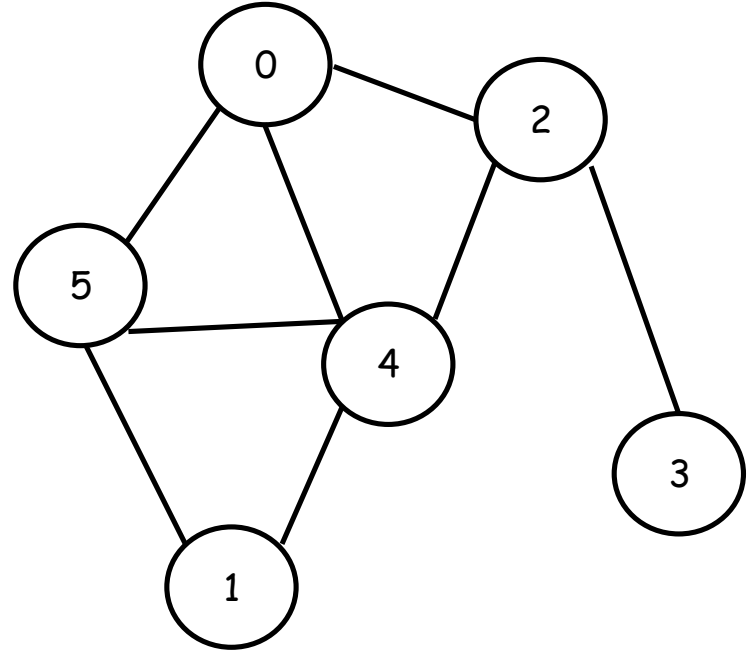
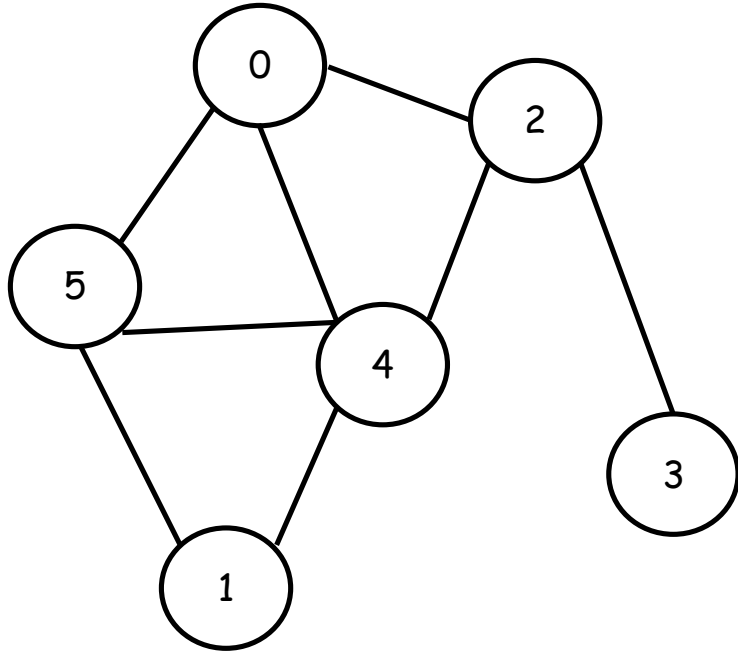


Trees

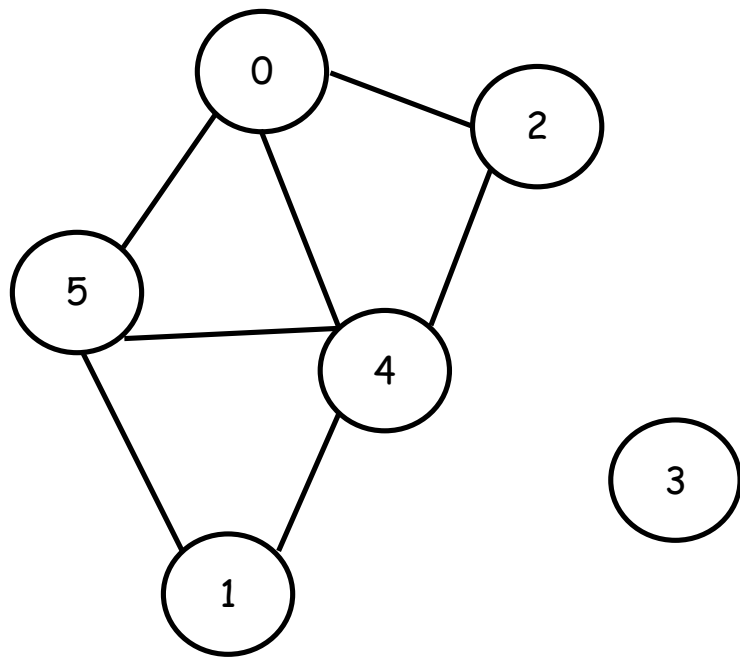
Introduction to Trees

a connected undirected graph with no simple circuits

Connected Graph



Connected Graph

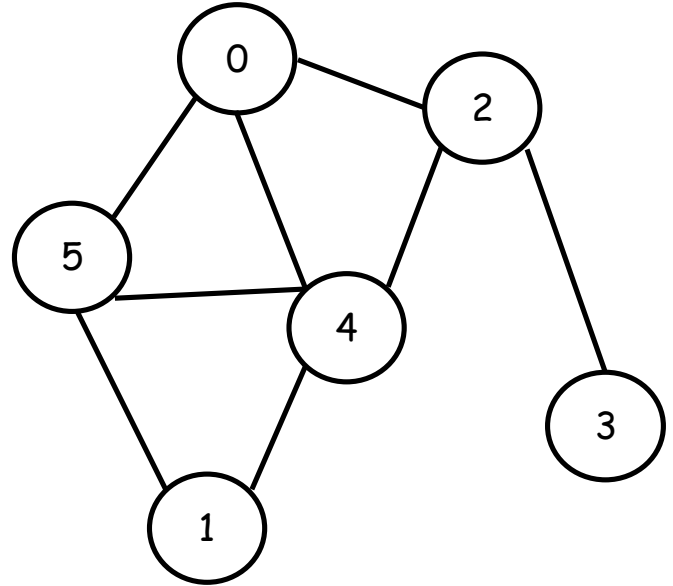


Not a tree

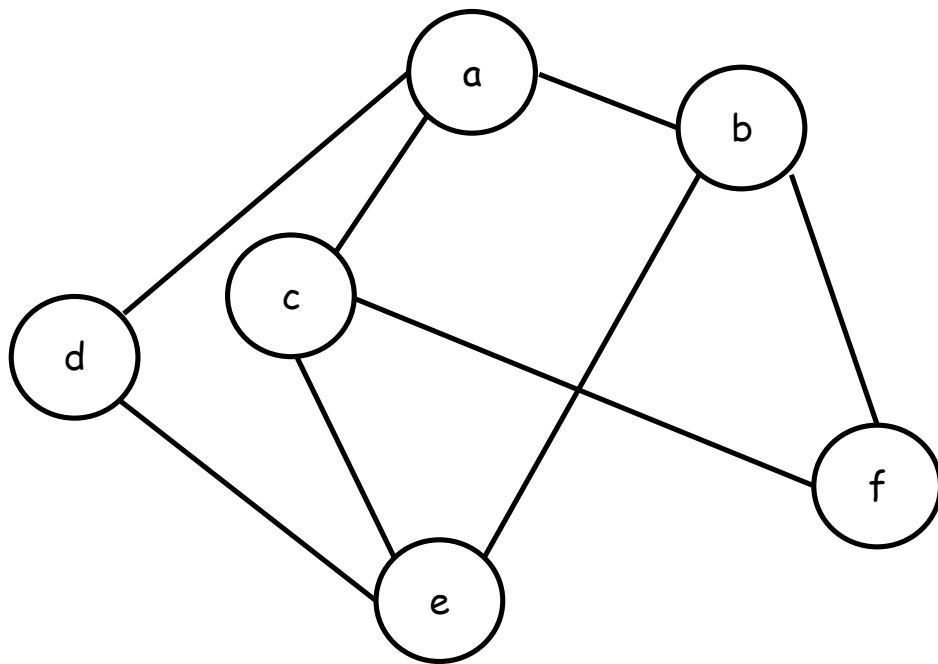
Simple Circuits

Simple path that begins and ends at the same vertex.

