

Predicate Logic

Predicates

Proposition with variables.

Example:

$$P(x) ::= [x > 3]$$

$$P(x, y) ::= [x + 2 = y]$$

Quantifiers

$\forall x$ For ALL x

$\exists y$ There EXISTS some y

\forall is like AND

Let x range over set $\{1, 2, 3, 4, 5, 6\}$

$P(x) ::= [x > 3]$

$\forall x. P(x)$

same as $P(1)$ AND $P(2)$ AND $P(3)$ AND $P(4)$ AND
 $P(5)$ AND $P(6)$

\exists is like OR

Let x range over set $\{1, 2, 3, 4, 5, 6\}$

$P(x) ::= [x > 3]$

$\exists x. P(x)$

same as $P(1) \text{ OR } P(2) \text{ OR } P(3) \text{ OR } P(4) \text{ OR } P(5) \text{ OR } P(6)$

Nested Quantifier1

$$x, y \in \mathbb{R}$$

$$Q(x, y) ::= [x + y = 0]$$

$$\forall x \exists y Q(x, y) = ?$$

Nested Quantifier2

$$x \in \mathbb{R}, y \in \mathbb{N} = \{1, 2, \dots\}$$

$$Q(x, y) ::= [x + y = 0]$$

$$\forall x \exists y Q(x, y) = ?$$

Nested Quantifier3

$$x \in \mathbb{R}, y \in \mathbb{N} = \{1, 2, \dots\}$$

$$Q(x, y) ::= [x + y = 0]$$

$$\exists x \forall y Q(x, y) = ?$$

Nested Quantifier4

$$x \in \mathbb{R}, y \in \mathbb{N} = \{1, 2, \dots\}$$

$$Q(x, y) ::= [x * y = 0]$$

$$\exists x \forall y Q(x, y) = ?$$

Nested Quantifier5

$$x \in \mathbb{R}, y \in \mathbb{R}$$

$$Q(x, y) ::= [(x * y)^2 \geq 0]$$

$$\forall x \forall y Q(x, y) = ?$$

Nested Quantifier6

$$x \in \mathbb{R}, y \in \mathbb{R}$$

$$Q(x, y) ::= [x^2 + y^2 = 13]$$

$$\exists x \exists y Q(x, y) = ?$$

Nested Quantifier7

$$x \in \mathbb{R}, y \in \mathbb{R}$$

$$Q(x, y) ::= [x^2 + y^2 = -6]$$

$$\exists x \exists y Q(x, y) = ?$$

Nested Quantifier8

$$x \in \mathbb{R}, y \in \mathbb{R}, z \in \mathbb{R}$$

$$Q(x, y, z) ::= [x + y = z]$$

$$\forall x \forall y \exists z Q(x, y, z) = ?$$

$$\forall x \forall y \exists z [x + y = z]$$

$$x = 1$$

$$y = 1$$

$$z = 2, 1 + 1 = 2: \top$$

$$\forall x \forall y \exists z [x + y = z]$$

$$x = 1$$

$$y = 1$$

$$z = 2, 1 + 1 = 2: \top$$

What's the quantifier before z ?

$\exists z$

$$\forall x \forall y \exists z [x + y = z]$$

$$x = 1$$

$$y = 1$$

$$z = 2, 1 + 1 = 2: \top$$

What's the quantifier before z ?

$\exists z$

$$x = 1$$

$$\boxed{\top} \left[\begin{array}{l} \rightarrow y = 1 \\ \rightarrow \end{array} \right.$$

