

# Graphs

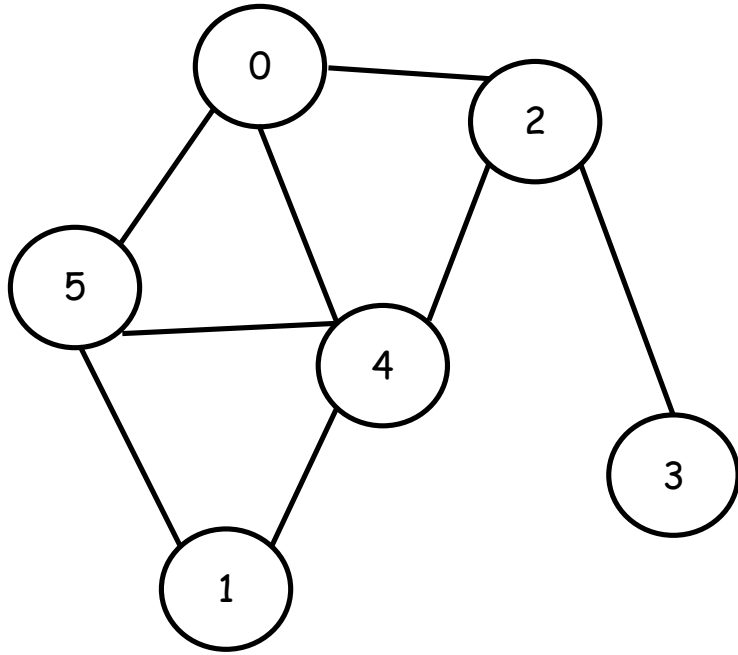
# Undirected Graph: Vertices and edges



No Direction

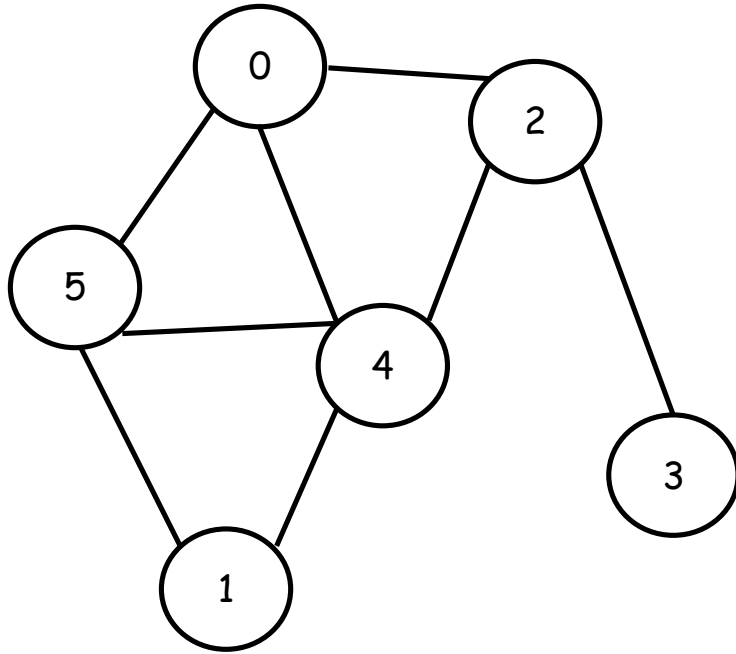
Representation: 02  
20

# Undirected Graph: Vertices and edges



Vertices = {0, 1, 2, 3, 4, 5}

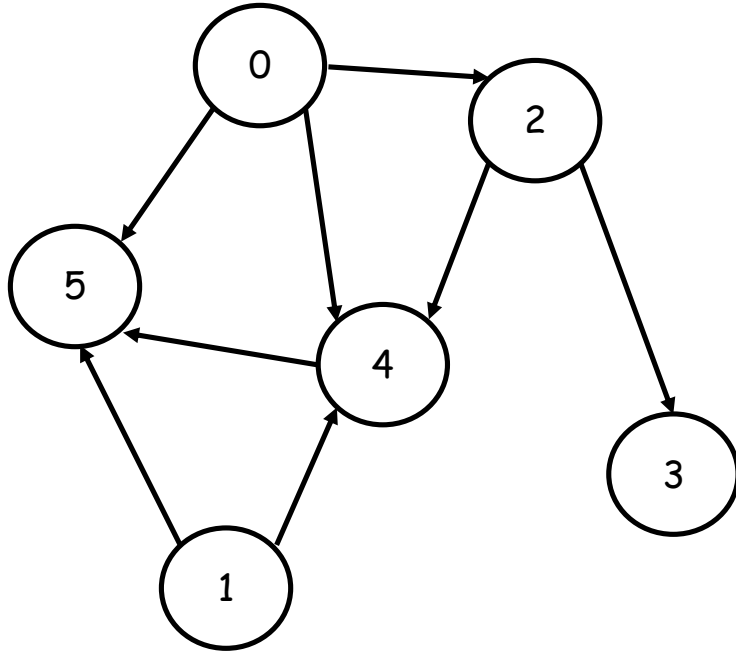
# Undirected Graph: Vertices and edges



Edges = {02, 05, 04, 15,  
14, 24, 23, 45 }

No convention  
Which vertex to write first.

