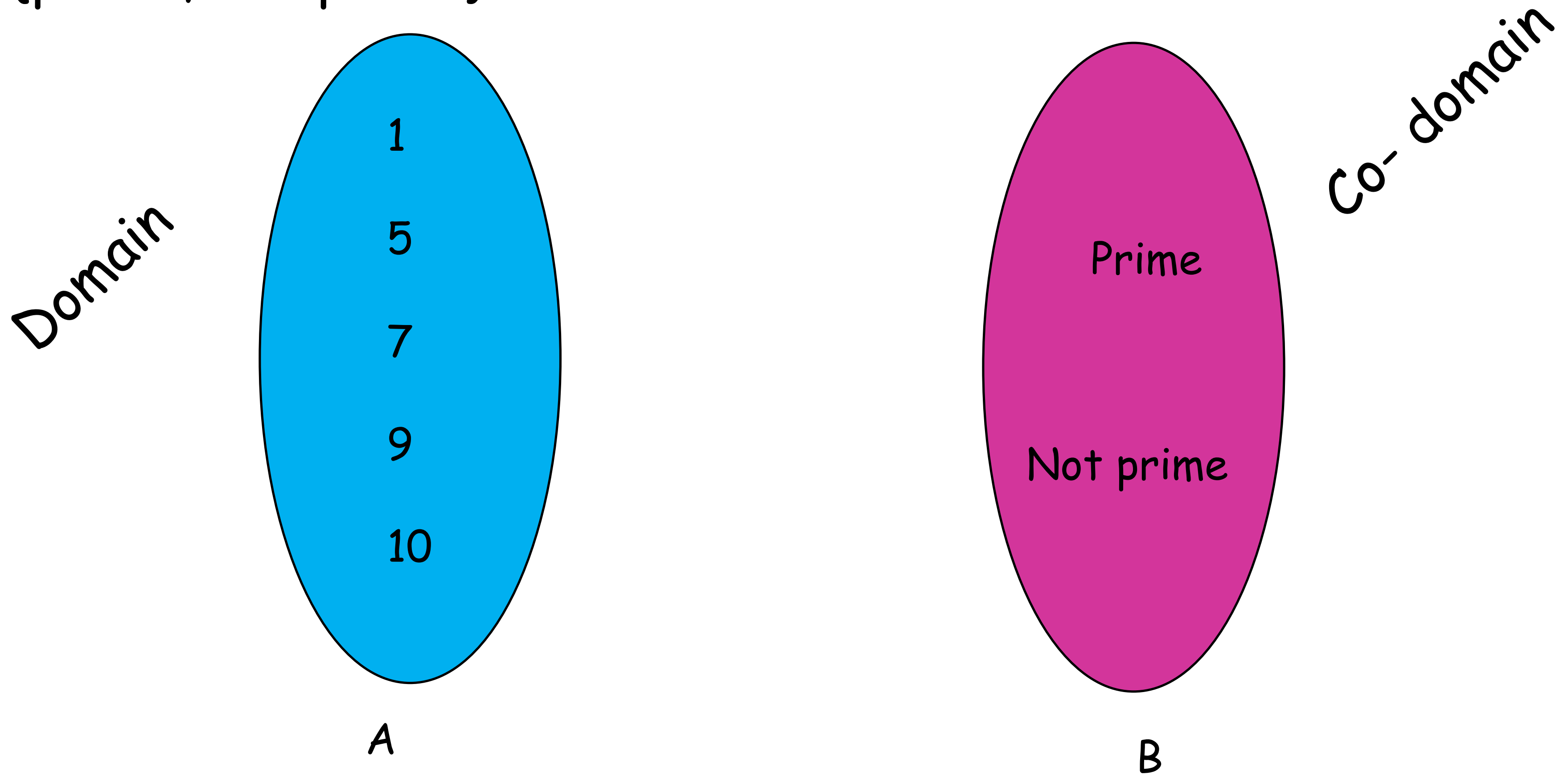
Binary Relation

$A = \{1, 5, 7, 9, 10\}$: Domain

