Bracketing Methods

Bisection Method

Bisection Method: Algorithm1 (Finding x)

1. Choose lower x_l and upper x_u such that

$$f(x_l) * f(x_u) < 0$$

2. Root estimate

$$x_r = \frac{x_u + x_l}{2}$$

3. $if\,(f(x_l) * f(x_r) < 0)$

$$x_u = x_r \text{ and go to Step 2}$$

$if\,(f(x_l) * f(x_r) > 0)$

$$x_l = x_r \text{ and go to Step 2}$$

$if\,(f(x_l) * f(x_r) = 0)$

$$x = x_r \text{ and Stop}$$

Bisection Method: Algorithm1 (Finding x)

Termination Criteria

$$if\,(f(x_l) * f(x_r) = 0)$$

$$x = x_r \text{ and Stop}$$

Sometimes it may take thousands of iterations to stop the algorithm

Setting Termination Criteria: Terminologies

Approximate Relative error, ε_a

General: $\varepsilon_a = \frac{x_r^n - x_r^{n-1}}{x_r^n}$ (Applicable for any method)

For Bracketing methods only: $\varepsilon_a = \frac{x_u - x_l}{x_u + x_l}$

$n = \text{current step}, n - 1 = \text{previous step}$

$x_r = \text{Root estimate}, x_u = \text{Lower estimate}$

$x_u = \text{upper estimate}$

Stopping Criterion, ε_s

Given as input

Number of iterations, $maxitr$

Given as input

Bisection Method: Algorithm2 (Finding x)

With external termination criteria

1. Choose lower x_l and upper x_u such that

$$f(x_l) * f(x_u) < 0$$

2. Root estimate

$$x_r = \frac{x_u + x_l}{2}$$

Bisection Method: Algorithm2 (Finding x)

With external terminal criteria

3. While (Number of iterations have not been performed or Approximate relative error(ε_a) < Stopping Criterion (ε_s))

{

$if\,(f(x_l) * f(x_r) < 0)$

$x_u = x_r$ and go to Step 2

$if\,(f(x_l) * f(x_r) > 0)$

$x_l = x_r$ and go to Step 2

$if\,(f(x_l) * f(x_r) = 0)$

$x = x_r$ and Stop

}

Bisection Method: Mathematical Problem

1. $$f(x) = -0.5x^2 + 2.5x + 4.5$$

Using three iterations of bisection method and a stopping criterion of 0.02 determine the root of the given equation with $x_u = 5$ and $x_l = 10$. Also determine the approximate relative error in each step.

False Position Method

False Position Method: Algorithm (Finding x)

1. Choose lower x_l and upper x_u such that

$$f(x_l) * f(x_u) < 0$$

2. Root estimate

$$x_r = x_u - \frac{f(x_u) * (x_l - x_u)}{f(x_l) - f(x_u)}$$

False Position Method: Algorithm (Finding x)

3. While (Number of iterations have not been performed or
Approximate relative error(ε_a) < Stopping Criterion (ε_s))

{

$$if\,(f(x_l) * f(x_r) < 0)$$

$$x_u = x_r \text{ and go to Step 2}$$

$$if\,(f(x_l) * f(x_r) > 0)$$

$$x_l = x_r \text{ and go to Step 2}$$

$$if\,(f(x_l) * f(x_r) = 0)$$

$$x = x_r \text{ and Stop}$$

}

False Position Method: Mathematical Problem

1. $$f(x) = -0.5x^2 + 2.5x + 4.5$$

Using three iterations of false postion method and a stopping criterion of 0.02 determine the root of the given equation with $x_u = 5$ and $x_l = 10$. Also determine the approximate relative error in each step.