# Directed Graph: Vertices and edges



Direction: From Node 0 - Node 2

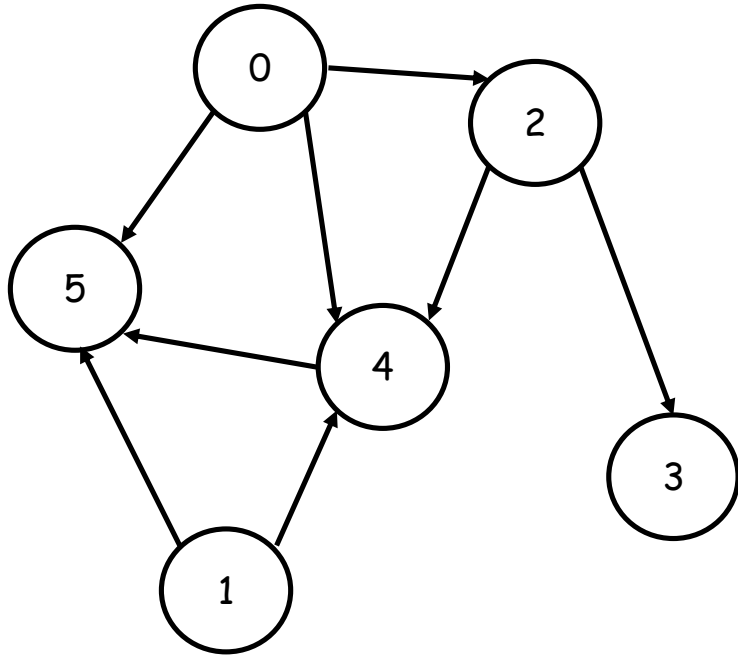
Node 0: Initial Vertex

Node 2: Terminal Vertex

Direction: Initial Vertex - Terminal Vertex

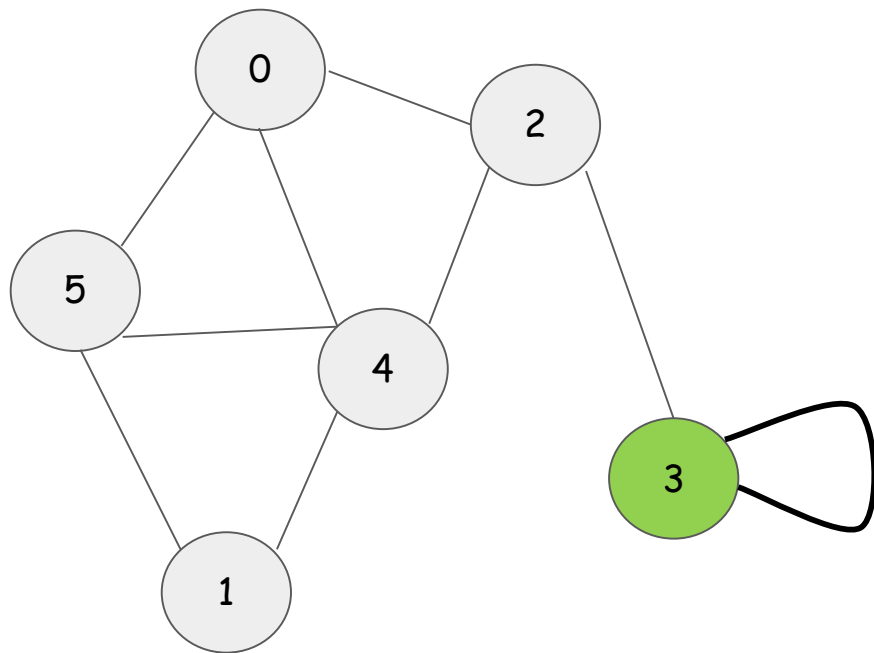
Representation:  $\frac{02}{20}$

# Directed Graph: Vertices and edges

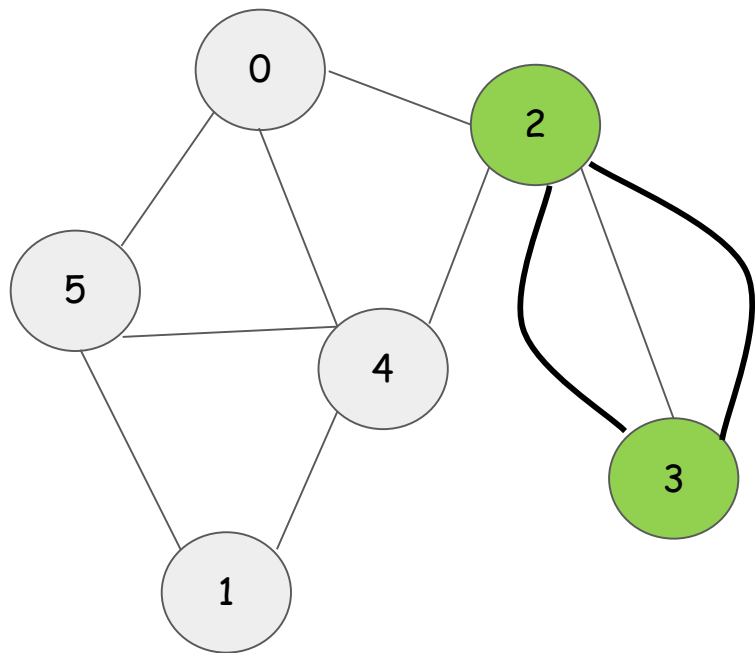


Edges = {05, 04, 02,  
24, 45, 15, 14, 23}

# Loops in Graphs



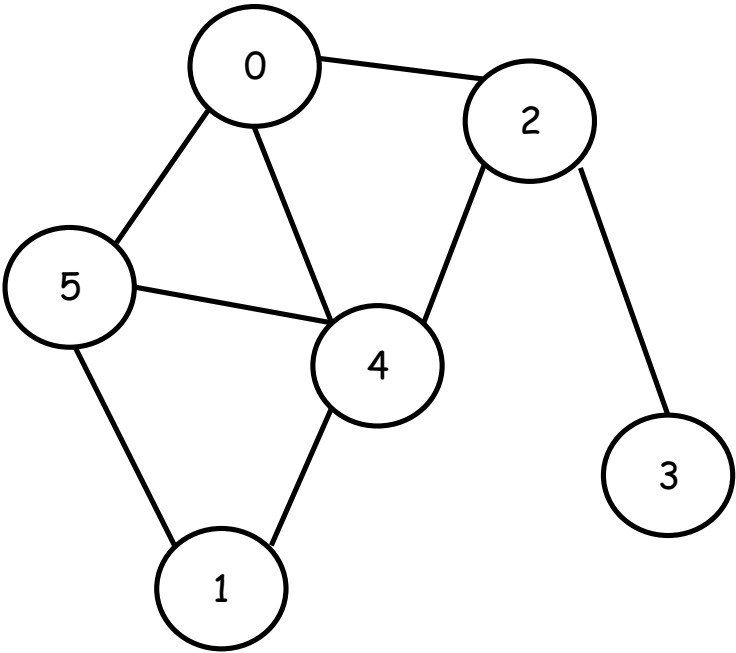
# Multiple edges in Graphs





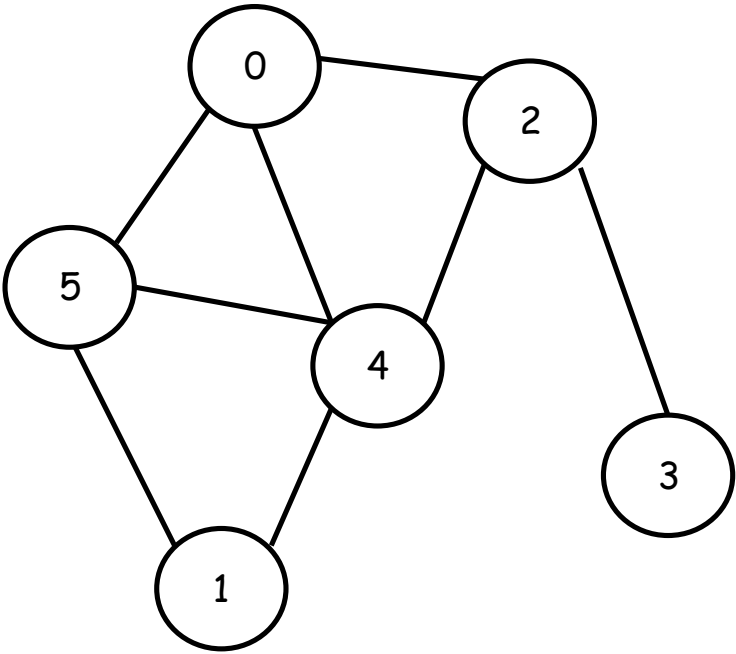
# Adjacency Matrix Representation: Undirected Graph

	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						



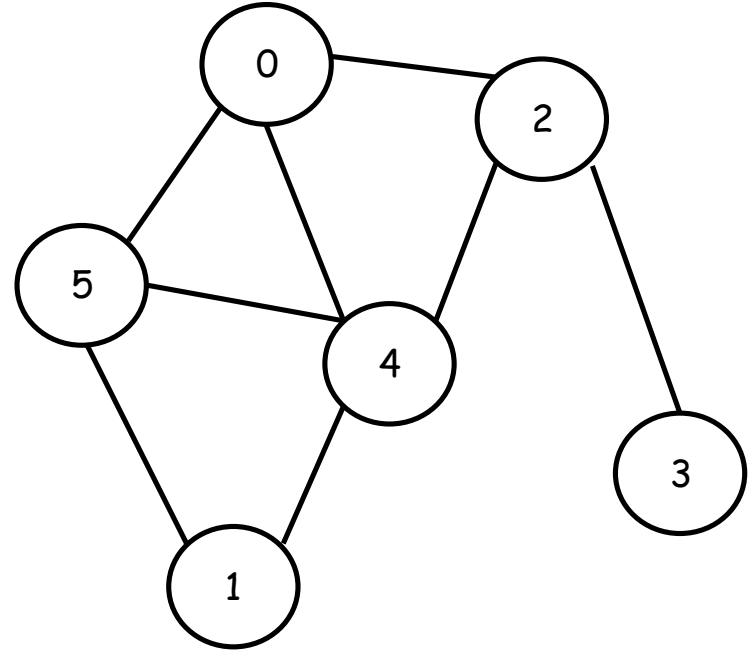
# Adjacency Matrix Representation: Undirected Graph