Simple path: a sequence of vertices without repetition

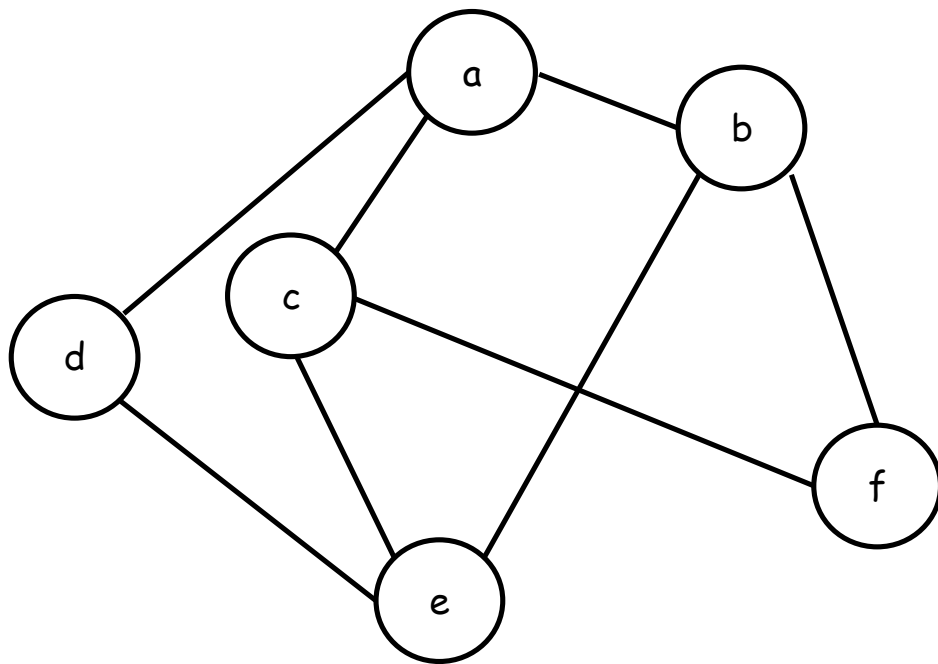


Simple Circuit



Not a tree

Simple Circuit

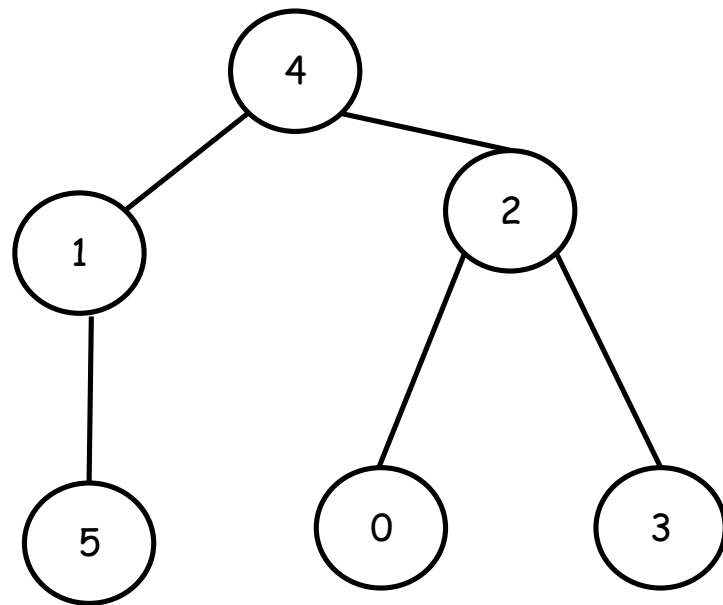
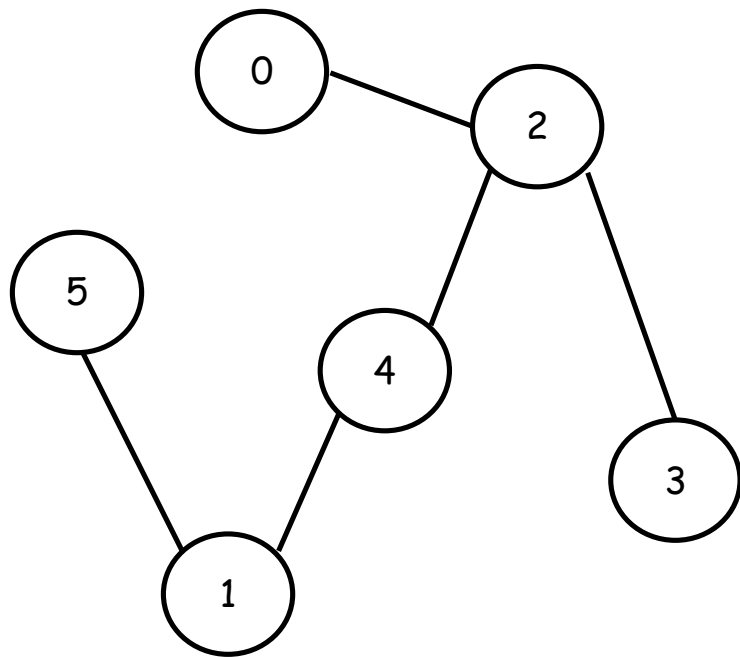


Not a tree

Rooted Tree

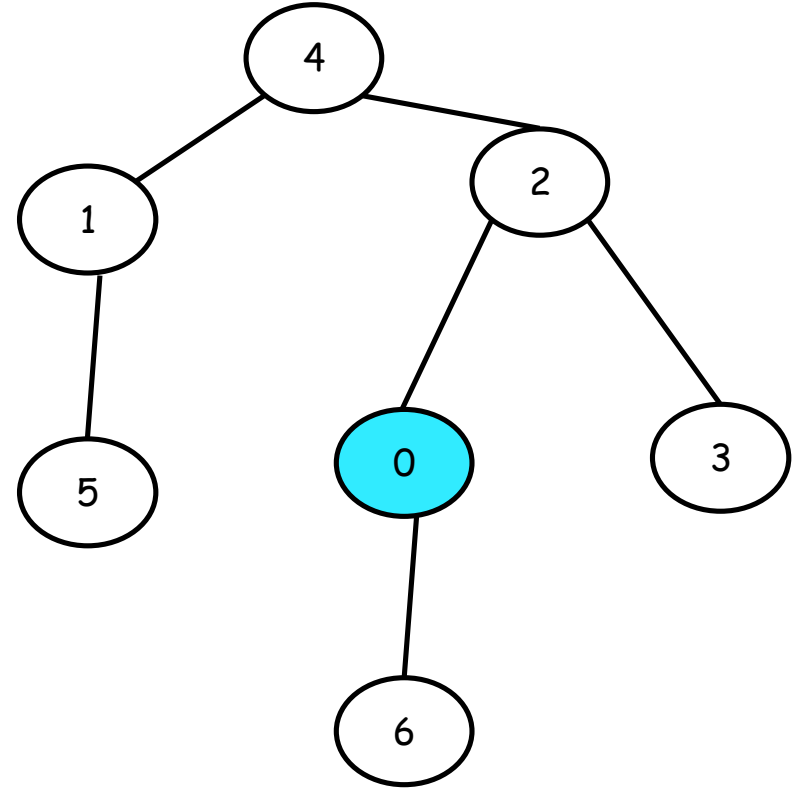
one vertex is designated as the root and every edge is directed away from the root.

