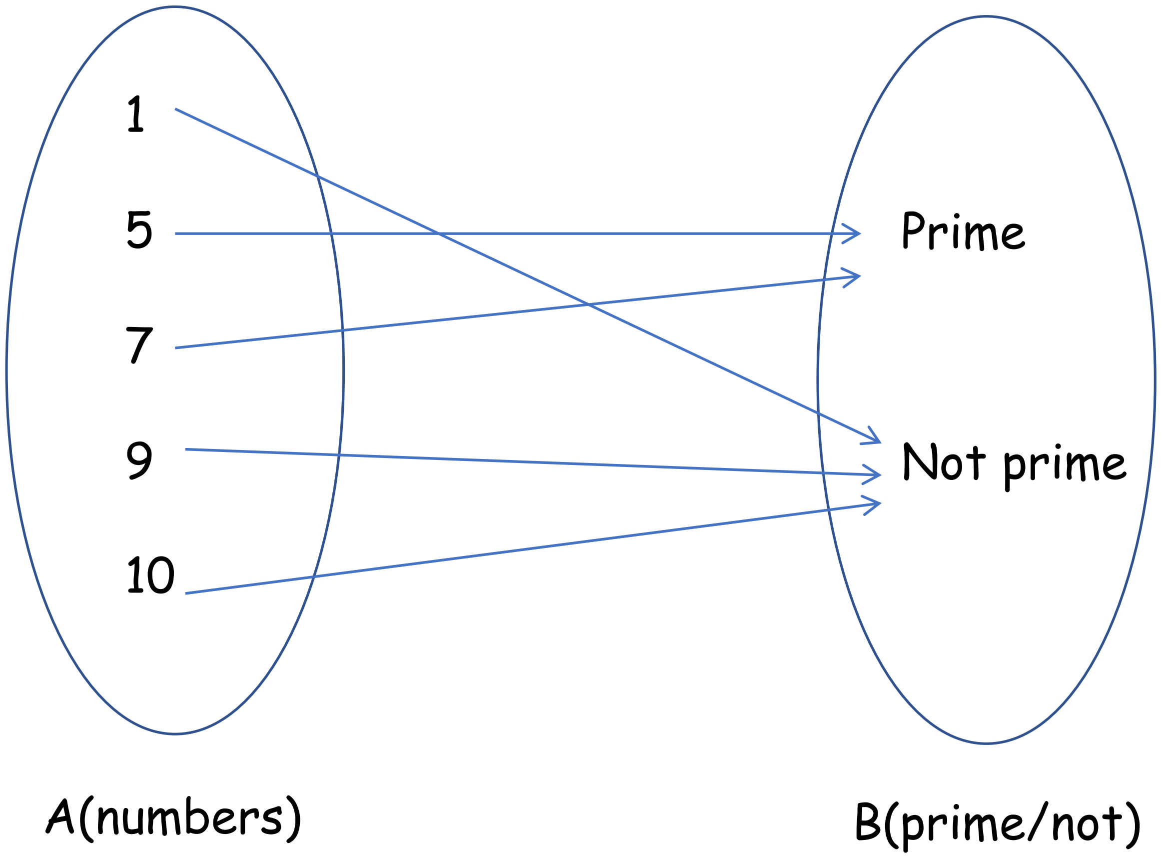
Functions

What are functions?