	0	1	2	3	4	5
0	0	0	1	0	1	1
1						
2						
3						
4						
5						



# Adjacency Matrix Representation: Undirected Graph

	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2						
3						
4						
5						



# Adjacency Matrix Representation: Undirected Graph

	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2	1	0	0	1	1	0
3	0	0	1	0	0	0
4	1	1	1	0	0	1
5	1	1	0	0	1	0



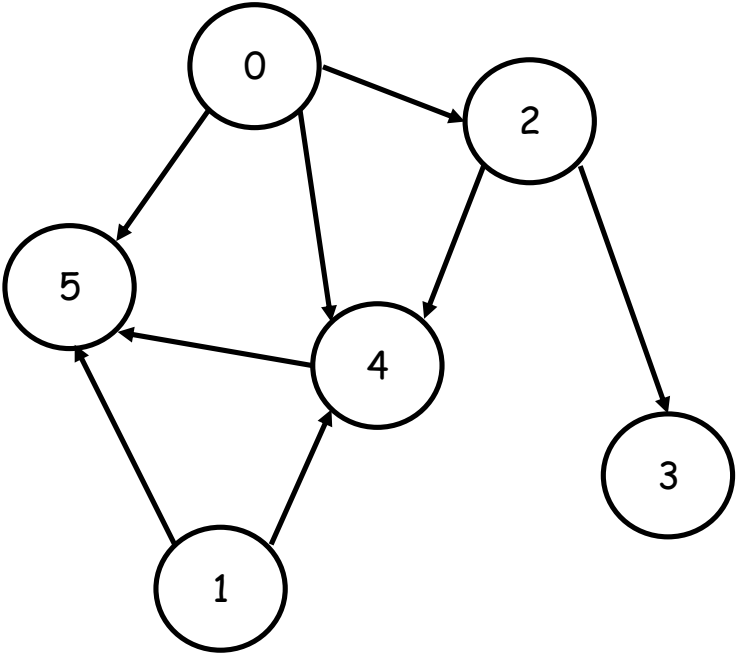
A 2D array:  
 $a[\text{Number of vertices}]$   
 $[\text{Number of vertices}]$

$$a[i][j] = a[j][i]$$

# Adjacency Matrix Representation: Directed Graph

Initial Vertex

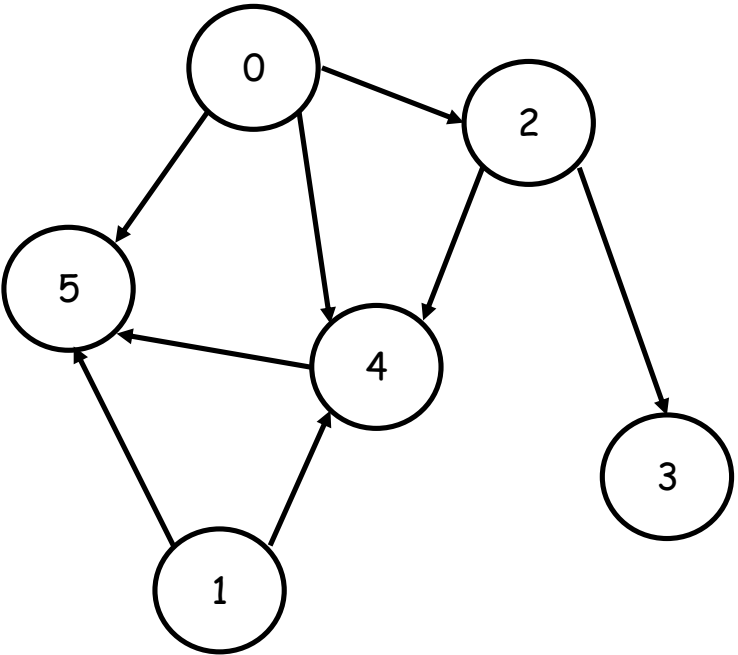
	Terminal Vertex					
	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						



# Adjacency Matrix Representation: Directed Graph

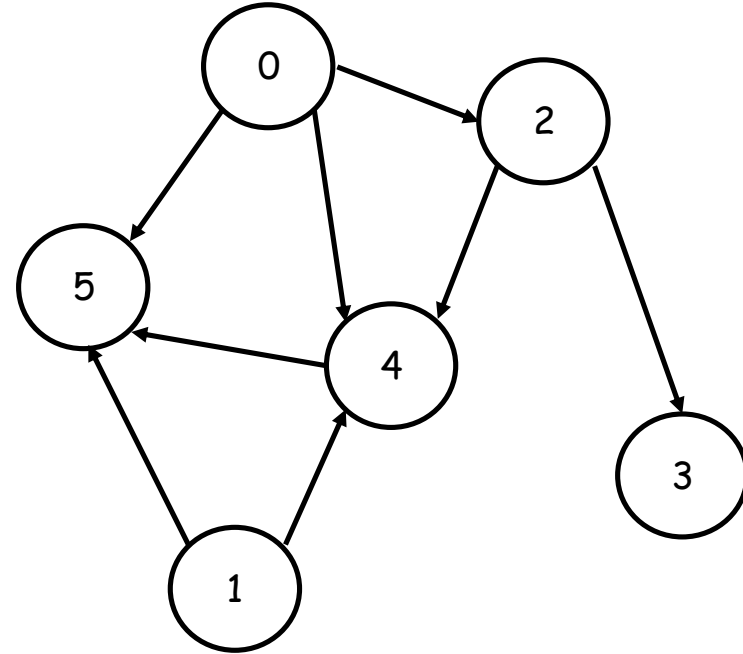
Initial Vertex

	Terminal Vertex					
	0	1	2	3	4	5
0	0	0	1	0	1	1
1						
2						
3						
4						
5						



# Adjacency Matrix Representation: Directed Graph

Initial Vertex	Terminal Vertex					
	0	1	2	3	4	5
	0	0	0	1	0	1
	1	0	0	0	0	1
	2					
	3					
	4					
	5					



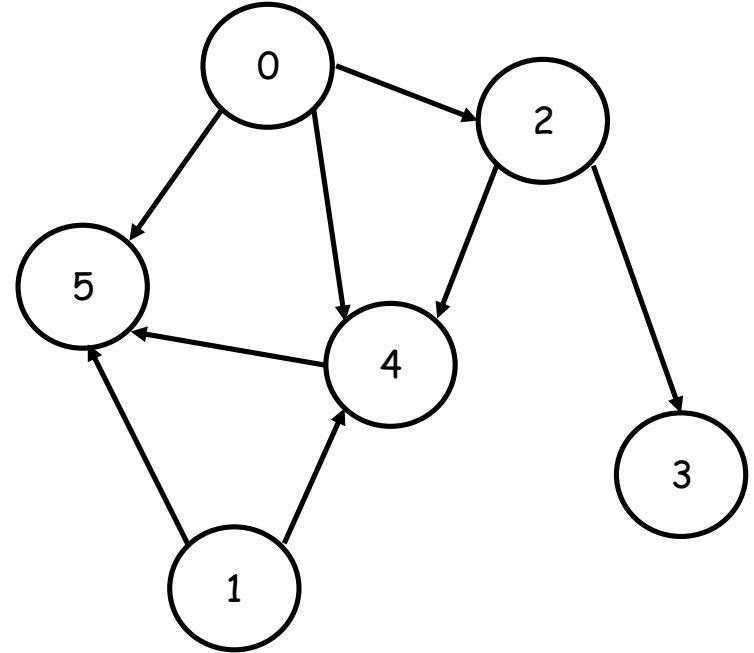
# Adjacency Matrix Representation: Directed Graph

Graph

Terminal Vertex

	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2	0	0	0	1	1	0
3						
4						
5						