$$z = 2, 1 + 1 = 2: \top$$

$$\forall x \forall y \exists z [x + y = z]$$

What's the quantifier before y ? $\forall y$

$$\forall x \forall y \exists z [x + y = z]$$

What's the quantifier before y ? $\forall y$

$$x = 1$$

\boxed{T}

$\left[\begin{array}{l} \rightarrow \\ \rightarrow \end{array} \right]$

$$y = 1$$

$$z = 1, 1 + 1 = 2: T$$

\boxed{T}

$\left[\begin{array}{l} \rightarrow \\ \rightarrow \end{array} \right]$

$$y = 2$$

$$z = 3, 1 + 2 = 3: T$$

$$\forall x \forall y \exists z [x + y = z]$$

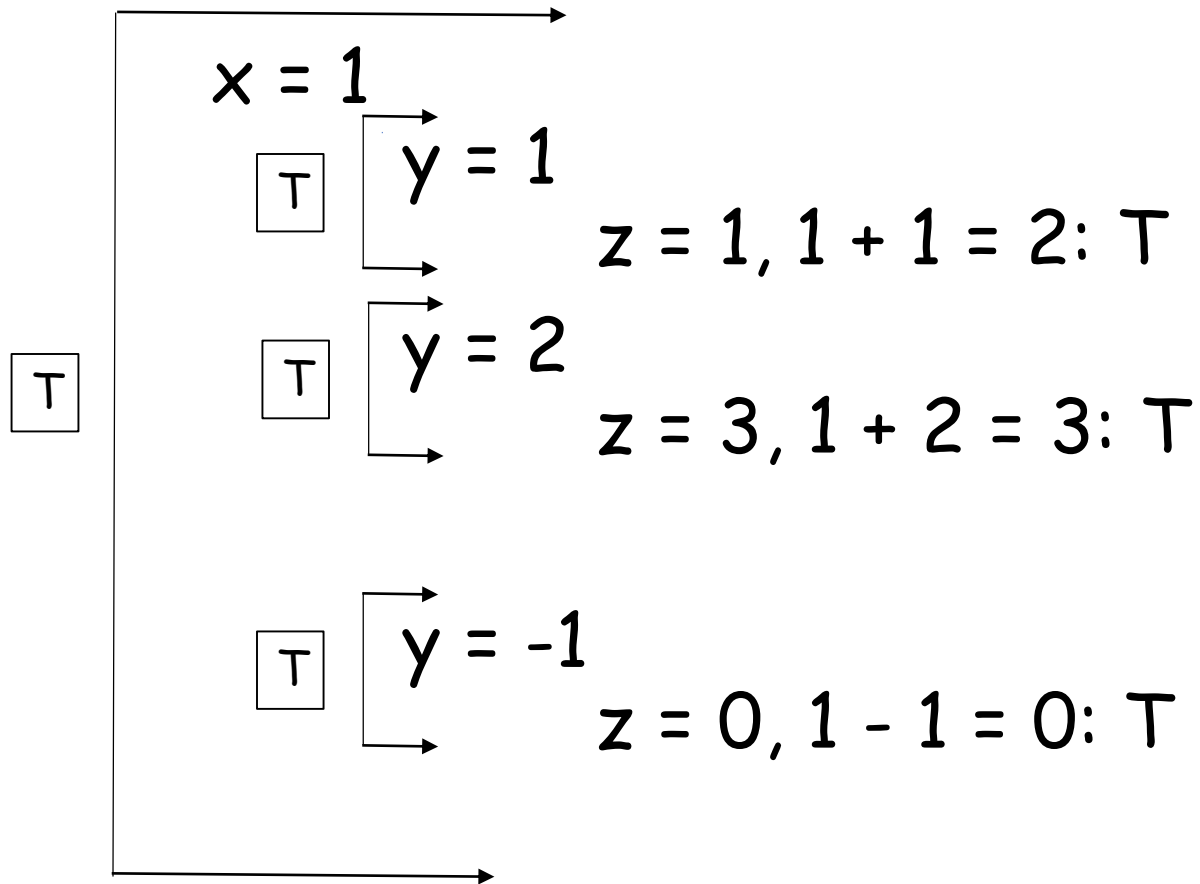
$$x = 1$$

$$\boxed{\top} \left[\begin{array}{l} \rightarrow y = 1 \\ \rightarrow \end{array} \right. \quad z = 1, 1 + 1 = 2: \top$$

$$\boxed{\top} \left[\begin{array}{l} \rightarrow y = 2 \\ \rightarrow \end{array} \right. \quad z = 3, 1 + 2 = 3: \top$$

$$\boxed{\top} \left[\begin{array}{l} \rightarrow y = -1 \\ \rightarrow \end{array} \right. \quad z = 0, 1 - 1 = 0: \top$$