Informally, mapping between two sets



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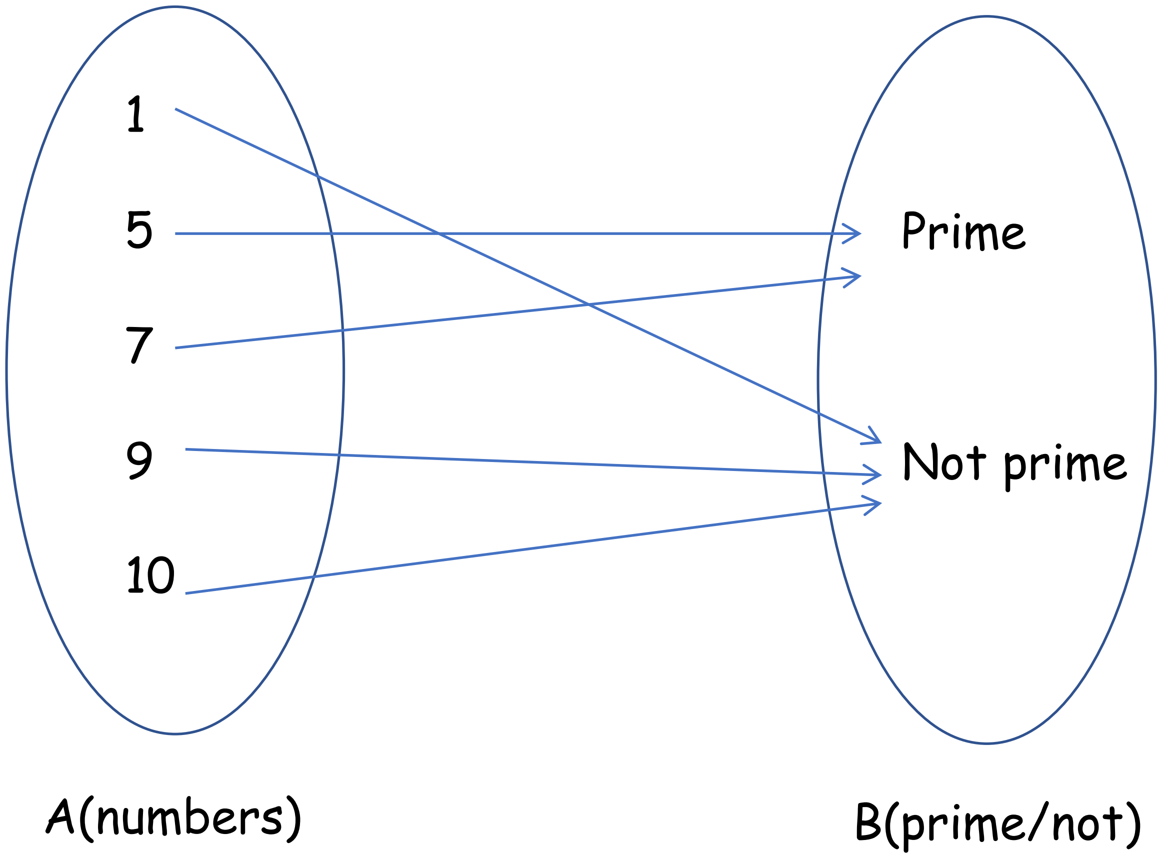