Initial Vertex





# Adjacency Matrix Representation: Directed Graph

Initial Vertex	Terminal Vertex					
	0	1	2	3	4	5
	0	0	0	1	0	1
	1	0	0	0	0	1
	2	0	0	0	1	1
	3	0	0	0	0	0
	4	0	0	0	0	1
	5	0	0	0	0	0

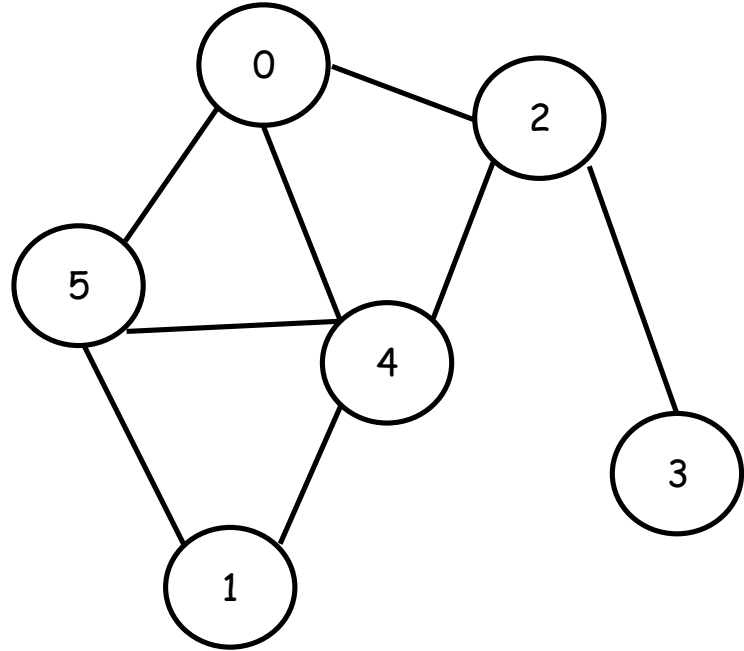
A 2D array:  
 $a[\text{Number of vertices}][\text{Number of vertices}]$



$a[i][j] = a[j][i]$   
 $a[i][j] = a[j][i]$   
No Restriction

# Degree of a vertex: Undirected Graph

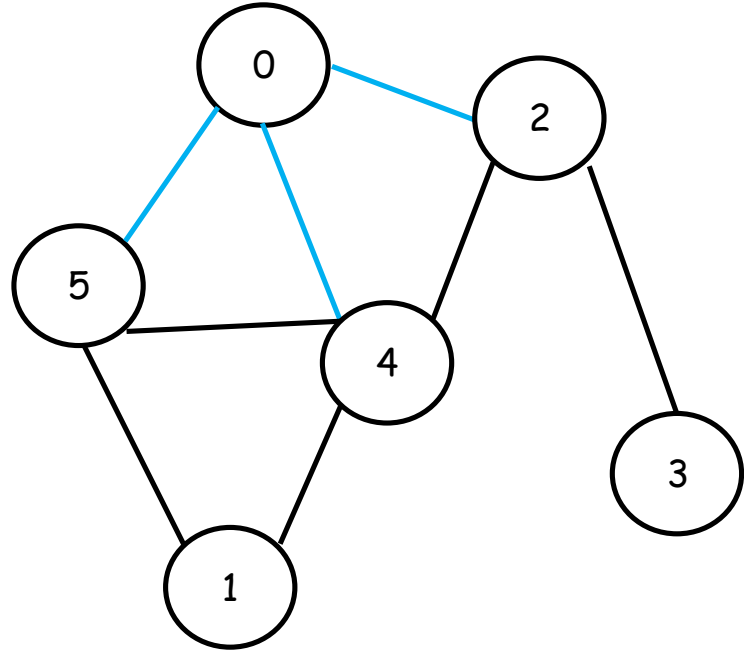
- Number of edges incident with it
- In case of Loop add +2