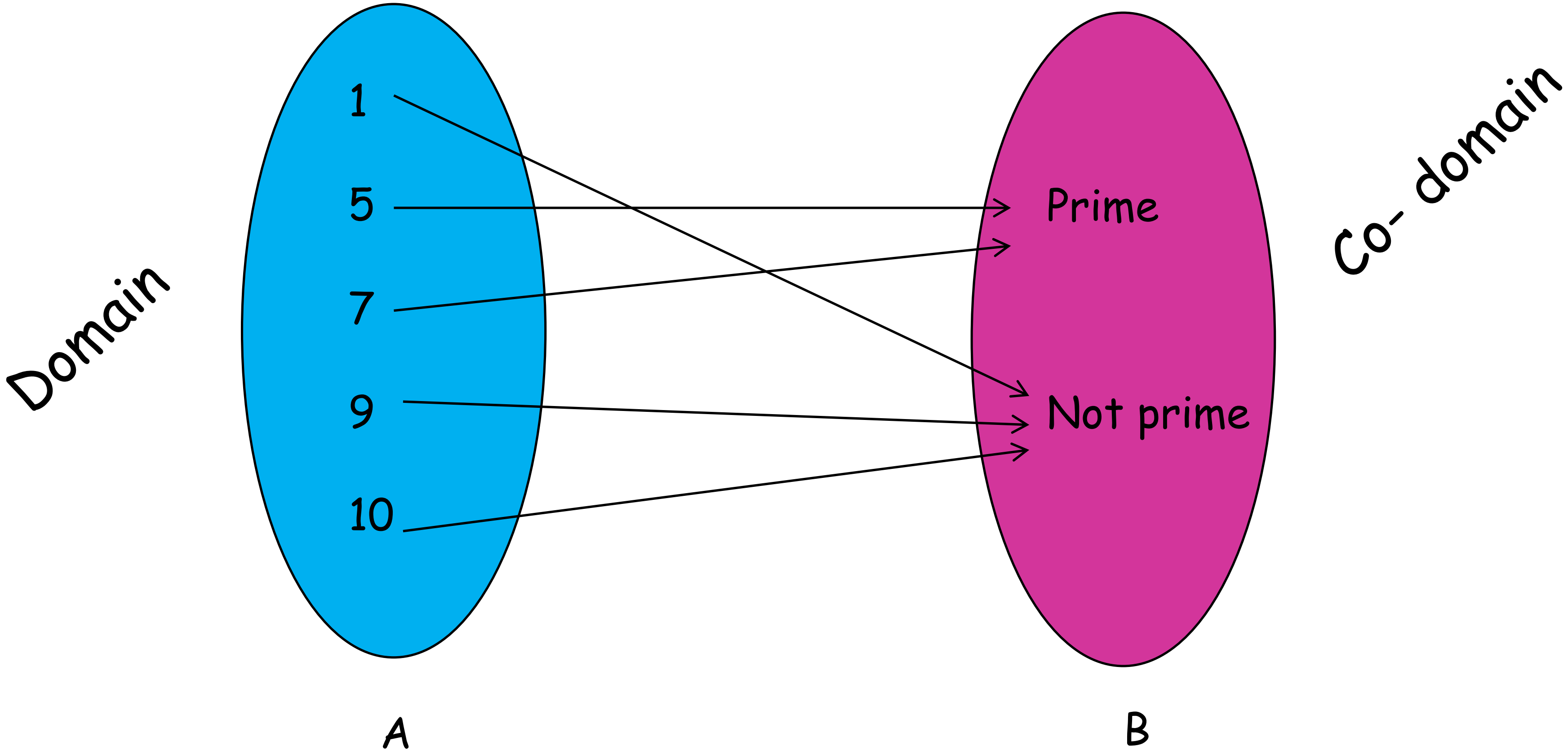


Binary Relation

B = {prime, not prime}: Co-domain

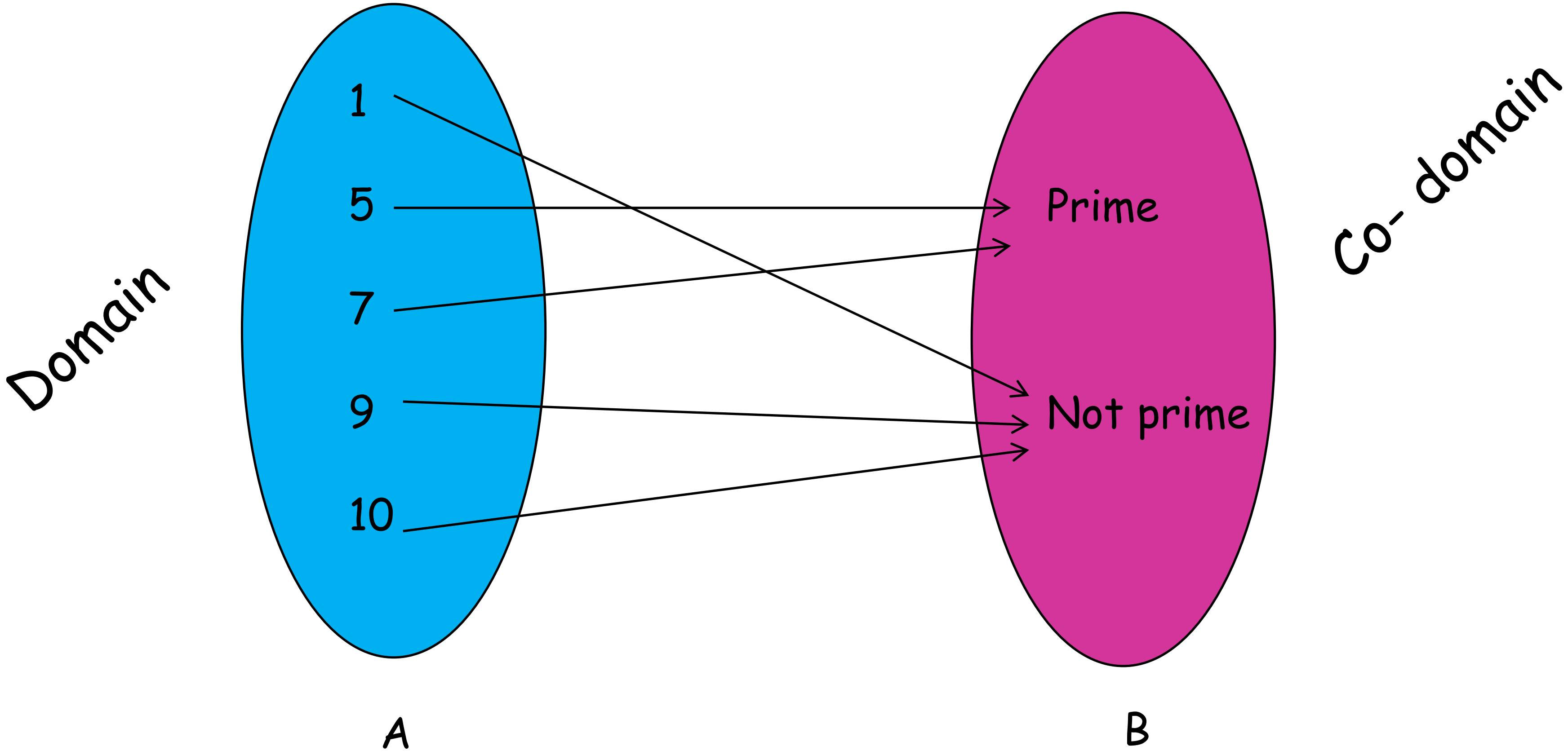


