

Iterative Methods for solving Linear Algebraic Equations

Problem Set 4

November 14, 2019

1 Problem

Estimate solution of a system of linear equations using Gauss-Seidal and Jacobian method. Stop iterations if maximum number of iterations are already completed.

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

2 Input

n = Number of equations : integer

A = Coefficient Matrix : a 2D-NumPy Array

b = Constant Column Vector : a 1D-NumPy Array

XO = Initial values of $[x_1, x_2, \dots, x_n]$: a 1D-NumPy Array

k = Maximum number of iterations : integer

3 Output

Print the iteration number, $x = [x_1, x_2, \dots, x_n]$, which is a NumPy Array for each iteration.

4 Algorithm

Jacobian Method

`function JACOBIAN`

```

 $k \leftarrow 1$ 
while Convergence not reached do
  for  $i \leftarrow 1$  to  $n$  do
     $Sum \leftarrow 0$ 
    for  $j \leftarrow 1$  to  $n$  do
      if  $i \neq j$  then
         $Sum \leftarrow Sum + a_{ij}x_j$ 
      end if
    end for
     $x_i \leftarrow \frac{1}{a_{ii}}(b_i - Sum)$ 
    save it temporarily, DO NOT UPDATE
  end for
  UPDATE x
   $k \leftarrow k + 1$ 
end while
end function

```

Gauss-Seidel Method

Almost same as Jacobian Method. The only difference lies in the time to update the array x .

5 Sample Input-Output

Jacobian Method

Input

```

 $n = 4$ 
 $A = [[10, -1, 2, 0], [-1, 11, -1, 3], [2, -1, 10, -1], [0, 3, -1, 8]]$ 
 $b = [6, 25, -11, 15]$ 
 $XO = [0, 0, 0, 0]$ 
 $k = 25$ 

```

Output

```

iteration = 1,  $x = [0.6, 2.27272, -1.1, 1.875]$ 
iteration = 2,  $x = [1.04727, 1.7159, -0.80522, 0.88522]$ 
iteration = 3,  $x = [0.93263, 2.05330, -1.0493, 1.13088]$ 
iteration = 4,  $x = [1.01519, 1.95369, -0.9681, 0.97384]$ 
iteration = 5,  $x = [0.98899, 2.0114, -1.0102, 1.02135]$ 

```

6 Important Notes

- Implement using Python programming language

7 Marks Distribution

- Algorithm Implementation: 20(10 + 10)

8 Rules

- Any type of plagiarism is strongly forbidden. **NO/NEGATIVE** marks will be given to the students who will be found to be involved in plagiarism (from internet/senior/class- mates code etc.).

It does not matter who is the server and who is the client.

9 Deadline

Deadline is set at 19 July, 2019 8:00 am.