$f: A \rightarrow B$

A = Domain; B = Co-Domain

If $f(a) = b$

b = image of a

a = pre-image of b

Range, R = Images of A

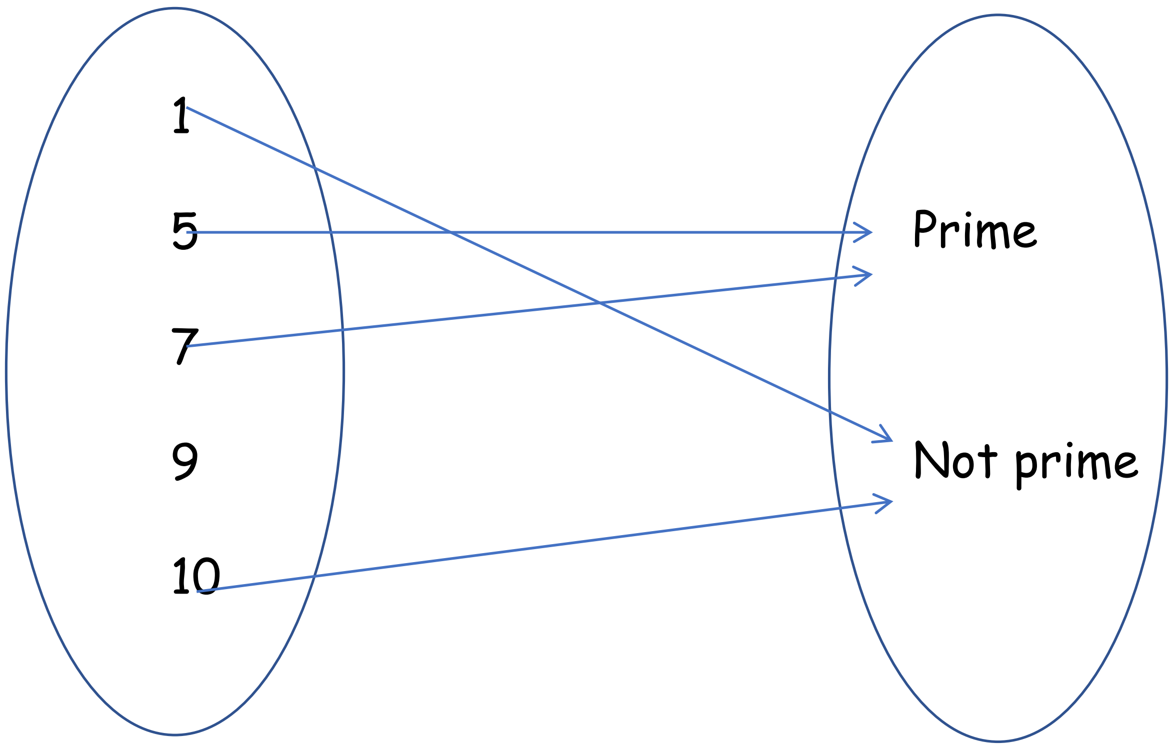
Examples of functions

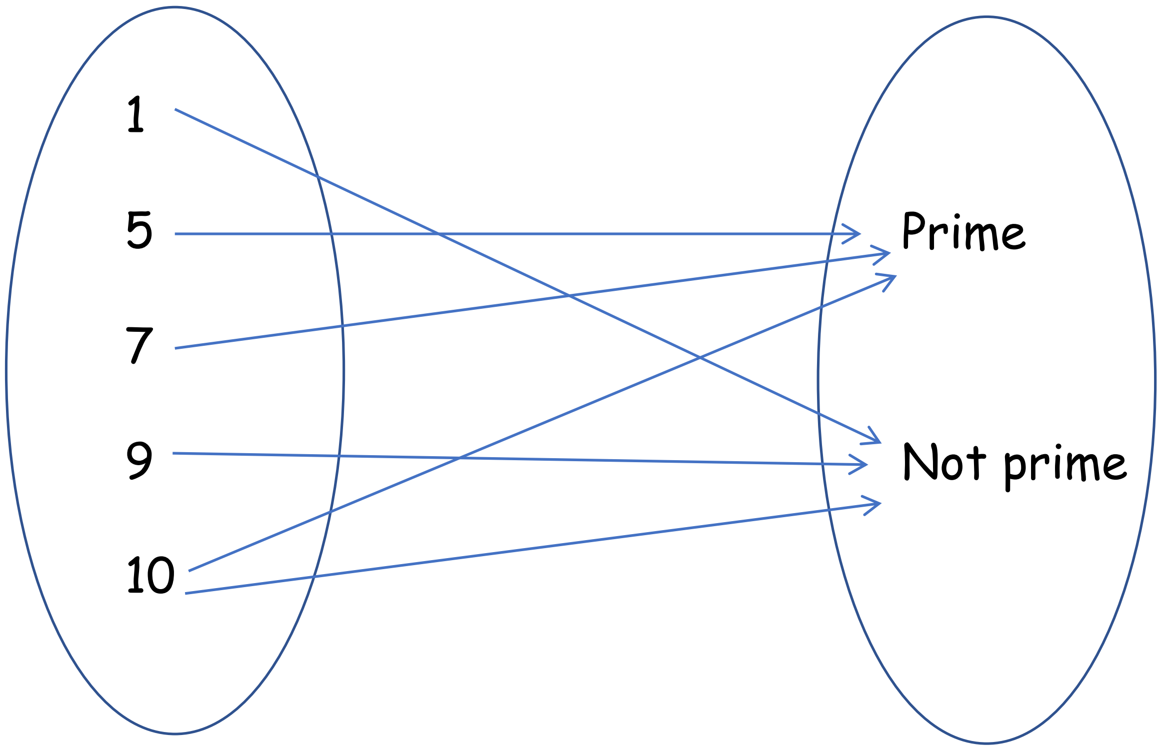
Suppose that each student in a discrete mathematics class is assigned a letter grade from the set $\{A, B, C, D, F\}$. And suppose that the grades are A for Adams, C for Chou, B for Goodfriend, A for Rodriguez, and F for Stevens.