# Degree of a vertex: Undirected Graph

- Number of edges incident with it
- In case of Loop add +2

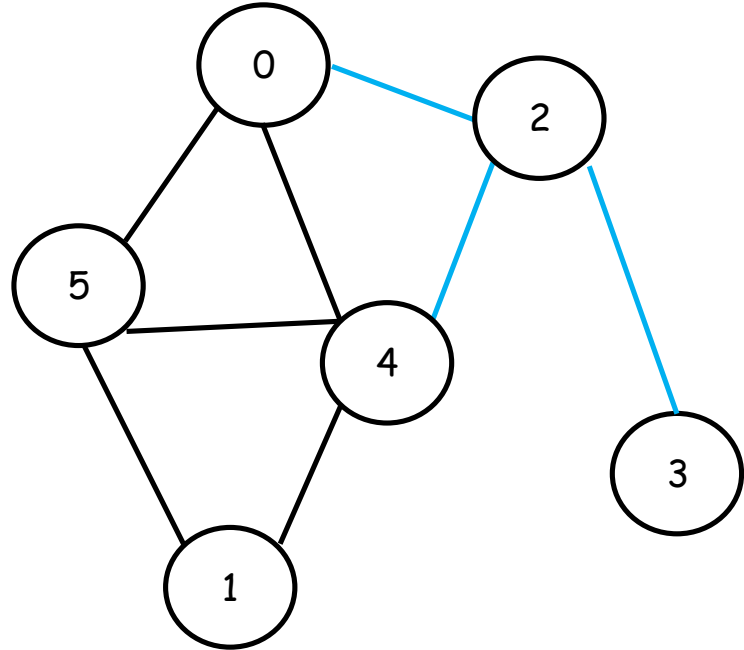
Vertex	Degree
0	3
1	
2	
3	
4	
5	



# Degree of a vertex: Undirected Graph

- Number of edges incident with it
- In case of Loop add +2

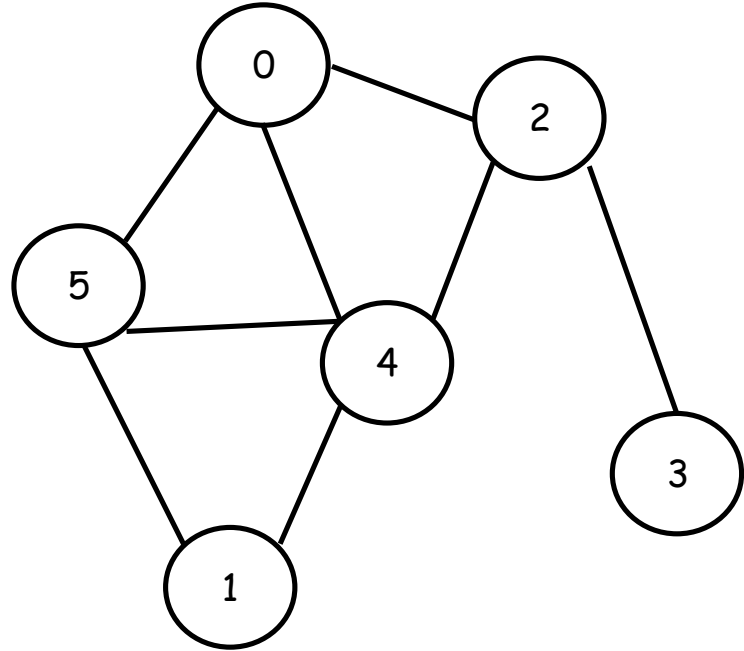
Vertex	Degree
0	3
1	2
2	
3	
4	
5	



# Degree of a vertex: Undirected Graph

- Number of edges incident with it
- In case of Loop add +2

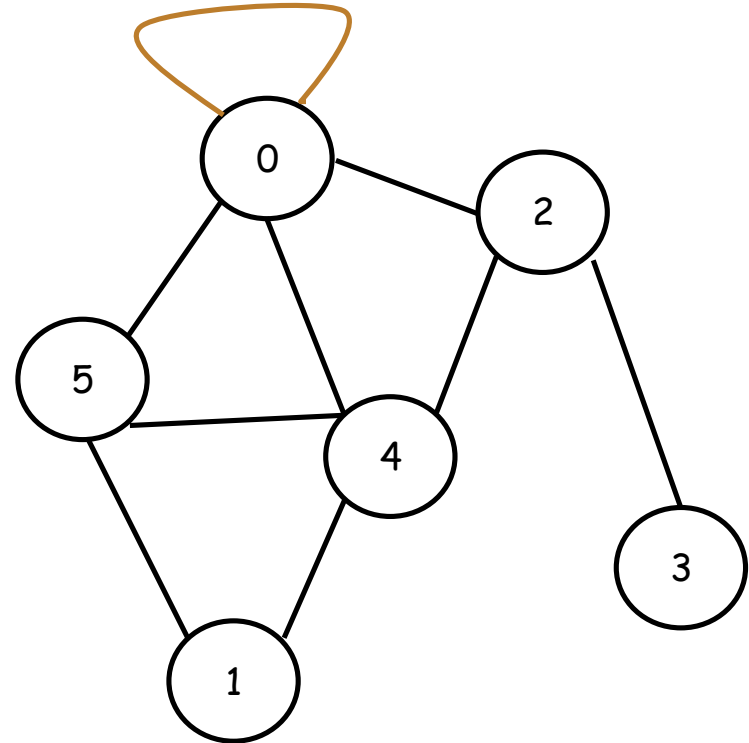
Vertex	Degree
0	3
1	2
2	3
3	1
4	4
5	3



# Degree of a vertex: Undirected Graph

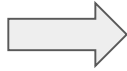
- Number of edges incident with it
- In case of Loop add +2

Vertex	Degree
0	5 (3 + 2)
1	2
2	3
3	1
4	4
5	3



# Degree of a vertex: Undirected Graph

Vertex	Degree
0	3
1	2
2	3
3	1
4	4
5	3



$\Sigma$  (1's)



$\Sigma$  (1's)

	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2	1	0	0	1	1	0
3	0	0	1	0	0	0
4	1	1	1	0	0	1
5	1	1	0	0	1	0

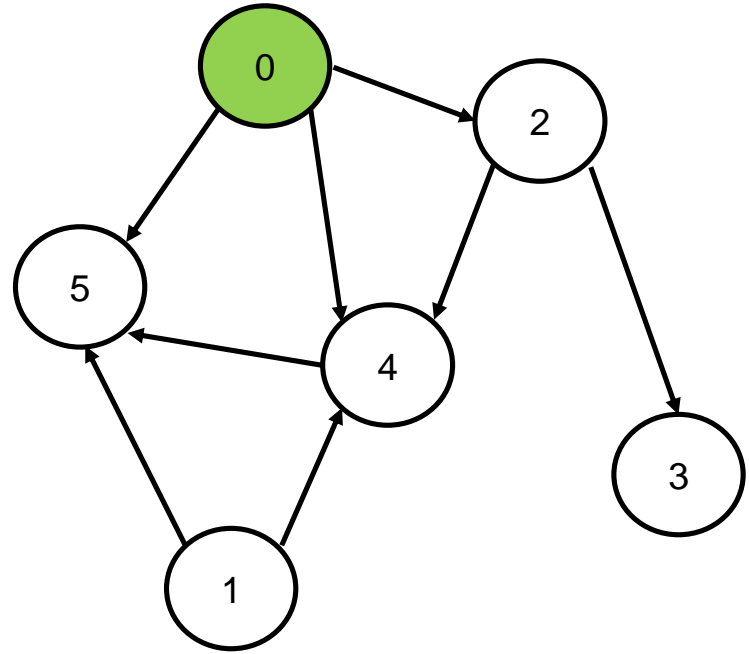
# Degree of a vertex: Directed Graph

- In Degree : No of **INCOMING** edges
- Out Degree : No of **OUTGOING** edges



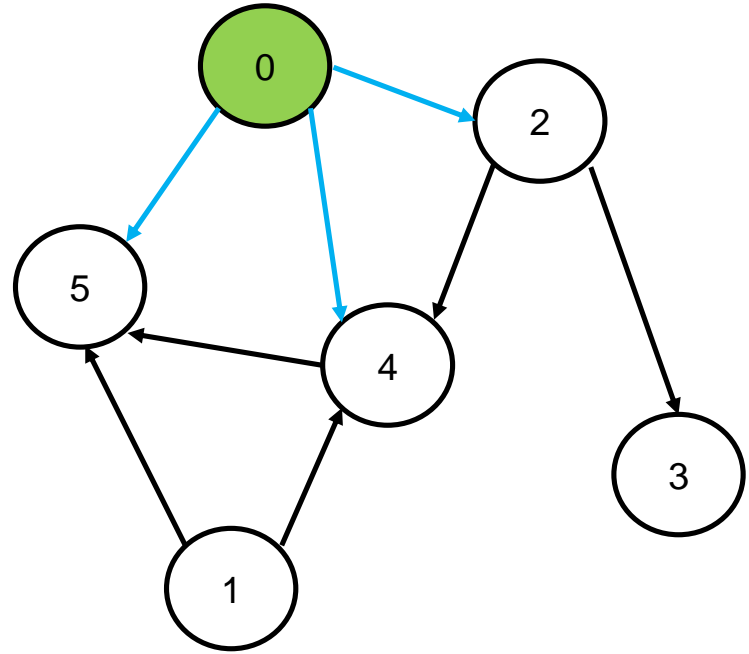
# Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	
1		
2		
3		
4		
5		



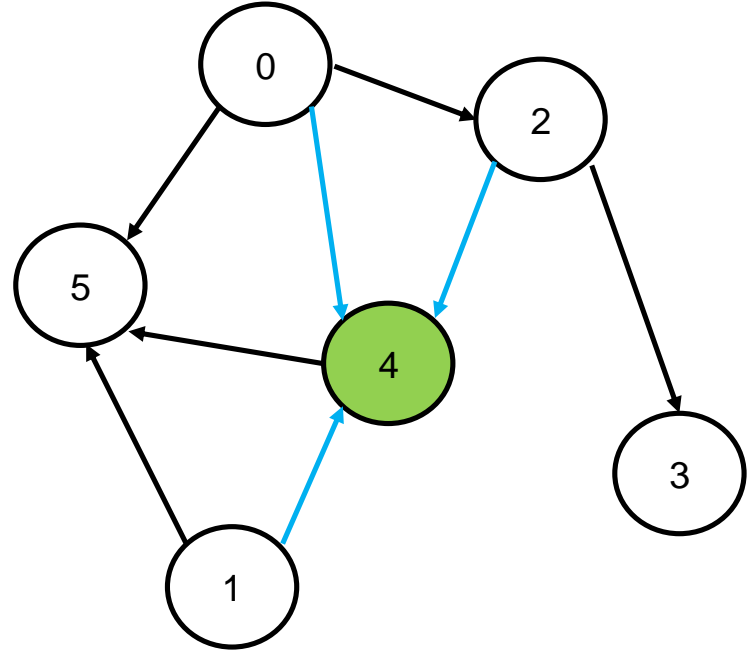
# Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	3
1		
2		
3		
4		
5		