Rooted Tree



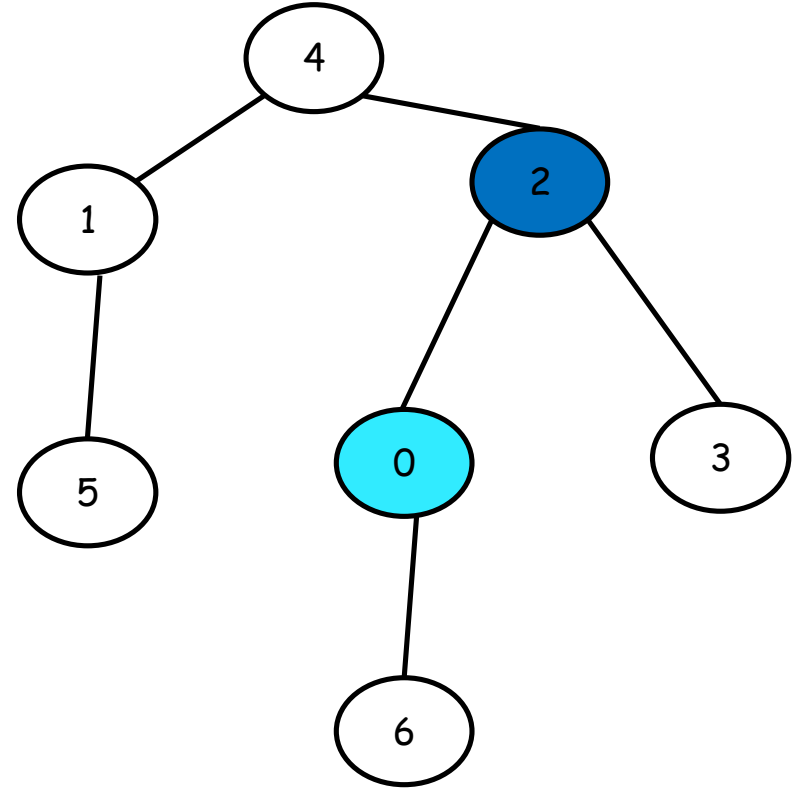
Parent

Parent of 0 =



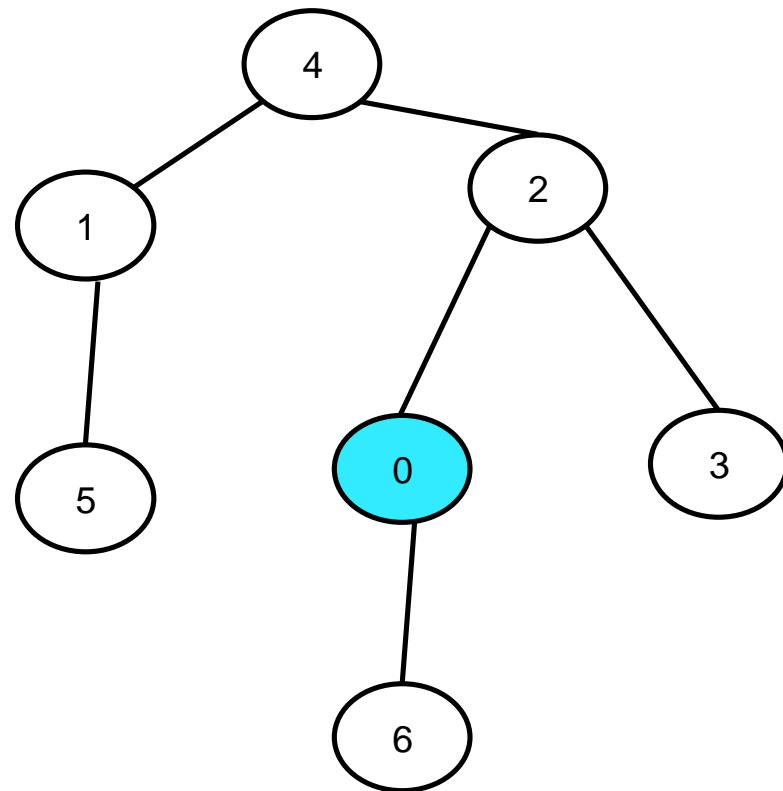
Parent

Parent of 0 = 2



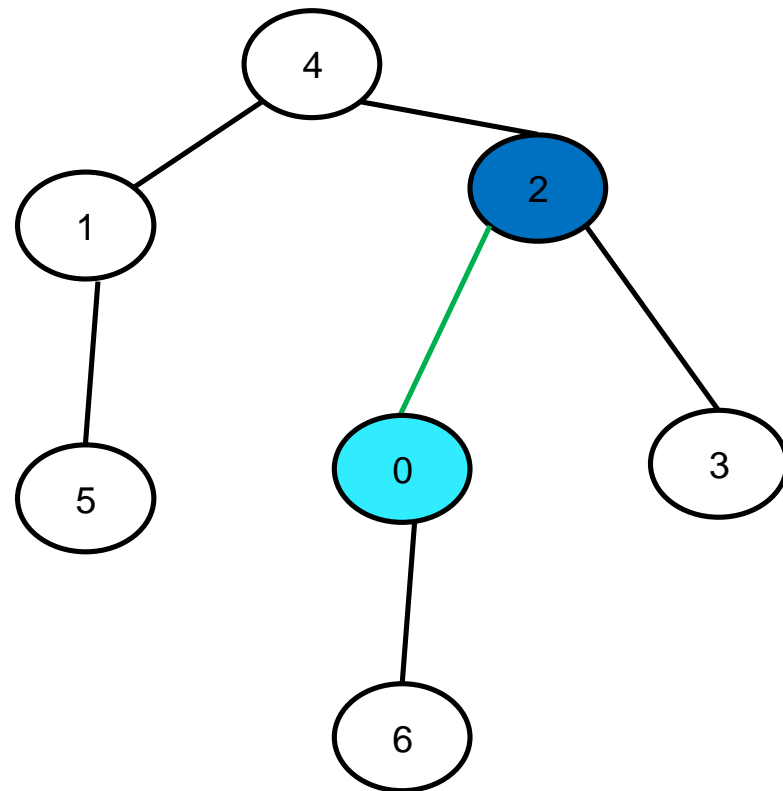
Siblings

Siblings of 0 =



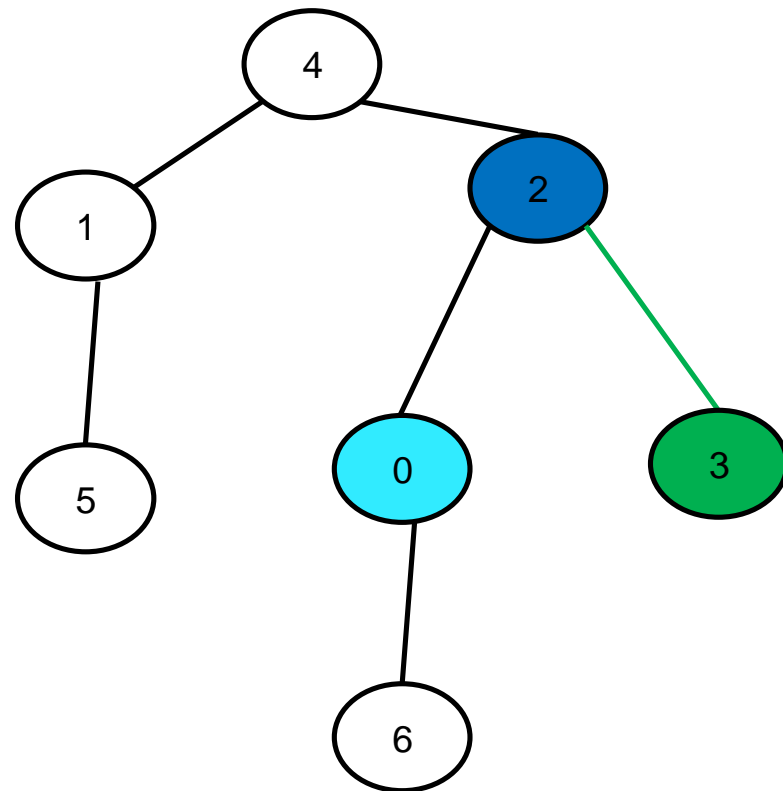
Siblings

Siblings of 0 =




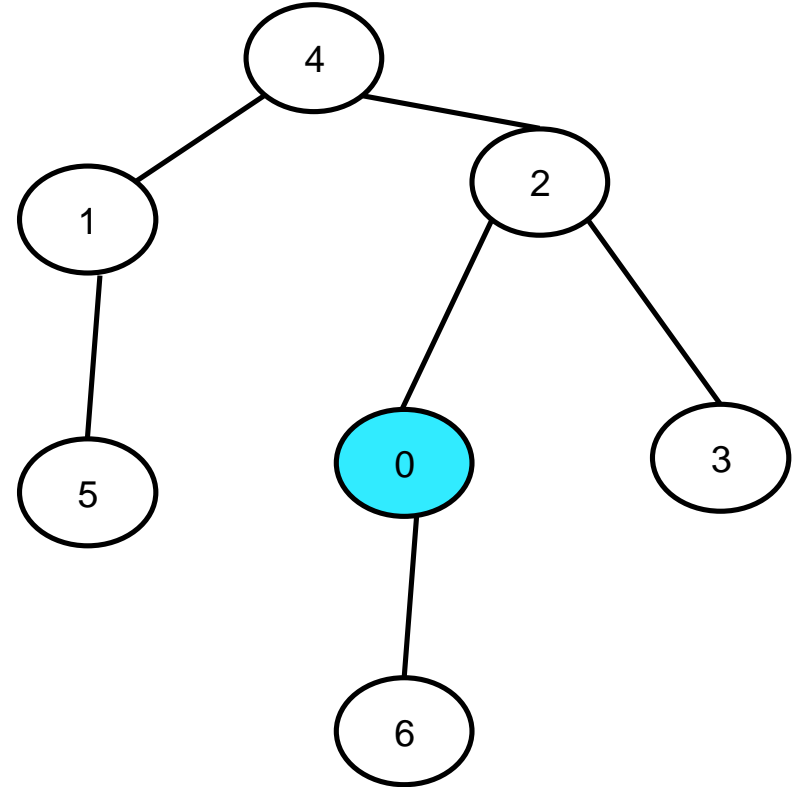
Siblings

Siblings of 0 = 3