$$\forall x \forall y \exists z [x + y = z]$$

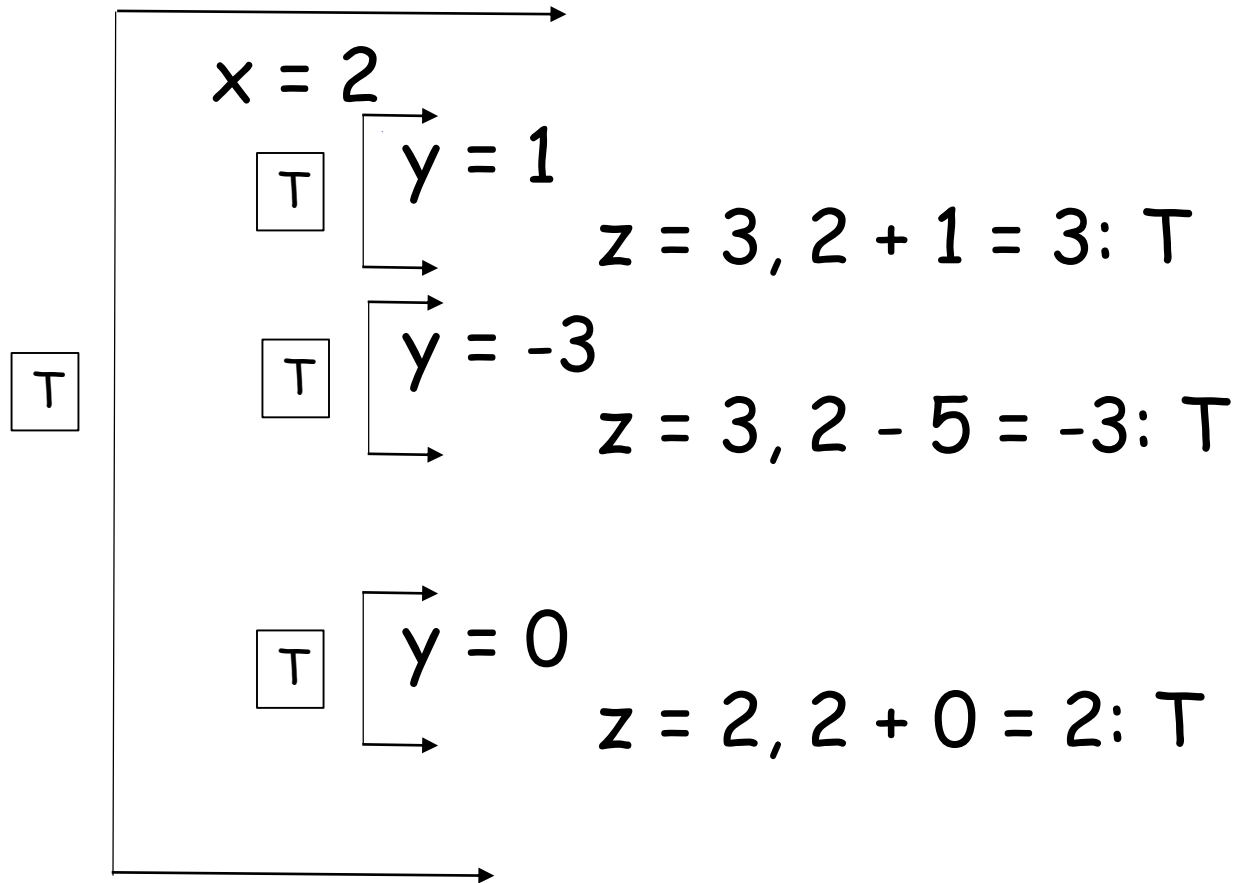


$$\forall x \forall y \exists z [x + y = z]$$

What's the quantifier before x? $\forall x$

$$\forall x \forall y \exists z [x + y = z]$$

What's the quantifier before x ? $\forall x$



Nested Quantifier9

$$x \in \mathbb{R}, y \in \mathbb{R}, z \in \mathbb{R}$$

$$Q(x, y, z) ::= [x + y = z]$$

$$\exists x \forall y \forall z Q(x, y, z) = ?$$

$$\exists x \forall y \forall z [x + y = z]$$

$$x = 1$$

$$y = 1$$

$$z = 2, 1 + 1 = 2: \top$$

$$\exists x \forall y \forall z [x + y = z]$$

$$x = 1$$

$$y = 1$$

$$z = 2, 1 + 1 = 2: \text{T}$$

What's the quantifier before z ?

$\forall z$

$$\exists x \forall y \forall z [x + y = z]$$

$$x = 1$$

$$y = 1$$

$$z = 2, 1 + 1 = 2: \text{T}$$

$$z = 1, 1 + 1 = 1: \text{F}$$

$$\exists x \forall y \forall z [x + y = z]$$

$$x = 1$$

$$y = 1$$

$$z = 2, 1 + 1 = 2: \text{T}$$

$$z = 1, 1 + 1 = 1: \text{F}$$

What's the quantifier before z ?