Binary Relation



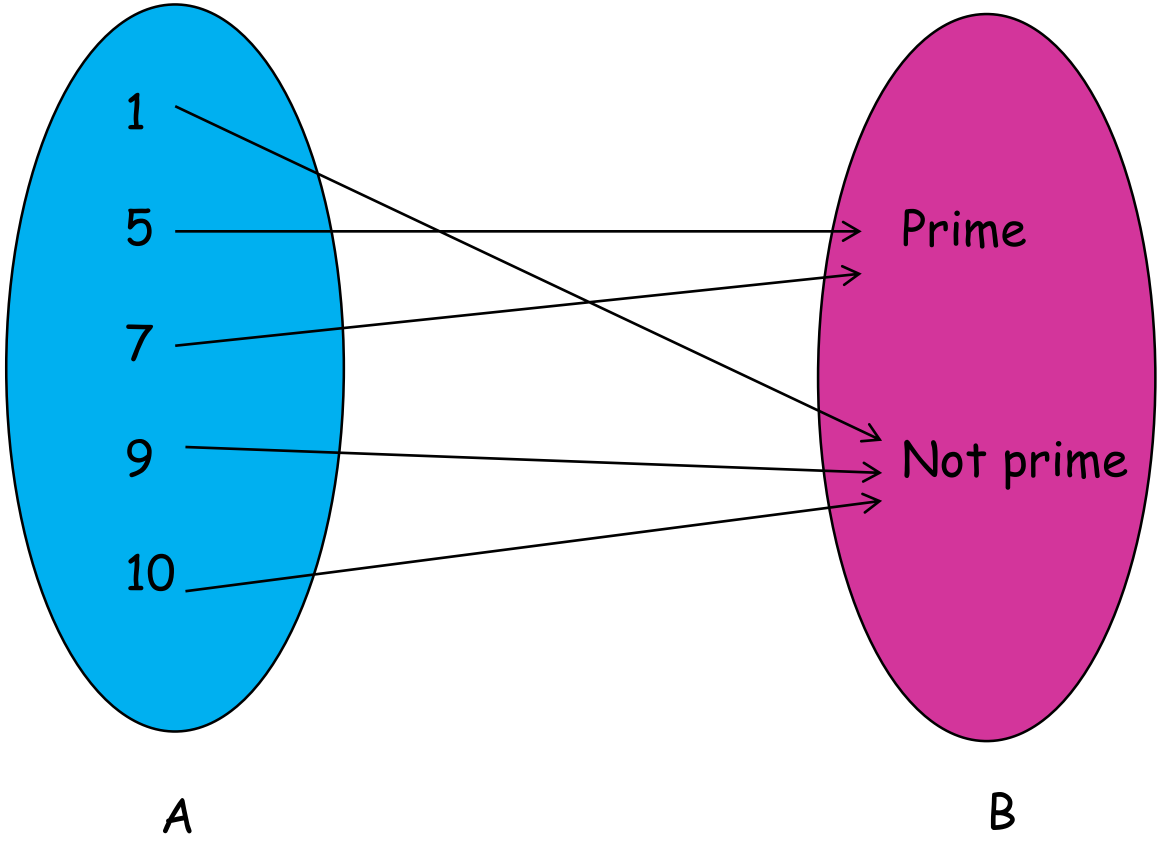
Binary Relation