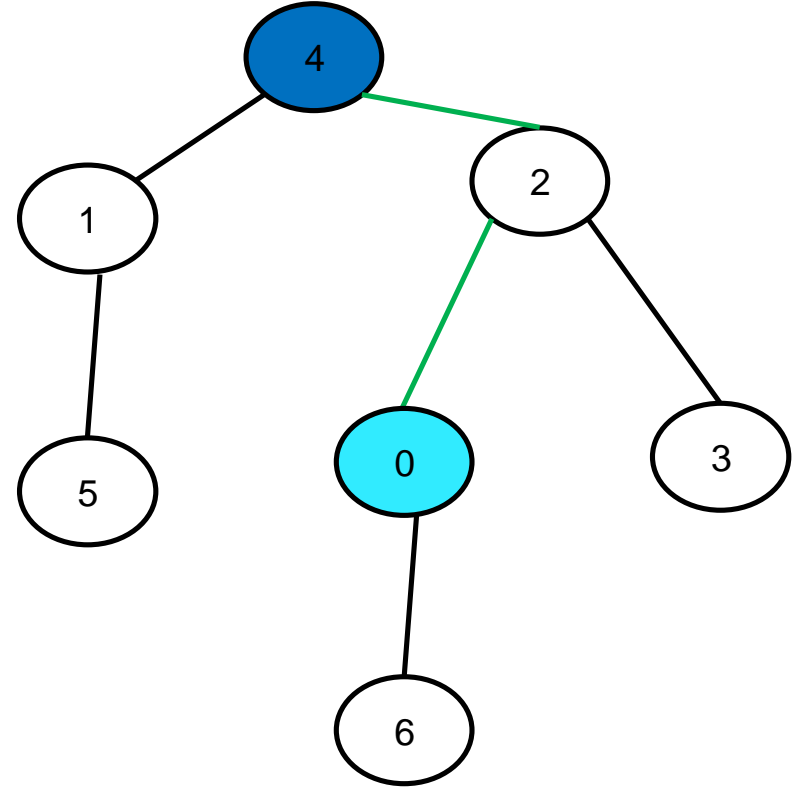
Ancestor of a vertex

Ancestor of 0 =
vertices in the path from
root  vertex 0
Excluding 0




Ancestor of a vertex

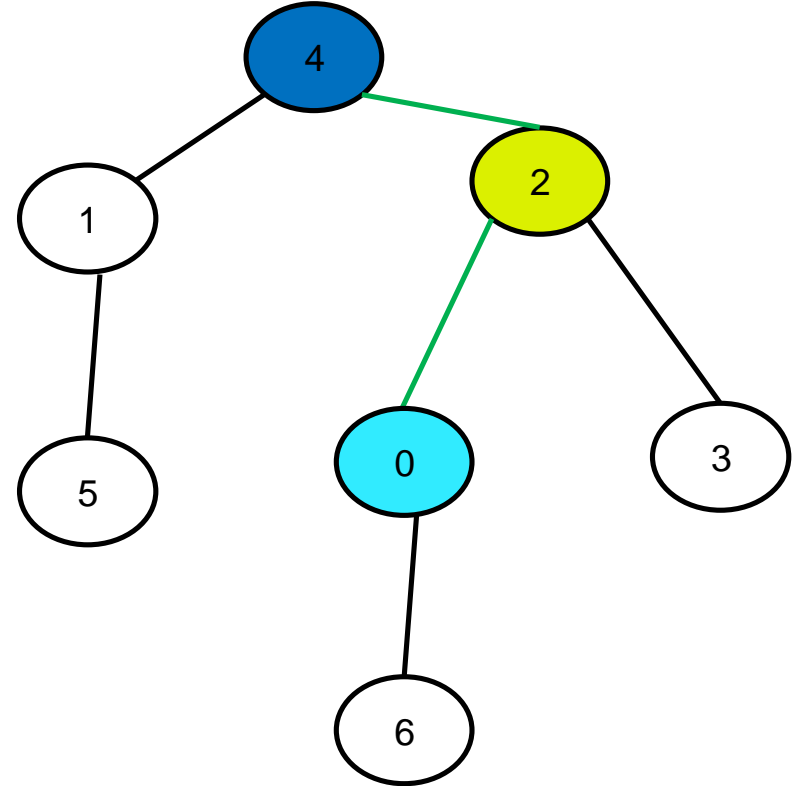
Ancestor of 0 = vertices in the path from
root  vertex 0 Excluding 0



Ancestor of a vertex

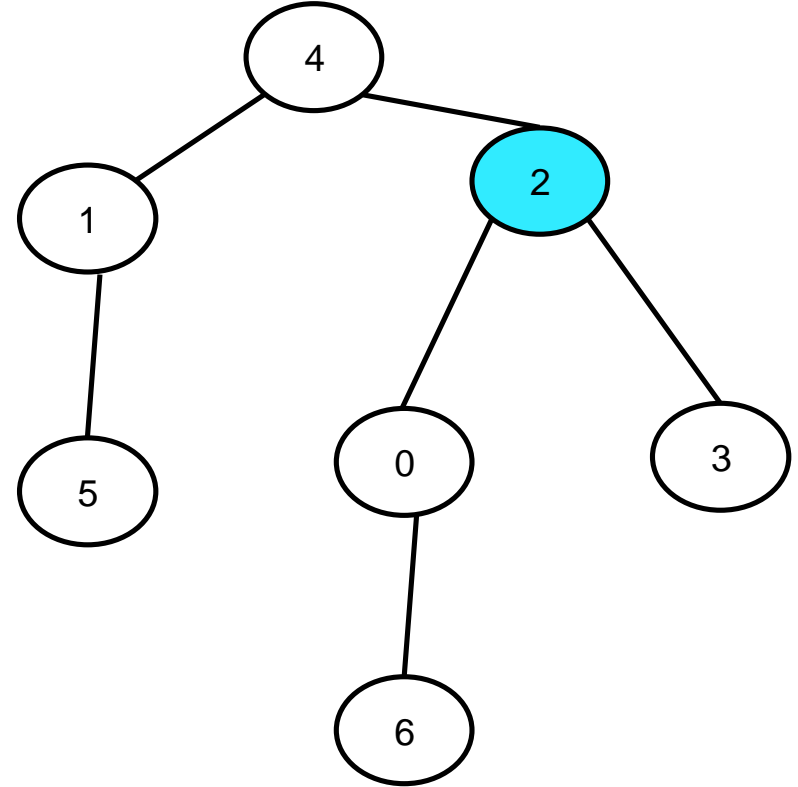
Ancestor of 0 = vertices in the path from
root  vertex 0 Excluding 0

