Counting

First rule of counting - The Product Rule

Suppose that a procedure can be broken down into a sequence of two tasks.

If there are n_1 ways to do the first task and there are n_2 ways to do the second task,

then there are $n_1 n_2$ ways to do the procedure.

Problem 1.1

How many 3-bit strings? Or
How many sequences of three bits from $\{0, 1\}$?

Problem 1.2

How many outcomes possible for k coin tosses?

Problem 1.3

How many 10 digit numbers?

Problem 1.4

How many k digit m-base numbers?

Problem 1.5

The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?

Problem 1.6

How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits (and no sequences of letters are prohibited, even if they are obscene)?

Problem 1.7

A telephone number has the form NYX-NNX-XXXX

Let X denote a digit that can take any of the values 0 through 9,
let N denote a digit that can take any of the values 2 through 9,
and let Y denote a digit that must be a 0 or a 1.

How many different North American telephone numbers are possible ?

The Sum Rule

If a task can be done either in one of $n1$ ways or in one of $n2$ ways, where none of the set of $n1$ ways is the same as any of the set of $n2$ ways (they do not overlap), then there are $n1 + n2$ ways to do the task.

Problem 2.1

In how many ways can we select two books from different subjects among 5 distinct CS books, 3 distinct math books, 2 distinct art books?

Problem 2.2

A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

Problem 2.3

Suppose that either a member of the mathematics faculty or a student who is a mathematics major is chosen as a representative to a university committee. How many different choices are there for this representative if there are 37 members of the mathematics faculty and 83 mathematics majors and no one is both a faculty member and a student?

Inclusion-Exclusion Rule

If a task can be done in either $n1$ ways or $n2$ ways, then the number of ways to do the task is $n1 + n2$ minus the number of ways to do the task that are common to the two different ways.

Problem 3.1

Suppose that either a member of the mathematics faculty or a student who is a mathematics major is chosen as a representative to a university committee. How many different choices are there for this representative if there are 37 members of the mathematics faculty and 83 mathematics majors and 2 of them are both a mathematical faculty and mathematical majors?

Problem 3.2

How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

Problem 4.1

Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit.

How many possible passwords are there?

Problem 4.2

A six person committee of Alice, Bob, Charlie, Dylan, Elle, Frank is to select a chairperson, secretary, treasurer from themselves.

a. How many ways this can be done?
b. If either Elle or Bob must be a chairperson?
c. If Elle must hold one position?
d. If both Dylan and Frank must hold positions

Problem 4.2

Problem 5.1: Solve using tree diagram

How many bit strings of length four do not have two consecutive 1s?

Pigeonhole Principle

If k is a positive integer and $k + 1$ or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.

Problem 6.1

How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points?

Generalized Pigeonhole Principle

If *N* objects are placed into *k* boxes, then there is at least one box containing at least ceiling(*N*/*k*) objects.

the minimum number of objects =?
such that at least *r* of these objects must be in one of *k* boxes when these objects are distributed among the boxes.

$N = k(r-1) + 1$

Problem 6.2

What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F?

Problem 6.3

A bag contains 10 red marbles, 10 white marbles, and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color?

Problem 6.4

A box contains 6 red, 8 green, 10 blue, 12 yellow and 15 white balls. What is the minimum no. of balls we have to choose randomly from the box to ensure that we get 8 balls of same color?