What are functions?

A function from A to B is an assignment of exactly one member of B to each element of A

What are functions?





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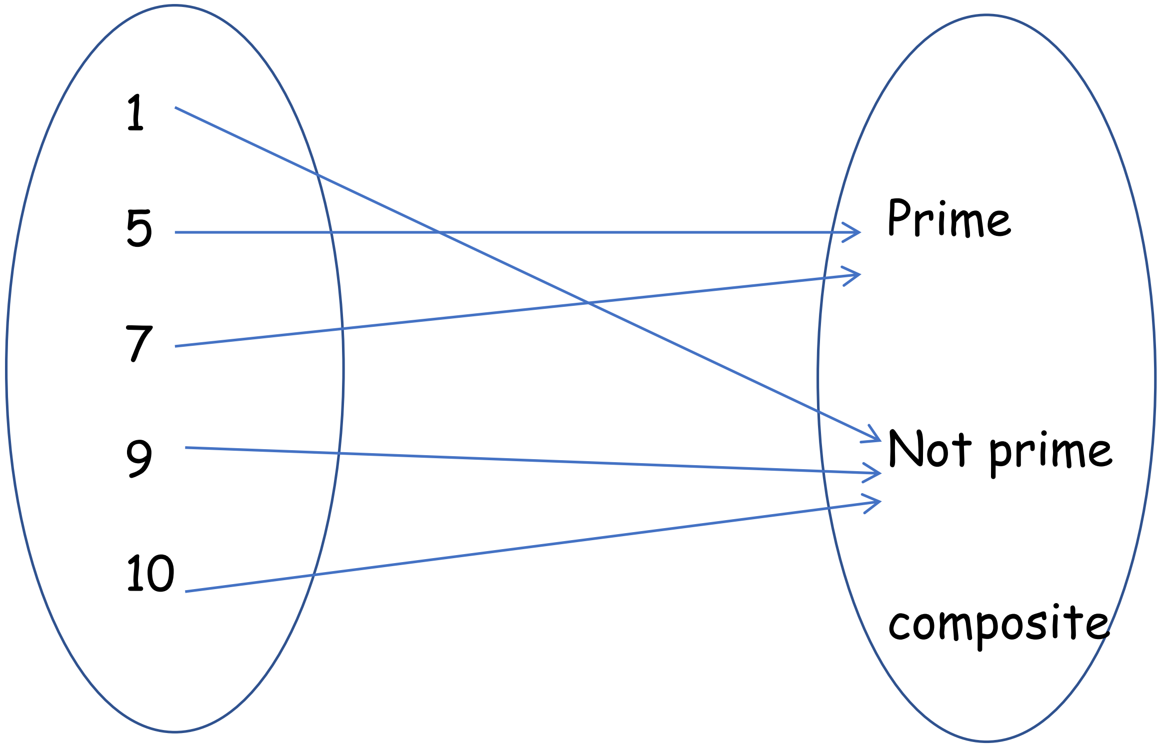

Multiplication and addition of functions

f1: $A \to \mathbb{R}$
f2: $A \to \mathbb{R}$

f1 f2: $A \to \mathbb{R}$
$(f1f2)(x) = f1(x)\, f2(x)$

f1 + f2: $A \to \mathbb{R}$
$(f1+f2)(x) = f1(x) + f2(x)$

One to One(Injective) functions

A function f is said to be one-to-one, or an injunction,
if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f

$$\forall a \forall b (f(a) = f(b)) \rightarrow a = b$$

One to One(Injective) functions

f(x) = x^2 from $\mathbb{N}$ to $\mathbb{Z}$

To show that f is injective Show that if f (x) = f (y) for arbitrary x, y ∈ A with x = y, then x = y.