4, 2



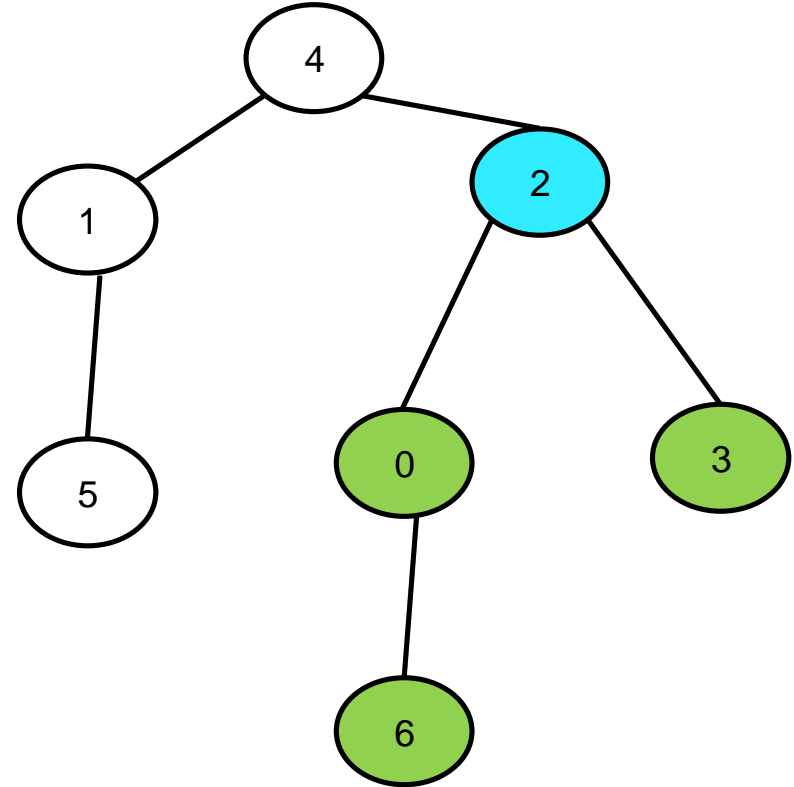
Descendant of a vertex

Descendants of 2 = vertices
whose ancestors are 2



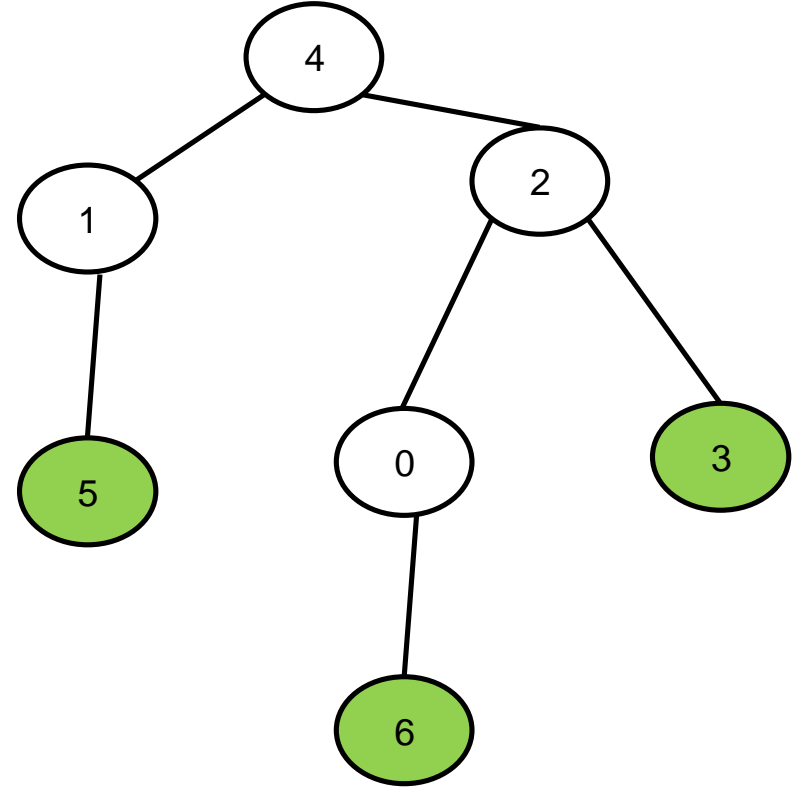
Descendant of a vertex

Descendants of 2 = 0,
3, 6



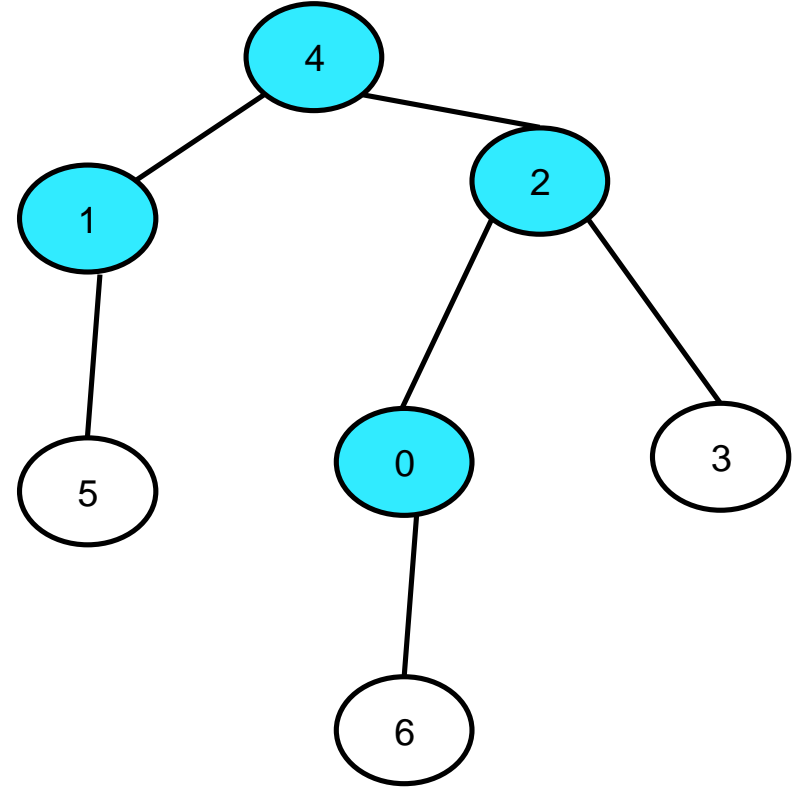
Leaf Nodes

Vertices that have no children

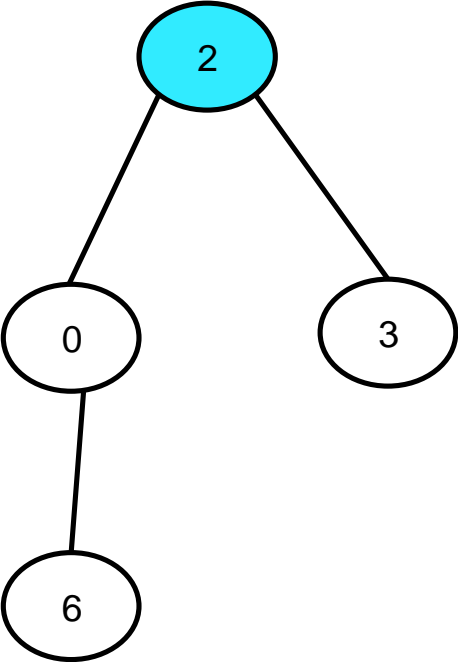
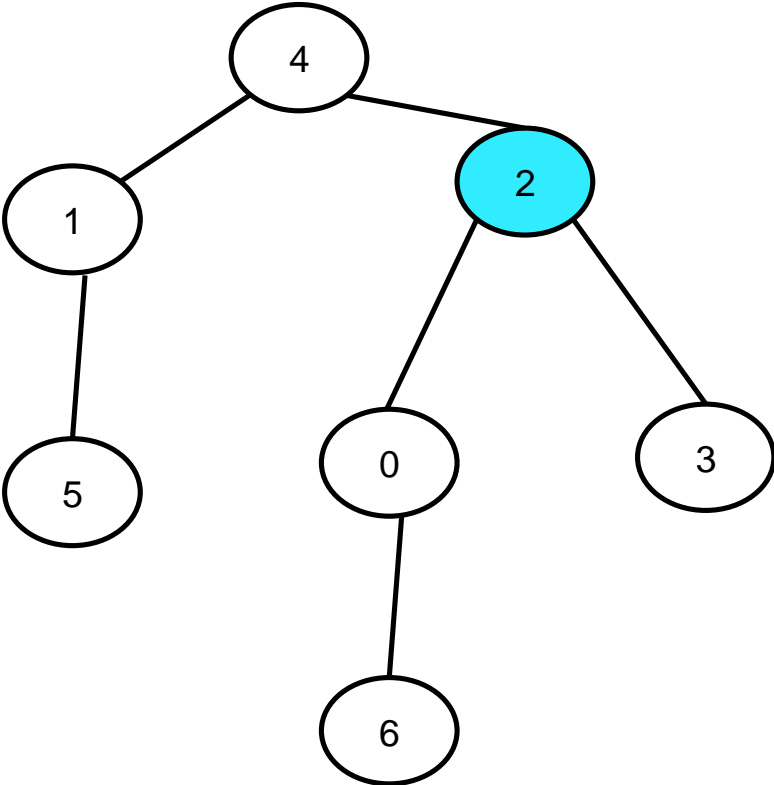