Is $a \in A$ prime or not?



Binary Relation

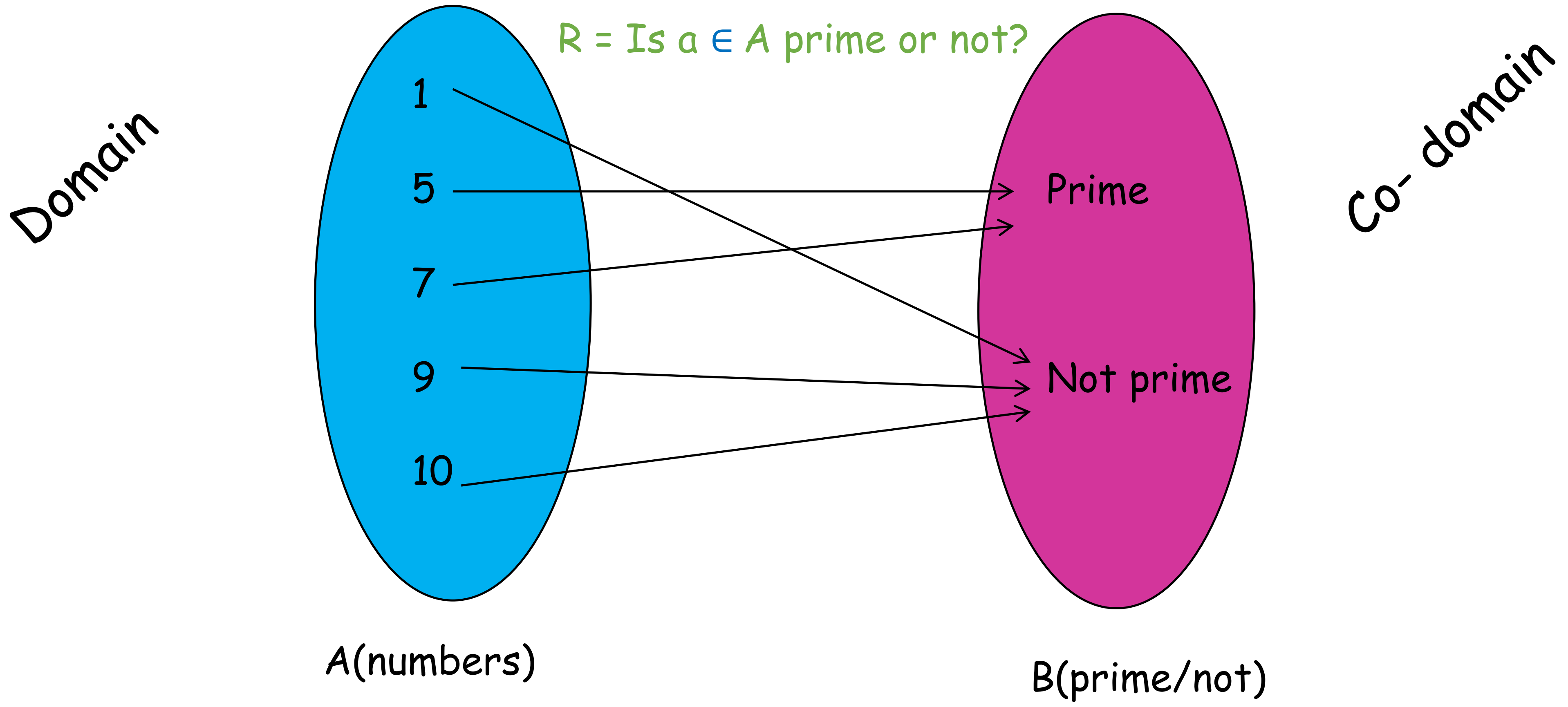
Is a ∈ A prime or not?