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Onto(Surjective) Functions

A function f from A to B is called onto if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$.

$$\forall y \exists x f(x) = y$$

Onto(Surjective) functions

f(x) = x^2 from $\mathbb{N}$ to $\mathbb{Z}$

To show that f is surjective Consider an arbitrary element y ∈ B and find an element x ∈ A such that f (x) = y.

Onto(Surjective) functions

$f(x) = x+1$ from $\mathbb{Z}$ to $\mathbb{Z}$

To show that f is surjective Consider an arbitrary element $y \in B$ and find an element $x \in A$ such that $f(x) = y$.

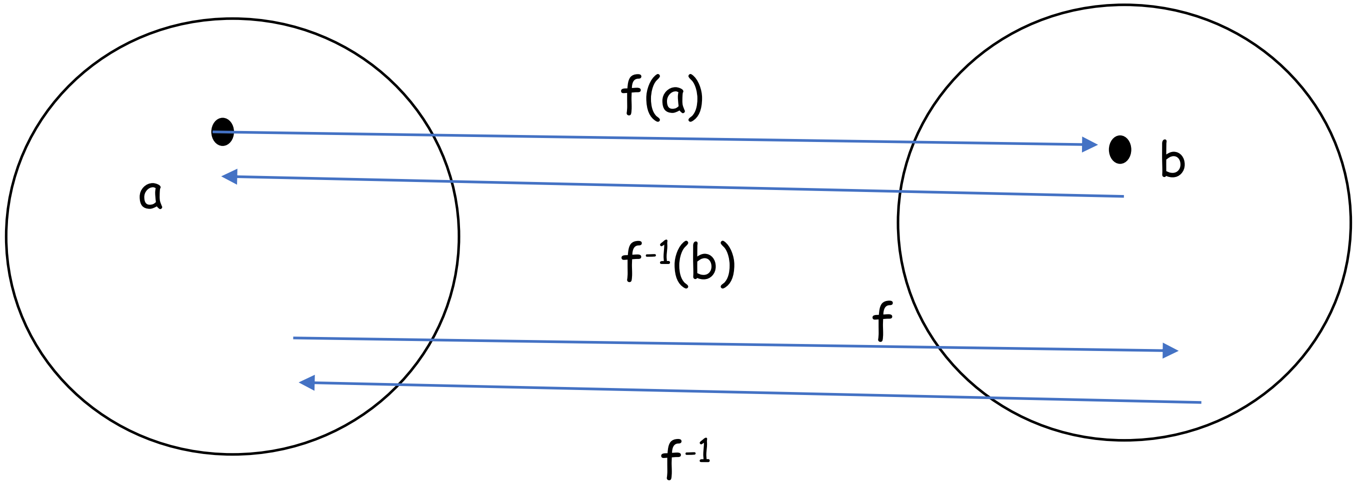
Bijective Functions

A function f is a one-to-one correspondence, or a bijection, if it is both one-to-one and onto.

Inverse Functions

Let f be a bijective function from the set A to the set B. The inverse function of f is the function that assigns to an element b belonging to B the unique element a in A such that f(a) = b.
The inverse function of f is denoted by f^{-1}. Hence, $f^{-1}(b) = a$ when f(a) = b.

Inverse Functions



Inverse functions

$f: \mathbb{Z}$ to $\mathbb{Z}$ such that $f(x) = x + 1$. Find if f is invertible and if invertible what's the inverse

Composition of functions

Let g be a function from the set A to the set B and let f be a function from the set B to the set C. The composition of the functions f and g, denoted for all $a \in A$ by $f \circ g$, is defined by

$$(f \circ g)(a) = f(g(a))$$