Internal Nodes

Vertices that have children

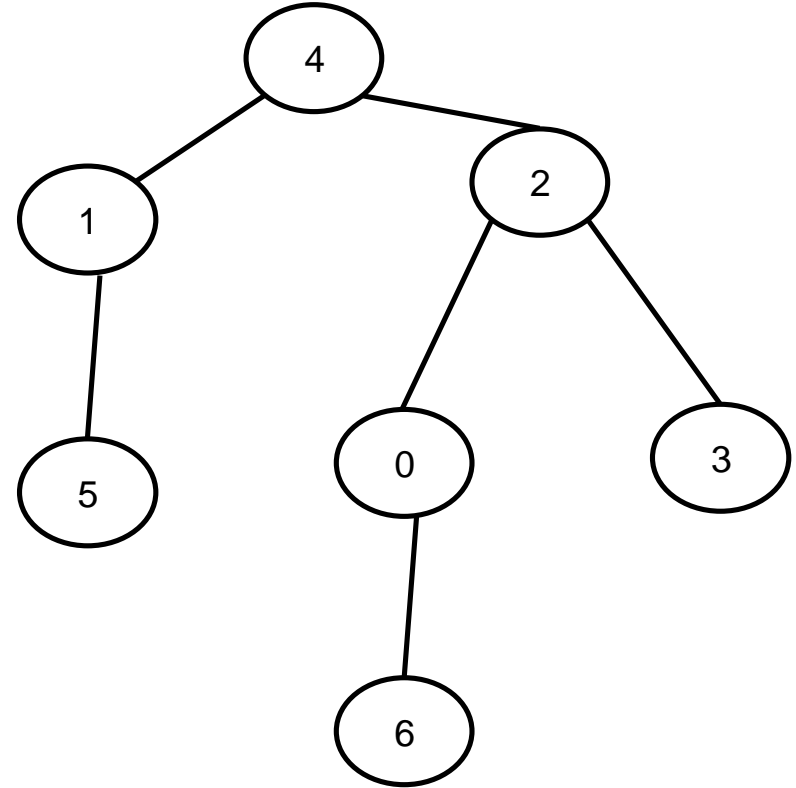


Subtree rooted at a vertex



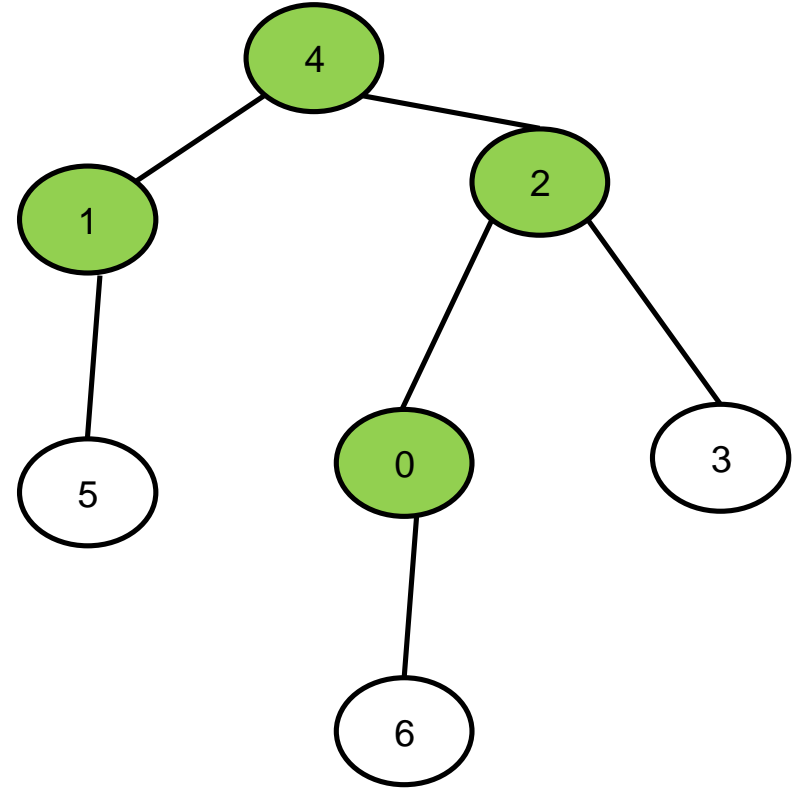
M-ary tree

No of children of **every** internal vertex $\leq m$



M-ary tree

No of children of **every** internal vertex $\leq m$



M-ary tree

Vertex 4 = 2

Vertex 1 = 1

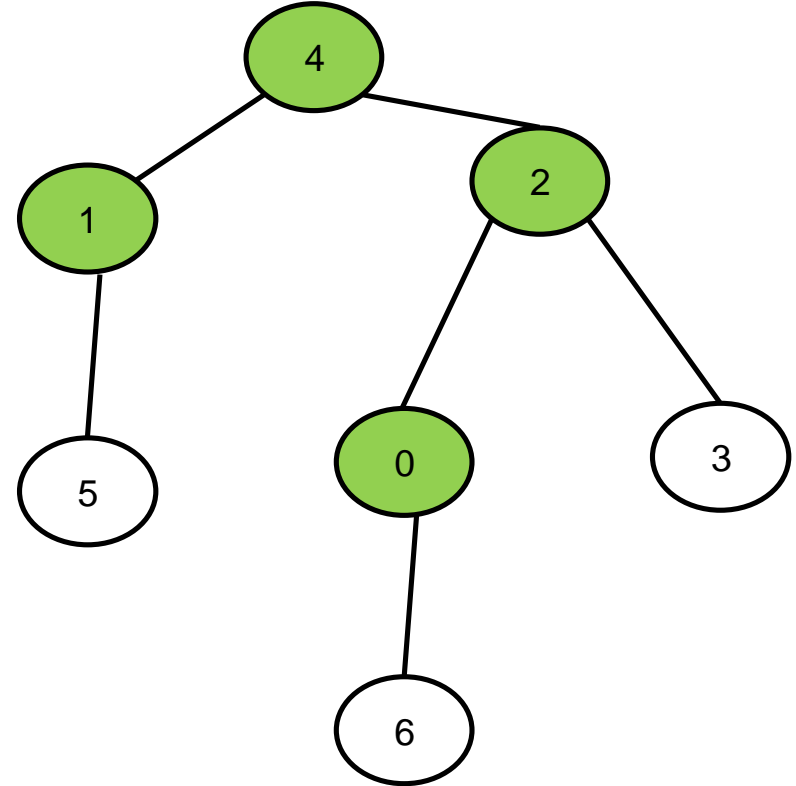
Vertex 2 = 2

Vertex 0 = 1

A 1- ary tree No

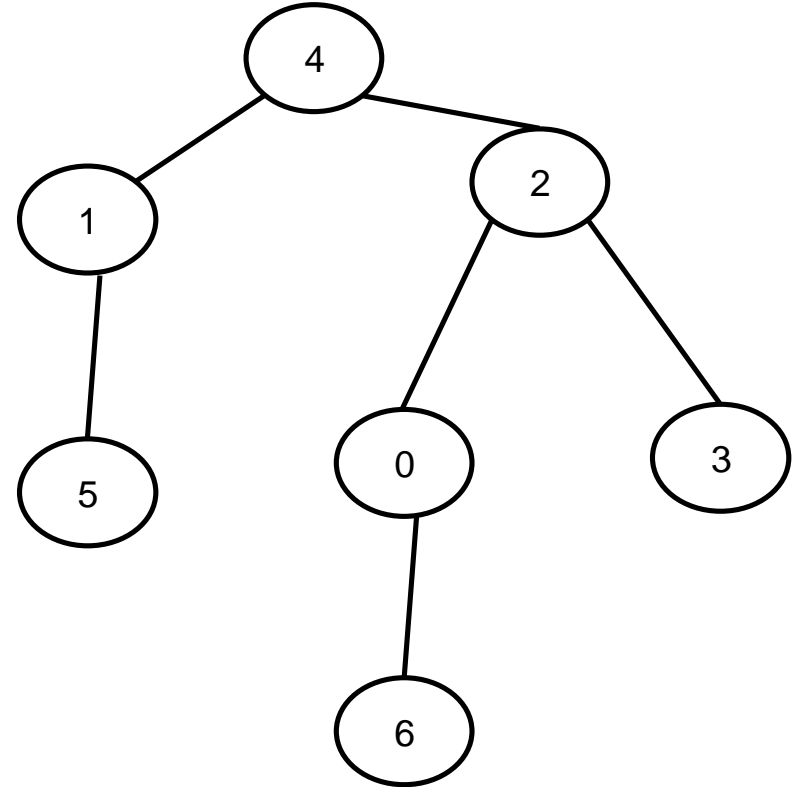
A 2- ary tree Yes

A 3- ary tree Yes



Full M-ary tree

No of children of **every** internal vertex = m



M-ary tree

Vertex 4 = 2

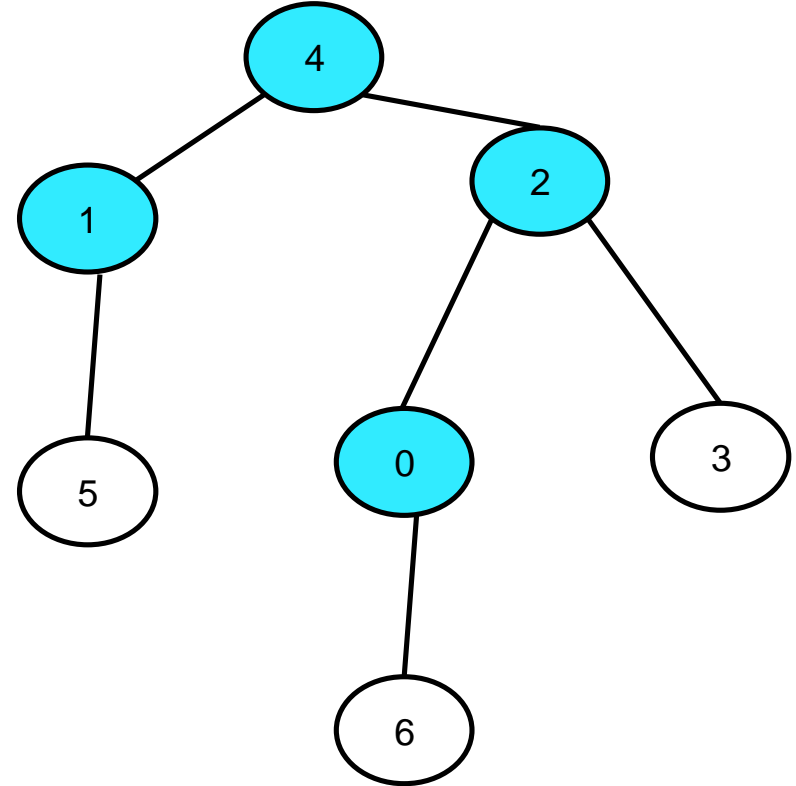
Vertex 1 = 1

Vertex 2 = 2