R = {(1, Not prime),
(5, prime),
(7, prime),
(9, Not prime),
(10, Not prime)}

Binary Relation

A binary relation associates elements of one set called domain, with element of another set called co-domain

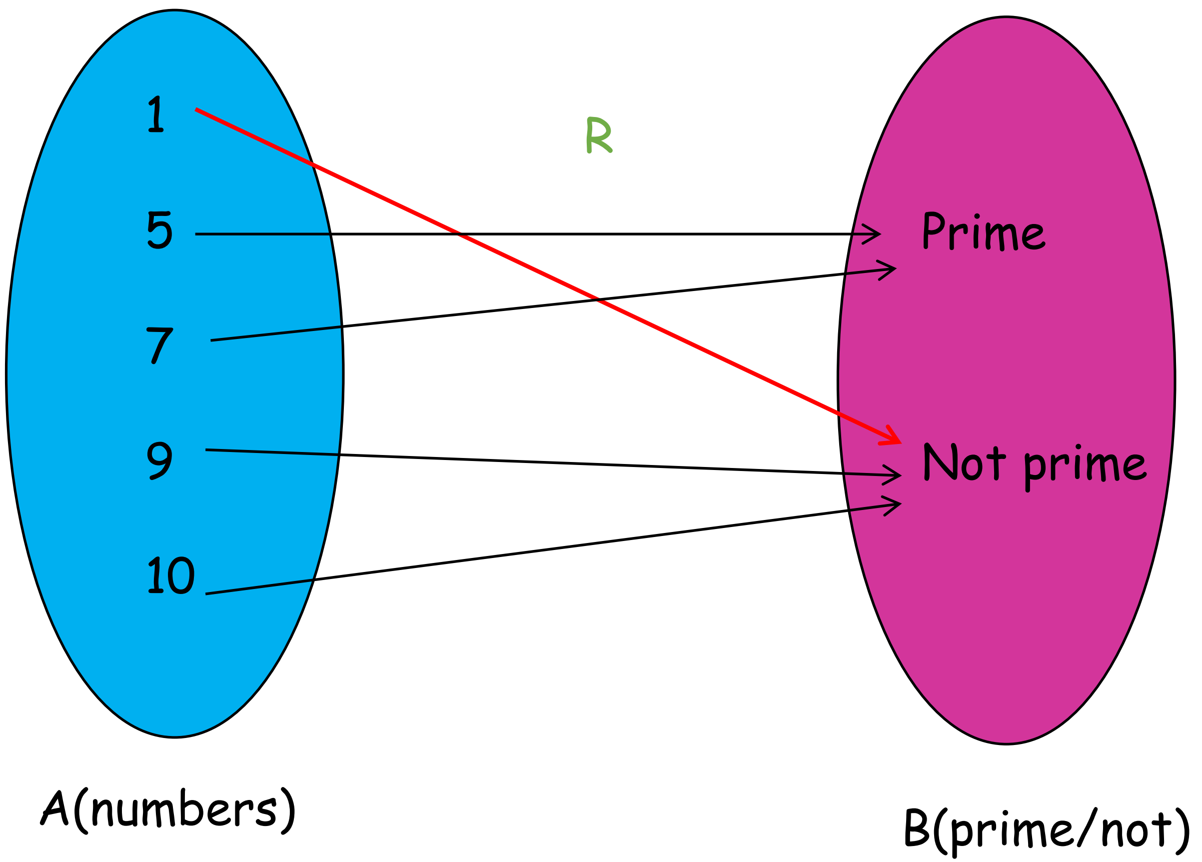


“Is prime/ not” relation R

Notation:

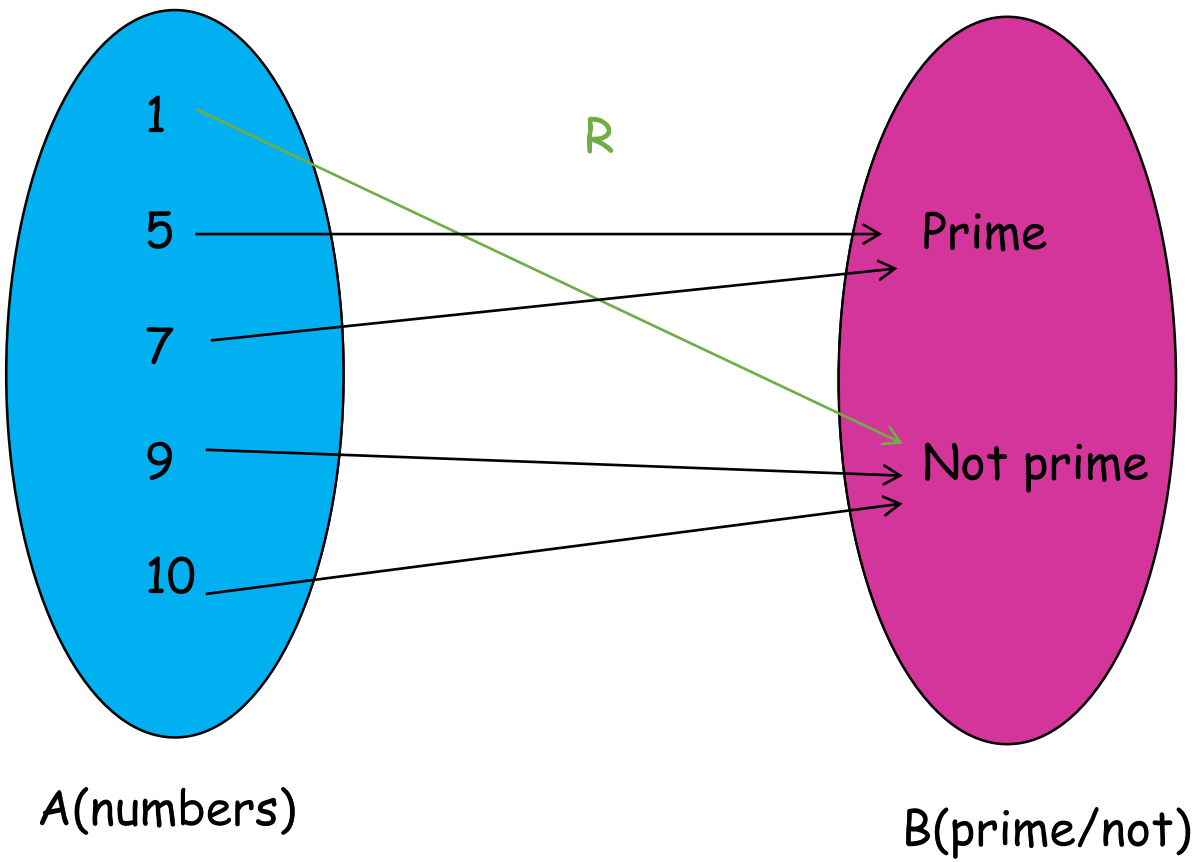
1 R not prime

R(1, not prime)



Images under R

$R(1) = \{\text{not prime}\}$



Binary Relation

A binary relation from set A (Domain) to B (Co-domain)is a subset of A x B

Relation on a set

A relation on a set A is a relation from A to A.