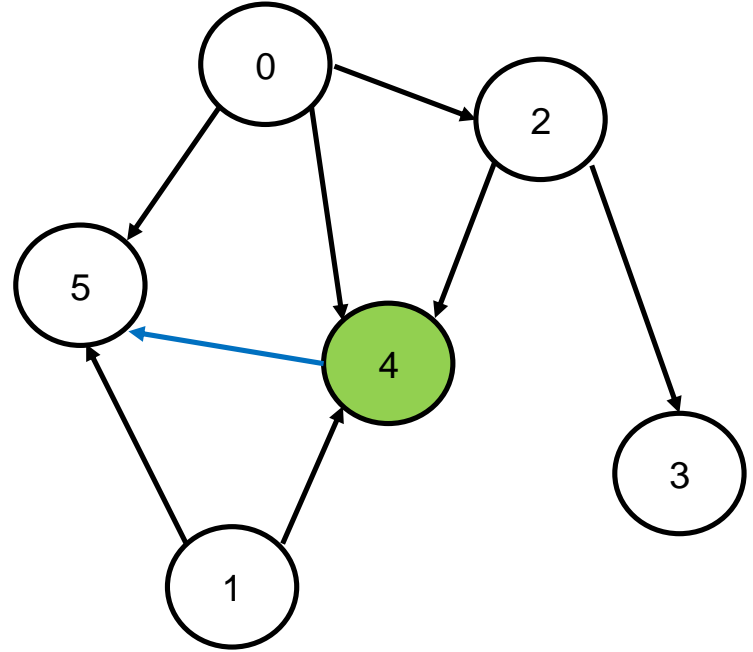
# Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	3
1		
2		
3		
4	3	
5		



# Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	3
1		
2		
3		
4	3	1
5		

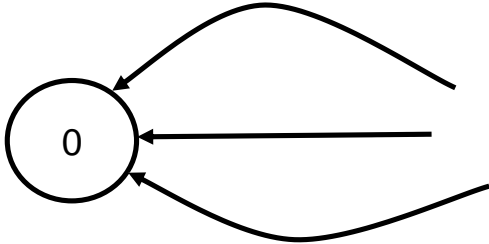


# Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	3

In degree = number of incoming edges

In degree of vertex 0 = number of incoming edges to 0



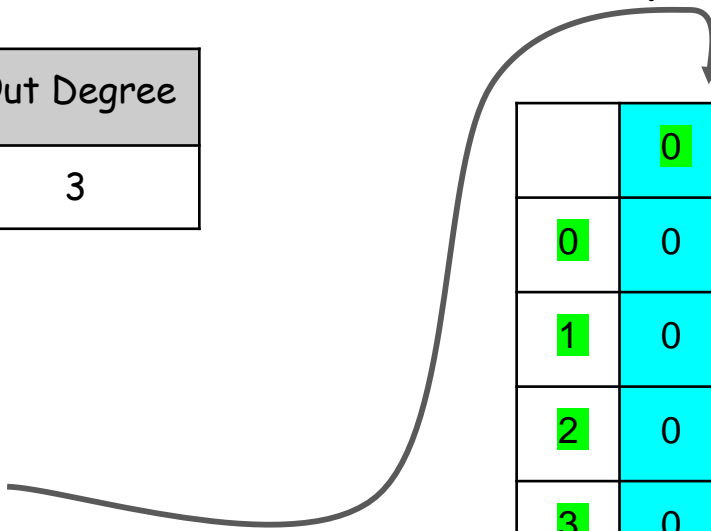
So, for counting in degrees of vertex 0  
0 must be a **TERMINAL VERTEX**

# Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	3



$\Sigma$   
(1's)



	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2	0	0	0	1	1	0
3	0	0	0	0	0	0
4	0	0	0	0	0	1
5	0	0	0	0	0	0

# Degree of a vertex: Directed Graph

Vertex	In Degree	Out Degree
0	0	3



$\Sigma$   
(1's)

	0	1	2	3	4	5
0	0	0	1	0	1	1
1	0	0	0	0	1	1
2	0	0	0	1	1	0
3	0	0	0	0	0	0
4	0	0	0	0	0	1
5	0	0	0	0	0	0

# Bipartite Graph

if its vertex set  $V$  can be partitioned into two disjoint sets  $V_1$  and  $V_2$  such that

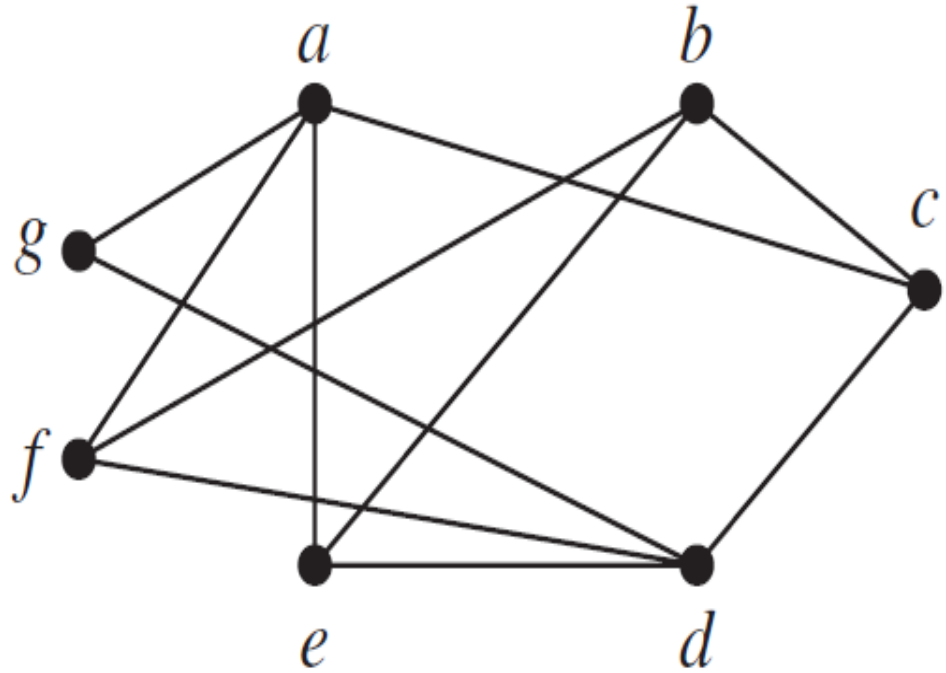
no edge in  $G$  connects either two vertices in  $V_1$  or two vertices in  $V_2$