$\forall z$

$$\exists x \forall y \forall z [x + y = z]$$

$$x = 1$$

$$y = 1$$

$$z = 2, 1 + 1 = 2: \text{T}$$

$$z = 1, 1 + 1 = 1: \text{F}$$

What's the quantifier before z?

$\forall z$

$$x = 1$$

$$y = 1$$

F

$$z = 2, 1 + 1 = 2: \text{T}$$

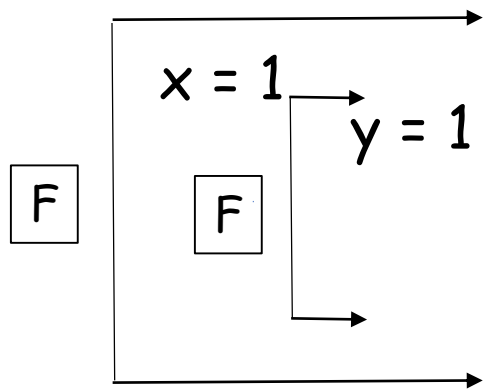
$$z = 1, 1 + 1 = 1: \text{F}$$

$$\exists x \forall y \forall z [x + y = z]$$

What's the quantifier before y ? $\forall y$

$$\exists x \forall y \forall z [x + y = z]$$

What's the quantifier before y ? $\forall y$



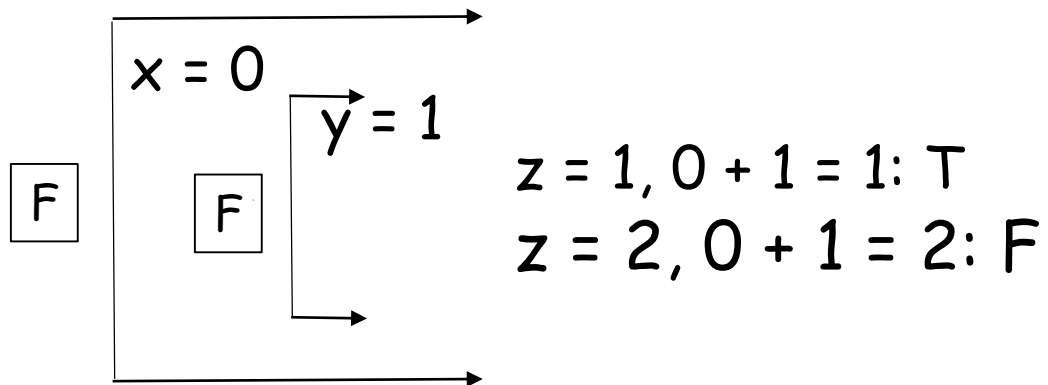
$z = 2, 1 + 1 = 2: T$
 $z = 1, 1 + 1 = 1: F$

$$\exists x \forall y \forall z [x + y = z]$$

What's the quantifier before x ? $\exists x$

$$\exists x \forall y \forall z [x + y = z]$$

What's the quantifier before x ? $\exists x$



Nested Quantifier10

$x \in \mathbb{N}, y \in \mathbb{N}, z \in \mathbb{N}$

$Q(x, y, z) ::= [x = yz]$

$\forall x \exists y \exists z Q(x, y, z) = ?$

$$\forall x \exists y \exists z [x = yz]$$

Negating Nested Quantifiers

Negating nested quantifiers

Find the negation of $\forall x \exists y \exists z Q(x, y, z)$, where $Q(x, y, z) ::= [x = yz]$

$$\neg (\forall x \exists y \exists z Q(x, y, z))$$

$$\equiv \exists x \neg (\exists y \exists z Q(x, y, z))$$

$$\equiv \exists x \forall y \neg (\exists z Q(x, y, z))$$

$$\equiv \exists x \forall y \forall z \neg Q(x, y, z)$$

Negating nested quantifiers

$$\equiv \exists x \forall y \forall z \neg (x = yz)$$

$$\equiv \exists x \forall y \forall z (x \neq yz)$$

Negating nested quantifiers

Find the negation of $\forall x \exists y \exists z Q(x, y, z) \vee P(x, y, z)$,
where

$$Q(x, y, z) ::= [x = yz]$$

$$P(x, y, z) ::= [x + y > z]$$

$$\exists x \forall y \forall z (x \neq yz) \wedge (x + y \leq z)$$