Relation on a set

Let A be the set {1, 2, 3, 4}. Which ordered pairs are in the relation R = {(a, b) | a divides b}?

$A \times B$ = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)}

R = { }

Relation on a set

Let A be the set {1, 2, 3, 4}. Which ordered pairs are in the relation R = {(a, b) | a less equal b}?

$A \times B$ = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)}

R = { }

Relation on a set

Let A be the set $\{1, 2, 3, 4\}$. Which ordered pairs are in the relation $R = \{(a, b) \mid a = b\}$?

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$

$R = \{ \qquad \}$

Properties of Relations

Reflexive Relation

A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.

Reflexive Relation

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$R3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4), (5, 5)\}$,

is a relation on $A = \{1, 2, 3, 4, 5\}$

Reflexive Relation

A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.

$R3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4), (5, 5)\}$,

is a relation on $A = \{1, 2, 3, 4, 5\}$

$(1, 1) \in R$

$(2, 2) \in R$

$(3, 3) \in R$

$(4, 4) \in R$

$(5, 5) \in R$

reflexive relation

Reflexive Relation

A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}
is a relation on $A = \{1, 2, 3, 4\}$

$(1, 1) \in R$
$(2, 2) \in R$
$(3, 3) \notin R$
$(4, 4)$

Not a reflexive relation

Symmetric Relation

A relation R on a set A is called symmetric
if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

Symmetric Relation

A relation R on a set A is called symmetric if (b, a) ∈ R whenever (a, b) ∈ R, for all a, b ∈ A.

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}
is a relation on A = {1, 2, 3, 4}

(a, b)	(b, a)
(1, 1)	
(1, 2)	
(2, 1)	
(2, 2)	
(3, 4)	
(4, 1)	
(4, 4)	

Symmetric Relation

A relation R on a set A is called symmetric if (b, a) ∈ R whenever (a, b) ∈ R, for all a, b ∈ A.

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)} is a relation on A = {1, 2, 3, 4}

(a, b)	(b, a)	
(1, 1)	(1, 1)	∈ R
(1, 2)	(2, 1)	∈ R
(2, 1)	(1, 2)	∈ R
(2, 2)	(2, 2)	∈ R
(3, 4)	(4, 3)	∉ R
(4, 1)		
(4, 4)		

Not symmetric

Symmetric Relation

R2 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4), (5, 5)}, is a relation on $A = \{1, 2, 3, 4, 5\}$

(a, b)	(b, a)

A relation R on a set A is called symmetric if (b, a) ∈ R whenever (a, b) ∈ R, for all a, b ∈ A.

Antisymmetric Relation

A relation R on a set A such that for all a, b ∈ A,
if (a, b) ∈ R and (b, a) ∈ R, then a = b
is called antisymmetric.

Antisymmetric Relation

A relation R on a set A such that for all a, b ∈ A, if (a, b) ∈ R and (b, a) ∈ R, then a = b is called antisymmetric.

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)} is a relation on A = {1, 2, 3, 4}

(a, b)	(b, a)
(1, 1)	
(1, 2)	
(2, 1)	
(2, 2)	
(3, 4)	
(4, 1)	
(4, 4)	

Antisymmetric Relation

A relation R on a set A such that for all a, b ∈ A, if (a, b) ∈ R and (b, a) ∈ R, then a = b is called antisymmetric.

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}
is a relation on A = {1, 2, 3, 4}

(a, b)	(b, a)
(1, 1)	
(1, 2)	(2, 1)
(2, 1)	
(2, 2)	
(3, 4)	
(4, 1)	
(4, 4)	

pass

∈ R

Not antisymmetric

Antisymmetric Relation

A relation R on a set A such that for all a, b ∈ A, if (a, b) ∈ R and (b, a) ∈ R, then a = b is called antisymmetric.