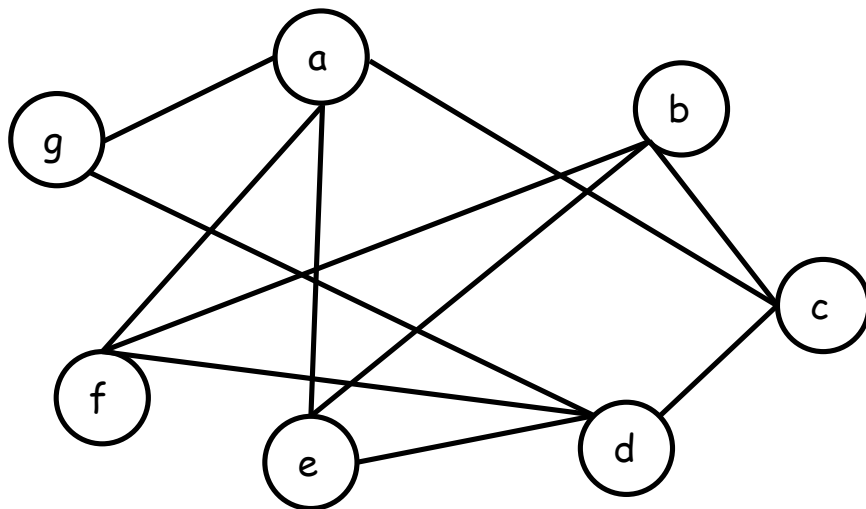
# Bipartite Graph

$V1 = \{a, b, d\}$

$V2 = \{c, e, f, g\}$

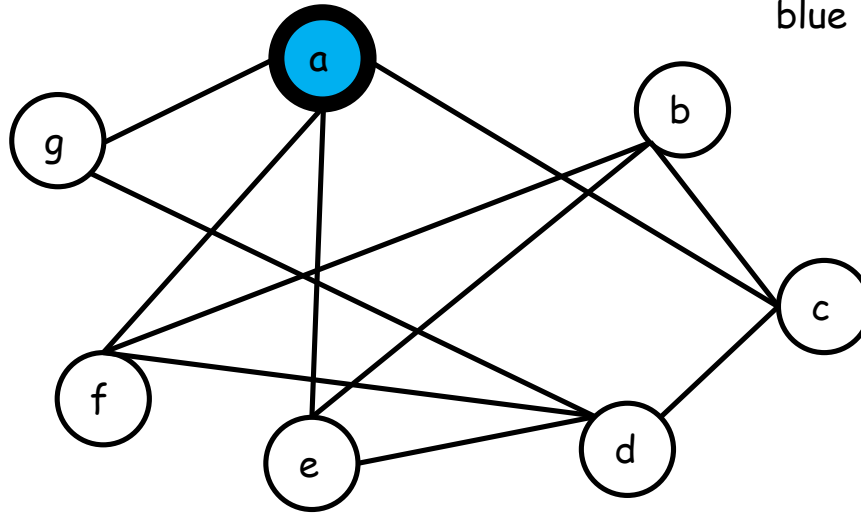


# How to decide if a graph is bipartite or not

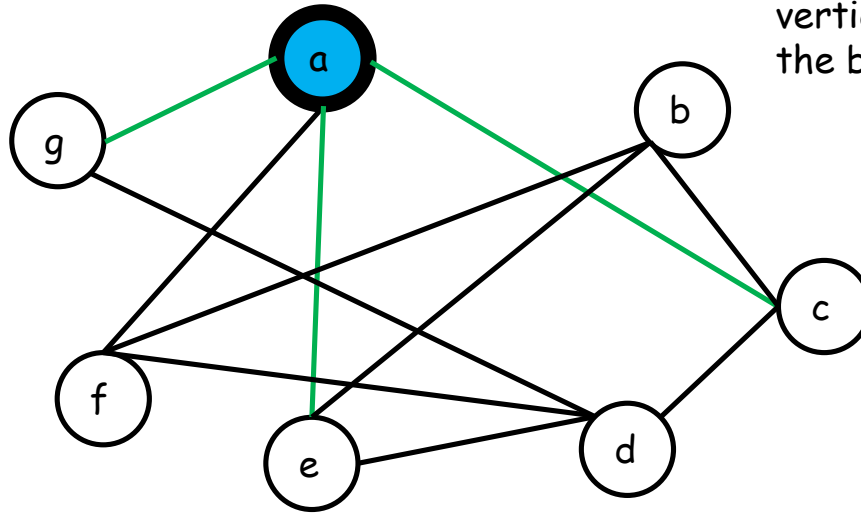


# How to decide if a graph is bipartite or not

1. Color any of the vertices blue

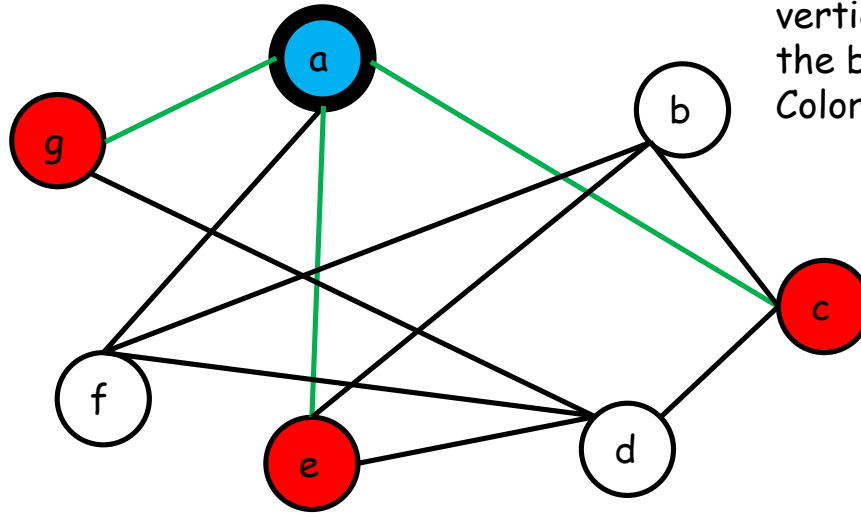


# How to decide if a graph is bipartite or not



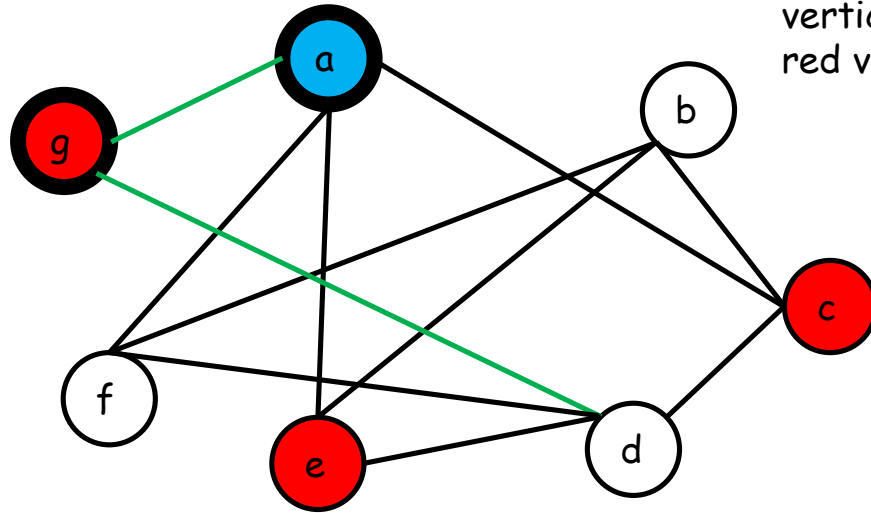
2. Identify all uncolored vertices that are adjacent to the blue vertex.

# How to decide if a graph is bipartite or not



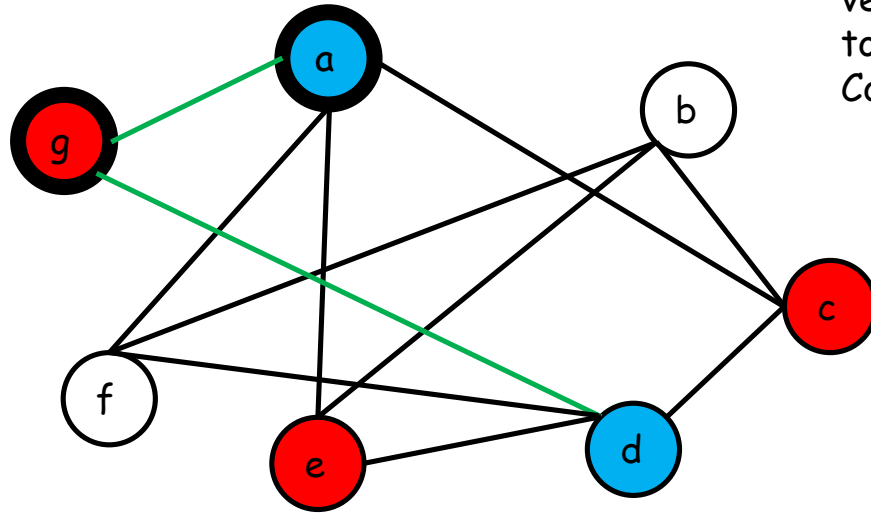
2. Identify all uncolored vertices that are adjacent to the blue vertex. Color them red