Vertex 0 = 1

A Full 1- ary tree No

A Full 2- ary tree No

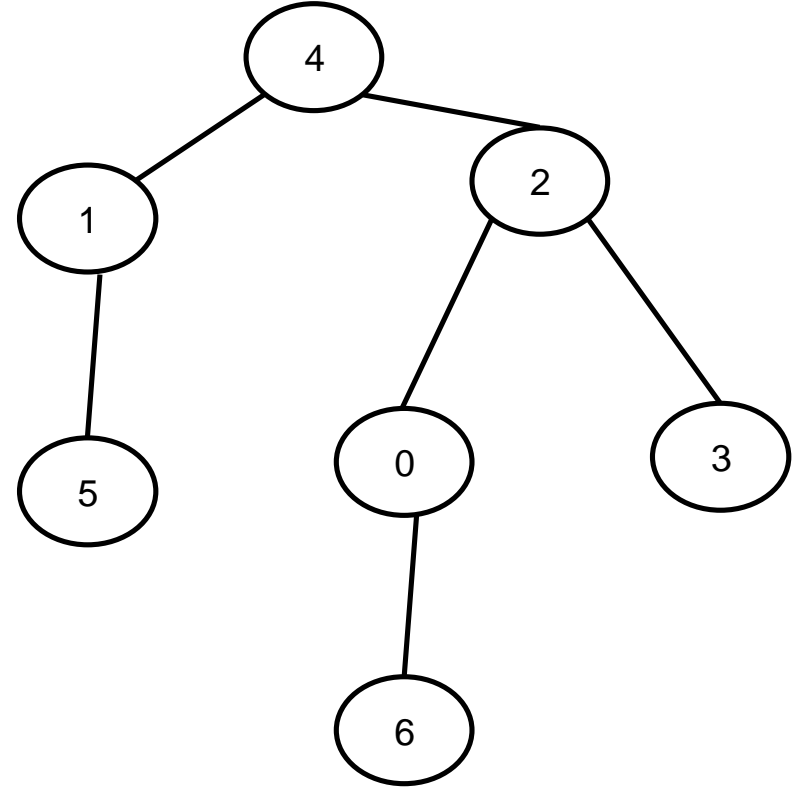


Ordered Rooted Tree

the children of each internal vertex are shown in ordered from left to right.

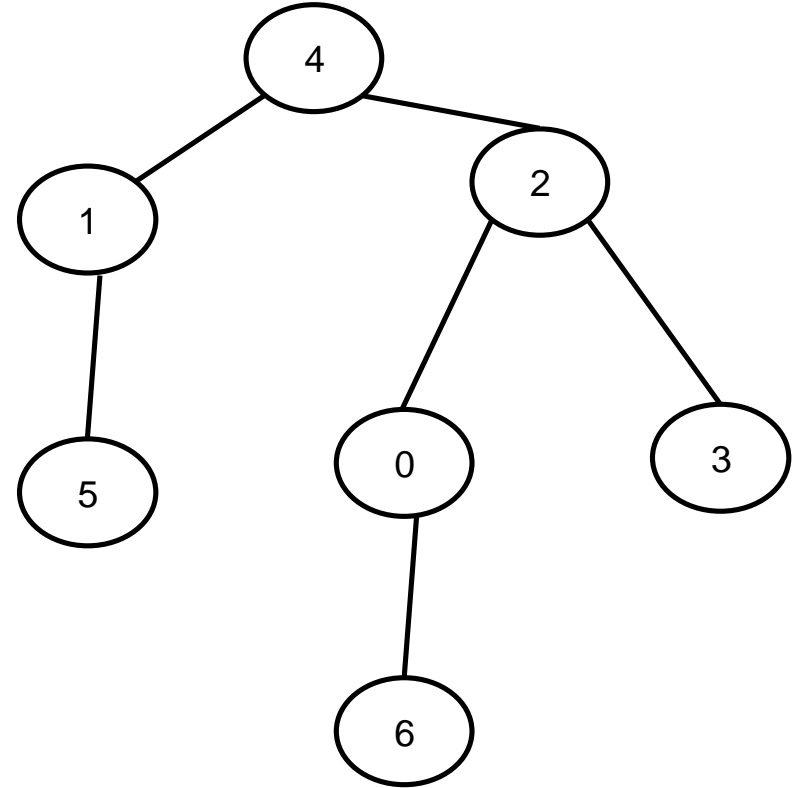
First child of 2 = 0 (Left child)

Second child of 2 = 3 (Right child)



Level of a vertex

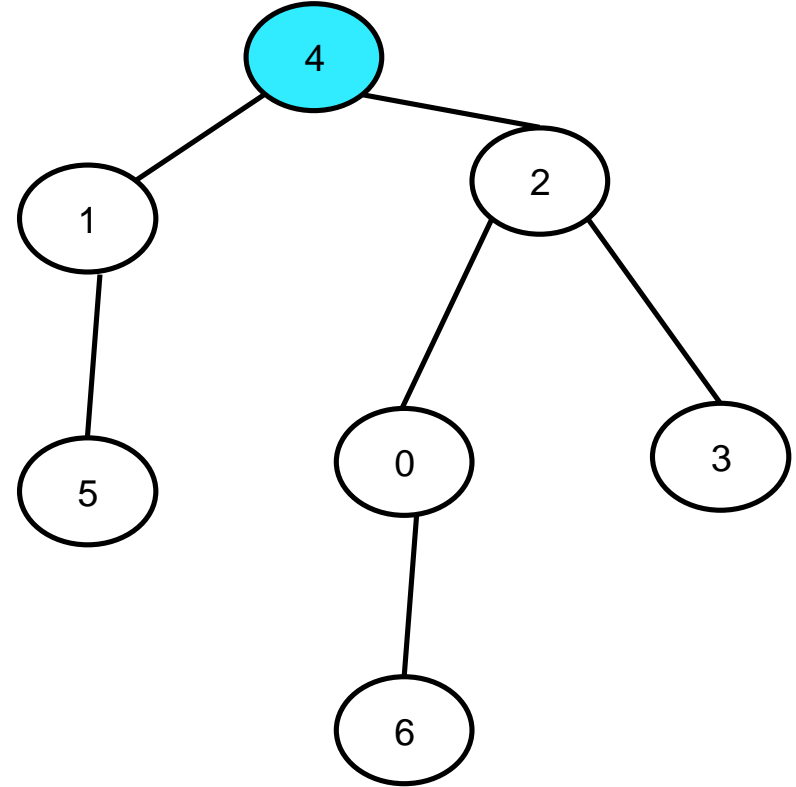
the length of the unique path from the root to this vertex.



Level of a vertex

the length of the unique path from the root to this vertex.

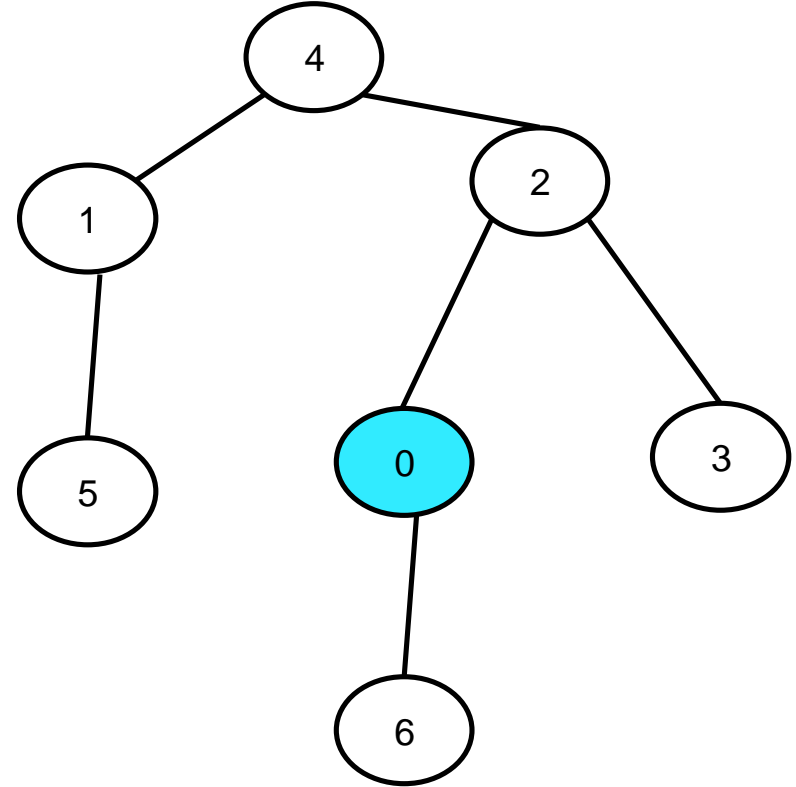
LEVEL OF ROOT = 0



Level of a vertex

the length of the unique path from the root to this vertex.