R2 = {(1, 1), (1, 2), (1, 4), (2, 2), (3, 3), (4, 4), (5, 5)}, is a relation on A = {1, 2, 3, 4, 5}

(a, b)	(b, a)	
(1, 1)		pass
(1, 2)	(2, 1)	∉ R
(1, 4)	(4, 1)	∉ R
(2, 2)		pass
(3, 3)		pass
(4, 4)		pass
(5, 5)		pass

antisymmetric

Transitive Relation

A relation R on a set A is called transitive if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

Transitive Relation

A relation R on a set A is called
transitive
if whenever $(a, b) \in R$ and $(b, c) \in R$,
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$R1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$
is a relation on $A = \{1, 2, 3, 4\}$

(a, b)	(b, c)	(a, c)
(1, 1)		

Transitive Relation

A relation R on a set A is called
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if whenever $(a, b) \in R$ and $(b, c) \in R$,
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R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}
is a relation on A = {1, 2, 3, 4}

(a, b)	(b, c)	(a, c)
(1, 1)	(1, 1)	
	(1,2)	

(1, 1) -> (a, b)

a = 1, b = 1

Find the (b, c)s -> find those tuples
that start with 1s

Transitive Relation

A relation R on a set A is called
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then $(a, c) \in R$, for all $a, b, c \in A$.

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}
is a relation on A = {1, 2, 3, 4}

(a, b)	(b, c)	(a, c)
(1, 1)	(1, 1)	(1,1)
	(1,2)	(1,2)

Transitive Relation

A relation R on a set A is called
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if whenever $(a, b) \in R$ and $(b, c) \in R$,
then $(a, c) \in R$, for all $a, b, c \in A$.

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}
is a relation on A = {1, 2, 3, 4}

(a, b)	(b, c)	(a, c)	
(1, 1)	(1, 1)	(1,1)	$\in$ R
	(1,2)	(1,2)	$\in$ R

Transitive Relation

A relation R on a set A is called
transitive
if whenever $(a, b) \in R$ and $(b, c) \in R$,
then $(a, c) \in R$, for all $a, b, c \in A$.

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}
is a relation on A = {1, 2, 3, 4}

(a, b)	(b, c)	(a, c)	
(1, 1)	(1, 1)	(1,1)	$\in$ R
	(1,2)	(1,2)	$\in$ R
(1,2)	(2,1)	(1, 1)	$\in$ R
	(2,2)	(1, 2)	$\in$ R

Transitive Relation

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if whenever $(a, b) \in R$ and $(b, c) \in R$,
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R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}
is a relation on A = {1, 2, 3, 4}

(a, b)	(b, c)	(a, c)	
(1, 1)	(1, 1)	(1,1)	$\in$ R
	(1,2)	(1,2)	$\in$ R
(1,2)	(2,1)	(1, 1)	$\in$ R
	(2,2)	(1, 2)	$\in$ R

Transitive Relation

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}
is a relation on A = {1, 2, 3, 4}

Not transitive

(a, b)	(b, c)	(a, c)	
(1, 1)	(1, 1)	(1,1)	$\in$ R
	(1,2)	(1,2)	$\in$ R
(1,2)	(2,1)	(1, 1)	$\in$ R
	(2,2)	(1, 2)	$\in$ R
(2,1)	(1,1)	(2,1)	$\in$ R
	(1,2)	(2,1)	$\in$ R

(a, b)	(b, c)	(a, c)	
(2, 2)	(2, 1)	(2,1)	$\in$ R
	(2,2)	(2,2)	$\in$ R
(3,4)	(4,1)	(3, 1)	$\notin$ R
	(4,4)	(1, 2)	

Is the “divides” relation on the set of positive integers reflexive?

A relation R on a set A is
called reflexive
if $(a, a) \in R$ for every
element $a \in A$.

Is the "divides" relation on the set of positive integers symmetric?

A relation R on a set A is called symmetric if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

Is the "divides" relation on the set of positive integers anti-symmetric?

A relation R on a set A such that for all a, b ∈ A, if (a, b) ∈ R and (b, a) ∈ R, then a = b is called antisymmetric.

Is the “divides” relation on the set of positive integers transitive?

A relation R on a set A is called transitive if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.