# How to decide if a graph is bipartite or not



3. Identify all uncolored vertices that are adjacent to a red vertex.

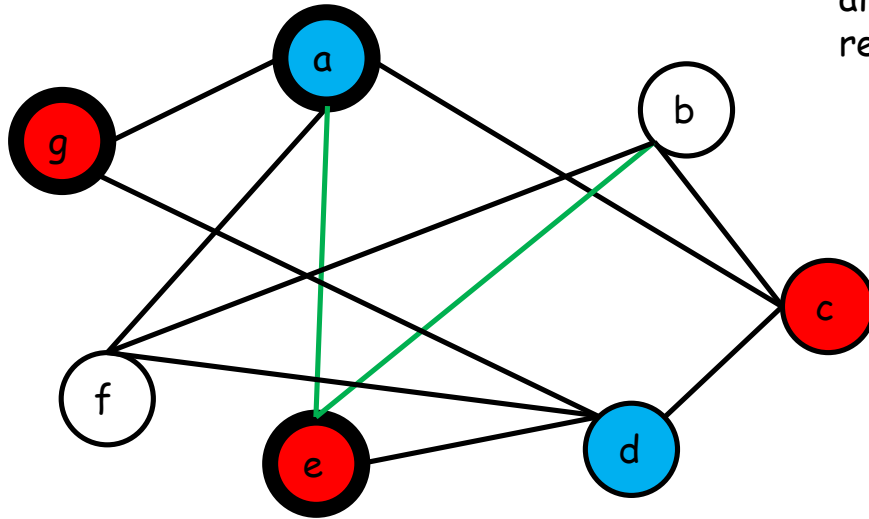
# How to decide if a graph is bipartite or not



3. Identify all uncolored vertices that are adjacent to a red vertex. Color them blue.

# How to decide if a graph is bipartite or not

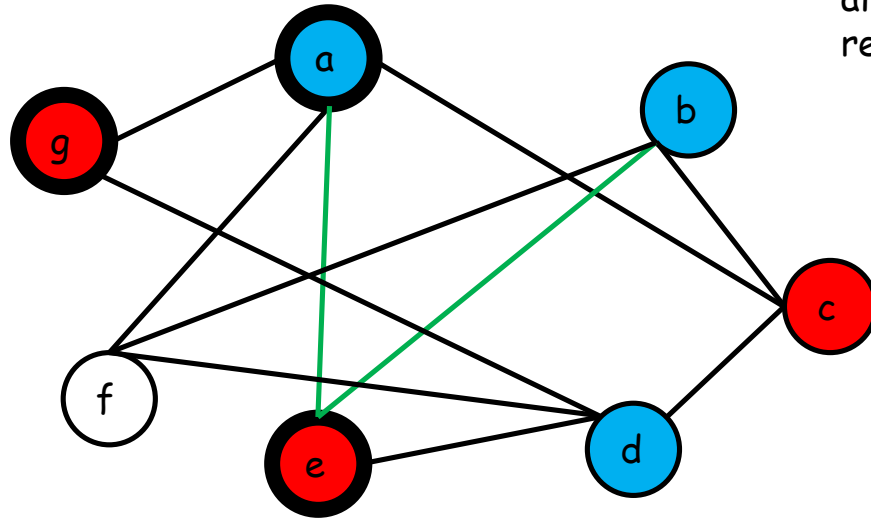
Repeat steps 2 and 3 until all the vertices are colored red or blue.





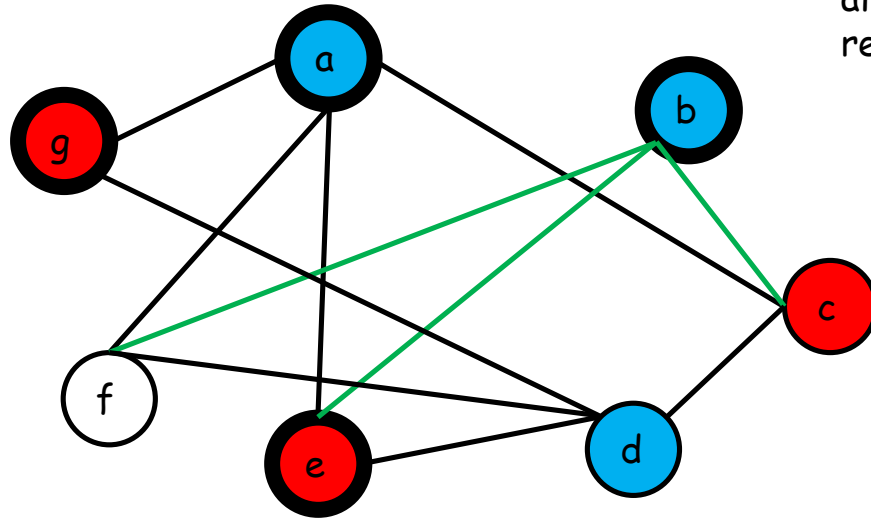
# How to decide if a graph is bipartite or not

Repeat steps 2 and 3 until all the vertices are colored red or blue.

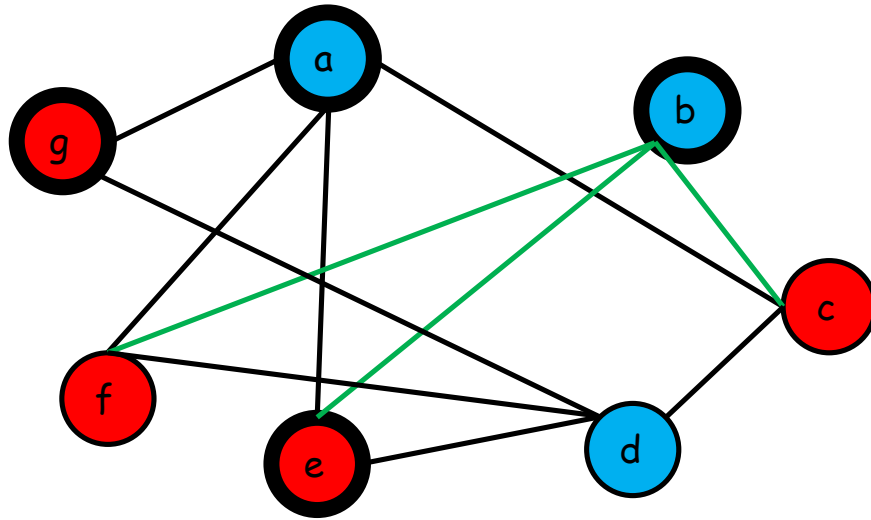


# How to decide if a graph is bipartite or not

Repeat steps 2 and 3 until all the vertices are colored red or blue.



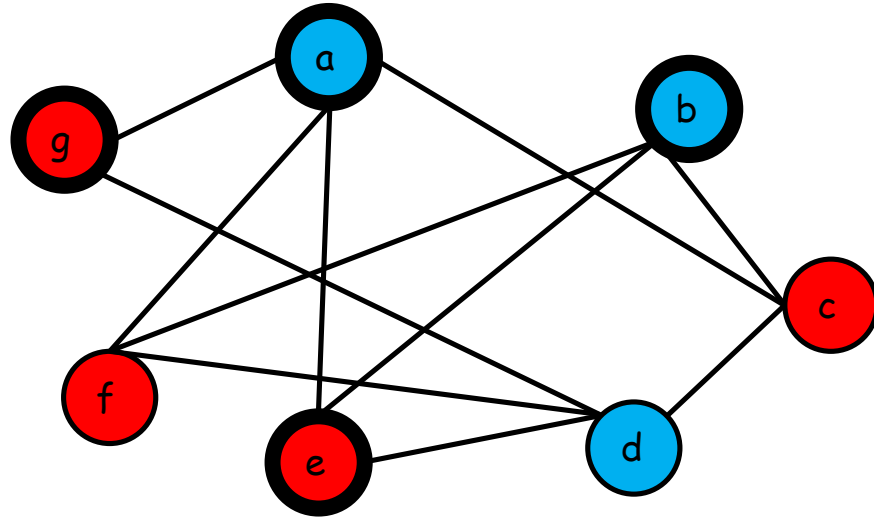
# How to decide if a graph is bipartite or not



If there are any two vertices adjacent of the same color, then your graph is not bipartite, otherwise it is bipartite

∴Bipartite graph

# How to decide if a graph is bipartite or not



Disjoint sets

$V1 = \{a, b, d\}$

$V2 = \{c, e, f, g\}$