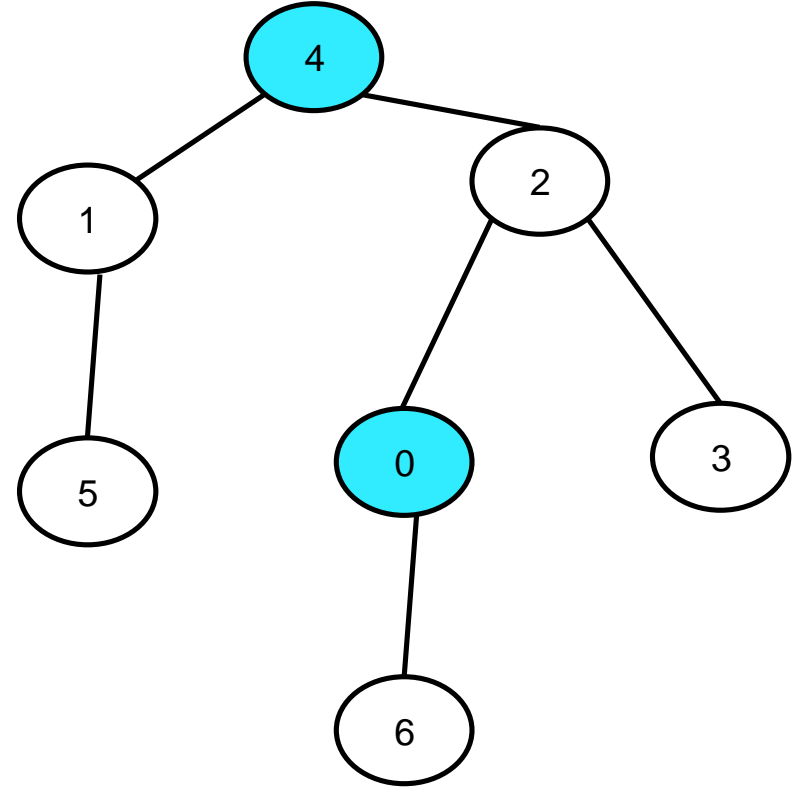
LEVEL OF 0 =



Level of a vertex

the length of the unique path from the root to this vertex.

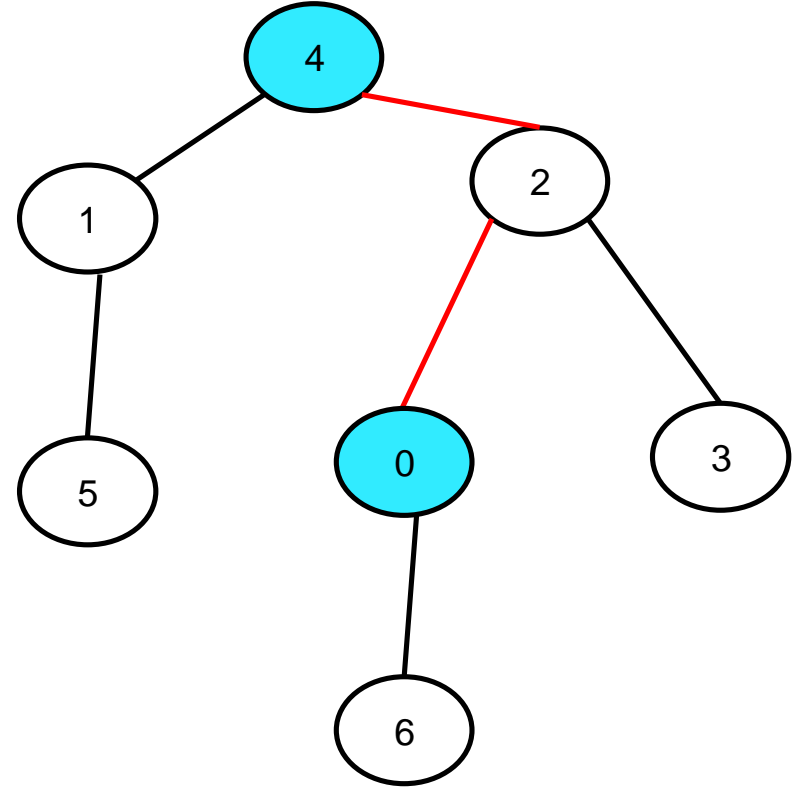
LEVEL OF 0 =



Level of a vertex

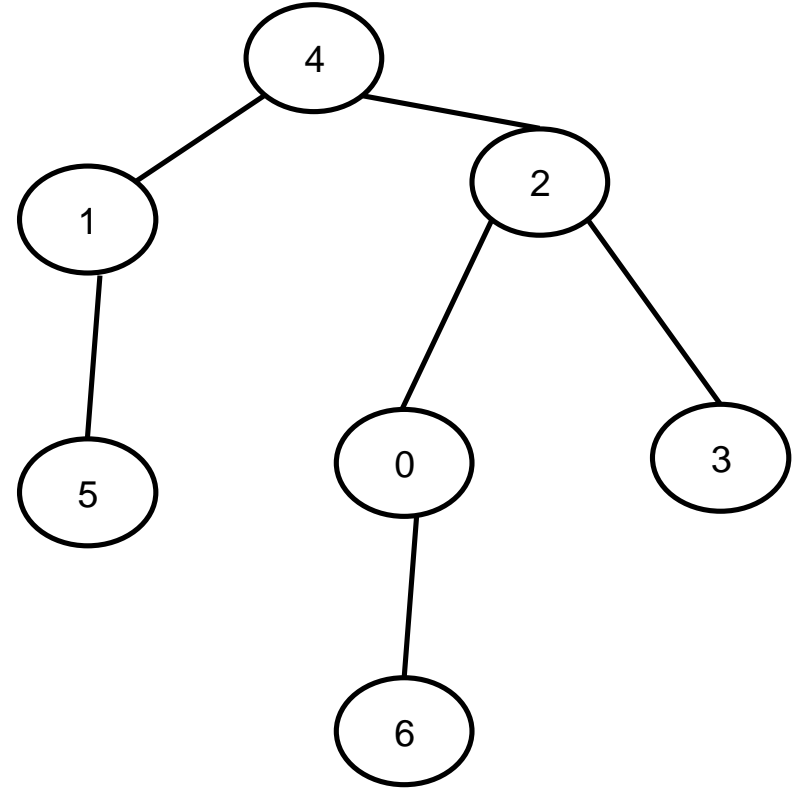
the length of the unique path from the root to this vertex.

LEVEL OF 0 = 2



Height of a tree

= MAX (Level of vertices)



Height of a tree

= MAX (Level of vertices)

Level of

Vertex 4 = 0

Vertex 1 = 1

Vertex 2 = 1

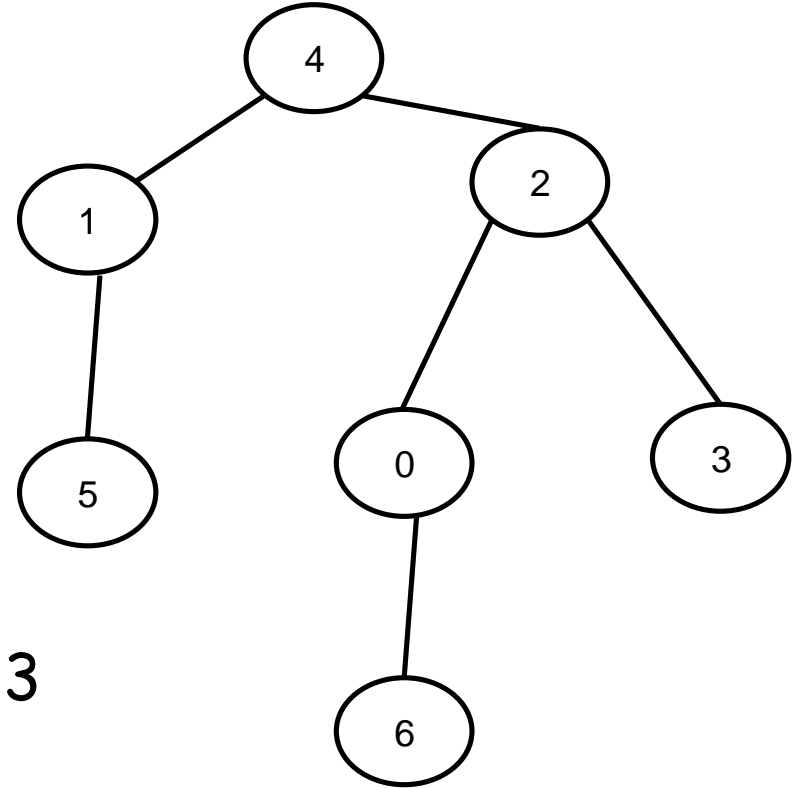
Vertex 5 = 2

Vertex 0 = 2

Vertex 3 = 2

Vertex 6 = 3

$$\text{Height} = \text{Max}(0, 1, 1, 2, 2, 3) = 3$$



Balanced Tree

If height = h , all the leaves are at Level h or $h-1$

Level of

Vertex 4 = 0

Vertex 1 = 1

Vertex 2 = 1

Vertex 5 = 2

Vertex 0 = 2

Vertex 3 = 2

Vertex 6 = 3

Height = 3

